Quasi-Bayesian estimation of time varying volatility in DSGE models [†]

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Abstract

We propose a novel quasi-Bayesian Metropolis-within-Gibbs algorithm that can be used to estimate drifts in the shock volatilities of a linearised DSGE model. The resulting volatility estimates differ from existing approaches in two ways. First, the time variation enters nonparametrically, so that our approach ensures consistent estimation in a wide class of processes, thereby eliminating the need to specify the volatility law of motion and alleviating the risk of invalid inference due to misspecification. Second, the conditional quasi-posterior of the drifting volatilities is available in closed form which makes inference straightforward and simplifies existing algorithms. We apply our estimation procedure to a standard DSGE model and find that the estimated volatility paths are smoother compared to alternative stochastic volatility estimates. Moreover, we demonstrate that our procedure can deliver statistically significant improvements to the density forecasts of the DSGE model compared to alternative methods.

1 Introduction

The presence of changing volatility in macroeconomic time series has been documented in the literature, among others, by Primiceri (2005) and Sims and Zha (2006). Allowing the volatility to change over time can lead to a better model fit when the sample considered contains periods characterised by changing volatility. Moreover, allowing for stochastic volatility can improve the quality of the model's forecasts, particularly the density forecasts. Estimation of time varying volatility in reduced form models such as vector autoregressions has become popular with papers such as Cogley and Sargent (2005), Primiceri (2005), Cogley and Sbordone (2008), Benati and Surico (2009), Gali and Gambetti (2009), Canova and Gambetti (2009) and Mumtaz and Surico (2009). On the other hand, estimation of drifting volatility in dynamic stochastic general equilibrium (DSGE) models has received less consideration due to the additional complexity that arises from the nonlinearities and rational expectations that feature in these models. There are, in general, two approaches to

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introducing changing volatility in a DSGE model. The first has been advocated by Fernandez-Villaverde, Rubio-Ramirez and Uribe (2011) and Fernandez-Villaverde and Rubio-Ramirez (2013). In these papers, a law of motion for the volatility is introduced before solving the nonlinear rational expectation model. For the stochastic volatility not to vanish, linearisation around the deterministic steady state is not appropriate and at least a third-order approximation is required. The resulting model's solution includes nonlinear terms and, as a result, nonlinear filters such as the particle filter are required to estimate the model. The advantage of this approach is that the resulting model is not 'certainty-equivalent' and shocks to volatility can have real effects on the decisions made by economic agents in the model. A drawback is that both solution algorithms and nonlinear filters can be computationally very demanding and the complexity increases with the size of the model; as a result, only small-sized DSGE models can be estimated in this way. The second approach has been proposed by Justiniano and Primiceri (2008) and applied recently in a forecasting exercise in Diebold, Schorfheide and Shin (2017). This augments the solution of a linearised DSGE model by adding stochastic volatility to the shocks in the state equation of the model and estimates the model's parameters and the drifting volatilities using a Metropolis-within-Gibbs algorithm. The main advantage is that estimation is computationally cheaper than nonlinear filters and can be applied to larger DSGE models such as those used by central banks. Both approaches discussed impose additional model structure by relying on the assumption that the law of motion for the volatility is correctly specified. This is typically of exogenous and reduced-form nature such as an AR(1) or a random walk, or, as in Liu, Waggoner and Zha (2011) or Bianchi (2013), a Markov-switching process.

This paper proposes a novel quasi-Bayesian Metropolis-within-Gibbs algorithm that can be used to jointly estimate the time variation in the volatilities of a DSGE model's shocks and the time invariant structural parameters. The proposed methodology shares similarity with Justiniano and Primiceri (2008) in the way the volatility enters the structural model but, instead of assuming a law of motion, our approach estimates the changing volatility with the help of a nonparametric estimator, building on previous work of Petrova (2017) and Giraitis, Kapetanios and Yates (2014). The resulting volatility estimates differ from those in Justiniano and Primiceri (2008) and Diebold et al. (2017) in two ways. First, the time variation enters nonparametrically, ensuring consistent estimation in a wide class of deterministic and stochastic processes for the volatility, and alleviating the risk of invalid inference due to misspecification of the state equation. This point is illustrated in Petrova (2017) who shows in a Monte Carlo exercise that treating the volatility as a state variable when the state equation is misspecified may result in asymptotically invalid estimates. Second, our Metropolis-within-Gibbs algorithm is based on analytic inverted-Wishart expressions obtained for the conditional quasi-posterior of the drifting volatilities. Moreover, the proposed algorithm is free of nuisance parameters (e.g. starting values required for the Kalman filter) and does not require increasing the dimension of: i) the state vector to include the latent volatilities; ii) the parameter vector to include additional coefficients from the latent volatility processes. As a result, our approach simplifies inference and makes existing algorithms more tractable and robust. It is the Bayesian treatment of this paper that facilitates the construction of such an algorithm and, to our best knowledge, this is the first procedure that can accommodate mixtures of time varying volatilities and time invariant parameters in a DSGE framework without imposing parametric assumptions on the volatility processes. The novelty is the facilitation of such mixtures: the frequentist method of Giraitis et al. (2014, 2016) cannot handle mixtures, while Petrova (2017) only deals with conjugate posterior mixtures where Metropolis steps are not required.

We apply the methodology proposed in this paper to a typical medium-sized DSGE model (Smets and Wouters (2007)) and report the documented changes in the volatility of the shocks. We compare our results to the approach of Justiniano and Primiceri (2008) and show that our nonparametric specification delivers smoother paths for the shock volatilities over time.

We also perform an out-of-sample forecasting exercise in order to evaluate the effect our method has on the forecasting performance of the structural model. To this end, we find that the version of the model with nonparametric volatility delivers statistically significant density forecast improvements.

2 Econometric Methodology

The starting point of our analysis is the linearised rational expectation model given by

$$A(\theta)\mathbb{E}_t[x_{t+1}] = B(\theta)x_t + C(\theta)\eta_t, \quad \Omega_t^{-1/2}\eta_t \sim \mathcal{N}(0, I_k). \tag{1}$$

where x_t is an $n \times 1$ vector of the model's variables, $A(\theta)$, $B(\theta)$ and $C(\theta)$ are matrix functions of the time invariant parameter vector θ and η_t is a vector of uncorrelated structural shocks with diagonal time varying covariance matrix Ω_t. The model's solution, when it exists, takes the form: $x_t = F(\theta)x_{t-1} + G(\theta)\eta_t$, where for most DSGE models, the $n \times n$ matrix F and $n \times k$ matrix G can be computed numerically for a given value of θ, using for example Blanchard and Kahn (1980) or Sims (2002) solution algorithms. The system is augmented with a measurement equation of the form $y_t = D(\theta) + Z(\theta)x_t$, where y_t is an $m \times 1$ vector of observables, typically of a smaller dimension than x_t.

We first assume that we observe a realisation from the history of the shocks η_t for $t \in \{1, ..., T\}$, which we denote $\widetilde{\eta}_{1:T}$. Conditional on such a draw, the model simplifies to $\widetilde{\eta}_t = \Omega_t^{1/2} v_t$, $v_t \sim \mathcal{N}(0, I_k)$. In this setting, Petrova (2017) proposes a quasi-Bayesian methodology for estimating Ω_t at each point $t \in \{1, ..., T\}$, which, for a wide class of time varying processes (see Remark 1 below for details) is consistent and asymptotically valid for inference. We outline the quasi-Bayesian methodology. First, at each point t, a Wishart prior distribution for the precision matrix Ω_t^{-1} is specified of the form $\Omega_t^{-1} \sim \mathcal{W}(\alpha_{0t}, \gamma_{0t}^{-1})$, where α_{0t} is a degrees of freedom prior parameter and γ_{0t}^{-1} is a $k \times k$ diagonal scale matrix. The kernel-weighted likelihood function of Giraitis et al. (2014, 2016), given the distributional assumption in (1), takes the form:

$$\mathcal{L}_t(\widetilde{\eta}_{1:T}|\Omega_t^{-1}) = (2\pi)^{-(k/2)\sum_{j=1}^{T} w_{tj}} \, |\Omega_t|^{-\sum_{j=1}^{T} w_{tj}/2} e^{-\frac{1}{2}\sum_{j=1}^{T} w_{tj}(\widetilde{\eta}_j{}' \Omega_t^{-1} \widetilde{\eta}_j)} \tag{2}$$

where w_{tj} are weights computed using a kernel function and normalised as:

$$w_{tj} = \left(\sum\nolimits_{j=1}^{T} \omega_{tj}^2\right)^{-1} \omega_{tj}, \; \omega_{tj} = \left(\tilde{w}_{tj} / \sum\nolimits_{j=1}^{T} \tilde{w}_{tj}\right), \; \tilde{w}_{tj} = \mathcal{K}\left(\frac{t-j}{H}\right) \text{ for } t, j \in \{1, ..., T\}. \tag{3}$$

The kernel function $\mathcal{K}$ is assumed to be non-negative, continuous and bounded function. The bandwidth parameter H satisfies $H \to \infty$ and $H = o(T/\log T)$. The normalisation of the kernel weights in (3) is proposed in Petrova (2017) and is required to ensure that the prior is asymptotically dominated by the data and the same rate of convergence as in Giraitis et al. (2014, 2016) is achieved.

Proposition 1 *Combining the prior distribution for* Ω_t^{-1} *with the local likelihood function in (2) delivers a quasi-posterior distribution for* Ω_t^{-1} *for each* $t \in \{1, ..., T\}$ *which, conditional on a realisation of the structural shocks* $\widetilde{\eta}_{1:T}$*, has a Wishart form:* $\Omega_t^{-1}|\widetilde{\eta}_{1:T} \sim \mathcal{W}(\widetilde{\alpha}_t, \widetilde{\gamma}_t^{-1})$ *with posterior parameters* $\widetilde{\alpha}_t = \alpha_{0t} + \sum_{j=1}^{T} w_{tj}$, $\quad \widetilde{\gamma}_t = \gamma_{0t} + \sum_{j=1}^{T} w_{tj} \widetilde{\eta}_j \widetilde{\eta}_j'$.

2.1 Remarks

1. The asymptotic validity of the method covers both deterministic and stochastic processes for the volatility (see Giraitis et al. (2014) and Petrova (2017) for further details and examples). More specifically, defining σ_t^2 to be a vector containing the diagonal elements of Ω_t, the quasi-

Bayesian methodology provides a consistent estimation for any one of the following processes: (i) a deterministic process given by $\sigma_t^2 = f\left(\frac{t}{T}\right)$, where $f(.)$ is a piecewise differentiable function; (ii) a vector-valued stochastic process satisfying $\sup_{j:|j-t|\leq h} ||\sigma_t^2 - \sigma_j^2||^2 = O_p\left(h/t\right)$ for $1 \leq h \leq t$ as $h \to \infty$, or (iii) any combination of (i) and (ii).

2. The time variation in the covariance matrix Ω_t is nonparametric: the sequence σ_t^2 needs only satisfy one of the "slow drift" conditions (i)-(iii) in Remark 1, which encompass, for example, constant volatility, breaks, deterministic or bounded random walk processes, without the need for imposing specific modelling restrictions on the volatility law of motion.

3. The existing approach of Justiniano and Primiceri (2008) adds σ_t to the state vector of latent variables and requires a stochastic process for it. The process used in Justiniano and Primiceri (2008) and Diebold et al. (2017) is a random walk for $\ln(\sigma_t)$. This assumption is convenient as it is a simple way to induce persistence in σ_t and, at the same time, reduces the number of additional coefficients needed for each state equation. However, under misspecification of the state space, the Kalman filter can provide invalid inference, even asymptotically, as illustrated by Petrova (2017).

4. Justiniano and Primiceri (2008) transform the model for $\widetilde{\eta}_{1:T}$ into conditionally linear and Gaussian state space using a procedure suggested by Kim, Shephard and Chib (1998) which approximates the resulting log-$\chi^2(1)$ distributed residuals with a mixture of Normal distributions. This requires an additional step in the resulting algorithm, which in redundant in our version based on Proposition 1.

5. For the choice of kernel in (3), the widely used Normal kernel weights are given by $\tilde{w}_{tj} = (1/\sqrt{2\pi})\exp((-1/2)((t-j)/H)^2)$, while the rolling window procedure results as a special case of flat kernel weights: $w_{tj} = \mathbb{I}(\,|t-j| \leq H)$.

Conditional on a draw from the history of the time varying volatilities $\Omega_{1:T}$, the model is a linear Gaussian state space with known heteroskedasticity; so the Kalman filter can be employed to recursively build the likelihood of the parameters θ and then a standard Metropolis-Hastings step can be used (see Metropolis et al. (1953), Hastings (1970) or Schorfheide (2000)) to draw from the conditional posterior of θ. Finally, conditional on θ, a disturbance smoother such as the one described in Carter and Kohn (1994) or Durbin and Koopman (2002) can be used to obtain a draw from the history of the structural shocks η_t. This conditioning argument can be used to construct a Metropolis-within-Gibbs algorithm to approximate the joint posterior of Ω_t, θ and η_t. The resulting algorithm can be found in the Appendix.

3 Empirical Application

In this section we apply our Metropolis-within-Gibbs algorithm to the Smets and Wouters (2007) model which is an extension of a small-scale monetary RBC model with sticky prices. We refer to this model as NPV-DSGE.

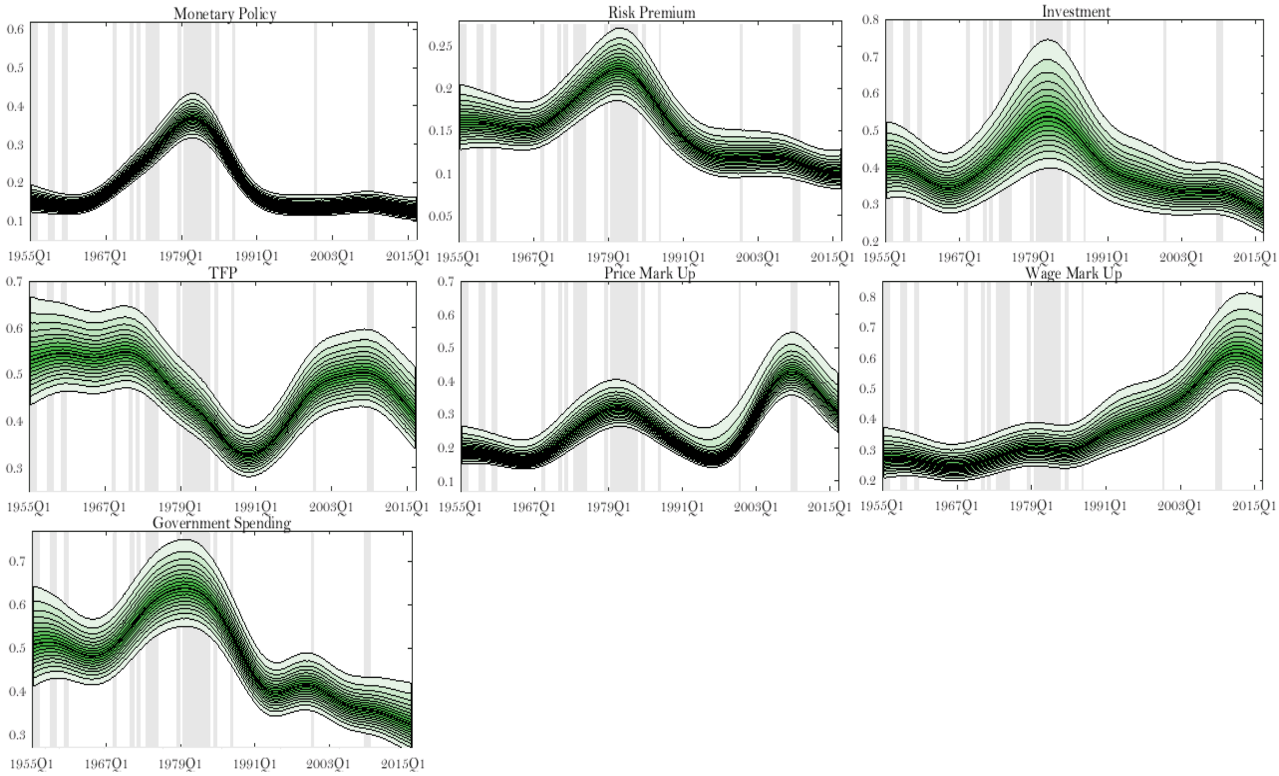

Figure 1: Nonparametric Volatility Estimates

In addition, we estimate three other specifications for the Smets and Wouters (2007) model: i) a standard constant volatility model (CV-DSGE) estimated with standard MCMC methods, ii) a model with stochastic volatility estimated with the algorithm of Justiniano and Primiceri (2008) (SV-DSGE), and iii) a Markov Switching volatility DSGE model (MSV-DSGE) with two volatility regimes as in Liu et al. (2011). The estimated parameters for all specifications as well as details on priors, algorithms and data used can be found in the Appendix. Figures 1 and 2 display the volatilities of the model's shocks over time and the corresponding 95% posterior bands. The different shades represent quantiles of the posterior distribution. The figures also display periods characterised by more than 50% probability of the high volatility regime, estimated with the MSV-DSGE model and shaded in light grey. The estimated volatilities for both specifications follow similar patterns over time; however, the NPV-DSGE model delivers smoother estimates over time and has narrowed posterior bands implying more precise estimates, while the SV-DSGE estimates are more noisy and ragged. All shock volatilities (with the exception of the wage shock) are high in the 1970s and early 1980s and fall thereafter, consistent with findings in the literature on the Great Moderation.

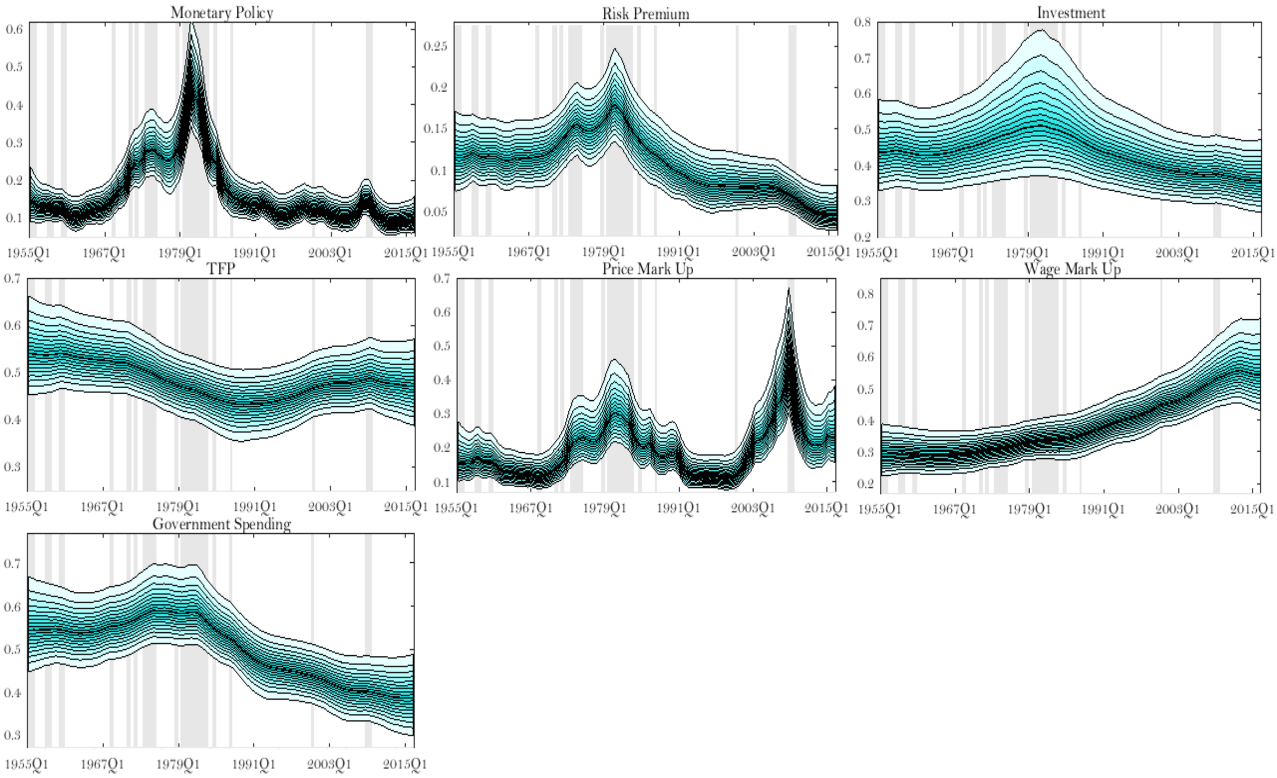

Figure 2: Stochastic Volatility Estimates

This finding is further supported by the MSV-DSGE model: the shaded areas during the 1970s and early 1980s indicate long periods characterised by the high volatility regime. We also find that some shock volatilities (TFP, price and wage mark up) increase during the recent financial crisis as a consequence of the increased uncertainty in this period. Our method uncovers considerably more time variation in the TFP shock volatility compared to the SV-DSGE model, and this turns out to have a positive effect on the quality of the model-implied density forecasts for output and investment growth, as demonstrated in the next section. The differences in the estimated TFP volatility between NPV-DSGE and SV-DSGE models can be explained by recalling that our NPV-DSGE does not restrict the volatility process to a geometric random walk and, as a consequence, is more robust.

4 Forecasting

In this section, we evaluate the relative forecasting performance of the NPV-DSGE specification applied to the Smets and Wouters (2007) model and estimated with our Metropolis-within-Gibbs algorithm. We generate density forecasts for the observables of the model and compare the forecasting record of NPV-DSGE against the constant volatility (CV-DSGE) model, as well as the stochastic volatility specification (SV-DSGE) estimated with Justiniano and Primiceri (2008)'s algorithm and a Markov Switching Volatility (MSV-DSGE) with a two volatility states. More details on the sample and forecasting origins can be found in the Appendix. The Appendix also contains the point forecasts for the different models, which perform very similarly and are rarely statistically different from each other at 95%. Table 1 evaluates the quality of the density forecasts measured by log predictive score (LPS). The table displays absolute LPS for the NPV-DSGE model and differences in LPS between the NPV-DSGE and the alternative CV-, SV- and MSV-DSGE models respectively, so positive numbers imply superior performance of our NPV-DSGE approach.

	Output	Cons	Inv	Wage	Hours	Inflation	Int Rate
Log Predictive Score - NPV-DSGE							
h=1	-1.03	-0.83	-1.95	-1.66	-0.87	-0.64	0.63
h=2	-1.13	-0.95	-2.12	-1.64	-1.53	-0.77	0.08
h=4	-1.17	-0.95	-2.20	-1.64	-2.35	-0.87	-0.47
h=8	-1.15	-0.90	-2.14	-1.58	-3.28	-0.84	-0.89
Relative LPS: NPV-DSGE against CV-DSGE							
h=1	0.09**	0.03	0.03*	-0.08*	0.02	0.01	0.25**
h=2	0.07**	0.00	0.03	-0.08*	0.00	-0.02	0.19**
h=4	0.06*	0.01	0.03	-0.11**	-0.06	-0.04	0.14*
h=8	0.08*	0.05*	0.02	-0.15*	-0.09	0.00	0.15*
Relative LPS: NPV-DSGE against SV-DSGE							
h=1	0.08**	0.01	0.07**	-0.02	0.04**	0.02	0.00
h=2	0.08**	0.02	0.10**	-0.01	0.04**	0.01	-0.01
h=4	0.09**	0.05**	0.09**	-0.01	0.04*	0.03	0.01
h=8	0.09**	0.02	0.08**	0.00	0.03	0.01	0.00
Relative LPS: NPV-DSGE against MSV-DSGE							
h=1	0.04	0.02	0.07*	0.03	0.10**	-0.01	-0.13*
h=2	0.12*	0.03	0.17**	0.03	0.18**	0.00	-0.04
h=4	0.15*	0.01	0.24**	0.03	0.24*	-0.07	0.10
h=8	0.12	-0.07	0.24*	0.04	0.16	-0.16*	0.28*

Table 1. Log Predictive Score. The figures in the top panel are the LPS of the NPV-DSGE model, the figures under the remaining models are differences between the LPS of the NPV-DSGE, and CV-, SV- and MSV-DSGE specifications respectively. '*' and '**' indicate rejection of the null of equal performance against a two-sided alternative at 5% and 1% significance level respectively, using a Diebold-Mariano test.

With the exception of the wage growth, our NPV-DSGE model delivers statistically significant improvements for most variables and horizons over both CV-, SV- and MSV-DSGE specifications. Outperforming the CV-DSGE model is expected, especially since the out-of-sample contains periods characterised by very different volatility dynamics (1987-2014). The differences in density forecast performance between the SV-DSGE and NPV-DSGE models can be accounted for by recalling that the NPV-DSGE model's volatility estimates are much smoother, as illustrated in Figure 1, while the SV-DSGE delivers more noisy and ragged estimates. In particular, the LPS performance of the NPV-DSGE model is statistically superior at 1% significance level than that implied by the SV-DSGE model for output and investment growth for all horizons, which are key variables to forecast. This is a consequence of the NPV-DSGE approach uncovering more time variation in the TFP shock's volatility which affects consumption, investment and output forecasts through the production function and the resource constraint of the Smets and Wouters (2007) model. The superior performance of our NPV-DSGE model over the MSV-DSGE specification is due to i) the oversimplified way in which volatility enters the MSV-DSGE model (i.e. two common volatility states for all seven shocks), and ii) the Markov switching nature of the volatility process being subject to abrupt changes rather than smooth time variation.

5 Conclusion

In this paper we propose a novel quasi-Bayesian Metropolis-within-Gibbs algorithm for the estimation of changing volatility in DSGE models. The estimation is based on previous work of Petrova (2017) and differs from existing approaches in being nonparametric and, as a consequence, valid under possible misspecification of the law of motion of the volatility, ensuring consistent estimation in a wide class of deterministic and stochastic processes. The proposed approach delivers a conditional quasi-posterior for the drifting volatilities of an inverse-Wishart form which gives rise to a novel Metropolis-within-Gibbs algorithm. The availability of a closed form expression for the quasi-posterior makes our algorithm computationally simpler than existing algorithms that are based on a combination of Kalman filtering and Kim et al. (1998)'s procedure.

We apply our estimation procedure to the Smets and Wouters (2007) model and show that the estimated volatilities of the structural shocks exhibit slower time variation compared to alternative approaches. In addition, we demonstrate that the algorithm developed in this paper delivers statistically significant improvements to the density forecasts of the model in comparison to the stochastic volatility and Markov switching approaches respectively.

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