A Financial Accelerator through Coordination Failure

Oliver de Groot

A Financial Accelerator through Coordination Failure[*]

Oliver de Groot[†]

University of St Andrews & European Central Bank

31 January 2019

Abstract

This paper studies the effect of liquidity crises in short-term debt markets in a dynamic general equilibrium framework. Creditors (retail banks) receive imperfect signals regarding the profitability of borrowers (wholesale banks) and, based on these signals and their beliefs about other creditors actions, choose whether to rollover funding, or not. The uncoordinated actions of creditors cause a suboptimal incidence of rollover, generating an illiquidity premium. Leverage magnifies the coordination inefficiency. Illiquidity shocks in credit markets result in sharp contractions in output. Policy responses are analyzed.

Keywords: Financial frictions, DSGE models, Global games, Bank runs, Unconventional monetary policy, Financial crises.

JEL Classification: D82, E32, E44, G12

1 Introduction

Creditors financing a project face a coordination problem. Fear of other creditors not rolling over funding may lead to preemptive action, undermining the project, and the chance of repayment to those creditors as well. Coordination problems impact the functioning of many types of the credit market but often the most visible manifestation of coordination failure is a bank run.[1] At least since Bagehot (1873)'s description of the 1866 financial panic, economists have acknowledged the inherent fragility of financial intermediaries. Table 1 lists notable bank runs during the 2007-08 financial crisis. The demise of Northern Rock (UK) and Lehman Brothers (US), for example, have been interpreted (see Brunnermeier (2009) and Shin (2009)) as events in which short-term interbank market creditors were unwilling to continue lending to these institutions, for fear that other creditors were doing likewise.[2]

This paper makes three contributions. First, it builds a rigorously micro-founded coordination problem in credit markets in a dynamic, general equilibrium macroeconomic model. Second, the model highlights the role that maturity- and liquidity-mismatch play in the propagation of shocks through the economy. Third, it provides a laboratory to study the effect of unconventional policy responses to a large scale liquidity dry up, as seen during the 2007-08 financial crisis.

To the best of my knowledge, this paper was the first to model liquidity and bank runs within a dynamic general equilibrium framework.[3] Recently, Gertler and Kiyotaki (2015) and Gertler et al. (2016) also studied the macroeconomic implications of bank runs. My contribution distinguishes itself from these in two important respects. First, in the aforementioned papers bank runs are themselves aggregate shocks to the economy in which the entire financial sector experiences a run. In reality, as Table 1 suggests, even during a major financial crisis, only a subset of all financial institutions experience a run. My model captures this feature. In particular, in any given period the proportion of borrowers in danger

---

[1]Coordination problems also impact the market for bank loans, commercial paper, and corporate bonds. For example, Hertzberg et al. (2011) exploited a natural experiment, which compelled banks to make public negative private assessments about their borrowers, to quantify the effect of coordination problems. Even the commercial paper market, primarily for low-default-risk firms, experienced liquidity dry ups, following the Penn Central bankruptcy (1970), the Russia/LTCM crisis (1998), and the Enron/WorldCom episode (2002). In 2008, the Fed introduced the Commercial Paper Funding Facility (CPFF) to prevent a market freeze. No issuers who used the CPFF defaulted on their debt obligations, suggesting the liquidity dry up was driven by coordination problems rather than increased insolvency risk. Disorderly liquidation of assets is economically inefficient. In the US, chapter 11 bankruptcy provision is designed to address the coordination problem among creditors (Jackson (1986)), giving legal protection to a firm to remain in business while being restructured.

[2]In practice, it is difficult to disentangle whether the unwillingness of creditors to continue lending hastened the bankruptcy of an insolvent bank or whether the run scuppered a sound institution. Morris and Shin (2016) disentangle the two in theory.

[3]The first version of this paper was circulated in 2010.

of experiencing a run is determined by the aggregate state, with the proportion of borrowers experiencing a run being endogenously countercyclical. Second, runs in the model are determined by a rigorously microfounded global game. In contrast, Gertler and Kiyotaki (2015) and Gertler et al. (2016) use an ad hoc threshold rule (based on the insights from the global games literature).

Table 1: Notable bank runs during the 2007-08 financial crisis

2007 Aug	Countrywide Financial	US	Wholesale funding run followed by a retail-deposit run.
2007 Sep	Northern Rock	UK	Wholesale funding run followed by a retail-deposit run.
2008 Mar	Bear Stearns	US	Wholesale funding run.
2008 Jul	IndyMac	US	Wholesale funding and brokered-deposit run.
2008 Sep	Lehman Brothers	US	Wholesale funding run.
2008 Sep	Washington Mutual	US	Retail deposit run.
2008 Sep	Wachovia	US	Retail- and brokered-deposit bank run.

In my model, wholesale banks (*the borrowers*) finance themselves with short-term debt from a multitude of retail banks (*the creditors*). I show that coordination problems generate an equilibrium trade off between the leverage of a borrower and the illiquidity premium paid on external finance, resulting from a suboptimal incidence of debt rollover. The coordination problems among creditors amplify business cycle dynamics and illiquidity shocks in credit markets generate a novel channel for additional volatility in economic activity. I study two potential policy instruments to dampen the effect of an illiquidity crisis: *direct lending* to and *equity injections* into wholesale banks. I show, for a given dollar amount committed, that equity injections dampen the contemporaneous effect on economic activity more than direct lending, but causes output to remain subdued for longer. This is because a dollar of debt intermediated by the policymaker improves the solvency of a wholesale bank (which largely determines the extent of the coordination problem) by less than a dollar of additional equity. However, equity injections disincentivize balance sheet adjustment, slowing the output recovery following the shock.

Coordination failure arises from two assumptions. One, borrowers have a balance sheet maturity mismatch with longer maturity (more illiquid) assets and shorter maturity (more liquid) liabilities. Two, multiple creditors per borrower who cannot coordinate their actions. When a borrower's debt matures, each creditor must decide whether to rollover, taking account of fundamentals and the actions of other creditors. Diamond and Dybvig (1983) study this coordination problem with depositors as the creditors and find the existence of multiple

equilibria, one of which is a bank run.[4] Such multiple equilibria, however, are problematic for a general equilibrium framework where the endogenous pricing of a debt contract requires ex ante knowledge about the probability distribution of ex post outcomes. Global games offer a solution.[5] Multiple equilibria in Diamond and Dybvig (1983) is a consequence of the assumption that creditors' information sets are perfectly symmetric. By introducing idiosyncratic noise into creditors' signal of a borrower's solvency, the global games literature shows that a unique set of self-fulfilling beliefs prevail in equilibrium. I thus model the coordination problem in credit markets as a sequence of micro-founded static global games, building on Morris and Shin (2004), Rochet and Vives (2004), and Goldstein and Pauzner (2005). The resulting dynamic general equilibrium model allows the study of the macroeconomic effects of illiquidity shocks and the efficacy of alternative policy interventions.[6]

Liquidity hoarding and interbank lending during the crisis has been studied Heider et al. (2015). However, theories of coordination problems in credit markets have received relatively little attention in the macroeconomics literature. The majority of the financial friction literature has broadly followed two paths which find their roots in the work of Bernanke and Gertler (1989) (building on the costly state verification (CSV) assumption from Townsend (1979), in which there is an informational asymmetry in a single borrower-creditor relationship) and Kiyotaki and Moore (1997) (building on the hold-up assumption of Hart and Moore (1994), introducing a binding collateral constraint on lending).[7] The distortion created by coordination problems among creditors is a distinct and important feature of credit markets, especially during the 2007-08 financial crisis. This paper therefore is an important complementary framework to analyze credit market interventions. Related papers modeling the policy response of the Fed, Treasury, and other central banks and governments to the financial crisis in the context of alternative financial frictions include Gertler and Kiyotaki (2010) and Gertler and Karadi (2011).[8]

The rest of the paper: Section 2 presents the model. Section 3 analyzes partial equilibrium comparative statics of the global game and the dynamic response of the model to illiquidity shocks. Section 4 analyzes direct lending and equity injections. Section 5 concludes.

---

[4]See Gorton and Winton (2003) for a review of the literature Diamond and Dybvig (1983) spawned.

[5]The seminal contribution is Carlsson and van Damme (1993). For an overview, see Morris and Shin (2003). Corsetti et al. (2004) apply global games to currency crises.

[6]Kurlat (2013) and Dang et al. (2012) explain why the market for some assets freeze during crises.

[7]Seminal contributions include Carlstrom and Fuerst (1997) and Bernanke et al. (1999); Iacoviello (2005) applied financial frictions to the housing market; Christiano et al. (2014) estimated the importance of financial risk shocks; Carlstrom et al. (2010), Fiore and Tristani (2013), and Cúrdia and Woodford (2016) analyze optimal monetary policy with financial frictions; Angeloni and Faia (2013) model macroprudential policy. de Groot (2016) reviews the financial accelerator literature.

[8]See also Sargent and Wallace (1982), Reis (2009b), and Cúrdia and Woodford (2010).

2   The model

The model is a canonical real business cycle model in which I embed a coordination problem facing retail banks in financing wholesale banks.

2.1   Households

A representative household consumes, C_t, supplies labor, L_t, and has preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma_C}}{1 - \sigma_C} - \chi \frac{L_t^{1+\rho}}{1 + \rho} \right). \tag{1}$$

The household maximizes (1) subject to its budget constraint given by

$$C_t \leq W_t L_t + R_{t-1} D_{t-1} - D_t + \Pi_t, \tag{2}$$

where W_t is the real wage, R_t is a predetermined risk-free rate on deposits held with retail banks, Π_t are profits (from owning goods producing firms and capital producers). The household's first-order conditions are given by

$$W_t = \chi L_t^{\rho} C_t^{\sigma_C}, \tag{3}$$

$$1 = \beta \mathbb{E}_t \left(C_{t+1}/C_t \right)^{-\sigma_C} R_t. \tag{4}$$

2.2   Capital producers

The production of new capital is subject to adjustment costs. A representative capital producer takes I_t consumption goods and transforms them into $\Psi \left(I_t/I \right) I$ units of capital that it sells at price Q_t (where I is the steady state level of investment). I assume the function Ψ is concave and $\Psi(1) = \Psi'(1) = 1$ and $\Psi''(1) = -\psi$. Profits are given by $Q_t \Psi \left(I_t/I \right) I - I_t$. The producer's first-order condition is given by

$$Q_t = \left(\Psi' \left(I_t/I \right) \right)^{-1}. \tag{5}$$

The aggregate capital stock at the end of period t is given by

$$K_t = (1 - \delta) K_t^* + \Psi \left(I_t/I \right) I, \tag{6}$$

where K_t^* is capital available for time t production. This is different from the capital stock at the end of $t - 1$ as some is lost due to the costly uncoordinated liquidation of failing banks.

2.3 Goods producers

Goods producing firms hire labor and rent capital as inputs to a constant returns to scale technology to produce output, Y_t, given by

$$Y_t = A_t \left(K_t^*\right)^\alpha L_t^{1-\alpha}, \tag{7}$$

where A_t is the stochastic level of technology. Output is sold in a perfectly competitive market at a unit price. The firm's first-order conditions are given by

$$MPK_t = \alpha Y_t / K_t^*, \tag{8}$$

$$W_t = (1-\alpha) Y_t / L_t, \tag{9}$$

where MPK_t is the marginal product of capital. In the absence of coordination failure, arbitrage ensures that the discounted expected return on capital equals the risk-free rate

$$\mathbb{E}_t \left((C_{t+1}/C_t)^{-\sigma_C} R_{K,t+1}\right) = \mathbb{E}_t \left((C_{t+1}/C_t)^{-\sigma_C}\right) R_t, \tag{10}$$

where $R_{K,t} \equiv (MPK_t + (1-\delta) Q_t) / Q_{t-1}$.

2.4 Retail and wholesale banks

The financial sector transforms household savings into financing for firms to rent capital. Two types of intermediary exist: retail and wholesale banks. Retail banks accept household savings and lend to wholesale banks who finance firms. By assumption, wholesale banks rely on short-term debt funding from a large number of retail banks. As a result, retail banks face a coordination problem in deciding whether to rollover maturing debt.

Wholesale banks Think of them as *capital managers*. They invest in firms' capital with the intention of augmenting the capital with their idiosyncratic productivity and earning a return from the augmented capital. Formally, there is a continuum of wholesale banks indexed by w with preferences linear in consumption. At the end of $t-1$, a wholesale bank purchases capital, K_{wt-1}, at price Q_{t-1} financed with net worth, N_{wt-1}, and external financing from the full continuum of retail banks. This external finance takes the form of a two-stage intertemporal loan: 1) At an intra-period stage of time t retail banks decide whether to rollover the loan. 2) Rolled over loans are scheduled for repayment at the end of t. Thus, a wholesale bank's balance sheet has a maturity mismatch; its assets (capital) earn

a return at the end of t but its liabilities (the debt) must be rolled intra-period.[9]

At the end of $t-1$ wholesale banks are homogeneous in all respects except net worth. Thus, wholesale bank leverage, $\ell_t = Q_t K_{wt}/N_{wt}$, and the terms of the debt contract (such as the loan rate) will be common across banks. At the beginning of t, wholesale bank w observes its idiosyncratic productivity that will transform its capital to $\omega_{wt} K_{wt-1}$, where ω_{wt} is iid across time and space, $\mathbb{E}(\omega_{wt}) = 1$, with cdf F, pdf f, and support on $[0, \infty)$. The distribution is assumed to be log-normal and common knowledge. The aggregate return per unit of productive capital, $\omega_{wt} K_{wt-1}$, is given by $R_{K,t}$. The contract specifies a non-default loan rate, $R_{L,t}$.[10] If a retail bank does not rollover at the intra-period stage, the loan must be repaid immediately with loan rate $\tilde{R}_{L,t} = R_{L,t}/R_{K,t}$.[11]

Table 2: Timing in a given period

1.	The aggregate state, $\{A_t, \lambda_t\}$, is realized and $\bar{\omega}_t$ is determined from a state-contingent menu.
2.	The idiosyncratic productivity, ω_{wt}, of each wholesale bank is realized.
3.	Retail banks receive signals ω_{rwt} and decide whether to rollover funding.
4.	Banks with capital use their idiosyncratic productivity, ω_{wt} or γ, to augment their capital.
5.	The augmented capital is rented to goods producers at rental rate MPK_t.
6.	After production, capital is sold to the capital producers at price Q_t.
7.	Wholesale banks repay retail banks and retail banks repay households.
8.	A measure, $1-v$, of wholesale banks exit, consume their profits and a new measure, $1-v$, enter.
9.	Wholesale banks buy capital, K_{wt}, at price Q_t using net worth, N_{wt}, and new external financing.

Suppose all retail banks rollover. It is then possible to define a wholesale bank's solvency condition in terms of a cutoff value of idiosyncratic productivity, denoted $\bar{\omega}_t$, such that the

---

[9]The maturity mismatch is not beneficial per se, as in, Diamond and Dybvig (1983) where some depositors have liquidity needs at the intra-period stage. Instead, the assumed market structure is motivated by Brunnermeier and Oehmke (2013)'s *maturity rat race* in which a negative externality induces borrowers to use excessively short-term financing. The externality arises because short maturity claims dilute the value of long maturity claims, causing a borrower to successively move towards short-term financing, with complete short-term financing the only stable equilibrium.

[10]Following Bernanke et al. (1999), the debt contract will have wholesale banks absorb the aggregate risk. Retail banks hold a portfolio of wholesale bank debt which perfectly diversifies idiosyncratic risk. Thus, retail banks hold perfectly safe portfolios with a predetermined return that can be passed on to depositors. Carlstrom et al. (2016) and Dmitriev and Hoddenbagh (2017) study the consequences of (optimal) state contingent deposit contracts.

[11]This assumption reduces the complexity of the debt contract and since $R_{L,t} > \tilde{R}_{L,t}$ does not materially change the behaviour of the model.

wholesale bank has just enough resources to repay the loan, given by

$$\bar{\omega}_t R_{K,t} \ell_{t-1} = R_{L,t} \left(\ell_{t-1} - 1 \right). \tag{11}$$

However, retail banks need not rollover. If a retail bank does not rollover, the wholesale bank pays $\tilde{R}_{L,t} \left(Q_{t-1} K_{wt-1} - N_{wt-1} \right)$ in units of capital (since the wholesale bank has no other assets at the intra-period stage), which is equivalent to $\bar{\omega}_t Q_{t-1} K_{wt-1}$. Transferring capital, however, incurs a cost: For every unit of capital transferred at the intra-period stage, the wholesale bank loses $1 - \lambda_t$ units of capital, where $\lambda_t \in (0,1)$ is an exogenous stochastic measure of the liquidity of wholesale banks' assets.

When one is forced to sell an asset quickly in an illiquid market, one must content oneself with a price for the asset below its fundamental value. It is this feature of credit markets which is captured by λ_t. Thus, wholesale banks' balance sheets not only exhibit a maturity mismatch, but also a liquidity mismatch. The intra-period value of a wholesale bank's capital is $\lambda_t K_{wt-1}$ while the claims on the wholesale bank if no retail banks rollover is $\bar{\omega}_t K_{wt-1}$. When the loan rate is such that $\bar{\omega}_t > \lambda_t$, the wholesale bank is illiquid and vulnerable to a credit run. Appendix A.2 provides conditions under which $\bar{\omega}_t$ (which is endogenous) is greater than λ_t in equilibrium. Thus, the coordination problem is an endogenous phenomenon.

Retail banks Retail banks are perfectly competitive, accept deposits from households (paying a risk-free return) and hold a fully diversified portfolio of wholesale banks' short-term debt. As described above, retail banks choose whether to rollover loans at an intra-period stage. By not rolling over, a retail bank receives a fraction of the wholesale bank's capital and earns a return by taking over the capital management process (i.e. augmenting the capital itself and renting the augmented capital to firms).

This section solves the retail banks' rollover decision rule, conditional on a given debt contract specified by $\{\ell_{t-1}, \bar{\omega}_t\}$. At the beginning of t, each retail bank, indexed by r, observes a noisy signal of each wholesale bank's idiosyncratic productivity, $\omega_{rwt} = \omega_{wt} + \varepsilon_{rwt}$. The noise term, $\varepsilon_{rwt} \sim U \left(-\bar{\varepsilon}, \bar{\varepsilon} \right)$, is iid across time and space. Given the signals, retail banks decide individually for each wholesale bank, whether to rollover funding.

A retail bank's payoff (to rolling over or not) depends on a wholesale bank's realized idiosyncratic productivity and on the actions of other retail banks. These payoffs are derived in Appendix A.2 and given in Table 3, where $1 - p_t$ is the proportion of retail banks that rollover. γ is the productivity of the retail bank (i.e. the retail banks' equivalent of ω_{wt}) and measures the retail banks' ability to do a wholesale bank's job. When $\gamma = 0$, retail and wholesale banks are complements in the financial sector—neither can do the others' job. When $0 > \gamma > 1$, there is some substitutability with retail banks able to earn a positive

return from capital management.[12]

Table 3: Payoffs to retail banks

Rollover	Not rollover				
$\bar{\omega}$	$\gamma\bar{\omega}$	when	$0 \leq p \leq \frac{\lambda}{\bar{\omega}}$	and	$\omega \geq \frac{\bar{\omega}(1-p)\lambda}{\lambda - p\bar{\omega}}$
$\frac{\omega}{1-p}\left(1 - \frac{p\bar{\omega}}{\lambda}\right)$	$\gamma\bar{\omega}$	when	$0 \leq p \leq \frac{\lambda}{\bar{\omega}}$	and	$\omega < \frac{\bar{\omega}(1-p)\lambda}{\lambda - p\bar{\omega}}$
0	$\frac{\gamma\lambda}{p}$	when	$\frac{\lambda}{\bar{\omega}} < p \leq 1$		

Note: Payoffs are normalized by $R_{K,t}Q_{t-1}K_{wt-1}$, and are for a given retail bank's action regarding a given wholesale bank. Since the problem is a static, symmetric game, all time and bank indexes have been dropped.

Table 3, top-left: When the fraction of retail banks not rolling over, p_t, is low and the wholesale bank's productivity, ω_{wt}, is high, a retail bank that rolls over receives the non-default return $\bar{\omega}_t$. Bottom-left: If however, the retail bank rolls over when a large fraction of retail banks do not, such that the wholesale bank runs out of capital, the rolled over retail bank gets zero. Middle-left: This is the intermediate outcome in which the wholesale bank survives the intra-period stage but cannot fully repay its debt obligation at the end of t.

The right column gives the payoffs from not rolling over. If a retail bank does not rollover, but few other retail banks do likewise so the wholesale bank does not lose all its capital, the retail bank receives $\bar{\omega}_t$, applies its own productivity, γ, and earns $\gamma\bar{\omega}_t$. Right-bottom: If the fraction of retail banks that do not rollover is high, the wholesale bank has insufficient liquid capital and the liquid capital, λ_t, is divided equally among the retail banks that do not roll over, earning $\gamma\lambda_t/p_t$.

The payoff structure exhibits *strategic complementarities*. Up to the critical p_t at which the wholesale bank fails (i.e. runs out of capital), the *net* payoff from rolling over (i.e. rollover minus not rolling over) decreases as the fraction of retail banks that do not rollover, p_t, rises. Thus, retail banks face a strategic environment in which higher-order beliefs regarding the actions of other retail banks are important.

Proposition 1 states that, under certain technical assumptions, a retail bank's action is uniquely determined by its signal: It only rolls over if its signal, ω_{rwt}, is above a threshold, denoted ω_t^*.

Proposition 1 *There is a unique (symmetric) equilibrium in which a retail banks does not rollover if it observes a signal ω_{rwt} below threshold ω_t^*, and rolls over otherwise.*

---

[12]When γ is sufficiently high, retail banks will have no incentive to lend to wholesale banks since they will be able to earn a higher return by directly buying and managing capital themselves.

The proof is given in Appendix A.2.[13] In computing the threshold, ω_t^*, observe that a retail bank with signal $\omega_{rwt} = \omega_t^*$, must be indifferent between rolling over and not. The retail bank's posterior distribution of ω_{wt} is uniform over the interval $[\omega_t^* - \varepsilon, \omega_t^* + \varepsilon]$. Moreover, the retail bank believes that the fraction of retail banks not rolling over, p_t, is given by

$$p(\omega_w, \omega^*) = \begin{cases} 1 & \omega^* - \varepsilon_{rw} > \omega_w \\ \frac{1}{2} + \frac{\omega^* - \omega_w}{2\varepsilon_{rw}} & \text{if} \quad \omega^* - \varepsilon_{rw} \leq \omega_w \leq \omega^* + \varepsilon_{rw} \\ 0 & \omega_w < \omega^* + \varepsilon_{rw} \end{cases}.$$

Thus, the posterior distribution of p_t is uniform over $[0,1]$. In the limit when $\bar{\varepsilon} \to 0$ the resulting indifference condition is given by[14]

$$\int_{p=\frac{\lambda}{\bar{\omega}}}^{1} -\frac{\gamma\lambda}{p}dp + \int_{p=0}^{\frac{\lambda}{\bar{\omega}}} \left(\frac{\omega^*}{1-p} \left(1 - \frac{p\bar{\omega}}{\lambda} \right) - \gamma\bar{\omega} \right) dp = 0. \tag{12}$$

where solving for ω_t^* leads to Proposition 2.

Proposition 2 *In equilibrium, with the noise component of retail banks' signals arbitrarily close to zero, the rollover threshold, ω_t^*, is given by*

$$\omega_t^* = \gamma\lambda_t x_t \left(1 - \ln(x_t) \right) \left(x_t + (1 - x_t)\ln(1 - x_t) \right)^{-1}, \tag{13}$$

where $x_t \equiv \lambda_t/\bar{\omega}_t$. It is a striking result of Proposition 1 that there exists a unique switching equilibrium, even when the noise term in retail banks' signals is arbitrarily close to zero. Thus, in equilibrium, wholesale banks never experience a partial run but experience a complete run and full rollover with probability $F(\omega_t^*)$ and $1 - F(\omega_t^*)$, respectively.

To see the inefficiency created by the coordination problem, consider the decision of a retail bank when it is the sole holder of a wholesale bank's debt (or, equivalently, a scenario in which retail banks can costlessly and credibly coordinate their actions).[15] If it does not

---

[13]The proof requires that there exist lower and upper dominance regions, $[0, \omega^L)$ and (ω^H, ∞), in which a retail bank would not rollover and rollover, respectively, regardless the actions of other retail banks. This requirement is discussed in Appendix A.2. When retail banks receive a noiseless signal on the interval $[\omega^L, \omega^H]$, there are multiple equilibria. But, a grain of doubt for retail banks (i.e. $\bar{\varepsilon}$ arbitrarily close to zero) leads to the starkly different (and very useful) result given in Proposition 1.

[14]This condition specifies ω^* for $\omega^* < \bar{\omega}$. The complete indifference condition is given in Appendix A.2.

[15]There are two benchmarks against which to gauge the inefficiency. The first, applied in the text, is to assume that creditors can perfectly coordinate their actions. The second is to assume long-term debt. This eliminates the rollover decision and thus eliminates the coordination problem. A standard RBC model is akin to this second benchmark. Under the first benchmark, it is efficient to coordinate and not rollover for wholesale banks with extremely low values of ω_{wt} (i.e. $\omega_{\text{eff}}^* > 0$). Even after accounting for the liquidation cost, retail banks' ability to manage capital means they are able to generate a higher return. It is therefore possible to rank the three scenarios: Short-term contracts with perfect coordination are preferred to long-term

rollover it gets $\gamma\lambda_t$ and if it rolls over it gets the lesser of $\bar{\omega}_t$ and ω_{wt}. The optimal action is to rollover when $\omega_{wt} > \gamma\lambda_t$ and not rollover otherwise. The efficient, full coordination, threshold is given by $\omega_{\text{eff}}^* = \gamma\lambda_t$ and is lower than ω_t^*, implying that, because of coordination problems, the probability of a run is higher than is optimal. This leads to Propositions 3.

Proposition 3 *i) The inefficiency wedge, $\omega^*/\omega_{\text{eff}}^*$, is increasing in the illiquidity, $x = \lambda/\bar{\omega}$, of the wholesale bank: $\partial\left(\omega^*/\omega_{\text{eff}}^*\right)/\partial x < 0$. ii) In the limit, when the wholesale bank is not illiquid, there is no inefficiency: $\lim_{x\to 1}\left(\omega^*/\omega_{\text{eff}}^*\right) = 1$.*

To get a sense of the dynamic implications, suppose the loan rate (and thus $\bar{\omega}_t$) moves countercyclically. This implies that in a recession, for a given λ_t, the distortionary effects of coordination problems in the credit market are magnified, generating a financial accelerator.

The contracting problem A debt contract can be characterized by the pair $\{\ell_t, \bar{\omega}_{t+1}\}$. Since retail banks are perfectly competitive and pay a predetermined return to depositors, R_t, the debt contract must satisfy a zero profit break-even condition for the retail banks. And since retail banks hold fully diversified portfolios of loans, a retail bank's expected (normalized) payoff is given by

$$\underbrace{\bar{\omega}\int_{\bar{\omega}}^{\infty}f\left(\omega\right)d\omega}_{\substack{\text{i. Rolled over}\\\text{and pay in full}}} + \underbrace{\int_{\omega^*}^{\bar{\omega}}\omega f\left(\omega\right)d\omega}_{\substack{\text{ii. Rolled over}\\\text{and default}}} + \underbrace{\gamma\lambda_t\int_{0}^{\omega^*}f\left(\omega\right)d\omega}_{\substack{\text{iii. Not rolled}\\\text{over}}}, \tag{14}$$

which is an integral over three possible outcomes:[16] The wholesale bank i) survives the intra-period stage and repays its loan in full; ii) survives the intra-period stage but is insolvent resulting in partial payment; or iii) does not survive and the the retail banks earn a return managing the capital themselves. We can rewrite (14) as $\Gamma_t - G_t$ where

$$\Gamma \equiv \bar{\omega}\int_{\bar{\omega}}^{\infty}f\left(\omega\right)d\omega + \int_{0}^{\bar{\omega}}\omega f\left(\omega\right)d\omega \quad \text{and} \quad G \equiv \int_{0}^{\omega^*}\left(\omega - \gamma\lambda_t\right)f\left(\omega\right)d\omega. \tag{15}$$

Thus, $1 - \Gamma_t$ and $\Gamma_t - G_t$ are the fraction of gross returns accruing to wholesale and retail banks, respectively, where G_t captures the deadweight loss resulting from coordination failure.

The contracting problem maximizes the wholesale banks' return subject to the retail banks' break even condition, given by

$$\max_{\ell_t, \bar{\omega}_{t+1}} \mathbb{E}_t\left(1 - \Gamma_{t+1}\right)S_{t+1}\ell_t \quad \text{s.t.} \quad \left(\Gamma_{t+1} - G_{t+1}\right)S_{t+1}\ell_t \geq \left(\ell_t - 1\right), \tag{16}$$

---

contracts, but long-term contracts are preferred to short-term contracts without coordination.

[16]This assumes $\omega_t^* < \bar{\omega}_t$. See Appendix A.2 for when this does not hold.

where $S_t \equiv R_{K,t}/R_{t-1}$. The constraint holds for all realizations of $R_{K,t+1}$ because the retail banks' return, R_t, is assumed to be predetermined. The combined first-order condition (substituting out the Lagrange multiplier) is given by

$$\mathbb{E}_t \left(\frac{d\Gamma_{t+1}}{d\Gamma_{t+1} - dG_{t+1}} \right) = \mathbb{E}_t \left(S_{t+1} \left(1 + \frac{\Gamma_{t+1}dG_{t+1} - d\Gamma_{t+1}G_{t+1}}{d\Gamma_{t+1} - dG_{t+1}} \right) \right), \tag{17}$$

where $d\Gamma_t \equiv \partial\Gamma_t/\partial\bar{\omega}_t$ and $dG_t \equiv \partial G_t/\partial\bar{\omega}_t$. The break-even condition from (16) and equation (17) combine to implicitly define a loan supply schedule that has the following form

$$\mathbb{E}_t S_{t+1} = \theta \left(\underset{(+)}{\ell_t}, \underset{-}{\lambda_t} \right), \tag{18}$$

where $\mathbb{E}_t S_{t+1}$ is denoted the *illiquidity premium*. The illiquidity premium is increasing in wholesale banks' leverage and decreasing in the intra-period liquidity of capital. Endogenous fluctuations in leverage and exogenous fluctuations in liquidity generate a time-varying wedge between the expected return on capital and the risk-free rate in the economy.

2.5   Closing the model

Aggregation is straightforward since, at the end of a period, wholesale banks are only heterogenous in net worth but choose a common leverage ratio and face the same interest rate on loans. Net worth of wholesale banks are its end-of-period profits. A fraction $1 - \upsilon$ of wholesale banks are the forced to exit the market and consume their profits while a measure $1 - \upsilon$ of new wholesale banks enter. In addition, all wholesale banks receive a positive but arbitrarily small lump-sum tranfer from households. These two assumptions ensure that i) wholesale banks do not accumulate sufficient net worth to no longer need external finance and ii) the net worth of every wholesale bank is positive and the contracting problem is well defined. It follows that the evolution of aggregate net worth, N_t, is given by

$$N_t = \upsilon \left((1 - G_t) S_t \ell_{t-1} - (\ell_{t-1} - 1) \right) R_{t-1} N_{t-1}. \tag{19}$$

The deadweight cost of coordination failure is in units of capital. Thus, while the aggregate stock of capital purchased in $t - 1$ for use in t is K_{t-1}, only $K_t^* = (1 - G_t) K_{t-1}$ is put to productive use. Aggregate wholesale bank consumption of exiting bankers is given by $C_{W,t} = \frac{1-\upsilon}{\upsilon} N_t$. Finally, the economy's aggregate resource constraint is given by $Y_t = C_t + C_{W,t} + I_t$.
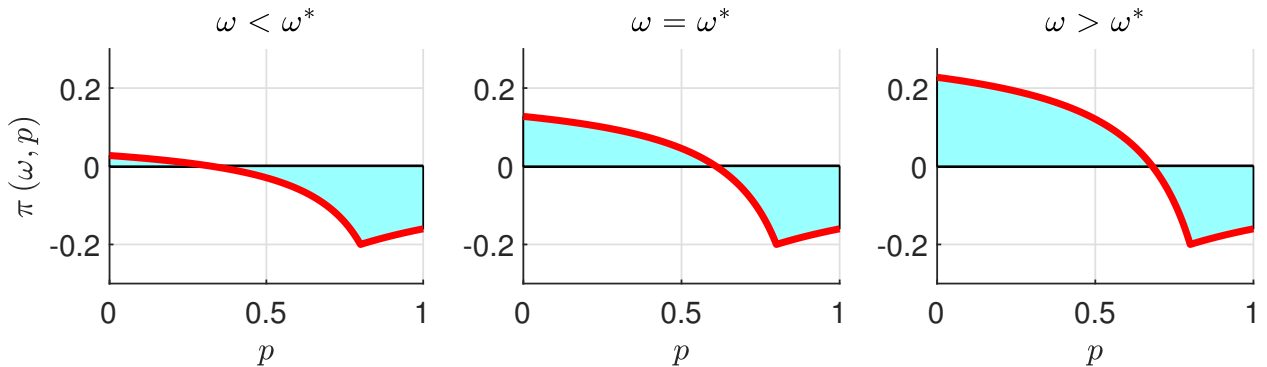
3 Results

3.1 Comparative statics

This section analyzes the comparative static properties of the static coordination failure model. In particular, it shows the interplay between liquidity, coordination problems, leverage and the illiquidity premium before the full dynamic properties of the model are explored.

In the next subsection, the full model is carefully parameterized to capture moments in US aggregate data. For the present comparative static exercises, the following stylized parameterization is used: $\gamma = \lambda = 0.4$, $\bar{\omega} = 0.5$, and $\sigma^2 = 0.35$, where $\ln(\omega) \sim N\left(-\frac{1}{2}\sigma^2, \sigma^2\right)$. These parameter values aid graphical illustration without altering the qualitative properties of the quantitatively parameterized model presented in the next section.

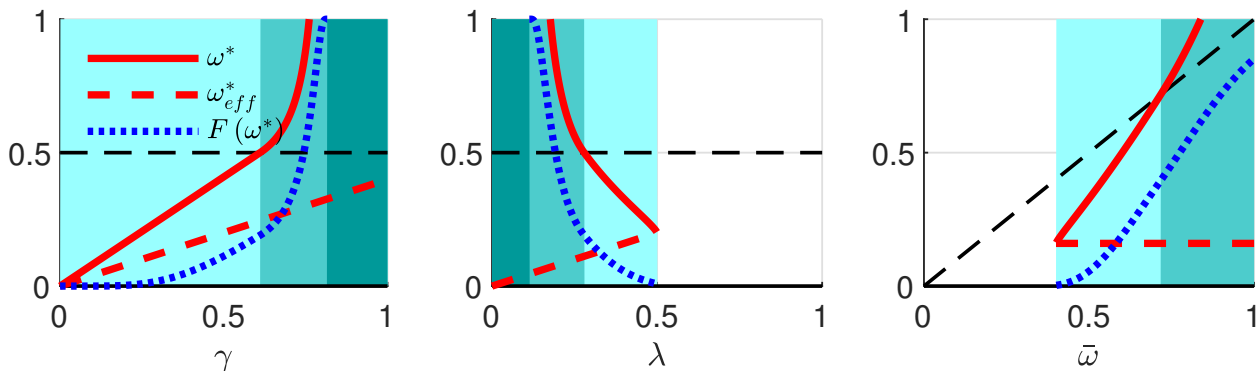Figure 1: Net payoff function

Note: Set $\gamma = \lambda = 0.4$ and $\bar{\omega} = 0.5$. This gives $\omega^* = 0.3275$. The left and right panels set $\omega = \omega^* \pm 0.1$. ω^* is the value of ω that equalized the shaded area above and below the line.

The red line in Figure 1 plots the net payoff function to rolling over for various ω values. The payoff function exhibits two important features: A single crossing where the net payoff to rolling over is zero, and a negative slope, which implies strategic complementarities. In the left-panel (low ω), the equilibrium action is to not rollover whereas in the right-panel (high ω), the equilibrium action is to rollover. The middle panel defines the threshold ω^* (solving the indifference condition in (12)), which is when the sum of the blue areas equals zero.

Figure 2 plots the behaviour of ω^* and ω^*_{eff} as γ, λ, and $\bar{\omega}$ are altered, respectively. In Section 2, I focused the model's derivation on a specific region of the parameter space, which I term the *mild fragility* scenario. However, as Appendix A.2 describes in detail, the model generates four scenarios: 1) No fragility, 2) Mild fragility, 3) Acute fragility, and 4) No rollover. These are represented in Figure 2 by the shaded areas from white to dark green, respectively. In the middle panel, when λ is sufficiently high (white region), there is no
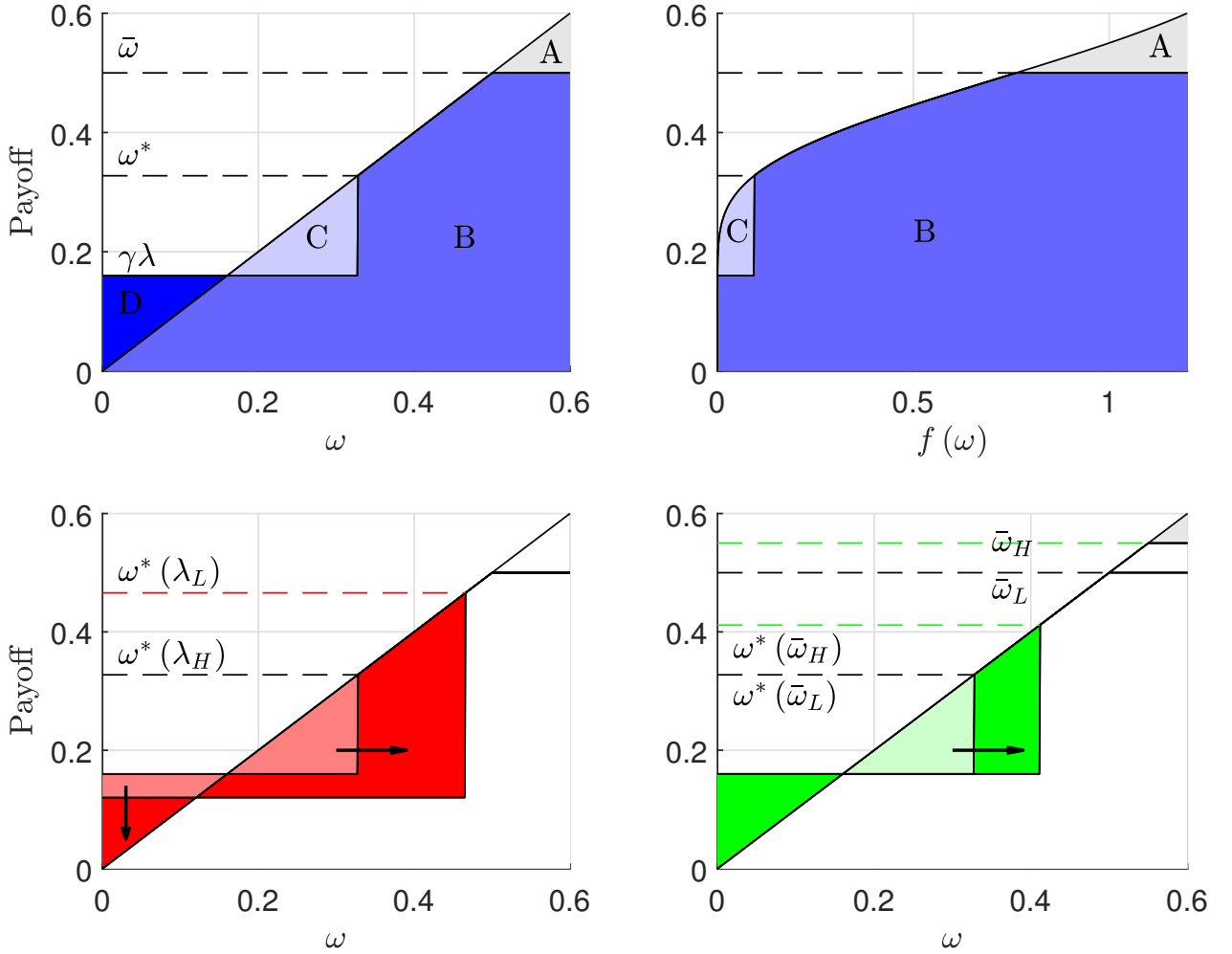
Figure 2: Comparative statics for ω^*

Note: Set $\gamma = \lambda = 0.4$ and $\bar{\omega} = 0.5$. The shaded areas from white to dark-green represent Scenario 1) No fragility; 2) Mild fragility; 3) Acute fragility; and 4) No rollover.

coordination problem: Retail banks base their rollover decision on their private signal alone and not on higher order beliefs about other retail banks' actions. The pale green region is the mild fragility scenario that is the focus of this paper. In this scenario, $\omega^* < \bar{\omega}$. A fall in liquidity, (i.e. a lowering of λ) raises the rollover threshold and lowers the efficient rollover threshold, thus increasing the inefficiency generated by the coordination problem. When λ falls further, the economy enters the acute fragility scenario in which $\omega^* > \bar{\omega}$. Notice the curvature of the red line. In this scenario, the rollover threshold becomes very sensitive to small changes in liquidity. And, as a result, the proportion of wholesale banks that experience a run, $F(\omega^*)$, also increases dramatically (blue-dot line). Finally, when the liquidity of the capital stock is extremely low, there is a no rollover scenario, in which case rollover never occurs and the market effectively fails to exist. Clearly, there are interesting nonlinearities in this model, but which are beyond the scope of the current paper to explore further. This paper focuses on perturbations of the model in the light-green (mild fragility) region. One can think of the model of Gertler and Kiyotaki (2015) and Gertler et al. (2016) as studying large jumps from the white (no fragility) region to the dark-green (no rollover) region.

Figure 3 plots the debt contract, showing how the return on capital is split between the borrower and creditor for different realizations of the wholesale bank's idiosyncratic ω. Top-left panel: A textbook debt contract has the creditor receiving the area $\Gamma(\cdot) = B + C$ and the borrower receiving the residual: $1 - \Gamma(\cdot) = A$. Short-term debt and coordination failure generates a deadweight cost given by $G(\cdot) = C - D$. C is a loss since retail banks would have received a higher return if all retail banks had rolled over for these realizations of ω. D is a gain resulting from the existence of short-term debt: The retail bank receives a higher return for these realizations of ω relative to the case in which all debt contracts were
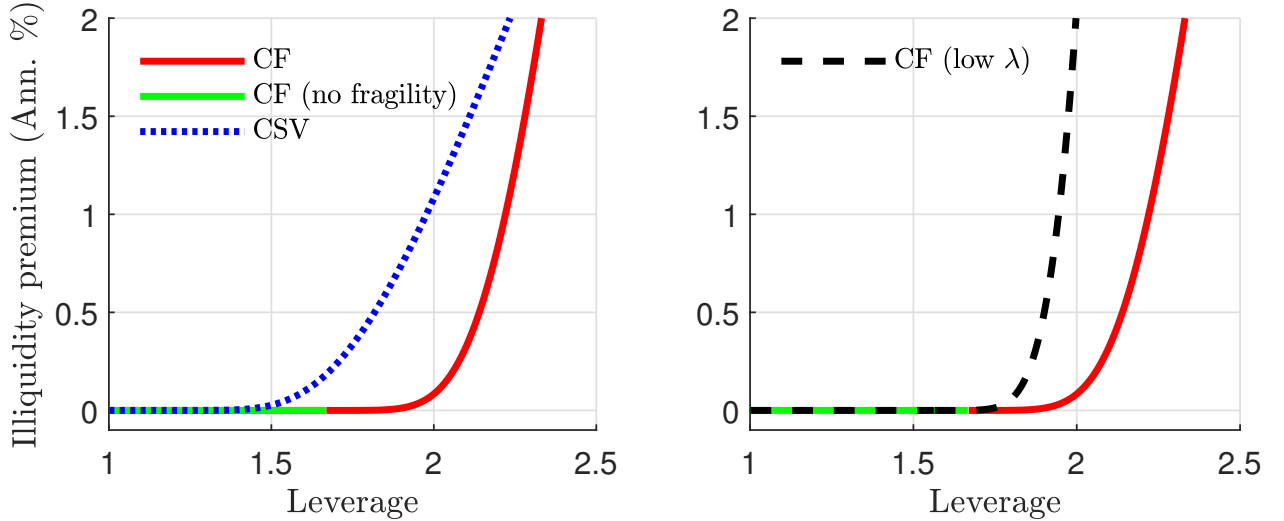
13

Figure 3: Debt contract

Note: Top-left: $\Gamma\left(\cdot\right) = B+C$ and $1-\Gamma\left(\cdot\right) = A$. Deadweight cost given by $G\left(\cdot\right) = C-D$. Top-right panel is probability-weighted. Bottom-left: Comparative static of a rise in illiquidity (i.e. fall in λ from λ_H to λ_L). The deadweight cost rises (i.e. area $C-D$ expands). Bottom-right: Comparative static of a rise in $\bar{\omega}$ from $\bar{\omega}_L$ to $\bar{\omega}_H$. The deadweight cost again increases.

long-term. The top-right panel shows the same panel but probability-weighted and shows that, given the distributional assumptions, area D is negligible. The bottom-left panel shows the effect of a rise in illiquidity caused by a fall in λ from 0.4 to 0.3. As a result, ω^* rises and the deadweight cost rises (i.e. the area $C-D$ expands). The bottom-right panel shows the effect of an exogenous rise in the lending rate, with $\bar{\omega}$ rising from 0.5 to 0.55. This also increases ω^* and the deadweight cost of coordination failure.

Figure 4 plots the supply curve of loanable funds by translating the deadweight costs of coordination failure studied above into a tradeoff between wholesale banks' leverage and the illiquidity premium they pay on external finance. This is done by endogenizing $\bar{\omega}$. The

Figure 4: Illiquidity premium and leverage

Note: Set $\gamma = \lambda = 0.4$, $\bar{\omega} = 0.5$ and $\sigma = 0.35$. CF denoted coordination failure model. CSV denotes the costly state verification model of Bernanke et al. (1999) with monitoring cost parameter $\mu = 0.166$.

left panel shows that if wholesale banks have low leverage (green line), then there is no coordination problem for retail banks and as a result the illiquidity premium is zero. However, as leverage rises above a critical value then the coordination problem appears (red line), generating an illiquidity premium which is increasing in leverage. An interesting comparison is with an alternative model of financial frictions, the costly state verification (CSV) model of Townsend (1979). In this model, costly state verification generates a *risk* premium even for low levels of leverage but grows more gradually (blue-dot line). The implication of this comparison is that the CSV-type risk premium is larger during normal times (when leverage is relatively low) than the CF-type illiquidity premium, but in periods of financial stress when borrower net worth falls and leverage sharply rises, then illiquidity premium rise rapidly and can even become larger than typical risk premia.

The right panel shows the effect of a fall in liquidity. When λ falls, the loanable funds supply schedule shifts inward and steepens, resulting in a higher illiquidity premium for a given leverage ratio. The next section studies the dynamic effect of such an illiquidity shock.

3.2 Parameterization

Table 4 list the calibrated and estimated parameters. The model parameters are partitioned into three groups. The first group of parameters are results from previous studies. The second group are calibrated using steady state relationships.

Following Basu and Bundick (2017), the third group of parameters are estimated. These

15

Table 4: Structural parameters

α	0.33	χ	$5.02^{\ddagger}$	σ_a	$0.0055^{\dagger}$	υ	$0.981^{\ddagger}$
β	0.99	ρ	3	ρ_a	$0.868^{\dagger}$	σ^2	$0.048^{\ddagger}$
δ	0.025	φ	$0.273^{\dagger}$	σ_λ	$0.0041^{\dagger}$	γ	$0.589^{\ddagger}$
σ_C	2.0			ρ_λ	$0.893^{\dagger}$	λ	$0.703^{\ddagger}$

Note: Parameters with $\ddagger$ are calibrated to match steady state moments. Parameters with $\dagger$ are estimated using simulated method of moments to match US aggregate data moments.

parameters are chosen to minimize the distance between the model moments and moments in US aggregate data. Formerly, the estimator is the solution of the following problem

$$J = \min_{\theta} \left(M^D - M(\theta) \right)' W^{-1} \left(M^D - M(\theta) \right), \tag{20}$$
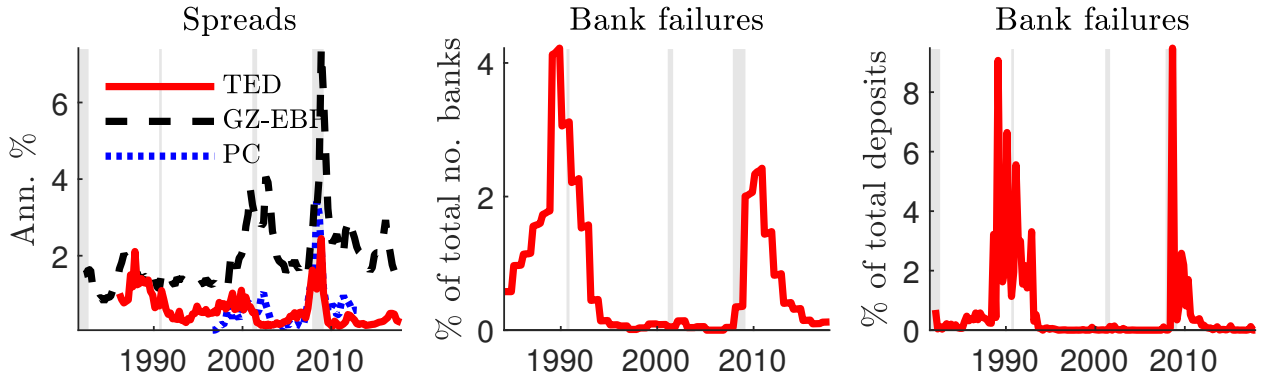
where M^D denotes a vector that comprises the unconditional variances and autocorrelations of output, consumption, investment, hours worked and the illiquidity premium in the data and $M(\theta)$ denotes its model counterpart and $\theta = \{\varphi, \sigma_a, \rho_a, \sigma_\lambda, \rho_\lambda\}$. W is a diagonal matrix that contains the standard errors of the data moment estimates.

Based on standard values in the literature at a quarterly frequency, the subjective discount factor is $\beta = 0.99$, the intertemporal elasticity of substitution, $1/\sigma_C = 2.0$, the inverse of the Frisch elasticity of labor supply is $\rho = 3$ and the depreciation rate of capital is 0.025. The weight on leisure in the utility function, χ is chosen such that steady state labor supply is normalized to 1.

The financial sector is governed by four structural parameters: The capital management productivity of retail banks, γ, the steady-state intra-period liquidity of capital, λ, the variance of the distribution of idiosyncratic shocks, σ^2, and the proportion of wholesale banks that survive each period, υ. The values are pinned down by four steady state moments of the model that approximately match long-run averages in US data, given in Table 5.

One, I use the TED spread, which is the spread between the 3-month LIBOR and the 3-month Treasury bill, as a proxy for the illiquidity premium. LIBOR is the rate banks charge each other for lending. Since the coordination problem in the lending relationship between retail and wholesale banks is the only wedge between the return on deposits and the return on capital in this model, the TED spread is a good proxy for the illiquidity premium. For comparison, Figure 5 plots the TED spread alongside the Gilchrist and Zakrajsek (2012a) excess bond premium, which is significantly larger but captures a much broader definition

Figure 5: Illiquidity premium and bank failure

Note: TED is the spread between 3-month LIBOR and the 3-month Treasury bill. GZ-EBP is the excess bond premium from Gilchrist and Zakrajsek (2012a). PC is a liquidity premium calculated from 7 spread series using principal components analysis following Bredemeier et al. (2018). Bank failure source: FDIC. Shaded areas denote NBER recession dates.

of credit spreads/risk premiums. The figure also plots an alternative measure of the liquidity premium, which takes the principal component of seven spreads between the return on Treasuries and illiquid assets of similar safety and maturity. This follows the methodology of Bredemeier et al. (2018) (see Appendix C for details).[17] Thus, I match the steady state illiquidity premium to the mean of the TED spread from 1986-2017, which is 58bp.

Two, Figure 5 also plots a time-series of bank failures, which I constructed from FDIC data. The series goes back to 1934 but for consistency with the other time-series, I use only the period 1986-2017. In the model, the fraction of banks that fail and the fraction of failures weighted by balance sheet size are equivalent. In the data, however, these series are quite different. In particular, at the peak of the 2007-08 financial crisis, 2% of banks covered by the FDIC failed but this accounted for 9% of total deposits. In addition, there were a large number of bank failures in the aftermath of the crisis in 2010-12 but these were mainly small banks. Thus, the series for bank failures in terms of total deposits appears much less persistent. For the empirical exercise, I use the measure of bank failures and as a share of deposits. The mean annualized failure rate from 1986-2017 was 0.67%.

Three, leverage in the wholesale bank sector varies widely across financial institutions, with some banks with leverage in excess of 10. I take a conservative estimate and calibrate the steady state leverage of the wholesale bank sector to be 4 following Gertler et al. (2012).[18]

---

[17]Robustness exercises using this series are available on request.

[18]While the deterministic steady state is, by definition, independent of uncertainty, it is likely that wholesale banks' (privately) optimal leverage is dependent on the risk they face on their balance sheet. Gertler et al. (2012) and de Groot (2014) investigate how leverage at the *risk-adjusted* steady state can rise (and hence amplify the financial accelerator) in a low risk environment.

Finally, the average recovery ratio of liquidated assets is set at 50% following estimates from Berger et al. (1996).

Table 5: Targeted steady state moments

Moment	Description	Value
$R_K - R$	Illiquidity premium†	58bp (ann.)
$F(\bar{\omega})$	Bank failure rate††	0.67% (ann.)
K/N	Capital-to-net worth ratio	4‡
$\int_0^{\omega^*} \frac{\gamma\lambda}{\omega} f(\omega)\, d\omega$	Average recovery ratio of liquidated assets	50%‡‡

Note: † TED spread; †† source: FDIC; ‡ Gertler et al. (2012). ‡‡ Berger et al. (1996).

The remaining five parameters are estimated using simulated method of moments. The following 10 moments are included: The standard deviation and autocorrelation of output, consumption, investment, hours, and the TED spread. Four of the five parameters are the persistence and standard deviation of the two exogenous shock series: Technology, A_t, and liquidity, λ_t. The final parameter, φ, controls investment-adjustments costs. This is a key parameter for determining the strength of the financial accelerator. When φ is zero, the financial accelerator is generally weak because investment demand adjusts rapidly to changes in financial conditions.[19] The success of the model in matching the business cycle moments is reported in Table 6. Given the parsimony of the model, it does a reasonable job of matching business cycle moments. Most notably, hours worked are too volatilite and consumption is too persistent. However, the model captures well the properties of the TED spread.

3.3 Impulse responses and crisis scenario

Technology shocks   Figure 6 shows the reaction to a 1% negative technology shock.[20] The blue-solid and green-dot lines show the reaction of the model with and without coordination failure, respectively. Coordination failure does not significantly alter the qualitative shapes of the responses, but does alter the magnitude of the responses. Notably, the drops in investment, asset prices and the capital stock are magnified.

---

[19]The model is solved to first-order in the neighborhood of the deterministic steady state. The model, however, admits interesting non-linearities as shown in Section 3. Although it would be a fruitful avenue for future research to solve the model using global methods, it is beyond the scope of the current paper.
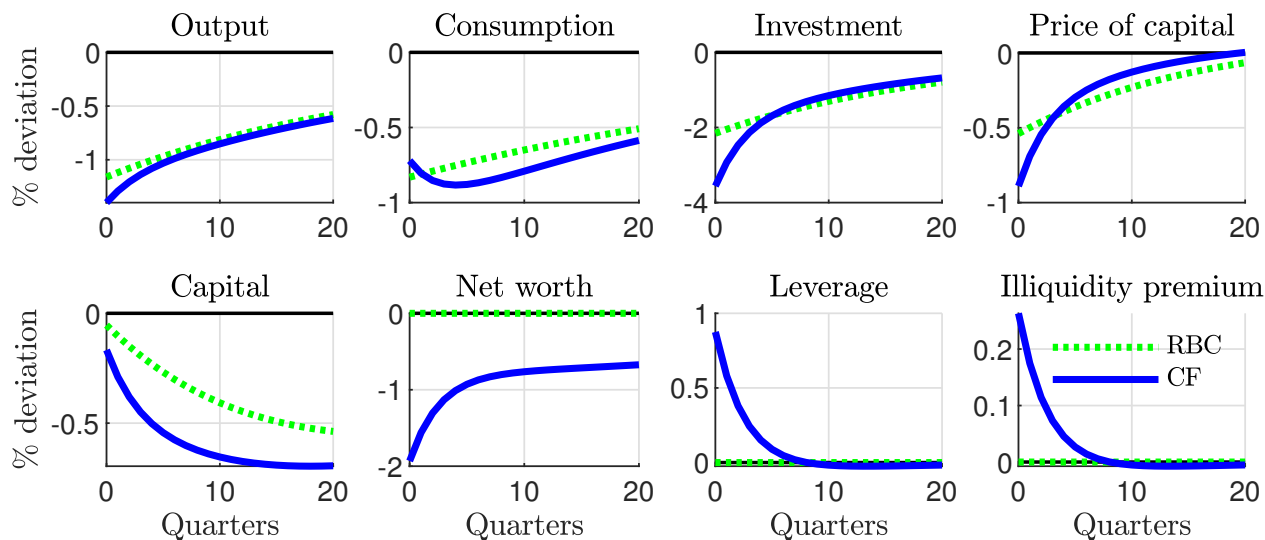
[20]Supplementary material on other shocks (e.g. capital quality and net worth shocks) and the behaviour of the rollover threshold, recovery, default, and rollover rates are available on request.

Table 6: Business cycle moments

Moment	Data	Model	Moment	Data	Model
std(GDP)	1.060	1.051	ac(GDP)	0.862	0.786
std(C)	0.864	0.686	ac(C)	0.820	0.965
std(I)	4.418	4.154	ac(I)	0.931	0.714
std(Hours)	0.478	0.794	ac(Hours)	0.792	0.676
std(TED)	0.410	0.359	ac(TED)	0.796	0.651

Note: Construction of data moments given in Appendix C.

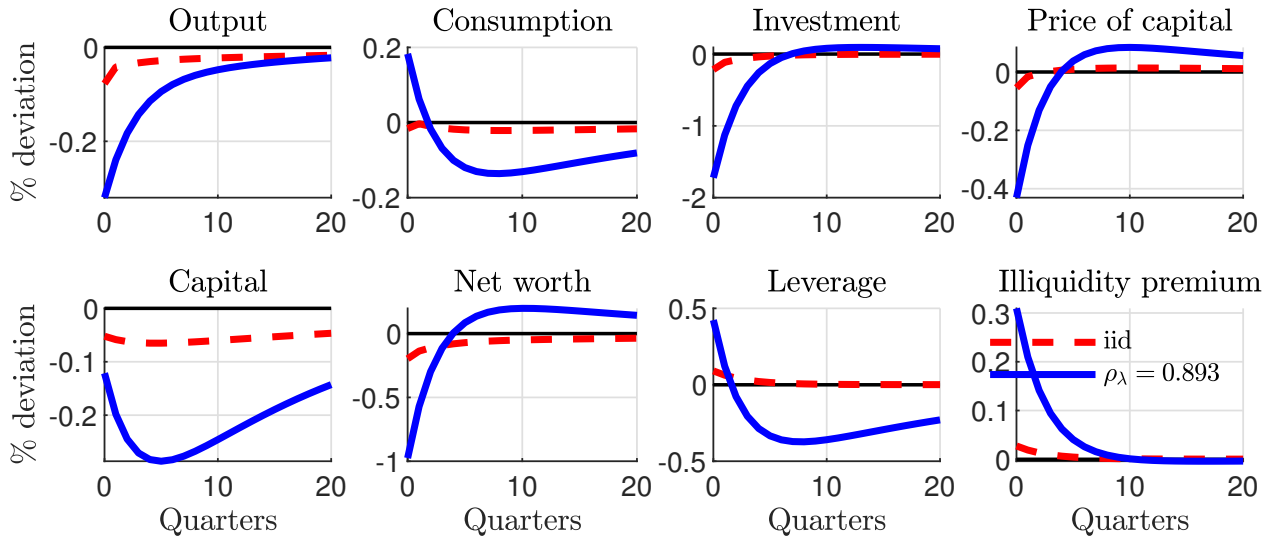Figure 6: Technology shock

Note: 1% negative technology shock

Coordination failure introduces three new aggregate variables of interest: Wholesale bank net worth and leverage and the illiquidity premium. The negative technology shock causes a drop in wholesale bank net worth and an increase in wholesale bank leverage. The risk-free (deposit) rate falls while the expected return on capital rises, leading to a sharp rise in the illiquidity premium.[21] These responses are the result of three features of coordination

---

[21] The countercyclical leverage ratio is a feature of most financial accelerator models. Adrian et al. (2013) criticize the financial accelerator models for failing to capture the procyclical leverage ratio in the US data. However, Gertler (2013) argues that the discrepancy is due to differences in the way net worth is measured in the data relative to the model. In the model, net worth is measured in terms of market values. In the data, in contrast, equity is measured using a mixture of book and fair value accounting, and even then, during periods of market illiquidity, even fair value accounting replaces market values with a "smoothed" value.

failure. First, the loan rate paid by wholesale banks is a function of the expected return on capital, which means that the wholesale banks absorb the aggregate risk. When the negative technology shock hits, the realized return on capital is below its expected return, which drives down the aggregate wholesale bank profits and hence their net worth. Second, wholesale bank net worth decreases faster than the demand for capital, implicitly causing leverage to rise. Third, higher leverage increases the severity of the coordination problem among retail banks, causing the illiquidity premium to rise. As a result, retail banks demand a higher loan rate (i.e. an increase in $\bar{\omega}_t$) following a negative productivity shock, which increases the illiquidity of wholesale banks yet further. This causes investment and the price of capital to fall further in response to a negative technology shock. Investment falls 3.8% on impact following a 1% technology shock in the coordination failure model, relative to a 2.0% in the frictionless case.

Illiquidity shock   A novel feature of the model is the ability to generate an exogenous rise in illiquidity in credit markets. Figure 7 shows the response to a 1% rise in intra-period capital illiquidity with an iid (red-dash) and a persistent shock (blue-solid line), respectively.

Figure 7: Illiquidity shock

Note: 1% illiquidity shock

A rise in the illiquidity of capital leads to a rise in the rollover threshold, ω^*, implying a higher incidence of bank runs. This causes a sharp rise in the illiquidity premium paid by wholesale banks on external finance. The rise in the illiquidity premium is the result of both a rise in the expected return on capital as well as a fall in the risk-free deposit rate. The retail banks cut back on the supply of loanable funds. However, the drop in the demand for

capital is insufficient to offset the fall in wholesale bank net worth. In order to break even, the retail banks lower the return paid on deposits. Households, facing a lower return on savings, generates a temporary consumption boom.[22] In the transition back to steady state, the fall in the demand for investment causes a decline in the capital stock. As the impact of the illiquidity shocks recedes, households cut consumption order to restore their steady state savings ratio. This requires a long period of deleveraging by wholesale banks.

The size and persistence in the response of capital (and hence output) is sensitive to the persistence of the exogenous rise in illiquidity. The evolution of capital is relatively gradual. A less persistent shock therefore gives less opportunity for capital to fall and do serious damage to potential output. Thus, the length and severity of a recession depends on how long the credit market remains illiquid.
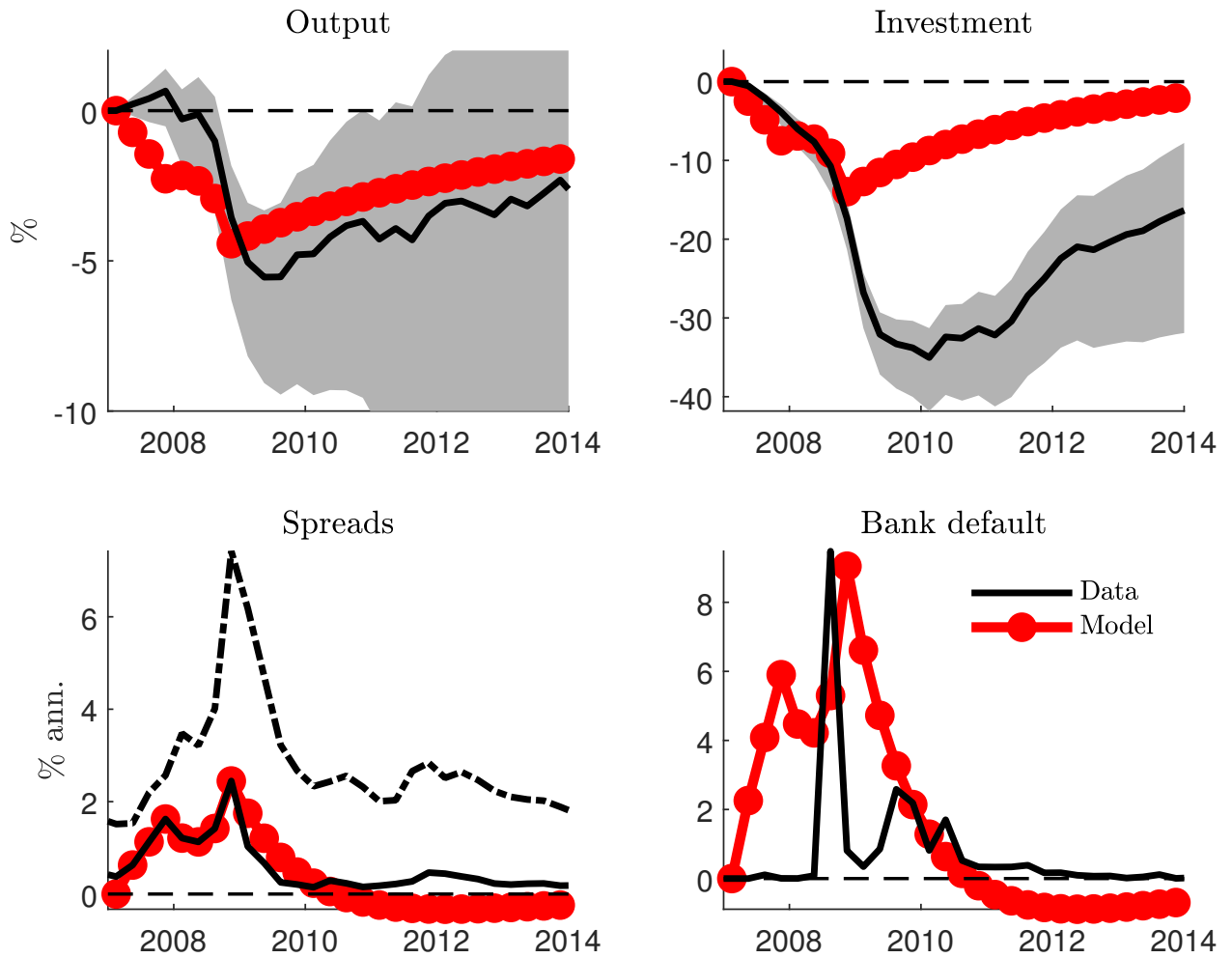
This model has several features unique to coordination failure. First, retail banks' are most likely to run when liquidity is most needed. Second, as the constraints on wholesale banks tighten countercyclically, the share of capital held by retail banks will move counter-cyclically. This occurred during the financial crisis with dealer banks in the shadow banking sector shrinking and commercial banks expanding their market share.

Illiquidity during the 2007-08 financial crisis   In this section, I provide an illiquidity shock narrative of the 2007-08 financial crisis as an external validation exercise for the model. Since the model is a parsimonous RBC model, chosen for clarity rather than quantitative accuracy, it is unlikely to provide a close fit to the data. However, the exercise provides a good way of assessing its strengths and weaknesses.

In this exercise, I suppose the economy was hit by a sequence of illiquidity shocks from 2007q2–2008q4 that match the evolution of the illiquidity premium (TED spread) in the data. Using simple projection methods, I estimate a distribution of counterfactual paths for output, investment, the illiquidity premium and bank failure had the financial crisis not occurred. The black line in Figure 8 represents the deviation between the actual path for these variables and the median of the counterfactual path. The methodology follows Christiano et al. (2015) and is described in Appendix C. The grey area captures the uncertainty in these counterfactual paths. The counterfactual evolution of output is most uncertain while the bank failure rate is estimated very precisely (i.e. without the financial crisis, the bank failure rate would have almost certainly remained at zero).

---

[22]The implication is that when an illiquidity shocks hits, consumption rises temporarily while output and investment fall. This is because of a high intertemporal elasticity of substitution for households which means that the substitution effect dominates the wealth effect. This result is shared by many other papers which incorporate shocks to preferences, investment goods prices or other financial frictions. A possible extension to the model to alleviate this result is to assume that technology shocks and illiquidity shocks are correlated. Indeed, during recessions, markets do seem to experience more illiquidity.

Figure 8: Illiquidity shocks during the 2007-08 crisis

Note: Black lines represent the data relative to an uncertain trend using following Christiano et al. (2015). Black-dash line is the Gilchrist and Zakrajsek (2012b) EBP. Red line is the model prediction with illiquidity shocks chosen to match exactly the TED spread from 2007q2–2008q4.

The model's illiquidity shocks captures well the fall in output during the financial crisis although only around a third of the drop in investment, suggesting that investment specific shocks also played an important role. The model captures well the peak bank failure rate of 9%. However, the model predicts that bank failures would have appeared as early as 2007 and would have persisted into 2010. Thus, when integrating over the total number of bank failures for the 2007-2010 period, the model overpredicts the number of bank failures.

Of course, there is a clear rationale for policymakers (monetary or fiscal) to offset the effect of illiquidity in the credit market, to which we turn in the next section.

4 Policy responses

The coordination failure model facilitates the analysis of two *unconventional* credit market policies adopted by the US Federal Reserve during the 2007-08 financial crisis: 1) Direct lending (or *liquidity injections*) in credit markets and 2) equity injections.[23]

4.1 Direct lending (DL)

In this scenario, the policymaker supplements private lending in credit markets with additional lending directly to wholesale banks with abnormal excess returns. Since wholesale banks in the model are funded by a continuum of retail banks, the retail banks cannot coordinate actions. The policymaker instead behaves as a single, large market participant with deep pockets. By committing to rollover, it is able to reduce the coordination problem.[24]

Let $\mathbf{n}_t$ denote the proportion of total lending to wholesale bank w provided by retail banks. Funding provided by the policymaker is therefore given by $(1 - \mathbf{n}_t)(Q_t K_{wt} - N_{wt})$. Retail banks can still choose not to rollover while the policymaker is assumed to always rollover.[25] The policymaker lends at the prevailing market rate $R_{L,t}$. However, by expanding the supply of run-free funds available in the market, it reduces the loan rate endogenously.

The new rollover threshold, augmented by direct lending, is given by

$$\omega^* = \gamma \lambda \mathbf{n} x \left(1 - \ln(x)\right) / \left(\mathbf{n} x + (1 - \mathbf{n} x) \ln(1 - \mathbf{n} x)\right), \qquad (21)$$

where $x \equiv \lambda / (\mathbf{n}\bar{\omega})$ is the new measure of endogenous illiquidity. The efficient rollover threshold becomes $\omega^*_{\text{eff}} = \gamma \lambda / \mathbf{n}$. It is straightforward to show that, all else equal, direct lending increases the efficient threshold while ω^* falls, thus reducing the market distortion.[26] Retail banks' break-even condition is unchanged except for the deadweight cost, G_t, given by

$$G = \int_0^{\omega^*} (\omega - \gamma \lambda / \mathbf{n}) f(\omega) \, d\omega. \qquad (22)$$

---

[23]Cúrdia and Woodford (2010), Reis (2009a) and Gertler and Kiyotaki (2010) also model the effect of credit market policies.

[24]In other respects, the government is likely less efficient at intermediating funds. Thus, think of this as only a crisis measure.

[25]In the crisis, the Fed implicitly lengthened the maturity structure of banks' borrowing by directly lending at longer maturities than private agents were willing to lend. See the Fed's press release: http://www.federalreserve.gov/newsevents/press/monetary/20081007c.htm

[26]In fact, a sufficiently large intervention will push the economy into the "no fragility" region described in Appendix A.2. This scenario is not considered here.

4.2 Equity injections (EI)

In this scenario, the policymaker acquires part ownership of wholesale banks.[27] I assume that government equity and private equity has the same characteristics. Let c_t be the proportion of total equity that is privately held by wholesale bankers, such that the funds injected by the government are given by $(1 - c_t) N_t$. The retail banks' break-even condition is unchanged but wholesale bank profits are split between the wholesale bankers and the government in proportion to c_t. The augment net worth equation is given by

$$N_t = \frac{c_{t-1}}{c_t} \upsilon \left((1 - G_t) S_t \ell_{t-1} - (\ell_{t-1} - 1) \right) R_{t-1} N_{t-1}.$$

Since equity injections expand wholesale bank net worth, this will expand credit creation, reduce leverage ratios, and lower the illiquidity premium.

4.3 The policy rule

The policymaker obtains funds by levying lump sum taxes on households.[28] Returns from direct lending and equity are also transferred lump sum to households. The aim is to assess the relative power of the two alternative instruments and so this simplification facilitates a clean comparison. The government flow budget constraint is given by

$$T_t = \underbrace{(1 - n_{t-1}) \left(\Gamma_t - \int_0^{\omega_t^*} \omega f(\omega) \, d\omega \right) R_{K,t} Q_{t-1} K_{t-1}}_{\text{Return on direct lending}}$$

$$+ \underbrace{(1 - c_{t-1}) (1 - \Gamma_t) R_{K,t} Q_{t-1} K_{t-1}}_{\text{Return on equity injections}} - \underbrace{(1 - n_t) (Q_t K_t - N_t)}_{\text{Direct lending}} - \underbrace{(1 - c_t) N_t}_{\text{Equity injections}}, \qquad (23)$$

where T_t are lump-sum transfers net of taxes. Note the policymaker does not receive the same return on direct lending as retail banks. Since the policymaker commits to always rollover, it loses $\gamma \lambda_t$ to banks that still experience a run.

The policymaker has two policy instruments available, c_t and n_t. I consider a simple and implementable reaction function that governs the use of these instruments given by

$$n_t = 1 - a_{DL} \left(\mathbb{E}_t S_{t+1}/S - 1 \right) \quad \text{and} \quad c_t = 1 - a_{EI} \left(\mathbb{E}_t S_{t+1}/S - 1 \right), \qquad (24)$$

---

[27]I abstract from efficiency costs associated with government acquisition of equity

[28]The lump sum taxes/transfers should more realistically be modelled as central bank reserves when comparing to developments during the financial crisis. However, the additional realism adds unnecessary complexity to the model without materially altering the comparison of the policy instruments.

where $a_{DL}, a_{EI} \geq 0$. Thus, the size of the policy intervention depends positively on the illiquidity premium. The rule is reasonable since the magnitude of the distortion the policymaker is trying to offset is proxied by the illiquidity premium. The rule is also practically implementable since credit spreads are observable (although disentangling illiquidity risk from credit risk may not be so easy). After all, it was the sharp rise in spreads during the financial crisis that pushed the Fed into introducing unconventional credit policies, even before the conventional tool of monetary policy, the nominal interest rate, reached the zero lower bound.

4.4   Illiquidity shock with policy responses

Figure 9 shows a 1% fall in the intra-period liquidity of the capital stock. To provide a clear comparison between policy instruments, the experiment ensures that both policies deliver the same initial cost to the government in term of GDP. This requires the parameters of the policy rules to be set at $a_{DL} = 3.00$ and $a_{EI} = 14.02$.[29]
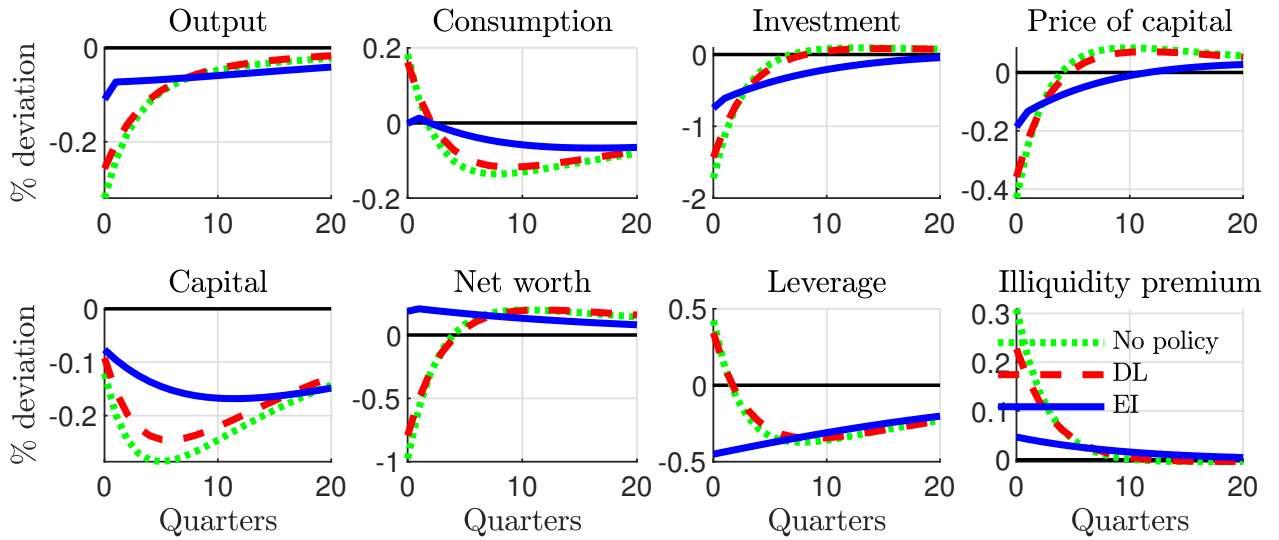
Figure 9 shows that equity injections mitigate the effects of the initial shock to liquidity better than direct lending. The initial fall in output with no policy intervention was 0.35%. Equity injections and direct lending reduced this to 0.11% and 0.27%, respectively. The reason is that equity injections directly offset the fall in net worth, actually causing leverage to fall. However, while equity injections reduce the initial impact of the illiquidity shock, they also cause the effects of the shock to persist for longer. The reason is as follows: Without coordination problems, the Modigliani-Miller theorem states the irrelevance between debt or equity financing. But, with coordination problems, the shadow price of equity is lower, exactly because equity avoids the coordination problem. Wholesale banks build up equity so that they don't require debt finance. Direct lending, even though it reduces the maturity mismatch, does not materially improve the solvency of the wholesale banks. Equity injections are therefore powerful in mitigating the problem in credit markets (because it lowers the need to access them). However, as the illiquidity premium recedes, the policymaker's withdrawal of equity offsets the recovery in net worth that the wholesale bank would have experienced in the counterfactual scenario without policy intervention.

5   Conclusion

This paper incorporates short-term uncoordinated creditors in credit markets in a dynamic, general equilibrium setting. The coordination problem generates a time-varying incidence of bank runs, rising during downturns and especially during periods of illiquidity. The resulting

---

[29]In terms of quantities, the size of the initial intervention in these experiments is 0.6% of annual GDP, or for the US economy, approximately \$90bn. The Fed's balance sheet expanded by \$1tn during the crisis.

Figure 9: Illiquidity shock and policy response

Note: 1% illiquidity shock with policy intervention

illiquidity premium provides a new lense through which to study the financial accelerator and the role of credit market interventions during financial crises. In particular, the model shows that equity injections are more powerful policies than direct lending.

Moreover, the microfounded coordination problem at the heart of this model provides rich nonlinearities that this current research has not yet fully explored and will likely be a fruitful avenue for future research.

References

Adrian, Tobias, Paolo Colla, and Hyun Song Shin (2013) 'Which financial frictions? Parsing the evidence from the financial crisis of 2007 to 2009.' *NBER Macroeconomics Annual* 27(1), 159–214

Angeloni, Ignazio, and Ester Faia (2013) 'Capital regulation and monetary policy with fragile banks.' *Journal of Monetary Economics* 60(3), 311–324

Bagehot, Walter (1873) *Lombard Street: A description of the money market* (London: Henry S. King & Co.)

Basu, Susanto, and Brent Bundick (2017) 'Uncertainty shocks in a model of effective demand.' *Econometrica* 85(3), 937–958

Berger, Philip G., Eli Ofek, and Itzhak Swary (1996) 'Investor valuation of the abandonment option.' *Journal of Financial Economics* 42(2), 257–287

Bernanke, Ben S., and Mark Gertler (1989) 'Agency costs, net worth, and business fluctuations.' *American Economic Review* 79(1), 14–31

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist (1999) 'The financial accelerator in a quantitative business cycle framework.' vol. 1C of *Handbook of Macroeconomics* (Amsterdam: Elsevier) chapter 21, pp. 1341–1393

Bredemeier, Christian, Christoph Kaufmann, and Andreas Schabert (2018) 'Interest rate spreads and forward guidance'

Brunnermeier, Markus K. (2009) 'Deciphering the liquidity and credit crunch 2007-2008.' *Journal of Economic Perspectives* 23(1), 77–100

Brunnermeier, Markus K., and Martin Oehmke (2013) 'The maturity rat race.' *Journal of Finance* 68(2), 483–521

Carlsson, Hans, and Eric van Damme (1993) 'Global games and equilibrium selection.' *Econometrica* 61(5), 989–1018

Carlstrom, Charles T., and Timothy S. Fuerst (1997) 'Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis.' *American Economic Review* 87(5), 893–910

Carlstrom, Charles T., Timothy S. Fuerst, and Matthias Paustian (2010) 'Optimal monetary policy in a model with agency costs.' *Journal of Money, Credit and Banking* 42, 37–70

— (2016) 'Optimal contracts, aggregate risk, and the financial accelerator.' *American Economic Journal: Macroeconomics* 8(1), 119–47

Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt (2015) 'Understanding the great recession.' *American Economic Journal: Macroeconomics* 7(1), 110–67

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno (2014) 'Risk shocks.' *American Economic Review* 104(1), 27–65

Corsetti, Giancarlo, Amil Dasgupta, Stephen Morris, and Hyun Song Shin (2004) 'Does one Soros make a difference? A theory of currency crises with large and small traders.' *Review of Economic Studies* 71(1), 87–113

Cúrdia, Vasco, and Michael Woodford (2010) 'Conventional and unconventional monetary policy.' *Federal Reserve Bank of St. Louis Review* 92(4), 229–64

— (2016) 'Credit frictions and optimal monetary policy.' *Journal of Monetary Economics* 84, 30–65

Dang, Tri Vi, Gary Gorton, and Bengt Holmström (2012) 'Ignorance, debt and financial crises.' *Unpublished manuscript*

de Groot, Oliver (2014) 'The risk channel of monetary policy.' *International Journal of Central Banking* 10(2), 115–160

— (2016) 'The financial accelerator.' In *The New Palgrave Dictionary of Economics,* ed. Steven N. Durlauf and Lawrence E. Blume (Basingstoke: Palgrave Macmillan)

Del Negro, Marco, Gauti Eggertsson, Andrea Ferrero, and Nobuhiro Kiyotaki (2017) 'The great escape? a quantitative evaluation of the fed's liquidity facilities.' *American Economic Review* 107(3), 824–57

Diamond, Douglas W., and Philip H. Dybvig (1983) 'Bank runs, deposit insurance, and liquidity.' *Journal of Political Economy* 91(3), 401–419

Dmitriev, Mikhail, and Jonathan Hoddenbagh (2017) 'The financial accelerator and the optimal state-dependent contract.' *Review of Economic Dynamics* 24, 43–65

Fiore, Fiorella D., and Oreste Tristani (2013) 'Optimal monetary policy in a model of the credit channel.' *The Economic Journal* 123(571), 906–931

Gertler, Mark (2013) 'Comment on "Which financial frictions? Parsing the evidence from the financial crisis of 2007 to 2009".' *NBER Macroeconomics Annual* 27(1), 215–223

Gertler, Mark, and Nobuhiro Kiyotaki (2010) 'Financial intermediation and credit policy in business cycle analysis.' vol. 3 of *Handbook of Monetary Economics* (Amsterdam: Elsevier) chapter 11, pp. 547–599

___ (2015) 'Banking, liquidity, and bank runs in an infinite horizon economy.' *American Economic Review* 105(7), 2011–43

Gertler, Mark, and Peter Karadi (2011) 'A model of unconventional monetary policy.' *Journal of Monetary Economics* 58(1), 17–34

Gertler, Mark, Nobuhiro Kiyotaki, and Albert Queralto (2012) 'Financial crises, bank risk exposure and government financial policy.' *Journal of Monetary Economics* 59, Supplement, 17–34

Gertler, Mark, Nobuhiro Kiyotaki, and Andrea Prestipino (2016) 'Wholesale banking and bank runs in macroeconomic modeling of financial crises.' vol. 2 of *Handbook of Macroeconomics* (Amsterdam: Elsevier) chapter 11, pp. 1345–1425

Gilchrist, Simon, and Egon Zakrajsek (2012a) 'Credit spreads and business cycle fluctuations.' *American Economic Review* 102(4), 1692–1720

___ (2012b) 'Credit spreads and business cycle fluctuations.' *The American Economic Review* 102(4), 1692–1720

Goldstein, Itay, and Ady Pauzner (2005) 'Demand–deposit contracts and the probability of bank runs.' *Journal of Finance* 60(3), 1293–1327

Gorton, Gary, and Andrew Winton (2003) 'Financial intermediation.' vol. 1A of *Handbook of the Economics of Finance: Corporate Finance* (Amsterdam: Elsevier) chapter 8, pp. 431–552

Hart, Oliver, and John Moore (1994) 'A theory of debt based on the inalienability of human capital.' *Quarterly Journal of Economics* 109(4), 841–79

Heider, Florian, Marie Hoerova, and Cornelia Holthausen (2015) 'Liquidity hoarding and interbank market rates: The role of counterparty risk.' *Journal of Financial Economics* 118(2), 336 – 354

Hertzberg, Andrew, José M. Liberti, and Daniel Paravisini (2011) 'Public information and coordination: Evidence from a credit registry expansion.' *Journal of Finance* 66(2), 379–412

Iacoviello, Matteo (2005) 'House prices, borrowing constraints, and monetary policy in the business cycle.' *The American Economic Review* 95(3), 739–764

Jackson, Thomas H. (1986) *The logic and limits of bankruptcy law* (Boston: Harvard University Press)

Kiyotaki, Nobuhiro, and John Moore (1997) 'Credit cycles.' *Journal of Political Economy* 105(2), 211–248

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen (2012) 'The aggregate demand for treasury debt.' *Journal of Political Economy* 120(2), 233–267

Kurlat, Pablo (2013) 'Lemons markets and the transmission of aggregate shocks.' *American Economic Review* 103(4), 1463–1489

Morris, Stephen, and Hyun Song Shin (2003) 'Global games: Theory and applications.' vol. 1 of *Advances in economics and econometrics: Theory and applications, Eighth world congress* (Cambridge: Cambridge University Press) pp. 56–114

— (2004) 'Coordination risk and the price of debt.' *European Economic Review* 48, 133–153

— (2016) 'Illiquidity component of credit risk.' *International Economic Review* 57(4), 1135–1148

Reis, Ricardo (2009a) 'Interpreting the unconventional US monetary policy of 2007-09.' *Brookings Papers on Economic Activity* pp. 119–165

— (2009b) 'Where should liquidity be injected during a financial crisis?' *Unpublished manuscript*

Rochet, Jean-Charles, and Xavier Vives (2004) 'Coordination failures and the lender of last resort: Was Bagehot right after all?' *Journal of the European Economic Association* 2(6), 1116–1147

Sargent, Thomas J., and Neil Wallace (1982) 'The real-bills doctrine versus the quantity theory: A reconsideration.' *Journal of Political Economy* 90(6), 1212–1236

Shin, Hyun Song (2009) 'Reflections on Northern Rock: The bank run that heralded the global financial crisis.' *Journal of Economic Perspectives* 23(1), 101–119

Townsend, Robert M. (1979) 'Optimal contracts and competitive markets with costly state verification.' *Journal of Economic Theory* 21(2), 265–93

A   ONLINE APPENDIX (NOT FOR PUBLICATION)

A.1   Financial frictions via costly state verification

As has been alluded to in several places above, the model with coordination failure generates a similar set of reduced form aggregate equilibrium relationships as the *financial accelerator* model of Bernanke et al. (1999) from very different microfoundations. Specifically, coordination problems among creditors, like Bernanke et al. (1999) generates a risk spread between internal and external finance related to the endogenous evolution of borrowers' (wholesale banks here, entrepreneurs in Bernanke et al. (1999)) leverage. However, the two models are quite distinct in terms of the understanding of the key features of credit market dynamics.

While it is beyond the scope of this paper to assess which of the credit market frictions— coordination failure or bankruptcy costs—is empirically more relevant, it is instructive to provide a rigorous comparison of the two models, and highlight how the empirical work could distinguish between these two mechanisms. The Bernanke et al. (1999) model offers a second benchmark (in addition to the frictionless real business cycle benchmark in Section 2) from which to assess the role of coordination failure.

To this end, a stripped down version of Bernanke et al. (1999)'s model appropriate for direct comparison is presented. The financial friction in Bernanke et al. (1999) is the result of a *costly state verification* (CSV) assumption, first introduced by Townsend (1979). In their model, debt also matures every period, but unlike the coordination failure model, there is no possibility of early foreclosure by retail banks. Instead, the friction is the result of an informational asymmetry between borrowers (entrepreneurs in this case) and retail banks (the creditor). At the end of a period, an entrepreneur knows his gross profits, $\omega_{t+1}(e) R_{t+1}^E Q_t K_{t+1}(e)$. The retail bank however, does not. As before, the non-default threshold for an borrower is given by $\bar{\omega}_{t+1}$. Suppose the borrower declares that his gross profits were less than the contractual amount owed to the retail bank, and therefore claims to only be able to pay a fraction of his debt obligation. Does the retail bank believe this? How does the retail bank respond? In the costly state verification model, the retail bank is able to pay a monitoring cost in order to observe the entrepreneur's gross profits. The monitoring cost is assumed to be a proportion of gross profits, $\mu \omega_{t+1}(e) R_{t+1}^E Q_t K_{t+1}(e)$, with $0 < \mu < 1$. It turns out that the borrower has the incentive to truthfully reveal his gross profits if the retail bank commits to monitoring the borrower whenever the borrower is insolvent: $\omega_{t+1}(e) < \bar{\omega}_{t+1}$. When monitoring is costless, $\mu = 0$, the model reduces to the frictionless real business cycle benchmark. At the other extreme, when $\mu = 1$, the credit market ceases to function. If

however $0 < \mu < 1$, the expected gross return to the retail bank from lending is given by:

$$\left(\underbrace{\bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) \, d\omega}_{\substack{\text{i. Returns on debt} \\ \text{that pays in full}}} + \underbrace{(1 - \mu) \int_{0}^{\bar{\omega}} \omega f(\omega) \, d\omega}_{\substack{\text{ii. Returns on debt that} \\ \text{doesn't pay in full net of} \\ \text{monitoring costs}}}\right) R_{t+1}^{E} Q_t K_{t+1} \, (e)$$

Rewrite the expected gross profits accruing to the retail bank as:

$$\left(\Gamma(\bar{\omega}) - G^{CSV}(\bar{\omega})\right) R_{t+1}^{E} Q_t K_{t+1} \, (e)$$

where $\Gamma(\cdot)$ is as in Section 2.4 and $G_{CSV}(\cdot)$ is:

$$G_{CSV}(\bar{\omega}) = \mu \int_{0}^{\bar{\omega}} \omega f(\omega) \, d\omega \tag{25}$$

The borrower's break-even condition under the coordination failure and costly state verification assumptions are clearly similar, except in the interpretation and functional form of the financial friction, $G_{CSV}(\bar{\omega})$ under CSV and $G_{CF}(\bar{\omega}(\omega^*))$ under coordination failure (CF). Under costly state verification, $G_{CSV}(\bar{\omega})$ is the expected monitoring (or agency) cost while in Section 2.4, $G_{CF}(\omega^*(\bar{\omega}))$ was the cost of coordination failure. The differences in the endogenous responses of these two frictions to the same aggregate shocks is what distinguishes the dynamics of the two model economies.

Setting up the borrower's Lagrangian similar to that in Section 2.4 to solve for the menu of state-contingent equilibrium debt contracts under CSV gives:

$$\frac{\mathbb{E}_t R_{t+1}^{E}}{R_{t+1}} = \Xi_{CSV}\left(\frac{Q_t K_{t+1}}{N_{t+1}}\right) \tag{26}$$

where $\Xi_{CSV}'(\cdot) > 0$ and $\Xi_{CSV}''(\cdot) > 0$. This equation exhibits the same functional form as equation (??); the external finance premium, $\frac{\mathbb{E}_t R_{t+1}^{E}}{R_{t+1}}$ is increasing and convex in the wholesale bank's capital to net worth ratio. The reason for this relationship under costly state verification is that the expected monitoring costs rise as the ratio of borrowing to net worth increases. In the next section, it will be shown that it is the *curvature* of this function that will be the key distinction between the two models, and their dynamics.

As a result of the asymmetry of information, competitive wholesale banks are able to earn informational rents, leading the evolution of aggregate net worth to follow:

$$N_{t+1} = \upsilon \left(R_t^{E} Q_{t-1} K_t - R_t \left(Q_{t-1} K_t - N_t\right)\right) + T^{E}$$

There is a small discrepancy between the net worth equations under coordination failure (see equation (19)) and costly state verification, above. This discrepancy results from the way in which the costs (whether coordination costs or agency costs) are realized. Under coordination failure, the cost was a loss in units of productive capital. Under costly state verification, the monitoring costs are paid in terms of units of output. Thus, under costly state verification, $K_t^* = K_t$ for all t, but the aggregate resource constraint becomes:

$$Y_t = C_t + G_t + I_t + C_t^E + \underbrace{\mu \left(\int_0^{\bar{\omega}_t} \omega f(\omega) \, d\omega \right) R_t^E Q_{t-1} K_t}_{\text{Deadweight cost of monitoring}}$$

A.2   The coordination game

This appendix retraces many of the technical aspects of Section 2.4 to ensure completeness by adding additional proofs, details and explanations. Subscripts and indexes have been dropped wherever possible to aid clarity.

A.2.1   Construction of payoffs in Table 3

This subsection explains the construction of the payoff matrix for retail banks in Table 3. Suppose a wholesale bank owns one unit of *raw* capital, of which only $0 < \lambda < 1$ units are liquid. Suppose a fraction $0 < p < 1$ of retail banks do not rollover. The debt contract offers a retail bank that does not rollover $\bar{\omega}$ units, leaving the wholesale bank with

$$\begin{cases} 1 - \frac{p\bar{\omega}}{\lambda} & \text{if} \quad p\bar{\omega} \leq \lambda \\ 0 & \text{if} \quad p\bar{\omega} > \lambda \end{cases}$$

units of raw capital. A retail bank that does not rollover then owns

$$\begin{cases} \bar{\omega} & \text{if} \quad p\bar{\omega} \leq \lambda \\ \frac{\lambda}{p} & \text{if} \quad p\bar{\omega} > \lambda \end{cases}$$

units of raw capital. Thus, the wholesale bank *fails* at the intra-period stage (i.e. looses all its capital to retail banks that do not rollover) if the fraction of retail banks that do not rollover, p, is in the interval $\left(\frac{\lambda}{\bar{\omega}}, 1\right]$. When $p \in \left(\frac{\lambda}{\bar{\omega}}, 1\right]$ the raw liquid capital, λ, is divided equally among the retail banks that do not rollover.

The wholesale and retail banks have productivity ω and γ, respectively. They use their productivity to transform the raw capital into *effective* capital (i.e. capital that can be rented

to firms). The wholesale banks and the retail banks that did not rollover therefore hold

$$\left\{ \begin{array}{c} \omega \left(1 - \frac{p\bar{\omega}}{\lambda}\right) \\ 0 \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{ll} \gamma\bar{\omega} & \text{if} \quad p\bar{\omega} \leq \lambda \\ \frac{\gamma\lambda}{p} & \text{if} \quad p\bar{\omega} < \lambda \end{array} \right.$$

The debt contract offers rolled over retail banks a non-default gross return on $\bar{\omega}$ units of effective capital. If the wholesale bank fails at the intra-period stage, a rolled over retail bank receives 0. If the wholesale bank survives the intra-period stage, a rolled over retail bank receives the gross return on the following units of effective capital:

$$\left\{ \begin{array}{ll} \bar{\omega} & \text{if} \quad \omega \left(1 - \frac{p\bar{\omega}}{\lambda}\right) \geq (1-p)\bar{\omega} \\ \frac{\omega}{(1-p)} \left(1 - \frac{p\bar{\omega}}{\lambda}\right) & \text{if} \quad \omega \left(1 - \frac{p\bar{\omega}}{\lambda}\right) < (1-p)\bar{\omega} \end{array} \right.$$

The second line states that if the wholesale bank generates a gross return lower than its debt obligation to the rolled over retail banks, the gross return will be shared equally among the rolled over retail banks. In Table 3, the *if* statement is rearranged as follows: $\omega \geq \frac{\lambda(1-p)\bar{\omega}}{(\lambda - p\bar{\omega})}$. Finally, the wholesale bank, as the residual claimant on the gross returns, receives the gross return on the following units of effective capital:

$$\left\{ \begin{array}{ll} \left(\omega \left(1 - \frac{p\bar{\omega}}{\lambda}\right) - (1-p)\bar{\omega}\right) & \text{if} \quad \omega \geq \frac{\lambda(1-p)\bar{\omega}}{(\lambda - p\bar{\omega})} \\ 0 & \text{if} \quad \omega < \frac{\lambda(1-p)\bar{\omega}}{(\lambda - p\bar{\omega})} \end{array} \right.$$

if it survives the intra-period stage. If the wholesale bank fails at the intra-period stage it earns zero gross return. This completes the description of the payoff matrix in Table 3.

A.2.2   Proof of Proposition 1

When ω is not common knowledge, the game played by retail banks each period in deciding whether to rollover is a *global game*. A general proof that the game described in Section 2.4 has a unique (symmetric) switching equilibrium (given in Proposition 1) is provided in Morris and Shin (2003). This subsection simply identifies the characteristics of the model that fit the conditions for Morris and Shin's proof.

The game described in Section 2.4 has a continuum of retail banks. Each retail bank receives a private signal, x, and has to choose an action, $a \in \{\mathtt{n}, \mathtt{r}\}$ (not rollover or rollover). All retail banks have the same payoff function, u, where $u(a, p, \omega)$ is a retail bank's payoff if it chooses action a, if a fraction, p, of the other retail banks choose to not rollover, and the state is ω. To analyze best responses, it is enough to know the net payoff of rollover vs. not

rollover. The net payoff function, π, is given by

$$\pi(p,\omega) \equiv u(\mathtt{r},p,\omega) - u(\mathtt{n},p,\omega).$$

The state, ω, is drawn from a continuously differentiable and strictly positive density. Importantly, the payoffs in Table 3 satisfy conditions 4-9 given below:

Condition 4 *State monotonicity:*
The net payoff function, $\pi(p,\omega)$, is nondecreasing in ω.

Condition 5 *Action single crossing:*
For each $\omega \in \mathbb{R}$, there exists a p^* such that $\pi(p,\omega) < 0$ if $p < p^*$ and $\pi(p,\omega) > 0$ if $p > p^*$.

Condition 6 *Uniform limit dominance:*
There exist $\omega^L \in \mathbb{R}$, $\omega^H \in \mathbb{R}$, and $\varepsilon \in \mathbb{R}_{++}$, such that 1) $\pi(p,\omega) \leq -\varepsilon$ for all $p \in (0,1]$ and $\omega \leq \omega^L$; and 2) $\pi(p,\omega) > \varepsilon$ for all $p \in (0,1]$ and $\omega \geq \omega^H$.

Condition 7 *Monotone likelihood property:*
If $\bar{x} > \underline{x}$, then $h(\bar{x} - \omega)/h(\underline{x} - \omega)$ is increasing in ω, where $h(\cdot)$ is the distribution of the noise term.

Condition 8 *Continuity:*
$\int_{p=0}^{1} g(p)\pi(p,\omega)\,dp$ is continuous with respect to the signal x and density $g(\cdot)$.

Condition 9 *Strict Laplacian state monotonicity:*
There exists a unique ω^* solving $\int_{p=0}^{1} \pi(p,\omega^*)dp = 0$.

Morris and Shin (2003) (see page 67-70 and Appendix C) prove the following result which can be applied to this setting: Let ω^* be defined as in Condition 9. The coordination game played by retail banks has a unique (symmetric) switching strategy equilibrium, with a retail bank choosing rollover if $x > \omega^*$ and foreclosure if $x < \omega^*$.

Condition 4 states that the incentive to rollover is increasing in ω. Thus, a retail bank's optimal action will be increasing in the state, given the other retail banks' actions. Condition 5 states that the net payoff function should only cross zero once. Thus, the payoff matrix does not need to exhibit strategic complementarities (i.e. exhibit action monotonicity) across the full range of p. It does however have to satisfy this weaker single crossing condition (referred to by Goldstein and Pauzner (2005) as one-sided strategic complementarities). It is clear from Table 3 that the net payoff, $\pi(p,\omega)$, is decreasing in $p \in (0,p^\dagger)$, where $p^\dagger = \frac{\lambda}{\bar{\omega}}$, is the

critical mass of foreclosing retail banks at which the wholesale bank fails at the intra-period stage. Above $p^\dagger$ the net payoff is increasing in p but remains negative.

Condition 6 requires that not rolling over is a dominant strategy for sufficiently low states, and rollover is a dominant strategy for sufficiently high states. In other words, there must be ranges of extremely good and extremely bad realizations of ω, for which a retail bank's best action is independent of its beliefs concerning other retail banks' behaviour. The lower region is when ω is so low that it is better for a retail bank to not rollover, even if all other retail banks rollover, and occurs when $\omega < \omega^L = \gamma\bar{\omega}$. Thus, $\left[0, \omega^L\right)$ denotes the *lower dominance region*. Similarly, I assume an *upper dominance region* $(\omega^H, \infty]$ in which a retail bank would rollover, independent of its beliefs about other retail banks' actions. Strictly speaking, the payoff matrix in Table 3 does not exhibit an upper dominance region. To implement the upper dominance region, I assume that there exists an external large economic agent (private or public) that would be willing to buy the wholesale bank out and pay its liabilities when ω is within the upper dominance region. The two dominance regions are just extreme ranges of the fundamentals at which retail banks' behaviour is known. This is important because in the choice of an equilibrium action for a given signal, retail banks must take into account the equilibrium actions at nearby signals. Again, these actions depend on the equilibrium actions taken at further signals, and so on. Eventually, the equilibrium must be consistent with the known behaviour at the dominance regions. Importantly though, the position of the equilibrium threshold point, ω^* does not depend on the exact specifications of the two regions. It is therefore possible to be agnostic about the exact details of the upper dominance region, with ω^H arbitrarily high. Although the payoff matrix in Table 3 does not have an upper dominance region, a number of natural economic stories can justify the assumption that if ω was sufficiently large, all retail banks would have a dominant strategy to rollover.

Condition 7 is a technical restriction on the noise distribution, which is satisfied by the uniform distribution assumed. Condition 8 is a weak continuity property that is satisfied despite a discontinuity in the payoffs at $p^\dagger = \frac{\lambda}{\omega}$. Finally, Condition 9 is used to find the unique threshold equilibrium.

A.2.3 The rollover decision

Section 2.4 explains the rollover decision for an equilibrium debt contract under the baseline parameterization of the model. The description in Section 2.4, however, is an incomplete characterization of the decision rule for retail banks for all theoretically feasible values of $\gamma, \lambda \in (0, 1)$ and $\bar{\omega} \in (0, \infty)$. It is possible to separate the decision rule into four scenarios,

depending on the values of γ, λ, and $\bar{\omega}$.[30] 1) The *no fragility* case when $\frac{\lambda}{\bar{\omega}} < 1$; 2) the *mild fragility* case when $\frac{\lambda}{\bar{\omega}} < 1$ but $\omega^* < \bar{\omega}$; 3) the *acute fragility* case when $\frac{\lambda}{\bar{\omega}} < 1$ and $\omega^* > \bar{\omega}$; and 4) the *no rollover* case when no rollover occurs with probability 1.

Scenario 1: No fragility, $\lambda \geq \bar{\omega}$ In this scenario, the wholesale bank is never illiquid at the intra-period stage. Even if $p = 1$ (i.e. no retail banks rollover) every retail bank is guaranteed the contractual $\bar{\omega}$ units of raw capital, and the wholesale bank always has a positive level of raw capital with which to continue operating after the intra-period stage. The payoff matrix in this scenario is given in Table 7.

<div align="center">

Table 7: No fragility scenario

</div>

Rollover	Not rollover		
$\bar{\omega}$	$\gamma\bar{\omega}$	if	$\omega \geq \frac{\bar{\omega}(1-p)\lambda}{\lambda - p\bar{\omega}}$
$\frac{\omega}{(1-p)}\left(1 - \frac{p\bar{\omega}}{\lambda}\right)$	$\gamma\bar{\omega}$	if	$\omega < \frac{\bar{\omega}(1-p)\lambda}{\lambda - p\bar{\omega}}$

If $\omega > \gamma\bar{\omega}$ it is optimal for all retail banks to rollover. If $0 < \omega < \gamma\bar{\omega}$, all retail banks can guarantee a return $\gamma\bar{\omega}$ by not rolling over. However, $p = 1$ is not the equilibrium. If all but one retail bank does not rollover the return to the retail bank that rolled over is $\bar{\omega}$. Instead, there is a mixed equilibrium. Retail banks will not rollover up to the point at which the payoff to rolling over and not is equalized. The equilibrium fraction that do not rollover, $p^{\ddagger}$, implicitly solves

$$\frac{\omega}{(1-p^{\ddagger})}\left(1 - \frac{p^{\ddagger}\bar{\omega}}{\lambda}\right) = \gamma\bar{\omega} \quad \rightarrow \quad p^{\ddagger} = \frac{\lambda}{\bar{\omega}}\frac{(\gamma\bar{\omega} - \omega)}{(\gamma\lambda - \omega)} < 1.$$

This means that when the wholesale bank is not fragile to the possibility of a run, there is no symmetric rollover threshold. In terms of payoffs though, it means that all retail banks are guaranteed the no rollover return $\gamma\bar{\omega}$ (although some retail banks will earn this gross return by rolling over).

---

[30]The decision rule is conditional on $\bar{\omega}$. When deciding whether to rollover, a retail bank takes $\bar{\omega}$ as given. Thus, we consider the full range of $\bar{\omega} \in (0, \infty)$ at this stage. However, $\bar{\omega}$ is an endogenous object. In equilibrium, a large subset of possible $\bar{\omega}$ values are never chosen by optimizing agents.

Scenario 2 & 3: Mild and acute fragility, $\lambda < \bar{\omega}$ In this case, the indifference condition in equation (12) generalizes to

$$-\int_{p=\frac{\lambda}{\bar{\omega}}}^{1} \frac{\gamma\lambda}{p}dp + \int_{p=0}^{\frac{\lambda}{\bar{\omega}}}\left(\left\{\begin{array}{ll} \bar{\omega} & \text{if } \omega^* > \frac{\lambda(1-p)\bar{\omega}}{(\lambda-p\bar{\omega})} \\ \frac{\omega^*}{(1-p)}\left(1-\frac{p\bar{\omega}}{\lambda}\right) & \text{if } \omega^* \le \frac{\lambda(1-p)\bar{\omega}}{(\lambda-p\bar{\omega})}\end{array}\right\} - \gamma\bar{\omega}\right)dp = 0, \qquad (27)$$

where the rollover threshold, ω^*, is the implicit solution to this indifference condition, which reduces to

$$0 = \gamma\frac{\lambda^2}{\bar{\omega}}\left(\ln\left(\frac{\lambda}{\bar{\omega}}\right)-1\right) + \left\{\begin{array}{ll} \frac{\omega^*}{\bar{\omega}}\lambda + \omega^*\left(1-\frac{\lambda}{\bar{\omega}}\right)\ln\left(1-\frac{\lambda}{\bar{\omega}}\right) & \text{if } \omega^* \le \bar{\omega} \\ \lambda + \omega^*\left(1-\frac{\lambda}{\bar{\omega}}\right)\ln\left(1-\frac{\lambda}{\omega^*}\right) & \text{if } \omega^* > \bar{\omega}\end{array}\right.$$

When $\omega^* = \bar{\omega}$, the two lines coincide. A rearranged version of the top line is equation (52) in Section 2.4. Thus, the main text presents the result where $\omega^* \le \bar{\omega}$, termed Scenario 2) *mild fragility*. Under this scenario there is an inefficiency due to the fact that $\omega^* > \omega^*_{\text{eff}}$. However, a wholesale bank that experiences a run is technically already insolvent.

Under Scenario 3) *acute fragility*, $\omega^* > \bar{\omega}$, even solvent wholesale banks face illiquidity and the risk of a run. While no closed-form solution exists for ω^* when $\omega^* > \bar{\omega}$, a comparison of the effect of $\bar{\omega}$ on ω^* at the shift from mild to acute fragility is possible. First, using (52), rewrite the inequality $\omega^* < \bar{\omega}$ as

$$\frac{1}{\gamma} < \frac{\left(\frac{\lambda}{\bar{\omega}}\right)^2\left(1-\ln\left(\frac{\lambda}{\bar{\omega}}\right)\right)}{\left(\frac{\lambda}{\bar{\omega}}+\left(1-\frac{\lambda}{\bar{\omega}}\right)\ln\left(1-\frac{\lambda}{\bar{\omega}}\right)\right)}.$$

The right-hand-side (rhs) as a function of $\lambda/\bar{\omega}$ can be shown to be negatively sloped with $\lim\left(\lambda/\bar{\omega}\to 0\right)\text{rhs} = +\infty$ and $\lim\left(\lambda/\bar{\omega}\to 1\right)\text{rhs} = 1$. Thus, when γ is close to 1 (i.e. the retail banks are good capital managers and close substitutes for the wholesale banks) the wholesale bank is acutely fragile even when its balance sheet is not very illiquid (i.e. when $\lambda/\bar{\omega}$ is close to 1). Second, ω^* is always increasing in $\bar{\omega}$:

$$\frac{\partial\omega^*(\bar{\omega})}{\partial\bar{\omega}} = \left\{\begin{array}{ll} \frac{\gamma\left(\frac{\lambda}{\bar{\omega}}\right)^2-\left(\frac{\lambda}{\bar{\omega}}+\ln\left(1-\frac{\lambda}{\bar{\omega}}\right)\right)\frac{\omega^*}{\bar{\omega}}}{\frac{\lambda}{\bar{\omega}}+\left(1-\frac{\lambda}{\bar{\omega}}\right)\ln\left(1-\frac{\lambda}{\bar{\omega}}\right)} > 0 & \text{if } \omega^* \le \bar{\omega} \\ \frac{1-\frac{\lambda}{\omega^*}}{1-\frac{\lambda}{\bar{\omega}}}\frac{\frac{\lambda}{\omega^*}\left(\gamma\frac{\lambda}{\bar{\omega}}-1\right)-\ln\left(1-\frac{\lambda}{\omega^*}\right)}{\frac{\lambda}{\omega^*}+\left(1-\frac{\lambda}{\omega^*}\right)\ln\left(1-\frac{\lambda}{\omega^*}\right)} > 0 & \text{if } \omega^* > \bar{\omega}\end{array}\right.$$

This means that the rollover threshold, ω^*, rises with solvency threshold, $\bar{\omega}$. In addition, it is possible to show that the rate at which ω^* increases with $\bar{\omega}$ is higher when $\omega^* > \bar{\omega}$. If the

decision rule in (52) is extrapolated to $\omega^* > \bar{\omega}$, then

$$\left. \frac{\partial \omega^* (\bar{\omega})}{\partial \bar{\omega}} \right|_{\text{mild}} < \left. \frac{\partial \omega^* (\bar{\omega})}{\partial \bar{\omega}} \right|_{\text{acute}} \quad \text{when} \quad \omega^* > \bar{\omega},$$

which states that the rollover threshold and the inefficiency cost of coordination failure rises more rapidly in $\bar{\omega}$ when there is acute fragility.

Scenario 4: No rollover, $\lambda < \bar{\omega}$ This is when Condition 9 is violated (i.e. $\lim_{\omega^* \to \infty} \int_{p=0}^{1} \pi(p, \omega^*) dp < 0$). The boundary between the acute fragility and no rollover scenarios solves

$$\lim_{\omega^* \to \infty} \gamma \lambda \frac{\lambda}{\bar{\omega}} \left(\ln \left(\frac{\lambda}{\bar{\omega}} \right) - 1 \right) + \lambda + \omega^* \left(1 - \frac{\lambda}{\bar{\omega}} \right) \ln \left(1 - \frac{\lambda}{\omega^*} \right) < 0,$$

which reduces to

$$1 < \gamma \left(1 - \ln \left(\frac{\lambda}{\bar{\omega}} \right) \right). \tag{28}$$

Since the term in brackets is greater than 1 and increasing in the wholesale bank's illiquidity, $\frac{\lambda}{\bar{\omega}}$, it implies that when γ is very high, there is no threshold solution, even for relatively low levels of illiquidity. In this scenario, retail banks do not rollover with probability 1.

A.2.4 Complete characterization of equilibrium payoffs

This section gives a complete characterization of the expected gross returns accruing to both the wholesale and the retail banks, normalized by $R_K Q K$. The expected payoff to a retail bank is given by

$$\begin{cases} \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega)\, d\omega + \int_{\gamma\bar{\omega}}^{\bar{\omega}} \omega f(\omega)\, d\omega + \gamma\bar{\omega} \int_{0}^{\gamma\bar{\omega}} f(\omega)\, d\omega & \text{if} \quad \text{no fragility} \\ \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega)\, d\omega + \int_{\omega^*}^{\bar{\omega}} \omega f(\omega)\, d\omega + \gamma\lambda \int_{0}^{\omega^*} f(\omega)\, d\omega & \text{if} \quad \text{mild fragility} \\ \bar{\omega} \int_{\omega^*}^{\infty} f(\omega)\, d\omega + \gamma\lambda \int_{0}^{\omega^*} f(\omega)\, d\omega & \text{if} \quad \text{acute fragility} \\ \gamma\lambda & \text{if} \quad \text{no rollover} \end{cases}$$

The expected payoff to the wholesale bank is given by

$$\begin{cases} \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}) f(\omega)\, d\omega & \text{if} \quad \text{no fragility} \\ \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}) f(\omega)\, d\omega & \text{if} \quad \text{mild fragility} \\ \int_{\omega^*}^{\infty} (\omega - \bar{\omega}) f(\omega)\, d\omega & \text{if} \quad \text{acute fragility} \\ 0 & \text{if} \quad \text{no rollover} \end{cases}$$

Using the notation, $\Gamma\left(\cdot\right) \equiv \bar{\omega} \int_{\bar{\omega}}^{\infty} f\left(\omega\right) d\omega + \int_{0}^{\bar{\omega}} \omega f\left(\omega\right) d\omega$, it is possible to rewrite the expected gross returns for the wholesale and retail banks as $1 - \Gamma\left(\cdot\right) - H\left(\cdot\right)$ and $\Gamma\left(\cdot\right) - G\left(\cdot\right)$, respectively, where $H\left(\cdot\right)$ and $G\left(\cdot\right)$ are defined as the expected cost of coordination failure for the wholesale and retail banks, respectively. The $G\left(\cdot\right)$ and $H\left(\cdot\right)$ functions are given as follows:

$$
G\left(\cdot\right) = \begin{cases}
\int_{0}^{\gamma\bar{\omega}} \left(\omega - \gamma\bar{\omega}\right) f\left(\omega\right) d\omega & \text{if } \text{no fragility} \\
\int_{0}^{\omega^{*}(\bar{\omega})} \left(\omega - \gamma\lambda\right) f\left(\omega\right) d\omega & \text{if } \text{mild fragility} \\
\left(\bar{\omega} - \gamma\lambda\right) \int_{\bar{\omega}}^{\omega^{*}} f\left(\omega\right) d\omega + \int_{0}^{\bar{\omega}} \left(\omega - \gamma\lambda\right) f\left(\omega\right) d\omega & \text{if } \text{acute fragility} \\
-\bar{\omega} \int_{\bar{\omega}}^{\infty} f\left(\omega\right) d\omega - \int_{0}^{\bar{\omega}} \omega f\left(\omega\right) d\omega + \gamma\lambda & \text{if } \text{no rollover}
\end{cases}
$$

and

$$
H\left(\cdot\right) = \begin{cases}
0 & \text{if } \text{no fragility} \\
0 & \text{if } \text{mild fragility} \\
\int_{\bar{\omega}}^{\omega^{*}(\bar{\omega})} \left(\omega - \bar{\omega}\right) f\left(\omega\right) d\omega & \text{if } \text{acute fragility} \\
\int_{\bar{\omega}}^{\infty} \left(\omega - \bar{\omega}\right) f\left(\omega\right) d\omega & \text{if } \text{no rollover}
\end{cases}
$$

with derivatives

$$
\frac{\partial G\left(\bar{\omega}\right)}{\partial \bar{\omega}} = \begin{cases}
-\gamma F\left(\gamma\bar{\omega}\right) & \text{if } \text{no fragility} \\
\left(\omega^{*} - \gamma\lambda\right) f\left(\omega^{*}\right) \frac{\partial \omega^{*}}{\partial \bar{\omega}} & \text{if } \text{mild fragility} \\
F\left(\omega^{*}\right) - F\left(\bar{\omega}\right) + \left(\bar{\omega} - \gamma\lambda\right) f\left(\omega^{*}\right) \frac{\partial \omega^{*}}{\partial \bar{\omega}} & \text{if } \text{acute fragility} \\
-1 + F\left(\bar{\omega}\right) & \text{if } \text{no rollover}
\end{cases}
$$

and

$$
\frac{\partial H\left(\bar{\omega}\right)}{\partial \bar{\omega}} = \begin{cases}
0 & \text{if } \text{no fragility} \\
0 & \text{if } \text{mild fragility} \\
-\left(F\left(\omega^{*}\right) - F\left(\bar{\omega}\right)\right) + \left(\omega^{*} - \bar{\omega}\right) f\left(\omega^{*}\right) \frac{\partial \omega^{*}}{\partial \bar{\omega}} & \text{if } \text{acute fragility} \\
-1 + F\left(\bar{\omega}\right) & \text{if } \text{no rollover}
\end{cases}
$$

A.3 The contracting problem

This section shows the debt contracting problem in Section 2.4 produced a positive monotonic relationship between the illiquidity premium and leverage ratio. The theory is developed for the case of no aggregate risk. The details in this section follow closely the contracting problem described in Section A.1. of Bernanke et al. (1999).

Let the gross rate of return on the value of a unit of effective capital equal R_{K}. Capital is subject to an idiosyncratic shock, $\omega \in [0, \infty)$ with $\mathbb{E}\left(\omega\right) = 1$. Assume $F\left(x\right) = \Pr\left(\omega < x\right)$

is a continuous probability distribution with $F(0) = 0$ and denote by $f(\omega)$ the *pdf* of ω. The equilibrium contract specifies $\bar{\omega}$. In equilibrium the retail banks earn an expected return equal to

$$(\Gamma(\cdot) - G(\cdot)) R_K K = R(K - N),$$

where $\Gamma(\cdot)$ is the gross share of the returns, $R_K K$, going to the retail banks. The net share of the returns going to the retail banks is $\Gamma(\cdot) - G(\cdot)$ and the share going to the wholesale banks is $1 - \Gamma(\cdot) - H(\cdot)$, where both $G(\cdot)$ and $H(\cdot)$ are expected deadweight costs of coordination failure. By definition, $0 < \Gamma(\cdot) < 1$. The assumptions made above imply that

$$\Gamma(\cdot) - G(\cdot) > 0 \quad \text{for all} \quad \bar{\omega} \in (0, \infty),$$

and

$$\lim_{\bar{\omega} \to 0} \Gamma(\cdot) - G(\cdot) = 0, \quad \lim_{\bar{\omega} \to \infty} \Gamma(\cdot) - G(\cdot) = \gamma\lambda.$$

Differentiating $\Gamma(\cdot) - G(\cdot)$ there exists an $\bar{\bar{\omega}}$ such that:

$$\Gamma'(\cdot) - G'(\cdot) \lesseqqgtr 0 \quad \text{for} \bar{\omega} \gtreqqless \bar{\bar{\omega}},$$

implying that the net payoff to the retail bank reaches a global maximum at $\bar{\bar{\omega}}$. It is also possible to show that

$$(\Gamma' + H') G'' + \Gamma' H'' - (\Gamma'' + H'') G' - \Gamma'' H' > 0 \quad \text{for} \quad \bar{\omega} < \bar{\bar{\omega}},$$

which will guarantee an interior solution. The contracting problem may now be written as

$$\max_{K, \bar{\omega}} (1 - \Gamma - H) R_K K \quad \text{s.t.} \quad (\Gamma - G) R_K K = R(K - N).$$

Next, denote the illiquidity premium as follows: $s \equiv \frac{R^E}{R}$. Owing to constant returns to scale, the the capital to net worth ratio, $k \equiv \frac{K}{N}$, can be used as the choice variable. Defining V as the Lagrange multiplier on the constraint, the FOCs are given by

$$
\begin{aligned}
\bar{\omega} &: \quad \Gamma' + H' - V(\Gamma' - G') = 0, \\
k &: \quad (1 - \Gamma + V(\Gamma - G)) s - V = 0, \\
V &: \quad (\Gamma - G) sk - (k - 1) = 0.
\end{aligned}
$$

Since $\Gamma - G$ is increasing on $(0, \bar{\bar{\omega}})$ and decreasing on $(\bar{\bar{\omega}}, \infty)$, the retail bank never chooses

$\bar{\omega} > \bar{\bar{\omega}}$. The FOC w.r.t. $\bar{\omega}$ implies the Lagrange multiplier, V, can be written as a function of $\bar{\omega}$, given by

$$V(\bar{\omega}) = \frac{\Gamma' + H'}{\Gamma' - G'}.$$

Taking derivatives gives

$$V' = \frac{(\Gamma' + H')\,G'' + \Gamma' H'' - (\Gamma'' + H'')\,G' - \Gamma'' H'}{(\Gamma' - G')^2} > 0 \quad \text{for} \bar{\omega} < \bar{\bar{\omega}},$$

and taking limits gives

$$\lim_{\bar{\omega} \to 0} V(\bar{\omega}) = 1 \quad \text{and} \quad \lim_{\bar{\omega} \to \bar{\bar{\omega}}} V(\bar{\omega}) = +\infty.$$

The FOCs then imply that $\bar{\omega}$ satisfies

$$s(\bar{\omega}) = \frac{V}{1 - \Gamma - H + V(\Gamma - G)}, \tag{29}$$

where s is the illiquidity wedge between the rate of return on capital and the risk-free demanded by retail banks. Taking derivatives gives

$$s' = s\frac{V'}{V}\left(\frac{1 - \Gamma - H}{1 - F - H + V(\Gamma - G)}\right) > 0 \quad \text{for} \quad \bar{\omega} < \bar{\bar{\omega}},$$

and taking limits gives

$$\lim_{\bar{\omega} \to 0} s(\bar{\omega}) = 1 \quad \text{and} \quad \lim_{\bar{\omega} \to \bar{\bar{\omega}}} s(\bar{\omega}) = \frac{1}{\Gamma(\bar{\bar{\omega}}) - G(\bar{\bar{\omega}}(\omega^*))} < \frac{1}{\gamma}.$$

Thus, this guarantees a one-to-one mapping between the optimal $\bar{\omega}$ and the illiquidity premium, s. I introduce the following additional assumption:

$$\frac{1}{\Gamma(\bar{\bar{\omega}}) - G(\bar{\bar{\omega}}(\omega^*))} < \frac{1}{\gamma}.$$

If this condition does not hold, then it is possible that the illiquidity premium is so high that retail banks could earn a higher return by cutting out the wholesale banks and buy and manage capital directly.[31]

---

[31]When aggregate risk is introduced, the restriction on the illiquidity premium is weakened, since retail banks still want to insulate households from the aggregate risk. Suppose we decompose R_K into $u\mathbb{E}R_K$, where u is iid over time and $\mathbb{E}u = 1$ and $\text{cov}(u, \mathbb{E}R_K) = 0$. Thus, the realization of R_K is decomposed into its expected value and its stochastic element. Suppose the support on u is $(u^{\min}, u^{\max})$. Then we need to

The FOCs also give

$$k\left(\bar{\omega}\right) = 1 + \frac{V\left(\Gamma - G\right)}{1 - F - H}.$$ (30)

Taking derivatives gives

$$k' = \frac{V'}{V}\left(k - 1\right) + \frac{\Gamma'}{1 - F - H}k > 0 \quad \text{for} \bar{\omega} < \bar{\bar{\omega}},$$

and taking limits gives

$$\lim_{\bar{\omega} \to 0} k\left(\bar{\omega}\right) = 1 \quad \text{and} \quad \lim_{\bar{\omega} \to \bar{\bar{\omega}}} k\left(\bar{\omega}\right) = +\infty.$$

Combining (29) and (30) expresses the illiquidity premium as an increasing function of the capital to net worth ratio:

$$s = \Xi\left(k\right) \quad \text{with} \quad \Xi'\left(\cdot\right) > 0.$$

Section A.3 of Bernanke et al. (1999) extend the derivation to show that the relationship between s and k is still monotonically increasing with the introduction of aggregate risk.

---

assume that $u^{\min}\mathbb{E}R_K/R < 1/\gamma$, which is a weaker condition.

B   ONLINE APPENDIX (NOT FOR PUBLICATION)

B.1   Model summary

This Section provides the details of the full DSGE model including the specific distribution assumptions for ω; the steady state of the system of equilibrium equations; and the full log-linearized system.

B.1.1   The distribution of ω

The distribution of ω is assumed to be log-normal. Specifically, $\ln(\omega) \sim N\left(-\frac{1}{2}\sigma^2, \sigma^2\right)$ with $\mathbb{E}(\omega) = 1$ and

$$\mathbb{E}(\omega \mid \omega > x) = \frac{1 - \Phi\left(\frac{1}{\sigma}\left(\ln x - \frac{\sigma^2}{2}\right)\right)}{1 - \Phi\left(\frac{1}{\sigma}\left(\ln x + \frac{\sigma^2}{2}\right)\right)}, \tag{31}$$

where $\Phi(\cdot)$ is the cdf of the standard normal. It follows that under the mild fragility scenario (see Section A.2), $\Lambda(\cdot)$ and $G(\cdot)$ are given by

$$\Gamma(\bar{\omega}) = \bar{\omega}(1 - \Phi(\bar{z})) + \Phi(\bar{z} - \sigma), \tag{32}$$

$$G(\bar{\omega}) = \Phi(z^* - \sigma) - \gamma\lambda\Phi(z^*), \tag{33}$$

and where $\bar{z}$ and z^* are related to $\bar{\omega}$ through $\bar{z} \equiv (\ln\bar{\omega} + \sigma^2/2)/\sigma$ and $z^* \equiv (\ln\omega^*(\bar{\omega}) + \sigma^2/2)/\sigma$, respectively. Differentiating with respect to $\bar{\omega}$ gives

$$\Gamma'(\bar{\omega}) = (1 - \Phi(\bar{z})) - \bar{\omega}\phi(\bar{z})\bar{z}' + \phi(\bar{z} - \sigma)\bar{z}', \tag{34}$$

$$G'(\bar{\omega}) = \phi(z^* - \sigma)z^{*'}\omega^{*'} - \gamma\lambda\phi(z^*)z^{*'}\omega^{*'}, \tag{35}$$

where $\bar{z}' = 1/(\sigma\bar{\omega})$ and $z^{*'} = 1/(\sigma\omega^*)$.

B.1.2 Equilibrium conditions

The full set of equilibrium conditions is given below. The non-financial block is given by

$$1 = \beta \mathbb{E}_t \left(C_{t+1}/C_t \right)^{-\sigma_C} R_t, \qquad \text{(Euler condition)} \quad (36)$$

$$\chi L_t^{1+\rho} = (1-\alpha) Y_t C_t^{-\sigma_C}, \qquad \text{(Labour market)} \quad (37)$$

$$Y_t = A_t \left((\phi_t K_{t-1}) \right)^\alpha L_t^{1-\alpha}, \qquad \text{(Production function)} \quad (38)$$

$$R_{K,t} = \frac{\alpha Y_t / (\phi_t K_{t-1}) + (1-\delta) Q_t}{Q_{t-1}}, \qquad \text{(Return on capital)} \quad (39)$$

$$K_t = (1-\delta) \phi_t K_{t-1} + \frac{(I_t/I)^{1-\varphi} - \varphi}{1-\varphi} I \qquad \text{(Capital accumulation)} \quad (40)$$

$$Q_t = (I_t/I)^\varphi \qquad \text{(Price of capital)} \quad (41)$$

$$Y_t = C_t + C_{W,t} + I_t, \qquad \text{(Aggregate resource constraint)} \quad (42)$$

In the frictionless model, $\phi_t = 1$, $C_{W,t} = 0$, and $1 = \beta \mathbb{E}_t \left(C_{t+1}/C_t \right)^{-\sigma_C} R_{K,t+1}$. The financial block is given by

$$\mathbb{E}_t \left(\frac{d\Gamma_{t+1}}{d\Gamma_{t+1} - dG_{t+1}} \right) = \mathbb{E}_t \left(S_{t+1} \left(1 + \frac{\Gamma_{t+1} dG_{t+1} - d\Gamma_{t+1} G_{t+1}}{d\Gamma_{t+1} - dG_{t+1}} \right) \right) \qquad \text{(FOCs)} \quad (43)$$

$$1 = (1 - (\Gamma_t - G_t) S_t) \ell_{t-1} \qquad \text{(Break-even)} \quad (44)$$

$$N_t = \upsilon \left(\phi_t S_t \ell_{t-1} - (\ell_{t-1} - 1) \right) R_{t-1} N_{t-1} \qquad \text{(Net worth)} \quad (45)$$

$$C_{W,t} = \frac{1-\upsilon}{\upsilon} N_t \qquad (46)$$

$$S_t = R_{K,t}/R_{t-1} \qquad (47)$$

$$\ell_t = Q_t K_{t-1}/N_{t-1} \qquad (48)$$

$$\Gamma_t = \bar{\omega}_t \left(1 - \Phi \left(\bar{z}_t \right) \right) + \Phi \left(\bar{z}_t - \sigma \right), \qquad (49)$$

$$G_t = \Phi \left(z_t^* - \sigma \right) - \gamma \lambda \Phi \left(z_t^* \right), \qquad (50)$$

$$\phi_t = 1 - G_t \qquad \text{(Deadweight)} \quad (51)$$

$$\omega_t^* = \gamma \lambda_t x_t \left(1 - \ln \left(x_t \right) \right) / \left(x_t + (1-x_t) \ln \left(1 - x_t \right) \right) \qquad (52)$$

$$x_t = \lambda_t / \bar{\omega}_t \qquad (53)$$

$$\bar{z}_t = \ln \left(\bar{\omega}_t \right) / \sigma + \sigma/2 \qquad (54)$$

$$z_t^* = \ln \left(\omega_t^* \right) / \sigma + \sigma/2 \qquad (55)$$

where all the derivatives are given by

$$dΓ_t = (1 - Φ(\bar{z}_t)) - \barω\phi(\bar{z}_t)\,d\bar{z}_t + \phi(\bar{z}_t - σ)\,d\bar{z}_t, \tag{56}$$

$$dG_t = \phi(z_t^* - σ)\,dz_t^* dω_t^* - γλ\phi(z_t^*)\,dz_t^* dω_t^*, \tag{57}$$

$$d\bar{z}_t = 1/(\barω_t σ) \tag{58}$$

$$dz_t^* = 1/(ω_t^* σ) \tag{59}$$

$$dω^* = \left(γx_t^2 - (x_t + \ln(1 - x_t))\,ω_t^*/\barω_t\right)/(x_t + (1 - x_t)\ln(1 - x_t)) \tag{60}$$

Finally, the exogenous stochastic process are given by

$$A_t = A_{t-1}^{ρ_A} e^{σ_A ε_{A,t}} \qquad \text{(Technology)} \tag{61}$$

$$λ_t = λ^{1-ρ_λ} λ_{t-1}^{ρ_λ} e^{σ_λ ε_{λ,t}} \qquad \text{(Liquidity)} \tag{62}$$

$$\tag{63}$$

B.1.3   Non-stochastic steady state

The non-stochastic steady state is given below. Equation (36) gives $R = \beta^{-1}$ and (41) gives $Q = 1$. χ is chosen to normalize $L = 1$. Note that $\phi \approx 1$. Equation (39) gives

$$\frac{Y}{K} = \phi \left(R_K - 1 + \delta \right) / \alpha, \tag{64}$$

and (38) gives $K = (Y/K)^{1/(\alpha - 1)}$. Equation (40) gives $\frac{I}{K} = 1 - \phi \left(1 - \delta \right)$. Next, $\frac{I}{Y} = \frac{I}{K} / \frac{Y}{K}$. Equation (42) gives $C/Y = 1 - \frac{1-\upsilon}{\upsilon} N - I/Y$. Equation (37) gives

$$\left(1 - \alpha \right) Y = \theta C^{-\sigma_C}. \tag{65}$$

The financial block, (43), (44), and (45), is given by

$$S = \frac{d\Gamma}{d\Gamma \left(1 - G \right) - \left(1 - \Gamma \right) dG},$$

$$S = \frac{1}{\Gamma - G} \frac{\ell - 1}{\ell},$$

$$1 = \nu \left(\phi S \ell - \ell + 1 \right) / \beta.$$

B.1.4 First-order approximation

The (log)-linear approximation of the model in the neighbourhood of the non-stochastic steady state is given below. The non-financial block is given by

$$c_t = \mathbb{E}_t c_{t+1} - (1/\sigma_C)\, r_t \qquad \text{(Euler condition)} \quad (66)$$

$$y_t = (1+\rho)\, l_t + \sigma_C c_t \qquad \text{(Labor market)} \quad (67)$$

$$y_t = a_t + \alpha\left(\tilde{\phi}_t + k_{t-1}\right) + (1-\alpha)\, l_t \qquad \text{(Production function)} \quad (68)$$

$$r_{k,t} = \frac{B_1}{B_1+B_2}\left(y_t - k_{t-1} - \tilde{\phi}_t\right) + \frac{B_2}{B_1+B_2} q_t - q_{t-1} \qquad \text{(Return on capital)} \quad (69)$$

$$k_t = \phi\,(1-\delta)\left(\tilde{\phi}_t + k_{t-1}\right) + (1 - \phi\,(1-\delta))\, i_t \qquad \text{(Capital accumulation)} \quad (70)$$

$$q_t = \psi i_t \qquad \text{(Price of capital)} \quad (71)$$

$$y_t = \frac{C}{Y} c_t + \frac{C_W}{Y} n_t + \frac{I}{Y} i_t \qquad \text{(Aggregate resource constraint)} \quad (72)$$

where $B_1 \equiv \alpha Y/(\phi K)$ and $B_2 = 1-\delta$. In the frictionless model, $\tilde{\phi}_t = n_t = 0$ and $c_t = \mathbb{E}_t c_{t+1} - (1/\sigma_C)\,\mathbb{E}_t r_{k,t+1}$. The financial block is given by

$$(\mathbb{E}_t r_{k,t+1} - r_t) = \frac{N}{K}\frac{\Gamma_{\bar\omega} G_{\bar\omega\bar\omega} - \Gamma_{\bar\omega\bar\omega} G_{\bar\omega}}{\Gamma_{\bar\omega}(\Gamma_{\bar\omega} - G_{\bar\omega})}\bar\omega \mathbb{E}_t\widetilde{\omega}_{t+1} + \left(\frac{N}{K}\frac{G_{\bar\omega\lambda}}{(\Gamma_{\bar\omega} - G_{\bar\omega})} + G_\lambda \beta R_K\right)\lambda \mathbb{E}_t\widetilde{\lambda}_{t+1}$$

$$+ \left(\frac{N}{K}\frac{G_{\bar\omega\mathbf{n}}}{(\Gamma_{\bar\omega} - G_{\bar\omega})} + G_{\mathbf{n}}\beta R_K\right)\mathbf{n}\mathbf{n}_{t+1}. \qquad (73)$$

Break even condition:

$$0 = \frac{\Gamma_{\bar\omega}\bar\omega}{1-\Gamma}\widetilde{\omega}_t - G_\lambda \beta R_K \frac{K}{N}\lambda\widetilde{\lambda}_t - G_{\mathbf{n}}\beta R_K \frac{K}{N}\mathbf{n}\mathbf{n}_t$$

$$+ \left(\frac{\Gamma - G}{1-\Gamma}\frac{\Gamma_{\bar\omega}}{\Gamma_{\bar\omega} - G_{\bar\omega}}\right)(r_{k,t} - r_{t-1}) - (q_{t-1} + k_{t-1} - n_{t-1}). \qquad (74)$$

The elasticity of the external finance premium with respect to the capital to net worth ratio, χ, calculated in Section 3.3 is derived by rolling forward equation (74) by one period and substituting into (73) and eliminating $\widetilde{\omega}_t$.

Net worth:

$$n_t = \upsilon\left(\frac{\phi R_K K}{N}\left(r_{k,t} + b_1\widetilde{\omega}_t + b_2\widetilde{\lambda}_t + b_3\mathbf{n}_t\right) - R\left(\frac{K}{N} - 1\right)r_t + \frac{(\phi R_K - R)\,K}{N}(q_{t-1} + k_{t-1}) + R n_{t-1}\right)$$

$$- \mathbf{c}_t + \mathbf{c}_{t-1}.$$

Price of capital: Technology and illiquidity shock processes:

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t} \quad \& \quad \widetilde{\lambda}_t = (1 - \rho_\lambda) \lambda + \rho_\lambda \widetilde{\lambda}_{t-1} + \sigma_\lambda \varepsilon_{\lambda,t}.$$

Government debt accumulation:

$$d_{g,t} = \frac{1}{\beta(1+x)} d_{g,t-1} - \frac{\tau' N}{1+x} \mathtt{c}_t + \frac{(1-\Gamma) R_K K}{1+x} \mathtt{c}_{t-1} - \frac{\tau'(K-N)}{1+x} \mathtt{n}_t + \frac{(\Gamma - pol) R_K K}{1+x} \mathtt{n}_{t-1}.$$

Policy rules:

$$\mathtt{n}_t = -a_{DL} \left(\mathbb{E}_t r_{k,t+1} - r_t \right) \quad \text{and} \quad \mathtt{c}_t = -a_{EI} \left(\mathbb{E}_t r_{k,t+1} - r_t \right).$$

C   Data

C.1   Illiquidity premium

As a robustness check, in addition to using the TED spread, I also construct a measure of the liquidity premium following Bredemeier et al. (2018). In particular, I collect quartlery data from 1982-2018 from `http://fred.stlouisfed.orf` to construct seven spreads between Treasuries and illiquid assets of similar safety and maturity:

1. 3m AA NFC commerical paper rate - 3m Treasury

2. BoA US corporate 1-3y - 3y Treasury

3. BoA US corporate AAA - 5y Treasury

4. Moody's Aaa corporate bond - 10y Treasury

5. Moody's Baa corporate bond - 10y Treasury

6. Certificates of deposit 3m - 3m Treasury

7. Certificates of deposit 6m - 6m Treasury

Next, I construct a factor model with all spreads to extract a common component over time using principle component analysis. Let f_t be the resulting factor and let LP_t be the liquidity premium. These are related by $LP_t = a + bf_t$, where a and b are unknown. To recover these parameters, I make the following two assumptions: 1) Following Krishnamurthy and Vissing-Jorgensen (2012), the liquidity premium in 2007q2 equalled 46bp. 2) Following Del Negro et al. (2017), the peak liquidity premium in 2008q4 was 342bp.

C.2   Calibration using simulated method of moments

Using Basu and Bundick (2017) code, a subset of the model parameters are estimated using simulated method of moments to match the unconditional volatility and persistence of output and its components, hours worked and the TED spread. Formely, the estimator is the solution of the following problem

$$J = \min_{\theta} \left(M^D - M\left(\theta\right) \right)' W^{-1} \left(M^D - M\left(\theta\right) \right), \tag{75}$$

where θ is a vector of parameters to be estimated, M^D denotes a vector that comprises the data moments and $M\left(\theta\right)$ denotes its model counterpart. W is a diagnoal matrix with the empirical standard errors of the estimated moments. The length of the moment vector must be weakly greater than the length of the parameter vector. In this case, I have 10 moments and 5 parameters.

C.3   Data transformations

For calculating data moments, I use the logged and HP-filtered transformation of the following variables

- **Output** is real GDP (FRED code: `GDPC1`) divided by civilian noninstitutional population, Pop_t (`CNP16OV`)

- **Consumption** is nominal personal consumption expenditure (`PCEC`), deflated by the the GDP implicit price deflator, P_t (`GDPDEF`) and divided by Pop_t.

- **Investment** is nominal fixed private investment (`FPI`), deflated by P_t and divided by Pop_t.

- **Hours worked** is the nonfarm business sector average weekly hours (`PRS85006023`), divided by Pop_t.

- **TED spread** (`TEDRATE`).

To calculate the bank failure rate time-series, I collected data from the FDIC website: `https://www5.fdic.gov/hsob/SelectRpt.asp?EntryTyp=30Header=1`. This provides a time-stamp and balance sheet data for every bank covered by the FDIC that has failed from 1934-2018. I construct two series: 1) counts the number of bank failures per quarter and the total number of banks in the US, to construct a failure rate. 2) aggregates the total deposits of the failing banks and the total deposits in the US banking system, to construct a failure

rate. This second measure better accounts for the heterogeneity in bank size across the US banking sector.

C.4   Counterfactual data projections

To examine how the US economy would have evolved absent the financial crisis, I adopt the approach of Christiano et al. (2015). For each variable, I fit a linear trend from date τ to 2007q2, where $\tau \in \{1985q1, 2003q1\}$. I then extrapolate the trend line for each variable. Then, for each τ, I calculate the difference between the projected value of each variable and its actual value. Figure 8 plots the min, max, and median of these difference paths.