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- A Bewley decision maker cannot be indifferent between distinct monetary acts whenever his set of priors is fully-dimensional.
- When there are only two states of the world, Bewley preferences are incomplete if and only if indifference is trivial in the above sense.

On the Indifference Relation in Bewley Preferences

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Abstract

Bewley's (2002) influential model of preferences over uncertain prospects features an incomplete preference relation and a set of priors over the states of the world such that one act is preferred to another if and only if its expected utility is higher under every prior in that set. This note shows that, under general conditions on preferences, the decision maker in the Bewley model cannot be indifferent between distinct monetary acts whenever the set of priors is fully-dimensional. In the special case of two states, in particular, such "objectively rational" preferences are incomplete if and only if the indifference relation is trivial in the above sense.

Keywords: Bewley preferences; incomplete preferences; monetary acts; trivial indifference relation.

JEL Classification: D1, D5, D7, D8, D10

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1 Introduction

Bewley's (2002) influential model features an incomplete strict preference relation and a set of priors over the states of the world such that the decision maker prefers one uncertain act to another if and only if its expected utility is strictly higher under every prior in the set. Since Ghirardato, Maccheroni, Marinacci, and Siniscalchi (2003), a natural alternative approach towards modelling this robustness/unanimity preference rule has been to take a possibly incomplete *weak* preference relation as the primitive and let it be represented instead by weak dominance with respect to every prior.

Bewley preferences of both types have been employed in general equilibrium theory (e.g. Rigotti and Shannon 2005; Kajii and Ui 2009; Danan and Riedel 2013; Ma 2015), financial economics (e.g. Easley and O'Hara 2010), contract theory (Lopomo et al 2011), mechanism design (e.g. Chiesa et al 2015; Lopomo et al 2014) and social choice theory (e.g. Danan et al 2016). In addition, they have motivated a number of decision-theoretic extensions (e.g. Gilboa, Maccheroni, Marinacci and Schmeidler, 2010; Ok, Ortoleva and Riella, 2012; Galaabaatar and Karni, 2013; Faro, 2015). Since Gilboa et al (2010), in particular, these preferences are commonly referred to as *objectively rational*.

This note shows that when Bewley preferences are defined on the Euclidean space of purely uncertain monetary acts and satisfy standard regularity assumptions such as strict monotonicity and, in addition, the set of priors is of full dimension, then the indifference relation associated with them is trivial/degenerate in the sense that there exist no distinct choice alternatives between which the decision maker is indifferent. This implies that, in the special case of two states (capturing, for example, situations where an investment is either profitable or not profitable, or where a contracting agent exerts either high or low effort), Bewley preferences are incomplete if and only if indifference is trivial in the above sense.

2 Fully Dimensional Priors and Trivial Indifference

There is a finite state space $S := \{1, \dots, n\}$ and the choice domain is the set of all acts $f : S \rightarrow \mathbb{R}$. This domain coincides with the Euclidean space \mathbb{R}^n . The zero vector in \mathbb{R}^n is identified with $\mathbf{0}$. The set of all strictly positive probability measures on S is $\text{rint } \Delta(S)$. A set $P \subset \text{rint } \Delta(S)$ is of *full dimension* if it contains n linearly independent elements.

In the spirit of Ghirardato, Maccheroni, Marinacci, and Siniscalchi (2003), it will be said that a reflexive binary relation \succsim on \mathbb{R}^n admits a *weak Bewley representation* if there exists a continuous, strictly increasing function $u : \mathbb{R} \rightarrow \mathbb{R}$ and a compact, convex set $P \subset \text{rint } \Delta(S)$ such that, for all $x, y \in \mathbb{R}^n$,

$$x \succsim y \iff \sum_{i=1}^n p_i u(x_i) \geq \sum_{i=1}^n p_i u(y_i) \text{ for all } p \in P, \quad (1)$$

where, in particular, the indifference relation \sim that is derived from such a representation is defined by

$$x \sim y \iff \sum_{i=1}^n p_i u(x_i) = \sum_{i=1}^n p_i u(y_i) \text{ for all } p \in P. \quad (2)$$

Following Bewley (2002), it will be said that an irreflexive binary relation \succ on \mathbb{R}^n admits a *strict Bewley representation* if there exists a continuous, strictly increasing function $u : \mathbb{R} \rightarrow \mathbb{R}$ and a

compact, convex set $P \subset \text{rint } \Delta(S)$ such that, for all $x, y \in \mathbb{R}^n$,

$$x \succ y \iff \sum_{i=1}^n p_i u(x_i) > \sum_{i=1}^n p_i u(y_i) \text{ for all } p \in P, \quad (3)$$

and with the indifference relation \sim that is derived from such a representation also defined in Bewley (2002) as in (2).

Proposition 1.

Suppose \sim is the indifference relation that is derived from a weak or strict Bewley representation in \mathbb{R}^n by means of a continuous and strictly increasing utility function $u : \mathbb{R} \rightarrow \mathbb{R}$ and a compact, convex set of priors $P \subset \text{rint } \Delta(S)$. If P is of full dimension, then, for any $x, y \in \mathbb{R}^n$,

$$x \sim y \iff x = y. \quad (4)$$

Proof.

Let $\{q^1, \dots, q^n\} \subset P$ be a set of n linearly independent strictly positive probability measures on S . These exist by assumption and form a basis of \mathbb{R}^n . Suppose $x, y \in \mathbb{R}^n$ are such that $x \sim y$ in the sense of (2) and define $\mathbf{z} := \mathbf{u}(x) - \mathbf{u}(y)$, where, for any $v \in \mathbb{R}^n$, $\mathbf{u}(v) := (u(v_1), \dots, u(v_n))$. To establish that $x = y$ it suffices to show that $p\mathbf{z} = \mathbf{0}$ for all $p \in P$ implies $\mathbf{z} = \mathbf{0}$. (Indeed, since u is continuous and strictly increasing in \mathbb{R} , it readily follows that $\mathbf{u}(x) = \mathbf{u}(y) \iff x = y$.) To prove the assertion, suppose $p\mathbf{z} = \mathbf{0}$ for all $p \in P$. It holds that

$$q^i \mathbf{z} = \mathbf{0}, \quad \text{for all } i = 1, \dots, n \quad (5)$$

$$\sum_{i=1}^n \lambda_i q^i \neq \mathbf{0}, \quad \text{for all } (\lambda_1, \dots, \lambda_n) \in \mathbb{R}^n \setminus \{\mathbf{0}\} \quad (6)$$

$$\left(\sum_{i=1}^n \lambda_i q^i \right) \mathbf{z} = \mathbf{0}, \quad \text{for all } (\lambda_1, \dots, \lambda_n) \in \mathbb{R}^n \setminus \{\mathbf{0}\} \quad (7)$$

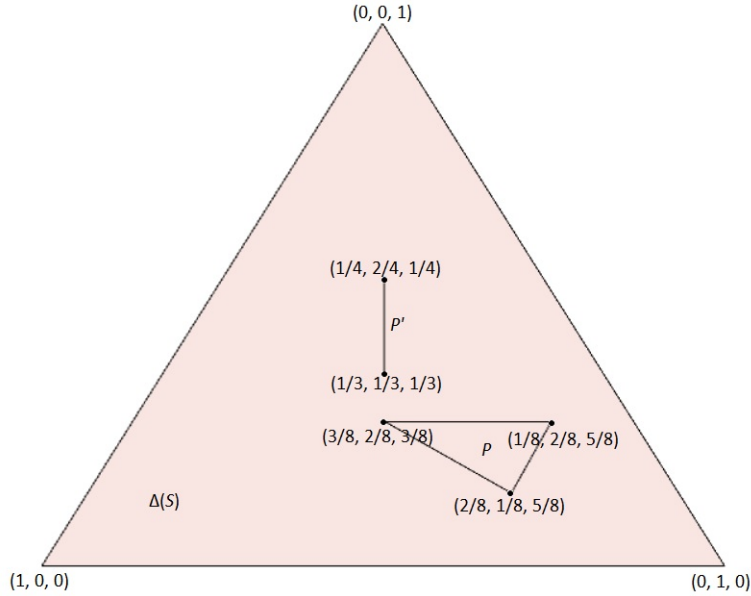
where (5) is true by assumption, (6) follows from the fact that $\{q^1, \dots, q^n\}$ is a basis of \mathbb{R}^n , and (7) is implied by (5). In particular, it follows from (6) and (7) that $\mathbf{z} = \mathbf{0}$, which, in view of the above, is equivalent to $x = y$. ■

In the special case of two states, Proposition 1 implies that the indifference relation in the Bewley model is trivial except when P consists of a single prior (in which case the decision maker is a subjective expected utility maximizer). This special case is relevant for both pedagogical and illustrative purposes. In view of the present analysis, for example, Bewley indifference curves for monetary acts over two states are literally non-existent under weak regularity assumptions on preferences (see also below).

Assuming three states and using the Marschak-Machina triangle, Fig. 1 presents two examples of compact and convex sets of priors P and P' that satisfy and violate the full dimensionality condition of Proposition 1, respectively. Indeed, the two-dimensional triangle P is the convex hull of the three linearly independent probability measures $p = (\frac{1}{8}, \frac{2}{8}, \frac{5}{8})$, $p' = (\frac{2}{8}, \frac{1}{8}, \frac{5}{8})$, $p'' = (\frac{3}{8}, \frac{2}{8}, \frac{3}{8})$ and, in light of Proposition 1, is associated with trivial indifference under both types of Bewley preferences. On the other hand, the one-dimensional linear segment P' is the convex hull of $q = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and $q' = (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$.

Since P' does not contain three linearly independent probability measures, it does not imply that indifference is trivial. Indeed, $px = py = 0$ holds for all $p \in P'$ whenever $x = (a, 0, -a)$ and $y = (b, 0, -b)$ are such that $a \neq b$.¹

Figure 1: Sets of priors with/without full dimensionality



As far as the underlying preferences' structure is concerned, the strict-preference version of Bewley's model that was asserted in Proposition 1 is characterized by axioms requiring \succ to be a strict partial order that also satisfies independence, strict monotonicity ($x \geq y$ and $x \neq y$ implies $x \succ y$) and open-graph continuity.² The weak-preference analogue of that model on the other hand is characterized by axioms requiring \succeq to be a preorder that also satisfies independence (as applied to such a relation), strict monotonicity and closed-graph continuity.³

It is presently unclear if intuitive axioms on preferences exist that characterize full dimensionality of the set of priors in the two versions of the Bewley model that are studied in this paper, and if the indifference-triviality conclusion carries over to more general domains of choice under uncertainty. It is also unclear if analogous results are obtainable in the extensions of the Bewley model that were proposed in Galaabaatar and Karni (2013) and Faro (2015). Answering these questions is left for future work.

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¹I thank a referee for suggesting this example.

²This follows from Theorem 1 in Bewley (2002).

³This follows from Proposition A.1 in Ghirardato, Maccheroni, and Marinacci (2004).

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