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A plea for KR

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Abstract
There is a strong case to be made for thinking that an obscure logic, KR, is better than classical logic and better than any relevant logic. The argument for KR over relevant logics is that KR counts disjunctive syllogism valid, and this is the biggest complaint about relevant logics. The argument for KR over classical logic depends on the normativity of logic and the paradoxes of implication. The paradoxes of implication are taken by relevant logicians to justify relevant logic, but considerations on the normativity of logic show that only some of the paradoxes of implication are genuine. KR avoids all the genuine paradoxes of implication, unlike classical logic. Overall, KR avoids the genuine paradoxes of implication and avoids the major objection to relevant logics. This combination of features provides strong reason to give KR a place in the conversation about the right logic(s).

Keywords Relevant logic · Paradoxes of implication · KR

1 Introduction

There is a certain obscure logic that deserves much more attention than it has received so far. I doubt that anyone who is not a logician has ever heard of it, and likely many logicians have not either. It is what we might call a *super relevant logic* because it is stronger than any relevant logic, but weaker than classical logic. The logic is KR, and there is a strong case to be made in favour of it.

KR was first mentioned in *Relevant Logics and Their Rivals* as being investigated independently by A. Abraham and by Routley and Meyer.[1] It results from adding axiom scheme (MECQ), $\vdash (\neg A \wedge A) \rightarrow \neg B$, to the relevant logic R. Andersen,

[1] Routley and Meyer (1982: pp. 378–379).

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Belnap, and Dunn write: "In KR, classical negation and relevant negation are *identified*. One's initial reaction to KR is that it is probably a trivial system, if it doesn't simply collapse into classical logic. As we shall see, this reaction could hardly be wider of the mark."[2] Andersen, Belnap, and Dunn go on to present some features of KR and note its connection to projective geometry. As far as I can tell, no one has ever advocated KR for any purpose, and the only significant roles it has played in the literature is in Urquhart's results on undecidability and interpolation, and Fallahi's recent work on pretabular logics.[3] KR also has a semantics that is considerably simpler than the standard Routley–Meyer semantics for R.[4]

The case for KR is twofold. One consideration pertains to the paradoxes of material implication, which provide the central motivation for relevant logics like R.[5] I argue that the debate about whether avoiding the paradoxes of implication provides a good reason to endorse relevant logic over classical logic rests on a mistake about the normativity of logic. The mistaken assumption is that if people do not infer according to a certain pattern, then the correct logic should not count that pattern as valid. Several decades ago, Gilbert Harman demonstrated that this assumption conflates the psychological process of *inference* with the semantic relation of *implication*. Instead of the mistaken assumption, I argue that we ought to adopt Greg Restall's view on the normativity of logic. However, one unnoticed consequence of Restall's view is that some of the paradoxes of implication are not problematic at all, while others are genuinely problematic. The correct logic ought to avoid only the genuine paradoxes of implication. It turns out that KR excludes all of them. As such, to adopt R over KR is to throw out some of the babies with the bathwater.[6]

Which babies? That is the second consideration in favour of KR: the status of disjunctive syllogism. Relevant logics are notorious for rejecting this inference rule, and the debate about it has been heated. I argue that there are plenty of considerations in favour of disjunctive syllogism, and no good reason to reject it. As such, disjunctive syllogism should be a rule of the correct logic or logics.

2 Anderson et al. (1992: p. 350).

3 See Urquhart (1983, 1993, 2017) and Fallahi (2018).

4 See Anderson et al. (1992: p. 350) for details on the semantics for KR and the soundness and completeness results.

5 Some philosophers and logicians argue for weak relevance logics (e.g., B) from considerations about the semantic paradoxes, but these are not the focus of this paper; see Beall (2009) for an example.

6 Arguing for a particular logic is a tricky business. There is currently a lively debate about logical pluralism—whether multiple distinct logics are equally correct. (See Beall and Restall (2006) and Cook (2010) for more on logical pluralism.) If logical pluralism is right, then there is no single universal standard for evaluating deductive arguments. For the most part, considerations for or against logical pluralism are independent of the case in favour of KR. My conclusion is: if there is one correct logic, then KR is it, and if there is not, then KR is among the correct ones. Indeed, there is a better case to be made for KR than for the logic of relevant implication (the strongest relevant logic, R) or classical logic (C), which are its two main competitors. There is, of course, a massive literature on intuitionistic logic (I), and the costs and benefits of adopting it over classical logic. This debate too is independent of the case for KR, which has a classical negation (in the sense that it obeys excluded middle and double negation elimination), but one could swap it out for an intuitionistic negation instead. One could intuitionize KR in the same way that one can intuitionize R; see Dosen (1981). Core logic is similar in that it relevantizes intuitionistic logic; see Tennant (2017). Intuitionistic KR (IKR?) has been studied in Robles and Méndez (2018). See Robles and Méndez (2007) for a different approach.

KR counts disjunctive syllogism valid. Among relevant logicians, the main reason for rejecting disjunctive syllogism has been that it is used in the derivation of some of the paradoxes of implication and so is guilty by association.[7] However, once one distinguishes between the genuine paradoxes of implication and the rest, it turns out that disjunctive syllogism is associated with none of the genuine ones. Thus, the main obstacle to accepting disjunctive syllogism is illusory.

Overall, the advantage of KR over relevant logics is its inclusion of disjunctive syllogism, and its benefit over classical logic is that it avoids the genuine paradoxes of implication. These two factors make KR a better choice than either a relevant logic or classical logic.

The paper is divided into four parts. In the first section, I introduce relevant logics, the common argument for them, and the superrelevant logics like KR. I explain how KR differs from R and CL. The second section concerns the normativity of logic; in this section, I discuss the views of Gilbert Harman, Florian Steinberger, and Greg Restall in order to establish which paradoxes of implication are genuine and which are illusory. In the third section, I offer an argument for KR over classical logic: KR has none of the genuine paradoxes of implication. In the fourth section, I offer an argument for KR over R: disjunctive syllogism should be counted valid and the major reasons given by relevant logicians for thinking it is invalid are not good. Hence, there is no reason to reject it and plenty of reason to accept it.

Remember, this is only a *plea* for KR to be part of the conversation. What follows are good reasons to think it has substantial benefits over its competitors and so should be taken seriously. However, I do not presume to give a conclusive case. In other words, KR should be considered when weighing the costs and benefits of various logics in the philosophical literature. The standards for success in showing that KR should be part of the conversation are considerably easier to meet than the standards for having conclusively shown that KR is the best logic.

2 Relevant logics, superrelevant logics, and classical logic

Understanding the relationship between KR, R, and C is the first order of business. We can begin with relevant logic and the primary argument for it.

2.1 Paradoxes of material implication

Historically, relevant logic developed out of concern for the following paradoxes of implication; they are valid in classical logic, but they are all invalid in every relevant logic.[8]

[7] Belnap recalls developing relevant logics from Ackermann's systems by eliminating disjunctive syllogism from its inference rules in order to have closer parallel between derivable sequents and conditional theorems. I return to this consideration in Section Four. See Belnap (1989).

[8] See Anderson and Belnap (Anderson and Belnap 1975: pp. 3–5) and Routley and Meyer (1982: pp. 1–10).

Material *Ex Contradictione Quodlibet* (MECQ):	$\vdash (A \wedge \neg A) \rightarrow B$
Material *Ex Falso Quodlibet* (MEFQ):	$\vdash \neg A \rightarrow (A \rightarrow B)$
Verum Ex Quodlibet (VEQ):	$\vdash A \rightarrow (B \vee \neg B)$
Positive Paradox (PP):	$\vdash A \rightarrow (B \rightarrow A)$
Negative Paradox (NP):	$\vdash A \rightarrow (B \rightarrow B)$
Linear Order (LO):	$\vdash (A \rightarrow B) \vee (B \rightarrow A)$
Unrelated Extremes (UE):	$\vdash (A \wedge \neg A) \rightarrow (B \vee \neg B)$

Each of these is a theorem of classical logic and none of them are theorems of relevant logic. It is commonly assumed in the debate that they stand or fall together. That assumption is exactly what I show is false.

If one thinks of the conditional symbol (→) as a model for the conditional construction of English (if …, then __), then one can find instances of these schemata that are intuitively unacceptable.[9] For example, ‘if snow is white, then if grass is green, snow is white’ is an instance of PP, but to many people, it sounds wrong. Relevant logics were originally designed to avoid these problems by using a conditional that does not have the paradoxes of implication as theorems.[10]

One does not just stipulate that the paradoxes of implication are not valid—one must formulate a logic in which it is impossible to derive them. There are multiple ways to achieve this but a natural one is to give up certain inference rules, the most obvious of which is *ex falso quodlibet*

(EFQ): $A, \neg A \vdash B$.

and the related *ex contradictione quodlibet*

(ECQ): $A \wedge \neg A \vdash B$.

Of course, it is not easy to eliminate all the axioms and inference rules that can be used to derive one of the paradoxes of implication without eliminating something people find intuitively acceptable. For example, one can use the inference rule of *disjunctive syllogism*

(DS): $A \vee B, \neg A \vdash B$

to derive any arbitrary sentence from a contradiction via what is known as the Lewis Argument.[11] Hence, if the logic is going to exclude (EFQ), then it has to exclude (DS) as well.[12] That is exactly why relevant logicians reject DS.[13] Keep in mind that in what follows, by ‘disjunctive syllogism’ and ‘DS’, I refer to the above inference

[9] I follow relevance logicians in assuming that the logical connectives should faithfully model their natural language counterparts. See Mares (2004: pp. 125–160) for discussion.

[10] See Tennant (2017) for an alternative motivation that emphasizes deducibility.

[11] Lewis and Langford (1932).

[12] This assumes that one does not follow Tennant (2017) in rejecting the structural rule of transitivity, which allows one to retain (DS) but not (EFQ). Comparison of KR with Tennant’s non-transitive systems is clearly needed, but my focus here is to get KR in the discussion before engaging in a complex defense like this.

[13] See Dunn and Restall (2002: pp. 30–34) and Mares (2004: p. 175) for further discussion of those who agree with Anderson and Belnap that one ought to reject DS because one rejects EFQ.

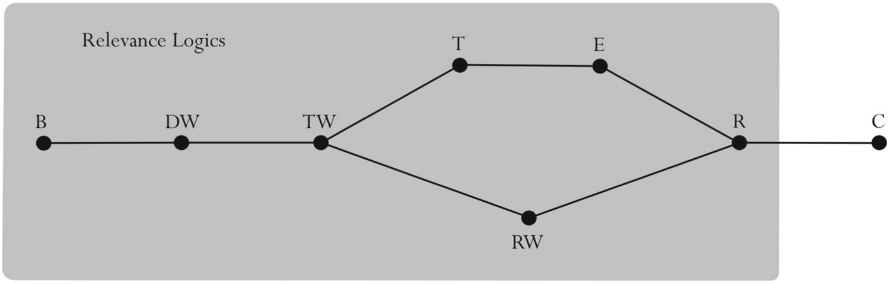

Fig. 1 Familiar relevant logics (this diagram is based on one in Read (1989: 60), with some differences; see Read (1989) for discussion. Logics like Core logic—see Tennant (2017)—are not included because this plea for KR focuses on showing that it is better than the traditional relevance logics like R. A close examination of the respective merits of KR and Classical Core Logic (as relevantizations of classical logic) is more complicated and has to be saved for future work.)

rule—we will see in a bit that these terms are interpreted in different ways so one must be careful to avoid confusion; more on this point later.

Relevant logicians aim to formulate a logic that is faithful to our reasoning processes, and to that end, they reject any inference rules that could be used to derive one of the paradoxes of implication. Their diagnosis of what has gone wrong in the reasoning for one of the paradoxes is that it involves a step where the relevant connection between the premises and the conclusion of an argument or between the antecedent and the consequent of a conditional is missing. As such, they think that (EFQ), where anything follows from a contradiction, is a mistake. As a result, they reject (DS) as well, which seems to many people like an intuitively acceptable inference rule [if only it could not be used to derive (EFQ)].

2.2 Relevant logics, superrelevant logics, and KR

In this section, I present some details of KR and place it in the context of the family of relevant logics and the logics that are stronger than R (and thus not relevant logics) but weaker than classical logic. Following Dunn and Restall, we can call these *super-relevant logics*, not because they are inspired by some relation, super-relevant, but rather because they are stronger than any of the relevant logics.[14]

Before we can appreciate my preferred logic, we need to get a clearer picture of the super-relevant logics. Consider the following figure that shows just a few of those commonly discussed:

In Figure 1, the logic, B, on the far left, is relatively weak and the logics increase in strength to the right. I begin by focusing on *axiomatic systems* of R and its extensions (issues related to natural deduction are discussed below). In an axiomatic system, a *derivation* consists of sequence of formulas where each one is either an axiom or

[14] Dunn and Restall (2002). Routley and Meyer (1982) call them para-relevant logics. What counts as a relevant logic is controversial, but many use the variable sharing property (e.g., antecedents and consequents of conditional theorems share a propositional variable); see Section Four for more discussion.

follows from earlier ones by an inference rule.[15] A *logic* is a set of sequents, where a *sequent* is a pair consisting of a set of sentences and a sentence.

Now we need to add to this diagram the superrelevant logics, which belong in the region between R and C. Begin with a standard version of R[16]:

Axiom Schemata:

(R1) $A \to A$

(R2) $(A \to B) \to ((B \to C) \to (A \to C))$

(R3) $A \to ((A \to B) \to B)$

(R4) $(A \to (A \to B)) \to (A \to B)$

(R5) $(A \wedge B) \to A$

(R6) $(A \wedge B) \to B$

(R7) $A \to (A \vee B)$

(R8) $B \to (A \vee B)$

(R9) $((A \to B) \wedge (A \to C)) \to (A \to (B \wedge C))$

(R10) $(A \wedge (B \vee C)) \to ((A \wedge B) \vee (A \wedge C))$

(R11) $\neg\neg A \to A$

(R12) $(A \to \neg B) \to (B \to \neg A)$

Rules:

($\wedge$Intro) $A, B \vdash A \wedge B$

(MP) $A \to B, A \vdash B$

It will be helpful to consider Eric Schechter's investigation into the results of adding some paradoxes of implication to R.[17] First, if we add positive paradox:

(PP) $\vdash A \to (B \to A)$,

to R, the result is CL, classical logic—all the other paradoxes of implication are provable once PP is added to R. Second, if we add the mingle principle:

(M) $\vdash A \to (A \to A)$,

the result is the logic RM. In this logic, PP is not valid, but both LO and UE are valid. Stronger than RM, but not as strong as classical logic, is RM_3, which results from adding M and the following formula:

(3) $\vdash A \vee (A \to B)$

to R. M is obviously valid in RM_3, and LO and UE are as well. Fourth, if only linear order:

(LO) $\vdash (A \to B) \vee (B \to A)$,

15 See Anderson and Belnap (1975), Anderson et al. (1992), Mares (2004), and Schechter (2004) for a list of the axioms of R.

16 Mares (2004: pp. 208–209).

17 Schechter (2004).

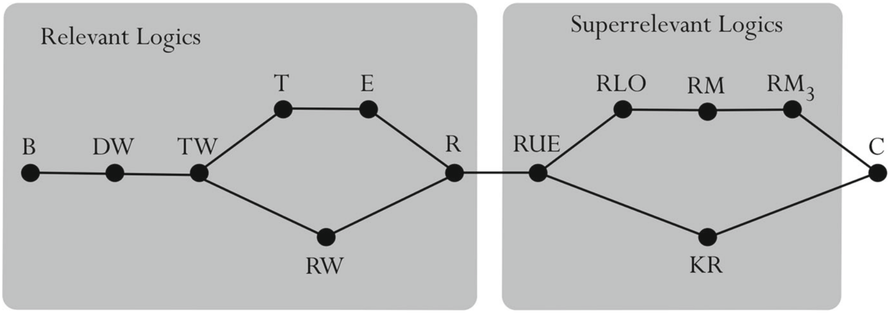

Fig. 2 Relevant and superrelevant logics

is added to R, the result is the logic RLO. In RLO, neither PP nor M are valid, but UE is valid. Finally, when UE is added to R, the result is the logic RUE. In RUE, PP, M, LO are all invalid.[18]

Figure 2 depicts the relationship between the four super-relevant logics discussed so far (i.e., RM_3, RM, RLO, and RUE).

It turns out that adding material ex falso quodlibet, material disjunctive syllogism, or verum ex quodlibet:

(MECQ) $\vdash (\neg A \wedge A) \rightarrow \neg B$

(MDS) $\vdash (\neg A \wedge (A \vee B)) \rightarrow B$

(VEQ) $\vdash A \rightarrow (B \vee \neg B)$

to R results in KR.

The place of KR among the superrelevant logics is justified by the following facts (proofs in footnotes).

1. (MECQ), (MDS), and (VEQ) are equivalent in R.[19]
2. (MECQ), (MDS), and (VEQ) are not theorems of RM_3.[20]
3. (MECQ), (MDS), and (VEQ) are not theorems of RM.[21]
4. (MECQ), (MDS), and (VEQ) are not theorems of RLO.[22]
5. (UE) is a theorem of KR.[23]
6. (LO), (PP), (NP), (MECQ), and (M) are not theorems of KR.[24]

18 For proofs of all these results, see Schechter (2004).

19 It is trivial to derive (VEQ) from (MECQ) and vice versa, and to derive (MDS) from (MECQ) and vice versa using only inference rules of R.

20 Let $\mathbf{MRM}_3$ be the matrix characteristic of the logic RM_3 with truth values 0, 1, and ½ where 1 and ½ are designated; see Priest (2001). Let A and B be distinct propositional variables and let $\Box$ be an interpretation defined in $\mathbf{MRM}_3$ such that $\Box(A)=½$ and $\Box(B)=0$. Then, $\Box((A\wedge \neg A) \rightarrow B) = 0$. Because (MECQ) is equivalent to (VEQ) and to (MDS), neither (VEQ) nor (MDS) is valid in RM_3 either.

21 RM_3 is an extension of RM; hence the result holds for RM.

22 RM_3 is an extension of RLO; hence the result holds for RLO.

23 It is trivial to derive (UE) from (MECQ) using only inference rules of R.

24 Using the standard Routley–Meyer ternary semantics for KR [see Anderson et al. (1992: p. 350)]. Consider the KR-frame where $D = \{0, 1\}$ and R = {<0, 0, 0>, <0, 1, 1>, <1, 0, 1>, <1, 1, 0>, <1, 1, 1>}.

	R	RUE	RLO	RM	RM_3	KR	CL
(PP) $\vdash A \rightarrow (B \rightarrow A)$	✗	✗	✗	✗	✗	✗	✓
(NP) $\vdash A \rightarrow (B \rightarrow B)$	✗	✗	✗	✗	✗	✗	✓
(MEFQ) $\vdash \neg A \rightarrow (A \rightarrow B)$	✗	✗	✗	✗	✗	✗	✓
(MECQ) $\vdash (A \wedge \neg A) \rightarrow B$	✗	✗	✗	✗	✗	✓	✓
(VEQ) $\vdash A \rightarrow (B \vee \neg B)$	✗	✗	✗	✗	✗	✓	✓
(M) $\vdash A \rightarrow (A \rightarrow A)$	✗	✗	✗	✓	✓	✗	✓
(LO) $\vdash (A \rightarrow B) \vee (B \rightarrow A)$	✗	✗	✓	✓	✓	✗	✓
(UE) $\vdash (A \wedge \neg A) \rightarrow (B \vee \neg B)$	✗	✓	✓	✓	✓	✓	✓

Fig. 3 Logics, theorems, and non-theorems

From these results, we acquire a decent understanding of the super-relevant logics and the place of KR among them. And Figure 3 illustrates which of the paradoxes of material implication are derivable in which logics.

2.3 KR

So far we have seen an axiomatization of KR, how it differs from R (one of the most popular relevant logics), how it differs from classical logic, and how it is related to the other super-relevant logics. We can say that KR is stronger than any relevant logic, but it is weaker than classical logic. All the arguments classified as valid by R are valid in KR too, but not vice versa. And all the arguments classified as valid by KR are valid in C too, but not vice versa.

3 The normativity of logic

Logic is often thought to be normative in the sense that it determines how one *ought* to reason or how one *may* reason. For example, it seems right to say that if some argument is valid, then it is permissible to reason from the premises of that argument to its conclusion. However, it turns out that the link between the validity of an argument and what people may or ought to do in reasoning is complex and subtle. In this section, I present the main argument in favour of relevance logic, show that it is an appeal to the normativity of logic, and present several problems for this sort of appeal.

Footnote 24 continued
There is KR-model on this KR-frame such that $0 \vDash A, \neg B$, and $1 \vDash \neg A, B$. Because $0 \nvDash A \rightarrow B$ and $0 \nvDash B \rightarrow A$, it follows that $0 \nvDash (A \rightarrow B) \vee (B \rightarrow A)$. Thus, (LO) is not a theorem of KR. Using the same KR-frame, a countermodel for (PP) is $0 \nvDash A \rightarrow (B \rightarrow A)$, $0 \vDash A$, and $1 \vDash \neg A, B$. On the same KR-frame, a countermodel for (NP) is $0 \nvDash A \rightarrow (B \rightarrow B)$, $0 \vDash \neg B$, and $1 \vDash A, B$. On the same KR-frame, a countermodel for (MECQ) is $0 \nvDash \neg A \rightarrow (A \rightarrow B)$, $0 \vDash \neg A$, and $1 \vDash A, \neg B$. And finally, a countermodel for (M) is $0 \nvDash A \rightarrow (A \rightarrow A)$, $0 \vDash \neg A$, and $1 \vDash A$.

3.1 The argument for relevant logic

All the most famous arguments for relevant logic are based on the paradoxes of implication, and these arguments are textbook examples of an appeal to the normativity of logic.[25] Here is Graham Priest on the what he calls the "anomaly" that in classical logic, anything follows from a contradiction [i.e., (ECQ)]:

> By a process that does not fall short of indoctrination most logicians have now had their sensibilities dulled to these glaring anomalies. However, this is possible only because logicians have also forgotten that logic is a normative subject: it is supposed to provide an account of correct reasoning. When seen in this light the full force of these absurdities can be appreciated.[26]

The line of argument is clear: because logic is supposed to capture correct reasoning, and we do not infer anything from a contradiction, logic should exclude (ECQ). For example, imagine a person who finds herself holding contradictory beliefs. Upon discovering this, is she likely to start randomly inferring lots of beliefs from this contradiction? Unless she is a particularly batty person, no, she will not. Instead, when one realizes that one holds contradictory beliefs, one usually attempts to eliminate the contradiction by belief revision. People simply do not behave as if anything follows from a contradiction.

Relevant logicians conclude that the correct logic ought not permit (ECQ). For example, Anderson and Belnap claim that (ECQ) is "self-evidently preposterous," their reason being that we do not and should not infer anything from a contradiction, our theory ought to not have inference rules permitting or obligating us to do so.[27] Edwin Mares and Robert Meyer make this kind of appeal as well.[28] The central point for these relevant logicians is that (ECQ) should not be an inference rule because it is not something we should follow.

Relevant logicians make the same appeal to common inference patterns when rejecting other inference rules that can be used to derive paradoxes of implication. For example, Mares discusses the rule form of (VEQ) and argues,

> Thus, if we set p to mean 'my dog barks at rubbish collectors' and q to mean 'it is raining in Bolivia right now' we find out that the inference
>
> My dog barks at rubbish collectors
> ∴ Either it is raining in Bolivia right now or it is not.

[25] This plea for KR engages only with the canonical arguments for relevant logic. New and fascinating arguments for relevant logics have recently been given by Tennant (2017: pp. 40–44) and Beall (2018). Beall in particular takes care to make the assumptions of his argument consistent with Harman's point, which I emphasize in this section. However, Beall's argument does depend on his shrieking manoeuvre, which leaves it open to criticisms like those in Scharp (2018). Further discussion and comparison is desirable, but beyond the scope of this paper.

[26] Priest (1979: p. 297). Although Priest is not himself a relevance logician, he is arguing for a paraconsistent logic, i.e., one that rejects (ECQ). Quoted in Steinberger (2016).

[27] Anderson and Belnap (1975: p. 165).

[28] Mares and Meyer (2001).

is valid. Since the premise is true, the argument is also sound. But, … it is a very bad argument.[29]

And, later, Mares writes about this same inference rule, "But according to our pre-theoretical logical intuitions, it does not seem that we are justified in inferring any logical truth from any proposition."[30] Again, we see relevant logic justified by an appeal to our intuitions about which ways of reasoning are right and which are not.

Relevant logicians argue that if such and such is not good reasoning, then the associated argument is invalid. This argument clearly appeals to one direction of the link between logical consequence and good reasoning.

3.2 Harman, MacFarlane, and Steinberger on the normativity of logic

I am going to argue that the relevant logicians' argument for relevant logic is unsound. However, to do so, I appeal to several developments in the normativity of logic debate.

The first major move is Gilbert Harman's celebrated criticism of the orthodox view on the normativity of logic. Harman distinguished between *inference*, a mental process and *implication*, a relation between propositions.[31] When one confuses these notions, one might think that inference rules are *laws* of *thought*. If one thinks of inference rules as laws of thought, then one might accept the following principle: *if one believes the premises of a valid inference, then one should (or is permitted to) believe the conclusion*. However, Harman argues that inference rules do not govern the process of inference—they do not determine how one may or ought to infer. That is, inference rules neither permit one to infer in a certain way nor obligate one to infer in a certain way. Rather, inference rules encode the relation of implication between propositions.

Consider the inference rule modus ponens: $p, p \rightarrow q \vdash q$. Assume, for a moment, that inference rules dictate what one *ought* to believe.[32] If one believes both p and $p \rightarrow q$ then one would be wrong or mistaken if one did not also believe q. Imagine that a person came to hold both the belief that p and the belief that $p \rightarrow q$ at very different times in her life such that she never really considered both beliefs in the same moment.[33] It does not seem that she has done anything wrong by not also holding the belief that q. It is important to make clear that there is not enough time in the day for one to determine and come to hold all the beliefs that follow (via inference rules) from the combinations of all the other beliefs one already holds. Inference rules do not dictate what one *ought* to believe. Therefore, the inference rule modus ponens does not tell us that if one believes p and $p \rightarrow q$ then one also *ought to* believe q.

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29 Mares (2004: p. 4).

30 Mares (2004: p. 8).

31 See Harman (1986). I acknowledge that the terms 'inference', 'implication', 'entailment' are often used to mean different things by different people even within the same debate. While it obviously would be far easier if philosophers were more consistent on these matters, this is (sadly) not something that I have any control over. Moreover, it is not my point here. In the text, I point to a conceptual distinction that I think is crucial for the discussion at hand, but what particular terms one uses to mark the distinction is not.

32 See Mares (2004) for example.

33 Throughout this paper, I use expressions like 'I hold the belief p' as shorthand for "I believe the content of the sentence that 'p' stands for".

Instead, one might think that inference rules dictate what one *may* believe. For the example of modus ponens, it would follow that one who holds the two beliefs (p and $p \rightarrow q$) is permitted to infer q. But, what if the person already holds the belief that $\neg q$? If we understand inference rules as functioning this way, then modus ponens would permit one to believe both q and $\neg q$. Clearly, the mere fact that one comes to believe p and $p \rightarrow q$ does not, by itself, give permission also to believe q—if it did, there could be cases in which inference rules provide permission to hold contradictory beliefs. Inference rules dictate neither what one is *permitted* to believe nor what one is *obligated* to believe. That is, despite their name, inference rules are not rules governing our mental process of inference.

The second major advance in reflection on the normativity of logic comes from John MacFarlane. He suggested that we examine the extent to which logic is normative by reflecting on bridge principles that connect logical consequence with the normative arena.[34] A *bridge principle* is any principle that successfully links these two domains in a way that upholds the significance of logical consequence for norms governing human reasoning. MacFarlane also provides a handy categorization scheme for bridge principles that cover a wide range of options.

The general form of all the bridge principles MacFarlane considers is as follows:

If P1, …, Pn ⊢ Q, then φ(P1, …, Pn, Q)

The overall form is a conditional. The antecedent is about logical consequence and the consequent is some claim about the propositions related in the antecedent. The nature of this claim varies in three possible ways: scope, normativity, and valance.

Scope:	(C) Narrow scope 'ought', only for the consequent: if A, then O(B)
	(W) Wide scope 'ought', for the whole conditional: O(if A, then B)
	(B) Both antecedent and consequent bound by 'ought': if O(A), then O(B)
Normativity:	(o) ought
	(p) permission
	(r) defeasible reason
Valence:	(+) positive—obligation to believe
	(−) negative—obligation to not disbelieve

There are twelve possible combinations and so twelve possible bridge principles. For example:

(Co+) If P1, …, Pn ⊢ Q, then if S believes the Pi, S ought to believe Q.

(Co-) If P1, …, Pn ⊢ Q, then if S believes the Pi, S ought to not-disbelieve Q.

are two of the twelve.

In Florian Steinberger's recent paper, "Explosion and the Normativity of Logic," he argues in detail that once one thinks about bridge principles in the way MacFarlane suggests, none of them justify the claim that (ECQ) is invalid. As Steinberger reconstructs the relevant logician's argument against (ECQ), it needs a bridge principle that is *both* plausible *and* strong enough to support the desired conclusion [i.e., that (ECQ)

[34] MacFarlane (2004).

is invalid or should be excluded from any legitimate logic]. Steinberger then argues that *none* of the bridge principles in MacFarlane's space of possibilities satisfies both criteria. Here is the argument against (EFQ) that Steinberger reconstructs on behalf of the relevantist:

(1) (ECQ) is valid.
(2) S believes each member of an inconsistent set of propositions Φ.
(3) If P1, …, Pn ⊢ Q, then if S believes the Pi, S ought to believe Q.
(4) Even if S's set of beliefs is inconsistent and any proposition Q whatsoever is entailed by it (courtesy of ECQ), there are Qs such that S ought not to believe Q.
(5) Φ ⊢ Q for some patently unacceptable Q that S ought not to believe (from 1 and 2).
(6) S ought not to believe Q (from 4).
(7) S ought to believe Q (from 2, 3 and 5 via modus ponens).
(8) Contradiction (from 6 and 7).
(9) (ECQ) is invalid (from 1 by reductio). □

Call this the *normative argument*.[35] A problem with the normative argument is that one might reject (3) instead of (1) because there are independent reasons to reject (3). (3) is a bridge principle that links up logical consequence with some normative aspect of human activity.

Steinberger provides us with not only a precise and detailed objection to the relevant logician's argument against ex falso, but also a general schema for making the same kind of point in all the other cases. That is, we can apply Steinberger's normative argument to the rest of the paradoxes of implication. Moreover, in the course of giving his argument, Steinberger gives us a comprehensive assessment of all the bridge principles in MacFarlane's categorization scheme.

> (Co+), (Co−), (Cp+), (Cp−) are implausible in that they fall prey to Harman's objections.
> (Cr+), (Cr−) are too weak in that they do not make the normative argument sound.
> (Wo+), (Wo−), (Wp+), (Wp−), (Wr+), and (Wr−) are too weak.
> (Bo+), (Bo−), (Bp+), (Bp−) are too weak.
> (Br+), and (Br−) are implausible and too weak.

Steinberger's conclusion is that *the relevant logicians' normative argument for relevance logic is unsound*; there is no way to fill in the bridge principle that is plausible and strong enough to make the argument go through.

3.3 A normative necessary condition for validity

Harman inspires us to question the sense in which logic is normative by criticizing the dominant view on the connection between validity and normativity at the time. This turned out to be just one bridge principle (Co+) among many, as MacFarlane pointed out. And Steinberger reconstructed one criticism of (ECQ) and showed that it does not go through for any of MacFarlane's bridge principles.

35 See Steinberger (2016).

However, there is another way to use bridge principles in the discussion about the normativity of logic: they state necessary conditions for validity. For example, (Co+), the bridge principle Harman originally criticized, is:

(Co+) If P1, …, Pn ⊢ Q, then if S believes the Pi, S ought to believe Q.

All the bridge principles we have seen are conditionals with a logical consequence claim as its antecedent and a claim about the normativity of some human states or processes as its consequent. (Co+), for example, links logical consequence and what ought to be believed. Of course, we have good reasons from Harman to think that (Co+) is false. But there are other more plausible bridge principles that do provide substantive necessary conditions on logical consequence. Like any necessary condition, they *cannot* be used to show that some entailment *holds*, but it *can* be used to show that some purported entailment *fails to hold*.

In what follows, I focus on a particularly popular bridge principle that seems plausible to me as well.[36] Consider what Greg Restall proposes in the following passage:

> To take an argument to be valid does not mean that when one asserts the premises one should also assert the conclusion (that way lies madness, or at least, making *many* assertions). No, to take an argument to be valid involves (at least as a part) the commitment to take the assertion of the premises to stand against the denial of the conclusion. In general, we can think of logical consequence as governing *positions* involving statements asserted and those denied. Logical validity governs positions in the following way:
>
> If A ⊢ B, then the *position* consisting of asserting A and denying B *clashes*.
> If B deductively follows from A, and I assert A and deny B, I have made a mistake.[37]

Inset in the quote is a bridge principle. Its antecedent is a single-premise entailment, and its consequent pertains to speech acts. We can generalize it in two ways: (i) to cover propositional attitudes as well, and (ii) to include multipremise deductions. In what follows, I use the term 'acceptance' as a propositional attitude associated with sincere assertion (one could also use 'belief', but 'acceptance' is more common in the logical literature) and 'rejection' as a propositional attitude associated with sincere denial.[38]

Restall's bridge principle is very similar to the following bridge principle from MacFarlane:

> (Wo−) If P1, …, Pn ⊢ Q, then it ought to be that: if S believes the Pi, S does not disbelieve Q.

I think it is more accurate to the Restall quote to formulate the consequent as a wide scope 'ought not' over a conjunction, like this:

[36] Although many assume that Harman was purely skeptical about the normativity of logic, he does offer something very much like what Restall suggests in the following quote; see Harman (1986: ch. 2).

[37] Restall (2013: p. 83). I have removed Restall's use/mention convention in this quote.

[38] One might think that to reject that p is just to accept that not p (and to deny that p is to assert that not p), but this claim is incompatible with many non-classical logics. For example, the intuitionist rejects the law of excluded middle, but does not accept its negation. Likewise, the dialetheist accepts the liar sentence and its negation, but does not reject either of them.

(Restall) If P1, …, Pn $\vdash$ Q, then it ought to be that not: S believes the Pi and S disbelieves Q.

If one could replace logical equivalents inside 'ought' operators, then (Wo-) and (Restall) would be equivalent, but there are good reasons to think that such moves are questionable.[39]

We can use (Restall) as a necessary condition for an argument being valid. If the consequent of (Restall) is false for some particular case, then that is not a valid argument. In other words, if for some sentences P1, …, Pn and Q, it is permissible that S believes all the Pi and S disbelieves Q, then Q is not a logical consequence of P1, …, Pn. It is exactly this sort of argument I propose in the next section.

I am not going to argue for (Restall) except to say that it holds up well under scrutiny, as in Steinberger's analysis. There is good reason to think that other bridge principles are also plausible, but thinking through their implications is beyond the scope of this paper.[40] Remember, this a *plea* for KR: there are good reasons to think that KR should be taken seriously. It is reasonable to appeal to a popular and plausible bridge principle to help make that case.

So far, I have provided some background on KR and argued that the primary argument for relevant logics involves a fallacy. I also advocated a particular way of thinking about the normativity of logic encoded in (Restall). Now we put these considerations to work.

4 KR is better than C

KR is better than classical logic because KR has none of the genuine paradoxes of implication, although KR does have some illusory paradoxes of implication.

It turns out that some of the paradoxes of implication, like (ECQ) are implausible when substituted in for (Restall), but others are not. For (ECQ), imagine an explicit contradiction, $\neg A \wedge A$, as a premise and any sentence, B, as the conclusion of the argument in question. Now we ask ourselves about the consequent of (Restall) in this case. We ask: is it permissible that: S believes $\neg A \wedge A$ and S disbelieves B? The answer is *no*, regardless of what B is. The reason is that it is impermissible to believe the contradiction, regardless of whatever else is going on. Therefore, in the instance under consideration, the consequent of (Restall) is true. We need a case that makes the consequent of (Restall) false to be able to use it at all. One might wonder whether there are any cases like this. There are.

Can we find a paradox of implication that would make the consequent of (Restall) false? If so, then we would have a good reason for saying it is invalid. Of course, each of the paradoxes of implication is formulated as a theorem rather than as an inference rule, but the case of an inference rule like (ECQ) and a theorem like (MECQ), there is an obvious connection that needs to be explored. If an inference rule is valid in some logic but the associated theorem invalid in that logic, then that logic cannot deem conditional proof valid in a natural deduction system, which is a serious cost. Yes,

[39] See McNamara (2010) for a survey.

[40] See Steinberger (2017) for a discussion.

some of the weaker relevant logics reject conditional proof (e.g., B), but there is strong intuitive support for it.[41] Moreover, any logic we are considering that includes modus ponens cannot include a conditional theorem but exclude the associated inference rule. Therefore, if we can show that the associated inference rule should not be deemed valid, then the theorem in question should not be deemed valid either.[42] Below are the results for the paradoxes of implication.

(MECQ), $\vdash (A \wedge \neg A) \rightarrow B$. In applying (Restall), the essential issue is whether it is a mistake to believe $A \wedge \neg A$ and not believe B. *Yes it is.* It is always a mistake because it is always a mistake to believe $A \wedge \neg A$.[43] Therefore, (MECQ) meets the necessary condition laid down by (Restall). It is not a genuine paradox.

(MEFQ) $\vdash \neg A \rightarrow (A \rightarrow B)$. Is it a mistake to believe not A and not believe if A then B? *No.* For example, it is not a mistake to believe that 0=0 and not believe that if 0≠0 then snow is white. Therefore, in the case of (MEFQ), the consequent of (Restall) is false. Hence, we have good reason to think that (MEFQ) should be *excluded* from any acceptable logic. This is our first clear example of a principle that fails to meet the necessary condition on validity laid down by (Restall). *It is a genuine paradox.*

(VEQ), $\vdash A \rightarrow (B \vee \neg B)$. Is it a mistake to believe A and not believe $B \vee \neg B$? *Yes*. From the beginning, we have been ignoring issues associated with intuitionism. This plea for KR consists entirely in showing that KR is superior to classical logic and that it is superior to relevant logic. Both R and C include excluded middle as theorem. As such, given the background assumptions in play for us, it is always a mistake to not believe $B \vee \neg B$. Therefore, (VEQ) meets the necessary condition laid down by (Restall). *It is not a genuine paradox.*

(PP), $\vdash A \rightarrow (B \rightarrow A)$. Is it a mistake to believe A and not believe if B then A? *No*. For example, it is not a mistake to believe that grass is green and to not believe that if snow is white, then grass is green. Therefore, (PP) fails to meet the necessary condition for validity expressed by (Restall). Consequently, (PP) should be *excluded* from any acceptable logic. *It is a genuine paradox.*

(NP), $\vdash A \rightarrow (B \rightarrow B)$. Is it a mistake to believe A and not believe if B then B? *Yes*. It is always a mistake to not believe if A then A. Therefore, (NP) meets the necessary condition laid down by (Restall). *It is not a genuine paradox.*

(LO), $\vdash (A \rightarrow B) \vee (B \rightarrow A)$. Is it a mistake to not believe if A then B or if B then A? *Yes*. Because this principle is a disjunction, it is the only one that is not associated with some non-trivial inference rule. However, to evaluate this one we do not need to evaluate a nested conditional. Instead, it is easy to come up with an example like: either if grass is green then snow is white, or if snow is white then grass is green. There is no mistake at all in rejecting this disjunction. Therefore, (LO) fails to meet the necessary condition. Consequently, (LO) should be *excluded* from any acceptable logic. *It is a genuine paradox.*

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[41] It was Belnap and Anderson's original reason to prefer the relevant logic E over Ackermann's systems; see Belnap (1989).

[42] See Anderson and Belnap's "Grammatical Propaedeutic" in Anderson and Belnap (1975: pp. 473–493) for an influential discussion of implication and entailment.

[43] Dialetheism is not an option under consideration in this paper; if dialtheism is required to defend relevant logic, then that is a massive cost for relevant logic given the myriad worries about dialetheism.

(UE), $\vdash (A \wedge \neg A) \rightarrow (B \vee \neg B)$. Is it a mistake to believe $A \wedge \neg A$ and not believe if $B \vee \neg B$? *Yes*. It is always a mistake to believe $A \wedge \neg A$ and it is always a mistake to not believe $B \vee \neg B$. Therefore, (UE) meets the necessary condition laid down by (Restall). *It is not a genuine paradox.*

Overall, the results of this inquiry into the normative argument with Steinberger's bridge principle are mixed. Some of the paradoxes of implication are genuine and should be excluded from any legitimate logic, while others are illusory and may be included in legitimate logics. The results are summarized below:

(MECQ)	$\vdash (A \wedge \neg A) \rightarrow B$	Illusory—*may be included* in legitimate logics
(MEFQ)	$\vdash \neg A \rightarrow (A \rightarrow B)$	Genuine—*must be excluded* from legitimate logics
(VEQ)	$\vdash A \rightarrow (B \vee \neg B)$	Illusory—*may be included* in legitimate logics
(PP)	$\vdash A \rightarrow (B \rightarrow A)$	Genuine—*must be excluded* from legitimate logics
(NP)	$\vdash A \rightarrow (B \rightarrow B)$	Illusory—*may be included* in legitimate logics
(LO)	$\vdash (A \rightarrow B) \vee (B \rightarrow A)$	Genuine—*must be excluded* from legitimate logics
(UE)	$\vdash (A \wedge \neg A) \rightarrow (B \vee \neg B)$	Illusory—*may be included* in legitimate logics

Overall, (MEFQ), (PP), and (LO) are the genuine paradoxes of material implication and the relevant logicians were right about them: they really ought to be excluded from any legitimate logic. However, the rest are illusory. They are not paradoxical at all and so relevant logicians were wrong about them. It is permissible for any legitimate logic to count these principles as valid.[44]

I am not the first to draw a substantive distinction among the paradoxes of material implication. Indeed, Anderson, Belnap, and Dunn write:

> To those who have taken the trouble to read the literature on relevance logic rather than fulminate against it, it has been a familiar fact since the early 70s that there are two conceptually distinct classes of "paradoxes of material implication." The archetype of the first class (paradox of consistency) is $(A \wedge \neg A) \rightarrow B$. The archetype of the second (paradox of relevance) is $A \rightarrow (B \rightarrow A)$. It is easy to devise systems of entailment that omit one but not the other.[45]

This distinction gets mentioned occasionally in the literature but rarely, and the two classifications are never discussed in detail.[46]

One problem for the view expressed in the quotation is how to draw the distinction between paradoxes of consistency and paradoxes of relevance. (MECQ) and (PP) are mentioned explicitly, so they are easy. The following seems like a reasonable classification, but there is no explicit classification as far as I know.

[44] Notice that, although (Restall) is only a necessary condition for logical consequence, I have used it to provide necessary and sufficient conditions for the *genuineness* of a paradox of implication. That is, if a formula expressing a putative paradox of implication meets the (Restall) condition—the instance is true—then it is *illusory*. If it fails to meet the (Restall) condition, then it is *genuine*.

[45] Anderson et al. (1992: p. 249). They suggest that KR includes all the paradoxes of consistency and none of the paradoxes of relevance, but they never state this claim explicitly.

[46] See Robles and Méndez (2007) and Urquhart (2017) for example.

(MECQ)	$\vdash (A \wedge \neg A) \rightarrow B$	Consistency
(MEFQ)	$\vdash \neg A \rightarrow (A \rightarrow B)$	Both?
(VEQ)	$\vdash A \rightarrow (B \vee \neg B)$	Consistency
(PP)	$\vdash A \rightarrow (B \rightarrow A)$	Relevance
(NP)	$\vdash A \rightarrow (B \rightarrow B)$	Consistency
(LO)	$\vdash (A \rightarrow B) \vee (B \rightarrow A)$	Relevance
(UE)	$\vdash (A \wedge \neg A) \rightarrow (B \vee \neg B)$	Consistency

One problem is how to classify (MEFQ).[47] It seems very much like (MECQ), which is a paradox of consistency. But it also looks like a paradox of relevance in that it allows one to derive an irrelevant conditional via modus ponens from any negated formula. Hence, I have listed it as both. Notice that this classification lines up pretty well with classification above of these into genuine and illusory based on (Restall). Indeed the (Restall) classification is more precise since it has an explicit test.

And now for the ultimate question: are there any logics that exclude (MEFQ), (PP), (LO)? Yes! One of them is KR! The fact that classical logic counts the genuine paradoxes of implication as valid and KR does not constitutes the primary reason for thinking that KR is better than classical logic. KR does a better job of respecting the normativity of logic, at least the link between validity and normativity that is expressed by (Restall). In other words, one should choose KR over classical logic because KR is compatible with (Restall) and classical logic is not. Of course, all the relevant logics are compatible with (Restall) too. They avoid all the genuine paradoxes of implication as well, but they exclude far more of classical logic than KR does. We shall visit the other side of the argument and see why KR is superior to relevant logics in the next section.

Consider first an objection to my case in favour of KR over classical logic. This objection comes from John Burgess, who, in 1981, noted that one might think relevant logicians are guilty of conflating inference with implication. He quotes Harman making such a claim:

> By reasoning or inference I mean a process by which one changes one's view, adding some things and subtracting others. There is another use of the term 'inference' to refer to what I will call 'argument', consisting in premises, intermediate steps, and a conclusion. It is sometimes said that each step of an argument should follow from the premises or prior steps in accordance with a 'rule of inference'. I prefer to say 'rule of implication', since the relevant rules do not say how one may modify one's views in various contexts. Nor is there a different direct connection between rules of logical implication and principles of inference. We cannot say, for example, that one may infer anything one sees to be logically implied by one's prior beliefs. Clearly one should not clutter up one's mind with many of the obvious consequences of things one believes.

Furthermore, it may happen that one discovers that one's beliefs are logically inconsistent and therefore logically imply everything. Obviously, one ought not to respond

[47] Another problem is that (PP) and (NP) are logically equivalent (in R, KR, and classical logic). Hence, no extension of R can exclude one but not the other.

to such a discovery by believing as much as one can. Some philosophers and logicians [the reference is to Anderson and Belnap] have imagined that the remedy here is a new logic in which logical contradictions do not logically imply everything. But this is to miss the point that logic is not directly a theory of reasoning at all.[48]

However, Burgess dismisses this kind of objection in this paper.

Burgess considers the two readings of logic (i.e., the inference reading vs. the implication reading) as subjective (or psychological) and objective (or ontological). Then, he gives evidence that Anderson and Belnap presuppose the implication reading as their preferred reading of relevant logics (they focus on E and R primarily).[49] I agree with Burgess' interpretation—that is how Anderson and Belnap intend their logic to be interpreted. However, their arguments for the superiority of their logic over classical logic presuppose an inference reading (or psychological reading in Burgess' sense)—consider the evidence Anderson and Belnap provide (see Sect. 2.1). I do not claim that they intend a psychological reading; they do not. Burgess is correct about that. My argument is that the interpretation Anderson and Belnap give of their own logic (the objective one) is incompatible with the evidence they give for it (a psychological one). Moreover, this seems like the right way to interpret Harman's point. It is not the case that relevant logicians intend the psychological reading of their claims—so, it seems like Burgess' quick dismissal is based on a misinterpretation.

Furthermore, this issue goes beyond the point made in the quote. When one considers the kind of evidence that would support the objective interpretation—under the Steinberger bridge principle—it does not support classical logic. Instead, it supports something weaker, like KR. That does not seem to be an option that is even on Burgess' radar.

Here is another common objection: All the relevant logicians cited so far have been saying that their intuition is that certain implications do not hold (e.g., MECQ). This interpretation is more charitable and then the dispute between a defender of KR and a relevant logician is just a clash of intuitions about which sequents are logically valid.

My reply: If one looks back at each of the quotes above (and there are many more in the literature as well), one sees the relevant logician first citing considerations about what which ways of changing our beliefs would be reasonable or unreasonable. Then the relevant logician concludes that the associated implication is invalid. This is exactly the "no inference, no implication" fallacy I diagnose above.

Still, even if one rejects this diagnosis of where the relevant logicians arguments go wrong, it is crucial to remember that this section is not about preferring KR over R (that is the next section). It is about preferring KR over classical logic. The reason given is that KR avoids the genuine paradoxes of implication, but classical logic does not. Even a relevant logician can accept this argument. The primary case against relevant logic and for KR has nothing to with Harman's point. So even if one rejects the interpretation of Harman or the criticism of relevant logicians' arguments for relevant logic, the primary case for KR stands.

48 This unpublished writing of Harman's is quoted in Burgess (1981: p. 103).

49 Burgess (1981: pp. 103–104).

5 KR is better than R

It might be tempting to think that there is an argument for KR over R on the basis of the illusory paradoxes of implication. However, this is a mistake. There is no pressure to include the illusory paradoxes of implication in a logic. They *may* be included, but it is a mistake to think they *must* be included.[50]

Instead, the case for KR over R rests on the fact that KR includes disjunctive syllogism, R does not, there is no good argument for excluding disjunctive syllogism, and there is good reason for a logic to include it.

The rejection of DS is often thought of as "the most notorious feature of relevant logic."[51] Relevant logicians J. Michael Dunn and Greg Restall remark that it is

> typically the hardest thing to swallow concerning relevant logics. One starts off with some pleasant motivations about relevant implication and using subscripts to keep track of whether a hypothesis has actually been used, and then one comes to the point where one says 'and of course we have to give up disjunctive syllogism' and one loses one's audience.[52]

Similarly, Mares writes:

> The fact that disjunctive syllogism cannot be added to relevant logic is a problem. The original point of introducing relevant logic was to provide an intuitive characterization of deductive inference. But we use disjunctive syllogism all the time. … In short, disjunctive syllogism would seem to be one of our key deductive tools. So it looks like we have a problem.[53]

Obviously, this feature is commonly thought to be undesirable.

The fact is that going without disjunctive syllogism is probably the most unsavoury and counterintuitive aspects of relevant logic. This fact speaks volumes about the shear intuitive appeal of disjunctive syllogism. It is *not* one of the paradoxes of implication and so the main case in favour of relevant logic has nothing to do with disjunctive syllogism. It is such an important topic only because relevant logicians give it up as part of their misguided attempts to formulate logics that exclude *all* paradoxes of implication. However, as we have seen, disjunctive syllogism has been slandered. It is associated only with the *illusory* paradoxes of implication, and it is independent of the *genuine* paradoxes of implication. This consideration alone, to remove a major cost that comes with no corresponding benefit, is a reason to switch from R to KR as the right classification of logically valid arguments.

In addition, Burgess and others have argued that there are connections between entailment and practical rules that can underwrite an argument for disjunctive syllogism being *included* in any acceptable logic and the Harman point does not touch

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[50] Again, in R (and in KR), (PP) and (NP) are logically equivalent, even though one is illusory and one is genuine according to the (Restall). Hence, one would need a logic that is fundamentally different from KR if one wanted a logic that excludes *all and only* the genuine paradoxes of material implication.

[51] Dunn and Restall (2002: p. 30).

[52] Dunn and Restall (2002: p. 32).

[53] Mares (2004: pp. 176–177).

these connections.[54] For example, for each implication there is a rule of mathematical proof—e.g., associated with modus ponens is the rule of proof that one may conclude B from premises $A \rightarrow B$ and A in a deductive argument. And the contrary holds as well. We know from examples like those in Burgess's paper that it is a rule of deductive argumentation that one may conclude B from premises $\neg A$ and $A \vee B$. Thus, disjunctive syllogism is an implication. Moreover, disjunctive syllogism is an implication involving only logical sentential connectives. Therefore, it should be included in any acceptable logic. I think that is what Burgess's arguments show, and that is why KR is superior to any relevant logic.[55]

Here is an objection: relevant logics have been championed as the most familiar logics that have the *variable sharing property* (e.g., antecedents and consequents of conditional theorems share a sentential variable). This property is sometimes mentioned in the same context as a related property: the premise use property (i.e., all the premises are used to derive the conclusion in valid arguments).[56] However, KR loses both features. That seems to be a problem.

The variable sharing property and the criterion of premise use are good measures of relevance, but we already know that they are not sufficient for good arguments. No one thinks $A \vdash A$ is a good argument for A, even though it is valid on most relevance logics. It should not be shocking that they might not necessary either, especially when one sees the cost of accepting them (e.g., DS).[57]

The entailments picked out by relevant considerations do pick out interesting patterns of use. However, there are patterns of use that correspond to entailments that are not picked out by relevant considerations. In short, the case for KR being the right logic is ipso facto the case against variable sharing or premise use being a necessary condition for good arguments.

Now we move on to showing that there is no good argument for excluding disjunctive syllogism, or, at least, many of the popular arguments are not good. Relevant logicians have long argued that there is something wrong with disjunctive syllogism, or that they can provide some kind of substitute for it, or that it can be added without trouble to various relevant logics. If correct, these considerations would undermine one of the central arguments of this defence of KR. What are we to make of them?

First, consider some arguments that disjunctive syllogism should be excluded from any acceptable logic. One well-known argument of this kind comes from Jay Garfield, who claims that disjunctive syllogism can lead one astray if one is reasoning from misleading information. For example, I believe something false, but for good reason, like the proposition that the Earth is only 6000 years old, and I use disjunction intro-

54 See also Burgess (1981, 1983, 1984, 2005), Mortensen (1983, 1986), Read (1983), and Tennant (2005) for discussion.

55 Also motivated by the importance of disjunctive syllogism in various forms of reasoning, Tennant (2017) defends a substructural relevant logic—core logic—by rejecting unrestricted transitivity in an effort to keep disjunctive syllogism without the paradoxes of implication. Tennant's systems deserve their own comparison with KR, but this is beyond the scope of the paper.

56 Note that relevant logicians diverge on how to define 'variable sharing'. For example Read (1989) and Mares (2014) explicitly include premise use as part of variable sharing, but Méndez and Robles (2012) does not.

57 See Tennant (2015) for discussion of variable sharing and options for relevant logics.

duction to arrive at a disjunction with two false disjuncts like the following: the Earth is only 6000 years old or climate change is a hoax. Later, my initial false belief is corrected so that I now believe that the Earth is not only 6000 years old. If I now use disjunctive syllogism, I arrive at the false belief that climate change is a hoax.[58] Disjunctive syllogism has led me astray.

However, as Mares argues, the point is not specific to disjunctive syllogism. If one begins with false beliefs, no matter how well justified, *any* inference rule can lead one to other false beliefs, even after the initial belief has been corrected.[59] There is no reason to think that disjunctive syllogism is somehow singled out by Garfield's considerations.

Another attempt to show that disjunctive syllogism should be rejected because it can lead one astray comes from S. V. Bhave, who argues that in situations involving some kind of indeterminacy, disjunctive syllogism can take one from true premises to a false conclusion. If one allows sentences to have truth values other than truth and falsity (indicating some kind of indeterminacy), and one defines negation so that a sentence and its negation might each be true, then an instance of disjunctive syllogism can take one from a true sentence p together with a true disjunction (whose disjuncts are p's negation and some false sentence q), to a false conclusion q.[60] But the problem here should be obvious—the "counterexample" to disjunctive syllogism depends on reinterpreting negation so that a sentence and its negation can each be true.

Mares argues that disjunctive syllogism is unacceptable when reasoning about certain inconsistent fictions. Following a short story in which Jones inherits an estate and does not inherit an estate, Mares writes, "It seems we need situations in which it is both true that Jones inherits the estate and Jones does not inherit the estate. From this we would hardly want to infer that Jones hates ice cream. But we could do so if we are allowed disjunctive syllogism. … So it seems that we should reject disjunctive syllogism as a rule of inference." The problem with this argument is, again, that it conflates inference and implication—it is inconsistent with the Harman point. Mares claims that the argument from 'Jones inherits the estate' and 'Jones does not inherit the estate' to 'Jones hates ice cream' is intuitively unacceptable—people *should* not infer in this way. Thus, he concludes, the implication involved should be excluded from any acceptable logic. That is exactly the kind of argument we have seen again and again from relevant logicians, and it is one that is undermined by the Harman point. The move from 'such and such *inference* is unacceptable' to 'such and such *implication* is unacceptable' is illegitimate.

Instead of arguing that disjunctive syllogism should be rejected, some relevant logicians argue that we can make due with something *other than* disjunctive syllogism. For example, Read claims that there are some cases that appear as if someone is using disjunctive syllogism when they are in fact using intensional disjunctive syllogism. Intensional disjunctive syllogism is a rule governing a special connective call fission, $\oplus$, which is defined as: $p \oplus q =_{df} \neg p \rightarrow q$. Intensional disjunctive syllogism says that

[58] Garfield (1990).

[59] Mares (2004: pp. 177–178).

[60] Bhave (1997: p. 402).

q follows from $\neg p$ and $p \oplus q$. Given the definition of fission, intensional disjunctive syllogism is just a version of modus ponens.[61]

The problem with Read's suggestion is that he provides no evidence that we distinguish between something like disjunction and something like fission in natural language. The famous Lewis argument, for example, would contain an equivocation according to Read. That is, in the following deduction,

1. $A \wedge \neg A$	Assumption
2. A	1, $\wedge$E
3. $A \vee B$	2, $\vee$I
4. $\neg A$	1, $\wedge$E
5. B	3,4, DS

the formula on line 3 is, according to Read, ambiguous because the disjunction can be read as a normal extensional disjunction or as a fission. If it is extensional disjunction, then the transition to line 5 is unacceptable to the relevant logician, and if it is fission, then the transition to line 3 is unacceptable. Either way, the argument is invalid. But Read provides no evidence that there is even an ambiguity here. His proposal is that there is this connective that obeys something like disjunctive syllogism that can be defined in a relevant logic. But this proposal does nothing to quiet the worry that people have about disjunctive syllogism as an inference rule pertaining to extensional disjunction.

Another attempt like Read's is to interpret disjunctive syllogism as a *pragmatic* rule: if one accepts $p \vee q$, and one accepts $\neg p$, then one should accept q. Mares suggests that this rule follows from three others:

PDS: if one accepts $p \vee q$, and rejects p, then one should accept q.
PDS′: if one rejects $p \wedge q$, and accepts p, then one should reject q.
Contra: one should reject any contradiction.

From these we can reason in the following way: someone who accepts a disjunction $A \vee B$ and a negation, $\neg A$, should reject the contradiction $A \wedge \neg A$ (by Contra), and so should reject A (by PDS′), and so should accept B (by PDS).[62] However, there are several reasons to think that this pragmatic proposal is inadequate. First, Mares' pragmatic rules PDS and PDS′ are false as shown in the discussion of the Harman point in Sect. 2.2. For example, it is *false* that if one accepts a disjunction and rejects one of its disjuncts then one should accept the other disjunct. In a situation where the agent, say, already rejects both disjuncts, the rule councils the agent to do something irrational—i.e., accept inconsistent claims. Instead, an agent in this situation should revise his or her acceptances and rejections, and logic has little to say about how the agent should go about this task. Therefore, Mares' pragmatic version of disjunctive syllogism runs afoul of the Harman point again.

Finally, relevant logicians have devoted plenty of energy to showing that disjunctive syllogism is *admissible* in various relevant logics. Why are these results not good enough to satisfy those who want to use disjunctive syllogism? The answer is that these

61 Read (1982).

62 Mares (2004: pp. 184–186).

logicians are not talking about whether disjunctive syllogism is *valid*. For example, in two of the most recent such publications, Robles and Méndez provide a semantics for a particular relevant logic that shows disjunctive syllogism is admissible.[63] However, 'admissible' is a term of art that is distinct from having disjunctive syllogism as an inference rule or axiom. Instead, they are referring to the rule: if $\vdash$ A and $\vdash \neg$ A $\vee$ B, then $\vdash$ B. This rule is *not* disjunctive syllogism. The fact is that *neither* the inference rule of disjunctive syllogism (i.e., $A \wedge (\neg A \vee B) \vdash B$) *nor* the axiom of material disjunctive syllogism (i.e., $\vdash A \wedge (\neg A \vee B) \rightarrow B$) is logically valid in any relevant logic.

In sum, the major unsolved problem with every relevant logic is that they exclude disjunctive syllogism—if any relevant logic is correct, then disjunctive syllogism is invalid. There are, however, good reasons to think that disjunctive syllogism is valid and should be included in any acceptable logic. Proponents of relevant logics have not been able to identify anything problematic about disjunctive syllogism that distinguishes it from other inference rules, and none of their alternatives to disjunctive syllogism are workable. In short, disjunctive syllogism is as good an inference rule as modus ponens or any of the others. There is no reason to exclude it from an acceptable logic, and there is no reason to think that its role can be filled by something else.

One final objection: some logics other than KR include disjunctive syllogism and exclude the genuine paradoxes of implication (e.g., Ackermann's systems Π' and Π").[64] Why is KR better than them?

My reply is: yes, this plea for KR is also a plea for any other logic that in this category. Is that a problem? If I were arguing for KR as the only legitimate logic instead of offering a plea for KR, then it would be. I do think KR has other virtues as well, and these support it over other logics in this category, but these considerations go beyond what is required for a successful plea.

6 Conclusion

The preceding considerations constitute a *plea* for the logic KR. It should be part of the vast conversation about what is (or are) the right logic(s). Logicians today are arguing for a bewildering array of logics.[65] Surely there is a place for a logic that has a somewhat more relevant conditional than classical logic but avoids what is undisputed as the biggest problem facing relevance logics.

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63 Robles and Méndez (2010, 2011).

64 See Ackermann (1956). See also Tennant (2017) whose system of classical core logic also satisfies this description.

65 Even logics in which contraction fails are taken to be competitors (very roughly, for example, in these logics, it might be that p & p entails q, even though p does not entail q. See Zardini (2011).

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