Indecisiveness, Undesirability and Overload Revealed Through Rational Choice Deferral*

Rational Choice Deferral (short title)

Georgios Gerasimou

January 9, 2017

Abstract

Three reasons why decision makers may defer choice are *indecisiveness* between various feasible options, *unattractiveness* of these options, and *choice overload*. This paper provides a choice-theoretic explanation for each of these phenomena by means of three deferral-permitting models of decision making that are driven by preference incompleteness, undesirability and complexity constraints, respectively. These models feature *rational* choice deferral in the sense that whenever the individual does not defer, he chooses a most preferred feasible option. Active choices are therefore always consistent with the Weak Axiom of Revealed Preference. The three models suggest novel ways in which observable data can be used to recover preferences as well as their indecisiveness, desirability and complexity components or thresholds. Several examples illustrate the relevance of these models for empirical and theoretical work.

Keywords: Choice deferral; incomplete preferences; indecisiveness; unattractiveness; choice overload; revealed preference; rational choice.

JEL Classification: D01, D03, D11

Consider an individual who must decide between keeping his existing insurance policy and replacing it with some other policy from those available to him. The status quo associated with his decision problem is his current policy, i.e. some explicit and concrete market alternative. Consider a second individual who has no insurance policy and is presented with the same options that the first one is. The status quo associated with this person's decision problem is the state of being uninsured, i.e. a non-market outside option that may also be considered objectively

*This paper supersedes unpublished manuscripts titled *"Incomplete preferences and rational choice avoidance"* (January 2012) and *"Rational indecisive choice"* (October 2010) which originated in a chapter of my doctoral dissertation that was submitted to the University of Cambridge. I thank Miguel A. Ballester, Miguel Costa-Gomes, Carlos Cueva, Robert Evans, Özgür Evren, Paola Manzini, Marco Mariotti, Mauro Papi, John Quah, Levent Ülkü and especially two referees of this journal and the editor, Martin Cripps, for useful comments and suggestions. I also acknowledge helpful feedback from seminar/conference audiences at Cambridge, Amsterdam, Bielefeld, Technion, Aberdeen, Alicante, Cyprus, ESEM 2011 (Oslo), BRIC 2014 (Barcelona) and FUR 2014 (Rotterdam). Any errors are my own.
Mailing address: School of Economics & Finance, Castlecliffe, The Scores, KY16 9AR, St Andrews, UK. Email address: gg26@st-andrews.ac.uk

inferior to *all* available insurance policies.[1] When the first individual chooses his status quo, this choice could reflect a genuine preference for that option over the other feasible ones or, possibly, a *status quo bias*.[2] By contrast, when the second individual maintains his status quo, he is choosing *none* of the market alternatives available to him.[3] A term that is often used by psychologists to describe this distinct type of behaviour is *choice deferral*.[4]

Several models that aim to describe an individual's choices in decision problems with an explicit and concrete status quo option have been proposed.[5] Most of them assume that whenever a decision problem does not feature such a status quo, the individual chooses one of the feasible market alternatives. In so doing, these models rule out by construction the possibility of choice deferral. The present paper complements this theoretical literature by focusing on the large class of decision problems where the individual's status quo is a non-market outside option.

Experimental evidence, casual empiricism and introspection suggest that among the reasons for the occurrence of choice deferral are the following:

1. *Unattractiveness/undesirability*: An individual may defer when faced with decision problems that feature options which are simply "not good enough". This source of deferral is well-documented in the consumer psychology literature.[6] Economists too have long been familiar with this type of deferral, e.g. in the context of principal-agent contracts where the participation constraints are not satisfied. Importantly, as long as the decision maker's desirability criterion is well-defined and stable across contexts/menus, this source of deferral poses no challenge to the utility maximisation paradigm. Despite this "folk wisdom", however, it appears that no choice-theoretic foundations for this kind of decision making exist.

2. *Indecisiveness/decision conflict*: Deferral may occur when the relevant menu contains multi-attribute, trade-off-generating options that are difficult to compare. Many studies in psychology and economics have demonstrated this tendency in consumer, savings, medical and managerial decision making,[7] occasionally also rejecting alternative explanations for deferral such as rational search for more options. When this type of deferral occurs in a menu with two alternatives, for example, it suggests that the decision maker cannot compare them, contrary to what the fundamental axiom of completeness would require from his preferences. Therefore, such instances of deferral are irreconcilable with utility-maximising behaviour.

3. *Complexity/choice overload*: Decision problems that feature a large number of alternatives may cause deferral due to the increased complexity that is associated with them. Such behaviour has also been documented in consumer, savings as well as in voting decisions.[8] This source of deferral too is incompatible with utility

[1]See also Spiegler (2015) for a terminological distinction between "market defaults" and "outside-option defaults", and for the interpretation of the latter as being inferior to market alternatives.

[2]Samuelson and Zeckhauser (1988); Knetsch (1989).

[3]It was precisely this difference in the nature of the status quo associated with these two types of decision problems that was utilised by Samuelson and Zeckhauser (1988) to establish a status quo bias in the health-care plan choices of Harvard University employees in the 80s. This difference was also exploited in a similar way by Knetsch (1989) when this author provided further evidence for this bias in his mug/chocolate bar exchange experiments. In both cases, a comparison was made between the choices of decision makers whose status quo did not coincide with an explicit and concrete option and those of decision makers whose status quo did take such a form.

[4]See, for instance, Tversky and Shafir (1992); Shafir *et al.* (1993); Dhar (1999); Anderson (2003); Krijnen *et al.* (2015). We will also use this term despite the fact that it is slightly misleading given the static nature of the decision problems that the experimental subjects in these studies were presented with, as well as the static nature of most of our analysis in this paper.

[5]Tversky and Kahneman (1991); Munro and Sugden (2003); Mandler (2004); Masatlioglu and Ok (2005, 2015); Dean (2008); Ortoleva (2010); Apesteguia and Ballester (2009, 2013); Gerasimou (2016).

[6]See, for instance, Zakay (1984), Dhar and Sherman (1996) and Mochon (2013).

[7]See, for instance, Greenleaf and Lehmann (1995); Redelmeier and Shafir (1995); Dhar (1997); Luce (1998); Madrian and Shea (2001); Dhar and Simonson (2003); Sawers (2005); Danan and Ziegelmeyer (2006); Bhatia and Mullett (2016); Costa-Gomes *et al.* (2016).

[8]See, for instance, Iyengar and Lepper (2000), Iyengar *et al.* (2004), Augenblick and Nicholson (2016) and the meta-analysis in Chernev *et al.*

maximisation. For example, a certain option may be chosen over many others in a number of "simple" binary menus, while choice may be deferred in the "complex" menu that is formed by taking the union of these binary ones: All preferences revealed in the small menus are contradicted by deferral in the large menu.

In this paper we propose and axiomatically characterise three models of choice deferral, one for each of the above potential sources of this phenomenon. The common feature in all three models is that deferral is *rational* in the sense that whenever the agent decides to choose a feasible option rather than to defer, his choices are consistent with the Weak Axiom of Revealed Preference. Thus, in these three models deferral is neither due to some inconsistency in the agent's underlying preferences nor to some context-dependent decision rule. Indeed, whenever the agent chooses a non-outside option, he always does so by maximising his menu-independent preferences, similar to a standard utility maximiser. Moreover, in the tradition of revealed preference theory, all three models build on simple and easily falsifiable axioms on observable behaviour, which in this case includes choice *and* deferral data. Therefore, an agent's conformity with either model allows an outside observer to recover the agent's preferences and other relevant entities such as his unattractiveness or complexity criteria. Moreover, as we show in Appendix A, all three models include rational choice as a special case.

The model of undesirability-driven deferral features constrained utility maximisation in the sense that the agent chooses his most preferred option at a menu if and only if this is strictly preferred to his desirability threshold, and defers otherwise. The model of indecisiveness-driven deferral portrays an agent with transitive but possibly incomplete preferences who follows the decision rule whereby an alternative is chosen if and only if it is preferred to all other feasible alternatives. This model suggests a novel criterion for distinguishing between the concepts of indifference and indecisiveness using behavioural data. Finally, in the model of overload-driven deferral the agent defers at a menu if and only if its perceived complexity exceeds the agent's constant (menu-independent) complexity threshold, and chooses as a utility maximiser otherwise.

In Section 7 we argue that the three models are informative and applicable in a variety of economic domains, such as the empirical analysis of choice datasets with deferral observations, industrial organisation, robust social choice and choice under uncertainty. As far as the latter domain is concerned, in Appendix B we show how a two-period extension of the indecisiveness-driven deferral model can be used to provide choice-theoretic foundations for "objectively rational" or Bewley (2002) preferences which have found many applications in economics.

1 Preliminaries

The set of all possible choice alternatives is X and is assumed finite. The set $\mathcal{M}$ denotes the collection of all subsets of X, which will be called *menus*. A choice correspondence C is a possibly multi- and empty-valued mapping from $\mathcal{M}$ into itself that satisfies $C(A) \subseteq A$ for all $A \in \mathcal{M}$.

Our interpretation of the situation in which $C(A) = \varnothing$ for some menu $A \neq \varnothing$ is that the agent *defers choice* at A by opting for his unmodelled outside option.[9] Therefore, we will generally *not* assume the following property, which will be treated as an explicit axiom instead:

(2015).

[9]In Section 6 we outline the main implications of modelling the outside option as an explicit feasible option.

Nonemptiness
If $\emptyset \neq A \in \mathcal{M}$, then $C(A) \neq \emptyset$.

Surprisingly, this assumption has been relaxed (in varying degrees) in very few works that we are aware of, such as Hurwicz (1986), Clark (1995), Gaertner and Xu (2004), Kreps (2012) and Gerasimou (2016). In Section 6 we provide arguments in favour of allowing C to be possibly empty-valued.

Contrary to Nonemptiness, the *Weak Axiom of Revealed Preference* (WARP) which we state next is part of (or implied by) the axiomatic system of each of the three models that we present below.

WARP
If $x \in C(A)$, $y \in A \setminus C(A)$ and $y \in C(B)$, then $x \notin B$.

When Nonemptiness is also assumed, WARP can be written in a number of equivalent ways. Without it, however, such equivalences are no longer valid. The above statement of WARP is in the spirit of Samuelson's (1938) original version of the axiom: *If an alternative x is chosen over another alternative y in some menu, then there is no menu where y is choosable and x is feasible.* The axiom's status as a core principle of choice consistency remains intact (if not strengthened) in an environment where deferral is allowed. Intuitively, when the decision maker chooses x over y from a menu A even though she had the opportunity to defer, this may be a stronger indication that x is preferred to y relative to the case where choice from A was forced.[10] To the extent that this is so, it becomes even more plausible to expect that a rational decision maker will not choose y from any menu where x is also feasible.

It may be useful to remark at this point that, by ruling out the possibility of deferral, most choice-theoretic models essentially allow for the terms "decision" and "choice" to be considered synonymous. In this paper where deferral is assumed to be possible we take the view that "decision" is a more general term, as it may indicate either deferral or *active* choice of some feasible option. In light of this remark, we also emphasise from the outset that we limit the range of applicability of WARP to the individual's (active) choices and not across all his decisions, which may well violate this axiom. We come back to this point in Section 6 with a justification and more detailed discussion.

As far as notation is concerned, a binary relation $\succsim$ on X denotes a possibly incomplete *preorder* on X, i.e. a reflexive and transitive binary relation. When this relation is assumed complete, it will be referred to as a *weak order*. The set of greatest/maximum elements of $\succsim$ is defined and denoted by

$$B_{\succsim}(A) := \{x \in A : x \succsim y \text{ for all } y \in A\}.$$

This set is always nonempty when the preorder $\succsim$ is complete, but generally not otherwise.

[10] Costa-Gomes *et al.* (2016) provide evidence from an incentivised lab experiment which suggests that the average number of WARP violations (either absolute or relative to the number of the subjects' active choices) is indeed significantly lower when choice is not forced.

2 Indecisiveness and Maximally Dominant Choice

We open this section with an anecdote that illustrates well some key elements of the indecisiveness-driven deferral model that we develop below:

> *"At the bookstore, [Thomas Schelling] was presented with two attractive encyclopedias and, finding it difficult to choose between the two, ended up buying neither – this, despite the fact that had only one encyclopaedia been available he would have happily bought it."*
>
> Shafir, Simonson and Tversky (1993, p.21)

In line with this story and with the many experimental studies that have documented similar behaviour, in this section we propose and analyse a model in which deferral is rooted in the agent's inability to compare some alternatives due to the incompleteness of his preferences.

Our first axiom here imposes a special upper bound on the empty-valuedness of C and plays a very important role conceptually.

Desirability
If $x \in X$, then $C(\{x\}) = \{x\}$.

This is compatible with the interpretation that whenever the decision maker is faced with only one alternative, the latter is sufficiently good for him to choose it. Under this interpretation, Desirability rules out unattractiveness of the alternatives as a potential reason for deferral.[11] Hence, it is also compatible with the interpretation that if the agent was to perceive the act of deferring as an explicit option, in the present timeless decision setting he would consider this option to be inferior to all other alternatives. We note, however, that despite this fact and the presently static nature of the decision environment, modelling an individual who satisfies Desirability as occasionally opting for deferral does not amount to portraying him as irrational. To the extent that choice deferral serves a relevant purpose (e.g. regret avoidance; economizing on time or cognitive resources) and the agent expects to be given the opportunity to make an active choice in the future, such decision making can be viewed as compatible, in principle, with maximization of his self-interest.

Contraction Consistency
If $x \in C(A)$ and $x \in B \subset A$, then $x \in C(B)$.

This standard axiom is weaker than WARP when Nonemptiness is assumed, but logically distinct from it otherwise. For example, $x \in C(A)$, $x \in B \subset A$ and $C(B) = \emptyset$ is consistent with WARP but violates Contraction Consistency. Conversely, if $\{x\} = C(\{x, y, z\})$ and $\{y\} = C(\{w, x, y\})$, then Contraction Consistency is satisfied but WARP is violated. However, ruling out this kind of behaviour is normatively appealing. Indeed, when choice reveals preference, x being choosable at A suggests that x is at least as good as every other alternative in A, hence

[11] The logical complement of this axiom, Undesirability, is introduced in the context of unattractiveness-driven deferral in the next section.

in $B \subset A$ too. Therefore, to allow for the possibility that nothing is chosen from B amounts to saying that the most preferred option in the menu is not chosen, which is irrational.

Strong Expansion
If $x \in C(A)$, $y \in A$ and $y \in C(B)$, then $x \in C(A \cup B)$.

This axiom was introduced as "Property $\gamma+$" in Salant and Rubinstein (2008). Like Contraction Consistency, it is implied by WARP when Nonemptiness is assumed and is distinct from WARP otherwise. For example, $x \in C(A)$, $y \in A$, $y \in C(B)$ and $C(A \cup B) = \varnothing$ is compatible with WARP but violates Strong Expansion. Conversely, $\{x\} = C(\{w, x, y\})$ and $\{y\} = C(\{x, y, z\})$ violates WARP but not Strong Expansion. This axiom strengthens the well-known "Property γ" (Sen, 1971) which states that an alternative x that is choosable in both A and B is also choosable in $A \cup B$. Indeed, Strong Expansion requires that if x is chosen in the presence of y in some menu A and y is chosen in B, then x is chosen in $A \cup B$. This obviously reduces to "Property γ" in the special case where $x = y$. Intuitively, if a rational agent chooses x in the presence of y at menu A when deferral is possible, this suggests that he finds x at least as good as y and everything else in A. Likewise, the fact that y is chosen in B suggests that y is at least as good as everything else in B. Such an agent should therefore also consider x to be at least as good as everything else in $A \cup B$ and hence would choose x from this expanded menu, as required by the axiom.

Importantly, even though Contraction Consistency and Strong Expansion are logically distinct from WARP individually, together they do imply this axiom even if C does not satisfy Nonemptiness. Indeed, suppose to the contrary that $x \in C(A)$, $y \in A \setminus C(A)$, $y \in C(B)$ and $x \in B$ for distinct menus A and B and alternatives x and y. Strong Expansion implies $y \in C(A \cup B)$, while Contraction Consistency then implies $y \in C(A)$, which is a contradiction. However, we note that if these two axioms are weakened to "$x \in C(A)$ *and* $x \in B \subset A$ *implies* $C(B) \neq \varnothing$" and "$x \in C(A)$, $y \in A$ *and* $y \in C(B)$ *implies* $C(A \cup B) \neq \varnothing$", respectively, then their joint implications do not include WARP.

Proposition 1
The following are equivalent for a choice correspondence $C : \mathcal{M} \to \mathcal{M}$:
1. C satisfies Desirability, Contraction Consistency and Strong Expansion.
2. There exists a unique preorder $\succsim$ on X such that

$$C(A) = \begin{cases} \varnothing, & \textit{iff } B_{\succsim}(A) = \varnothing \\ B_{\succsim}(A), & \textit{otherwise} \end{cases} \tag{1}$$

All proofs are in Appendix C. The tightness of all axiomatic systems is established in Appendix D.

We will refer to (1) as the model of *maximally dominant choice* (MDC). This portrays the decision maker as choosing one of the feasible market options if and only if the relevant menu contains a most preferred such option, and as deferring whenever this is not the case. In particular, despite the incompleteness of his preferences, such an individual makes active choices by following the decision rule that is prescribed by utility maximisation. As a result, his active choices across menus are fully consistent. This consistency hedges him against the theoretical possibility of manipulation that comes about when choice is context-dependent[12] or via some money-pump exploitation that is generally possible when preferences are incomplete and the agent chooses some option that is merely undominated (Mandler, 2009; Danan, 2010). When the agent defers, he opts for his non-market outside option which, from the point of view of static choice, is essentially considered to be inferior to *every* option (cf the Desirability axiom). There are plausible psychological explanations for such behaviour. Luce (1998), for instance, shows that some of the variation in the occurrence of deferral is due to the anticipation of negative emotions such as regret and self-blame that may follow once a decision maker who makes an active choice in a conflict situation realises later that he has made his choice was suboptimal.

The model's predictions are compatible in a straightforward way with experimental findings on choice from conflict-inducing multi-attribute alternatives such as those reported by Tversky and Shafir (1992) (including the anecdote in the quote that opened this section). These findings suggest that, given two alternatives x and y that dominate each other in some important attribute, many people are willing to choose x when x is the only feasible option and y when y is the only feasible option, but nothing when both x and y (and only them) are feasible. If preferences are let to coincide with the usual partial ordering on attribute space, then one would have $x \succsim x$, $y \succsim y$ and $x \not\succsim y$, $y \not\succsim x$, in which case (1) would indeed predict $C(\{x\}) = \{x\}$, $C(\{y\}) = \{y\}$ and $C(\{x,y\}) = \emptyset$, consistent with the findings.

Given that incomparability/indecisiveness is the only source of deferral in the MDC model, one may expect that such an individual may be able to eventually resolve his indecision by completing his preferences in the future (possibly after acquiring information about the alternatives), and then to choose as a utility maximiser from any menu where he had originally deferred. In Appendix B we extend the model in this direction and use this extension to provide a choice-theoretic foundation for "objectively rational" Bewley (2002) preferences under uncertainty.

2.1 Revealed Preference, Indifference and Indecisiveness

MDC suggests a purely behavioural and generally applicable criterion to disentangle indifference and indecisiveness from a given set of decision observations that are consistent with it. Specifically, for an agent who has generated such data, the psychological state with respect to two alternatives x and y is

indifference *iff for every menu* $A \in \mathcal{M}$ *such that* $x, y \in A$, *it holds that* $y \in C(A)$ *whenever* $x \in C(A)$;

indecisiveness *iff for every menu* $A \in \mathcal{M}$ *such that* $x, y \in A$ *it holds that* $x, y \notin C(A)$;

preference (for x **over** y**)** *iff there exists a menu* $A \in \mathcal{M}$ *such that* $x \in C(A)$ *and* $y \in A \setminus C(A)$.

As in Eliaz and Ok (2006), and consistent with any model of rational choice, the agent here is revealed to be indifferent between x and y if and only if both options are always choosable whenever both are feasible and one is choosable. However, unlike the Eliaz-Ok criterion for revealed indecisiveness which necessitates one alternative to be chosen *over* the other in some menu and is therefore associated with a violation of WARP, the agent in the MDC model *is revealed to be indecisive between x and y if and only if neither of these options is ever chosen in the presence of*

[12] Dhar and Simonson (2003), for instance, report on the strengthening of the famous "attraction effect" when deferral is allowed.

the other. This novel criterion is the first to allow for a distinction between revealed indifference and indecisiveness that applies to WARP-consistent agents with incomplete preferences. Moreover, unlike Eliaz and Ok (2006), no a priori restrictions on the agent's preferences are necessary for this distinction to be made (a condition called "regularity" must be satisfied by the agent's preferences in the Eliaz-Ok model).

Mandler (2009) proposed a distinction between the two concepts that is based on data from sequential pairwise trades of options that are not ranked by strict preference. Specifically, if such sequential trades eventually result in the decision maker owning an alternative that is either strictly better or strictly worse to the one he started off with, then Mandler's criterion suggests that he must have been indecisive at some point along the sequence of pairs that were involved in the trades. On the other hand, in the absence of such a strict preference ranking it cannot be ruled out that the agent was indifferent throughout. Clearly, the MDC distinction between indifference and indecisiveness is compatible with Mandler's distinction because the revealed indifference relation is transitive whereas the incomparability relation is generally not.

An interesting relationship exists between the properties of the MDC model and the revealed-preference analysis in Bernheim and Rangel (2009). These authors suggested that, in a model-free world, an alternative x may be thought of as being strictly preferred to another alternative y if there is no menu in which y is chosen and x is feasible. In the MDC model, x is revealed preferred to y if, in addition to this, there is a menu in which x is chosen *over* y. Thus, although the Bernheim-Rangel condition is necessary for x to be revealed preferred to y under the MDC model, it is not sufficient. In fact, as is evident from the preceding discussion, this condition is also necessary for the agent to be revealed indecisive between x and y in the MDC model. Moreover, it holds that *x and y are not comparable by the Bernheim-Rangel revealed preference relation if and only if they are revealed incomparable in the context of the MDC model*. Given that the Bernheim-Rangel revealed-preference criterion was proposed in a different context where an individual's behavioural dataset was not assumed to include deferral observations (although, clearly, it need not be restricted in this way), the link established between the revealed indecisiveness relations in the two models is at least somewhat surprising.

2.2 A Dominance-Based Explanation of the Choice Overload Effect

The MDC model is also compatible with evidence suggesting the occurrence and disappearance of the so called *"choice overload"* effect. This refers to the phenomenon whereby decision makers defer choice significantly more often when faced with large/complex menus that contain many alternatives than when they are faced with smaller/simpler ones. The first evidence for this effect came from the field and was reported in Iyengar and Lepper (2000).

One of the proposed explanations for choice overload is that deferral in large menus is caused by the lack of familiarity and the absence of a most preferred option.[13] Intuitively, the higher the degree of incompleteness in the agent's preferences, the more likely it is that, as the size of the menu increases, the number of incomparable pairs will become so large that no alternative is preferred to all others. This explanation is compatible with (1). However, the model is also compatible with the explanation of how the effect breaks down when a dominant option is added to a large menu at which choice would have otherwise been deferred (Scheibenhenne *et al.* (2010);

[13]Iyengar and Lepper (2000); Scheibenhenne *et al.* (2010); Chernev *et al.* (2015).

Chernev *et al.* (2015)). Indeed, as explained above, this happens when $C(A) = \varnothing$, regardless of the size of A, and when an alternative x is added to A and is such that $x \succsim y$ for all $y \in A$, in which case $C(A \cup \{x\}) = \{x\}$. Thus, Proposition 1 offers incompleteness-driven explanations for both the occurrence and the disappearance of the choice overload effect.

3 Desirability-Constrained Rationality

The next source of choice deferral that we study is inferiority of all feasible alternatives relative to an agent-specific desirability threshold. As we mentioned in the introduction, this is by no means a novel choice procedure. However, it appears that a precise choice-theoretic formulation does not exist, and therefore the revealed-preference implications associated with the behaviour prescribed by it are unknown. We pin down such a set of necessary and sufficient conditions in this section and formalise some other relevant aspects in this procedure.

Our first axiom here is the logical complement of Desirability:

Undesirability
There exists $x \in X$ such that $C(\{x\}) = \varnothing$.

Indeed, if an option x is not considered to be "good enough", then it is intuitive that the decision maker will not choose it whenever this is the only feasible option. In contract theory, for example, the Undesirability axiom manifests itself through the agent's participation constraint: unless a contract's expected utility exceeds the agent's reservation utility, the agent will not choose it even if it's the only feasible one. Some experimental evidence that are compatible with this axiom was provided by Zakay (1984), who documented a behavioural tendency to choose (defer) at singletons depending on whether these were perceived as being "close to" ("far from") a hypothetical ideal alternative.[14]

Contractive Undesirability
If $C(A) = \varnothing$ and $B \subset A$, then $C(B) = \varnothing$.

This condition is also necessary for modelling unattractiveness-driven deferral. Intuitively, if an individual considers nothing to be sufficiently good in some menu A, then this must also be true in every submenu B of A. This is in sharp contrast to the MDC model of indecisiveness-driven deferral where removing an option from a menu at which no choice was made may result in a dominant feasible option to arise and therefore in a choice to be made.

It is worth stressing that these two axioms ensure that if x is considered undesirable in one menu, then this is true in every menu. Thus, they rule out the possibility that the agent's desirability threshold is menu-dependent. This restriction, however, is intuitive in the context of rational decision making. Indeed, if the agent's threshold depended on the menu, this could render him vulnerable to a special kind of money-pump manipulations. For example, if the agent was somehow endowed with x and was also presented with a menu where everything

[14]See also Mochon (2013) for some related findings.

(including x) was undesirable, he would be willing to pay a small $\epsilon_1 > 0$ in order to "lose" x. A theoretical possibility then is that the agent could also be presented with a menu where x is not only desirable but also the most preferred option. In this case, the agent would be willing to pay some $\epsilon_2 > 0$ to re-obtain x. In so doing, he would be back to his original state and poorer by the amount $\epsilon_1 + \epsilon_2$.

We will say that a preference relation $\succsim$ on X *quasi-rationalizes a choice correspondence* $C : \mathcal{M} \to \mathcal{M}$ if $C(A) = B_{\succsim}(A)$ whenever $C(A) \neq \emptyset$. We will also say that a weak order $\succsim$ quasi-rationalizes C *uniquely up to an element* $x^* \in X$ if for any weak order $\succsim'$ on X where $x \succsim' y$ holds whenever $x \succsim y$ and $x, y \succ x^*$ hold, the weak order $\succsim'$ also quasi-rationalizes C.

Proposition 2

The following are equivalent for a choice correspondence $C : \mathcal{M} \to \mathcal{M}$:

1. C satisfies WARP, Contraction Consistency, Undesirability and Contractive Undesirability.

2. There exists a weak order $\succsim$ on X and an alternative $x^* \in X$ such that, for all $A \in \mathcal{M}$,

$$C(A) = \begin{cases} \emptyset, & \text{iff } z \in B_{\succsim}(A) \text{ implies } x^* \succsim z \\ B_{\succsim}(A), & \text{otherwise} \end{cases} \tag{2}$$

Moreover, $\succsim$ quasi-rationalizes C uniquely up to x^*.

In this model the agent behaves as if he had complete and transitive preferences over the set X and as making decisions like a utility maximiser as long as his most preferred feasible option in the given menu is strictly preferred to some pre-specified alternative x^*. The latter determines his context-independent *undesirability threshold* in the sense that the agent defers choice from a menu if and only if the best alternative in that menu is weakly inferior to x^*. Since the behaviour captured in (2) deviates from rational choice only in that some alternatives are never chosen because they are undesirable, we will refer to it as the model of *desirability-constrained rationality* (DCR).

With regard to preference revelation in the DCR model, quasi-rationalizability of the choice correspondence C by the weak order $\succsim$ uniquely up to x^* suggests that such revelation is possible for the part of the agent's preference order that ends on the (possibly degenerate) indifference set where x^* belongs. This part of the agent's preferences is completely and uniquely recoverable from his choices. The "usefulness" of his deferring behaviour in this respect is limited to the identification of those alternatives in X that are undesirable in the sense that they will never be chosen. Clearly, the agent's preference ranking of these alternatives cannot be recovered using decision data. This is the reason for the potential multitude of weak orderings that are compatible with C in the quasi-rationalizability sense that is claimed in Proposition 2. Finally, given the lack of choice data that might indicate a preference between such options, there is no way for an outside observer to know that the agent can in fact rank them in the first place. Proposition 2 merely states that such a ranking is *possible*.

It is worth emphasising that the option x^* which sets the agent's desirability threshold will not be feasible to him in general but influences his behaviour even in decision problems where it is not. In this regard the DCR model is similar to the Tversky and Kahneman (1991) model of reference-dependent preferences with loss aversion, where an option can act as the agent's reference point even if it is infeasible.

A slightly more compact way in which the DCR model can be written is that the agent has a utility function u on X and a desirability threshold captured by some alternative x^*, and that his behaviour at menu A is such that

$$C(A) = \begin{cases} \arg\max_{x \in A} u(x), & \text{if } \max_{x \in A} u(x) > u(x^*) \\ \varnothing, & \textit{otherwise} \end{cases}$$

We conclude this section by noting that an additional benefit from identifying the behavioural axioms that characterise undesirability-driven *rational* choice deferral is that this identification can serve as the benchmark for other models where deferral is caused by *context-dependent* undesirability. For example, Dhar and Sherman (1996) provided experimental evidence suggesting that the same two alternatives can induce deferral due to unattractiveness in one context (e.g. when their unique "bad" features are emphasised) but not in another (e.g. when their unique "good" features are emphasised). Such behaviour is generally incompatible not only with Undesirability and Contractive Undesirability, but also with Contraction Consistency.

4 Overload-Constrained Rationality

The anecdote below illustrates nicely a special case of the behaviour that we are interested in explaining in this section, as well as the mechanism through which our proposed model does so:

> *"As a neophyte shoe salesman, I was told never to show customers more than three pairs of shoes. If they saw more, they would not be able to decide on any of them."*
>
> New York Times, 26 January 2004[15]

In line with the substance of the preceding quote and related experimental findings, in our last model the agent defers in decision problems that he considers to be complex. As in Iyengar and Lepper (2000), we will refer to this third source of deferral as *choice overload*. In a previous section we argued that the MDC model of indecisiveness-driven deferral is compatible with one of the suggested explanations for this phenomenon. Here we propose a different explanation that builds on a model of utility maximisation that is constrained by *complexity thresholds*.

The first additional axiom that we introduce here is standard.

Binary Choice Consistency
If $x \in C(\{x, y\})$ and $y \in C(\{y, z\})$, then $x \in C(\{x, z\})$.

Similar to our preceding discussion of WARP, transitivity of binary choices may be considered even more appealing in deferral-permitting decision environments for individuals who aim to behave rationally in the sense that, when they do make active choices, these reflect maximisation of a transitive preference relation. In some models

[15] Letter to the editor of the New York Times (p.A22) by Milton Waxman, originally quoted in Kuksov and Villas-Boas (2010).

where the revealed weak preference relation is defined by choices in binary menus, this axiom ensures transitivity of that relation. In the model below it has a slightly more intricate role that will become clear shortly.

Deferral Monotonicity
If $B \supset A \neq \varnothing$ and $C(A) = \varnothing$, then $C(B) = \varnothing$.

To our knowledge, this axiom is novel. It suggests that if the individual finds a menu A to be complex, then he finds every menu $B \supset A$ to be complex as well, so that overload-driven deferral is monotonic with respect to set inclusion. While the intuitive appeal of this axiom is obvious at one level, we note that it can fail descriptively when insertion of a clearly superior alternative in a complex menu may override the complexity of the decision problem and nudge the individual towards that alternative. On the other hand, one may argue that the axiom is "normatively appealing" for a cognitive- or resource-constrained decision maker on the grounds that the complexity criterion that such an individual may employ incorporates a break-even cost-benefit analysis, so that once a menu is complex according to this criterion, then any menu that includes it must be at least as complex even if it contains stand-out options, possibly because their discovery may be too costly.

Proposition 3
The following are equivalent for a choice correspondence $C : \mathcal{M} \to \mathcal{M}$:
1. C satisfies Desirability, WARP, Deferral Monotonicity and Binary Choice Consistency.
2. There exist a unique preorder $\trianglerighteq$ on X, a completion $\succsim$ of this preorder, a function $\psi : \mathcal{M} \to \mathbb{R}$ and an integer n such that, for all $A, B \in \mathcal{M}$ and all $x, y, z \in X$

$$C(A) = \begin{cases} \varnothing, & \textit{iff } \psi(A) > n \\ B_{\succsim}(A), & \textit{otherwise} \end{cases} \tag{3a}$$

$$|A| = 1 \implies \psi(A) \leq n \tag{3b}$$

$$B \supset A \implies \psi(B) \geq \psi(A) \tag{3c}$$

$$\psi(\{x,y\}) \leq n \;\; \& \;\; \psi(\{y,z\}) \leq n \implies \psi(\{x,z\}) \leq n. \tag{3d}$$

The decision maker here is portrayed as a utility maximiser who is non-standard in that he employs a complexity/overload criterion that determines whether he will engage in utility maximisation at some menu or whether he will *defer* at that menu instead. It will therefore be referred to as the model of *overload-constrained rationality* (OCR). Specifically, the individual's complexity criterion here is captured by the *complexity function* ψ and *complexity threshold* n. These may correspond to the number of elements in a menu and a cut-off menu size, respectively, or to the time it takes the decision maker to make a choice from a menu and the total time he has available. Whichever the case, (3a) suggests that the agent defers when and only when the decision problem is sufficiently complex according to the pair (ψ, n).

It follows directly from (3c) that the complexity function is increasing with respect to menu inclusion, which is a consequence of the Deferral Monotonicity axiom. It also follows from (3a) and (3b) that a decision problem consisting of a singleton menu is never complex. Finally, (3d) suggests that if the binary menus $\{x, y\}$ and $\{y, z\}$ are not complex, then the same will be true for menu $\{x, z\}$. When ψ is the cardinality function, for example, then non-complexity of one binary menu implies that all such menus must also be non-complex, consistent with (3d). Other interpretations of ψ are also compatible with this axiom. For instance, if $\psi(A)$ counts the number of seconds that the decision maker believes are required for him to go through each alternative in A and n stands for the number of seconds that he is willing to devote to this task, then (3d) is again satisfied.

The OCR model is very much in the spirit of Herbert Simon's research programme on the incorporation of bounded rationality into mainstream economics, particularly in relation to his call for formally acknowledging the role of humans' limited computational and information-processing abilities in models of individual decision making.[16] The specific way in which the OCR model does so is also very close conceptually to the main conclusions in Miller's (1956) work, which has been very influential among psychologists. Miller's experiments built on one-dimensional stimuli and suggested specific numerical bounds on people's short-term memory, hence also on their capacity to process information.

Unlike the models of indecisiveness- and undesirability-driven deferral that were presented above and where the agent's preferences are fully and almost fully recoverable from decision data, respectively, in the OCR model such recovery is only partially possible. Indeed, behaviour compatible with the four axioms of Proposition 3 uniquely pin down a generally incomplete preference relation $\trianglerighteq$ which is defined by the rule $x \trianglerighteq y$ iff $x \in C(A)$ and $y \in A$ for some menu A. Given the complexity function ψ and its associated threshold n there may be alternatives x and y for which $\psi(\{x, y\}) > n$ and $x, y \notin C(A)$ for every menu A with $\psi(A) \leq n$. For such alternatives it holds that $x \not\trianglerighteq y$ and $y \not\trianglerighteq x$. While the model is well-defined in terms of a completion of this preorder, since completions are generally not unique this fact restricts the extent to which preferences can be recovered in the OCR model.

In the special case where the complexity function ψ captures the number of alternatives in a menu, i.e. where $\psi(A) = |A|$ for all A, the model suggests a second explanation of the choice overload phenomenon that was first discussed in Section 2. Specifically, in this case the model's interpretation is that the decision maker always counts the number of alternatives in a menu and if this exceeds his pre-specified menu-size threshold, he defers. This prediction is compatible with the choice-overload findings in Iyengar and Lepper (2000). Moreover, minimisation of cognitive effort of the kind that is featured in the OCR model has been recognised as one of the four main drivers of choice overload in the very extensive recent meta-analysis of this phenomenon that was carried out by Chernev *et al.* (2015). On the other hand, unlike the explanation that is suggested by the MDC model, OCR cannot account for the observed breakdown of choice overload when a clearly dominant option is added to a menu that is considered complex.

As a corollary to Proposition 3 which may facilitate the use of the OCR model in applications we note that the stated four axioms characterise the existence of a utility function u on X, a complexity function ψ on $\mathcal{M}$ that satisfies (3c)–(3d), and a number n (where both u and the pair (ψ, n) are unique up to strictly increasing

[16]See Simon (1957), for example.

transformations) such that, for all $A \in \mathcal{M}$,

$$C(A) = \begin{cases} \emptyset, & \textit{iff } \psi(A) > n \\ \\ \arg\max_{x \in A} u(x), & \textit{otherwise} \end{cases} \tag{4}$$

Indeed, once the preference relation $\trianglerighteq$ is completed by some weak order $\succsim$, the latter is representable by means of an ordinally unique utility function u, and, for any given pair (ψ, n), (4) holds regardless of which u is chosen to represent $\succsim$. Similarly, for any choice of u that represents the given completion $\succsim$ of $\trianglerighteq$, and any initial pair (ψ, n), applying a strictly increasing transformation ϕ on the pair (ψ, n) results in the pair $(\phi, \phi(n))$ under which (4) still holds but for a different complexity scale.

5 Between the Three Models

In all three models the decision maker's choices satisfy WARP, Contraction Consistency and Binary Choice Consistency. In particular, since OCR predicts $C(B) \neq \emptyset$ whenever $C(A) \neq \emptyset$ and $B \subset A$, Contraction Consistency is implied by WARP. Compatibility of MDC with Binary Choice Consistency is also obvious in view of the fact that preferences in this model are transitive. To see that this axiom is also compatible with DCR, observe that $x \in C(\{x, y\})$ and $y \in C(\{y, z\})$ here suggest that both x and y are above the desirability threshold. Moreover, since preferences are transitive and the "desirable" option x must be preferred to z, we also have $x \in C(\{x, z\})$, as required.

The Desirability axiom is clearly satisfied in MDC and OCR (in the case of the latter this is due to (3b)). Its logical complement, Undesirability, is satisfied by DCR only. It is intuitive that Contractive Undesirability too is satisfied only by DCR. Indeed, moving from A where $C(A) = \emptyset$ to $B \subset A$ in MDC can render some option in B a most preferred one in that menu, and may therefore result in $C(B) \neq \emptyset$. In OCR one may have $\psi(A) > n$ but $\psi(B) \leq n$, in which case too one obtains $C(B) \neq \emptyset$.

Like MDC, the DCR model is compatible with Strong Expansion. The logic behind this is analogous to the logic that shows conformity of this model with Binary Choice Consistency. On the other hand, OCR may lead to violations of Strong Expansion. For example, if $\psi(\cdot) = |\cdot|$ and $n = 2$ we would have $x \in C(\{x, y\})$, $y \in C(\{x, z\})$ and $C(\{x, y, z\}) = \emptyset$. Finally, neither MDC nor DCR obey Deferral Monotonicity in general. In the former case this can happen when $C(A) = \emptyset$ and the menu $B \supset A$ includes an option $x \in B \setminus A$ that is weakly preferred to all other options in B. In the latter case we have $C(A) = \emptyset$ when A consists of "undesirable" options, and $C(B) \neq \emptyset$ for every $B \supset A$ that contains at least one "desirable" alternative.

Table 1 summarises the axiomatic similarities and differences between the three proposed models of rational choice deferral by checking conformity of each model with each of the axioms that were presented above.

Although the two models are generally distinct, we note that there are non-trivial datasets which are explainable by both MDC and OCR. For example, consider the case where $X = \{w, x, y, z\}$ and assume that choices are made from all singletons. Suppose now that $C(\{w, x\}) = \{w\}$, $C(\{y, z\}) = \{y\}$ and $C(A) = \emptyset$ for all remaining menus $A \subseteq X$. The MDC model is compatible with these choices when the preorder $\succsim$ on X is

Table 1: Axiom Compatibility

	Indecisiveness (MDC)	Unattractiveness (DCR)	Overload (OCR)
WARP	✓	✓	✓
Binary Choice Consistency	✓	✓	✓
Contraction Consistency	✓	✓	✓
Strong Expansion	✓	✓	×
Desirability	✓	×	✓
Undesirability	×	✓	×
Contractive Undesirability	×	✓	×
Deferral Monotonicity	×	×	✓

such that $w \succ x$, $y \succ z$ and no other comparisons between distinct alternatives are possible. Yet, the OCR model is also compatible with them when: i) the complexity threshold is $n = 1$; ii) the complexity function is $\psi(\{w\}) = \psi(\{x\}) = \psi(\{y\}) = \psi(\{z\}) = \psi(\{w,x\}) = \psi(\{y,z\}) = 1$ and $\psi(A) = 2$ for all other $A \subseteq X$; iii) $\succsim$ is any weak order on X such that $w \succ x$ and $y \succ z$.

On the other hand, suppose Desirability is satisfied and $C(A) = \{w\}$ whenever $w \in A$ while $C(A) = \emptyset$ for every other non-singleton menu A. Then, the MDC model explains the dataset by means of the preorder $\succsim$ where $w \succ x, y, z$ and no other comparisons between distinct options are possible, while the OCR model is incompatible with it. Conversely, suppose choices are made at singletons and also that $C(\{w,x\}) = C(\{w,y\}) = C(\{w,z\}) = \{w\}$, $C(\{x,y\}) = C(\{x,z\}) = \{x\}$, $C(\{y,z\}) = y$ and $C(A) = \emptyset$ for every other $A \neq \emptyset$. These choices are incompatible with MDC but are explained by the the OCR model when $\psi(\cdot) \equiv |\cdot|$ and $n = 2$.

The DCR model is not compatible with the choices in the preceding examples because the Desirability axiom was satisfied in all of them. Yet, the following example presents choices that violate this axiom and are compatible with this model only. Suppose, indeed, that $X = \{w,x,y,z\}$ and $C(\{w\}) = \emptyset = C(\{x\})$, $C(\{y\}) = \{y\}$, $C(\{z\}) = \{z\}$, $C(\{w,x\}) = \emptyset$, $C(\{w,y\}) = \{y\} = C(\{x,y\})$, $C(\{w,z\}) = \{z\} = C(\{x,z\})$, $C(\{y,z\}) = \{z\}$, $C(\{w,x,z\}) = C(\{w,y,z\}) = C(\{x,y,z\}) = \{z\}$, $C(\{w,x,y\}) = \{y\}$ and $C(X) = \{z\}$. The DCR model explains these choices, for example when preferences are such that $z \succ y \succ x \succsim w$ and $x^* = x$.

6 Does the Approach to Modelling Deferral Matter?

We now raise the question of whether the adoption of a possibly empty-valued choice correspondence is essential for modelling rational choice deferral in the context of the three models that we analysed above. In particular, we compare this with the approach where deferral is modelled by an explicit and always feasible option. To this end, suppose now that a decision problem comprises a menu $A \in \mathcal{M}$ together with an explicit deferral/outside option d that is also an element of the grand choice set X. Moreover, assume that the collection of all decision problems now becomes $\mathcal{M}^* := \{A \cup \{d\} : A \in \mathcal{M}\}$. Also, let a choice correspondence here be a mapping C^* from $\mathcal{M}^*$ into itself.

Let us first consider the effects of this alternative modelling approach on the MDC model. Modifying (1) along these lines results in the choice procedure whereby, for all $A \in \mathcal{M}$,

$$C^*(A \cup \{d\}) = \begin{cases} \{d\}, & \textit{iff } B_{\succsim}(A \cup \{d\}) = \emptyset \\ \\ B_{\succsim}(A \cup \{d\}), & \textit{otherwise} \end{cases} \tag{5}$$

where $\succsim$ is a possibly incomplete preorder on X. In order to bring (5) as close as possible to the MDC model which explains deferral when the default option associated with it may be thought of as being objectively inferior to all market alternatives (subject to the caveats discussed in Section 2) but may still be chosen when some of these superior alternatives are incomparable, one may assume that $x \succ d$ for all $x \in X \setminus \{d\}$. Unlike the original MDC model, (5) predicts WARP violations whenever the agent defers at a problem $A \cup \{d\}$ in which A contains at least two options. Indeed, $\{x\} = C^*(\{x\} \cup \{d\})$ for all $x \in X$ and $\{d\} = C^*(\{x, y\} \cup \{d\})$ whenever $x \not\succsim y$ and $y \not\succsim x$.[17]

Modifying the OCR model to also make it compatible with this alternative approach, and assuming that $\succsim$ is a weak order such that $x \succ d$ for all $x \neq d$, we obtain the choice procedure whereby for all $A \in \mathcal{M}$,

$$C^*(A \cup \{d\}) = \begin{cases} \{d\}, & \textit{iff } \psi(A \cup \{d\}) > n \\ \\ B_{\succsim}(A \cup \{d\}), & \textit{otherwise} \end{cases} \tag{6}$$

Similar to (5) but unlike the original OCR model that was laid out in (3), this procedure necessarily leads to WARP violations whenever d is chosen in some decision problem $A \cup \{d\}$.

Finally, modifying the DCR model along the same lines we obtain

$$C^*(A \cup \{d\}) = \begin{cases} \{d\}, & \textit{iff } B_{\succsim}(A \cup \{d\}) = \{d\} \\ \\ B_{\succsim}(A \cup \{d\}), & \textit{otherwise} \end{cases} \tag{7}$$

for all $A \in \mathcal{M}$. Unlike (5) and (6), and consistent with the motivation for the DCR model, the deferral option here is not necessarily inferior to everything else, while it naturally takes the role of the desirability threshold x^* in (2). For these reasons, and unlike the MDC and OCR procedures, no WARP violations are induced by (7). This fact makes DCR robust with respect to the way in which deferral is modelled.

Given the MDC and OCR models' sensitivity to the approach with which deferral is modelled when it comes to discussing WARP consistency, and given the distinction between "decision" and "choice" that we drew in Section 1 to highlight the WARP-consistent (active) choices that are predicted by these models, it is natural to ask which of the empty-set and d-option approach is more appropriate. While the question is to a certain extent philosophical, we provide four arguments in support of our adoption of the empty-set approach that are driven by practical, intuitive, empirical and methodological considerations, respectively:

1. The very sizeable and most interesting behavioural component that involves active choices between market options is in fact fully rational in both models, and, unlike the empty-set approach, the d-option approach

[17] We note that this choice procedure is distinct from the standard incompleteness-based explanation of status quo bias that goes back to Bewley (2002), according to which the status quo option is chosen if it is merely undominated.

to deferral prevents this from being transparent to the analyst. In particular, by adopting the latter approach one would be hiding this fully consistent behavioural component "under the rug", which in turn would also make the task of recovering the underlying preference ordering over market alternatives harder than it actually is.

2. If an indecisive individual's three decisions from menus $\{x\}$, $\{y\}$ and $\{x, y\}$ were as in the quote that opened Section 2 (i.e. choice at singletons and deferral at the binary menu), the d-set approach to the MDC model would number two WARP violations while the empty-set approach none. If the individual always conforms with what is essentially the most demanding generalisation of the utility-maximisation principle in this context, is it intuitive to "charge" him with two counts of inconsistency?

3. As we explain below, the empty-set approach allows for a straightforward extension of the well-known and widely applied Houtman-Maks (1985) method of empirically analysing the degree of consistency in a given choice dataset. This generalisation of the method that also covers cases of *choice and deferral* datasets is impossible with the d-option approach.

4. The primitive definition of a choice correspondence requires $C(A)$ to be a subset of A for every menu A, and hence encompasses the case where $C(A)$ is empty. Building on this simple fact, our analysis shows that the basic analytical tools of revealed preference theory are flexible enough to formally accommodate the viewpoint that certain sources of deferral can be thought of as being compatible with preference-maximising and WARP-consistent choice.

Turning to the relation between this alternative approach to modelling deferral and existing models where the decision problem is assumed to feature an explicit status quo option, the one closest to (5) is Dean's (2008) *Conflict Decision Avoidance* model. This also builds on an incomplete preference relation $\succsim$ over the set X when the latter includes the deferral option (in that model, $\diamond$ stands for d). Like MDC, the CDA agent chooses a most preferred option from a menu A if one exists. If such an option does not exist, however, then d is chosen if and only if it is dominated by nothing else in the menu according to a *secondary* preference criterion (which is captured by a choice correspondence T on $\mathcal{M}^*$) and the requirement that $d \in A \setminus T(A)$. If this is not the case either, then an option that maximises a *completion* of his primary preferences $\succsim$ is chosen. Importantly, the formulation of the MDC procedure that is captured by (5) differs from the CDA model by relying on a single and uniquely recoverable preference relation $\succsim$. Hence, it does not necessitate the introduction of a secondary criterion. Moreover, with the goal of explaining behaviour of the kind described in the quote that opened Section 2, this version of the MDC model assumes that, from the perspective of static decision making, deferral is inferior to all options. Therefore, unlike CDA, it does not require a mechanism through which completion of the agent's primary preferences takes place and choice of some non-deferral option is made.

7 Applicability and Relevance

7.1 Empirical Analysis of Datasets with Deferral Observations

A frequently-employed criterion in empirical studies that assess the rationality of a given choice dataset is the so-called Houtman and Maks (1985) index.[18] This counts the minimum number of changes that need to be made in a given dataset for it to be as if it was generated by maximisation of some weak order.[19] A generalisation of this rationality index that makes it particularly relevant and useful in cases where datasets include deferral observations is proposed and applied in Costa-Gomes *et al.* (2016). The generalised index counts the minimum number of changes that need to be made for the dataset to be as if it was generated by maximisation of a possibly incomplete preorder. The MDC model provides a choice-theoretic foundation for this generalisation of the Houtman-Maks index in the sense that the number of required changes in a given dataset is zero if and only if the dataset conforms perfectly with MDC. Costa-Gomes *et al.* (2016) also propose and apply an analogous extension of the Houtman-Maks index that can detect the degree of proximity of a given dataset with the DCR model of undesirability-constrained utility maximisation. More generally, Table 1 provides guidance on the kinds of axiomatic combinations one needs to look out for in datasets with choice deferral observations in order to check their degree of conformity with the above three models, and also to distinguish between them.

7.2 Choice under Uncertainty

Bewley's (2002) model of "objectively rational" preferences under uncertainty is based on a possibly incomplete preference relation and a set of priors over the states of the world such that one act is preferred to another if and only if its expected utility is higher under every prior in the set. In Appendix B we show how a two-period extension of the MDC model can be used to provide choice-theoretic foundations for the Bewley model where any first-period deferral that is a result of applying the above decision rule is resolved in the second period when the agent becomes a subjective expected utility maximiser by narrowing down his beliefs to a single prior.

7.3 Robust Social Choice

The possibility of a social welfare function that produces transitive but generally incomplete societal preferences when the choice domain contains general, risky and uncertain objects has been investigated in Fishburn (1974), Danan *et al.* (2015) and Danan *et al.* (2016), respectively. This approach allows for analysing *robust* social decisions in the sense that the society's incomplete preference ranking can be made to agree with *every* agent's preferences, including (where relevant), their beliefs. In such contexts, if one insists that the social ranking satisfy this notion of Pareto efficiency, the transition from social welfare to social *choice* functions or correspondences comes about in the way dictated by the MDC model: Given a preference profile, the social planner chooses a social alternative at some menu if and only if this is weakly preferred to every alternative in the menu according to society's preferences (hence according to every agent's preferences), and defers otherwise. A real-world example of high-level collective

[18] See, for instance, Choi *et al.* (2007) and Choi *et al.* (2014).

[19] Apesteguia and Ballester (2015) propose an alternative rationality index and also provide a detailed comparative discussion of this and other rationality criteria, including Houtman-Maks's.

decision making of this kind is the European Council's unanimity requirement, which currently applies to a variety of important issues (e.g. the common foreign and security policy).[20]

7.4 Industrial Organisation

A recent direction taken in behavioural industrial organisation is motivated by the presence of potentially choice-deferring individuals in the consumer population. Specifically, in the "opt-in" version of the oligopolistic model introduced in Bachi and Spiegler (2015)[21] where consumers are not endowed with some alternative, they are assumed to defer whenever they are unable to detect a preference-dominant item in the set of two-attribute products offered by the firms. The behaviour of these consumers coincides with the one predicted by the MDC model. In addition, motivated by the interesting trade-off between increasing and decreasing the variety of the products that firms offer when they are selling to a consumer population that is heterogeneous both in terms of their preferences and their overload thresholds, Gerasimou and Papi (2016) introduce a duopolistic model where otherwise rational consumers are overloaded as in the special case of the OCR procedure where menu complexity is identified with cardinality.

8 Related Literature

In addition to Dean's (2008) CDA model, the MDC decision rule shares features with the *Extended Partial Dominance* (EPD) procedure (Gerasimou, 2016). In all three models the agent has incomplete preferences, but even when one compares their predictions in decision problems that belong to the same class, these generally differ even when preferences coincide. The similarities and differences between MDC and CDA were discussed in Section 6. The EPD model allows for the possibility of WARP-inconsistent active choices that are driven by the joint criteria of an alternative being totally undominated *and* partially dominant according to a single acyclic preference relation. Unlike MDC, this model is compatible with both choice deferral and status quo bias, and provides a formal distinction between them. The context-dependent nature of this model, however, results in WARP-inconsistent choices, which are ruled out by MDC.

Frick (2016) proposed the *monotone threshold representation* (MTR) model which is similar to OCR in that both feature a menu function (called the *departure threshold* in MTR) which is weakly increasing with respect to set inclusion. The MTR model predicts increasingly inconsistent behaviour as the menu size increases but, unlike OCR, does not allow for choice deferral. The two models are not nested. Indeed, unless it satisfies Nonemptiness (in which case it reduces to rational choice), OCR violates Frick's *Occasional Optimality* axiom, which weakens WARP but implies Nonemptiness. On the other hand, MTR generally violates WARP.

Dean (2008) and Dean *et al.* (2014) explain the incidence of status quo bias (but not choice deferral) in "large" menus.[22] The former paper does so by means of the *information overload* model which generalises the CDA model mentioned above and predicts that the agent's preferences are less complete in larger menus. The latter paper attributes these instances of status quo bias to the agent's limited attention, which results in a subset of the original

[20] See http://www.consilium.europa.eu/en/council-eu/voting-system/unanimity/ for more details.
[21] See also Spiegler (2015).
[22] The strengthening of status quo bias in large menus was first documented in Samuelson and Zeckhauser (1988).

menu ultimately being considered.

In the menu-preference literature, Sarver (2008) and Ortoleva (2013) provide preference representation theorems in the domain of menus of lotteries where individuals are modelled as having a preference for smaller menus due to regret anticipation and thinking aversion, respectively. Buturak and Evren (2016) offer a choice-theoretic foundation of Sarver's model that permits choice deferral. In stochastic choice, Fudenberg and Strzalecki (2015) generalise the dynamic logit model in a way that allows the decision maker to have a preference for smaller menus.

We also note that the DCR and OCR models can be embedded in the "choice with frames" environment of Salant and Rubinstein (2008) once the latter is suitably generalised to allow for choice deferral. Specifically, letting a decision problem be described by a pair (A, f) where A is a menu and f is a frame that influences decision at A, as these authors proposed, the DCR model would suggest that f is constant across all problems and coincides with the agent's desirability threshold x^*. The OCR model would also suggest that f is constant across all problems. Yet, here f would coincide with the decision maker's "complexity" pair (ψ, n) instead. It is not clear, however, if the MDC model can also be embedded in this framework.

Finally, we emphasise that there are many potential sources of choice deferral for which we have not accounted in this paper. Some of those studied by economists include consumer search, dynamically inconsistent preferences and procrastination (O'Donoghue and Rabin, 1999), procedural reasons such as choice refusal considerations (Gaertner and Xu, 2004), contextual inference and the informational content of a menu (Kamenica, 2008), limited attention (Manzini and Mariotti, 2014) and, as already noted, rational regret anticipation (Buturak and Evren, 2016). The reader is referred to the cited works for more details on these alternative approaches.

School of Economics & Finance, University of St Andrews

References

Anderson, C.J. (2003). 'The psychology of doing nothing: Forms of decision avoidance result from reason and emotion', *Psychological Bulletin*, vol. 129, pp. 139–167.

Apesteguia, J. and Ballester, M. (2009). 'A theory of reference-dependent behavior', *Economic Theory*, vol. 40(3), pp. 427–455.

Apesteguia, J. and Ballester, M. (2013). 'Choice by sequential procedures', *Games and Economic Behavior*, vol. 77(1), pp. 90–99.

Apesteguia, J. and Ballester, M. (2015). 'A measure of rationality and welfare', *Journal of Political Economy*, (6), pp. 1278–1310.

Arrow, K.J. (1959). 'Rational choice functions and orderings', *Economica*, vol. 26(102), pp. 121–127.

Augenblick, N. and Nicholson, S. (2016). 'Ballot position, choice fatigue and voter behaviour', *Review of Economic Studies*, vol. 83(2), pp. 460–480.

Bachi, B. and Spiegler, R. (2015). 'Buridanic competition', *mimeo*.

Bernheim, B.D. and Rangel, A. (2009). 'Beyond revealed preference: Choice-theoretic foundations for behavioral welfare economics', *Quarterly Journal of Economics*, vol. 124(1), pp. 51–104.

Bewley, T.F. (2002). 'Knightian decision theory. Part I', *Decisions in Economics and Finance*, vol. 25(2), pp. 79–110.

Bhatia, S. and Mullett, T.L. (2016). 'The dynamics of deferred decision', *Cognitive Psychology*, vol. 86, pp. 112–151.

Buturak, G. and Evren, O. (2016). 'Choice overload and asymmetric regret', *Theoretical Economics*, forthcoming.

Chernev, A., Böckenholt, U. and Goodman, J. (2015). 'Choice overload: A conceptual review and meta-analysis', *Journal of Consumer Psychology*, vol. 25(2), pp. 333–358.

Choi, S., Fisman, R., Gale, D. and Kariv, S. (2007). 'Consistency and heterogeneity of individual behavior under uncertainty', *American Economic Review*, vol. 97(5), pp. 1921–1938.

Choi, S., Kariv, S., Müller, W. and Silverman, D. (2014). 'Who is (more) rational?', *American Economic Review*, vol. 104(6), pp. 1518–1550.

Clark, S.A. (1995). 'Indecisive choice theory', *Mathematical Social Sciences*, vol. 30(2), pp. 155–170.

Costa-Gomes, M., Cueva, C., Gerasimou, G. and Tejiscak, M. (2016). 'Choice, deferral and consistency', *mimeo*.

Danan, E. (2010). 'Randomization vs selection: How to choose in the absence of preference?', *Management Science*, vol. 56(3), pp. 503–518.

Danan, E., Gajdos, T., Hill, B. and Tallon, J.M. (2016). 'Robust social decisions', *American Economic Review*, vol. 106(9), pp. 2407–2425.

Danan, E., Gajdos, T. and Tallon, J.M. (2015). 'Harsanyi's aggregation theorem with incomplete preferences', *American Economic Journal: Microeconomics*, vol. 7(1), pp. 61–69.

Danan, E. and Ziegelmeyer, A. (2006). 'Are preferences complete? An experimental measurement of indecisiveness under risk', *mimeo*.

Dean, M. (2008). 'Status quo bias in large and small choice sets', *mimeo*.

Dean, M., Kibris, O. and Masatlioglu, Y. (2014). 'Limited attention and status quo bias', *mimeo*.

Dhar, R. (1997). 'Consumer preference for a no-choice option', *Journal of Consumer Research*, vol. 24(2), pp. 215–231.

Dhar, R. (1999). 'Choice deferral', in (P. E. Earl and S. Kemp, eds.), *The Elgar Companion to Consumer Research and Economic Psychology*, Cheltenham: Edward Elgar Publishing.

Dhar, R. and Sherman, S.J. (1996). 'The effect of common and unique features in consumer choice', *Journal of Consumer Research*, vol. 23(3), pp. 193–203.

Dhar, R. and Simonson, I. (2003). 'The effect of forced choice on choice', *Journal of Marketing Research*, vol. 40(2), pp. 146–160.

Eliaz, K. and Ok, E.A. (2006). 'Indifference or indecisiveness? Choice-theoretic foundations of incomplete preferences', *Games and Economic Behavior*, vol. 56(1), pp. 61–86.

Fishburn, P.C. (1974). 'Impossibility theorems without the social completeness axiom', *Econometrica*, vol. 42(4), pp. 695–704.

Frick, M. (2016). 'Monotone threshold representations', *Theoretical Economics*, vol. 11(3), pp. 757–772.

Fudenberg, D. and Strzalecki, T. (2015). 'Dynamic logit with choice aversion', *Econometrica*, vol. 83(2), pp. 651–691.

Gaertner, W. and Xu, Y. (2004). 'Procedural choice', *Economic Theory*, vol. 24(2), pp. 335–349.

Gerasimou, G. (2016). 'Asymmetric dominance, deferral and status quo bias in a behavioral model of choice', *Theory and Decision*, vol. 80(2), pp. 295–312.

Gerasimou, G. and Papi, M. (2016). 'Oligopolistic competition with choice-overloaded consumers', *mimeo.*

Gilboa, I., Maccheroni, F., Marinacci, M. and Schmeidler, D. (2010). 'Objective and subjective rationality in a multiple prior model', *Econometrica*, vol. 78(2), pp. 755–770.

Greenleaf, E.A. and Lehmann, D.R. (1995). 'Reasons for substantial delay in consumer decision making', *Journal of Consumer Research*, vol. 22(2), pp. 186–199.

Houtman, M. and Maks, J.A. (1985). 'Determining all maximal data subsets consistent with revealed preference', *Kwantitatieve Methoden*, vol. 19(1), pp. 89–104.

Hurwicz, L. (1986). 'On the implementation of social choice rules in irrational societies', in (W. P. Heller, R. M. Starr and D. A. Starett, eds.), *Essays in Honor of Kenneth J. Arrow*, pp. 75–96, vol. I, chap. 4, Cambridge: Cambridge University Press.

Iyengar, S.S., Huberman, G. and Jiang, W. (2004). 'How much choice is too much? Contributions to 401(k) retirement plans', in (O. S. Mitchell and S. P. Utkus, eds.), *Pension Design and Structure: New Lessons from Behavioral Finance*, pp. 83–96, chap. 5, New York: Oxford University Press.

Iyengar, S.S. and Lepper, M.R. (2000). 'When choice is demotivating: Can one desire too much of a good thing?', *Journal of Personality and Social Psychology*, vol. 79(6), pp. 995–1006.

Kamenica, E. (2008). 'Contextual inference in markets: On the informational content of product lines', *American Economic Review*, vol. 98(5), pp. 2127–2149.

Knetsch, J.L. (1989). 'The endowment effect and evidence of nonreversible indifference curves', *American Economic Review*, vol. 79(5), pp. 1277–1284.

Kopylov, I. (2009). 'Choice deferral and ambiguity aversion', *Theoretical Economics*, vol. 4, pp. 199–225.

Kreps, D.M. (1979). 'A representation theorem for 'preference for flexibility'', *Econometrica*, vol. 47, pp. 565–577.

Kreps, D.M. (2012). *Microeconomic Foundations I: Choice and Competitive Markets*, Princeton NJ: Princeton University Press.

Krijnen, J.M.T., Zeelenberg, M. and Breugelmans, S.M. (2015). 'Decision importance as a cue for deferral', *Judgement and Decision Making*, vol. 10(5), pp. 407–415.

Kuksov, D. and Villas-Boas, J.M. (2010). 'When more alternatives lead to less choice', *Marketing Science*, vol. 29(3), pp. 507–524.

Luce, M.F. (1998). 'Choosing to avoid: Coping with negatively emotion-laden consumer decisions', *Journal of Consumer Research*, vol. 24(4), pp. 409–433.

Madrian, B.C. and Shea, D.F. (2001). 'The power of suggestion: Inertia in 401(k) participation and savings behavior', *Quarterly Journal of Economics*, vol. 116(4), pp. 1149–1187.

Mandler, M. (2004). 'Status quo maintenance reconsidered: Changing or incomplete preferences?', *Economic Journal*, vol. 114(499), pp. F518–F535.

Mandler, M. (2009). 'Indifference and incompleteness distinguished by rational trade', *Games and Economic Behavior*, vol. 67(1), pp. 300–314.

Manzini, P. and Mariotti, M. (2014). 'Stochastic choice and consideration sets', *Econometrica*, vol. 83(3), pp. 1153–1176.

Masatlioglu, Y. and Ok, E.A. (2005). 'Rational choice with status quo bias', *Journal of Economic Theory*, vol. 121(1), pp. 1–29.

Masatlioglu, Y. and Ok, E.A. (2015). 'A canonical model of choice with initial endowments', *Review of Economic Studies*, vol. 81(2), pp. 851–883.

Miller, G.A. (1956). 'The magical number seven, plus or minus two: Some limits on our capacity for processing information', *Psychological Review*, vol. 63(2), pp. 81–97.

Mochon, D. (2013). 'Single-option aversion', *Journal of Consumer Research*, vol. 40(3), pp. 555–566.

Munro, A. and Sugden, R. (2003). 'On the theory of reference-dependent preferences', *Journal of Economic Behavior & Organization*, vol. 50(4), pp. 407–428.

O'Donoghue, T. and Rabin, M. (1999). 'Doing it now or later', *American Economic Review*, vol. 89(1), pp. 103–124.

Ortoleva, P. (2010). 'Status quo bias, multiple priors and uncertainty aversion', *Games and Economic Behavior*, vol. 69(2), pp. 411–424.

Ortoleva, P. (2013). 'The price of flexibility: Towards a theory of thinking aversion', *Journal of Economic Theory*, vol. 148(3), pp. 903–934.

Pejsachowicz, L. and Toussaert, S. (2015). 'Choice deferral, indecisiveness and preference for flexibility', *mimeo*.

Redelmeier, D.A. and Shafir, E. (1995). 'Medical decision making in situations that offer multiple alternatives', *Journal of the American Medical Association*, vol. 273(4), pp. 302–305.

Richter, M.K. (1966). 'Revealed preference theory', *Econometrica*, vol. 34(3), pp. 635–645.

Salant, Y. and Rubinstein, A. (2008). '(A, f): Choice with frames', *Review of Economic Studies*, vol. 75(4), pp. 1287–1296.

Samuelson, P.A. (1938). 'A note on the pure theory of consumer's behaviour', *Economica*, vol. 5(17), pp. 61–71.

Samuelson, W. and Zeckhauser, R. (1988). 'Status quo bias in decision making', *Journal of Risk and Uncertainty*, vol. 1(1), pp. 7–59.

Sarver, T. (2008). 'Anticipating regret: Why fewer options may be better', *Econometrica*, vol. 76(2), pp. 263–305.

Sawers, K.M. (2005). 'Evidence of choice avoidance in capital-investment judgements', *Contemporary Accounting Research*, vol. 22(4), pp. 1063–1092.

Scheibenhenne, B., Greifeneder, R. and Todd, P.M. (2010). 'Can there ever be too many options? A meta-analytic review of choice overload', *Journal of Consumer Research*, vol. 37(3), pp. 409–425.

Sen, A. (1971). 'Choice functions and revealed preference', *Review of Economic Studies*, vol. 38(3), pp. 307–317.

Sen, A. (1997). 'Maximization and the act of choice', *Econometrica*, vol. 65(4), pp. 745–779.

Shafir, E., Simonson, I. and Tversky, A. (1993). 'Reason-based choice', *Cognition*, vol. 11(1-2), pp. 11–36.

Simon, H. (1957). *Models of Man*, New York: Wiley.

Spiegler, R. (2015). 'On the equilibrium effects of nudging', *Journal of Legal Studies*, vol. 44(2), pp. 389–416.

Stoye, J. (2015). 'Choice theory when agents can randomize', *Journal of Economic Theory*, vol. 155, pp. 131–151.

Szpilrajn, E. (1930). 'Sur l'extension de l'ordre partiel', *Fundamenta Mathematicae*, vol. 16(1), pp. 386–389.

Tversky, A. and Kahneman, D. (1991). 'Loss aversion in riskless choice: A reference-dependent model', *Quarterly Journal of Economics*, vol. 106(4), pp. 1039–1061.

Tversky, A. and Shafir, E. (1992). 'Choice under conflict: The dynamics of deferred decision', *Psychological Science*, vol. 3(6), pp. 358–361.

Zakay, D. (1984). "'To choose or not to choose": On choice strategy in face of a single alternative', *American Journal of Psychology*, vol. 97(3), pp. 373–389.

Appendix

A Rational Choice as a Special Case

When preferences are complete in the MDC model of Proposition 1, the latter clearly reduces to utility maximisation. From the axiomatic point of view this is equivalent to Nonemptiness being satisfied on top of the other axioms. The result below provides a novel decomposition of Nonemptiness + WARP that characterises utility maximisation in this context. This decomposition is in terms of the other two consistency axioms that are involved in Proposition 1. It follows from this and the other statements below that the DCR model of Proposition 2 also reduces to rational choice over the entire set X when Undesirability is ruled out by Nonemptiness. Finally, Proposition 3 too collapses to utility maximisation once this axiom is imposed.

Before stating the result we recall that a choice correspondence C satisfies *Expansion Consistency* or *Sen's* β if $x, y \in C(A)$*,* $B \supset A$ *and* $x \in C(B)$ *implies* $y \in C(B)$. Also, we let $\widehat{\mathcal{M}}$ here stand for the collection of all nonempty subsets of X.

Proposition 4

The following are equivalent for a choice correspondence $C : \widehat{\mathcal{M}} \to \widehat{\mathcal{M}}$*:*

1. C satisfies Nonemptiness and WARP.

2. C satisfies Nonemptiness, Contraction Consistency and Expansion Consistency.

3. C satisfies Nonemptiness, Contraction Consistency and Strong Expansion.

4. There exists a unique weak order $\succsim$ *on X such that, for all* $A \in \widehat{\mathcal{M}}$*,*

$$C(A) = B_{\succsim}(A). \tag{A.1}$$

Proof of Proposition 4

We will show that $2 \Leftrightarrow 3$. For the $\Rightarrow$ direction, suppose $x \in C(A)$, $y \in A$ and $y \in C(B)$. Assume to the contrary that $x \notin C(A \cup B)$. Let $z \in C(A \cup B)$. From Contraction Consistency we have $z \in C(\{x, z\})$ and $z \in C(\{y, z\})$. Consider first the case where $y \in C(A \cup B)$. In view of the above, Contraction Consistency implies $C(\{y, z\}) = \{y, z\}$ and $C(\{x, y\}) = \{x, y\}$. Since $A \cup B \supset \{x, y\}$, it follows from $C(\{x, y\}) = \{x, y\}$, Expansion Consistency and $y \in C(A \cup B)$ that $x \in C(A \cup B)$, a contradiction. Now consider the case where $y \notin C(A \cup B)$. If $z \in A$, then $x \in C(A)$, $z \in C(A \cup B)$ and Contraction Consistency imply $C(\{x, z\}) = \{x, z\}$. Expansion Consistency now again implies $x \in C(A \cup B)$, a contradiction. If $z \in B$, then $y \in C(B)$, $z \in C(A \cup B)$ and Contraction Consistency imply $C(\{y, z\}) = \{y, z\}$. Expansion Consistency again implies $y \in C(A \cup B)$, a contradiction. This proves that Strong Expansion holds.

For the converse implication, suppose $x, y \in C(A)$, $B \supset A$ and $x \in C(B)$. We must show that $y \in C(B)$. Since $x, y \in C(A)$, Contraction Consistency implies $C(\{x, y\}) = \{x, y\}$. Since $y \in C(\{x, y\})$ and $x \in C(B)$, it follows from Strong Expansion and $y \in B$ that $y \in C(B)$. ■

The third statement in Proposition 4 is novel and clarifies that Expansion Consistency and Strong Expansion are equivalent under Nonemptiness and Contraction Consistency. It also suggests that either of these axiomatic combinations characterises rational choice. The equivalence between the first and fourth statements is due to Arrow (1959), while that between the second and fourth statements is due to Sen (1971).

B Sequential Choice and the Resolution of Indecisiveness

B.1 Maximally Dominant Choice over Two Periods

Intuition suggests that indecisiveness between alternatives can be resolved over time, for example after reflection or suitable information acquisition (this would correspond to what Sen (1997) referred to as *tentative* preference incompleteness). As a result, choice deferral that is driven solely by this kind of indecisiveness should vanish too. To model this, our primitive here is a *pair* of choice correspondences $C_1, C_2 : \mathcal{M} \to \mathcal{M}$. We interpret C_i as capturing the agent's behaviour in period $i = 1, 2$. Given that the domain of both C_1 and C_2 is the same collection of menus, and each menu in this collection represents a decision problem, we are implicitly assuming that first-period problems neither disappear nor change in the second period.

The axioms that follow impose some structure on the pair C_1, C_2 and are mainly normative.

Eventual Nonemptiness
If $\varnothing \neq A \in \mathcal{M}$, then $C_2(A) \neq \varnothing$.

This axiom precludes indefinite deferral by requiring the decision maker to choose an alternative from every menu in the second period, regardless of whether he initially deferred at this menu or not. This restriction is relevant in decision problems where the individual must ultimately make a choice, e.g. when deciding among job offers. When the agent defers at some menu A initially but chooses from it eventually, we can think of this choice as being the result of the agent receiving additional information about the alternatives in A during the interim stage via some unmodelled process.

Sequential Choice Consistency
If $x \in C_1(A)$, then $x \in C_2(A)$.

This axiom requires the agent's second-period choice at every menu A to be consistent with the choice (if any) that would have been made by him had he faced that menu in the first period.

Proposition 5
The following are equivalent for two choice correspondences $C_1, C_2 : \mathcal{M} \to \mathcal{M}$:
1. C_1 and C_2 satisfy Desirability, Contraction Consistency, Strong Expansion, Eventual Nonemptiness and Sequential Choice Consistency.
2. There exists a unique preorder $\succsim_1$ and a completion $\succsim_2$ of this preorder such that, for all $A \in \mathcal{M}$

$$C_i(A) = \{x \in A : x \succsim_i y \text{ for all } y \in A\}, \qquad i = 1, 2. \tag{B.1}$$

Proof of Proposition 5:

We only prove that $1 \Rightarrow 2$ (the proof of the converse implication is straightforward). Let C_1, C_2 satisfy the axioms and define the relation $\succsim_1$ on X by $x \succsim_1 y$ if there exists $A \in \mathcal{M}$ such that $x \in C_1(A)$ and $y \in A$. Since all singletons are included in $\mathcal{M}$, it follows from Desirability that $\succsim_1$ is reflexive. Suppose now that $x \succsim_1 y$ and $y \succsim_1 z$ for some $x, y, z \in X$. There exist $A, B \in \mathcal{M}$ such that $x \in C_1(A)$, $y \in A$ and $y \in C_1(B)$, $z \in B$. From Strong Expansion, $x \in C_1(A \cup B)$. Since $\{x, y, z\} \in \mathcal{M}$ by assumption and $\{x, y, z\} \subseteq A \cup B$, it follows from Contraction Consistency that $x \in C_1(\{x, y, z\})$. Hence, $x \succsim_1 z$. This shows that $\succsim_1$ is transitive, hence a preorder.

We must first prove that

$$C_1(A) = \{x \in A : x \succsim_1 y \text{ for all } y \in A\}. \tag{B.2}$$

Suppose that (B.2) is not satisfied. Assume that there is $A \in \mathcal{M}$ such that $x \in A$, $x \succsim_1 y$ for all $y \in A$ and $x \notin C_1(A)$. If $y \in C_1(A)$ for some $y \neq x$, then this fact and $x \succsim_1 y$ together contradict WARP, which, as noted in the text, is implied by Contraction Consistency and Strong Expansion. Suppose $C_1(A) = \varnothing$ instead. The postulate that $x \succsim_1 y$ for all $y \in A$ is equivalent to the postulate that for each $y \in A$ there exists $B_y \in \mathcal{M}$ such that $y \in B_y$ and $x \in C_1(B_y)$. Let $S := \bigcup\{B_y : y \in A\}$. Clearly, $A \subseteq S$. Moreover, the full domain assumption ensures that $S \in \mathcal{M}$. Repeated application of Strong Expansion gives $x \in C_1(S)$. Since $A \subseteq S$, $x \in A$ and $A \in \mathcal{M}$, it follows from Contraction Consistency that $x \in C_1(A)$. Conversely, if $x \in C_1(A)$ and $y \in A$, then, by definition of $\succsim_1$, $x \succsim_1 y$. Thus, (B.2) holds. Uniqueness of $\succsim_1$ follows from the fact that all binary menus are included in $\mathcal{M}$.

Now define $\succsim_2$ on X by $x \succsim_2 y$ if there is $A \in \mathcal{M}$ such that $x \in C_2(A)$ and $y \in A$. Since Eventual Nonemptiness holds, we know from Proposition 4 that $\succsim_2$ is a weak order that satisfies

$$C_2(A) = \{x \in A : x \succsim_2 y \text{ for all } y \in A\}. \tag{B.3}$$

Finally, from Sequential Choice Consistency we have $x \succsim_2 y$ whenever $x \succsim_1 y$. Hence, $\succsim_2$ is indeed a completion of $\succsim_1$. ■

This can be interpreted as a two-period model in which deferral is possible in the first period but an active choice is necessarily made in the second. Moreover, the choice that is made in the second period is by maximisation of the agent's first-period preferences which have meanwhile been completed through some unmodelled process (e.g. via information acquisition). Following Gilboa Gilboa *et al.* (2010), one may think of $\succsim_1$ as capturing the "objective"but incomplete component of the agent's preferences and of $\succsim_2$ as capturing his "subjective" component that completes them (see also below). Being objective in the sense that the agent can convince others about the validity of the preference statements included in $\succsim_1$, this part of the agent's preferences is stable and not subject to change after any additional information acquisition. This provides a theoretical justification for the preclusion of cross-period choice reversals that is featured in this model.

B.2 A Choice-Theoretic Foundation for Bewley Preferences

A specific domain in which the sequential MDC model of Proposition 5 is potentially illuminating is that of choice under uncertainty. In this domain, the finite set of outcomes X is coupled by a pair (S, Σ), where S is a set of possible states of the world and Σ a σ-algebra of events derived from S. Moreover, the objects of choice are acts that belong to the set $\mathcal{F}$ of all Σ-measurable functions mapping S into $\Delta(X)$, the latter being the set of probability distributions on X. Let $\mathcal{F}$ and $\Delta(S)$ be endowed with some suitable topologies and let $\mathcal{D}$ denote the set of all convex, compact subsets of $\mathcal{F}$ (the empty set is trivially both convex and compact). Finally, assume that the choice correspondences C_i, $i = 1, 2$, map $\mathcal{D}$ into itself.

The benchmark model of incomplete preferences under uncertainty is Bewley (2002). In the weak-preference analogue of this model that was axiomatised in Gilboa *et al.* (2010), the decision maker is portrayed as having "objectively rational" preferences over $\mathcal{F}$ that are captured by a preorder $\succsim$ (which is necessarily complete only in the subdomain of constant acts), and also as having beliefs over S that are captured by a non-singleton closed, convex set $\Pi \subseteq \Delta(S)$ of probability measures on S. In this version of the Bewley model the agent compares two acts f and g by the following, partially applicable rule:

$$f \succsim g \iff \int_S \mathbb{E}_{f(s)} u d\pi(s) \geq \int_S \mathbb{E}_{g(s)} u d\pi(s) \qquad \text{for all } \pi \in \Pi. \tag{B.4}$$

Here, $u : X \to \mathbb{R}$ is a von Neumann-Morgenstern utility function, which exists due to the completeness of $\succsim$ in the subdomain of constant acts, $\Delta(X)$, while $\mathbb{E}_{f(s)} u = \sum_{x \in X} f(s)(x) u(x)$.

Being derived from a model of *preference* and not *choice* under uncertainty, the Bewley rule does not directly specify what the agent's decision is in those cases where no feasible act is preferred to all others in a given menu. However, Bewley (2002) argued that the agent *preserves the status quo* unless he can find an act that is preferred to it. Choosing from a set of insurance policies when already endowed with one of them is an example of a problem where this is possible. Yet, as we've been arguing throughout this paper, in many decision problems the status quo does not take the form of an explicit and concrete market alternative such as this. An example would again be the problem of choosing an insurance policy when not already endowed with one. Assuming that every option is acceptable in principle, the status quo here does not actually enter the decision problem. One way forward for the decision maker then is to choose among the preference-undominated acts, as studied in Stoye (2015). Another possibility, however, is that the lack of a partially or totally dominant option in such cases can lead to deferral.

In light of the above, we can use Proposition 5 to provide an alternative choice-theoretic interpretation of the Bewley model for the case of decision problems in which the original Bewley decision rule may be inapplicable due to the different nature of the status quo and also due to the agent being unwilling to choose an option that is merely undominated. Specifically, given the above primitives one can model an agent faced with a menu A in $\mathcal{D}$ as possibly deferring à la Bewley (2002) in the first period and as choosing like a subjective expected utility maximiser in the second period, as follows:

$$C_1(A) = \bigcap_{\pi \in \Pi} \arg\max_{f \in A} \int_S \mathbb{E}_{f(s)} u d\pi(s) \tag{B.5a}$$

$$C_2(A) = \arg\max_{f \in A} \int_S \mathbb{E}_{f(s)} u dp(s). \tag{B.5b}$$

Under (B.5) the individual chooses a most preferred act in the first period if and only if one exists. When such an act does not exist, choice is deferred to the second period, by which time he has narrowed down his beliefs to a *single* prior p from the original set Π, and chooses as a subjective expected utility maximiser according to the same, first-period utility function u that dictates his unchanged tastes.

In the same vein, one can use Proposition 5 to provide a choice-theoretic foundation for the objective vs subjective rationality model of preferences under uncertainty that was axiomatised in Gilboa *et al.* (2010) by attaching a temporal dimension to the associated objective and subjective preference relations. Specifically, these authors provided necessary and sufficient conditions on a pair of preference relations $\succsim_1$ and $\succsim_2$ for there to exist a set of priors Π and a utility function u such that the former is represented by these two objects as in (B.4) and the latter according to the maxmin expected utility rule. Our proposed behavioural/choice-theoretic counterpart of this preference model is

$$C_1(A) = \bigcap_{\pi\in\Pi} \arg\max_{f\in A} \int_S \mathbb{E}_{f(s)} u d\pi(s) \tag{B.6a}$$

$$C_2(A) = \arg\max_{f\in A} \min_{\pi\in\Pi} \int_S \mathbb{E}_{f(s)} u d\pi(s) \tag{B.6b}$$

The difference between (B.5) and (B.6) lies in the second-period choice criterion, where subjective expected utility maximisation in (B.5b) has been replaced by *maxmin* subjective expected utility maximisation in (B.6b). We note that a similar choice-theoretic interpretation can also be given to the dual-preference model of Kopylov (2009).[23]

C Proofs of Propositions 1–3

Proof of Proposition 1:

This is included in the proof of Proposition 5 above. ∎

Proof of Proposition 2:

Necessity of the axioms is straightforward to check. We prove sufficiency. To this end, we note first that the four axioms in the statement of the proposition jointly imply Strong Expansion. Suppose not. We have $x \in C(A)$, $y \in A$, $y \in C(B)$ and $C(A \cup B) = \emptyset$ (WARP is violated if $x \notin C(A \cup B) \neq \emptyset$). It now follows from Contractive Undesirability and $C(A \cup B) = \emptyset$ that $C(A) = C(B) = \emptyset$, a contradiction.

Now let $X^* := \{x \in X : x \in C(A) \text{ for some } A \in \mathcal{M}\}$. Let $Y = X \setminus X^*$. From Undesirability, $Y \neq \emptyset$. Define the relation $\rhd$ on X by $x \rhd y$ if $x \in C(A)$, $y \in A$ for some $A \in \mathcal{M}$, and the relation $\succsim^*$ by

$$\succsim^* = \rhd \cup \{(x, x) : x \in Y\}.$$

In view of Contraction Consistency and the definition of $\succsim^*$, this relation is reflexive. We will also show that the restriction of $\succsim^*$ on X^* is complete and transitive, hence a weak order. Consider $x, y, z \in X^*$ and suppose $x \succsim^* y \succsim^* z$. Employing the same Strong-Expansion argument that was used in the proof of Proposition 5 shows that $x \succsim^* z$, which establishes transitivity. Now suppose to the contrary that $x \not\succsim^* y$ and $y \not\succsim^* x$ for some

[23] We also refer the reader to Pejsachowicz and Toussaert (2015) for an analysis of preferences over menus of lotteries where the primitive is also a pair of an incomplete preference relation and a completion of that relation, in which case the latter necessarily exhibits preference for flexibility in the sense of Kreps (1979).

$x, y \in X^*$. It follows that $C(\{x, y\}) = \emptyset$. From Contractive Undesirability we then obtain $C(\{x\}) = C(\{y\}) = \emptyset$. Since $x, y \in X^*$, there exist $A, B \in \mathcal{M}$ such that $x \in C(A)$ and $y \in C(B)$. In light of this, $C(\{x\}) = C(\{y\}) = \emptyset$ contradicts Contraction Consistency. Therefore, $\succsim^*$ is a reflexive relation on X and a weak order on $X^* \subset X$.

Now order the $\sim^*$-equivalence classes of X^* by $[x_i] \gg [x_j]$ if $x \succ^* x'$ for $x \in [x_i]$ and $x' \in [x_j]$. Since X^* is finite, there is some integer k such that $[x_1] \gg [x_2] \gg \ldots \gg [x_k]$. Next, let R be an arbitrary weak order on Y with its symmetric and asymmetric parts denoted by I and P, respectively, and order the I-equivalence classes of Y by $[y_i]P[y_j]$ if yPy' for $y \in [y_i]$ and $y' \in [y_j]$. Again, there exists an integer n such that $[y_1]P[y_2]P\ldots P[y_n]$. Finally, define the relation $\succsim$ by

$$\succsim = \succsim^* \bigcup R \bigcup_{i \leq k,\, j \leq n} ([x_i] \times [y_j]) \bigcup_{i \leq k} ([x_i] \times [x_i]) \bigcup_{j \leq n} ([y_j] \times [y_j]).$$

where

$$[x_i] \times [y_j] = \bigcup \{(x, y) : x \in [x_i],\ y \in [y_j]\}.$$

It is easy to check that $\succsim$ is a weak order that extends $\succsim^*$ from X^* to X.

Let $x^* \in Y$ be such that $x^* \succsim y$ for all $y \in Y$. To establish the first part in (2), let $C(A) = \emptyset$ for some $A \in \mathcal{M}$. Suppose to the contrary that $z \succ x^*$ for some $z \in B_{\succsim}(A)$. Since, by definition, $z \succ x^*$ implies $z \succ^* x^*$, it follows from the above that $z \in X^*$ and therefore that $z \in C(D)$ for some $D \in \mathcal{M}$. In view of Contraction Consistency, this implies $C(\{z\}) = \{z\}$. Since $\{z\} \subset A$, this and $C(A) = \emptyset$ together contradict Contractive Undesirability. Thus, $C(A) = \emptyset$ implies $x^* \succsim z$ for $z \in B_{\succsim}(A)$. Conversely, suppose $x^* \succsim z$ for $z \in B_{\succsim}(A)$. This implies $x^* \succsim z$ for all $z \in A$. In view of the definition of x^* and Y, this further implies $A \subseteq Y$. It follows then that $z \notin C(A)$ for all $z \in A$, and therefore that $C(A) = \emptyset$. To establish the second part, let $z \succ x^*$ for $z \in B_{\succsim}(A)$. From above, $C(A) \neq \emptyset$. Suppose $z \notin C(A)$ and $y \in C(A)$. It holds that $y \succ z$. But since $z \in B_{\succsim}(A)$ implies $z \succsim x$ for all $x \in A$, a contradiction obtains. Therefore, $C(A) \neq \emptyset$ implies $C(A) = B_{\succsim}(A)$.

Finally, consider a weak order $\succsim'$ on X such that $x \succsim y \Leftrightarrow x \succsim' y$ for all $x, y \in X^*$ such that $x, y \succ x^*$. Clearly, $B_{\succsim}(A) = B_{\succsim'}(A)$ for all $A \in \mathcal{M}$ such that $A \cap X^* \neq \emptyset$. Hence, $C(A) \neq \emptyset$ implies $C(A) = B_{\succsim'}(A)$. Therefore, $\succsim$ quasi-rationalises C uniquely up to x^*. ■

Proof of Proposition 3.

For some $n \in \mathbb{N}$, define the function $\psi : \mathcal{M} \rightarrow \mathbb{R}$ by

$$\psi(A) := \begin{cases} n, & \text{if } A = \emptyset \text{ or } C(A) \neq \emptyset \\ n+1, & \text{otherwise} \end{cases} \tag{C.1}$$

By construction, $C(A) = \emptyset$ iff $\psi(A) > n$. Moreover, Desirability, Deferral Monotonicity and Binary Choice Consistency ensure that ψ also satisfies (3b), (3c) and (3d), respectively.

For $A \neq \emptyset$ we have $C(A) \neq \emptyset$ iff $\psi(A) = n$. We must show that there exists a unique preorder $\trianglerighteq$ and a completion $\succsim$ of $\trianglerighteq$ such that $C(A) \neq \emptyset$ implies $C(A) = B_{\succsim}(A)$ for all $A \in \mathcal{M}$. The proof of this relies on a suitable adaptation of the argument in Richter (1966, p. 640). Define the relation $\trianglerighteq$ on X by $x \trianglerighteq y$ if there is $A \in \mathcal{M}$ such that $x \in C(A)$ and $y \in A$. From Desirability, $\trianglerighteq$ is reflexive. Now suppose $x \in C(A)$, $y \in A$ and $y \in C(B)$, $z \in B$. Since all binary menus are in $\mathcal{M}$ by assumption, in view of WARP and (the contrapositive of) Deferral Monotonicity, these assumptions imply $x \in C\{x, y\})$ and $y \in C(\{y, z\})$, respectively. It now follows from Binary Choice Consistency that $x \in C(\{x, z\})$, which implies $x \succsim z$. Therefore, $\trianglerighteq$ is transitive, hence a preorder. Since all binary menus are in $\mathcal{M}$, $\trianglerighteq$ is uniquely defined.

Define the relation P on X by xPy if $x \trianglerighteq y$ and $y \ntrianglerighteq x$. Since P is the asymmetric part of the preorder $\trianglerighteq$, P is a strict partial order. Define the relation J on X by xJy if $x \trianglerighteq y$ and $y \trianglerighteq x$. J is the symmetric part of the preorder $\trianglerighteq$, hence an equivalence relation on X. Let the J-equivalence class of $x \in X$ be denoted by $[x]$. Let $\mathcal{X}$ be the quotient set derived from X by J. Let $\mathcal{P}$ be a relation on $\mathcal{X}$ defined by $[x]\mathcal{P}[y]$ if xPy. The relation $\mathcal{P}$ is a strict partial order on $\mathcal{X}$. From Szpilrajn's (1930) theorem, it admits an extension into a strict linear order $\mathcal{R}$ on $\mathcal{X}$. Now define the relation $\succsim$ on X by $x \succsim y$ iff xJy or $[x]\mathcal{R}[y]$. It is easy to show that $\succsim$ is a complete preorder on X.

Suppose $C(A) \neq \emptyset$. Let $x \in C(A)$. For all $y \in A$, it holds that xPy or xJy. Hence, $x \in C(A)$ implies $x \succsim y$ for all $y \in A$. Conversely, suppose $C(A) \neq \emptyset$, $x \in A$ and $x \succsim y$ for all $y \in A$. Let $z \in C(A)$. If $x = z$, there is nothing to prove. Suppose $z \neq x$. We have zJx or $[z]\mathcal{R}[x]$. At the same time, $x \succsim z$ implies xJz or $[x]\mathcal{R}[z]$. Suppose $x\mathcal{R}y$ and $y\mathcal{R}x$. Since $x \neq z$, this violates the asymmetry of $\mathcal{R}$. Hence, xJz. This implies $x \in C(B)$ and $z \in B$ for some $B \in \mathcal{M}$. Suppose $x \notin C(A)$. Since $z \in C(A)$, this contradicts WARP. Therefore, $x \in C(A)$. This completes the proof that (3a) holds.

With regard to necessity of the axioms, it follows from (3a) that WARP is satisfied. It also follows from (3a) and (3c) that Deferral Monotonicity is satisfied, and from (3a) and (3b) that Desirability is satisfied. Finally, it follows from (3a) and (3d) that Binary Choice Consistency is satisfied too. ∎

D Axiom Independence

Let $X = \{w, x, y, z\}$. Each of the three examples below presents behaviour that satisfies all but one of the axioms involved in Proposition 1.

Not Desirability

$$
\begin{array}{lll}
C(\{w\}) = \{w\},\ C(\{x\}) = \{x\}, & C(\{y\}) = \{y\}, & C(\{z\}) = \emptyset \\
C(\{w, x\}) = \{w, x\}, & C(\{w, y\}) = \{w\}, & C(\{x, y\}) = \{x\}, \\
C(\{w, z\}) = C(\{x, z\}) = C(\{y, z\}) = \emptyset, & & \\
C(\{w, x, z\}) = C(\{w, y, z\}) = C(\{x, y, z\}) = \emptyset, & C(\{w, x, y\}) = \{w, x\}, & \\
C(\{w, x, y, z\}) = \emptyset & &
\end{array}
$$

Not Contraction Consistency

$$\begin{array}{lll} C(\{w\}) = \{w\},\ C(\{x\}) = \{x\}, & C(\{y\}) = \{y\},\ C(\{z\}) = \{z\} & \\ C(\{w,y\}) = C(\{w,z\}) = \{w\}, & & \\ C(\{w,x\}) = C(\{x,y\}) = \emptyset, & C(\{x,z\}) = C(\{y,z\}) = \emptyset, & \\ C(\{w,x,y\}) = \{w,x\}, & C(\{w,x,z\}) = C(\{x,y,z\}) = \emptyset, & C(\{w,y,z\}) = \{w\}, \\ C(\{w,x,y,z\}) = \{w,x\} & & \end{array}$$

Not Strong Expansion

$$\begin{array}{ll} C(\{w\}) = \{w\},\ C(\{x\}) = \{x\}, & C(\{y\}) = \{y\},\ C(\{z\}) = \{z\} \\ C(\{w,x\}) = C(\{w,y\}) = C(\{w,z\}) = \{w\}, & C(\{x,y\}) = C(\{x,z\}) = C(\{y,z\}) = \emptyset, \\ C(\{w,x,y\}) = \{w\}, & C(\{w,x,z\}) = C(\{w,y,z\}) = C(\{x,y,z\}) = \emptyset \\ C(\{w,x,y,z\}) = \emptyset & \end{array}$$

Next, each of the four examples below presents behaviour that satisfies all but one of the axioms involved in Proposition 2.

Not WARP

$$\begin{array}{llll} C(\{w\}) = C(\{z\}) = \emptyset, & C(\{x\}) = \{x\}, & C(\{y\}) = \{y\}, & \\ C(\{w,x\}) = \{w,x\}, & C(\{w,y\}) = \{w,y\}, & C(\{w,z\}) = \{w\}, & \\ C(\{x,y\}) = C(\{x,z\}) = \{x\}, & C(\{y,z\}) = \{y\}, & & \\ C(\{w,x,y\}) = \{w,x\}, & C(\{w,x,z\}) = \{x\}, & C(\{w,y,z\}) = \{y\}, & C(\{x,y,z\}) = \{x\} \\ C(\{w,x,y,z\}) = \{x\} & & & \end{array}$$

Not Contraction Consistency

$$\begin{array}{lll} C(\{w\}) = \{w\}, & C(\{x\}) = C(\{y\}) = C(\{z\}) = \emptyset & \\ C(\{w,x\}) = \{w,x\}, & C(\{w,y\}) = C(\{w,z\}) = \{w\}, & \\ C(\{x,y\}) = C(\{x,z\}) = C(\{y,z\}) = \emptyset & & \\ C(\{w,x,y\}) = C(\{w,x,z\}) = \{w,x\}, & C(\{w,y,z\}) = \{w\}, & C(\{x,y,z\}) = \emptyset, \\ C(\{w,x,y,z\}) = \{w,x\} & & \end{array}$$

Not Undesirability

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\}, \quad C(\{y\}) = \{y\},\ C(\{z\}) = \{z\}$$
$$C(\{w,x\}) = \{w,x\}, \quad C(\{w,y\}) = C(\{w,z\}) = \{w\},$$
$$C(\{x,y\}) = C(\{x,z\}) = \{x\}, \quad C(\{y,z\}) = \{y,z\}$$
$$C(\{w,x,y\}) = C(\{w,x,z\}) = \{w,x\}, \quad C(\{w,y,z\}) = \{w\}, \quad C(\{x,y,z\}) = \{x\}$$
$$C(\{w,x,y,z\}) = \{w,x\}$$

Not Contractive Undesirability

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\},\ C(\{y\}) = \{y\}, \quad C(\{z\}) = \emptyset$$
$$C(\{w,x\}) = \{w,x\}, \quad C(\{w,y\}) = \{w\},$$
$$C(\{x,z\}) = C(\{y,z\}) = C(\{w,z\}) = \emptyset, \quad C(\{x,y\}) = \{x\},$$
$$C(\{w,x,y\}) = \{w,x\}, \quad C(\{w,x,z\}) = C(\{w,y,z\}) = C(\{x,y,z\}) = \emptyset$$
$$C(\{w,x,y,z\}) = \emptyset$$

Finally, each of the four examples below presents behaviour that satisfies all but one of the axioms involved in Proposition 3.

Not WARP

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\}, \quad C(\{y\}) = \{y\},\ C(\{z\}) = \{z\}$$
$$C(\{w,x\}) = \{w,x\}, \quad C(\{w,y\}) = \{w\}, \quad C(\{x,y\}) = C(\{x,z\}) = \{x\},$$
$$C(\{w,z\}) = C(\{y,z\}) = \emptyset$$
$$C(\{w,x,y\}) = \{w,x\}, \quad C(\{w,x,z\}) = C(\{w,z\}) = \emptyset, \quad C(\{x,y,z\}) = \{x\},$$
$$C(\{w,x,y,z\}) = \emptyset$$

Not Desirability

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\}, \quad C(\{y\}) = \{y\},\ C(\{z\}) = \emptyset,$$
$$C(\{w,x\}) = \{w,x\}, \quad C(\{w,y\}) = \{w\},$$
$$C(\{x,y\}) = \{x\}, \quad C(\{w,z\}) = C(\{x,z\}) = C(\{y,z\}) = \emptyset,$$
$$C(\{w,x,y\}) = \{w,x\}, \quad C(\{w,x,z\}) = C(\{w,y,z\}) = C(\{x,y,z\}) = \emptyset,$$
$$C(\{w,x,y,z\}) = \emptyset$$

Not Binary Choice Consistency

$$\begin{aligned}
&C(\{w\}) = \{w\},\ C(\{x\}) = \{x\}, && C(\{y\}) = \{y\},\ C(\{z\}) = \{z\}\\
&C(\{w,y\}) = \{w\}, && C(\{y,z\}) = \{y\}\\
&C(\{w,x\}) = C(\{w,z\}) = \emptyset, && C(\{x,y\}) = C(\{x,z\}) = \emptyset,\\
&C(\{w,x,y\}) = C(\{w,x,z\}) = \emptyset, && C(\{w,y,z\}) = C(\{x,y,z\}) = \emptyset\\
&C(\{w,x,y,z\}) = \emptyset
\end{aligned}$$

Not Deferral Monotonicity

$$\begin{aligned}
&C(\{w\}) = \{w\},\ C(\{x\}) = \{x\}, && C(\{y\}) = \{y\},\ C(\{z\}) = \{z\}\\
&C(\{w,x\}) = C(\{w,y\}) = C(\{w,z\}) = \{w\}, && C(\{x,y\}) = C(\{x,z\}) = C(\{y,z\}) = \emptyset\\
&C(\{w,x,y\}) = \{w\}, && C(\{w,x,z\}) = C(\{w,y,z\}) = C(\{x,y,z\}) = \emptyset\\
&C(\{w,x,y,z\}) = \emptyset
\end{aligned}$$