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Highlights for MS#EL41967: “Information Disclosure and Asymmetric Speed of Learning in Booms and Busts”

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- We analyse how information disclosure affects learning speed in booms and busts
- We give a formal condition on what type of information leads to asymmetric learning
- Learning is faster in a bust when only the winning bids are disclosed
Information Disclosure and Asymmetric Speed of Learning in Booms and Busts

Francesco Palazzo¹, Min Zhang²,∗

Abstract
We consider a model in which agents gradually learn about the aggregate market conditions — ‘boom’ or ‘bust’ — from the information disclosed after a trading round. The disclosure rules can generate asymmetric learning and affect the degree of asymmetry. In particular, when only winning bids are publicly disclosed, learning is more rapid in a bust.

JEL classification: D82, D83.
Keywords: Asymmetric Learning; Information Disclosure.

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1. Introduction

In many markets there is limited disclosure of price offers and it is common to only observe the final transaction prices; for example, a real-estate buyer may collect information on past prices but he may not have any information on unsuccessful offers. The existence of informational limitations may come from different origins: the absence of an organized trading platform, practical limitations on information disclosure, legal restrictions on transparency, etc.

This paper considers a model where agents gradually learn about the aggregate market conditions — ‘boom’ or ‘bust’ — from the information publicly disclosed after a trading round. We show how the type of information disclosed may not be equally informative when the aggregate market is in a ‘boom’ or in a ‘bust’. Our main result provides a relationship between the statistical properties of the disclosed information and the different speeds of learning in booms and busts. In particular, when only the winning offers are disclosed — a common feature in many markets — beliefs tend to adjust more rapidly when the aggregate state is low, i.e., in a ‘bust’. Indeed, the winning offer discloses the highest private valuation among the bidders, and a low realization of it provides strong evidence in favor of the low state because it implies that all bidders have low private valuations. On the contrary, a high realization may merely come from a single outlier with a particularly high valuation, hence bringing weaker evidence in favor of the high state. In other words, competition among buyers introduces orderedness in the formation of trading prices, and in turn generates asymmetric informativeness about the underlying states.

This learning pattern is likely to affect the resulting price dynamics in relevant markets, especially their responsiveness to cyclical changes. For instance, in Palazzo and Zhang [10] we present an auction model with resale in which bids endogenously depend on bidders’ beliefs over the aggregate market conditions. Asymmetric learning implies a more rapid decline of prices during a bust relative to the corresponding rise during a boom.

Our model shares the feature of learning about underlying states from public observables with the literature on observational learning. The literature mainly studies whether public beliefs converge to the truth, i.e., complete learning, which is not an issue here with trading prices as observables. We instead focus on the relative speed of learning between the underlying states, and how it depends on the type of information disclosed by the public observables. In this respect, our work is also related to the literature on procyclical learning as a source of business cycles. Van Nieuwerburgh and Veldkamp [13], Chalkley and Lee [4] and Veldkamp [14] share the idea that agents enjoy more precise signals during a boom: in the former paper learning is faster in a boom because of an increase in the signal precision; in the latter two papers the precision of public signals changes along the cycle due to differences in the endogenous composition of informed agents. In these models the dynamics of beliefs and aggregate activities are characterized by fast declines and slow recoveries.

Relative to this strand of literature, the asymmetric speed of learning in this paper arises from the orderedness of the information disclosed to the public. On the other hand, information disclosure in our model is determined exogenously by the trading mechanism, as opposed to the literature on information revelation in auctions where disclosure is endogenous and strategic. Similarly, our work also differs from the literature on learning in decentralized markets where information revelation again is through strategic trading behavior of sellers and/or buyers.

Section 2 provides the main result on asymmetric learning. Section 3 concludes. All proofs are in the Appendix.

2. The Model

2.1. Setup

Consider a sequential market for a durable object. Time is discrete, \( t \in \{0, 1, 2, \ldots \} \), and there is an unobservable state \( \theta \in [H, L] \), \( \pi_0 \equiv P_0(\theta = H) = \frac{4}{7} \) denotes the common prior, and \( \pi_t \equiv P_t(\theta = H) \) denotes the public posterior at the beginning of period \( t \).

In every period \( t \) an identical object is offered on sale and \( N \geq 2 \) buyers are randomly drawn from a population of infinitely many agents. The object is sold according to a trading mechanism \( \mathcal{M} \), and each buyer reports a message \( m_t \) to the mechanism based on her private valuation \( v \) of the object.

The private valuations generated in each period are identically and independently distributed according to a cumulative distribution function (cdf) \( F_\theta \) across the \( N \) buyers and over time. Both \( F_H \) and \( F_L \) are absolutely continuous on the common support \([0, 1]\) and continuously differentiable with probability density functions (pdf) \( f_H \) and \( f_L \) that satisfy strict monotone likelihood ratio (MLR) property.

By the revelation principle (Myerson [9]), there exists a direct mechanism to which all buyers in period \( t \) truthfully report their valuations, \( V_t \equiv (v_{1,t}, v_{2,t}, \ldots, v_{N,t}) \). We abstract from modeling any specific informational limitations, and directly characterize the information publicly revealed by the (direct) trading mechanism with a reduced-form disclosure rule (a measurable statistic), \( T : [0, 1]^N \rightarrow \mathbb{R}^M, M \leq N \). For instance, if trades take place through a sequence of first-price auctions, public disclosure of the winning bid is equivalent to public disclosure of the highest valuation in \( V_t \).

2.2. Asymmetric Learning

Denote with \( S_T \) the support of disclosed information \( T(V) \), and \( f_T^2 \) the pdf of \( T(V) \) under state \( \theta \). We describe the evolution of the public beliefs in terms of log-likelihood ratio, \( \pi_T(\theta = H | T(V)) \).

\[ \frac{\pi_T(\theta = H | T(V))}{\pi_T(\theta = L | T(V))} = \frac{f_T^2(\theta = H | T(V))}{f_T^2(\theta = L | T(V))} \]

The focus there is often on the revelation of seller’s information for the purpose of revenue maximization. The seminal paper by Milgrom and Weber [8] shows that full revelation is optimal, known as the ‘linkage principle’. More recent papers by Benoit and Dubra [2] and Tan [12] consider buyers’ information revelation in auctions with both private and common values.

This literature studies whether trading process implies full information revelation when the decentralized market becomes approximately frictionless. Wolinsky [16] and Blouin and Serrano [3] propose negative answers in either stationary or non-stationary settings, while Serrano and Yoshio [11] confirm the existence of fully revealing equilibria under one-sided informational asymmetry in the stationary setting. In a setting with finitely many sellers and a continuum of buyers, Gottardi and Serrano [5] show that information is fully and immediately revealed under intense competition among sufficiently many sellers.

With a slight abuse of notation, time subscript for \( V \) is omitted due to its i.i.d. property over time.

\[ \frac{\pi_T(\theta = H | T(V))}{\pi_T(\theta = L | T(V))} = \frac{f_T^2(\theta = H | T(V))}{f_T^2(\theta = L | T(V))} \]

1As Lee [6] shows, learning is complete with sufficiently ‘rich’ action spaces, (e.g., prices), and nondegenerate payoffs. See also earlier work by Milgrom [7] on information aggregation in auctions.

A related work by Acemoglu et al. [1] studies the rate of convergence of learning when agents observe either the most recent action or a random action from the past.
updated with a new disclosure $y_t \in S_T$ in period $t$:

$$l_{t+1}(h_t, y_t) = \ln \frac{\pi_{t+1}}{1 - \pi_{t+1}} = \ln \frac{\pi_{t+1} f_{H}^{f_{L}}(y_{t+1})}{\pi_{t+1} f_{L}^{f_{L}}(y_{t+1})} - l_t + \ln \frac{f_{H}^{f_{L}}(y_{t+1})}{f_{L}^{f_{L}}(y_{t+1})}$$  \hspace{1cm} (1)

We add a superscript $T$ to the log-likelihood ratios in equation (1) to stress their dependence on the disclosure rule $T$, and simply denote the belief update from period $t$ to $t + 1$ by

$$\Delta l_{t+1}^T(y_t) \equiv l_{t+1}^T - l_t^T = \ln \frac{f_{H}^{f_{L}}(y_{t+1})}{f_{L}^{f_{L}}(y_{t+1})}.$$  \hspace{1cm} The information generated by $T$ is assumed bounded but non-trivial:

$$\exists M > 0 \text{ s.t. } 0 < |\Delta l^T(y_t)| < M \text{ for almost every } y \in S_T. \hspace{1cm} (2)$$

For any integer $q \geq 1$, equation (1) generalizes into:

$$l_{t+q}^T = l_t^T + \sum_{m=1}^{q} \Delta l_{t+m}^T(y_{t+m-1}).$$  \hspace{1cm} (3)

Taking the expected value:

$$E_{\pi_{t}, \theta} l_{t+q}^T = l_t^T + \sum_{m=1}^{q} E_{\pi_{t}, \theta} \Delta l_{t+m}^T(y_{t+m-1}) = l_t^T + q E_{\pi_{t}} |\Delta l^T|.$$  \hspace{1cm} (4)

The last equation exploits the fact that, conditional on $\theta$, samples are i.i.d. across all periods.

Proposition 1 links the information revealed by the trading mechanism to the relative speed of convergence of public beliefs to the high or low state.

**Proposition 1. Let:**

$$\tau^H \equiv \inf \{t \geq 0 : \pi_t \geq 1 - \epsilon\}, \hspace{0.5cm} \tau^L \equiv \inf \{t \geq 0 : \pi_t \leq \epsilon\}. \hspace{1cm} (5)$$

*For a measurable statistic $T : [0, 1]^N \rightarrow \mathbb{R}^M$, we have*

$$\lim_{\epsilon \to 0} \frac{E_{T} [\tau^H]}{E_{T} [\tau^L]} \geq \frac{E_{\pi_{t}} |\Delta l^H|}{E_{\pi_{t}} |\Delta l^L|} > 1 \text{ if } E_{\pi_{t}} |\Delta l^H| + E_{\pi_{t}} |\Delta l^L| < 0; \hspace{1cm} (6)$$

$$\lim_{\epsilon \to 0} \frac{E_{T} [\tau^H]}{E_{T} [\tau^L]} \geq \frac{E_{\pi_{t}} |\Delta l^H|}{E_{\pi_{t}} |\Delta l^L|} > 0 \text{ if } E_{\pi_{t}} |\Delta l^H| + E_{\pi_{t}} |\Delta l^L| > 0. \hspace{1cm} (7)$$

The ratio $\frac{E_{\pi_{t}} |\Delta l^H|}{E_{\pi_{t}} |\Delta l^L|}$ determines, in every period, whether the information revealed by $T$ is more or less informative between the two states. Proposition 1 highlights that, in our dynamic setup with gradual information revelation, this ratio also serves as a lower bound for the ratio of the expected times to (almost) learn the truth under the two states.

The next corollary points out that the speed of learning is asymmetric when the trading mechanism reveals an order times to (almost) learn the truth under the two states. Proposition 1.

**Corollary 1. For $T(V) = v^{(1,N)}$ with sufficiently large $N$, equation (6) holds, i.e., learning is faster under state $L$.**

The number of buyers $N$ has two effects on learning. On the one hand, with a larger $N$, a low realization of $v^{(1,N)}$ becomes stronger evidence in favor of state $L$ as it suggests all buyers must have low private valuations, while a high realization of $v^{(1,N)}$ becomes weaker evidence in favor of state $H$ as it could just come from a single buyer with an ‘abnormally’

high valuation. Such informational asymmetry toward state $L$ is amplified as $N$ increases. On the other hand, increasing the number of buyers $N$ makes $v^{(1,N)}$ more likely to have relatively high rather than low realizations, i.e., people are more likely to observe evidence in favor of state $H$. Nevertheless, such distributional advantage toward state $H$ diminishes as $N$ gets large, because any further increase of $N$ does not shift much the distribution of the order statistic. Therefore, the first effect dominates the second one for sufficiently large $N$, and it eventually leads to different speeds of learning under the two states.\(^8\)

**3. Conclusion**

This paper provides a formal condition to characterize how partial information disclosure of trading histories can generate a difference in the speed of learning about the aggregate market conditions. In particular, when only winning offers are publicly disclosed, learning is relatively faster and prices thus respond more quickly during a bust than during a boom. A natural way to limit such difference in price responsiveness along the cycle is to grant full disclosure of past price offers.

**Appendix**

**Proof of Proposition 1**

As $\pi_0 = \frac{1}{2}$, for every $t \geq 1$ we have $l_{t}^T = \sum_{i=0}^{t-1} \Delta l_{i+1}^T$, where $\Delta l_{i+1}^T = \ln \frac{f_{H}^{f_{L}}(y_{i})}{f_{L}^{f_{L}}(y_{i})}$, $i = 0, 1, \ldots, t$, are i.i.d. across periods. The learning times $\tau_{H}^T$ and $\tau_{L}^T$ can be expressed in terms of $l_{t}^T$:

$$\tau_{H}^T = \inf \{t > 0 : l_{t}^T \geq \ln \frac{1 - \epsilon}{\epsilon}\}, \hspace{0.5cm} \tau_{L}^T = \inf \{t > 0 : l_{t}^T \leq \ln \frac{\epsilon}{1 - \epsilon}\} \hspace{1cm} (8)$$

Applying Wald [15]’s lemma to the i.i.d. sequence $\{\Delta l_{i+1}^T\}_{i=0}^t$:

$$E_{\pi_t}[\tau_{H}^T] = E_{\pi_t}[\tau_{L}^T] + E_{\pi_t}[|\Delta l^T|] \quad \forall \theta \in \{H, L\} \hspace{1cm} (9)$$

By Gibbs’ inequality,

$$E_{\pi_t}[|\Delta l^T|] = \int_{S_T} \ln \frac{f_{H}^{f_{L}}(y_{i})}{f_{L}^{f_{L}}(y_{i})} f_{H}^{f_{L}}(y_{i}) d(y_{i}) > 0, \hspace{1cm} (10)$$

and similarly $E_{\pi_t}[|\Delta l^L|] < 0$.\(^9\) When $E_{\pi_t}[|\Delta l^H| + E_{\pi_t}[|\Delta l^L|] < 0,$

$$E_{\pi_t}[|\Delta l^H|] = - \left( E_{\pi_t}[|\Delta l^H|] + c \right) \hspace{1cm} (11)$$

where $c = - \int_{S_T} \ln \frac{f_{H}^{f_{L}}(y_{i})}{f_{L}^{f_{L}}(y_{i})} (f_{H}^{f_{L}}(y_{i}) + f_{L}^{f_{L}}(y_{i})) d(y_{i}) > 0$. Notice that $c$ only depends on the pdfs and is independent of $\epsilon$. Substituting in (9):

$$E_{\pi_t}[\tau_{H}^T] = E_{\pi_t}[\tau_{L}^T] = E_{H}[\tau_{H}^T], \hspace{1cm} E_{L}[\tau_{H}^T] + c = -E_{L}[\tau_{L}^T] \hspace{1cm} (12)$$

Hence:

$$E_{H}[\Delta l^H](E_{H}[\tau_{H}^T] - E_{L}[\tau_{L}^T]) = E_{H}[\tau_{H}^T] + E_{L}[\tau_{L}^T] + E_{L}[\tau_{L}^T] c \hspace{1cm} (13)$$

\(^8\)With the same intuition, Corollary 1 can be naturally extended to the case of $T(V) = v^{(k,N)}$ with a general $k$.

\(^9\)The inequalities are strict as $|\Delta l^T| = |\ln \frac{f_{H}^{f_{L}}}{f_{L}^{f_{L}}}| > 0$ a.s. by assumption.
Rearranging:

\[
\mathbb{E}_H[\Delta^T] \left( \frac{\mathbb{E}_H[\tau^T] + \mathbb{E}_L[\tau^T]}{\mathbb{E}_L[\tau^T]} - 1 \right) = \frac{\mathbb{E}_H[\Delta^T] + \mathbb{E}_L[\Delta^T]}{\mathbb{E}_L[\tau^T]} + c
\]  \hspace{1cm} (14)

Notice that \(\Delta^T < M\) implies \(\mathbb{E}_L[\tau^T] \geq \ln \frac{M}{1-c} - M\) and by definition \(\mathbb{E}_H[\tau^T] \geq M\), so:

\[
\mathbb{E}_H[\Delta^T] \left( \frac{\mathbb{E}_H[\tau^T] + \mathbb{E}_L[\tau^T]}{\mathbb{E}_L[\tau^T]} - 1 \right) \geq \frac{M}{\mathbb{E}_L[\tau^T]} + c
\]  \hspace{1cm} (15)

On the other hand, by equation (9):

\[
\mathbb{E}_L[\tau^T] = \frac{\mathbb{E}_L[\tau^T]}{\mathbb{E}_L[\tau^T]} \geq \ln \frac{M}{1-c} > 0
\]  \hspace{1cm} (16)

\[
\Rightarrow \mathbb{E}_H[\Delta^T] \left( \frac{\mathbb{E}_H[\tau^T] + \mathbb{E}_L[\tau^T]}{\mathbb{E}_L[\tau^T]} - 1 \right) \geq -\frac{M^2}{\ln \frac{M}{1-c}} + c
\]  \hspace{1cm} (17)

Since \(0 < \mathbb{E}_H[\Delta^T] < M\), we have:

\[
\frac{\mathbb{E}_H[\tau^T]}{\mathbb{E}_L[\tau^T]} - 1 > -\frac{M}{\ln \frac{M}{1-c}} + \frac{c}{\mathbb{E}_L[\tau^T]}
\]  \hspace{1cm} (18)

Similarly, because \(\Delta^T\) is bounded a.s. by assumption and \(F^\theta(1,N+1)(\cdot) - F^\theta(1,N)(\cdot) < 0\) as \(N \to \infty\), for \(\theta \in \{H,L\}\):

\[
\text{By the mean value theorem for integrals:}
(1y) + (1L) = \ln \frac{F_H(y)}{F_L(y)} + \ln \frac{F_H(y)}{F_L(y)} < 0\text{ for some } y, \bar{y} \in (0, 1). \hspace{1cm} (22)
\]

where \(F_H(\cdot) < F_L(\cdot)\) due to the MLR property.

Meanwhile, \(\Delta^T\) is bounded a.s. by assumption and \(F^\theta(1,N+1)(\cdot) - F^\theta(1,N)(\cdot) \to 0\) as \(N \to \infty\), for \(\theta \in \{H,L\}\). \hspace{1cm} (23)

Hence, \((2_H), (2_L) \to 0\) as \(N \to \infty\). Then for sufficiently large \(N\), the expression \(\mathbb{E}_H[\Delta^T] + \mathbb{E}_L[\Delta^T] - (\mathbb{E}_H[\tau^T] + \mathbb{E}_L[\tau^T])\) has the same sign of \((1_H + 1_L)\), i.e., negative. \blacksquare

**References**


\[\text{11Both } (2_H)\text{ and } (2_L)\text{ are in fact positive diminishing terms, because } \Delta^T(\cdot) \text{ is monotone due to the MLR property while } F^\theta(1,N+1)(\cdot) < F^\theta(1,N)(\cdot) \text{ for } \theta \in \{H,L\}.\]