Accuracy and precision of dolphin group size estimates

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Abstract: Estimating the number of dolphins in a group is a challenging task. To assess the accuracy and precision of dolphin group size estimates, observer estimates were compared to counts from large-format vertical aerial photographs. During 11 research cruises, a total of 2,435 size estimates of 434 groups were made by 59 observers. Observer estimates were modeled as a function of the photo count in a hierarchical Bayesian framework. Accuracy varied widely among observers, and somewhat less widely among dolphin species. Most observers tended to underestimate, and the tendency increased with group size. Groups of 25, 50, 100, and 500 were underestimated by <1%, 16%, 27%, and 47%, respectively, on average. Precision of group size estimates was low, and estimates were highly variable among observers for the same group. Predicted true group size, given an observer estimate, was larger than the observer estimate for groups of more than about 25 dolphins. Predicted group size had low precision, with coefficients of variation ranging from 0.7 to 1.9. Studies which depend on group size estimates will be improved if the tendency to underestimate group size and the high uncertainty of group size estimates are included in the analysis.

Keywords: group size estimation, abundance estimation, aerial photography, Bayesian hierarchical model, random-effects model, reverse jump MCMC
Estimation of group size is an important component of ecological and behavioral studies of animals which occur in groups. However, estimation of group size in wildlife studies can be difficult. Replicate counts of birds showed high variation (Ryan and Cooper 1989), the number of birds was undercounted in aerial surveys (Bayliss and Yeomans 1990), and known group sizes of elk were underestimated from a helicopter (Cogan and Diefenbach 1998). Even counting the number of birds in photographs had a negative bias (Erwin 1982). Experiments in visual perception have shown a tendency to underestimate the size of large groups of objects (Krueger 1972), apparently related to distortions produced by saccadic (“jerky”) eye movements (Binda et al. 2011). Determining the size of a group of cetaceans is particularly challenging because of several characteristics that make group size estimation difficult: (1) the animals are moving; (2) an unknown fraction of the group is underwater at any moment; (3) the fraction underwater changes with behavior; (4) groups can be large; and (5) the distribution of group sizes is usually skewed, with a few groups much larger than the mean.

Accurate estimation of group size is necessary for unbiased estimation of abundance. In standard distance sampling (e.g., line transects), the density of groups is estimated and then multiplied by an estimate of expected group size (Buckland et al. 2001). Alternatively, group size may be a covariate of the detection process and expected group size is not estimated explicitly (Borchers and Burnham 2004). In either case it is assumed that group sizes are measured accurately. Using earlier subsets of the photographic calibration data presented here, some line-transect analyses have used group size estimates corrected by observer-specific calibration factors (Gerrodette and Forcada 2005, Barlow and Forney 2007). In most studies, however, correction factors for group size estimation are not available.
Assessing precision of group size estimates is equally important. Even if group sizes were to be estimated accurately on average, there is measurement error associated with each group size estimate. Including the variability associated with group size estimates is necessary for proper assessment of uncertainty. If measurement error is not included, variance of estimates of abundance and other quantities that depend on group size estimates will be too small. In other words, an important source of uncertainty will not be included in the analysis, and conclusions may appear to be more precise than they should be.

In this large field study, we measured the accuracy and precision of dolphin group size estimates. True group size was assessed with counts from high-quality vertical aerial photographs, and ship-based observer estimates were calibrated against these counts. The tendencies of different individual observers to under- or over-estimate group size were estimated in a hierarchical Bayesian framework, for different group sizes, species, and sea-state conditions. The performance of a new (out-of-sample) observer was predicted by integrating over observer and/or species effects. Given an observer estimate, we inferred true group size by sampling posterior distributions.

Methods

Field methods

Photographs of dolphin groups were collected during 11 research cruises between 1987 and 2006 in the eastern tropical Pacific Ocean. During all cruises except the last, the NOAA vessel *David Starr Jordan* carried a Hughes 500D helicopter equipped with two large-format military reconnaissance cameras mounted below the fuselage. During the 2006 cruise, images
were collected with the same camera systems mounted in a NOAA Twin Otter fixed wing aircraft. Under conditions of sun angle (generally mid-morning and mid-afternoon) and sea state (generally Beaufort 0-4) that allowed dolphins to be clearly visible from above, vertical photographs of dolphin groups were taken from an altitude of 200-300m (Gilpatrick 1993). The camera recorded images on 114mm negatives, and had a motion-compensation system that moved the film at the same speed that the image was moving within the camera, thus eliminating blurring due to the forward motion of the aircraft. The cycle rate of the camera was adjusted to achieve 80% overlap between adjacent frames during a photographic pass over a dolphin group. The number of photographic passes of each dolphin group varied with group size, configuration and behavior.

After a group of dolphins had been photographed, the group was approached by the ship in a way to give the marine mammal observers on the ship the best possible view of the whole group, considering wind, swell, and sun angle. All observers who had adequate views of the group, usually all six observers on the ship, made their best estimates of group size. We refer to these estimates as the “observer estimates.” Observers usually first detected dolphins with 25X binoculars, but switched to 7X binoculars and then to naked eye as the ship approached the group. The minimum approach distance varied with group size and behavior, but typically was 10-50m. Observers made group size estimates independently and did not discuss their estimates with each other, either during the sighting or afterward. Independence in this context refers to the behavioral independence of the observers, not to the statistical independence of their estimates. All observers had previous experience in cetacean field work. Before each cruise, observers were given training on group size estimation, including tests with known numbers of
static objects, computer simulations of moving, intermittently visible objects, and instruction on counting by subgroups (e.g., by tens or fifties) for more consistent estimation.

Laboratory methods

The aerial photographs of dolphin groups were reviewed on light tables equipped with dissection microscopes (Gilpatrick 1993). Photographs were compared with notes recorded during the photographic passes to ensure that the entire group was captured within the series of images that made up a photograph pass. For groups that were successfully photographed, the best pass was selected, and three readers independently counted the number of dolphins in the group from the series of images. If the CV among counts was > 0.1, or if notes by aerial and shipboard observers indicated that there was confusion over the identity of the group, the group was not included in the data analyzed here (Gilpatrick 1993).

To qualify as a “calibration school” for this analysis, the whole group had to be photographed from the air with a series of overlapping photographs, the photo counts of the three independent readers had to agree closely, and the shipboard observers had to view the whole group for a sufficient time to make good estimates. Calibration schools were thus not a random sample of all dolphin groups, but rather a selected set for which we were confident that true group size could be accurately determined. We omitted as outliers eight cases for which there was a large (greater than a factor of four) discrepancy between mean photo count and mean observer estimate, probably a result of undetected splitting or coalescence of groups after photography but before observer estimates. A total of 434 groups met these criteria as calibration schools, with 2,435 estimates of group size by 59 observers.

Statistical model
To evaluate observer estimates of group size, we used the mean of the counts by the three
photograph readers for each calibration school, and refer to this as the “photo count.” This
measure of true group size had some error (variation among the three readers), but this variation
(mean photo count CV over all groups = 0.047) was much smaller than the variation among
observer estimates of the same groups (mean CV = 0.42). Preliminary exploration of the data
suggested that, on a log-log scale, observer estimates could be linearly related to photo counts
and that variance was approximately constant over a large range of group sizes (Fig. 1A). In
addition, observers varied widely in the accuracy of their group size estimates (Fig. 1B). We
evaluated a variety of linear and nonlinear models in a frequentist setting, with both fixed and
random effects, with R function lmer, and used likelihood ratio tests, information criteria such as
AIC and DIC, and visual examinations of residual and q-q plots to identify a reasonable set of
candidate models. We found that dolphin species and Beaufort sea state could possibly affect the
accuracy of group size estimates, and that a linear model of the logarithm of photo counts
provided a more parsimonious fit to the data than a quadratic model.

Let $y_{ij}$ be the observer estimate of the size of group $i$ by observer $j$, and let $x_i$ be the photo
count of group $i$. We modeled differences among observers as random effects, and dolphin
species and wind conditions as fixed additional effects that might affect group size estimates.
The full hierarchical model may be written as

$$
\log(y_{ij}) = \alpha_{0j} + (\alpha_{1j} + \beta_j) \log(x_i) + \sum_{k=2}^{7} \beta_k S_k + \beta_{k2} B_k + \epsilon_{ij}
$$

$$
\left(\begin{array}{c}
\alpha_{0j} \\
\alpha_{1j}
\end{array}\right) \sim N\left(\begin{array}{c}
0 \\
0
\end{array}\right), \left(\begin{array}{cc}
\sigma_{\alpha0}^2 & \rho \sigma_{\alpha0} \sigma_{\alpha1} \\
\rho \sigma_{\alpha0} \sigma_{\alpha1} & \sigma_{\alpha1}^2
\end{array}\right)
$$

$$
\epsilon_{ij} \sim N(0, \sigma^2_\epsilon),
$$

(1)
where $\beta_i$ was the coefficient associated with the log of the photo counts, $\beta_k$ were coefficients associated with six species $S_{ik}, k=2,\ldots,7$, and $\beta_8$ was the coefficient associated with Beaufort sea state $B_i$. Two random effects, $\alpha_{0j}$ and $\alpha_{1j}$, allowed the relationship between $\log(y_{ij})$ and $\log(x_i)$ to vary among observers, $\alpha_{0j}$ in terms of the intercept and $\alpha_{1j}$ in terms of the slope coefficient $\beta_1$. The two sets of random-effects coefficients had means of zero, variances $\sigma_{\alpha_0}^2$ and $\sigma_{\alpha_1}^2$, and correlation $\rho$. The assumption was that the 59 observers were a random selection from a larger pool of possible observers whose group size estimation tendencies were normally distributed.

Species $S_{ik}$ entered the model as an indicator variable, with a value of 1 if group $i$ was species $k$ and 0 otherwise. Species were recorded in the field at the lowest possible taxonomic level, including subspecies. We combined the field identifications into six species categories: pantropical spotted dolphins ($Stenella attenuata$, 51 groups), spinner dolphins ($S. longirostris$, 40 groups), mixed spotted-spinner dolphin groups (78 groups), striped dolphins ($S. coeruleoalba$, 114 groups), common dolphins ($Delphinus delphis$ and $D. capensis$, 87 groups) and other (64 groups). “Other” was a heterogeneous category including Risso’s dolphins ($Grampus griseus$), common bottlenose dolphins ($Tursiops truncatus$), rough-toothed dolphins ($Steno bredanensis$), short-finned pilot whales ($Globicephalus macrorhynchus$), and other groups which did not fit into the previous categories, such as mixed common-striped dolphin groups. In the eastern tropical Pacific Ocean, mixed spotted-spinner dolphin groups are common, so we included these as a distinct category. Sea state $B$ was recorded on the Beaufort scale as an integer from 0 to 5; however, only one of the 434 calibration schools occurred in Beaufort 5 conditions, so the effective range of the model was Beaufort 0-4. Because the Beaufort scale is ordered, we modeled sea state as a continuous variable with a single linear coefficient. Models with sea state
as a categorical variable are addressed in the Discussion. $S_i$ and $B_i$ were the same for all observers for a given group $i$, so we omitted subscript $j$ for these covariates in Eq. 1.

We considered four variants of Eq. 1 as candidate models: model 1, without species or sea-state effects ($\beta_k=0$ and $\beta_8=0$); model 2, with species but without sea-state effects ($\beta_8=0$); model 3, with sea-state but without species effects ($\beta_k=0$); and model 4, the full model with both species and sea-state effects. All four models included observers as a random effect.

Bayesian inference

To include model selection in a Bayesian framework, we fitted the models in R using reversible jump Markov Chain Monte Carlo (RJMCMC) methods (King et al. 2009, Oedekoven et al. 2014). In this approach, the model itself was treated as an additional parameter to be estimated, and the joint posterior distribution included both parameters and models (Appendix 1). A uniform discrete prior was specified for the four models, and uniform continuous priors were specified for all coefficients $\beta$ and standard deviations $\sigma$ in Eq. 1. Model probabilities were calculated as the fraction of iterations of the RJMCMC chain in each model after burn-in (Appendix 1).

The four models were also fitted in the BUGS language (Lunn et al. 2000) and compared with the Watanabe-Akaike (or Widely Applicable) Information Criterion (WAIC) (Watanabe 2010). WAIC can be viewed as an improvement to the Deviance Information Criterion (DIC) (Spiegelhalter et al. 2002), which has some shortcomings for hierarchical models (Plummer 2008, Millar 2009, Lunn et al. 2013). WAIC was calculated using pointwise predictive density at the observer level from the MCMC posterior samples for each model (Gelman et al. 2014, Vehtari et al. 2016). We used standard procedures to assess burn-in, autocorrelation, and
convergence of the MCMC samples (Appendix 2). BUGS code is given in Appendix 3 and R
code for the RJMCMC analysis in Appendix 4.

For prediction, we sampled from the BUGS posterior samples of model 2, which had the
most support (see Results). We made two kinds of predictions: an observer estimate given true
group size and true group size given an observer estimate. For each, we predicted conditionally
and unconditionally on both observer and species. A conditional prediction for an observer or
species meant a prediction given that it was made by a particular observer or given that it was
made of a group of a particular species. An unconditional prediction was calculated to infer
estimation tendencies for a new (out-of-sample) observer and/or species – that is, estimation
tendencies integrated over observers or species effects. Unconditional predictions were
approximated by sampling observers and/or species randomly. We sampled the MCMC chain
50,000 times with replacement, each time also randomly sampling an observer and a species for
unconditional inference. For the model error term $\sigma^2$, we made random draws from normal
distributions with the MCMC sample variances. To preserve the covariance structure, we used
the whole set of parameter values for each selected MCMC iteration, and computed observer
estimate $y$ given group size, or group size $x$ given observer estimate, based on Eq. 1. We
checked the accuracy of our predictions by comparing them to the photo counts (Appendix 5).
For each of the 2,435 observer estimates, we determined if the central 95% credibility interval of
predicted size included the photo count.

Results

Calibration schools
Group sizes of calibration schools ranged from 5 to 6,012 (Fig. 1). The set of 434 calibration schools represented about 8% of dolphin groups of the same species detected during the 11 surveys. On average, calibration schools were larger in size (because we did not photograph groups containing only a few dolphins) and were photographed in lower Beaufort sea states (because it was harder to obtain clear images in windy conditions) than for all dolphin groups. Importantly, the variation among independent observer estimates for a dolphin group was similar for calibration schools (mean CV 0.42, interquartile range 0.29-0.50) and all detected groups (mean CV 0.39, interquartile range 0.24-0.51). The number of calibration schools per observer ranged from 6 to 159, with a median of 33 and a mean of 41.3.

**Observer estimates of dolphin group size**

The raw data indicated that observers generally tended to underestimate dolphin group size; 69% of observer estimates were less than the photo count (Fig. 1). Both model selection methods indicated that the accuracy of observer estimates was affected by the species of the group but less so by Beaufort sea state. Posterior model probabilities indicated by the RJMCMC chain were 0.0, 0.984, 0.0, and 0.016 for models 1-4, respectively (Fig. 2). With proper selection of proposal distributions, models 2 and 4 had stationary distributions throughout the history of the chain (Fig. A1 in Appendix 1). WAIC scores showed a similar pattern favoring model 2 but with some support for model 4, with values of 3766.0, 3596.0, 3769.3, and 3598.5 for models 1-4, respectively.

Marginal posterior distributions of parameters for models 2 and 4 were similar (Table 1). For model 4, the sea-state coefficient $\beta_8$ was small in absolute value and the 95% credibility interval included 0, further indications that wind conditions in the range Beaufort 0-4 had little
effect on the accuracy of group size estimation. The coefficient for log(photo count), $\beta_1$, was $<1.0$ (mean 0.80, 95% credibility interval 0.76 to 0.83 for model 2), which meant that the tendency to underestimate increased with group size. Species coefficients decreased in the order mixed spotted-spinner, common, spotted, spinner, striped, and other (Fig. 3A). However, the posterior distributions of species coefficients overlapped (Table 1), indicating that the differences among species were modest. The random-effects coefficients were negatively correlated (mean $\rho = -0.79$, 95% credibility interval -0.59 to -0.91).

Observers differed in accuracy of group size estimation (Fig. 3B). Among the 59 observers, some tended to underestimate and others tended to overestimate. For spotted dolphin groups of 25, 50, 100, and 500 animals, the observers with the lowest estimation tendency had mean posterior estimates of 18, 29, 45 and 132, respectively, while the observers with the highest estimation tendency had mean posteriors of 42, 72, 125, and 585 (Table 2). The “average observer” (actually four different observers, one for each of the four group sizes in Table 2) had estimates of 25, 44, 78, and 290 for spotted dolphin groups of 25, 50, 100, and 500, respectively.

Thus, over all observers, groups of 25 spotted dolphins were estimated accurately on average, but the range among observers was from underestimation by 29% to overestimation by 66% (Table 2). There were similarly large ranges in accuracy among observers for larger groups: for groups of 50, -43% to +45%; for groups of 100, -55% to +25%; and for groups of 500, -74% to +17%. The “average observer” underestimated spotted dolphin groups of 50, 100, and 500 animals by 11%, 22%, and 42%, respectively. We chose spotted dolphins for these numerical comparisons because spotted dolphins were near the middle of the species effect (Table 1).

There would be less underestimation of group size for common dolphins and mixed groups of spotted-spinner dolphins, and more underestimation of group size for spinner, striped, and other
dolphins. Averaged over all species, the mean figures of underestimation were <1%, 16%, 27%, and 47% for groups of 25, 50, 100, and 500 animals.

The random-effects model allowed intercept and slope parameters to be estimated for each observer (Fig. 4), constrained by the hierarchical assumptions of normal distributions and correlation between slope and intercept. Visually, the greater importance of the observer effect relative to the species effect can be judged by comparing Fig. 3B with Fig. 3A. Numerically, the range of plausible values for observer intercepts ($1.5 \approx \pm 2 \sigma_0$) was greater than the range of species effects ($\approx 0.4$), based on the mean posterior values in Table 1.

Accuracy decreased with group size (Fig. 5). Groups of 25 spotted dolphins were slightly underestimated, but groups of 500 were severely underestimated. For groups of 25, 50, 100, and 500 dolphins, posterior means for an out-of-sample observer (gray lines in Fig. 5) were 24.2, 42.2, 73.1, and 264.4, respectively. To show conditional estimates, we used observer #53 as an example. The black lines in Fig. 5 for observer #53 were slightly to the left of the gray lines unconditioned on observer, indicating that this observer tended to underestimate more than the average over all observers.

The posterior distributions of observer estimates were approximately normal on a natural logarithmic scale (Fig. 5). The distributions were quite wide, illustrating the high uncertainty (or low precision) in observer estimates of group size. Conditional estimates had higher precision than unconditional estimates. Estimates made by a particular observer (observer #53) for a particular species (spotted dolphins) had slightly higher precision (less uncertainty) than estimates by the same observer for an unknown species (compare thin dashed with thick solid black lines in Fig. 5). Unconditional estimates for any observer or species had the least precision.
The differences between conditional and unconditional estimates were small, however, in the context of the overall high variability of group size estimates.

Predictions of dolphin group size from observer estimates

Conversely, given an observer estimate, predicted true group size was usually larger than the estimate, especially for larger groups (Fig. 6). For observer estimates of 25, 50, 100, and 500 dolphins, posterior means were 26.0, 63.5, 154.0, and 1,194.5, respectively, for an out-of-sample observer (gray lines in Fig. 6). As with posterior distributions of observer estimates given group size, predicted group sizes conditional on observer and species had higher precision than unconditional estimates (compare black and gray lines in Fig. 6). Because observer #53 tended to underestimate more than average, predicted group size was larger for this observer than for the average over all observers.

Dolphin group size predicted from an observer estimate had high uncertainty.

Coefficients of variation for predicted group size conditional on species ranged from approximately 0.7 to 0.9 (Table 3). Coefficients of variation for unconditional predictions were even larger, ranging from 0.9 to 1.9, due to the additional uncertainty of predicting group size for an unknown species. Given an observer estimate of 100 dolphins, the 95% credibility interval for the true size of the group ranged from 43 to 621 for a group of spotted dolphins, and from 37 to 776 for a group unconditional on species. Posterior distributions accurately captured the uncertainty in predicting dolphin group size from an observer estimate (Appendix 5, Fig. A4).

The degree to which an observer estimate was increased to estimate true group size depended on species. For an observer estimate of 25 dolphins, for example, the median predicted group size was smaller than 25 for mixed spotted-spinner and common groups, and
larger than 25 for spotted, spinner, striped, and other groups (Table 3). Because the exponentiated posterior distributions were lognormal, means were larger than medians. Therefore, we used the median (50% quantile) as the best measure of central tendency for these distributions, because there was equal probability of a value being higher or lower than the median. Integrated over species and observer effects, estimates of 25, 50, 100, and 500 were increased by 4%, 24%, 47%, and 122%, respectively, to obtain the medians of the posterior distributions of predicted group size (Table 3). In other words, given an observer estimate of 500 dolphins, the most probable true size of the group would be more than twice that number.

Discussion

Accuracy and precision

The discrepancy between an observer estimate of dolphin group size and the true number can be discussed in terms of two components: accuracy and precision. Accuracy is measured by the difference between the true number and the mean of repeated observations. Inaccurate measurement of group size leads to biased results. Precision is assessed by the random error among repeated observations. Random error will be positive for some observations and negative for others, but with a mean of zero. Low precision means high variance and greater uncertainty in results.

We found that accuracy of dolphin group size estimates depended on group size, observer, and species. Within the Beaufort 0-4 range of the calibration schools, Beaufort sea state had less effect on accuracy, once group size and observer effects had been accounted for.
There was a general tendency to underestimate dolphin group size, and this tendency increased with group size. The coefficient of the log of photo count ($\beta_1$, Table 1) was < 1.0, which meant that large groups were underestimated more than small groups. Observer estimates were accurate (on average) for dolphin groups of 25 animals, but were too low by 16% for groups of 50, too low by 27% for groups of 100, and too low by 47% for groups of 500 (Fig. 5). These estimates of accuracy averaged over all observers do not measure the accuracy of a particular observer, nor the discrepancy between an observer estimate and true group size for a particular group. Accuracy of dolphin group size estimation in this study applies within the range of calibration school sizes with a reasonable number of samples, roughly between 10 and 1000 animals (Fig. 3).

These results were broadly consistent with previous studies which showed that humans tend to underestimate group sizes in wildlife studies (Caughley 1974, Bayliss and Yeomans 1990, Cogan and Diefenbach 1998). The rate of decline in accuracy with group size ($\beta_1 = 0.80$, Table 1) falls in the range of perceptual experiments measuring underestimation of the number of dots on paper (Krueger 1972). Underestimation of large groups may have a physiological basis related to eye movement; estimation of small groups (about 10 or fewer objects) does not have this negative bias and seems to involve a different perceptual mechanism (Binda et al. 2011).

The degree of underestimation also varied by species. For the six species categories in this study, dolphin group size estimates were lower in the order: mixed spotted-spinner, common, spotted, spinner, striped, and other (Table 1, Fig. 3A). This order of species coefficients corresponded roughly to mean group size among the six species groups, with mixed spotted-spinner and common dolphin groups being largest, and striped and other dolphin groups
smallest. This correspondence suggests that the effects of group size and species were somewhat confounded.

Accuracy varied among the 59 observers. While there was an overall tendency to underestimate dolphin group size, some observers had a stronger tendency to underestimate, while others had a tendency to overestimate (Table 2). The random-effects model allowed the estimation of separate effects for each observer (Fig. 3B), but connected the observers as a group and allowed the tendency of all observers together to support estimation for each single observer (Fig. 4). A random-effects model is often understood in terms of “partial pooling.” It represents an intermediate approach between complete pooling (treating all observers as a single group, Fig. 1A) and no pooling (treating each observer independently, Fig. 1B). The random-effects approach spans a range of models between these extremes, and includes complete pooling and complete separation as special cases at the limits (Gelman and Hill 2007). The degree of pooling is related to the amount of shrinkage of individual effects toward the mean (Gelman and Pardoe 2006).

Precision of observer estimates of dolphin group size was strikingly low (Fig. 5). For a group of 100 dolphins, for example, estimates could range from about 30 to 200 with 95% probability. Regardless of an observer’s accuracy, it was common for the observer to estimate 50% high for one group and 50% low for the next. As a consequence, there was high variability among the independent observer estimates of group size, both for calibration schools as well as for non-calibration dolphin groups. The mean CV among observer estimates was 0.4 across a wide range of group sizes. Clearly, estimating the size of a dolphin group is a challenging task.

Statistical issues
As a measure of true group size, we used the mean of photo counts by three independent readers. A binomial moment estimator has been proposed for repeated counts with imperfect detection, *i.e.*, false negatives (DasGupta and Rubin 2005, Walsh *et al.* 2009), but in our study variation among counts of the three readers was also due to false positives. Large tuna, which frequently accompany dolphin groups in the eastern tropical Pacific, can be mistaken for a submerged dolphin in the photographs. Splashes and reflections might also be counted as a partially hidden dolphin.

RJMCMC and WAIC are two fully Bayesian approaches to model selection (Hooten and Hobbs 2015). RJMCMC treats the model itself as an additional unknown parameter to be estimated, while WAIC is a score function based on the predictive ability of the model. Both indicated that the accuracy of dolphin group size estimates varied by observer and species (model 2). There was little posterior support for model 4, which included Beaufort sea state (Fig. 2). The posterior odds of models 2 and 4 (the Bayes factor, Kass and Raftery 1995) was 60.6, indicating strong support for model 2 over model 4. The WAIC difference of 2.5 also indicated support of model 2 over model 4. If sea state was modeled as a categorical variable, model 4 had a posterior probability of zero (it was never selected in the RJMCMC algorithm), but if sea state was modeled as a continuous variable, model 4 was selected 2% of the time (Fig. 2, Fig. A1 in Appendix 1). Thus, it appeared that modeling sea state as a continuous variable rather than as separate factor variables was a more parsimonious approach. As there was little support for model 4, and because parameter estimates were similar for models 2 and 4 (Table 1), we focused on model 2 for inference and did not use model-averaged estimates.

Because Bayesian inference is based on conditional probabilities, it was possible to make inference regardless of observer and/or species, by integrating over observer and species effects.
The estimation tendency of a new, out-of-sample observer included the uncertainty of not knowing which observer, out of the “universe” of possible observers with different estimation tendencies, might be chosen. Such estimates unconditional for observer and species are shown as gray lines in Figs. 5 and 6. The greater uncertainty of the unconditional estimates is indicated by the wider probability distributions in those figures, relative to the conditional estimates shown with black lines.

*Application of results*

To obtain the best estimates of group size, we can use the estimation tendencies revealed in this study to adjust observer estimates of dolphin group size. We wish to predict true group size, given an observer estimate. The Bayesian approach allowed us to solve this inverse problem with proper accounting of uncertainty. Since, for groups larger than about 25 dolphins, there was a tendency to underestimate group size, predictions of true group size tended to be larger than the estimate (Fig. 6). Because the degree of underestimation depended on group size, species, and observer, the amount that a group size estimate had to be increased to predict true group size also depended on group size, species, and observer (Table 3). The amount that an estimate had to be increased could be substantial. For example, a group size estimate of 100 dolphins had to be increased by 47% to obtain the unconditional best (median) estimate of true group size.

Because an estimate of group size had low precision, predicted group size based on an estimate also had low precision. Posterior distributions had CVs of approximately 0.7 to 0.9 for groups of known species, and 0.9 to 1.9 for groups of any species (Table 3). For an out-of-sample observer estimate of 25 dolphins, for example, median predicted true group size was 25.9
animals (accuracy was good), but the 50% credibility interval extended from 16 to 42 dolphins, and the 95% credibility interval from 6 to 111 dolphins (Table 3). This source of uncertainty is usually ignored in distance sampling analyses, although Gerrodette and Forcada (2005) included uncertainty in group size through a bootstrap procedure. Most line-transect analyses compute the variance in expected group size from the sizes of the observed groups.

On cetacean line-transect surveys conducted by the Southwest Fisheries Science Center, three independent estimates of group size are recorded for each sighting. For the best estimate of group size, Gerrodette and Forcada (2005) used an average of the three calibration-adjusted observer estimates, weighted by the inverse of the group size estimation variance of each observer. The value of making several independent estimates of group size will be examined in a future paper.

Given our findings of inaccuracy for groups larger than 25 dolphins and low precision for groups of all sizes, it is worth noting that the estimates of group size in this study were a selected set of estimates made in optimal circumstances. Each group was approached with the specific objective of obtaining group size estimates, the observers had good views of the entire group, and the ship remained with the group until the observers had made their best possible estimates. Almost certainly the behavior of dolphin groups affects the accuracy and precision of group size estimates, but our set of calibration schools consisted of well-behaved groups that could be observed and photographed in their entirety.

Accuracy and precision may be lower for groups estimated in less optimal conditions. Schwarz et al. (2010) found that estimates of delphinid group sizes were 58% lower when the ship did not approach groups (passing mode) than when it did (closing mode). Barlow et al.
(1997) also found that group size estimates were smaller in passing mode. Barlow and Taylor (2005) found that an extended 90-min period of observation improved group size estimates of asynchronously diving sperm whales (*Physeter microcephalus*). The position of the observer may also matter. The estimates of group size in this study were made from a platform approximately 10 m above the water. The estimation tendencies reported here may not apply to other situations, such as estimates made from higher or lower platforms on a ship, or estimates made from land at various elevations and distances to sightings. Caughley *et al.* (1976) found that the accuracy of aerial counts varied with aircraft speed, height, and observer.

We conclude with two recommendations for studies that depend on estimates of cetacean group size. First, we recommend training to improve group size estimation. Although we were not able to measure how our pre-cruise training affected observers’ estimates, we believe that the training had a positive effect. Training may include displays of groups of objects of known size, and instructions on estimating group size by counting subgroups of multiple animals. Second, we recommend assessment of accuracy and precision of group size estimation under the particular conditions of a study. The large budget of this study is unlikely to be replicated, but digital photography by drones is a more economical and much safer option today. Laake *et al.* (2012) used two observer teams to assess the accuracy of pod size estimates for migrating gray whales (*Eschrichtius robustus*). Although pod size was usually only one or two animals, correcting pod size estimates had an important effect on abundance estimates and inferred population trajectory.

If a study is unable to assess accuracy of group size estimates, the results of this study can be applied with appropriate caution. We have noted that biases might be different for group size estimates made under other conditions, such as greater distances. One of our central results was
that people varied widely in their group size estimation tendencies; therefore, the ideal is to calibrate particular individual observers. However, the random-effects model for the observer effect allowed inference for observers outside this study. Table 3 and Figures 5 and 6 show posterior distributions for a new, out-of-sample observer – that is, accuracy and precision of group size estimates which include the uncertainty of not knowing which observer, out of the large number of possible observers with different estimation tendencies, might have been chosen. Unless more specific information can be obtained, it would be reasonable to assume that the estimation tendencies of the 59 observers in this study are representative of all observers.
Acknowledgments

We thank all the marine mammal observers who made the group size estimates, the aerial photographers and NOAA helicopter pilots, the captains and crews of the NOAA research vessels, the support staff at the Southwest Fisheries Science Center, and the Government of Mexico for flight permission in 2006. The photo counts were made by Jim Gilpatrick, Robin Westlake, Morgan Lynn, and Katie Cramer of the SWFSC. The manuscript benefitted from comments by Jim Gilpatrick, Jeffrey Moore, and two anonymous reviewers.


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Table 1. Marginal posterior distributions of parameters for the two models with posterior support. Distributions are summarized by means, standard deviations (SD) and three quantiles. All parameters had uniform prior distributions. RE = random effects for observers. See Eq. 1 for definitions of parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 2</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td><strong>SD</strong></td>
<td><strong>2.5%</strong></td>
</tr>
<tr>
<td>Log(photo count), $\beta_1$</td>
<td>0.796</td>
<td>0.018</td>
</tr>
<tr>
<td>Sp: spotted-spinner, $\beta_2$</td>
<td>0.805</td>
<td>0.087</td>
</tr>
<tr>
<td>Sp: common, $\beta_3$</td>
<td>0.757</td>
<td>0.084</td>
</tr>
<tr>
<td>Sp: spotted, $\beta_4$</td>
<td>0.656</td>
<td>0.082</td>
</tr>
<tr>
<td>Sp: spinner, $\beta_5$</td>
<td>0.603</td>
<td>0.088</td>
</tr>
<tr>
<td>Sp: striped, $\beta_6$</td>
<td>0.513</td>
<td>0.075</td>
</tr>
<tr>
<td>Sp: other, $\beta_7$</td>
<td>0.423</td>
<td>0.076</td>
</tr>
<tr>
<td>Beaufort sea state, $\beta_8$</td>
<td>na</td>
<td>na</td>
</tr>
<tr>
<td>SD intercept RE, $\sigma_{a0}$</td>
<td>0.382</td>
<td>0.073</td>
</tr>
<tr>
<td>SD slope RE, $\sigma_{a1}$</td>
<td>0.104</td>
<td>0.017</td>
</tr>
<tr>
<td>Correlation RE, $\rho$</td>
<td>-0.792</td>
<td>0.085</td>
</tr>
<tr>
<td>SD model, $\sigma_\epsilon$</td>
<td>0.497</td>
<td>0.008</td>
</tr>
</tbody>
</table>
Table 2. Summary of estimation tendencies among observers. For each group size, the entries in the table show the distribution of the means of the posteriors of the 59 observers for estimates of a group of spotted dolphins. “Mean diff.”, “Min diff.” and “Max diff.” are the differences between the mean (or minimum or maximum) of the mean observer estimates and true group size, expressed as percentages of group size.

<table>
<thead>
<tr>
<th>Group size</th>
<th>Distribution of means of observer estimates</th>
<th>Mean diff.</th>
<th>Min diff.</th>
<th>Max diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Min</td>
<td>25%</td>
<td>50%</td>
</tr>
<tr>
<td>25</td>
<td>25.4</td>
<td>17.7</td>
<td>21.2</td>
<td>25.9</td>
</tr>
<tr>
<td>50</td>
<td>44.4</td>
<td>28.5</td>
<td>35.5</td>
<td>45.0</td>
</tr>
<tr>
<td>100</td>
<td>77.7</td>
<td>44.9</td>
<td>60.3</td>
<td>78.3</td>
</tr>
<tr>
<td>500</td>
<td>289.7</td>
<td>131.7</td>
<td>201.8</td>
<td>293.0</td>
</tr>
</tbody>
</table>
Table 3. Predicted dolphin group sizes given observer estimates of 25, 50, 100, and 500 animals by a new (out-of-sample) observer, for six dolphin species and integrated over species (“any species”). Posterior distributions have been exponentiated to show values on the scale of the number of dolphins. Distributions of predicted group size are approximately lognormal, and are summarized by means, standard deviations (SD), coefficients of variation (CV) and five quantiles. “Difference” is the difference between the median (the 50% quantile) of predicted group size and observer estimate, expressed as a percentage of the observer estimate.
<table>
<thead>
<tr>
<th>Observer estimate</th>
<th>Dolphin species</th>
<th>Predicted group size</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>25</td>
<td>spotted-spinner</td>
<td>26.4</td>
<td>19.6</td>
</tr>
<tr>
<td></td>
<td>common</td>
<td>28.2</td>
<td>21.1</td>
</tr>
<tr>
<td></td>
<td>spotted</td>
<td>32.3</td>
<td>24.4</td>
</tr>
<tr>
<td></td>
<td>spinner</td>
<td>34.6</td>
<td>26.3</td>
</tr>
<tr>
<td></td>
<td>striped</td>
<td>39.0</td>
<td>29.1</td>
</tr>
<tr>
<td></td>
<td>other</td>
<td>43.8</td>
<td>33.3</td>
</tr>
<tr>
<td></td>
<td>any species</td>
<td>34.1</td>
<td>30.0</td>
</tr>
<tr>
<td>50</td>
<td>spotted-spinner</td>
<td>66.6</td>
<td>50.4</td>
</tr>
<tr>
<td></td>
<td>common</td>
<td>70.7</td>
<td>53.2</td>
</tr>
<tr>
<td></td>
<td>spotted</td>
<td>81.1</td>
<td>61.6</td>
</tr>
<tr>
<td></td>
<td>spinner</td>
<td>87.3</td>
<td>66.7</td>
</tr>
<tr>
<td></td>
<td>striped</td>
<td>98.6</td>
<td>75.2</td>
</tr>
<tr>
<td></td>
<td>other</td>
<td>110.6</td>
<td>84.1</td>
</tr>
<tr>
<td></td>
<td>any species</td>
<td>84.5</td>
<td>79.6</td>
</tr>
<tr>
<td>100</td>
<td>spotted-spinner</td>
<td>167.2</td>
<td>128.0</td>
</tr>
<tr>
<td></td>
<td>common</td>
<td>179.5</td>
<td>137.4</td>
</tr>
<tr>
<td></td>
<td>spotted</td>
<td>206.2</td>
<td>160.0</td>
</tr>
<tr>
<td></td>
<td>spinner</td>
<td>222.2</td>
<td>175.1</td>
</tr>
<tr>
<td></td>
<td>striped</td>
<td>247.3</td>
<td>191.0</td>
</tr>
<tr>
<td></td>
<td>other</td>
<td>281.7</td>
<td>219.6</td>
</tr>
<tr>
<td></td>
<td>any species</td>
<td>211.2</td>
<td>241.4</td>
</tr>
<tr>
<td>500</td>
<td>spotted-spinner</td>
<td>1466.7</td>
<td>1215.5</td>
</tr>
<tr>
<td></td>
<td>common</td>
<td>1581.4</td>
<td>1335.3</td>
</tr>
<tr>
<td></td>
<td>spotted</td>
<td>1801.7</td>
<td>1518.8</td>
</tr>
<tr>
<td></td>
<td>spinner</td>
<td>1941.8</td>
<td>1651.7</td>
</tr>
<tr>
<td></td>
<td>striped</td>
<td>2181.9</td>
<td>1972.4</td>
</tr>
<tr>
<td></td>
<td>other</td>
<td>2483.6</td>
<td>2145.0</td>
</tr>
<tr>
<td></td>
<td>any species</td>
<td>1944.2</td>
<td>3600.3</td>
</tr>
</tbody>
</table>
Fig. 1: Dolphin group size calibration data plotted on logarithmic scales. (A) Photo counts and observer estimates of group size for 434 calibration schools. The size of each group was estimated independently by multiple (usually 6) shipboard observers. The dashed line is a regression of log(observer estimate) on log(photo count), while the solid gray line is a 1:1 relationship. (B) Regressions of log(observer estimate) on log(photo count) for each of the 59 observers.

Fig. 2. Prior and posterior probabilities of four models of dolphin group size estimation based on RJMCMC. Differences among observers were modeled as random effects (RE) in all four models; species and sea state were fixed effects.

Fig. 3. Estimates of (A) species and (B) observer effects on dolphin group size estimation. Regression lines are based on means of posterior distributions.

Fig. 4. Posterior distributions of random effects for each observer for (A) intercept $\alpha_0$ and (B) slope $\alpha_1$ (see Eq. 1). Points are means and lines are central 95% credibility intervals.

Fig. 5. Posterior distributions of observer estimates for dolphin groups of 25, 50, 100, and 500 animals. Thin dashed lines are the distributions of estimates for a given observer (#53) whose tendencies were estimated in this study, for a given species (spotted dolphins). Thick black lines are the distributions for the same observer for any species (integrated over species). Thick gray lines are the distributions for a new, out-of-sample observer with unknown tendencies, for any species (integrated over observers and species). The probability densities (vertical scale) of all distributions are scaled relative to the maximum value.
Fig. 6. Predicted dolphin group sizes given observer estimates of 25, 50, 100, and 500 animals.

Thin dashed lines are the distributions of group size for a given observer (#53) whose tendencies were estimated in this study, for a given species (spotted dolphins). Thick black lines are the distributions for the same observer for any species (integrated over species). Thick gray lines are the distributions for a new, out-of-sample observer with unknown tendencies, for any species (integrated over observers and species). The probability densities (vertical scale) of all distributions are scaled relative to the maximum density value.
Fig. 1

Fig. 2
Fig. 3

Fig. 4
Fig. 5

Fig. 6

---

observer #53, spotted
observer #53, any sp.
new observer, any sp.

---

RELATIVE PROBABILITY

OBSERVER ESTIMATE

PREDICTED GROUP SIZE

---

estimate = 25
estimate = 50
estimate = 100
estimate = 500

---

group size = 25
group size = 50
group size = 100
group size = 500
Random-effects models such as Eq. (1) can be implemented in a Bayesian framework using hierarchical models where each parameter, including the random-effects standard deviations, are assumed to have a distribution. Markov Chain Monte Carlo (MCMC) simulation can be used to obtain summary statistics of the posterior distributions of the parameters given the data. To include model selection in our analysis, we treated the model itself as a parameter and formed the joint posterior distribution of both parameters and models. An RJMCMC algorithm (Green 1995) explored this posterior distribution. The RJMCMC algorithm represented a random walk, where each iteration consisted of two steps: (1) the reversible jump (RJ) step where we proposed to move to a different model (the between-model move), and (2) the Metropolis-Hastings (MH) step where we updated the parameters from the current model (the within-model move). We placed uniform priors on all parameters with an upper bound of 1, and a lower bound of -1 for coefficients and a lower bound of 0 for standard deviations.

All models included in the analysis contained the intercept and the log of the photo counts as well as their corresponding random-effects coefficients (Eq. 1). Hence, the RJ step at each iteration consisted of proposing to add or delete each of the two remaining covariates (sea-state and species) in turn, depending on whether the covariate was in the current model or not. Four different models were possible that differed only in the inclusion or exclusion of species ($\beta_k$, with $k = 2, \ldots, 7$) and sea state ($\beta_8$) coefficients in Eq.(1): for model 1, $\beta_k=0$ and $\beta_8=0$; for model 2, $\beta_8=0$; for model 3, $\beta_k=0$; and for model 4, both species and sea-state coefficients were non-zero (full model). A proposal to add a covariate to the model involved drawing random samples from the respective proposal distributions for the parameters and accepting this proposal based on the calculated acceptance probability (see, e.g., King et al. 2009 on how to obtain the...
A proposal to delete a covariate from a model involved setting its coefficients to zero and accepting this proposal based on the calculated acceptance probability. The four models were considered equally likely a priori.

The MH step at each iteration consisted of updating the parameters that were currently in the model using an MH update (Metropolis et al. 1953, Hastings 1970). This included the coefficient associated with the log of the photo counts, the standard deviations associated with the random effects and model errors as well as the coefficients for species and sea-state if these covariates were in the current model. Furthermore, all random-effects coefficients were updated during each iteration. In particular, this update involved a random walk single-update with normal proposal distributions, where the mean was equal to the current value of the parameter (or random-effects coefficient) and the standard deviations were fine-tuned during pilot tuning to achieve appropriate acceptance rates (Gelman et al. 1996).

The chain was started with the full model and completed 210,000 iterations. We discarded the first 10,000 as burn-in and thinned the chain by retaining every 50th value, thus obtaining a posterior sample of 4000 values. Posterior model probabilities were the fraction of iterations that the chain spent in the respective model. Models 1 and 3 were never selected; model 4 was selected 1.6% of the time consistently through the history of the chain (Fig. A1). Similar results were obtained regardless of which model was used to initiate the chain.
Fig. A1. Sequence of RJMCMC jumps among models after burn-in. Results were similar regardless of which model was chosen to initiate the chain. To show separate points, random values have been added to each point (jittering).
Each of the four variants of Eq. 1 was implemented in the BUGS language. Uniform priors were specified for all parameters except the random-effects coefficients, which were latent. Due to the large amount of data, specification of other priors, such as normal distributions (lognormal distributions for variance parameters) with means far from values supported by the data, had no effect on posterior distributions. For each model, we ran three chains of 120,000 iterations each, discarding the first 20,000 as burn-in from different random initial starting values. For the remaining 100,000 iterations, we retained every 100\textsuperscript{th} value (thinning) to reduce autocorrelation. Thus the final sample consisted of 1000 values for each of three chains. The effective sample size for each parameter, calculated with R package coda, was near 1000, indicating that autocorrelation was low. The chains were well-mixed for all parameters (Fig. A2), and converged to similar values (Fig. A3).
Fig. A2. Traces of posterior samples. Green, red and blue lines show three independent MCMC chains of 1000 iterations each, with different initial values. See Eq. 1 and Table 1 for definitions of parameters.
Fig. A3. Marginal posterior probability density distributions for parameters. Green, red and blue lines show three independent MCMC chains with different initial values. The histogram is the total sample of all three chains. See Eq. 1 and Table 1 for definitions of parameters.
BUGS model <- function() {
  for (i in 1:n) {
    y[i] ~ dnorm(y.hat[i], tau.model)
    y.hat[i] <- a0[obs[i]] + a1[obs[i]]*x[i] + b.sp[sp[i]]  # model 2
    # y.hat[i] <- a0[obs[i]] + a1[obs[i]]*x[i] + b.sp[sp[i]] + b.bf*bf[i]  # model 4
  }
  tau.model <- pow(sigma.model, -2)
  sigma.model ~ dunif(sigma.min, sigma.max)  # prior
  for (i in 1:6) {b.sp[i] ~ dunif(b.min, b.max)}  # 6 species factor levels
  # b.bf ~ dunif(b.min, b.max)  # sea state
  for (j in 1:n.obs) {
    a0[j] <- A[j,1]
    a1[j] <- A[j,2]
    A.hat[j,1] <- 0  # mean of intercept random effects
    A.hat[j,2] <- b1  # mean of slope random effects
  }
  b1 ~ dunif(b.min, b.max)  # prior
  Sigma.A[1,1] <- pow(sigma.a0, 2)
  Sigma.A[2,2] <- pow(sigma.a1, 2)
  Sigma.A[1,2] <- rho*sigma.a0*sigma.a1
  sigma.a0 ~ dunif(sigma.min, sigma.max)  # prior
  sigma.a1 ~ dunif(sigma.min, sigma.max)  # prior
  rho ~ dunif(-1,1)  # prior
}
Appendix 4. R code for RJMCMC analysis

# RJMCMC calibration analysis for ETP dolphin school size estimation
#
library(tcltk2)   # for progress bar
## Proposal distributions for parameters for RJ step
rjprop.mean.sp <- rep(0,5)
rjprop.mean.bft <- 0
rjprop.sd.bft <- 0.1
rjprop.sd.sp <- rep(0.3,5)
## Proposal distributions for parameters for MH step
mhprop.sd.int <- 0.035
mhprop.sd.ph <- 0.007
mhprop.sd.sp <- rep(0.04,5)
mhprop.sd.bft <- 0.005
mhprop.sd.sd.model <- 0.01
mhprop.sd.sd.obs.int = 0.01
mhprop.sd.sd.obs.ph = 0.01
mhprop.sd.params <- c(mhprop.sd.int, mhprop.sd.ph, mhprop.sd.sp, mhprop.sd.bft,
mhprop.sd.sd.model, mhprop.sd.sd.obs.int, mhprop.sd.sd.obs.ph)
names(mhprop.sd.params) <- c('sd.int','sd.ph',rep('sd.sp',5),'sd.bft',
'sd.sd.model','sd.sd.obs.int','sd.sd.obs.ph')

 ############# model set-up
## Starting values for the parameters
# fixed effects
int.0 <- 0.7   # intercept
ph.0 <- 0.8    # slope for photo
sp.0 <- rjprop.mean.sp # factor covariate with 6 levels (first level absorbed in the intercept)
bft.0 <- rjprop.mean.bft # beaufort coefficient
sd.model.0 <- 0.5   # standard deviation of model errors
# random effects for observers
sd.obs.int.0 <- 0.2      # intercept for regression
re.obs.int <- rnorm(n.obs,0,sd.obs.int.0)
names(re.obs.int) <- sort(unique(observers))
sd.obs.ph.0 <- 0.05
re.obs.ph <- rnorm(n.obs,0,sd.obs.ph.0)
names(re.obs.ph) <- sort(unique(observers))
params <- c(int.0,ph.0,sp.0,bft.0,sd.model.0,sd.obs.int.0,sd.obs.ph.0)
names(params) <-
c('int','ph',paste("sp",levels(species)[2:6],sep="."),'bft','sd.model','sd.obs.int','sd.obs.ph')
param.list <- matrix(0,4,8)
param.list[1,c(1,2)] <- 1
param.list[2,c(1:7)] <- 1
param.list[3,c(1,2,8)] <- 1
param.list[4,1:8] <- 1

# choose the model
cur.mod <- 1
# which parameters are switched on
cur.p <- param.list[cur.mod,]
params[1:8] <- params[1:8]*cur.p

## Prior limits for parameters
prior.params.lo <- -1
prior.params.hi <- 1
prior.sd.lo <- 0
prior.sd.hi <- 1

# number of iterations, about 3000 per hour
n.iter <- 3000*70  # total number of iterations
n.thin <- 10       # thinning; number of posterior samples will be floor(n.iter/n.thin) + 1

# setting up matrices that will store the posterior samples
nr <- round(n.iter/n.thin,0)+1  # number of rows is thinned no. of updates + starting value
params.mat <- matrix(NA,nr,length(params))
colnames(params.mat) <- names(params)
priors.mat[1,] <- params
re.obs.int.mat <- matrix(NA,nr,n.obs)
colnames(re.obs.int.mat) <- paste("obs",levels(observers),".int",sep="")
re.obs.int.mat[1,] <- re.obs.int
re.obs.ph.mat <- matrix(NA,nr,n.obs)
colnames(re.obs.ph.mat) <- paste("obs",levels(observers),".ph",sep="")
re.obs.ph.mat[1,] <- re.obs.ph

# vector for storing model choices
model <- array(NA,nr)
# the predictor
x <- l.photo
# the response
y <- l.best

###### the likelihood equations
log.lik <- function(y = y, x = x, params = params, re.obs.int = re.obs.int, re.obs.ph = re.obs.ph){
  sp.params<-params[3:7]  # these will be zero if beaufort is not included in the model
  bft.params<-params[8]   # these will be zero if species is not included in the model
  mu <- params['int'] + re.obs.int[observers] + (params['ph'] + re.obs.ph[observers]) * x +
c(0,sp.params)[match(species,levels(species))]*beaufort
log.lik <- sum(log(dnorm(y,mu,params['sd.model'])))) +
sum(log(dnorm(re.obs.ph,0,params['sd.obs.ph'])))) +
sum(log(dnorm(re.obs.int,0,params['sd.obs.int']),na.rm=T))
log.lik

# test
log.lik(y = l.best, x = l.photo, params = params, re.obs.int = re.obs.int, re.obs.ph = re.obs.ph)

#########################################################################
# progress bar
pb <- tkProgressBar(title = "progress bar", min = 0,max = n.iter, width = 200)

# the RJMCMC algorithm
isave <- 1              # set the counter; first value is starting value
for (b in 2:n.iter){
  newparams <- params

  ###### the RJ step
  if(cur.p[3]==0){  # if species is currently not in the model, propose to add it
    newparams[3:7] <- rnorm(5,rjprop.mean.sp,rjprop.sd.sp)   ################ changed from
    new.lik <- log.lik(y = y, x = x, params = newparams, re.obs.int = re.obs.int, re.obs.ph =
    re.obs.ph)
    cur.lik <- log.lik(y = y, x = x, params = params, re.obs.int = re.obs.int, re.obs.ph = re.obs.ph)
    num <- new.lik + sum(log(dunif(newparams[3:7],prior.params.lo,prior.params.hi))) # add
    den <- cur.lik + sum(log(dnorm(newparams[3:7],rjprop.mean.sp,rjprop.sd.sp)))   # add
    proposal densities for new parameters
    A<-min(1,exp(num-den))
    V<-runif(1)
  }else{ # if species is currently in the model, propose to delete it
    newparams[3:7] <- 0
    new.lik <- log.lik(y = y, x = x, params = newparams, re.obs.int = re.obs.int, re.obs.ph =
    re.obs.ph)
    cur.lik <- log.lik(y = y, x = x, params = params, re.obs.int = re.obs.int, re.obs.ph = re.obs.ph)
    num <- new.lik + sum(log(dnorm(params[3:7],rjprop.mean.sp,rjprop.sd.sp)))   # add proposal
    den <- cur.lik + sum(log(dunif(params[3:7],prior.params.lo,prior.params.hi))) # add priors for
    current parameters
    A<-min(1,exp(num-den))
    V<-runif(1)
  }
}

if (cur.p[8]==0) {  # if beaufort is currently not in the model, propose to add it
    newparams[8] <- rnorm(1,rjprop.mean.bft,rjprop.sd.bft)
    new.lik <- log.lik(y = y, x = x, params = newparams, re.obs.int = re.obs.int, re.obs.ph =
                      re.obs.ph)
    cur.lik <- log.lik(y = y, x = x, params = params, re.obs.int = re.obs.int, re.obs.ph = re.obs.ph)
    num <- new.lik + sum(log(dunif(newparams[8],prior.params.lo,prior.params.hi))) # add priors
    for new parameters
    den <- cur.lik + sum(log(dnorm(newparams[8],rjprop.mean.sp,rjprop.sd.sp)))   # add proposal densities for new parameters
    A<-min(1,exp(num-den))
    V<-runif(1)
}  # if beaufort is currently in the model, propose to delete it
else{
    newparams[8] <- 0
    new.lik <- log.lik(y = y, x = x, params = newparams, re.obs.int = re.obs.int, re.obs.ph =
                      re.obs.ph)
    cur.lik <- log.lik(y = y, x = x, params = params, re.obs.int = re.obs.int, re.obs.ph = re.obs.ph)
    num <- new.lik + sum(log(dnorm(params[8],rjprop.mean.bft,rjprop.sd.bft)))   # add proposal densities for current parameters
    den <- cur.lik + sum(log(dunif(params[8],prior.params.lo,prior.params.hi))) # add priors for current parameters
    A<-min(1,exp(num-den))
    V<-runif(1)
}

# which model did we end up with?
cur.mod<-match(sum(cur.p),apply(param.list,1,sum))

##### the MH step
newparams <- params
new.re.obs.int <- re.obs.int
new.re.obs.ph <- re.obs.ph
# updating the parameters
# the first level of species coefficients or beaufort coefficients are always zero, don't need updating
for (p in which(cur.p==1)) {  # parameters which can be negative
    u <- rnorm(1,params[p],mhp.prop.sd.params[p])
    newparams[p] <- u
    new.lik <- log.lik(y = y, x = x, params = newparams, re.obs.int = re.obs.int, re.obs.ph =
                       re.obs.ph)
    cur.lik <- log.lik(y = y, x = x, params = params, re.obs.int = re.obs.int, re.obs.ph = re.obs.ph)
num <- new.lik + log(dunif(newparams[p], prior.params.lo, prior.params.hi))
den <- cur.lik + log(dunif(params[p], prior.params.lo, prior.params.hi))
A <- min(1, exp(num-den))
V <- runif(1)
}
for (p in 9:11) {  # st dev cannot be negative
  u <- rnorm(1, params[p], mhprop.sd.params[p])
  newparams[p] <- u
  new.lik <- log.lik(y = y, x = x, params = newparams, re.obs.int = re.obs.int, re.obs.ph = re.obs.ph)
  cur.lik <- log.lik(y = y, x = x, params = params, re.obs.int = re.obs.int, re.obs.ph = re.obs.ph)
  num <- new.lik + log(dunif(newparams[p], prior.sd.lo, prior.sd.hi))
  den <- cur.lik + log(dunif(params[p], prior.sd.lo, prior.sd.hi))
  A <- min(1, exp(num-den))
  V <- runif(1)
}

# random effects coefficients - no priors on the coefficients
for (r in 1:n.obs) {
  new.re.obs.int[r] <- rnorm(1, re.obs.int[r], mhprop.sd.sd.obs.int)
  num <- log.lik(y = y, x = x, params = params, re.obs.int = new.re.obs.int[r], re.obs.ph = re.obs.ph)
  den <- log.lik(y = y, x = x, params = params, re.obs.int = re.obs.int, re.obs.ph = re.obs.ph)
  A <- min(1, exp(num-den))
  V <- runif(1)
  ifelse(V <= A, re.obs.int[r] <- new.re.obs.int[r], new.re.obs.int[r] <- re.obs.int[r])
}
for (r in 1:n.obs) {
  new.re.obs.ph[r] <- rnorm(1, re.obs.ph[r], mhprop.sd.sd.obs.ph)
  num <- log.lik(y = y, x = x, params = params, re.obs.int = re.obs.int, re.obs.ph = new.re.obs.ph)
  den <- log.lik(y = y, x = x, params = params, re.obs.int = re.obs.int, re.obs.ph = re.obs.ph)
  A <- min(1, exp(num-den))
  V <- runif(1)
  ifelse(V <= A, re.obs.ph[r] <- new.re.obs.ph[r], new.re.obs.ph[r] <- re.obs.ph[r])
}

# each "n.thin-th" iteration, store the parameter values in matrices
if (b %% n.thin < 1) {
  isave <- isave + 1
  params.mat[isave,] <- params
  re.obs.int.mat[isave,] <- re.obs.int
  re.obs.ph.mat[isave,] <- re.obs.ph
  model[isave] <- cur.mod
497 }  
498 # display progress  
499 Sys.sleep(1)  
500 setTkProgressBar(pb, b, label=paste(round(b/n.iter*100),"% completed",sep=""))  
501 } ### end of iteration loop  
502 close(pb); date()  
503  
504 ########################### end of RJMCMC sampling  
505  
506
Appendix 5. Coverage of predicted group sizes

For each group size estimate for each observer, we predicted group size using Eq. 1 and sampling the MCMC chains from model 2 as described in Methods. For each of the 2,435 observer estimates, we determined if the 95% credibility interval of predicted size included the photo count (our measure of true group size). Coverage of the 95% interval, measured as the fraction of intervals which included the photo count, was 0.955. We note that this procedure was an inverse prediction – that is, although the model fitted $y$ to $x$, we predicted $x$ given $y$. We also note that this procedure was not cross-validation, since the model was not refit for each of the 2,435 observer estimates. Therefore, since the value being predicted (photo count) was included in the model fitting, coverage was expected to be positively biased. Given the large sample size, however, we believe the positive bias due to the inclusion of a single datum would be small, as indeed it seemed to be. Fig A4 shows observer estimates and posterior distributions of predicted group size plotted against photo count for a selection of the 59 observers.
Fig. A4. Observer estimates (x), and group sizes predicted from those estimates, plotted relative to photo count (gray line) for selected observers. Circles are the means and vertical line segments the 95% credibility intervals of predicted group sizes. Cases for which the 95% credibility interval did not include the photo count are shown in red.