ESSAYS ON CARTEL POLICY WITH ENDOGENOUS CARTEL SIZE

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University of
St Andrews

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Abstract

This thesis examines the role of endogenous size processes in the stability and price setting decisions of cartels. Chapter One analyses how the stability of cartels depends on the level of horizontal product differentiation and on costs of collusion under the premise that a cartel can consist of less than all firms in an industry. It is shown that when the size of the cartel is determined endogenously, it is possible that increased costs of collusion make a cartel more stable.

Chapter Two analyses how the price setting of firms in collusive industries is affected by three different penalty regimes: i) profits, ii) overcharge, and iii) revenue based penalties. It is found that penalties influence price setting in two ways: directly, by affecting the industry price for a given cartel size and indirectly by affecting cartel size and thereby the price charged. When the penalties are equally tough, in the sense that they deter cartels over the same group of products, overcharge based penalties always lead to the lowest prices, followed by prices computed under profits based penalties and then revenue based penalties. For very few combinations of product differentiation and market size, revenue based penalties lead to lower prices than profits based penalties.

Finally, Chapter Three presents a model in which collusive stability is analysed in a dynamic setting of free entry, exit and mergers. Contrary to the previous literature it shows that stable and profitable collusion is possible under free entry, without the need for cartels to play entry deterring strategies. Furthermore, the empirical evidence that a breakdown of collusion can lead to increased merger activity is replicated. An additional contribution of this model is that it defines a new notion of a long run sustainable competitive market size under merger and entry.

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1 Introduction

Collusive practices or cartels are defined by the European Commission (EC) as "a group of similar, independent companies which join together to fix prices, to limit production or to share markets or customers between them".[1] Although collusive practices are illegal in most developed jurisdictions, multiple cases are detected each year, ranging from the German automotive industry to UK banks or Canadian car parts manufacturers. In order to detect and prosecute cartels, competition authorities (CAs) devote a significant amount of their resources into detecting and prosecuting cartels. For example, Joaquin Almunia, the then Vice President of the EC and responsible for Competition Policy, said in a speech on 19th of September 2014: "I have said in many occasions that cartels are my top enforcement priority. In fact, they have been a priority for the European Commission since the late 90's(...)"[2].

Nevertheless, CAs have limited resources and can therefore only investigate a limited number of industries and firms. Hence, in order to most effectively use their resources, CAs need to channel activities into those industries and firms that are the most prone to collusion. In order for this strategy to succeed, a well-founded understanding of the characteristics of industries at risk of collusion is crucial. At the same time, it is important to understand how firms react to actions imposed by CAs.

This PhD contributes to this understanding by expanding the existing industrial economics literature on cartels in three chapters. Specifically, this thesis challenges two standard assumptions that are made in much of the existing literature on cartels. Firstly, in Chapters One and Two, the standard assumption that all firms in an industry have to be part of the cartel is confronted. Secondly, in Chapter Three, the assumption that the number of firms in a collusive industry is fixed is challenged.

In particular, Chapter One analyses the relationship between horizontal product differentiation and the stability of costly collusion. Whilst there exists a wide range of academic research on this topic, a typical assumption inherent in the existing literature is that all firms inside the industry are part of the cartel. In parallel to this, there are also a series of models that predict that cartels can consist of less than all firms in the industry. However, none of the existing models explicitly analyse how the

[1]ec.europa.eu/competition/cartels/overview, last access on 07.09.2017

[2]Speech given at the IBA 18th Annual Competition Conference, text available under http://europa.eu/rapid/press-release_SPEECH-14-608_en.htm, last access on 07.09.2017

stability of collusion is affected by the degree of product differentiation, or how costs of collusion may affect cartel stability. The model presented in Chapter One is the first to combine the analysis of product differentiation and cartel stability alongside the assumptions that cartels can consist of less than all firms in the industry and that collusion may be costly.

In doing so, the model contributes to the existing literature in three specific ways. Firstly, through a comparison of how the stability of differently sized cartels depends on the degree of horizontal product differentiation when there are no costs of collusion, it finds that those cartels which encompass all firms in the industry tend to become less stable when products become more homogeneous, while for cartels that contain less than all firms in an industry (small cartels), the opposite is true. At the same time, it is shown that small cartels are more stable than their larger counterparts. Secondly, the model demonstrates how fixed costs of collusion can affect cartel stability when the number of firms inside the cartel is given exogenously. It is found that the result that small cartels are more stable than large ones is not upheld when costs are taken into account, but that collusive stability has to be compared on a case-by-case basis. Finally, the model illustrates how the stability of cartels depends on fixed costs of collusion when the number of firms in a cartel is determined endogenously. It is shown that when endogenous size processes are taken into account, in some cases it is possible for an increase in the costs of collusion to make a cartel more stable. Overall, the results in Chapter One demonstrate that decisions about collusive agreements can depend heavily on the number of firms involved and on the costs imposed on upholding collusion.

The findings from Chapter One motivate Chapter Two, which presents a structured comparison of how different penalty regimes affect the overall prices charged in a collusive industry. Three penalty regimes are considered. Those are, penalties based on overcharges, profits, and revenues. It is shown that penalties imposed by CAs affect the price setting of cartels in two ways: directly, by changing the price set by a cartel of a given size, and indirectly, by affecting the incentives to form a cartel and thereby changing the cartel size and with it, its market power and ability to raise prices. Both effects have been identified and studied in the previous literature. **?** compare the same penalty regimes under the implicit assumption that all firms have to be part of the cartel. Their results characterize the direct effects, but do not include a discussion of possible indirect effects. The indirect effect has been identified and compared to a direct effect by **?**. Their model discusses profits based penalties only, and thus a comparison of the relative effects of different penalty regimes is not undertaken. As a result, the model presented in Chapter Two is the first to compare the direct and indirect effect for different penalty regimes. In a repeated Bertrand competition model over true substitutes, it finds that the overall price effect of penalties based on profits is weakly negative, while overcharge based penalties always decrease the overall price compared to no penalties. Additionally, it is shown that the price effect of revenue based penalties is ambiguous. When comparing penalties on the basis that they all deter cartels over the same group of products, the model predicts that overcharge based penalties always lead to the lowest average price, followed by profits and revenue based penalties.

Whilst Chapters One and Two discuss how the number of firms inside a cartel may vary, it is assumed that the number of firms in a given industry is fixed. This is a typical assumption in traditional collusive models. Those models considering dynamic processes in a collusive framework commonly argue that any entry into a collusive industry would break up collusion (compare for example **?**) or that entry will not occur due to incumbent cartelists playing entry deterring strategies (e.g. **?**). Nevertheless, many industries in the real world are characterized by dynamic processes of entry, exit, and merger, which can lead to the number of firms participating in the cartel changing. This is what motivates Chapter Three. It contributes to the current literature by providing a framework to analyse how dynamic forces of entry, exit and merger may influence collusive stability. In contrast to previous literature, it is found that stable cartels can form in markets with free entry and these cartels can earn strictly positive profits without playing entry-deterring strategies. Additionally, the model mimics an empirical observation that the breakdown of collusive agreements can be followed by increased merger activity and that mergers increase the risk of an industry turning collusive. Finally, the model predicts a new notion of a long run stability point under merger and entry in which firms earn positive profits.

Overall, the three chapters presented in this PhD thesis contribute to the research on cartels by helping to gain a deeper understanding of how endogenous size processes affect the stability and price setting of cartels. In the first two chapters it is shown that the number of firms inside a cartel is an important factor to consider when deriving results about collusive stability or the prices charged by cartels.

Chapter Three then shows that collusion, entry, exit, and mergers are not distinct phenomena but that these dynamic processes can occur in the same industry and lead it towards a sustainable long run stability point.

Cartel Size, Costs of Collusion, and the Stability of Collusive Agreements with Differentiated Products

Jonas Kalb[1]

PhD Thesis: Chapter 1

Abstract

A standard assumption in the analysis of how collusive stability depends on the level of product differentiation, or on costs associated with collusion, is that all firms in an industry are inside the cartel. However, there is both empirical and theoretical evidence that cartels consisting of less than all firms in the industry can form. This is the first paper to research how the stability of these small cartels depends on both the level of product differentiation and costs of collusion. Three main results are identified. Firstly, small cartels tend to become more stable when products are more homogeneous, whilst cartels encompassing all firms in the industry tend to become less stable. Secondly, although small cartels are more stable compared to large cartels when costs are not considered, there is no clear ranking of collusive stability when positive costs are introduced. The relative stability of a cartel then needs to be compared on a case by case basis. Finally, when the number of firms inside a cartel is determined endogenously, higher costs of collusion can make cartels more stable.

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1 Introduction

The detection and prosecution of illegal price fixing conspiracies remains an important topic for competition authorities (CAs). This is reflected in multiple statements by top officials from agencies such as the European Commission (EC). For example, Joaquin Almuni, at the time Vice President of the EC responsible for Competition Policy, said in a speech on September 19th 2014 that cartels were his "top enforcement priority", and furthermore that "they have been a priority for the European Commission since the late 90's(...)".[2] In order to effectively enforce cartel law, it is vital to gain a deeper understanding of the circumstances required for the formation of collusive agreements. Only then can CA activity be concentrated towards those industries which are most prone to collusion. In order to evaluate what market environments facilitate collusion, the underlying economic forces determining when a cartel is stable have to be identified first.

The standard economic approach to determining if collusive agreements can be sustained follows that of a Prisoner's Dilemma. That means that while coordinated conduct would increase the firms' total profits, each cartel member has an incentive to deviate from the agreed strategy and gain additional profits. Therefore, firms always have an incentive to deviate from the collusive agreement in a one shot game. Under the assumption that all firms anticipate the deviation, this means that collusion is not an equilibrium strategy and therefore firms would compete against each other. However, it follows from Friedman (1971) that playing coordination strategies in Prisoner's Dilemma style games can be a Subgame Perfect Nash Equilibrium, when interaction is infinitely repeated, the players use grim trigger strategies against defectors, and they value future profits sufficiently high.[3,4] When applied to cartels, this holds when the long term profits of being a cartel member outweigh the short term gains of deviating from the collusive agreement and earning low competitive profits in the future. A standard result is that this condition holds when the cartel firms' discount factor is above some critical value. Any factor increasing the gains

[2]Speech given at the IBA 18th Annual Competition Conference, text available under http://europa.eu/rapid/press-release_SPEECH-14-608_en.htm, last access on 07.09.2017

[3]That is, firms play coordination strategies in the initial period and then continue doing so if all other firms played coordination strategies in all past periods. If at any point one firm deviates from playing these strategies, all firms in the cartel revert to playing competition strategies, and the market returns to a competitive equilibrium.

[4]Additional to infinitely repeated games, collusion has been shown to be sustainable under finite horizons, e.g. Benoit and Krishna (1987) or Harrington (1987). Similarly, punishment strategies other than the grim trigger have been shown to support collusive equilibria, e.g. Porter (1983).

from collusion relative to the competitive equilibrium decrease this critical value, and thereby make collusion more stable, ceteris paribus. Contrary to this, any factor increasing the expected overall profits from deviating also increase the critical value for the discount factor, which means that cartels become less stable, ceteris paribus. It is possible to analyse different dimensions of market characteristics, with respect to the degree in which they influence the profits of staying in the cartel, relative to the gains from deviating. Two such market characteristics are the degree of product differentiation, and the level of costs associated with being inside a cartel.

Regarding the degree of product differentiation, CAs mostly expect product homogeneity to increase collusive stability. For example, in their 2010 Horizontal Merger Guidelines, the Department of Justice outlines that “coordinated conduct” is more likely “if products in the relevant market are relatively homogeneous”, (DOJ-FTC (2010), paragraph 7.2). While there is some empirical evidence linking product homogeneity and collusion (e.g. Levenstein and Suslow (2006)), there are also examples of cartels forming over goods with very heterogeneous product characteristics, such as the European Elevator Cartel.[5] Similarly, there is theoretical evidence supporting the two opposing views that that collusion is more likely when products are more homogeneous and that it is more likely when products are more differentiated. For example, Raith (1996) argues that the ability of firms to monitor competitor behaviour is incomplete, but that the correlation between own output and competitor prices is higher when products are closer substitutes. Therefore, defection from the collusive agreement is more likely to be detected and punished when products are homogeneous. Thus, defection is less attractive and collusion more stable when products are more homogeneous.[6] On the other hand, in a quadratic utility model, Deneckere (1983) and Ross (1992) find that collusion is most stable when goods are either highly differentiated or very close substitutes. At the same time, Ross (1992) and Chang (1991) show in spatial models that cartels can be more stable when products are more differentiated. All of the aforementioned theoretical models consider duopoly settings only. In this way, they are somewhat limited in that they don’t allow for the formation of cartels which don’t include all firms in the industry.

However, empirical and theoretical evidence suggests that cartels can consist of less than all firms in the industry. For example, d’Aspremont et al. (1983) shows

[5]EC press release, IP/07/209, http://europa.eu/rapid/press-release_IP-07-209_en.htm?locale=en, last accessed 08.09.2017.

[6]A similar argument is brought forward byPorter (1983).

that in a price leadership model, small cartels can form which are stable in the sense that no firm has an incentive to leave the cartel and no firm has an incentive to join it. Hirth (1999) and Posada (2001) apply the same stability definitions to a differentiated goods markets and show that some stable cartels can form for different levels of product differentiation. Whilst the authors show that these cartels can be smaller than the entire industry, they don't analyse collusive stability. In a similar model, Posada (2000) provides the first analysis of how the stability of cartels, with different exogenously given sizes, varies. He finds that small cartels are more stable than larger ones, although they earn lower profits. The question of how the stability of differently sized cartels is affected by product differentiation is not considered. Additionally, none of the papers mentioned take into account that collusion can be costly for cartel members.

These costs of collusion represent a second market characteristic which has to be considered. Thomadsen and Rhee (2007) discuss that cartel agreements can be associated with organisational and potential legal costs. They argue that the setting up and running of a cartel requires resources. Similarly, collusion is illegal and therefore members of a cartel are at risk of having to pay penalties if detected and prosecuted. Assuming that firms are rational profit maximisers implies that they should take both legal and organisational costs of collusion into account when considering whether or not to form a cartel. The authors impose fixed costs of collusion on cartels forming in differentiated goods markets. Their findings are twofold. Firstly, higher costs of collusion decrease cartel stability. Secondly, when costs of collusion are sufficiently high, more product homogeneity always makes cartels more stable. Similarly, the results of Katsoulacos et al. (2015) in homogeneous goods markets imply that when expected penalties increase, the stability of collusion decreases. Once again, both papers consider only cartels which include all firms in the industry. However, Bos and Harrington (2015) find that when cartel size is endogenised, costs of collusion can alter the incentives of forming a collusive agreement and thereby change the cartel size. This implies that collusive stability is affected via two channels when costs are imposed. Firstly, the direct costs of collusion affect the profitability of collusion relative to competition. Secondly, changes in cartel size affects both profitability of collusion and the gains from deviating. No literature has yet analysed how these effects influence collusive stability.

The identified gaps in the literature are what motivates the research presented here. In summary, the contributions of this paper to the existing literature are threefold. Firstly, it is the first paper to analyse how the stability of differently sized cartels changes with respect to the degree of product differentiation. Secondly, it shows how the stability of cartels is affected when collusion is costly, given that cartels can be of different, exogenously given, sizes. Finally, it shows how collusive stability is affected by costs when it is taken into account that the number of firms inside a cartel is determined endogenously.

Three main results are identified. Firstly, small cartels are likely to become more stable as products become more homogeneous, while fully cartelised markets tend to become less stable as products become more homogeneous. Secondly, the result that small cartels are more stable than large cartels does not hold when costs are considered. In fact, the relative stability depends heavily on the degree of product differentiation and the number of firms in the industry. Therefore, there is no clear ordering of cartel sizes and collusive stability. Finally, when the size of the cartel is endogenously defined, there are special cases in which introducing costs of collusion can make cartels more stable.

The paper is structured as follows. Section 2.1 sets out the general model assumptions and derives results for exogenously given cartel sizes when costs are not considered. Section 2.2 then introduces costs into the exogenous cartel size model and analyses their effects on collusive stability. Section 2.3 endogenises the number of firms in a cartel and considers the effect that costs of collusion have on the resulting cartels. Finally, Section 3 concludes.

2 Model

2.1 No Cost, Exogenous Cartel Size

Consider a representative consumer who solves

$$\max_{q_1,\dots,q_n} u(q_1,\dots,q_n) - \sum_{i=1}^{n} p_i q_i, \tag{1}$$

where $\sum_{i=1}^{n} p_i q_i$ is the total spending on the bundle of goods $(q_1,\dots,q_n)$ at prices $(p_1,\dots,p_n)$ and $u(q_1,\dots,q_n)$ is the utility derived from consuming this bundle. The functional form of the utility is given by

$$u(q_1,...,q_n) = \sum_{i=1}^{n} q_i - \frac{1}{2}\Big(\sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{i\neq j}^{n} q_i q_j\Big) \tag{2}$$

where $\gamma \in (0,1)$ measures the degree of product substitutability, which can also be interpreted as the degree of product homogeneity. Larger values of γ correlate with a higher degree of substitutability or homogeneity. For $\gamma \to 0$ goods become perfectly independent, for $\gamma \to 1$ they become perfect substitutes.[7]

Setting the first order partial derivative of (1) with respect to q_i equal to zero and rearranging slightly results in

$$q_i = 1 - p_i - \gamma \sum_{i\neq j} q_j. \tag{3}$$

Summing this over all firms leads to

$$\sum_{i=1}^{n} q_i = n - \sum_{i=1}^{n} p_i - \gamma(n-1)\sum_{i=1}^{n} q_i, \tag{4}$$

After some rearranging and using that from the first order condition it also follows that $\sum_{j\neq i} q_j = \frac{1-p_i-q_i}{\gamma}$, the demand for good i is follows as[8]

$$q_i(p_i, p_{j\neq i}, \gamma, n) = \frac{(1-\gamma) + \gamma \sum_{j\neq i} p_j - [1+\gamma(n-2)]p_i}{(1-\gamma)\Big(1+\gamma(n-1)\Big)}. \tag{5}$$

This demand function has the properties that demand for good i is decreasing in its own price p_i, but increasing in any competitors price p_j, $j \neq i$. Thus, all goods $j \neq i$ are substitutes for good i. Furthermore, the own price reaction is stronger than the combination of competitor price reactions. This means that if all firms were to increase their price by the same amount, the demand for each good would decrease. Analytically,

$$\texttt{i)}\ \frac{\partial q_i}{\partial p_i} < 0 \qquad \texttt{ii)}\ \frac{\partial q_i}{\partial p_j} > 0 \qquad \texttt{iii)}\ |\frac{\partial q_i}{\partial p_i}| > \sum_{j\neq i} \frac{\partial q_i}{\partial p_j}. \tag{6}$$

[7]Both cases are excluded from the analysis as they don't provide an interesting enough framework for collusive stability. When $\gamma = 0$ prices are independent so collusion, as defined in this paper, is not attractive to firms. When $\gamma = 1$, the price setting game becomes the homogeneous goods Bertrand case. The results close to this corner solution converge to the correct outcomes, but technically the demand function would need to be redefined. The resulting need for additional notation and case differentiation doesn't add to the analysis but increases complexity.

[8]Step by step derivation of demand function in Section 4

Assume now that each good is produced by one of n symmetrical firms, who produce at zero cost and compete in prices. When the firms compete against each other without coordinating on prices, any firm's maximisation problem is given by

$$\max_{\{p_i\}} \pi_i^*(p_i) = \max_{\{p_i\}} p_i q_i = \max_{\{p_i\}} p_i \frac{(1-\gamma)+\gamma\sum_{j\neq i} p_j - [1+\gamma(n-2)]p_i}{(1-\gamma)\big(1+\gamma(n-1)\big)}. \tag{7}$$

Solving the first order condition leads to the best price for firm i as a function of the price vector of competitor prices p_{-i}

$$p_i^R = \frac{(1-\gamma)+\gamma\sum_{j\neq i} p_j}{2\big(1+\gamma(n-2)\big)}. \tag{8}$$

As all firms are symmetrical, they all face the same reaction function p_i^R. Therefore, $p_i = p_j = p \ \forall i,j \in [1,n]$. Thus, the optimal industry price under competition p^* is

$$p^*(\gamma) = \frac{1-\gamma}{2+\gamma(n-3)}. \tag{9}$$

Substituting this into the demand function, it follows that the equilibrium quantities q^* are

$$q^*(\gamma) = \frac{1+\gamma(n-2)}{[1+\gamma(n-1)][2+\gamma(n-3)]}. \tag{10}$$

Finally, the competitive equilibrium profits are given by

$$\pi^*(\gamma) = \frac{(1-\gamma)\big(1+\gamma(n-2)\big)}{[1+\gamma(n-1)][2+\gamma(n-3)]^2}, \tag{11}$$

which is a decreasing function of γ.[9] This confirms the traditional economic intuition that firms in price competition oligopoly markets earn less when products are closer substitutes. That is because consumers are more willing to switch consumption from one good to the other when prices differ. Therefore, when γ is large, high prices could not be sustained, because any firm would have an incentive to undercut the competition.

Assume now that of the n firms, the first $k \in [2,n]$ decide to coordinate their price setting behaviour and agree to set one common price $p^c(k)$, which maximises joint profits. Call these firms the cartel firms and the other firms the fringe firms.

[9] Proof in Section 4.

Any cartel firm then takes into account in its maximisation problem that $(k-1)$ other firms set the same coordinated price as it does. All other firms are expected to set individual prices p_i^f. This means, cartel members solve

$$\begin{aligned}\max_{\{p^c\}} \pi^c(k) &= \max_{\{p^c\}} p^c q^c \\ &= \max_{\{p^c\}} p^c \Big(\frac{(1-\gamma)+\gamma \sum_{i=k+1}^{n} p_i^f - [1+\gamma(n-k-1)]p^c}{(1-\gamma)[1+\gamma(n-1)]} \Big),\end{aligned} \tag{12}$$

where setting the first derivative equal to zero, and solving for the optimal cartel price as a function of the fringe firms' prices lead to

$$p^{c,R} = \frac{(1-\gamma)+\gamma \sum_{i=k+1}^{n} p_i^f}{2\Big(1+\gamma(n-k-1)\Big)}. \tag{13}$$

This is the cartel's reaction function to the fringe prices. At the same time, the maximisation problem for fringe firms hasn't changed as they still compete against each other, but also against the cartel. Therefore, a fringe firm's best response to all prices in the market is given by

$$\begin{aligned} p_i^{f,R} &= \frac{(1-\gamma)+\gamma \sum_{j\neq i} p_j}{2\Big(1+\gamma(n-2)\Big)} \\ \Rightarrow p^{f,R} &= \frac{(1-\gamma)+\gamma k p^c}{2+\gamma(n+k-3)} \end{aligned} \tag{14}$$

where the second line comes from the fact that all fringe firms are symmetrical and therefore set the same price $p^f(k)$ and all firms $i \in [1,k]$ set prices p^c. As can be seen, both the cartel's and the fringe's reaction function are increasing in the prices of the other group respectively. This indicates that the price setting game is one of strategic complements: if the cartel raises prices, it is the best response of the fringe to raise prices too and vice versa.

The combination of both reaction functions then gives the equilibrium cartel and fringe prices

$$p^c(k,\gamma) = \frac{(1-\gamma)\Big(2+\gamma(2n-3)\Big)}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k}, \text{ and} \tag{15}$$

$$p^f(k,\gamma) = \frac{(1-\gamma)\Big(2+\gamma(2n-k-2)\Big)}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k}. \tag{16}$$

Given this, the cartel and fringe quantities are computed by substituting $p^c(k)$ and $p^f(k)$ into the demand function:

$$q^c(k,\gamma) = \frac{(1-\gamma)\Big(1+\gamma(n-k-1)\Big)\Big(2+\gamma(2n-3)\Big)}{\Big(1+\gamma(n-1)\Big)\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big)} \tag{17}$$

and

$$q^f(k,\gamma) = \frac{\Big(2+\gamma(2n-k-2)\Big)\Big(1+\gamma(n-2)\Big)}{\Big(1+\gamma(n-1)\Big)\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big)}. \tag{18}$$

Finally, the equilibrium profits for a given cartel size k are given by

$$\pi^c(k,\gamma) = \frac{(1-\gamma)\Big(1+\gamma(n-k-1)\Big)\Big(2+\gamma(2n-3)\Big)^2}{\Big(1+\gamma(n-1)\Big)\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big)^2} \tag{19}$$

and

$$\pi^f(k,\gamma) = \frac{(1-\gamma)\Big(2+\gamma(2n-k-2)\Big)^2\Big(1+\gamma(n-2)\Big)}{\Big(1+\gamma(n-1)\Big)\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big)^2}. \tag{20}$$

As the fringe only exists when not all firms are inside the cartel, the price, quantity and profit functions for the fringe are only defined for $k \in [2,(n-1)]$. The comparative statics of the prices and profits regarding the cartel size k have the same sign for the fringe and the cartel: when there are more firms inside the cartel all firms in the industry set higher prices and earn higher profits.[10] Analytically,

$$\begin{aligned} &\texttt{i)} \quad \frac{\partial p^f}{\partial k} > 0 &\qquad &\texttt{ii)} \quad \frac{\partial \pi^f}{\partial k} > 0 \\ &\texttt{iii)} \quad \frac{\partial p^c}{\partial k} > 0 &\qquad &\texttt{iv)} \quad \frac{\partial \pi^c}{\partial k} > 0. \end{aligned} \tag{21}$$

Intuitively, the reason behind this is that when more firms are inside the cartel, the competitive pressure in the industry decreases. It also reflects the fact that the price setting game in this model follows a game of strategic complements. That

[10] Analytical proof in Section 4.

means that when one firm increases prices, the best response of all other firms is to raise prices as well. Therefore, when the cartel forms and raises the price, the best response for the fringe is to raise prices too. When more firms are part of the price raising cartel, the overall incentive to increase prices is higher.

With respect to the degree of product homogeneity, numerical simulations suggests that the prices and profits of both cartel and fringe are decreasing as products become more homogeneous. This holds for all cartel sizes.

Finally, it is possible to show[11] that both the cartel and the fringe charge prices above the competitive level, but that the fringe undercuts the cartel. Therefore, the ranking of prices is such that

$$p^c(k,\gamma) > p^f(k,\gamma) > p^*(\gamma). \tag{22}$$

This leads to the ranking of profits, which show that the fringe firms earn the highest profits, followed by the cartel firms which earn above the competitive level[12]:

$$\pi^f(k,\gamma) > \pi^c(k,\gamma) > \pi^*(\gamma). \tag{23}$$

Thus, for any given cartel size and degree of product homogeneity, firms prefer to be part of the fringe over being inside the cartel. However, at the same time, each firm in the market profits from the existence of a cartel. This relationship will become important again at a later point, when the cartel size is endogenised. For now, as the cartel size is exogenously given, the focus lays on the fact that $\pi^c(k,\gamma) > \pi^*(\gamma)$ and that therefore firms profit from being a member of the cartel, compared to the competitive outcome.

Given that collusion is illegal in most jurisdictions, it is a common assumption that firms are not able to set up binding collusive contracts. This implies that every firm, which agreed to be part of a cartel, can at any point in time begin setting prices which maximise own profits, without notifying the other members. The optimal price such a deviator would set follows from maximising profits given that all other $(k-1)$ cartel members still set collusive prices $p^c(k,\gamma)$ and all $(n-k)$ fringe firms set prices $p^f(k)$. To begin with the analysis of the deviator price setting, consider in a first step those cases in which negativity constraints on the quantity of

[11] Proof in Section 4

[12] Proof in Section 4

other firms in the market are not relevant. The price derived for these cases is called p_u^d. In a second step, the cases in which negativity constraints become relevant will be analysed. The price derived for these cases is p_c^d.

To find the optimal price p_u^d, a deviating firm solves

$$\begin{aligned}\max_{\{p_u^d\}} \pi^d(k) &= \max_{\{p_u^d\}} p_u^d q^d \\ &= \max_{\{p_u^d\}} p_u^d \left(\frac{(1-\gamma)+\gamma(n-k)p^f(k)+\gamma(k-1)p^c(k)-[1+\gamma(n-2)]p_u^d}{(1-\gamma)[1+\gamma(n-1)]} \right),\end{aligned} \tag{24}$$

where the first order condition leads directly to

$$p_u^d(k,\gamma) = \frac{(1-\gamma)+\gamma(n-k)p^f(k)+\gamma(k-1)p^c(k)}{2[1+\gamma(n-2)]}. \tag{25}$$

Combining this with the functional forms of $p^c(k)$ and $p^f(k)$ it follows that

$$\begin{aligned}p_u^d(k,\gamma) &= \frac{(1-\gamma)[2+\gamma(2n-k-3)][2+\gamma(2n-3)]}{2[1+\gamma(n-2)]\Big[2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big]}, \text{ or} \\ &= p^f(k,\gamma) - \frac{\gamma^2(1-\gamma)(k-1)}{2[1+\gamma(n-2)]\Big[2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big]},\end{aligned} \tag{26}$$

where the second line is only defined for $k < n$, i.e. for those cases when the fringe price is defined. This shows that a deviating firm undercuts both cartel and fringe firms. A direct implication of this is that some consumers will substitute away from goods produced by the cartel or the fringe and towards those produced by the deviator. In principle this may also mean that a cartel is left with zero demand in the defecting period. It can be shown that this is only the case when $n = k$ and when γ is high.[13] In these cases, the cartel sets high prices relative to the competitive level, leaving the deviating firm a wider range for undercutting its price. At the same time, when goods are more homogeneous, consumers are more willing to substitute away from cartel goods and towards deviator goods.

In the next step, the cases with relevant non-negativity constraints, in which firms set the price p_c^d, is considered. In these cases, a deviating firm expects to cater for the entire market demand. As mentioned before, the other firm's demand given

[13] Proof in Appendix

deviation can only become zero when $n = k$. Thus, p_c^d follows from the maximum price a deviator can set given that the cartel is left with zero demand. It follows from the cartel's demand function given that $n = k$ and one firm has defected,

$$q^c(p^d) = \frac{\frac{1}{2} - \gamma(1-p^d)}{(1-\gamma)[1+\gamma(n-1)]}. \tag{27}$$

Setting this equal to zero and solving for the deviator price p_c^d gives

$$p_c^d(\gamma) = 1 - \frac{1}{2\gamma}. \tag{28}$$

For any price below this, the cartel quantity becomes negative which is unfeasible. From this, it follows that in the case of $n = k$ the deviator price is described by the function $p^d(n,\gamma) = max[p_u^d(n,\gamma), p_c^d(\gamma)]$, where p_c^d is as defined above and $p_u^d(n)$ is given by

$$p_u^d(n,\gamma) = \frac{2+\gamma(n-3)}{4[1+\gamma(n-2)]}. \tag{29}$$

Finally, the degree of product homogeneity γ_c above which $p_u^d \leq p_c^d$ is given by

$$\gamma_c = \frac{n-3}{3n-5} + \sqrt{\frac{n^2-1}{(3n-5)^2}} \qquad \in [\frac{2}{3}, 0.732] \quad \forall n \geq 2. \tag{30}$$

This leads to the overall definition of the deviator pricing function as

$$p^d(k,\gamma) = \begin{cases} 1 - \frac{1}{2\gamma} & \text{when } k = n \text{ and } \gamma \geq \gamma_c \\ \frac{(1-\gamma)[2+\gamma(2n-k-3)][2+\gamma(2n-3)]}{2[1+\gamma(n-2)]\left[2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\right]} & \text{otherwise.} \end{cases} \tag{31}$$

The resulting deviator profits follow from substituting the relevant prices into the profit function as

$$\pi^d(k,\gamma) = \begin{cases} \frac{2\gamma-1}{4\gamma^2} & \text{when } k = n \text{ and } \gamma \geq \gamma_c \\ \frac{(1-\gamma)\left[[2+\gamma(2n-k-3)][2+\gamma(2n-3)]\right]^2}{4[1+\gamma(n-2)][1+\gamma(n-1)]\left[2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\right]^2} & \text{otherwise.} \end{cases} \tag{32}$$

It is then straightforward to show that the deviator profits exceed the cartel profits,[14] which brings up a standard result of collusive behaviour: in a one shot game collusion can't be sustained because every firm in the cartel has an incentive to deviate from the collusive agreement. Therefore, collusion is not a Nash Equilibrium in a one shot game. However, as discussed in the introduction, it has been established that playing collusive strategies can be sustained as a Nash Equilibrium when interaction between firms is repeated, firms value future profits high enough and the cartel plays a grim trigger strategy against defectors.[15] A grim trigger strategy means that firms set the collusive price in the initial period and then continue doing so if all other firms played collusive strategies in all past periods. If at any point a firm deviates, firms revert to setting competitive prices forever from the next period on.

Under these assumptions, collusion can be sustained if the present value of staying inside the cartel exceeds the one time profits of deviating, followed by earning competitive profits for all future periods. Assume that one unit of money received in the next period is worth $\delta \in (0,1)$ in this period. Thus, a cartel is called stable if

$$\begin{aligned} &\sum_{t=0}^{n} \pi^c(k,\gamma)\delta^t \geq \pi^d(k,\gamma) + \sum_{t=1}^{n} \pi^*(\gamma)\delta^t, \text{ which simplifies to} \\ &\quad \frac{\pi^c(k,\gamma)}{1-\delta} \geq \pi^d(k,\gamma) + \delta\frac{\pi^*(\gamma)}{1-\delta}. \end{aligned} \tag{33}$$

This can be rewritten to

$$\delta \geq \frac{\pi^d(k,\gamma) - \pi^c(k,\gamma)}{\pi^d(k,\gamma) - \pi^*(\gamma)} := \delta_0^*(k,\gamma), \tag{34}$$

where $\delta_0^*(k,\gamma)$ is the critical discount factor above which setting collusive prices is a Nash Equilibrium. From this it follows that cartel stability is inversely related to the value of the critical discount factor. High $\delta_0^*(k,\gamma)$ make it less likely that any given value of δ is above it, while low values of $\delta_0^*(k,\gamma)$ make it more likely that $\delta \geq \delta_0^*(k,\gamma)$.

At the same time, for any $\delta \in (0,1)$ to exist which fulfils $\delta \geq \delta_0^*(k,\gamma)$, it has to

[14] Proof in Appendix

[15] Compare for example Friedman (1971)

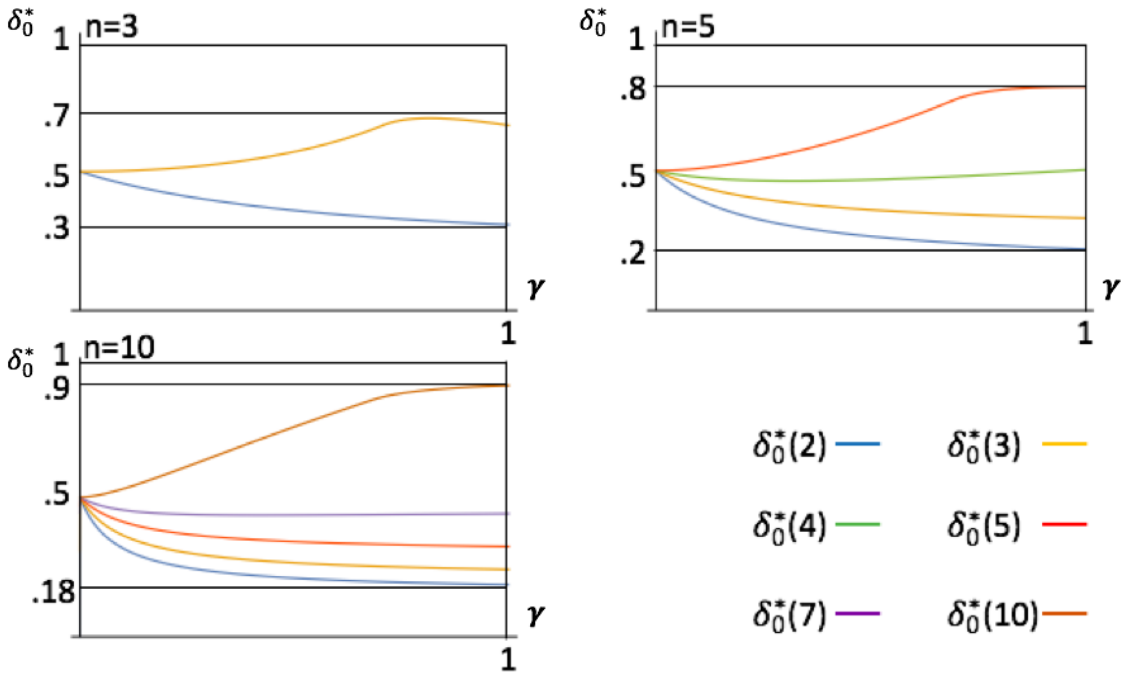

Figure 1: *The graphs plot the critical discount factors as a function of the degree of product homogeneity for varying cartel sizes.*

be that $\delta_0^*(k, \gamma) < 1$. This implies that collusive stability requires

$$\pi^c(k, \gamma) > \pi^*(\gamma). \tag{35}$$

Thus, only cartels which are profitable relative to the competitive outcome can form for some discount factor δ. For the demand structure defined in this model, $\pi^c(k, \gamma) > \pi^*(\gamma)$ has been established as a result. Therefore, it follows directly that there is some $\delta \in (0, 1)$ for any tuple (k, γ) such that a stable cartel can form. However, the likelihood that the firms' δ is high enough to fulfil $\delta \geq \delta_0^*(k, \gamma)$ is higher when $\delta_0^*(k, \gamma)$ is lower. This is important because it means that it is possible to take any two cartels, characterised by their size and degree of product homogeneity, and rank them according to the likelihood that they can be sustained for a given discount factor.

To do so, the following is defined: consider two tuples (k_1, γ_1) and (k_2, γ_2). If $\gamma_0^*(k_1, \gamma_1) < \gamma_0^*(k_2, \gamma_2)$ then collusion is more stable for the tuple (k_1, γ_1) than for the tuple (k_2, γ_2).

It is then possible to evaluate the stability of cartels by determining the level of the critical discount factor for different cartel sizes or levels of product homogeneity. To begin with, collusive stability for different cartel sizes is considered. The derivative of $\delta_0^*(k,\gamma)$ with respect to k is given by

$$\begin{aligned}\frac{\partial\delta_0^*}{\partial k} = \frac{A}{B}\Big[&4[n+k(k-2)]\\ &+2\gamma[(n-2)[3n+3k(k-2)]+k(k^2-3k+3)-1]\\ &+\gamma^2(n-2)[2n(n-2)+(k-2)[(k-3)+n]+1]-\gamma^2(n^2-3)\Big]>0\end{aligned} \tag{36}$$

where A and B are both positive functions[16] of γ, n and k. Each of the components of the derivative is positive $\forall n \geq 2$, $k \in [2,n]$ and $\gamma(0,1)$. Thus, collusion is harder to sustain for larger cartels. This might seem surprising, given that these cartels have more market power, are able to set higher prices, and earn higher profits. However, an increased price also implies that any firm considering to deviate has a wider range of prices to chose from when undercutting the cartel price. In fact, the difference between the cartel price and the deviator price is increasing in k:[17]

$$\frac{\partial[p^c-p_u^d]}{\partial k} = \frac{\gamma(1-\gamma)[2+\gamma(2n-3)]\Big[4+6\gamma(n-2)+\gamma^2[2(n-2)^2+(k-1)^2-(n-1)]\Big]}{2[1+\gamma(n-2)]\Big[2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big]^2} > 0. \tag{39}$$

As a result, more consumers switch from buying cartel products to buying deviator products when k is high. This increases deviating profits relative to cartel profits. Hence, the overall attractiveness of adhering to the collusive agreement decreases, rendering the cartel less stable.

Next, the degree of product homogeneity and how it affects δ_0^* is considered. Fig. 1 plots the critical discount factor as a function of γ for $n=3$, $n=5$, and

[16]

$$A = 4\gamma[2+\gamma(n-3)]^2[1+\gamma(n-2)][2+\gamma(2n-3)]^2 > 0 \tag{37}$$

and

$$\begin{aligned}B = \Big[&16\gamma[k^2+k(12-7n)-32(k-1)+6(n-2)]\\ &+4\gamma^2[107+k^3-108n+26n^2+k^2(-23+10n)+k(-105+128n-36n^2)]\\ &+4\gamma^3[-105+2k^3(-2+n)+161n-78n^2+12n^3+k^2(44-38n+8n^2)+k(98-191n+111n^2-20n^3)]\\ &+\gamma^4[4k^3(-2+n)^2+4k^2(-2+n)^2(-7+2n)+(-3+n)^2(17-24n+8n^2)\\ &+\gamma(370n-129-337n^2+124n^3-16n^4)]\Big]^2 > 0\end{aligned} \tag{38}$$

[17] The derivative uses the unconstrained price p_u^d because the constraint price p_c^d is only defined for the point $n=k$. Hence, the derivative of p_c^d with respect to k is not defined.

$n = 10$ exemplarily for varying cartel sizes k. The graphs also show the approximate supremum and infimum of the critical discount factor for each of these industry sizes. Three observations stand out. Firstly, when $k < n$, more homogeneous goods make collusion easier to sustain. In these cases, any cartel that forms has to take into account in their price setting that the fringe firms set a lower price than the cartel to attract more customers. When γ is large and therefore consumers are very willing to substitute goods for another, a high cartel price could then allow the fringe firms to gain all market demand because it could undercut the cartel by more. Taking this into account, any cartel consisting of $k < n$ members will only increase prices by a small amount compared to competition. The fringe firms then price below this already relatively low price. For a deviating firm which undercuts the fringe price, this implies that the gains from deviating are very low because the price is low. Therefore, more homogeneity decreases the incentive to cheat on the collusive agreement.

Secondly, consider the case when $k = n$. In these case, an increase in γ almost always decreases the stability of collusion. Similarly to the explanation above, this is due to the gains a potential deviator can earn. When $k = n$, there is no competitive pressure on the cartel and therefore it sets the high monopoly price of $p^c(n) = 1/2$. At the same time, a deviating firm earns more additional demand by undercutting the cartel when consumers are more willing to substitute away from more expensive products, i.e. when γ is large. This means that the deviator profits increase relative to the cartel profits, making it more attractive to deviate. Thus, collusion becomes less stable. The special case to this argument is when the number of firms in the industry is very small, which is reflected in the graph for $n = 3$. In this case, the critical discount factor is increasing in γ up to some maximum value and then it decreases slightly again. For these very small markets, the gains of being inside a cartel are very large compared to the competitive outcome, especially when γ is high. Therefore, the cartel's ability to punish deviation by returning to collusion is high, which increases the incentive to stay inside the cartel.

Finally, the difference between the infimum and the supremum of the discount factors is increasing in n. Economically, this means that the difference between the most stable and the least stable cartel, characterised by some (k, γ), is increasing in the number of firms in the industry. This has two reasons. Firstly, the infimum is always a point on the $\gamma^*(2)$ function, as the critical discount factor is increasing in k.

This implies that $k < n$ as long as $n \geq 3$ and therefore that a fringe exists. Thus, when more firms are in the industry, the number of firms that the cartel competes against increases as well, making it difficult for the cartel to increase prices. As a result, both the cartel and the fringe firms set prices close to the competitive equilibrium, making deviation highly unprofitable. Secondly, the supremum is always a point on $\gamma_0^*(n)$ and will typically be set in a region of relatively homogeneous goods. For any increase in n, the cartel profits relative to the deviator profits decrease, rendering collusion less stable.

Overall, this section has discussed the stability of collusive agreements in differentiated goods market for differently sized cartels. In doing so, $\delta_0^*(k,\gamma)$ has been identified as an inverse measure of collusive stability. This means that small values of $\delta_0^*(k,\gamma)$ are associated with relatively stable cartels, while large values of $\delta_0^*(k,\gamma)$ are associated with a relatively low cartel stability. It was found for a given industry size that, while small cartels become more stable as products become more homogeneous, the stability of large cartels decreases. In the next section, costs of collusion are introduced into the model.

2.2 Costs Included, Exogenous Cartel Size

It is now assumed that being part of a cartel is associated with some periodically reoccurring fixed costs $C > 0$. Adding these costs to the model allows for the possibility that forming a cartel may involve additional organisational or jurisdictional costs, which is likely in the real world. For example, firms need to monitor the behaviour of their co-conspirators, produce reports, hold secretive meetings and risk being fined for colluding. While these costs are not relevant for firms competing against each other, they are relevant for both firms inside the cartel and deviators.[18] The resulting cartel and deviator profits are denoted by $\pi_c^c(k,\gamma)$ and $\pi_c^d(k,\gamma)$

[18]It is assumed here that a deviating firm cannot forgo the costs of collusion. That is because deviating from the collusive agreement results in the highest pay-off when no other firms inside the cartel anticipate deviation. Terminating visible efforts into the running of the cartel may spark suspicion about a possible deviation and thus render it unprofitable. As a result, it is expected that any firm aiming to deviate will continue to partake in organisational activities for the cartel and incur the resulting costs. Similarly, being prosecuted and fined for being part of a cartel continues to be a risk in a deviation period.

respectively. They follows as

$$\pi_c^c(k,\gamma) = \pi^c(k,\gamma) - C$$
$$= \frac{(1-\gamma)\Big(1+\gamma(n-k-1)\Big)\Big(2+\gamma(2n-3)\Big)^2}{\Big(1+\gamma(n-1)\Big)\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big)^2} - C, \tag{40}$$

and

$$\pi_c^d(k,\gamma) = \pi^d(k,\gamma) - C$$
$$= \begin{cases} \frac{2\gamma-1}{4\gamma^2} - C & \text{when } k=n \text{ and } \gamma \geq \gamma_c \\ \frac{(1-\gamma)\Big[[2+\gamma(2n-k-3)][2+\gamma(2n-3)]\Big]^2}{4[1+\gamma(n-2)][1+\gamma(n-1)]\Big[2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big]^2} - C & \text{otherwise.} \end{cases} \tag{41}$$

Thus, a deviating firm continues to earn profits above those they can earn in a cartel: $\pi_c^d(k,\gamma) - \pi_c^c(k,\gamma) = \pi^d(k,\gamma) - \pi^c(k,\gamma) > 0$. The fringe profits and the competitive profits are the same as defined in the previous section.

Applying these profits to the cartel stability condition defined in (34) leads to the critical discount factor $\delta_c^*(k,\gamma)$ above which costly collusion can be sustained. It is given by

$$\delta_c^*(k,\gamma) = \frac{\pi_c^d(k,\gamma) - \pi_c^c(k,\gamma)}{\pi_c^d(k,\gamma) - \pi^*(\gamma)} = \frac{\pi^d(k,\gamma) - \pi^c(k,\gamma)}{\pi^d(k,\gamma) - C - \pi^*(\gamma)} \tag{42}$$

A cartel can then be called stable when

$$\delta \geq \delta_c^*(k,\gamma). \tag{43}$$

Similarly to before, and higher value of $\delta_c^*(k,\gamma)$ corresponds with lower cartel stability, while lower values imply more stable cartels. Three results immediately follow. Firstly, given an exogenous cartel size k, collusion is harder to sustain when collusion is more costly. This is a direct implication of

$$\frac{\partial \delta_c^*(k,\gamma)}{\partial C} = \frac{\pi^d(k,\gamma) - \pi^c(k,\gamma)}{[\pi^d(k,\gamma) - C - \pi^*(\gamma)]^2} > 0. \tag{44}$$

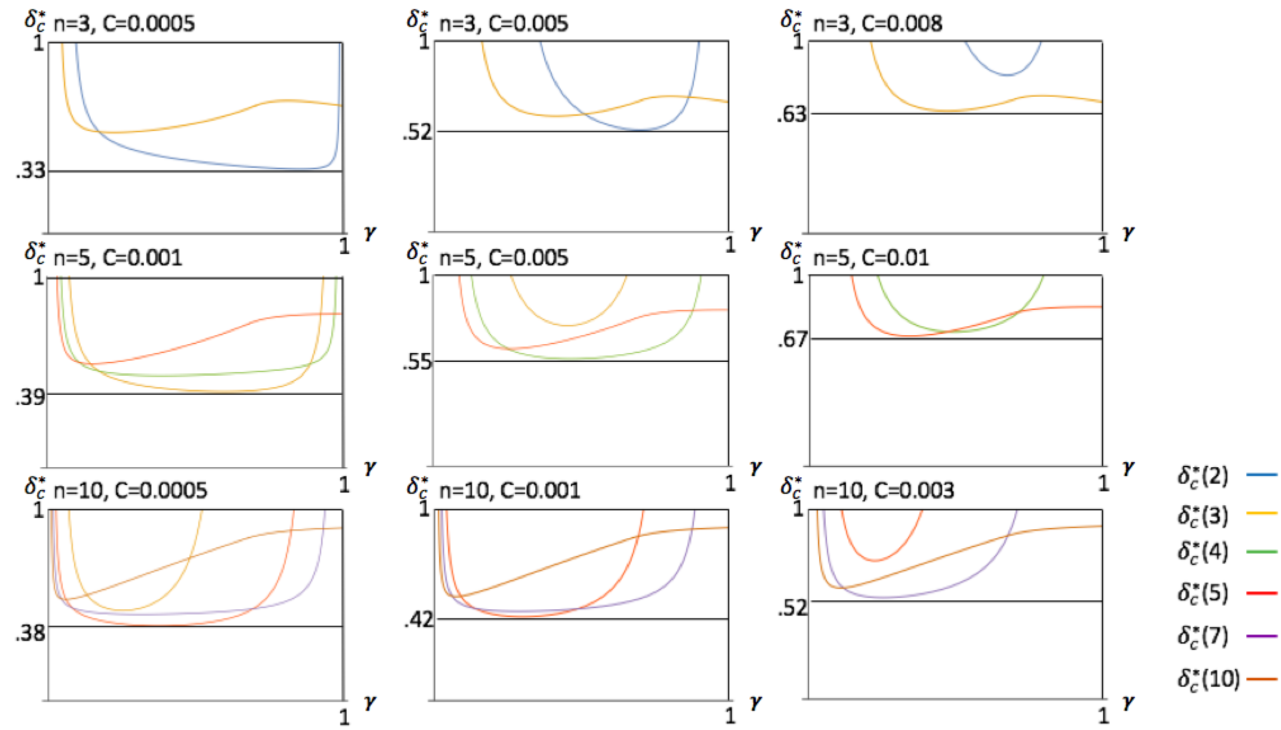

Figure 2: *The graphs plot the critical discount factors as a function of the degree of product homogeneity for varying cartel sizes and different cost levels.*

This also implies that $\delta_c^*(k,\gamma) > \delta_0^*(k,\gamma)\ \forall\, C > 0$. Intuitively, the result is clear: costs of collusion don't influence the gains of defecting relative to the cartel profits. At the same time, they make collusion less profitable compared to competition. Thus, the cartel's ability to punish defectors by returning to the competitive equilibrium is decreased. As a result, defection becomes more attractive.

Secondly, as cartel stability requires $\delta \geq \delta_c^*(k,\gamma)$ and $\delta \in (0,1)$, it must be that $\delta_c^*(k,\gamma) < 1$. This implies that when costs are sufficiently high, some cartel sizes are not stable any more. Specifically, collusion can only be sustained for some cartel size k when the costs C are such that

$$\begin{aligned} &\delta_c^*(k,\gamma) < 1 \\ \iff\ & C < \pi^c(k,\gamma) - \pi^*(\gamma), \end{aligned} \tag{45}$$

i.e. when the costs of collusion are smaller than the gains of collusion relative to competition. If this inequality wasn't fulfilled, firms would be better off reverting to the competitive equilibrium instead of colluding.

Thirdly, the result that cartels need to be profitable compared to competition is combined with the result that larger cartels earn higher profits: for any given discount factor and level of costs, it is more likely that a large cartel is stable than that a small cartel is stable. Analytically, that follows from the comparative statics of the inequality in (45) in which the right hand side is increasing in k, while the left hand side is constant. This means that larger cartels are more likely to earn profits which make up for the costs of collusion. Fig. 2 gives a graphic example of this. It plots the critical discount factor as a function of γ for three different market sizes ($n = 3$,$n = 5$, and $n = 10$) and varying levels of costs of collusion. It can be seen that when costs are included, there are ranges of γ for which the discount factor is not within the defined range of $\gamma \in (0,1)$ any more. In all of these cases, $\delta_c^* \geq 1$ and therefore, collusion is not sustainable. As an example, consider the case of $n = 3$ and $C = 0.008$. One can see that the critical discount factor for the cartel size $k = n = 3$ is not in the graph for small γ. Furthermore, a cartel with $k = 2$ members is only stable for a small range of relatively homogeneous, but not very homogeneous products.

However, this result only describes the general possibility of a cartel emerging for a given level of costs and does not have a direct carry over on the level of stability. To make inferences about the level of stability, the values of the critical discount factor need to be compared. For the same example, i.e. $n = 3$, $C = 0.008$, and $k = 2$ or $k = 3$, one can see that given a cartel forms, the larger cartel is always more stable than the smaller one. This is not a general result, though. For example, when $n = 5$ and $C = 0.005$, it can be seen that for very small γ, no cartel is stable. As γ increases, the first cartel to become stable is the one consisting of all firms in the market. For slightly larger γ, $k = 4$ also becomes stable. There is then a range of γ for which $\delta_c^*(5) < \delta_c^*(4)$ and therefore the larger cartel is more stable. At some point the two critical discount factors cross and the smaller of the two cartels becomes more stable. For very large γ, this is reversed again. Additionally, there is some mid range of γ for which $k = 3$ is also stable, but less stable than the other two cartel sizes. This observation stresses that the relative stability of large and small cartels cannot be generalised, but that it depends on the individual case. That is because a fixed level of costs of collusion decrease a small cartel's profits by a higher percentage, compared to a larger cartel. At the same time, it doesn't influence the one shot gains of deviation compared to collusion. This means that the critical discount factor of small cartels increases quicker in the level of costs than that of large cartels. Therefore, smaller cartels become less stable quicker when costs increase. In some cases this may mean that collusion for smaller cartels is less stable than for large cartels. In others it may not.

Furthermore, the relationship between product differentiation and collusive stability is considered. Without costs, it was seen that, while smaller cartels become more stable as products become more homogeneous, larger cartels become less stable. For small cartels, this relationship can't be replicated in the presence of costs at all. Instead, one can see that the critical discount factor is a strictly convex function of the degree of homogeneity which lies above 1 for both very homogeneous and very differentiated goods. This implies that, when costs are taken into account, small cartels are the most stable when products are characterized by moderate homogeneity, and that both more differentiation and more homogeneity decrease collusive stability. When cartels consist of all firms in the market and costs are considered, collusion is not sustainable for very differentiated goods. As γ increases, collusion first becomes more stable before the critical discount factor increases again. For further increases in γ the critical discount factor function turns concave. In the cases displayed, it does not cross unity again as γ converges towards one.

To summarize, including costs into the model has three main effects on collusive stability. Firstly, collusion is always harder to sustain for a given industry size, which can imply that some cartels can't form at all. Secondly, the result that smaller cartels are generally more stable than larger ones can't be supported any more. Depending on the level of costs and the degree of product differentiation, the relative stability of differently sized cartels has to be analysed on a case by case basis. Finally, the relationship between product differentiation and collusive stability for small cartels becomes non-monotonic when costs are considered. It is observed that in most cases, collusion is most stable for some moderate level of product differentiation, but that both extreme differentiation and homogeneity make collusion unstable. For cartels that include the whole market, collusion is the most stable for an intermediate value of product differentiation and becomes harder to sustain for more similar products. However, these cartels will also be stable for very differentiated products, given that the costs of collusion allow for any cartels in the market.

The discussion in this section then implies that when costs are taken into account, some collusion which would have been stable without costs, is not stable any more. Furthermore, the relative stability of collusive agreements compared with others changes when costs are considered. One can then ask the question, if cartels react to changes in costs either by including or excluding new members and what effect this has on the cartel's stability. This is done in the next section.

2.3 Costs Included, Endogenous Cartel Size

While the number of firms in collusive agreements was exogenously given in the previous sections, this section explores the effects of costly collusion when the number of firms inside the cartel is endogenously derived. To do so, cartel stability conditions following the work of d'Aspremont et al. (1983), are defined and applied to the firms in this model. The authors point out that while all firms would prefer being part of the fringe over being inside the cartel, larger cartels also imply higher profits for every firm in the industry. Therefore, for a given distribution of firms into fringe and cartel firms it could be beneficial for each firm to stay in their assigned roles. That is because, if a firm left the cartel to earn fringe profits, the number of firms inside the cartel would decrease which would also decrease the fringe profits that the leaving firm would earn. Contrary, it could pay off for fringe firms to join the cartel when an additional member would increase the cartel profits enough. Following this logic, a cartel size k^* is called the membership stable size when two conditions are fulfilled.

Firstly, it is inside stable. This means that no firms should have an incentive to openly leave the cartel and join the fringe, where openly means that the other firms are given the chance to reevaluate their price setting. Formally, an inside stable cartel (ISC) fulfils the condition

$$\pi^c(k,\gamma) - C \geq \pi^f(k-1,\gamma) \quad \forall k \geq 3, \text{ and} \tag{46}$$

$\pi^c(2,\gamma) - C \geq \pi^*(2,\gamma)$ for $k = 2$, where the differentiation in cases is due to the fact that when a firm leaves a cartel with 2 members, the cartel dissolves and the industry returns to competition.

Secondly, a collusive agreement needs to be outside stable. This means that no fringe firm should have an incentive to join the cartel. Formally, an outside stable cartel (OSC) fulfils the condition

$$\pi^f(k,\gamma) \geq \pi^c(k+1,\gamma) - C \quad \forall k \leq (n-1), \text{ and} \tag{47}$$

for $k = n$ the condition is always fulfilled because there are no firms on the outside

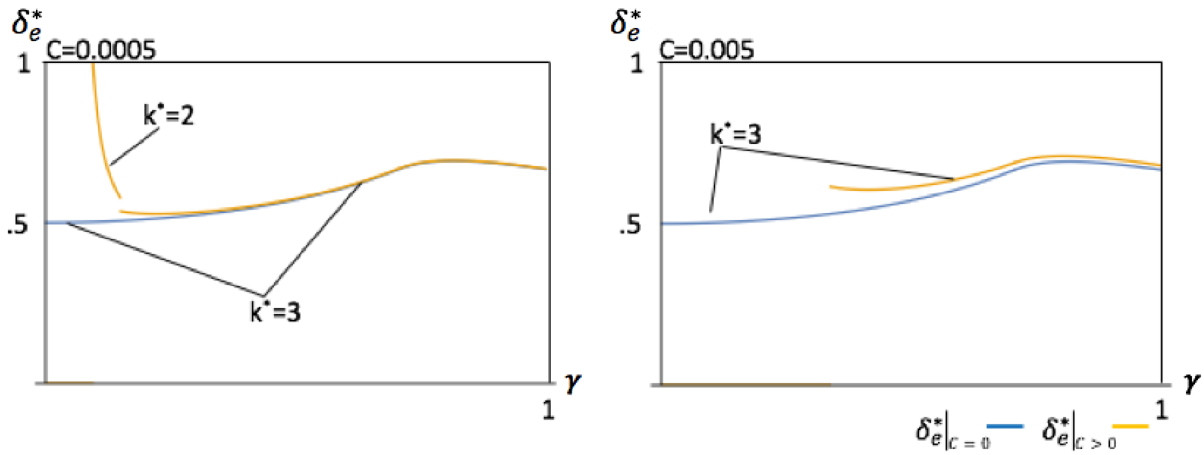

Figure 3: *The graphs plot the critical discount factors as a function of the degree of product homogeneity for the case of $n = 3$ and two different costs levels, assuming that the cartel size is determined endogenously.*

which could join the cartel.

While both conditions need to be fulfilled for the membership stable cartel size, they both serve a different purpose. Inside stability is required for the existence of stable cartels. That is because if there is no cartel size for which firms want to stay inside the cartel, all firms would leave collusive agreements and the cartel couldn't sustain. Outside stability on the other hand is important to determine the size of a collusive agreement. To see this in detail, a brief discussion of membership stable cartel sizes follows.

In this case of zero costs of collusion, it is easy to see that each industry has some inside stable cartel. This follows directly from the results that $\pi^c(2,\gamma) > \pi^*(\gamma)$ and $\frac{\partial \pi^c(k,\gamma)}{\partial k} > 0$. A direct implication of this is that any industry has a membership stable cartel size when $C = 0$. That is because, given the smallest inside stable cartel $k = 2$, there are two possible outcomes. Either, it doesn't pay off for a fringe firm to join the cartel. This would mean the membership stable cartel size is $k^* = 2$. Alternatively, when the cartel is not an OSC, another firm joins, thereby increasing the cartel size to $k = 3$. As it was profitable for the firm to join the cartel, it can't be profitable to leave it again. Thus, the $k = 3$ is definitely inside stable. Again, two outcomes are possible: either another firm joins or the cartel is already outside stable. This mechanism will repeat until either no firm wants to join, or all firms are inside the cartel.

Taking costs of collusion into account affects the incentives to both join or leave the cartel. From (46) it follows that for positive costs, some industries may not

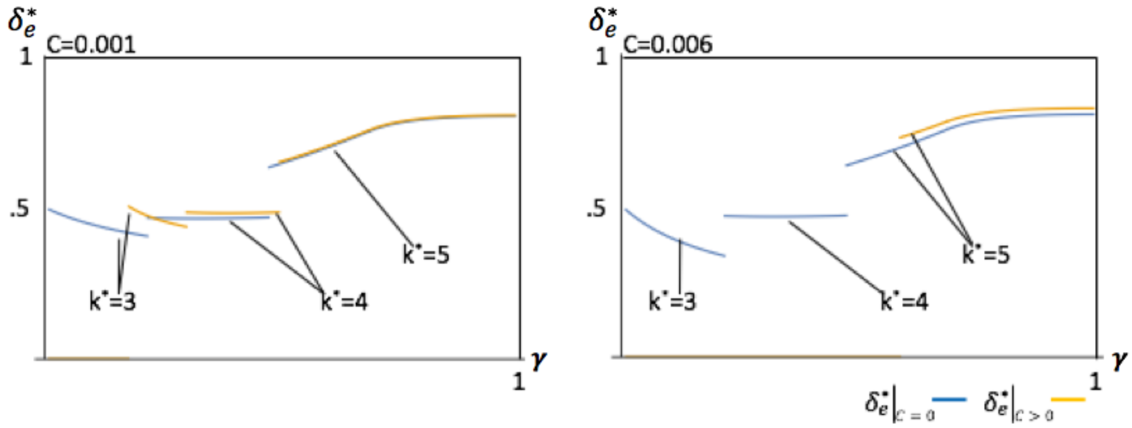

Figure 4: *The graphs plot the critical discount factors as a function of the degree of product homogeneity for the case of $n = 5$ and two different costs levels, assuming that the cartel size is determined endogenously.*

have inside stable cartels any more. Therefore, it is possible that no cartel forms. Furthermore, for higher C it is more likely that a cartel becomes OSC. This follows directly from the fact that the RHS of (47) is decreasing in C, making the condition more likely to hold when C is large.

In summary, without costs of collusion, any industry has some membership stable cartel size. Imposing fixed costs of collusion may mean that in some industries no cartel can form. In those industries in which a cartel can form, the incentives to join it are lower. This implies that the membership stable cartel size is likely to be lower when costs increase.

It is then possible to numerically determine the resulting membership stable cartel size k^* for any given market size, degree of product substitutability and level of costs. Given k^*, one can then determine the condition under which no cartel member has an incentive to deviate from the collusive agreement by using the critical discount factor. Similar to the case of exogenous cartel sizes, any cartel with a membership stable size k^* is then called stable if

$$\delta \geq \frac{\pi^d(k^*,\gamma) - \pi^c(k^*,\gamma)}{\pi^d(k^*,\gamma) - C - \pi^*(\gamma)} := \delta_e^*(\gamma), \tag{48}$$

where the subscript e denotes that the critical discount factor is determined for endogenously determined membership stable cartel sizes.

To see the effects of endogenising the cartel size and imposing costs on the stability of collusion, consider two examples. Firstly, Fig. 3 shows the case of $n = 3$. For this market size, when costs are equal to zero, the membership stable cartel will always consist of all firms in the industry. When $C = 0.0005$, cartels over very differentiated goods ($\gamma \leq 0.0943$) cannot be sustained, because they are not profitable. For $\gamma \in (0.0943, 0.14073)$ the membership stable cartel size is $k^* = 2$. This shows that $k*$ can decrease when costs are introduced. In this product range, collusion is harder to sustain than in the no cost case. For $\gamma \in [0.1473, 1)$, $k^* = 3$ and the critical discount factor is marginally greater than in the no cost case. Similarly, when $C = 0.005$ there are no stable cartels for $\gamma < 0.3282$. When $\gamma \in [0.3282, 1)$, the membership stable cartel size is equal to $k^* = 3$ and the critical discount factor is above the one computed under the no cost case. These two examples show cases in which introducing costs of collusion make collusion harder to sustain. Furthermore, cartels over some differentiated products can't be sustained any more.

Secondly, consider the case of $n = 5$ which is depicted in Fig. 4. When costs are equal to zero, the membership stable cartel sizes k^* are such that $k^* = 3$ $\forall\gamma \in (0, 0.2188]$. When $\gamma \in (0.2188, 0.47662]$ the cartel size is $k^* = 4$ and when $\gamma \in (0.4766, 1)$ it is $k^* = 5$. Introducing a cost level of $C = 0.001$ leads to no cartels being stable for $\gamma \leq 0.1791$. When $\gamma \in (0.1791, 0.3022)$, the membership stable cartel size is $k^* = 3$. Comparing this range with the no cost case shows that for all $\gamma \in (0.2188, 0.3022)$ the membership stable cartel size is lower when costs are included. Furthermore, the critical discount factor is higher in the no costs industry. This implies that including costs into the model has made collusion easier to sustain in this product range. That is because for these cartels, the decrease in size from 4 to 3 has decreased the relative gains of deviating more than the introduction of costs has decreased the profitability of collusion. When γ increases further and is in the interval $\gamma \in [0.3022, 0.4994)$, the membership stable cartel size with costs is $k^* = 4$ and finally for $\gamma \in [0.4994, 1)$, it is $k^* = 5$. For these ranges, the inclusion of costs increases the critical discount factor and therefore makes collusion harder to sustain, compared to the no costs case.

Overall, this section has provided conditions to determine the membership stable cartel size. It was then shown for two exemplary market sizes, how cartel stability changes when costs are introduced into the model. While for the most part, taking into account that collusion is costly makes collusion harder to sustain, it is possible that in special cases, costs make collusion easier to sustain. That is because the incentives to join a cartel are lower when collusion is costly and therefore, the membership stable cartel size can decrease. This can lower the potential gains of deviating more than the profitability of collusion, thereby rendering a cartel more stable. This contrasts the results defined by Thomadsen and Rhee (2007) and Katsoulacos et al. (2015) who find that larger costs of collusion always decrease collusive stability.

3 Conclusion

This paper has discussed how the stability of collusive agreements depend on two dimensions: the degree of product differentiation and the costs associated with forming a cartel. While both dimensions have previously received attention in the academic literature, a standard assumption within this literature is that all firms of the industry are inside the cartel. However, empirical and theoretical evidence suggest that cartels can consist of less than all firms in the industry. Nonetheless, there is no previous research linking this finding to the question of how collusive stability depends on product differentiation or on the costs of collusion.

This gap in the literature was addressed in this paper. In a first step, how the stability of cartels with an exogenously given number of firms depends on the degree of product differentiation was analysed. The stability of small cartels, i.e. those which consist of less than all firms in the industry, was compared to the stability of large cartels, i.e. those which encompass all firms in the industry. Two results were derived. Firstly, small cartels are more stable than large cartels. Secondly, it was found that large cartels tend to become less stable as products become more homogeneous. Contrary to this, small cartels tend to become more stable when products become more homogeneous. The intuitive reason behind this is that when products are relatively homogeneous, small cartels are forced to set low prices because they compete against fringe firms. Therefore, deviating from the cartel agreement doesn't pay off as it requires undercutting the already low cartel price. Contrary to this, large cartels set monopoly prices which implies that firms considering to deviate from the agreed on price can gain high profits.

In a second step, costs of collusion were introduced into the model. It was found that larger cartels are no longer strictly less stable than small cartels. In fact, introducing costs means that ranking the stability of cartels with respect to their size is not possible and that the relative stability of differently sized cartels needs to be compared on a case by case basis. As such, it can be the case that either smaller cartels are more stable than larger ones or vice versa. Furthermore, it was shown that the introduction of costs renders some cartel sizes unprofitable and that it is possible that no cartel size can be profitable for some levels of product differentiation. As unprofitable cartels cannot be sustained, this means that introducing costs into the model deters some cartels.

Finally, the number of firms inside the cartel was endogenised. A given cartel size was defined as membership stable when no firm has an incentive to leave or to join the cartel. It was then shown that when costs are not included in the model, there is a membership stable cartel size in any industry. Contrary to this, when costs of collusion were taken into account, there are some industries in which there is no membership stable cartel size. Furthermore, it was discussed that higher costs of collusion can decrease the number of firms in a given cartel because larger costs decrease the incentive to join a collusive agreement. In special cases, this can mean collusion is rendered more stable when costs are increased. That is because in a smaller cartel, the incentive to deviate is lower than in larger cartels. When then the incentives to deviate decrease more than the cartel profits, collusion can become more stable through an increase in costs.

Overall, this paper is the first to undertake an analysis of how cartel stability depends on the degree of product differentiation and the level of costs associated with collusion, given that a cartel can consist of less than all firms in the industry. While this paper provides a first intuition regarding these questions, further research into the topic is necessary. Some possible areas for this could be to introduce a more elaborate notion of the costs associated with collusion. For example, costs could be linked to the size of collusive agreements, or to the actual expected penalties that firms operating in regulated markets face. Another possible direction of research could be the question of how the costs of collusion depend on the degree of product differentiation.

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4 Appendix Chapter 1: Proofs

4.1 Denominator of cartel and fringe price positive

The denominator is equal to

$$\begin{aligned}&2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\\=&4+2\gamma(3n-k-5)+2g^2[(n-k-1)(n-3)-k]+\gamma^2(n-k)k\end{aligned} \tag{49}$$

which is a negatively signed quadratic function of k with a maximum at $k = \frac{\gamma(4-n)-2}{2\gamma} < 0\ \forall n \geq 3$ and hence is decreasing in k. Therefore, the function will be at its lowest when k is the highest value possible. For $k = (n-1)$, the denominator takes the value

$$4[1+\gamma(n-2)]-\gamma^2(n-1) > 0\ \forall n \geq 2 \text{ and } \gamma \in (0,1) \tag{50}$$

4.2 Functional form of demand function

$$\begin{aligned}&\max_{\{q_i\}_{i=1,2,\ldots,n}} \sum_{i=1}^{n} q_i - \frac{1}{2}\Big(\sum_{i=1}^{n} q_i^2 + 2\gamma\sum_{i\neq j} q_i q_j\Big) + I\\&\text{such that } I + \sum_{i=1}^{n} p_i q_i \leq m.\end{aligned} \tag{51}$$

Combining budget constraint and utility function, setting the first order partial derivative with respect to q_i equal to zero and rearranging slightly results in

$$q_i = 1 - p_i - \gamma\sum_{i\neq j} q_j. \tag{52}$$

Summing this over all firms leads to

$$\begin{aligned}&\sum_{i=1}^{n} q_i = n - \sum_{i=1}^{n} p_i - \gamma(n-1)\sum_{i=1}^{n} q_i, \text{ and hence}\\\Rightarrow&[1+\gamma(n-1)]\sum_{i=1}^{n} q_i = n - \sum_{i=1}^{n} p_i.\end{aligned} \tag{53}$$

Using that $\sum_{i=1}^{n} q_i = q_i + \sum_{j\neq i} q_j$ it follows then that

$$[1+\gamma(n-1)]q_i = n - \sum_{i=1}^{n} p_i - [1+\gamma(n-1)]\sum_{j\neq i} q_j. \tag{54}$$

combining this with

$$\sum_{j \neq i} q_j = \frac{1 - p_i - q_i}{\gamma}. \tag{55}$$

from the first order condition above, it follows after some rearranging that

$$q_i(p_i, p_{j \neq i}, \gamma, n) = \frac{(1-\gamma) + \gamma \sum_{j \neq i} p_j - [1 + \gamma(n-2)] p_i}{(1-\gamma)\Big(1 + \gamma(n-1)\Big)}. \tag{56}$$

4.3 Competitive profits decreasing in degree of product homogeneity

The derivative of the price function with respect to δ is given by

$$\frac{\partial \pi^*}{\partial \gamma} = -\frac{(n-1)\Big[2 + \gamma(4n-10) + 2\gamma^2[(n-3)(n-2)+1] - \gamma^3(n-3)(n-2)\Big]}{[1+\gamma(n-1)]^2[2+\gamma(n-3)]^3}, \tag{57}$$

where of the fraction, the denominator is positive $\forall n \geq 2$. The numerator is clearly positive $\forall n \geq 3$ because $2 > \gamma$. Therefore, the derivative is negative when $n \geq 3$. When $n = 2$, the whole expression becomes $-\frac{2[1-\gamma(1-\gamma)]}{(1-\gamma)^2(2+\gamma)^3}$, which is negative because $\gamma \in (0,1)$. Thus, $\frac{\partial \pi^*}{\partial \gamma} < 0 \ \forall n \geq 2$.

4.4 Prices increasing in number of cartel firms

The first order derivative of the cartel price with respect to the cartel size is given by

$$\frac{\partial p^c}{\partial k} = \frac{(1-\gamma)\gamma[2+\gamma(n+2k-4)][2+\gamma(2n-3)]}{(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)] - \gamma^2(n-k)k)^2} > 0, \tag{58}$$

where the denominator is always positive. As $n \geq k \geq 2$, and γ is positive but below one, the derivative is positive.

The first order derivative of the fringe price with respect to the cartel size is given by

$$\frac{\partial p^f}{\partial k} = \frac{(1-\gamma)\gamma^2[4k-2+\gamma\Big[(4k-2)(n-1) - k^2\Big]}{(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)] - \gamma^2(n-k)k)^2} > 0, \tag{59}$$

where the denominator is always positive. The only way the numerator can be negative is, when k grows and the negative squared cartel size outweighs the rest. The highest value of k for which the function is defined is $k = (n-1)$. For this cartel size, the numerator is still positive. Hence, overall both cartel and fringe price are increasing in k.

4.5 Profits increasing in number of cartel firms

The first order derivative of the cartel profits with respect to the cartel size is given by

$$\frac{\partial \pi^c}{\partial k} = \frac{(1-\gamma)\gamma^2[2+\gamma(2n-3)]^2[2(k-1)+\gamma(3k[n-k]-2[n-1])]}{[1+\gamma(n-1)](2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k)^3} > 0, \quad (60)$$

where the only term that could make this negative is the last term of the numerator $[2(k-1)+\gamma(3k[n-k]-2[n-1])]$. This term is at its lowest when the second part of the term is the most negative. That is when $(3k[n-k]-2[n-1]) = -2[n-1] < 0$, i.e. when $n = k$. For this, the whole term is positive though $[2(k-1)+\gamma(3k[n-k]-2[n-1])]|_{n=k} = (2n-1)-\gamma(2n-1) > 0$ because $\gamma \in (0,1)$. Therefore $\frac{\partial \pi^c}{\partial k} > 0$ is true.

For the fringe profits, it is already known that the price is increasing in k. The first order derivative of the fringe quantity is given by

$$\frac{\partial q^f}{\partial k} = \frac{\gamma^2[1+\gamma(n-2)]\Big[2(2k-1)+\gamma[k(n-k)+2k(n-2)n(k-2)+2]\Big]}{[1+\gamma(n-1)](2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k)^2} > 0. \quad (61)$$

Thus, both quantity and price are increasing in k. Therefore, the fringe profits are also increasing in k.

4.6 Ranking of prices

It is to be shown that $p^c(k) > p^f(k) > p^*$. Start with showing that $p^c(k) > p^f(k)$.

$$\begin{aligned} p^c(k) &= \frac{(1-\gamma)[2+2\gamma n-3\gamma]}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k} \\ p^f(k) &= \frac{(1-\gamma)[2+2\gamma(n-1)-\gamma k]}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k} \end{aligned} \quad (62)$$

$$\begin{aligned} p^c(k)-p^f(k) &= \frac{(1-\gamma)\gamma(k-1)}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k} \\ &> 0 \ \ \forall \ \ 2 \le k \le (n-1) \end{aligned} \quad (63)$$

because both numerator and denominator are always positive for these values of k and n.

Now show that $p^f(k) > p^*$.

$$p^f(k) > p^* = \frac{(1-\gamma)\gamma^2(k-1)k}{[2+\gamma(n-3)][2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k]} > 0, \tag{64}$$

because the numerator is clearly positive $\forall k > 2$ and the denominator is clearly positive $\forall n \geq 2$.

4.7 Ranking of Profits

The ranking of the profits is

$$\pi^f(k) > \pi^d(k) > \pi^c(k) > \pi^*. \tag{65}$$

Firstly, $\pi^c(k) > \pi^*$ is shown. As it is known that $\pi^c(k)$ is increasing in k, this inequality only has to be shown for $k = 2$. For all $k > 2$ it will then also hold.

$$\pi^c(2) - \pi^* = \frac{(1-\gamma)\gamma^2[4+16\gamma(n-2)+\gamma^2(17n^2-70n+69)+\gamma^3(5n^3-33n^2+67n-43)]}{4[2+\gamma(n-3)]^2[1+\gamma(n-1)][2+\gamma(3n-7)+\gamma^2(n^2-5n+5)]^2} \tag{66}$$

Where all parts of this fraction are weakly positive for $n \geq 2$ apart from $\gamma^2(17n^2 - 70n+69)$ which is only positive for $n \geq 3$ and $\gamma^3(5n^3-33n^2+67n-43)]$ which is only positive for $n \geq 4$. This means that for all $n \geq 4$, all parts of the fraction are positive and therefore $\pi^c(k) - \pi^* > 0$ for $n \geq 4$. It then has to be shown that $\pi^c(2) - \pi^* > 0$ for $n = 2$ and $n = 3$.

For $n = 2$ it follows that

$$\pi^c(2) - \pi^* = \frac{\gamma^2}{4(2-\gamma)^2(1+\gamma)} > 0 \tag{67}$$

For $n = 3$ it follows that

$$\pi^c(3) - \pi^* = \frac{(1-\gamma)\gamma^2[1-\gamma^3+4\gamma+3\gamma^2]}{4[1+2\gamma][2(1-\gamma)-\gamma^2]^2} > 0 \tag{68}$$

Hence, $\pi^c(k) > \pi^*$.

Secondly, $\pi^f(k) > \pi^c(k)$ is shown.

$$\pi^f(k)-\pi^c(k)=\frac{(1-\gamma)(k-1)[k+1+\gamma(n-1+k(n-2))]}{[1+\gamma(n-1)][2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k]^2}>0\forall n\geq k\geq 2 \tag{69}$$

Hence, $\pi^f(k) > \pi^c(k)$ holds.

Thirdly, $\pi^f(k) > \pi^d(k)$ is shown.

$$\pi^f(k)-\pi^d(k)=\frac{\gamma^2(1-\gamma)(k-1)\Big[8+4\gamma[(n-k)+(3n-6)]+\gamma^2[8(n-1)(n-2)+1-k(4n-7)]\Big]}{[1+\gamma(n-1)][2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k]^2}>0 \tag{70}$$

which is greater than zero because the denominator is positive and the only term that can be negative for high k in the numerator could be $[8(n-1)(n-2)+1-k(4n-7)]$. For the highest possible k, $k=(n-1)$, this is still positive though.

Finally, $\pi^d(k,\gamma) > \pi^c(k,\gamma)$ is shown. Two cases have to be differentiated.

- The unrestricted profits when $k<n$ or $k=n$ and $\gamma<\gamma_c$. In this case:

$$\pi^d(k)-\pi^c(k)=\frac{\gamma^2(1-\gamma)(k-1)^2[2+\gamma(2n-3)]}{4[1+\gamma(n-2)][1+\gamma(n-1)]\Big[2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big]^2}>0, \tag{71}$$

  where both the numerator and the denominator are clearly positive $\forall\ n\geq 2$ and $\gamma\in(0,1)$.

- The restricted case when $k=n$ and $\gamma\geq\gamma_c$. Then:

$$\pi^d(n)-\pi^c(n)=\frac{\gamma^2(2n-1)+\gamma(n-1)-1}{\gamma^2[1+\gamma(n-1)]}, \tag{72}$$

  where the denominator is always positive and the numerator increasing in γ and negative for very small γ but positive for large γ. Therefore, if this fraction is positive for $\gamma=\gamma_c$ it is also positive $\forall\ \gamma>\gamma_c$. Evaluating the fraction at the point $\gamma=\gamma_c$ leads to

$$\pi^d(n)-\pi^c(n)|_{\gamma=\gamma_c}=\frac{3(n-1)(n-2)+n-1}{8[(n^2-2)+n(\sqrt{n^2-1}-1)-\sqrt{n^2-1}]}>0\ \forall n\geq 2 \tag{73}$$

Therefore, firms always earn more when they deviate compared to when they set the collusive price.

4.8 Cartel demand in case of defection

Call the demand of any given cartel firm, given that one firm has defected $q^{c,-d}$. It follows from the demand for a cartel firm, evaluated for the case that one of the cartel firms has defected and prices at $p^d(k)$, $(n-k)$ firms set the fringe price $p^f(k)$ and the firm in question as well as $(k-2)$ remaining firms set the cartel price $p^c(k)$.

$$q^{c,-d} = \frac{[2+\gamma(2n-3)][2+2\gamma(2n-k-3)+\gamma^2[5-k(2n-3)+2n(n-3)]]}{2[1+\gamma(n-2)][1+\gamma(n-1)]\Big[2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big]}. \tag{74}$$

It has to be shown that this is positive $\forall\, \gamma \in (0,1)$, as long as $k < n$. Setting $q^{c,-d} = 0$ and solving for $g \in (0,1)$ under the restrictions that $n \geq 2$ and $k \in [2,n]$, the only potential root is given by

$$\hat{\gamma} = \frac{2n-3-k+\sqrt{k^2-1}}{(3n-5)-(2n-3)(n-k)}. \tag{75}$$

It will be shown that there is no $k < n$ for which $\hat{\gamma} \in (0,1)$ and therefore, there the cartel quantity is never equal to zero when $k < n$.

- The numerator of $\hat{\gamma}$ is always weakly positive when $k \in [2,(n-1)]$.
- The denominator is negative when $(3n-5) < (2n-3)(n-k)$, which is the least likely for the highest possible $k = (n-1)$. For this value, $\hat{\gamma} = \frac{(n-2)]\sqrt{n(n-2)}}{(n-2)} > 1$. For any k larger than this, the denominator is negative while the numerator is positive and therefore $\hat{\gamma} < 0$.
- Hence $\hat{\gamma} \notin (0,1)$ for $k < n$.

The Effect of Penalty Regimes on Prices in Markets with Endogenous Cartel Size

Jonas Kalb[1]

PhD Thesis: Chapter 2

Abstract

This article analyses the effect of three different penalty regimes on the price setting in collusive industries. The regimes analysed are i) profits, ii) overcharge, and iii) revenue based penalties. In this model, firms compete in prices over true substitutes in an infinitely repeated market. It is shown that penalties influence prices in two ways. Firstly, directly by changing the price for a given cartel size and secondly, indirectly by influencing the cartel size and thereby the price charged. The direct effect of these different regimes has been studied previously by Katsoulacos et al. (2015) for perfectly homogeneous goods. The indirect effect has been identified as ambiguous for profit based penalties in a capacity constraint homogeneous goods model by Bos and Harrington (2015). However, this is the first paper which analyses and compares these effects for different penalty regimes when goods are differentiated. It is found that the total price effect of profit based penalties is weakly negative. Overcharge based penalties always lead to a lower price compared to no penalties. However, increasing the penalty toughness may in some cases slightly increase prices. Finally, when penalties are based on revenue, the total price effect is ambiguous. When the penalties are set such that they deter cartels over the same products, overcharge based penalties always lead to the lowest price, followed by prices computed under the profit based penalties and then revenue based penalties. For very few levels of product differentiation and market size, profit based penalties lead to the highest prices.

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Contents

1 Introduction

In 2012 the European Commission (EC) imposed a record fine of 1.49 billion EUR on a group of banks.[2] Although collusion is illegal in most jurisdictions,[3] the convicted parties had formed a cartel to manipulate interest rates in their favour. Multiple cases of illegal collusive agreements are detected each year in all sectors and industries. For example, in January 2014 the German competition authority (CA) Bundeskartellamt fined 5 breweries. In the same year, Apple settled in a case of price fixing for e-books. The company had to pay a total of 450 million USD.[4] Further cases range from canned fruits, to car glass and candle wax. These recent examples show that forming a cartel is still a common business practice and it raises the question of how a CA can define a regulatory framework in which firms obey the laws imposed.

Most CAs base their penalties on the revenue gains with cartelised goods. Consider as an example both the US and the EU, where the CAs set out in their sentencing guidelines the affected volume of commerce will be used to calculate base penalties, which can then be adjusted upwards or downwards.[5] While the revenue earned by a cartel is a widely used measure to calculate penalties, some recent academic work sees evidence that it is not an optimal practice. Bageri et al. (2013) show that cartels have an incentive to set higher prices when revenue based penalties are imposed. Furthermore, the authors show that cartels with a higher revenue to profit ratio face higher penalties than cartels with a lower revenue to profit ratio, even if the harm induced is the same. Katsoulacos and Ulph (2013) analyse a Bertrand competition model over homogeneous goods with constant and symmetrical marginal cost in which collusion is detected with a constant probability. They too find that the cartel price can be increasing in the toughness of the revenue based regime and can even exceed the monopoly price.

Katsoulacos et al. (2015) elaborate on this idea and perform a structured comparison of four different penalty regimes (profits, revenue and overcharge based as well as fixed penalties) in terms of their effect on cartel price, cartel formation and their welfare effects. They define an infinitely repeated Bertrand competition model over homogeneous

[2] http://europa.eu/rapid/press-release_MEMO-13-1090_en.htm, last accessed on 13.03.2018

[3] For example, at the moment price fixing activities are classified as illegal under article 101 TFEU (Treaty for the Functioning of the European Union) in the EU and under the Sherman Act 1890 section 1 in the US.

[4] https://www.reuters.com/article/us-usa-court-ebooks/supreme-court-rejects-apple-e-books-price-fixing-appeal-ic last accessed 18.03.2018

[5] Compare United States Sentencing Commission Guideline Manual (DoJ (2005)) and European Commission Sentencing Guidelines (Commisson (2006))

goods with constant, symmetrical marginal costs. This has two important implications. Firstly, the non-cooperative outcome is that firms set the price equal to marginal cost. Secondly, any cartel that wants to sustain prices above marginal cost has to contain all members of the market. The authors show that the different penalty regimes produce cartel prices which can be ranked from the highest to lowest. Revenue based regimes produce higher prices than profit based regimes, which produce higher prices than overcharge based regimes. Furthermore, they show that when penalties have the same effect on cartel deterrence, overcharge based penalties lead to lower average prices and a higher total consumer welfare than profit based and revenue based penalties. Overall, they show that overcharge based penalty regimes welfare dominate profit based penalty regimes, which in turn welfare dominate revenue based penalty regimes. The authors conclude that CAs should switch to an overcharge based approach when calculating penalties. In an extension of this model, Katsoulacos et al. (2016) define a penalty regime based on revenues, but with a penalty rate based on overcharges. This regime can result in similar welfare improvements as the overcharged based penalty regime and requires less effort in implementing.

In both the aforementioned papers, the authors' assumptions imply that all firms in a market have to be inside the cartel to sustain prices above marginal costs. However, there are many cases in the real world where this is not fulfilled and cartels comprise of only a few firms in the market. Some academic work models this feature of firm behaviour. For example, Posada (2000), Hans et al. (1999), Posada (2001) and Eaton and Eswaran (1998)replicate it in differentiated goods markets with Bertrand competition, while Escrihuela-Villar et al. (2011) use homogeneous goods in a Cournot framework. This literature consistently finds that larger cartels charge higher prices.

However, none of this literature includes penalties in the model. This is done by Bos and Harrington (2015), who analyse the effect that profit based penalties have on cartel size. In a homogeneous goods market with price competition and capacity constraints they also show that bigger cartels set higher prices. The authors also show that the profit based penalties can either increase or decrease the cartel size. Contrary to the findings of Katsoulacos et al. (2015), this implies that a profit based penalty regime can have an ambiguous effect on the price charged in the market instead of having no effect.

Overall, the academic evidence suggests that the cartel size positively influences the price set by a cartel, but that the penalty regime can also influence the cartel size. This suggests that penalties have two effects on the cartel price. Firstly, the direct price effect, which is the change in price for a given cartel size. While this effect has been studied in the literature for different penalty regimes, the analysis was concentrated on the homogeneous goods case. Secondly, the indirect price effect, which comes through influencing the cartel size by alternating the incentive to form a cartel. This effect has been considered for profit based penalties by Bos and Harrington (2015), but no comparison to other penalty regimes has been made.

This paper aims to bridge the gap in the existing literature by analysing the price effect of different penalty regimes in the differentiated goods case, with endogenous cartel size. The novelty of the research is twofold. Firstly, this is the first paper comparing the direct price effect of different penalty regimes in differentiated goods industries. Secondly, this is the first paper which analyses how the different penalty regimes influence cartel size and thereby the indirect price effect. Both of these steps are important in evaluating the effects of penalty regimes because it is possible that the overall effect is ambiguous. Specifically, a penalty could directly decrease the cartel price and indirectly increase it, or the other way around. If this was the case, it is possible that the findings in the previous literature regarding the direct price effects cannot be generalised for markets in which indirect price effects are possible.

The model in this paper defines a market in which firms compete in prices over substitutable goods. Three penalty regimes are analysed: profits, overcharge, and revenue based penalties. For each of these regimes, the direct, indirect and overall price effects are derived. It is shown that, while the direct effect of profit based penalties is zero, the indirect effect can decrease the cartel price by decreasing the cartel size. With respect to overcharge based penalties it is found that they have a negative direct effect, but ambiguous indirect price effect. Moreover, compared to no penalties, the overall price effect is negative. The results for revenue based penalties are that they directly increase the market price but lead to weakly smaller cartels, which implies a weakly negative indirect price effect. In contrast to Katsoulacos et al. (2015), it is shown that the overall effect of revenue based penalties is ambiguous and that it can be the case that the overall price effect is negative. Further analysis is carried out to compare the penalties with each other. It is found that when all penalty regimes deter cartels over the same group of products, overcharge based penalties lead to significantly lower prices than profit based penalties, which in turn lead to slightly lower prices than revenue based penalties in most cases. Some exceptions exist in which profit based penalties lead to prices just above those computed under the revenue based penalty regime.

The paper is structured as follows. In Section 2.1, the general framework of the model is explained and in Section 2.2 a competitive baseline outcome is defined. In Section 2.3 firms are allowed to collude without having to face law enforcement. Following this, law enforcement is introduced into the model and penalties based on profits, overcharges, and revenue are analysed in Section 2.4, Section 2.5, and Section 2.6. Section 3 compares the penalty regimes. Finally, Section 4 concludes.

2 Model

2.1 General Set Up

Suppose a market in which consumers choose to buy horizontally differentiated goods which are supplied by $n \geq 2$ firms. A consumer's maximisation problem takes the form

$$\max_{q_1,\dots,q_n} u(q_1,...,q_n) - \sum_{i=1}^{n} p_i q_i, \tag{1}$$

where $\sum_{i=1}^{n} p_i q_i$ is the total spending on the bundle of goods $(q_1,...,q_n)$ at prices $(p_1,...,p_n)$ and $u(q_1,...,q_n)$ is the utility derived from consuming this bundle. The functional form of the utility is given by

$$u(q_1,...,q_n) = \sum_{i=1}^{n} q_i - \frac{1}{2}\Big(\sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{i \neq j}^{n} q_i q_j\Big) \tag{2}$$

where $\gamma \in (0,1)$ measures the degree of product substitutability, which can also be interpreted as the degree of product homogeneity. As γ goes closer to zero, goods become more independent of one another, if γ is closer to one, goods become more substitutable. The cases of perfect substitutes ($\gamma = 1$) and perfectly independent goods ($\gamma = 0$) are excluded in the analysis. The former has been analysed by Katsoulacos et al. (2015) already and the latter is the case in which firms pricing strategies are independent of each other and therefore price fixing is not of concern.[6]

From the first order conditions of the consumer's maximisation problem and some rearranging it is easily shown that the demand for good i is

$$q_i(p_i, p_{j\neq i}, \gamma, n) = \frac{(1-\gamma) + \gamma \sum_{j \neq i} p_j - [1+\gamma(n-2)]p_i}{(1-\gamma)\Big(1+\gamma(n-1)\Big)} \tag{3}$$

[6]Technically, it is possible that γ is negative, where $\gamma < 0$ corresponds to the case of complements. This paper however focuses on substitutes and therefore the degree of product substitutability is restricted to positive values.

This implies that the demand for firm i is decreasing in its own price p_i and increasing in the price of any competitor firm $j \neq i$.

$$\texttt{i)}\ \frac{\partial q_i}{\partial p_i} < 0 \qquad\qquad \texttt{ii)}\ \frac{\partial q_i}{\partial p_j} > 0 \tag{4}$$

This demand function ensures the existence of meaningful results as the own price effect on demand is stronger than the cross price effects on demand. In essence, this means that the demand for good i depends more on its own price than it depends on the sum of price changes in the market. If that wasn't the case, an increase in the price of all goods at the same time would lead to more demand and higher profits, which is not economically sensible. [7]

In this market $n \geq 2$ firms compete in prices and produce products i at constant marginal cost c. Firm decisions follow a two stage decision process. In the first stage, firms decide to either engage in collusion or in competition against each other. In the second stage, firms simultaneously set prices. When all firms choose to compete against each other, the outcome is the competitive equilibrium. Otherwise, the outcome is the collusive equilibrium. To solve for an equilibrium, the outcomes for the competitive and collusive case for a given number of firms n should be characterized. This is done in the following section.

2.2 Competition

This section discusses the outcome that emerges when no collusion takes place and firms maximise individual competitive profits π_i^*. Each firm solves

$$\max_{\{p_i\}} \pi_i^*(p_i) = \max_{\{p_i\}} (p_i - c)q_i = \max_{\{p_i\}} (p_i - c)\frac{(1-\gamma) + \gamma \sum_{j \neq i} p_j - [1 + \gamma(n-2)]p_i}{(1-\gamma)\Big(1 + \gamma(n-1)\Big)} \tag{6}$$

From the first order conditions price reaction functions directly follow. These determine for each firm i how it will set its price, given the prices of the other $(n-1)$ firms $j \neq i$ in

[7] Technically, this holds because

$$|\frac{\partial q_i}{\partial p_i}| > (n-1)\frac{\partial q_i}{\partial p_j} \tag{5}$$

the market:

$$p_i^R = \frac{1}{2}c + \frac{(1-\gamma)+\gamma\sum_{j\neq i}p_j}{2\big(1+\gamma(n-2)\big)} \tag{7}$$

As all firms are symmetrical, they have the same reaction function. Hence, the sum of prices can be rewritten as $\sum_{j\neq i}p_j = (n-1)p_{-i}.$, where the notation p_{-i} denotes all prices except the price of firm i.

$$p_i^R = \frac{1}{2}c + \frac{(1-\gamma)+\gamma(n-1)p_{-i}}{2\big(1+\gamma(n-2)\big)} \tag{8}$$

Given that all firms are symmetrical and therefore $p_i = p^* \ \forall \ i = 1,2,...,n$, the optimal prices in the competitive market are

$$p^* = \frac{1+\gamma(n-2)}{2+\gamma(n-3)}c + \frac{1-\gamma}{2+\gamma(n-3)} \tag{9}$$

which, in combination with the demand function leads to competitive quantities

$$q^* = \frac{1+\gamma(n-2)}{[1+\gamma(n-1)][2+\gamma(n-3)]}. \tag{10}$$

Finally, competitive equilibrium profits follow as

$$\pi^* = \frac{(1-\gamma)\big(1+\gamma(n-2)\big)}{[1+\gamma(n-1)][2+\gamma(n-3)]^2}(1-c)^2 \tag{11}$$

In terms of directional comparative statics, the optimal firm prices and profits have the same results and very similar intuition. They are decreasing in both the degree of product substitutability γ and market size n.[8] Specifically, very low values of γ, firms have close to monopoly power in their demand and set a price close to the monopoly price. In these cases, the firms profits will also be close to the monopoly profits. For higher γ, products become closer substitutes which means that any firm can increase demand by a higher amount when undercutting the price of the other firms in the market. As a result, competitive pressure on firms increases and prices decrease. This means that for high values of γ, the competitive price is closer to marginal costs and hence the profits are closer to zero. In term of market size, for small n, few firms compete against each other and prices, as well as profits, are high. For larger n, the competitive pressure on

[8] Follows directly from first order partial derivatives with respect to n and γ respectively

firms increases and prices and profits decrease. These are standard results of Bertrand competition with horizontally differentiated goods and symmetrical costs.

When marginal costs increase, firms partially pass the cost on to consumers by raising the price. As firms don't pass on all changes in costs, profits are decreasing in cost. When $c \geq 1$, the market price is above the cut-off price where demand is zero. It is therefore assumed that $0 < c < 1$.

To establish the effects of penalty regimes on the cartel formation process and the prices charged by the market firms, it is necessary to firstly outline the process of cartel formation without any penalties in place. This is carried out in the following sections.

2.3 Collusion: No Penalties

This section discusses the collusive outcome in the absence of penalties. Before endogenising the number of firms that choose to collude, assume that of the n firms in the market, the first exogenously chosen $k \in [2, n]$ firms decide to collude will maximise their joint profits. Call these firms the cartel. The remaining $(n-k)$ firms will continue to compete against each other and against the cartel firms. Call these firms the fringe.

As the fringe firms maximise individual profits, they have the same general maximisation approach as outlined in (6) so that from the first order condition of a fringe firm the reaction will be equivalent to (12). The sum of competitor prices $\sum_{j \neq i} p_j$ can be rewritten to account for the fact that the first k firms are inside the cartel, setting a joint price, and the other $(n-k-1)$ fringe firms set prices individually. Denote the cartel price with p^c and the fringe price as p^f. As fringe firms are symmetrical and hence in equilibrium set the same price $p_j = p^f \;\; \forall j = (k+1), (k+2), ..., n$. A fringe firm's optimal reaction to the cartel price then follows as

$$p^{f,R} = \frac{\Big(1 + \gamma(n-2)\Big)c + (1-\gamma) + \gamma k p^c}{2 + \gamma(n+k-3)}. \tag{12}$$

The cartel firms set the price such that it maximises the overall profits of its members. This means that cartel firms take into account that they set a combined price when maximising profits. Therefore, substituting $p_j = p^c \;\; \forall j = 1, 2, ..., k$ and $p_j = p^f \;\; \forall j = (k+1), (k+2), ..., n$ into the demand functions for the cartel, leads to the maximisation problem

$$\begin{aligned}\pi^c(k) &= \max_{\{p^c\}}(p^c - c)q^c \\ &= \max_{\{p^c\}}(p^c - c)\Big(\frac{(1-\gamma)+\gamma(n-k)p^f - [1+\gamma(n-k-1)]p^c}{(1-\gamma)[1+\gamma(n-1)]}\Big)\end{aligned} \tag{13}$$

From the first order condition and the symmetry of all firms, the cartels reaction function to the fringe price follows as

$$p^{c,R} = \frac{1}{2}c + \frac{(1-\gamma)+\gamma(n-k)p^f}{2\Big(1+\gamma(n-k-1)\Big)}, \tag{14}$$

Both the fringe's and the cartel's price reaction function are increasing in the competitor's price which implies that price setting in this model follows a game of strategic complements. When a competitor raises the price, the best response is to follow and raise the price as well, but less than the competitor did. This follows directly from the derivative of the reaction functions with respect to the competitor's price, where in both cases the derivative is positive but smaller than one.

Using the reaction functions computed, equilibrium prices as well as profits can be computed for a given cartel size k. For the prices, it follows that

$$\begin{aligned}p^c(k) =& \frac{\Big(1+\gamma(n-k-1)\Big)\Big(2+\gamma(n+k-3)\Big)+\gamma(n-k)\Big(1+\gamma(n-2)\Big)}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k}c \\ &+ \frac{(1-\gamma)\Big(2+\gamma(2n-3)\Big)}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k},\end{aligned} \tag{15}$$

and

$$\begin{aligned}p^f(k) =& \frac{\Big(1+\gamma(n-k-1)\Big)\Big(2+\gamma(2n+k-4)\Big)}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k}c \\ &+ \frac{(1-\gamma)\Big(2+\gamma(2n-k-2)\Big)}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k},\end{aligned} \tag{16}$$

where the denominator $2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k > 0 \;\forall\; n \geq k > 2$. [9]. From this, quantities are

$$q^c(k) = \frac{(1-c)(1-\gamma)\Big(1+\gamma(n-k-1)\Big)\Big(2+\gamma(2n-3)\Big)}{\Big(1+\gamma(n-1)\Big)\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big)} \tag{17}$$

[9]Analytical proof is in Appendix A

and

$$q^f(k) = \frac{(1-c)\Big(2+\gamma(2n-k-2)\Big)\Big(1+\gamma(n-2)\Big)}{\Big(1+\gamma(n-1)\Big)\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big)}, \tag{18}$$

which leads directly to profits

$$\begin{aligned}\pi^c(k) &= (p^c-c)q^c(p^c,p^f)\\ &= \frac{(1-c)^2(1-\gamma)\Big(1+\gamma(n-k-1)\Big)\Big(2+\gamma(2n-3)\Big)^2}{\Big(1+\gamma(n-1)\Big)\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big)^2}\end{aligned} \tag{19}$$

and

$$\begin{aligned}\pi^f(k) &= (p^f-c)q^f\\ &= \frac{(1-c)^2(1-\gamma)\Big(2+\gamma(2n-k-2)\Big)^2\Big(1+\gamma(n-2)\Big)}{\Big(1+\gamma(n-1)\Big)\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big)^2}.\end{aligned} \tag{20}$$

In the special case in which the cartel consists of all firms in the market, and hence $k = n$, the fringe doesn't exist and the above defined functional forms for the fringe are meaningless. For $k = n$ the cartel sets monopoly prices and earns monopoly profits.[10]

However, as long as $k < n$, one can compare the fringe and the cartel prices and profits. This comparison reveals that the fringe undercuts the cartel but still sets a price above the competitive level. Therefore, both the cartel and the fringe prices are strictly higher than the competitive equilibrium price. Furthermore, the cartel price is strictly higher than the fringe price.[11] Formally, this means:

$$p^c(k) > p^f(k) > p^*. \tag{21}$$

For profits, it can be shown that the cartel profits exceed the competitive profits, but that the fringe firms earn more than cartel firms. Formally:

$$\pi^f(k) > \pi^c(k) > \pi^*. \tag{22}$$

At this point, it is important to note that the analysis of stable collusive agreements usually requires the definition of deviating profits. That is, given that collusive agreements

[10] $p^c(k=n) = \frac{c+1}{2}$ and $\pi^c(k=n) = \frac{(1-c)^2}{4\big(1+\gamma(n-1)\big)}$ respectively

[11] Analytical proof in Appendix A

are non-binding contracts, they cannot be enforced and therefore some firm inside the cartel could have an incentive to deviate from the agreement to gain additional profits. This paper focusses on the price setting behaviour of cartel and fringe firms and therefore no formal analysis of deviating behaviour is carried out. However, deviator profits have been shown to exceed the collusive profits in previous papers, for example Posada (2000) who use a similar demand structure. Therefore, in order to ensure consistency with the literature, $\pi^d(k)$ is defined as the deviator profits given k firms are inside the industry and assume that $\pi^d(k) > \pi^c(k)\ \forall k$.

Having defined the relevant profits, two fundamental problems for firms who contemplate forming a cartel are identified.

1. Given that a cartel forms and there is a fringe, the fringe firms earn higher profits than the cartel firms. This implies that even though all firms profit from the existence of a cartel, no firm wants to be inside the cartel and would rather be part of the fringe. This follows from $\pi^f(k) > \pi^c(k)$.

2. Given that a cartel forms, any firm inside it has an incentive to deviate from the agreed price in a one shot game. This follows from $\pi^d(k) > \pi^c(k)$.

In order to derive solutions for these fundamental problems, its necessary to investigate the conditions under which collusion is sustainable.

The intuitive solution for the first problem follows from the comparative statics of the price and profit functions for the cartel and fringe respectively.[12] Here, it can be shown that both fringe and cartel prices and profits are increasing in the number of firms in the cartel:[13]

$$
\begin{array}{llll}
\texttt{i)} & \dfrac{\partial p^f}{\partial k} > 0 & \texttt{ii)} & \dfrac{\partial \pi^f}{\partial k} > 0 \\
\texttt{iii)} & \dfrac{\partial p^c}{\partial k} > 0 & \texttt{iv)} & \dfrac{\partial \pi^c}{\partial k} > 0.
\end{array}
\tag{23}
$$

This means that any firm, which has been assigned the role of a cartel firm exogenously, contemplates leaving the cartel, it has to take into account that by leaving the cartel it decreases k. As a result, all profits earned in the industry decrease. This is especially true for the profits of the fringe firms that it wants to join. Formally, instead of comparing its

[12]The definitions on cartel stability follow directly from d'Aspremont et al. (1983) and have been used in the academic literature in this, or in a very similar form. Some examples include, but are not limited to Posada (2000), Bos and Harrington (2015), Eaton and Eswaran (1998)

[13]Analytical proof in Appendix A

current profits $\pi^c(k)$ to the current fringe profits $\pi^f(k)$, the firm has to compare them to $\pi^f(k-1) < \pi^f(k)$. Therefore, it is possible that the switch from cartel to fringe doesn't pay off when $\pi^c(k) \geq \pi^f(k-1)$. This leads to the following definition

Inside Stable Cartel (ISC): A cartel is said to be inside stable, when

$$\pi^c(k) \geq \pi^f(k-1) \text{ for } k \geq 3, \tag{24}$$

and for $k = 2$, the condition is $\pi^c(k) \geq \pi^*$ because if any firm left a cartel with $k = 2$ firms, the cartel would not exist and the industry would return to competition.

This definition then leads to the following lemma.

Lemma 1: In a model with no law enforcement, (24) is always fulfilled for some value of $k = 2$.

This follows directly from the application of the ordering of profits in (22) to the (24). As $\pi^c(k) > \pi^*$ $\forall k \geq 2$, any cartel with $k = 2$ firms is inside stable.

At the same time, there is the possibility that firms currently outside of the cartel want to join it. The reason behind this is the inverted argument to the inside stability: firms have to compare the current fringe profits $\pi^f(k)$ to $\pi^c(k+1)$ instead of $\pi^c(k)$ because by joining the cartel they increase its size. Given that larger cartels earn more money, the additional gain from having more firms in a cartel could outweigh the loss of changing from being a fringe firm to being a cartel firm. That is the case, when $\pi^f(k) \leq \pi^c(k+1)$. This implies that a cartel will only stop growing when either there are no more firms in the fringe that can join, i.e. when $k = n$, or when joining the cartel doesn't pay off, i.e. when $\pi^f(k) \geq \pi^c(k+1)$. This leads to the following definition:

Outside Stable Cartel (OSC): A cartel is said to be outside stable, when

$$\pi^f(k) \geq \pi^c(k+1) \text{ for } 2 \leq k \leq (n-1). \tag{25}$$

By definition, any cartel with $k = n$ is outside stable.

If a cartel is both inside and outside stable for some cartel size k it means that no firm has an incentive to either leave or join the cartel openly. Call a cartel like this membership stable. The question that remains to answer is: is there some cartel size k^* for which a cartel is membership stable? To answer this, consider both conditions.

It follows directly from the comparison of (24) and (25) that any firm joining the cartel wants to stay inside of it. That is, because if a firm joins a cartel with $\tilde{k}$ firms, it means that (25) was not fulfilled for $\tilde{k}$ and therefore it is clear that $\pi^f(\tilde{k}) < \pi^c(\tilde{k}+1)$. After the firm joined, the cartel size is then $(\tilde{k}+1)$. The condition for a cartel to be an ISC at this size is $\pi^c(\tilde{k}+1) \geq \pi^f(\tilde{k})$, which is fulfilled because $\pi^f(\tilde{k}) < \pi^c(\tilde{k}+1)$. This has some important implications.

Lemma 2: If for a given k a cartel is not an OSC, it will be an ISC for $(k+1)$.

Proposition 1: In the case where no CA is active, every market has some cartel size k^* for which no firm has an incentive to change from being in the cartel to being in the fringe and vice versa.

Proposition 1 follows directly from the combination of Lemma 1 and Lemma 2. Consider the smallest possible cartel size $k = 2$. For this size, any cartel is inside stable (Lemma 1). This means that no firm has an incentive to leave the cartel. Two possible scenarios arise at this point: either the cartel is outside stable and hence $k^* = 2$, or (25) is not fulfilled. In the latter case, a firm will join the cartel increasing its size to $k = 3$. From Lemma 2 it then follows that after the firm has joined, the cartel is inside stable. Again, this means that there are two scenarios: either $k = 3$ is the stable cartel size k^*, or more firms will join. This mechanism works until (25) is fulfilled for some k. The largest possible cartel is $k = n$. Overall, this shows that there is always some cartel size k^* for which a cartel is both inside and outside stable. Call k^* the membership stable cartel size.

However, from the second fundamental cartel problem, it is known that any firm inside a cartel might have an incentive to cheat on the collusive agreement to earn higher profits. Therefore, being inside a cartel cannot be a Nash Equilibrium in any one shot game even if the inside and outside stability conditions are fulfilled. While this is true in non-repeated games, the majority of firm interactions take place repeatedly and it has long been established that collusion might be sustained in repeated games.[14] Therefore, assume that the price setting game is infinitely repeated and that a unit of profits earned in the next period is worth $\delta \in (0,1)$ units of profit today. From this it follows that the present value of sustaining collusion for all future periods is equal to $\sum_{t=0}^{\infty} \delta^t \pi^c(k) = \frac{\pi^c(k)}{1-\delta}$. Furthermore, assume that firms inside the cartel play a grim trigger strategy against defecting firms. This means that if one firm ever deviated from the collusive agreement, the industry would return to the competitive equilibrium for all future periods. For a firm considering to deviate, the present value of doing so is then $\pi^d(k) + \sum_{t=1}^{\infty} \delta^t \pi^* = \pi^d(k) + \delta \frac{\pi^*}{1-\delta}$. If the present value of staying inside the cartel outweighs those of deviating,

[14] This argument follows directly from Friedman (1971) and is featured in much of the academic literature on collusive agreements.

it is optimal for all cartel firms to not deviate. Formally, the condition is given by

$$\frac{\pi^c(k)}{1-\delta} \geq \pi^d(k) + \delta \frac{\pi^*}{1-\delta} \tag{26}$$

Solving this for δ leads to the following definition.

Stable Cartel (SC): A cartel size is said to be stable when setting the cartel price is a Nash Equilibrium. That is the case when:

$$\delta \geq \frac{\pi^d(k) - \pi^c(k)}{\pi^d(k) - \pi^*} := \delta^*(k). \tag{27}$$

Call $\delta^*(k)$ the critical discount factor above which collusion can be sustained. As long as the firms' discount factor is above $\delta^*(k)$, the cartel size k can be sustained as a Nash Equilibrium. For this to be possible, it must be that $\delta^*(k) < 1$ because otherwise $\delta \geq \delta^*(k)$ is not feasible. This implies that

$$\begin{aligned} &\delta^*(k) < 1, \text{ which means that} \\ &\pi^c(k) > \pi^* \end{aligned} \tag{28}$$

is necessary for a cartel to be stable. Therefore, any cartel that is stable must earn more than it would under competition. This also implies that any cartel size for which the cartel is profitable relative to the competitive outcome can be sustained as a Nash Equilibrium if the discount factor is sufficiently high. This argument will be used in the following to determine if a given membership stable cartel size can be called stable.

Lemma 3: For any cartel that is profitable relative to the competitive outcome, there is some δ such that staying inside the cartel is a Nash Equilibrium.

In the no CA case, the ordering of profits is such that $\pi^d(k) > \pi^c(k) > \pi^*$ and therefore it follows that $\delta^*(k) < 1$ can be established as a result. This directly implies that in the no CA case any cartel size k can be sustained for some discount factor δ.

Proposition 2: Given there is no law enforcement, any membership stable cartel size k^* can be sustained as a Nash Equilibrium for some discount factor $\delta \in (0.1)$. Thus, any membership stable cartel is also a stable cartel in the sense that no cartel member has an incentive to deviate from the collusive agreement.

In summary, this discussion has provided three conditions that have to be fulfilled for the existence of a collusive agreement. Firstly, firms inside the cartel should not have an incentive to leave. Secondly, firms outside the cartel should not have an incentive to

Table 1: *Membership stable cartel sizes in the no penalty case*

γ	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
n	**stable cartel size**								
3	3	3	3	3	3	3	3	3	3
4	3	3	3	4	4	4	4	4	4
5	3	3	4	4	5	5	5	5	5
7	3	4	4	5	5	6	7	7	7
9	4	4	5	5	5	6	6	6 *	9

* *indicates that the membership stable cartel size is non-unique and that $k = n$ is also membership stable.*

join. Finally, given that the first two conditions are fulfilled, the industry's valuation of future profits has to be high enough so that setting the cartel price is a Nash Equilibrium strategy. In this discussion it has become clear that the existence of a cartel is crucially related to the inside stability condition. If there was no k for which firms want to stay inside a cartel, there will never be a membership stable cartel. Furthermore, the outside stability condition has been linked to the cartel size: given any cartel size is inside stable, it will grow until it is also outside stable. Once a membership stable cartel is reached, it will also be stable in the sense that collusion can be sustained as a Nash Equilibrium if future profits are valued high enough. The outcomes of numerically determining these stable cartel sizes and market prices are reported in Table 1.

Table 1 shows the stable cartel sizes for ranges of $\gamma = 0.1$ to $\gamma = 0.9$, in steps of 0.1, and for some industry sizes between $n = 3$ and $n = 9$. Marginal costs have no effect on the stable cartel sizes, therefore they are not reported. Those cases in which all firms are inside the cartel ($k^* = n$) are highlighted in yellow. One can see that this is the case when the degree of product substitutability γ is high or when the industry is small. When γ is high, the competitive price level is close to marginal costs and competitive profits are low. In these cases, any cartel not consisting of all firms in the industry could raise prices only by a very small margin. That is because as long as there are firms outside the cartel, they would gain a large portion of the overall demand in the industry. However, when all firms are in the cartel they can set monopoly prices and share monopoly profits. When n is low, any firm joining or leaving the cartel has a large effect on the cartel's market power and thereby on the ability to raise prices. Therefore, it is more likely to pay off to join the cartel and increase the profits up to the monopoly level.

Table 2 shows the resulting cartel and market prices when costs are $c = 0.25$. While costs don't affect the stable cartel size, they positively affect the prices charged. Through-

Table 2: *Average market prices in the no penalt case ($c = 0.25$).*

γ	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
n	**Average market price**								
3	0.625	0.625	0.625	0.625	0.625	0.625	0.625	0.625	0.625
4	0.597	0.568	0.537	0.625	0.625	0.625	0.625	0.625	0.625
5	0.575	0.529	0.531	0.501	0.625	0.625	0.625	0.625	0.625
7	0.542	0.496	0.432	0.431	0.403	0.452	0.625	0.625	0.625
9	0.524	0.456	0.421	0.381	0.349	0.338	0.312	0.289*	0.625

* *indicates that stable cartel size is non unique and that $k = n$ is an additional stable cartel. In these cases, both the market and the cartel price are 0.625.*

out the numerical calculations in this paper, it is therefore assumed that $c = 0.25$ to ensure comparability. The market price $\bar{p}(k^*))$ is defined as the average per firm price charged in the industry and is calculated as

$$\bar{p}(k^*)) = \frac{k^* p^c(k^*) + (n - k^*) p^f(k^*)}{n}. \tag{29}$$

For example, take the case of $\gamma = 0.4$ and $n = 5$. In this case, the stable cartel size is equal to $k^* = 4$ and hence, four firms set the cartel price $p^c(4)$ while one firm sets the fringe price $p^f(4)$. The average price is then computed as $(4p^c(4) + p^f(4))/5$. Trivially, for the case $n = k$, the cartel and market price are equivalent and are equal to the monopoly price. All cases where $n = k$ are highlighted in yellow. It can be seen that the degree of product substitutability affects the cartel price in two ways. Firstly, for higher values of γ the competitive pressure between cartel and fringe firms leads to decreasing prices. Secondly, the cartel size increases when γ goes up as only large cartels are stable under the increased competitive pressure. This leads to higher prices. Overall, the prices firstly decrease in γ and then for higher values of γ increase again. In most cases, this increase comes from the fact that the cartel size increased to the maximum size $k^* = n$ and therefore firms can charge the monopoly price.

Overall, this section has provided the baseline framework to determine (membership) stable cartel sizes and the resulting market prices. It has been shown that in the absence of CA, every market has some stable cartel size which is also profitable relative to the competitive case and therefore sustainable for some discount factor. The cartel sizes and market prices determined in this section will be used as a reference in the following sections and to determine the sign of the direct and indirect price effects of the penalty regimes.

2.4 Collusion: Profit Based Penalties

Now assume the presence of a competition authority (CA) which detects and fines cartels. It cannot perfectly monitor cartel behaviour but instead detects and penalises a cartel with probability $\beta \in (0,1)$. The penalty will be based on some penalty function $F(\bullet)$ which is the same for each cartel. The CA doesn't fine firms which have not been colluding and does not impose negative penalties. It does penalise deviating firms if they are detected during the deviation period. All of this is common knowledge in the market.

Assume the CA bases the cartel penalties on the profits earned by the cartel members. In this case, the penalty function takes the form $F(\pi^c) = \phi\pi^c$ where $\phi > 0$ is the penalty rate. In this case, any firm associated with the cartel faces the expected penalty of $\beta\phi\pi^c = \tau_\pi\pi^c$ where $\tau_\pi = \beta\phi$ is the expected share of profit that is taken away from a cartel member. Parallel to Katsoulacos et al. (2015), τ_π can be interpreted as the toughness of the penalty regime. Throughout this section, the subscript π will denote that a variable is determined under a profit based penalty regime.

Similarly to the no CA case, cartel firms aim to maximise the overall profits of all cartel members. To achieve this, they take into account the price effect on the combined cartel demand. For each individual firm inside the cartel profits are given by $\pi^c_\pi(k) = (1-\tau_\pi)[p^c_\pi(k) - c]q^c_\pi(k)$. So for any given cartel size k, the cartel firms solve

$$\max_{\{p^c_\pi\}} \pi^c_\pi(k) = \max_{\{p^c_\pi\}} (1-\tau_\pi)(p^c_\pi - c)q^c_\pi = \max_{\{p^c_\pi\}} (1-\tau_\pi)\pi^c_\pi \tag{30}$$

Compared to a situation with no penalties, the profits are lowered by a fraction τ_π. When $\tau_\pi \geq 1$ the expected penalty exceeds the profits and a cartel makes negative expected profits. In this case, it would never form. For the analysis to be meaningful and interesting it will therefore be assumed that $\tau_\pi \in (0,1)$. Given these parameter restrictions, (30) is a positive monotonic transformation of (13) and hence, the optimal price set by a cartel in the profit based penalty regime is the same as in the no law-enforcement case, given k. In contrast to the cartel, fringe firms are not subject to penalisation and therefore, their maximisation problem is the same as in the no penalty case. Furthermore, fringe profits will not change. However, the defector price and profits follow the same logic as the cartel: as the profit function is a monotonous transformation of the no CA case, the price and quantity traded are the same as in the no CA case, given k. The cartel and deviator profits are then equal to the optimal profits computed in the no penalty case minus the expected penalty $\tau_\pi\pi^c$ and $\tau_\pi\pi^d$ respectively. Given the cartel size k, it follows that:

$$\begin{aligned}
&\texttt{ii)} \quad p^f_\pi(k) = p^f(k) &&\texttt{ii)}\, \pi^f_\pi(k) = \quad \pi^f(k)\\
&\texttt{iii)} \quad p^c_\pi(k) = p^c(k) &&\texttt{iv)}\, \pi^c_\pi(k) = \quad (1-\tau_\pi)\pi^c(k)\\
&\texttt{v)} \quad p^d_\pi(k) = p^d(k) &&\texttt{vi)}\, \pi^d_\pi(k) = \quad (1-\tau_\pi)\pi^d(k),
\end{aligned} \tag{31}$$

where the comparative statics regarding k hold as outlined in the no penalty case, i.e. all prices and profits are increasing in k.

In the following, the effect of the penalty regime on prices charged in the industry are analysed. To do so, two dimensions will be differentiated. Firstly, the direct price effect which is the penalty's price effect for a given cartel size k. Secondly, the indirect price effect (or size effect) which is the change in price that is due to the change in cartel size, incentivised by imposing a penalty. The direct effects are considered first. As follows directly from (31), the profit based penalty does not lead to a different price setting strategy for firms in the industry, given any cartel size k.

Proposition 1_π: The profit based penalty regime does not have a direct price effect.

This finding is in line with the findings of Katsoulacos et al. (2015) in the homogeneous goods case. Furthermore, a direct implication of it is that if profit based penalties have any impact on equilibrium prices it would have to be through the cartel size. Therefore, the indirect effects are determined by finding the stable and sustainable cartel sizes and comparing them to the no CA case. As discussed before, for any cartel to form it must overcome two problems. Firstly, a cartel size needs to be stable in the sense that no firm has an incentive to change its status from cartel to fringe and vice versa. Secondly, a stable cartel size needs to be sustainable in the sense that no firm has an incentive to deviate from the collusive agreement.

To begin with, a membership stable cartel is found by analysing how profit based penalties influence the inside stability of cartel. To do so, $\pi^c_\pi(k)$ and $\pi^f_\pi(k)$ are substituted into (24).

ISC: Under the profit based penalty regime, a cartel is inside stable for $k \geq 3$, when

$$(1-\tau_\pi)\pi^c(k) \geq \pi^f(k-1), \tag{32}$$

and for $k = 2$ when $(1-\tau_\pi)\pi^c \geq \pi^*$. Here, for any $\tau_\pi > 0$, the LHS of the expression is strictly lower than in the case with no CA. That is because cartel profits are strictly decreasing in the penalty toughness while fringe profits are unaffected. This implies that the condition is less likely to hold when profit based penalties are imposed and that therefore firms inside the cartel have a higher incentive to leave the cartel and join the fringe when penalties get tougher. Furthermore, it is now possible that a cartel with $k=2$ firms is not inside stable any more. That is the case when $(1-\tau_\pi)\pi^c(2) < \pi^*$, i.e. when the expected profits of forming a small cartel are lower than the competitive profits. This implies that the strict existence result for cartels from the no CA case doesn't hold any more and that therefore, it can be possible that in some given industry no cartel can form. This will be discussed again with cartel sustainability further below.

Consider now the cases in which at least one cartel size k is an ISC. In these cases, the outside stability determines the actual cartel size. Applying $\pi^c_\pi = (1-\tau_\pi)\pi^c$ and $\pi^f_\pi = \pi^f$ to (25) leads to

OSC: A cartel is said to be outside stable for any $k \in [2,(n-1)]$ when

$$\pi^f(k) \geq (1-\tau_\pi)\pi^c(k+1), \tag{33}$$

while any cartel with $k=n$ is outside stable.
In this case, it is clear that the RHS is decreasing in τ_π, which directly implies that for any $\tau_\pi > 0$, the condition is more likely to hold than in the no penalty case. By penalising only the cartel firms without altering the price setting incentives and thereby leaving fringe profits unaffected, the CA disincentivises joining the cartel. For a fringe firm, the potential gains of being part of a cartel with one additional member are traded off against the risk of paying a penalty. For high penalties that trade off is more likely to be decided in favour of staying outside the cartel. As a result, less firms want to join the cartel, which means that the cartel is more likely to be outside stable.

Combining the analysis of the latter two conditions, it is possible to state that when penalties are imposed on profits, firms are more likely to leave a collusive agreement and are less likely to join it. Numerical examples suggest that this means that profit based penalties lead to weakly smaller membership stable cartel sizes k^*_π, compared to the no penalty case.[15] As the cartel price is a decreasing function of k, this implies that profit based penalties have a weakly negative price effect on the cartel and market price.

Proposition 2_π: Profit based penalty regimes have weakly negative price effects on

[15] The reason for the effect to be weakly and not strictly is twofold. Firstly, when either n small or γ high, the fundamental reason to form a cartel with $k=n$ is that if any firm was to form a fringe, this would reduce cartel profits by too much for a cartel to remain stable. In these cases, the optimal cartel size remains $k=n$. Secondly, it is assumed that k is an integer. An incremental change in τ_π might then not be big enough to change k from one integer to the next.

Table 3: *Membership stable cartel sizes under the profit based penalty regime.*

γ / n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	τ_π
Stable cartel size										
3	–	–	3	3	3	3	3	3	3	0.015
3	–	–	3	3	3	3	3	3	3	0.03
3	–	–	–	–	3	3	3	3	3	0.09
4	–	–	3	4	4	4	4	4	4	0.015
4	–	–	–	3	4	4	4	4	4	0.03
4	–	–	–	–	–	4	4	4	4	0.09
5	–	–	3	4	4	5	5	5	5	0.015
5	–	–	–	3	4	5	5	5	5	0.03
5	–	–	–	–	–	4	5	5	5	0.09
7	–	–	3	4	5	5	7	7	7	0.015
7	–	–	–	–	4	5	7	7	7	0.03
7	–	–	–	–	–	–	7	7	7	0.09
9	–	–	–	4	5	5	5	6 *	9	0.015
9	–	–	–	–	–	–	4*	5 *	5 *	0.03
9	–	–	–	–	–	–	–	9	9	0.09

* *indicates that membership stable cartel size is non unique and that $k = n$ is an additional membership stable cartel to the one displayed.*

the cartel price.

The combination of Proposition 1_π and 2_π lead to the overall price effect of profit based penalties.

Result 1_π: The overall price effect of profit based penalties is weakly negative.

As the direct effect on the price is neutral but the indirect effect is weakly negative, the imposition of profit based penalties can decrease the price set in an industry. In those cases in which an increase of τ_π actually decreases k, the stable cartel consists of less firms and therefore, it sets a lower price.

Having found a membership stable cartel, it is crucial to determine if that cartel is actually sustainable as a Nash Equilibrium. From applying the profit functions for this penalty regime to the Stable Cartel condition it follows that any cartel size k is sustainable when

$$\delta \geq \frac{\pi^d(k) - \pi^c(k)}{\pi^d(k) - \frac{\pi^*}{(1-\tau_\pi)}} := \delta_\pi^*(k), \tag{34}$$

where δ_π^* is the critical discount factor above which collusion can be sustained under a profit based penalty regime. It follows immediately that δ_π^* is strictly increasing $\forall \tau(0,1)$ which implies that collusion is harder to sustain for higher penalties. Therefore, the penalty regime has two channels through which it deters collusion. Firstly, for any cartel to be sustainable it must be that $\delta^*_\pi < 1$ and therefore $(1-\tau_\pi)\pi^c(k) > \pi^*$. This means that only cartels which are profitable compared to the competitive outcome can form. This reflects the findings from analysing (32) for the case of $k = 2$, but generalises it for the case of $k \in [2, n]$. Therefore collusion is deterred especially in those industries in which products are more differentiated, as for in these industries the cartel's ability to raise the price compared to the competitive price is limited. Secondly, as $\delta^*_\pi(k) < \delta^*(k)\ \forall\ \tau \in (0, 1)$ cartels now require a higher discount factor to sustain. That means that some cartel sizes which would have been sustainable under a no penalty regime are no longer sustainable.

Proposition 3_π: The profit based penalty regime has a deterrence effect through two channels. Firstly, it makes some cartels unprofitable compared to the competitive outcome and therefore unsustainable. Secondly, cartels require a higher discount factor compared to the no penalty case.

To illustrate the overall effect of the profit based penalty regime, Table 3 and Table 4 depict the membership stable cartel size and average market prices for three arbitrarily chosen toughness level $\tau_\pi = \{0.015, 0.03, 0.09\}$. Given an estimate of the detection rate[16] of $\beta = 0.15$, these toughness levels correspond to $10, 20$ and 60 percent respectively.[17] Market sizes and degrees of product substitutability are the same as in the forgone tables to allow for comparison.

From the comparison of Table 3 and Table 1 one can see that $k^*_\pi \leq k^*$. This is highlighted by the colour coding in Table 3, which indicates that a cartel size has decreased compared to the no CA case if the cell is green. Yellow cells indicate that the stable cartel consists of all firms in the market. A dash indicates that no cartel could form for the given parameter combination of n, γ and τ_π. One can see that cartels over very differentiated goods are deterred first. This follows from the intuition that underlines Proposition 3_π: because these are the industries in which the cartel's ability to raise prices compared to the competitive level is limited, the additional gains from collusion are low. When collusion becomes costly, cartels in these industries are the first to become unprofitable. Similarly, more cartels are being deterred when n goes up. That is because the relative gains of collusion compared to competition are lower when n is large, often because the

[16]An estimated detection rate around the level of $\beta = 0.15$ is widely used in the academic literature on cartel law enforcement. Bryant and Eckard (1991) provides an estimate of 13 to 17 percent for the US while Combe et al. (2008) estimates a detection rate around 13 percent in the EU.

[17]In the US the maximum fine is set at 'no more than twice the gross gain' of collusion.(18 U.S. Code § 3571 (d))

Table 4: *Average market prices under the profit based penalty regime* ($c = 0.25$).

γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	τ_π
n					**Average market price**					
	–	–	0.625	0.625	0.625	0.625	0.625	0.625	0.625	0.015
3	–	–	–	0.625	0.625	0.625	0.625	0.625	0.625	0.03
	–	–	–	–	0.625	0.625	0.625	0.625	0.625	0.09
	–	–	0.537	0.625	0.625	0.625	0.625	0.625	0.625	0.015
4	–	–	–	0.504	0.625	0.625	0.625	0.625	0.625	0.03
	–	–	–	–	–	0.625	0.625	0.625	0.625	0.09
	–	–	0.487	0.501	0.467	0.625	0.625	0.625	0.625	0.015
5	–	–	–	0.447	0.467	0.625	0.625	0.625	0.625	0.03
	–	–	–	–	–	0.430	0.625	0.625	0.625	0.09
	–	–	0.429	0.408	0.403	0.368	0.625	0.625	0.625	0.015
7	–	–	–	–	0.372	0.368	0.625	0.625	0.625	0.03
	–	–	–	–	–	–	0.625	0.625	0.625	0.09
	–	–	–	0.368	0.349	0.323	0.300	0.289*	0.625	0.015
9	–	–	–	–	–	–	0.294*	0.281*	0.265*	0.03
	–	–	–	–	–	–	–	0.625	0.625	0.09

* *indicates that stable cartel size is non unique and that $k = n$ is an additional stable cartel. In these cases, both the market and the cartel price are 0.625.*

stable cartel size doesn't include all firms of the industry. Therefore, for a given level of penalties, it is more likely that collusion doesn't pay off. In smaller industries, the stable cartel size is likely to be such that $k_\pi^* = n$ and hence firms set monopoly prices. In these cases, cartels can sustain higher penalties and are less likely to be deterred.

Table 4 shows the overall price effect of the profit based penalty regime on the average price charged in the market, where the definition of the average market price is equivalent to the no CA case:

$$\bar{p}_\pi = \frac{k_\pi^* p^c(k_\pi^*) + (n - k_\pi^*) p^f(k_\pi^*)}{n}. \tag{35}$$

The colour code in this table is the same as in Table 3: those cases in which the indirect effect is negative are highlighted in green and blue respectively. Comparing the table to Table 2 reveals that the profit based penalty can effectively decrease the average market price in those cases in which it leads to smaller cartel sizes.

Overall, the discussion of the profit based penalty regime has resulted in two main findings. Firstly, the result that the overall price effect on cartels that do form is weakly negative. That is because although the direct price effect is neutral, it can be that the penalty results in smaller stable cartels. As these charge lower prices, the overall penalty effect on cartels is negative. Secondly, it was found that the penalty regime can deter collusion either by making it unprofitable to collude or by raising the discount factor necessary to sustain collusion. While this latter finding is in line with the results of Katsoulacos et al. (2015), their homogeneous goods model does not capture the negative indirect price effect found in this model.

2.5 Collusion: Overcharge Based Penalties

This section discusses the case when CAs base penalties on overcharges, which are defined as the amount by which a cartel price exceeds the competitive price. Apart from the change in penalty base, the CA operates equivalently to the profit based penalty case. That means it detects and fines collusion with probability $\beta \in (0,1)$, does not impose negative penalties when the penalty base becomes negative[18] and does not convict non-offenders wrongfully. Define formally the overcharge as $O = (p_o^c - p^*)$, where the subscript O denotes that a variable is determined under the overcharge based penalty regime. To determine the level of penalty, the CA multiplies the overcharge O by the penalty rate $\varphi > 0$. To overcome that in this set up the penalty would be computed in different units to the profits, the CA then multiplies $O\varphi$ with the competitive equilibrium units q^* to get the final penalty.[19] This means that the final fining function is $F(O) = \varphi O q^*$.[20] Define the toughness of the penalty regime as $\tau_0 = \beta\varphi$ and substitute $\sum_{j=k+1}^{n} p_j = (n-k)p_o^f$, the maximisation problem of a cartel firm follows as

$$\begin{aligned}\max_{\{p_o^c\}} \pi_o^c &= \max_{\{p_o^c\}} \Big((p_o^c - c)q_o^c - \tau_o q^* (p_o^c - p^*) \Big) \\ &= \max_{\{p_o^c\}} \Big((p_o^c - c)\Big[\frac{(1-\gamma) + \gamma(n-k)p_o^f - [1+\gamma(n-k-1)]p_o^c}{(1-\gamma)[1+\gamma(n-1)]}\Big] - \tau_o q^* (p_o^c - p^*) \Big)\end{aligned} \tag{36}$$

[18] While it might seem trivial to mention this, there are situations in which this distinction as to whether a penalty is positive or negative makes a clear difference in the equilibrium outcome.

[19] As has been discussed in KMU (2015) this leads to the profits and the penalty being computed in the same unit: while the profits are denoted in monetary units, price multiplied with penalty rate results in a penalty denoted in monetary units per unit of a good. This is technically unfeasible.

[20] Previous private damage actions have applied the actual cartel quantity traded to the overcharge instead of using the competitive quantity. However, using the competitive quantity q^* greatly simplifies the analysis and does not change the direction of the results. A small difference in outcomes between the two approaches would be that the negative price effects derived later in this chapter would be weaker when the actual cartel quantity traded was used. That is because the cartel quantity is a decreasing function of the cartel price. In this case, if a cartel increased the price over the competitive level that would decrease the cartel quantity, thereby making the penalty relatively less severe than in the case when the competitive quantity was used.

The first order condition then follows as

$$\frac{\partial p_o^c q_o^c}{\partial p_o^c} = c\frac{\partial q_o^c}{\partial p_o^c} + \tau_o q^*, \tag{37}$$

where the left hand side is the marginal revenue of increasing the cartel price (MR) and the right hand side denotes the marginal cost of increasing the cartel price (MC). The MC of increasing the cartel price are increasing in τ_o. Intuitively that is because the expected penalty increases linearly in the price charged by the cartel. Therefore, when $\tau_o > 0$ the cartel needs to reduce the price charged compared to the no CA case to achieve the optimality condition $MC = MR$.

Solving the first order condition for the cartel price gives the cartels best response function to fringe prices as

$$\begin{aligned} p_o^{c,R}(p^f) = &\frac{\Big(1+\gamma(n-k-1)\Big)\Big(2+\gamma(n+k-3)\Big)+\gamma(n-k)\Big(1+\gamma(n-2)\Big)}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k}c \\ &+\frac{(1-\gamma)\Big(2+\gamma(2n-3)\Big)}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k} \\ &-\tau_o q^* \frac{(1-\gamma)\Big(1+\gamma(n-1)\Big)}{2\Big(1+\gamma(n-k-1)\Big)}, \end{aligned} \tag{38}$$

which can be rewritten to

$$p_o^{c,R}(p^f) = p^{c,R}(p^f) - \tau_o q^* \frac{(1-\gamma)\Big(1+\gamma(n-1)\Big)}{2\Big(1+\gamma(n-k-1)\Big)} \tag{39}$$

This shows that compared to the no CA case, cartel firms set a lower price for any given fringe price. At the same time, the fringe firms' maximisation problem remains the same as it is without a CA. Hence, their reaction function is equivalent to before:

$$p_o^{f,R}(p_o^c) = p^{f,R}(p^c) = \frac{\Big(1+\gamma(n-2)\Big)c + (1-\gamma) + \gamma k p_o^c}{2+\gamma(n+k-3)}. \tag{40}$$

Combining both reaction functions leads to optimal cartel prices

$$\begin{aligned} p_o^c(k) &= p^c(k) - \tau_o q^* \frac{(1-\gamma)[2+\gamma(n+k-3)][1+\gamma(n-1)]}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k} \\ &= p^c(k) - \tau_o \tilde{p}_o^c(k) \end{aligned} \tag{41}$$

and for the fringe

$$
\begin{aligned}
p_o^f(k) &= p^f(k) - \tau_o q^* \frac{kg(1-\gamma)[1+\gamma(n-1)]}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k} \\
&= p^f(k) - \tau_o \tilde{p}_o^f(k)
\end{aligned}
\tag{42}
$$

where $[-\tau_o \tilde{p}_o^i(k)] < 0$, $i \in \{c, f\}$ is the direct price effect of the penalty on the cartel and fringe respectively.[21] [22]

Proposition 1_o**:** Overcharge based penalties have a negative direct price effect on both the cartel and the fringe.

This follows directly from the pricing functions. Intuitively, the overcharge based penalty increases the MC of raising the price. To achieve the optimality condition that MC=MR, cartel firms have to increase the MR which they do by decreasing the price. Fringe firms then evaluate their reaction function at lower cartel prices which leads them to decrease their prices as well.

The functional form of $p_o^c(k)$ shows that the cartel price is linearly decreasing in the toughness of the penalty regime τ_o. This implies that for some maximum penalty toughness τ_o^{max} the cartel price is pushed below the competitive equilibrium price p^*. For these values of $\tau_o \geq \tau_o^{max}$, the cartel would not form as profits would not exceed competitive profits. τ_o^{max} follows from $p_o^c(k) = p^*$ as

$$
\tau_o^{max} = \frac{\gamma(k-1)}{1+\gamma(n-2)} \tag{43}
$$

Assume therefore in the following that $\tau_o < \tau_o^{max}$ to derive the functional forms.[23]

With the given prices, it is then possible to determine the quantities traded as

$$
q_o^f(k) = q^f(k) - \tau_o \tilde{q}_o^f(k) \tag{44}
$$

and

$$
q_o^c(k) = q^c(k) + \tau_o \tilde{q}_o^c(k), \tag{45}
$$

[21] Proof that $[-\tau_o \tilde{p}^{i,*}(k)] < 0$ in Appendix B.

[22] For all cases in which the cartel forms, both the cartel and the fringe price are above the competitive price level p^*. This follows directly from the discussion of (43).

[23] It follows directly that $p_o^f(k) > p^*$ holds $\forall\ \tau_o < \tau_o^{max}$.

where

$$
\begin{aligned}
[\tau_o \tilde{q}_o^c(k)] &= \tau_o q^* \frac{[1+\gamma(n-k-1)][2+\gamma(n+k-3)] - \gamma^2(n-k)k}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)] - \gamma^2(n-k)k} && > 0 \\
&\text{and} \\
[-\tau_o \tilde{q}_o^f(k)] &= [-\tau_o q^*] \frac{k\gamma[1+\gamma(n-2)]}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)] - \gamma^2(n-k)k} && < 0,
\end{aligned} \tag{46}
$$

are the quantity effects of the overcharge based penalty regime on the fringe and cartel respectively. As can be seen, the quantity effect is positive for the cartel and negative for the fringe. This difference in sign follows from the relative magnitude of the price effects. It can be shown that the cartel's price reaction to overcharge based penalties is stronger than the fringe's[24], i.e. $\mid -\tau_o \tilde{p}_o^c(k) \mid > \mid -\tau_o \tilde{p}_o^f(k) \mid$. This means that for any increase in τ_o the cartel decreases their price more than the fringe firms do, meaning that some purchases will move to cartel products. Compared to a situation without penalties a cartel therefore sells more units and the fringe sells less units. In the direct comparison though, the fringe quantity still exceeds the cartel quantity: $q_o^f(k) > q_o^c(k)$.[25]

The cartel profits are computed as

$$
\begin{aligned}
\pi_o^c(k) &= \Big(p_o^c(k) - c\Big) q_o^c(k) - \tau_o q^* (p_o^c - p^*) \\
&= \Big(p^c - \tau_o \tilde{p}_o^c - c\Big)\Big(q^c + \tau_o \tilde{q}_o^c\Big) - \tau_o q^* (p^c - \tau_o \tilde{p}_o^c - p^*) \\
&= \pi^c - \tau_o \Big([p^c - p^*]q^* + \tilde{q}_o^c q^c - [p^c - c]\tilde{q}_O^c\Big) + \tau_o^2 \tilde{q}_o^c (q^* - \tilde{q}_o^c)
\end{aligned} \tag{47}
$$

which is shown to be a decreasing function of toughness parameter τ_o $\forall$ $\tau_o \in [0, \tau_o^{max})$ in Appendix B. The fringe profits are given by

$$
\begin{aligned}
\pi_o^f(k) &= \Big(p_o^f - c\Big) q_o^f(k) \\
&= \Big(p^f - \tau_o \tilde{p}_o^f - c\Big)\Big(q^f - \tau_o \tilde{q}_o^f\Big),
\end{aligned} \tag{48}
$$

which is decreasing in $\tau_o \in [0, \tau_o^{max})$ because both quantity and price are decreasing functions of τ_o.

Similarly to before, some firms inside the cartel have an incentive to break the cartel agreement and maximise individual profits instead. As a structured analysis of the deviator behaviour is not part of this paper, define the deviator profits under the overcharge based regime as $\pi_o^d(k)$. It is assumed that $\pi_o^d(k) > \pi_o^c(k)$ $\forall k$, which would render collusion

[24] Proof in Appendix B
[25] Proof in Appendix B

unstable for any one shot game.

However, as discussed before, collusion can be sustained when interaction is repeated. Therefore, the profits calculated under the overcharge based regime are applied to the Stable Cartel condition defined in (27). This leads to

$$\delta \geq \frac{\pi_o^d(k) - \pi_o^c(k)}{\pi_o^d(k) - \pi^*} := \delta_o^*, \tag{49}$$

where δ_o^* is the critical discount factor above which collusion can be sustained. For all $\delta \geq \delta_o^*$, setting the collusive prices is a Nash Equilibrium. For $\delta < \delta_o^*$ collusion is unstable.

A direct implication of this is that there is some level of penalties for which collusion can't be sustained. Specifically, for the Stable Cartel condition to hold for some $\delta \in (0,1)$, it must be that $\delta_o^* < 1$. This only holds when the cartel is profitable relative to the competitive outcome: $\pi_o^c(k) > \pi^*$.

Proposition 2_o: When penalties are high enough, the overcharge based penalty regime can deter collusion. This is the case for $\tau_o > \tau_o^{max}$.

Consider now how the membership stable cartel size k_o^* is affected by the penalty regime. Firstly, inside stability is analysed. Applying $\pi_o^c(k)$ and $\pi_o^c(k)$ to (24) leads to the following.

ISC: Under an overcharge based penalty regime, a cartel is inside stable when

$$\pi_o^c(k) \geq \pi_o^f(k-1) \text{ for } k \geq 3, \tag{50}$$

and $\pi_o^c(2) \geq \pi^*$ for $k = 2$.

As was established, the functions on both sides of the inequality are decreasing in τ_o. When penalties increase, there are two forces in opposite directions at work. Firstly, relative to the fringe, the cartel is affected more by an increase in the penalty. That is because it decreases the optimal price set and at the same time increases the expected penalty, ceteris paribus. Contrary to this, the fringe does not pay a penalty and it decreases its price less than the cartel. These effects increase the fringe profit relative to the cartel profits and make it less profitable to be inside a cartel. However, secondly, the cartel reacts more to changes in τ_o than the fringe. Therefore, the cartel quantity increases, while the fringe decreases. This makes it more profitable to be inside a cartel. Depending on the relative strength of these two forces, a cartel might then be more or less stable relative to the no CA case. This makes a comparison to the no CA case ambiguous and highly dependent on the degree of product substitutability and market size.

However, it is possible to conduct an intuitive comparison to the profit based penalty regime. While a formal approach of this would require some equivalence rule to ensure that both penalties are equally tough, a comparison of the general underlying intuition of argument does not require this. On the one hand, in the profit based penalty regime any increase in toughness τ_π decreases only the cartel profits, making it less attractive to stay inside a cartel. On the other, in the overcharge based penalty regime, an increase in τ_o affects both the cartel and the fringe negatively. This indicates that compared to the profit based penalties, it is more attractive to be inside a cartel when penalties are based on overcharges.

Consider now, how outside stability is affected. Similar to above, applying the corresponding profit functions to (25) leads to a new formulation of an OSC.

OSC: Under an overcharge based penalty regime, a cartel is outside stable when

$$\pi_o^f(k) \geq \pi_o^c(k+1) \text{ for } (n-1) \geq k \geq 2, \tag{51}$$

while a cartel with $k = n$ is always outside stable.
The outside stability condition has an analogous, though reverse, analysis to the inside stability condition. Hence, it is expected that relative to the profit based penalties more firms are willing to join the cartel when penalties are imposed on overcharges. In relation to the no CA case, the outcomes are expected to be ambiguous.

Overall, it is then expected from the intuitive comparison that the overcharge based penalty regime leads to more inside stable, and less outside stable cartels than the profit based penalty regime. This implies that the membership stable cartel size k_o^* is weakly larger than k_π^*. Finally, the expected ambiguity in the comparison with the no penalty stable cartel size k^* can be confirmed numerically.

Table 5 shows the membership stable cartel sizes that follow for the same cartel sizes n and degrees of products substitutability γ as used before. The penalty toughness is tested for $\tau_o = \{0.1, 0.2, 0.3\}$. Given a detection rate of $\beta = 0.15$, these levels of τ_o correspond to penalty rates of 66, 133 and 200 percent of the penalty base $q^*(p_o^c - p^*)$. In the US, the maximum public penalty applicable on a cartel is 200 percent of the illegal gains from collusion. While the overcharge multiplied with the competitive quantity can only be taken as a very rough estimate of these illegal gains, they can nevertheless function as an

Table 5: *Membership stable cartel sizes under the overcharge based penalty regime.*

γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	t
n	stable cartel size									
	3	3	3	3	3	3	3	3	3	0.1
3	–	3	3	3	3	3	3	3	3	0.2
	–	3	3	3	3	3	3	3	3	0.3
	3	3	3	4	4	4	4	4	4	0.1
4	4	3	3	4	4	4	4	4	4	0.2
	–	4	4	4	4	4	4	4	4	0.3
	3	3	4	4	4	5	5	5	5	0.1
5	4	4	4	4	4	5	5	5	5	0.2
	5	4	4	4	4	5	5	5	5	0.3
	4	4	4	4	5	5	7	7	7	0.1
7	5	4	4	4	5	5	5	5 *	7	0.2
	6	5	5	5	5	5	5	7	7	0.3
	4	4	4	5	5	5	5	6 *	6 *	0.1
9	5	5	5	5	5	5	5	5 *	5 *	0.2
	7	6	5	5	5	5	5	5 *	5 *	0.3

* *in these cases $k = n$ is also a membership stable cartel size.*

upper boundary.[26] Those cases in which the stable cartel size is above the stable cartel size in the no CA, i.e. $k_o^* > k^*$, case are highlighted in green, while $k_o^* < k^*$ is highlighted in light blue. One can see that for products that are relatively differentiated, the cartel size is bigger under the overcharged based penalty regime than without penalties. In these cases, the cartel's ability to set prices far above competitive level is limited as the competitive prices are already close to the monopoly level. Therefore, expected penalties are relatively low and being inside a cartel is not too costly. When products are moderate or close substitutes, the penalty either doesn't influence cartel size or it reduces it. In the latter cases, the imposition of the penalty makes smaller cartels more stable because they set lower prices and therefore expect lower penalties. Thus, it can be said that the size effect of the penalty relative to the no CA case are very ambiguous and depend on the degree of product substitutability.

Proposition 3_o: The indirect price effect of the overcharge based penalty regime is ambiguous and depends on the market size n, as well as on the degree of product substitutability γ.

The reaction to different levels of toughness can also be analysed. One can see that the stable cartel size is increasing in the toughness parameter τ_o as long as products are

[26] The illegal gains under any penalty regime are given by $(p_o^c - c)q_o^c - (p^* - c)q^*$ which is below the penalty basis $(p_o^c - p^*)q^*$. This follows from $(p_o^c - c)q_o^c - (p^* - c)q^* < (p_o^c - p^*)q^* \ \forall \ p_o^c > c$

Table 6: Market prices in the overcharge based penalty regime

γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	τ_o
n	stable cartel size									
3	0.604	0.603	0.601	0.599	0.597	0.595	0.593	0.591	0.589	0.1
	–	0.580	0.576	0.573	0.569	0.565	0.561	0.558	0.554	0.2
	–	0.558	0.552	0.546	0.541	0.535	0.529	0.524	0.518	0.3
4	0.582	0.552	0.521	0.597	0.595	0.593	0.592	0.590	0.589	0.1
	0.582	0.536	0.506	0.569	0.565	0.562	0.558	0.555	0.553	0.2
	–	0.553	0.547	0.541	0.535	0.530	0.525	0.521	0.516	0.3
5	0.563	0.518	0.517	0.485	0.451	0.592	0.591	0.590	0.588	0.1
	0.562	0.531	0.499	0.468	0.436	0.559	0.557	0.554	0.552	0.2
	0.559	0.513	0.482	0.452	0.421	0.527	0.522	0.519	0.515	0.3
7	0.543	0.486	0.439	0.399	0.392	0.360	0.590	0.589	0.588	0.1
	0.541	0.475	0.429	0.391	0.382	0.352	0.323	0.553	0.551	0.2
							(0.555)			0.2
	0.538	0.479	0.439	0.403	0.372	0.343	0.317	0.517	0.515	0.3
9	0.516	0.449	0.400	0.374	0.344	0.319	0.297	0.286	0.267	0.1
								(0.589)	(0.588)	0.1
	0.513	0.451	0.404	0.368	0.338	0.314	0.294	0.277	0.263	0.2
								(0.552)	(0.551)	0.2
	0.519	0.453	0.396	0.361	0.333	0.310	0.291	0.276	0.262	0.3
								(0.516)	(0.514)	0.2

Number is brackets show prices for the case of $k_o^* = n$ when k_o^* is non-unique.

relatively differentiated. For example, when the industry size is $n = 9$ and $\gamma = 0.2$, the stable cartel size goes from $k_o^* = 4$ at $\tau_o = 0.1$, to $k_o^* = 5$ at $\tau_o = 0.2$, and finally to $k_o^* = 6$ at $\tau_o = 0.3$. When products are relatively homogeneous, the stable cartel size is mostly independent of penalty toughness. Two special cases require mentioning. Firstly, when $n = 7$ and $\gamma = 0.7$ an increase in τ_o from 0.1 to 0.2, or to 0.3 leads to a lower stable cartel size. Secondly, there are three combinations of γ and n where for some τ_o two cartels are stable ($(n,\gamma) = \{(7,0.8),(9,0.8),(9,0.9)\}$). In these cases, some small cartel $k_o^* < n$ and a big cartel with $k_o^* = n$ are stable, and the smaller cartel size is decreasing in τ_o.[27] That is because small cartels sets lower prices. In this parameter range this means that they can sustain higher penalties. However, the most frequent observation regarding cartel size and penalty toughness is, that k_o^* is increasing in τ_o.

The question left to answer is what the overall effect of the overcharge based penalty is on the prices charged in the market. Table 6 shows the market prices as calculated in the no CA and the profit based penalty case. The cases in which all firms in the market

[27] In the case $(n,\gamma) = (7,0.8)$ an increase from $\tau = 0.1$ to 0.2 leads to $k_o^* = 5$. This remains the stable cartel size when $\tau_o = 0.3$.

collude ($k_o^* = n$) are highlighted in yellow and are the best example to see the direct price effect of overcharge based penalties. In these cases, the average market price in the no CA case would be equal to the monopoly price $p^c(n) = 0.625$ for $c = 0.25$. However, it can be seen that when penalties are imposed on overcharges, the cartel price is below the price set in the no CA case. Furthermore, it is decreasing in τ_o as well as in γ. That is because, ceteris paribus, higher levels of τ_o increase the expected penalty, which incentivises the cartel to set a lower price. Additionally, when γ is high the competitive price level is low, which implies a higher relative overcharge for the same cartel price. Thus, given any cartel price, the expected penalty is higher. Again, this incentivises the cartel to set a lower price. The indirect price effect is best observed in the case of $n = 4$ and $\gamma = 0.3$. In this case, the average price is increasing in τ_o because the cartel size k_o^* went from 3 to 4. However, compared to the no penalty case, the direct price effect outweighs the indirect price effect. In fact, for all cases analysed and displayed in this paper, the average market price in the overcharge based penalty regime is lower than in the no CA case. This leads to the following:

Result 1_o: When penalties are based on overcharges, the average price charged in the industry is lower than in the case of no penalties. Therefore, compared to no penalties, the overall price effect of the overcharge based penalty regime is negative. However, it is possible that tougher penalties lead to an increase of the average market price when products are relatively differentiated.

Overall, the analysis of the overcharge based penalty regime has lead to two main findings. Firstly, the overall price effect of the penalty is negative if compared to the no CA case. This comes from a strong negative direct price effect and an ambiguous but relatively weak indirect effect. When products are relatively differentiated, there are special cases in which an increase of the toughness τ_o results in an increase in price that is not sufficient to elevate the price above the no CA case. Secondly, for significantly high penalty rates $\tau_o > \tau_o^{max}$ collusion cannot be sustained and hence the penalty regime has a deterrence effect. However, for a penalty rate which corresponds roughly to the highest penalty currently allowed under US law this deterrence effect is small. A structured comparison with other penalty regimes follows in Section 3.

2.6 Collusion: Revenue Based Penalties

Assume now that the CA imposes penalties based on the revenue earned by the cartel firms. As discussed in the introduction, this is the current status quo for most CAs. The fining function then takes the form $F(R) = \kappa q_r^c p_r^c$, where $\kappa > 0$ is the penalty rate and the subscript r denotes that a variable is determined under the revenue based penalty regime. All other CA action is the same as before. Define $\tau_r = \beta\kappa$ as the toughness of the penalty regime. Taking into account that $\sum_{j=k+1}^{n} p_j = (n-k)p^f$, the cartel firm's maximisation problem then follows as

$$\begin{aligned} \max_{\{p_r^c\}} \pi_r^c &= \max_{\{p_r^c\}} \Big[(p_r^c - c)q_r^c - \tau_r p_r^c q_r^c\Big] = \max_{\{p_r^c\}} \Big[(1-\tau_r)p_r^c - c)\Big] q_r^c \\ &= \max_{\{p_r^c\}} \Big[(1-\tau_r)p_r^c - c)\Big] \Bigg[\frac{(1-\gamma) + (n-k)p^f - [1+\gamma(n-k-1)]p_r^c}{(1-\gamma)[1+\gamma(n-1)]}\Bigg]. \end{aligned} \tag{52}$$

The first order condition equating marginal revenue and marginal costs follows as

$$(1-\tau_r)\frac{\partial p_r^c q_r^c}{\partial p_r^c} = c\frac{\partial q_r^c}{\partial p_r^c}, \tag{53}$$

where it follows directly from the right hand side that in optimum both sides are negative. Therefore, any increase in the penalty toughness τ_r effectively increases the marginal revenue of raising the cartel price for any given price-output combination. To achieve the MR=MC condition again, the cartel firms need to raise the price, thereby decreasing the MR. From an intuitive perspective, the revenue based penalty regime incentivises firms to have lower revenues in order to keep potential penalties low. At the same time, optimal prices are increased to ensure high per unit profits.

To then derive the cartel's optimal price setting as a function of the fringe prices, the demand functions are applied to the first order condition and the resulting equation is solved for the cartel price.

$$p_r^{R,c}(p_r^f) = \frac{c}{2(1-\tau_r)} + \frac{(1-\gamma) + (n-k)\gamma p_r^f}{2[1+\gamma(n-k-1)]}. \tag{54}$$

Comparing this to the reaction function in the no CA case confirms the above intuition that cartels set higher prices: For any fringe price the cartel sets a higher price in this regime than in the no CA case as long as $\tau_r \in (0,1)$. Similar to the other penalty regimes, the fringe firms' general maximisation problem and hence, the best response function doesn't change. Therefore, it is given by (12). Combining both functions leads to optimal cartel and fringe prices

$$p_r^c(k) = \frac{\Big(1+\gamma(n-k-1)\Big)\Big(2+\gamma(n+k-3)\Big)+(1-\tau_r)\gamma(n-k)\Big(1+\gamma(n-2)\Big)}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k}\frac{c}{1-\tau_r} + \frac{(1-\gamma)\Big(2+\gamma(2n-3)\Big)}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k} \quad (55)$$

and

$$p_r^f(k) = \frac{[1+\gamma(n-k-1)][2+\gamma(2n-4)+\frac{gk}{(1-\tau_r)}]}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k}c + \frac{(1-\gamma)[2+\gamma(2n-k-2)]}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k}. \quad (56)$$

The comparative statics regarding k follow the same direction as for the other penalty regimes. Furthermore, it is easy to see that both prices are non-linearly increasing in the toughness of the penalty regime $\forall \tau_r \in (0,1)$.[28] As the cartel reacts to the penalty regime by increasing its price, the fringe firms' best response is to raise its price as well. This leads to the first proposition for this penalty base.

Proposition 1_r: The revenue based penalty regime has a positive direct price effect on the cartel and the fringe.

From this it follows immediately that under this regime, a cartel comprising of all firms in the industry, i.e. $k = n$, sets a price above the monopoly level. Furthermore, given that the cartel price is increasing in τ_r, there is some maximum value of penalty toughness τ_r^{max} for which the cartel price is equal to the choke price. That is the price above which the cartel demand is equal to zero and therefore, the cartel would earn zero profits. Before computing the functional form of τ_r^{max} it is then necessary to define the functional form for the cartel quantity demanded:

$$q_r^c(k) = \frac{[1+\gamma(n-k-1)](1-\gamma)(1-\tau_r)[2+\gamma(2n-3)]}{(1-\tau_r)(1-\gamma)[1+\gamma(n-1)]\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big)} - c\frac{[1+\gamma(n-k-1)][2+\gamma\Big(2n-5+\tau_r(n-k)\Big)-\gamma^2[2n-3+\tau_r(n-k)(2-n)]}{(1-\tau_r)(1-\gamma)[1+\gamma(n-1)]\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big)},$$

[28]For the fringe price it is trivial to see that $\frac{\partial p_r^f}{\partial \tau_r} > 0$. For the proof that the cartel price is increasing in τ follows from the fact that

$$\frac{\partial p_r^c - p_r^f}{\partial \tau_r} = \frac{\Big([1+\gamma(n-k-1)][2+\gamma(n-3)]\Big)}{\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big)}c > 0.$$

Given that $p_r^c - p_r^f$ is increasing in τ_r, it must be that p_r^c increases more than p_r^f.

(57)

The maximum penalty toughness τ_r^{max} then follows from $q_r^c(k) = 0$ as

$$\tau_r^{max} = \frac{(1-\gamma)(1-c)[2+\gamma(2n-3)]}{(1-\gamma)[2+\gamma(2n-3)] + c\gamma(n-k)[1+\gamma(n-2)]}. \tag{58}$$

For all $\tau \geq \tau_r^{max}$ the cartel would earn zero profits and therefore wouldn't form. Hence, it is assumed in the definition of the prices, quantities and profits that $\tau < \tau_r^{max}$. The fringe quantity is given by

$$\begin{aligned} q_r^f(k) = & \frac{\Big(1+\gamma(n-2)\Big)(1-\gamma)(1-\tau_r)[2+\gamma(2n-k-2)]}{(1-\tau_r)(1-\gamma)[1+\gamma(n-1)]\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)] - \gamma^2(n-k)k\Big)} \\ & - c\frac{\Big(1+\gamma(n-2)\Big)\Big[2[1-\gamma^2(n-1)](1-\tau_r) + \gamma[2(n-2)(1-\tau_r) - k] + \gamma^2[k+\tau_r k(k-n)]\Big]}{(1-\tau_r)(1-\gamma)[1+\gamma(n-1)]\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)] - \gamma^2(n-k)k\Big)}, \end{aligned} \tag{59}$$

which is increasing in τ_r.[29] Comparing both optimal quantities to the no CA case, it follows that cartel sells less units when penalties are imposed on revenue while the fringe sells more. That is because for any increase in the penalty toughness τ_r, the fringe increases its price less than the cartel. This leads to some consumers substituting cartel products for fringe products.

The cartel profits follow as

$$\pi_r^c(k) = D\frac{(A-\tau_r B)^2}{1-\tau_r}, \tag{60}$$

where[30]

$$\begin{aligned} A &= (1-c)(1-\gamma)[2+\gamma(2n-3)] && > 0 \\ B &= \Big((1-\gamma)[2+\gamma(2n-3)] + cg(n-k)[1+\gamma(n-2)]\Big) && \geq A \\ D &= \frac{1+\gamma(n-2)}{(1-\gamma)[1+\gamma(n-1)]\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)] - \gamma^2(n-k)k\Big)^2} && > 0. \end{aligned} \tag{61}$$

[29]Follows directly from

$$\frac{\partial q_r^f}{\partial \tau_r} = \frac{c\gamma k[1+\gamma(n-k-1)][1+\gamma(n-2)]}{(1-\tau_r)^2(1-\gamma)[1+\gamma(n-1)]\big[2[1+\gamma(n-k-1)][2+\gamma(n+k-3)] - \gamma^2(n-k)k\big]} > 0$$

[30]$A \leq B$ follows from $c \in [0,1)$ and $cg(n-k)[1+\gamma(n-2)] \geq 0$.

The comparative statics regarding k are the same as before: larger cartels earn higher profits. Furthermore, an increase in the penalty toughness τ_r decreases cartel profits $\forall \tau_r < \tau_r^{max}$.[31] The fringe profits are given by

$$\pi_r^f(k) = D\Big(\frac{(1-\tau_r)E + \gamma c[1-\gamma+\gamma\tau_r(n-k)]k}{(1-\tau_r)}\Big)^2, \tag{62}$$

where D is defined as above and

$$E = (1-\gamma)[2+\gamma(2n-k-2)] - c\Big((1-\gamma)[2+\gamma(2n-1)]-\gamma\Big). \tag{63}$$

Contrary to the cartel, the fringe profits are increasing in the toughness τ_r. That is because for fringe firms both the quantity and the price are increasing in τ.

Finally, a firm inside the cartel that aims to deviate from the collusive agreement earns profits $\pi_o^d(k)$. Similarly to the previous cases, it is assumed that $\pi_o^d(k) > \pi^c(k)$ which means that collusion can only be sustained when the price setting game is played repeatedly. More specifically, collusion can be sustained if setting the collusive price is a Nash Equilibrium.

As discussed before, this is the case when the cartel firms play grim trigger strategies against defectors and when they value future profits high enough. Applying the profit functions to (27), the Stable Cartel condition in the revenue based penalty regime is then

$$\delta \geq \frac{\pi_r^d(k) - \pi_r^c(k)}{\pi_r^d(k) - \pi^*} := \delta_r^*. \tag{64}$$

For this to hold for any δ it must be that $\delta_r^* < 1$ which only holds as long as $\pi_r^c(k) > \pi^*$. From the partial derivative of $\pi_r^c(k)$ with respect to τ_r and the result that $\pi_r^c|_{\tau_r=\tau_r^{max}}=0$, it is clear that there will be some value of $\tau_r \in (0, \tau_r^{max})$ for which cartels are not profitable, compared to the competitive outcome. In these cases, a cartel could not sustain. This implies that when the penalty regime gets tougher firms need a higher discount factor. Therefore, some cartels, which could have been stable for lower toughness parameters, are unstable. Hence, the revenue based penalty regime has a deterrence effect.

Proposition 2_r: When penalties are tough enough, the revenue based penalty can deter collusion by making cartels unprofitable.

The condition in (64) also implies that for any profitable cartel size k, there is some

[31] Proof in Appendix C

Table 7: *Membership stable cartel sizes under the revenue based penalty regime.*

γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	τ_r
n	stable cartel size									
	–	–	–	3	3	3	3	3	3	0.015
3	–	–	–	–	3	3	3	3	3	0.03
	–	–	–	–	3	3	3	3	3	0.045
	–	–	–	3	4	4	4	4	4	0.015
4	–	–	–	–	4	4	4	4	4	0.03
	–	–	–	–	–	4	4	4	4	0.045
	–	–	–	3	4	5	5	5	5	0.015
5	–	–	–	–	–	5	5	5	5	0.03
	–	–	–	–	–	–	5	5	5	0.045
	–	–	–	–	–	–	7	7	7	0.015
7	–	–	–	–	–	–	–	7	7	0.03
	–	–	–	–	–	–	–	7	7	0.045
	–	–	–	–	–	–	–	9	9	0.015
9	–	–	–	–	–	–	–	9	9	0.03
	–	–	–	–	–	–	–	9	9	0.045

discount factor $\delta \geq \delta_r^*$ such that collusion is in fact sustainable. Therefore, any membership stable cartel size k_r^* which leads to a profitable cartel can be sustained for some firms. To find k_r^*, the inside and outside stability conditions are considered again. To begin with, the profit functions derived under the revenue based penalty regime are applied to the definition of inside stable cartels.

ISC: A cartel is said to be inside stable, if no cartel firm has an incentive to leave the cartel and join the fringe, i.e. when

$$\pi_r^c(k) \geq \pi_r^f(k-1) \quad \forall \quad k \geq 3 \tag{65}$$

and $\pi_r^c(2) \geq \pi^*$ for $k = 2$.

In this condition, the cartel profits on the left hand side are decreasing in the toughness parameter. At the same time, the fringe profits are increasing in τ_r. That is because price setting follows a game of strategic (imperfect) complements: when the cartel raises its price, the optimal reaction of the fringe firms is to raise their price too, but to a lesser extend. Therefore, the fringe firms increase the demand for their products when the penalty increases. Thus, the fringe sells more units to a higher price compared to the no CA case. For (65) this means that for any increase in τ_r, the LHS decreases while the RHS increases. This makes it less likely that the condition holds. Economically, this means that cartels earn less while fringe firms earn more. Therefore, it is less attractive to be inside a cartel. As the inside stability condition is fundamental in the existence of a cartel, this also suggests that the deterrence effect of the penalty is strong.

While inside stability is required for the existence of any cartels, the outside stability condition is fundamental in determining cartel size. Applying the profit functions to the definition of an outside stable cartel results in:

OSC: A cartel is said to be outside stable, if no fringe firm has an incentive to join the cartel. That is when

$$\pi_r^f(k) \geq \pi_r^c(k+1) \quad \forall \quad 2 \leq k \leq (n-1), \tag{66}$$

and for $k = n$ every cartel is outside stable.

The comparative statics regarding τ_r of this equation follow the same explanation as for the inside stability, but inverted. This means that when penalties get tougher, fringe profits on the LHS increase while cartel profits on the RHS decrease. This implies that it is less attractive to join a cartel. Therefore, the condition is more likely to be fulfilled for a given k.

To summarise, the revenue based penalty regime then makes cartels less inside and more outside stable, while at the same time having strong deterrence effects. The former effect is expected to result in weakly smaller cartel sizes and thereby in a weakly negative indirect price effect.

Proposition 3$_r$: The revenue based penalty regime has a weakly negative indirect price effect.

A numerical example of the indirect price effect is given in Table 7, which shows the membership stable cartel size k_r^* for $c = 0.25$ and the same (γ, n)-tuples that were used in previous examples. The toughness is set at three different levels $\tau_r = \{0.015, 0.03, 0.045\}$. This corresponds to penalty rates of 10,20 and 30 percent, assuming a detection rate of $\beta = 0.15$. While a penalty rate of 30 percent of affected sales is the suggested maximum penalty rate in the EU fining guidelines.[32] At the same time, the fining guidelines also specify a maximum penalty rate of 10 percent of annual worldwide turnover. As the model presented here doesn't take into account that a firm may produce other goods, it doesn't differentiate between overall revenue and affected revenue. Because of this, the numerical

[32]Compare *Guidelines on the method of setting fines imposed pursuant to Article 23(2)(a) of Regulation No 1/2003, Section B.21*

Table 8: Market prices under the revenue based penalty regime.

γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	t
n	stable cartel size									
	–	–	–	0.627	0.627	0.627	0.627	0.627	0.627	0.015
3	–	–	–	–	0.629	0.629	0.629	0.629	0.629	0.03
	–	–	–	–	0.631	0.631	0.631	0.631	0.631	0.045
	–	–	–	0.506	0.627	0.627	0.627	0.627	0.627	0.015
4	–	–	–	–	0.629	0.629	0.629	0.629	0.629	0.03
	–	–	–	–	–	0.631	0.631	0.631	0.631	0.045
	–	–	–	0.448	0.469	0.627	0.627	0.627	0.627	0.015
5	–	–	–	–	–	0.629	0.629	0.629	0.629	0.03
	–	–	–	–	–	–	0.631	0.631	0.631	0.045
	–	–	–	–	–	–	0.627	0.627	0.627	0.015
7	–	–	–	–	–	–	–	0.629	0.629	0.03
	–	–	–	–	–	–	–	0.631	0.631	0.045
	–	–	–	–	–	–	–	0.627	0.627	0.015
9	–	–	–	–	–	–	–	0.629	0.629	0.03
	–	–	–	–	–	–	–	0.631	0.631	0.045

tests are conducted for the two maximum rates 10 and 30 percent and the arbitrarily chosen middle value of 20 percent. The table shows that compared to the no CA case, the stable cartel size k_r^* is lower in three cases: $(\gamma, n) = \{(0.4, 4), (0.4, 5), (0.5, 5)\}$. For all other cases in which a cartel forms, the cartel size is equal to $k_r^* = n$. This is the same outcome as in the no CA case. These cases are highlighted in yellow. Furthermore, it is likely that no cartel can form when either n or τ_r are high or when γ is low.

The combination of a weakly negative indirect, with a direct positive price effect imply that the overall price effect of the revenue based penalty regime is ambiguous for the general case. Numerical testing suggests that in the three cases in which the indirect effect is strong enough and the cartel size decreases, this outweighs the positive direct price effect. Therefore, in these cases the overall price effect of the penalty is negative. However, when the indirect price effect is neutral, the direct price effect increases prices. To illustrate this, Table 8 shows the average market prices for the same parameters as Table 7. The displayed average market price is computed equivalently to the other penalty regimes: $\bar{p}_r = [k * p_r^c(k) + (n-k)p_r^f(k)]/n$. In the direct comparison to Table 2, one can see that the three cases with $k_r^* < k^*$ also have average market prices below the ones in the no penalty regime. When $k_r^* = n$ the cartel sets a price above the monopoly price of 0.625 in all cases. This leads to the following.

Result 1_r: The overall price effect of the revenue based penalty regime is ambiguous.

In those cases in which the indirect price effect is strictly negative the penalty leads to decreasing prices. In those cases in which the indirect price effect is neutral the penalty leads to higher market prices.

Overall, this section has provided three main findings. Firstly, the indirect and direct price effect of the penalty have different signs. While the direct effect increases the cartel price and can elevate it above the monopoly level, the indirect or size effect decrease the cartel price. Secondly, the combination of the two effects lead to an ambiguous overall effect of the penalty. Numerically it is shown that whenever the cartel size is effectively reduced, the price charged in the market is lower compared to the no CA case. However, when the industry size remains at the same level as in the no CA case, the price charged under revenue based penalties is higher. Finally, the regime has a strong deterrence effect as it decreases cartel and increases fringe profits. Therefore, joining a collusive agreement or staying inside one becomes less attractive. A comparison to the other penalty regimes is undertaken in the next section.

3 Penalty Comparison

In this section, the size and overall price effect of the profits, overcharge and revenue based penalty regimes are compared. To do so, the results regarding price effects of the three regimes are considered again. The results of the previous sections suggested that relative to the prices that were computed in the absence of a CA,

- profit based penalties result in weakly lower prices;
- overcharge based penalties result in strictly lower prices; and
- revenue based penalties can result in higher or in lower prices.

This could imply that out of the three penalty regimes discussed, the overcharge based penalties lead to the most favourable outcomes. However, this approach lacks in substance for two reasons. Firstly, it doesn't take into account that the penalties have different deterrence effects. For example, from the cases considered before, one might come to the conclusion that while the overcharge based penalty regime leads to the lowest average prices, its deterrence effect appeared to be much lower than those of the revenue based penalty regime. This would follow from comparing Table 6 and Table 8. For the toughness parameters tested, the overcharge based penalty regime only deters all cartels

Table 9: Membership stable cartel sizes under penalty regimes deterring all cartels over $\gamma \leq 0.3$.

γ	0.301	0.4	0.5	0.6	0.7	0.8	0.9	
n			**Stable cartel size**					$\tau_i,\ i=\{r,\pi,o\}$
	2	3	3	3	3	3	3	$\tau_r = 0.0146$
3	2	3	3	3	3	3	3	$\tau_\pi = 0.0261$
	3	3	3	3	3	3	3	$\tau_o = 0.4165$
	3	3	4	4	4	4	4	$\tau_r = 0.0124$
4	3	3	4	4	4	4	4	$\tau_\pi = 0.0251$
	4	4	4	4	4	4	4	$\tau_o = 0.5625$
	3	4	4	5	5	5	5	$\tau_r = 0.0103$
5	3	4	4	5	5	5	5	$\tau_\pi = 0.0223$
	5	5	5	5	5	5	5	$\tau_o = 0.6316$
	3	4	5	5	7	7	7	$\tau_r = 0.0066$
7	3	4	5	5	7	7	7	$\tau_\pi = 0.0162$
	7	7	7	7	7	7	7	$\tau_o = 0.7200$
	3	4	4	4	4	9	9	$\tau_r = 0.0044$
9	3	4	5	5	6	6 *	9	$\tau_\pi = 0.0119$
	9	9	9	8	8	8	9	$\tau_o = 0.7742$

* *a cartel with $k = n$ is also membership stable.*

for three combinations of (γ, n, τ_o), while the revenue based penalty deters cartels over almost half the analysed range. However, it is possible that in the example provided, the only reason the overcharge based regime deters less cartels is because the toughness is lower relative to the revenue based penalties analysed. This leads to the next reason: the comparison doesn't take into account the penalty's relative toughness. How can a penalty rate of 10 percent of profits be compared to a penalty rate of 50 percent of quantity adjusted overcharges? Without a measure to make penalties equally tough in some sense, there is no way of ensuring comparability.

This issue is addressed at depth by Katsoulacos et al. (2015), who use two methods to equalise penalties. Firstly, they set the toughness in each regime such that it generates the same average penalty payments by cartels. Secondly, they set penalties such that they deter the same fraction of cartels in each regime. Under both equivalence rules, the authors find that the overcharge based penalties lead to the lowest average prices, followed by profit based penalties. Revenue based penalties lead to the highest average prices.

The method used in this paper to ensure that penalties are equally tough is inspired by the deterrence equivalence rule used by Katsoulacos et al. (2015). Similar to their paper, penalties are set such that they deter the same fraction of cartels. However, the approach differs in the following way. In their paper, the fraction of deterred firms is

Table 10: *Average market prices under penalty regimes deterring all cartels over $\gamma \leq 0.3$.*

γ	0.301	0.4	0.5	0.6	0.7	0.8	0.9	
n	**Average market price**							τ_i, $i = \{r, \pi, o\}$
3	0.5440	0.6269	0.6269	0.6269	0.6269	0.6269	0.6269	$\tau_r = 0.0146$
	0.5425	0.6250	0.6250	0.6250	0.6250	0.6250	0.6250	$\tau_\pi = 0.0261$
	0.5124	0.5038	0.4952	0.4865	0.4779	0.4692	0.4606	$\tau_o = 0.4165$
4	0.5379	0.5059	0.6266	0.6266	0.6266	0.6266	0.6266	$\tau_r = 0.0124$
	0.5366	0.5045	0.6250	0.6250	0.6250	0.6250	0.6250	$\tau_\pi = 0.0251$
	0.4781	0.4668	0.4563	0.4465	0.4375	0.4291	0.4213	$\tau_o = 0.5625$
5	0.4873	0.5027	0.6263	0.6263	0.6263	0.6263	0.6263	$\tau_r = 0.0103$
	0.4863	0.5015	0.4667	0.6250	0.6250	0.6250	0.6250	$\tau_\pi = 0.0223$
	0.4518	0.4389	0.4276	0.4178	0.4091	0.4013	0.3944	$\tau_o = 0.6316$
7	0.4295	0.4083	0.4036	0.3691	0.6258	0.6258	0.6258	$\tau_r = 0.0066$
	0.4290	0.4077	0.4028	0.3682	0.6250	0.6250	0.6250	$\tau_\pi = 0.0162$
	0.4139	0.4000	0.3888	0.3795	0.3719	0.3654	0.3598	$\tau_o = 0.7200$
9**	0.3962	0.3682	0.3382	0.3139	0.2939	0.6255	0.6255	$\tau_r = 0.0044$
	0.3959	0.3679	0.3494	0.3228	0.3120	0.2889 *	0.6250	$\tau_\pi = 0.0119$
	0.3880	0.3743	0.3637	0.3087	0.2915	0.2784	0.3386	$\tau_o = 0.7742$

* *$k = n$ is also a stable cartel size which leads to an market price of* 0.625.
** *No stable cartel in the revenue based penalty regime for $n = 9$ and* $\gamma \in (0.701218, 0.735432)$

defined as those with the same intrinsic difficulty of holding the cartel together. This measure consolidates information about the cartels' critical discount factor and market size. In this paper, the level of deterrence is measured by the penalties' ability to deter products over the same range of product substitutability γ. In particular, consider all the penalty regimes again and set the penalty for each market size n such that no cartels can form over products with $\gamma \leq 0.3$. This way, the penalties are equivalent in the sense that they all deter a same group of cartels. Differences in average prices can then be interpreted more easily. Table 9 shows the stable cartel sizes that follow under this deterrence equivalence for each of the three discussed penalty regimes. Levels of $\gamma \leq 0.3$ are not displayed, as for these values there are no cartels. All membership stable cartels are also stable cartels. The marginal costs are equal to $c = 0.25$. All the cases where $k_i^* = n$, $i = \{r, \pi, o\}$ are highlighted in yellow. This is especially likely when products are relatively homogeneous. For more differentiated goods, overcharge based penalties lead to larger cartels than the other two regimes. Comparing the profits and the revenue based regime, one can see that out of 15 cases in which $k < n$, 11 lead to the same industry size. In the remaining 4 cases, revenue based penalties result in a smaller cartel. In all of these cases, the stable cartel size under the overcharge based penalty regime are the largest. The only case in which the overcharge based penalty regime has the smallest cartel is $(\gamma, n) = (0.8, 9)$.

Following the stable cartel sizes, it is possible to compute the average market prices. These are depicted in Table 10. Again, the cases in which the full market colludes are shown in yellow. If that is the case for all three penalty regimes, it is particularly easy to compare the direct price effects. In all of these cases there is clear price ranking of $p_r^c > p_\pi^c > p_o^c$. This also follows directly from the direct price results established in the forgone sections. Comparing the cases in which $k < n$ for at least one of the penalty regimes shows that mostly the overcharge based penalty leads to the smallest price. That is true in 12 out of 16 of these cases. Taking into account that at the same time, overcharge based penalties often lead to larger cartels, this demonstrates the penalty's strong negative direct price effect. Furthermore it can be observed that for most parameters, the revenue based penalty regime leads to higher prices than the profit based regime. Out of the cases in which $n < k$ that is true for 12 out of 16 tuples and is only not true in those cases in which $k_r^* < k_\pi^*$. Therefore, apart from few exceptions, the order of the prices can be observed as $\bar{p}_r > \bar{p}_\pi > \bar{p}_o$. This would render the overcharge based penalties as the most favourable of the three.

However, a comparison of the toughness levels required to achieve the desired deterrence effect shows that τ_o is by far the highest of the three for all market sizes. Assuming a detection probability of $\beta = 0.15$, the penalty rate ranges from 278 to 516 percent of the penalty base $(p_o^c - p^*)q^*$. Under US law it is not possible to impose penalties of more than 200 percent of the illegal gains from collusion. As discussed in the section on overcharge based penalties, the penalty base $(p_o^c - p^*)q^*$ is strictly above these direct gains from collusion. Therefore, a penalty rate of 278 or even 516 percent would not be possible under the current sentencing guidelines. At the same time, the penalty rate under the revenue based regime requires a penalty rate between 2.9 and 9.7 percent of revenues, a range within the limits of maximum penalties in the EU of 10 percent of worldwide turnover.

Overall, this section has provided a first structured comparison of the profits, overcharge and revenue based penalty regimes. It was seen that when the three systems deter cartels over products with $\gamma \leq 0.3$, the overcharge based penalty regime leads to the lowest average prices.[33] With few exceptions, the second lowest price is observed under profit based penalties followed by revenue based penalties. This confirms the relative order of prices under deterrence equivalence found by Katsoulacos et al. (2015). However, as the deterrence effects of overcharge based penalties are weak compared to the other

[33]The results are expected to be robust, based on tests for random (γ, n)-tuples for other arbitrary deterrence levels than $\gamma \leq 0.3$.

two regimes, it was observed that the penalty rate necessary to achieve the same level of deterrence is significantly above those of the other regimes. They are also outside the limits set under current US law.

4 Conclusion

This paper analyses how the imposition of three different penalty regimes (profits, overcharge and revenue based penalties) affect the price setting of cartels differentiated goods markets. In an infinitely repeated price setting game in which firms compete over true substitutes, it is shown that membership stable and stable cartels can exists and that some of them may not contain all firms in the market. A membership stable cartel is one in which no firm has an incentive to either join or leave the cartel, given that the other firms have the chance to adjust their prices. In a stable cartel no firm has an incentive to cheat on the collusive agreement. It is shown that any cartel which is profitable compared to the competitive equilibrium in the market is stable. This implies that any membership stable cartel is also stable, given that firms value future profits high enough.

If there is no law enforcement, it is shown that every market as characterised by its size and degree of product substitutability has some stable cartel size. Furthermore, it was found that small cartels, i.e. those which consist of only a subgroup of all firms of the market, can exist in some markets. This can be observed especially when either products are relatively differentiated or when the number of firms in a market gets large. Full market collusion is more likely to be observed when products are closer substitutes or the industry is small. An additional finding is that, given the degree of product substitutability, the prices and profits in the market are increasing in the cartel size. Thus, smaller cartels charge lower prices.

In a second step, a CA which detects and fines cartels with some probability is introduced. Fringe firms are not prosecuted. Given a cartel is detected, a penalty is imposed on some pre-defined base. Three different bases are differentiated: profits, overcharge and revenues. It is found that penalty regimes influence prices through two channels which are compared in its relative strength. Firstly, the direct price effect is the price reaction to the penalty for a given cartel size. Secondly, the indirect effect is the change in price that is due to changes the stable cartel size incentivised by the penalty. Furthermore, it is shown that all penalties can render collusion unprofitable when the penalty rate is high enough. This means that all penalties have a deterrence effect.

In the profit based penalty regime it is shown that the direct price effect is neutral. As the maximisation problem of the cartel firms after the introduction of the penalties is a positive monotonic transformation of the maximisation problem without penalties, the optimal prices charged for any given cartel size do not change. However, the penalty has a weakly negative indirect price effect because it decreases the incentive to stay inside the cartel which can decrease the cartel size. As cartel and fringe price are decreasing in the number of firms in the industry, this means that profit based penalties can decrease the overall price charged in the industry.

When penalties are based on overcharge, the direct price effect is negative. The penalty targets directly the price charged by the cartel relative to the competitive price level. This disincentivises the cartel to raise the price by a large margin. The indirect effect is ambiguous, compared to the no penalty case. The overall effect is found to be negative when compared to the no penalty case. However, it is shown that there are some cases in which increasing the penalty toughness can also increase the cartel price. This effect does not elevate the prices above the no penalty cartel prices.

It is then shown that the revenue based penalty regime has a positive direct price effect. That is because it increases the marginal revenue of raising the cartel price. To counter this effect, the colluding firms set higher prices. At the same time, collusion is made less attractive as firms outside of the cartel are able to secure some additional demand. This leads to a weakly negative indirect price effect. Therefore, the total price effect of revenue based penalties is ambiguous. In those cases in which the indirect effect is neutral the penalty leads to higher prices. When the indirect effect is strictly negative, the overall effect is negative too.

To compare the overall price effects of the penalty regimes, they are made equally tough in the sense that they deter cartels over all products with $\gamma \leq 0.3$. Under the deterrence equivalent penalties, it is observed that the overcharge based penalties lead to the lowest average prices. For most parameters tested, the revenue based penalty regime leads to the highest prices. Some exceptions are found when the indirect effect outweighs the direct effect of the revenue based penalties significantly. In these few cases, the profit based penalty regime leads to the highest prices. It is furthermore found that the deterrence effect of the overcharge based penalty regime is a lot smaller than that of the other two regimes. Therefore, overcharge based penalties require much higher penalty rates than are currently permitted under US law to achieve a level of deterrence that can be reached with revenue based penalties in line with current sentencing guidelines.

The comparison with the existing literature shows that this is the first paper which analyses the price effect of different penalty regimes in differentiated markets, taking into account endogenous cartel formation. Furthermore, some of the previously found evidence on cartel behaviour was confirmed. As this paper uses the same underlying model to Posada (2000), who doesn't include penalties in the analysis, it is not surprising that the results on comparative statics of prices and profits regarding cartel sizes carry through. This is independent of the penalty regime in the market. Furthermore, in line with the findings of Bos and Harrington (2015), who show that endogenous cartel size may change when penalties are imposed, it was seen that cartel size is influenced by the penalty regime under which firms operate and the direction of the penalty effect was described.

Finally, while Katsoulacos et al. (2015) conduct the same general comparison of penalty regimes, their model is based on perfectly homogeneous goods under Bertrand competition which implies that any cartel that forms will consist of all firms in the market. Therefore, indirect price effects are excluded from their analysis. In the comparison to this paper, the results on direct price effect are confirmed. However, taking the indirect effects into account it was seen in this paper that there are cases in which both profit based and revenue based penalties can reduce the price charged in the market. This cannot happen in their paper. When the penalties in this paper are deterrence equivalent, Katsoulacos et al. (2015) 's result that overcharge based penalties lead to the lowest prices, followed by profits and revenue based penalties can be confirmed in most cases. However, there are very few exceptions in which profit based penalties lead to the highest price in the market. Furthermore, Katsoulacos et al. (2016) have undertaken the same analysis with a penalty based on revenues but where the penalty rate is a function of the cartel's overcharge. This comparison is not included in this paper, but is a worthwhile subject of future research.

Another area of potential future research is to further compare the penalty regimes under a different set of equivalence rule to make sure penalties are equally tough without violating current penalty maxima.

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5 Appendix A: No Law Enforcement

5.1 Proof that denominator of pricing function is positive

The denominator is equal to

$$\begin{aligned}&2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\\=&4+2g(3n-k-5)+2g^2[(n-k-1)(n-3)-k]+\gamma^2(n-k)k\end{aligned} \tag{67}$$

which is a negatively signed quadratic function of k with a maximum at $k=\frac{\gamma(4-n)-2}{2\gamma}<0$ $\forall n\geq 3$ and hence is decreasing in k. Therefore, the function will be at its lowest when k is the highest value possible. For $k=(n-1)$, the denominator takes the value

$$4[1+\gamma(n-2)]-\gamma^2(n-1)>0\ \forall n\geq 2 \text{ and } \gamma\in(0,1) \tag{68}$$

5.2 Proof that prices increase in cartel size

The first order derivative of the cartel price with respect to the cartel size is given by

$$\frac{\partial p^c}{\partial k}=\frac{(1-c)(1-\gamma)\gamma[2+\gamma(n+2k-4)][2+\gamma(2n-3)]}{(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k)^2}>0, \tag{69}$$

where the denominator is always positive. As $n\geq k\geq 2$, and c as well as γ positive but below one, the derivative is positive.

The first order derivative of the fringe price with respect to the cartel size is given by

$$\frac{\partial p^f}{\partial k}=\frac{(1-c)(1-\gamma)\gamma^2[4k-2+\gamma\Big[(4k-2)(n-1)-k^2\Big]}{(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k)^2}>0, \tag{70}$$

where the denominator is always positive. The only way the numerator can be negative is, when k grows and the negative squared cartel size outweighs the rest. The highest value of k for which the function is defined is $k=(n-1)$. For this cartel size, the numerator is still positive. Hence, overall both cartel and fringe price are increasing in k.

5.3 Proof that profits increase in cartel size

The first order derivative of the cartel profits with respect to the cartel size is given by

$$\frac{\partial \pi^c}{\partial k}=\frac{(1-c)^2(1-\gamma)\gamma^2[2+\gamma(2n-3)]^2[2(k-1)+\gamma(3k[n-k]-2[n-1])]}{[1+\gamma(n-1)](2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k)^3}>0, \tag{71}$$

where the only term that could make this negative is the last term of the numerator $[2(k-1)+\gamma(3k[n-k]-2[n-1])]$. This term is at its lowest when the second part of the term is the most negative. That is when $(3k[n-k]-2[n-1]) = -2[n-1] < 0$, i.e. when $n=k$. For this, the whole term is positive though $[2(k-1)+\gamma(3k[n-k]-2[n-1])]|_{n=k}=$ $(2n-1)-\gamma(2n-1) > 0$ because $\gamma \in (0,1)$. Therefore $\frac{\partial \pi^c}{\partial k} > 0$ is true.

The first order derivative of the fringe profits with respect to the cartel size is given by

$$\begin{aligned}\frac{\partial \pi^f}{\partial k} &= \frac{2(1-c)^2(1-\gamma)\gamma^2[1+\gamma(n-2)]\{4(2k-1)+\gamma[4(k-1)(n-1)+3k(n-k)-k(n-1)]}{[1+\gamma(n-1)](2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k)^3} \\ &+ \frac{2(1-c)^2(1-\gamma)\gamma^4[1+\gamma(n-2)][(n-1)(6k^2-2k[4n-3]+4)-k^3]}{[1+\gamma(n-1)](2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k)^3}\end{aligned} \tag{72}$$

which is suggested to be positive by numerical tests.

5.4 Proof of ordering of prices

It is to be shown that $p^c(k) > p^f(k) > p^*$. Start with showing that $p^c(k) > p^f(k)$. From the prices it follows directly that

$$\begin{aligned}p^c(k) - p^f(k) &= \frac{(1-c)(1-\gamma)\gamma(k-1)}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k} \\ &> 0 \;\; \forall \;\; 2 \leq k \leq (n-1)\end{aligned} \tag{73}$$

because both numerator and denominator are always positive for these values of k and n.

Now show that $p^f(k) > p^*$.

$$p^f(k) - p^* = \frac{(1-c)(1-\gamma)\gamma^2(k-1)k}{[2+\gamma(n-3)][2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k]} > 0, \tag{74}$$

because the numerator is clearly positive $\forall k > 2$ and the denominator is clearly positive $\forall n \geq 2$.

5.5 Proof of ordering of profits

The order of the profits is

$$\pi^f(k) > \pi^c(k) > \pi^*. \tag{75}$$

Firstly, $\pi^c(k) > \pi^*$ is shown. As it is known that $\pi^c(k)$ is increasing in k, this inequality only has to be shown for $k = 2$. For all $k > 2$ it will then also hold.

$$\pi^c(2) - \pi^* = \frac{(1-c)^2(1-\gamma)\gamma^2[4+16\gamma(n-2)+\gamma^2(17n^2-70n+69)+\gamma^3(5n^3-33n^2+67n-43)]}{4[2+\gamma(n-3)]^2[1+\gamma(n-1)][2+\gamma(3n-7)+\gamma^2(n^2-5n+5)]^2} \tag{76}$$

Where all parts of this fraction are weakly positive for $n \geq 2$ apart from $\gamma^2(17n^2-70n+69)$ which is only positive for $n \geq 3$ and $\gamma^3(5n^3-33n^2+67n-43)]$ which is only positive for $n \geq 4$. This means that for all $n \geq 4$, all parts of the fraction are positive and therefore $\pi^c(k) - \pi^* > 0$ for $n \geq 4$. It then has to be shown that $\pi^c(2) - \pi^* > 0$ for $n = 2$ and $n = 3$. For $n = 2$ and $k = 2$ it follows that

$$\pi^c - \pi^* = \frac{(1-c)^2\gamma^2}{4(2-\gamma)^2(1+\gamma)} > 0 \tag{77}$$

For $n = 3$ and $k = 2$ it follows that

$$\pi^c - \pi^* = \frac{(1-c)^2(1-\gamma)\gamma^2[1-\gamma^3+4\gamma+3\gamma^2]}{4[1+2\gamma][2(1-\gamma)-\gamma^2]^2} > 0 \tag{78}$$

Hence, $\pi^c(k) > \pi^*$.

Secondly, $\pi^f(k) > \pi^c(k)$ is shown.

$$\pi^f(k) - \pi^c(k) = \frac{(1-c)^2(1-\gamma)(k-1)[k+1+\gamma(n-1+k(n-2))]}{[1+\gamma(n-1)][2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k]^2} > 0 \forall n \geq k \geq 2 \tag{79}$$

Hence, $\pi^f(k) > \pi^c(k)$ holds.

6 Appendix B: Overcharge Based Penalties

6.1 Proof that price effect of the penalty is negative

The direct price effect for cartel and fringe are given by $[-\tau_o\tilde{p}^i(k)]$, $i \in \{c, f\}$ where

$$\tilde{p}^c_o(k) = q^* \frac{(1-\gamma)[2+\gamma(n+k-3)][1+\gamma(n-1)]}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k}$$

and (80)

$$\tilde{p}^f_o(k) = q^* \frac{k\gamma(1-\gamma)[1+\gamma(n-1)]}{2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k}.$$

For both cases, the denominator is positive as shown in Appendix A. Therefore, the price effects $[-\tau_o \tilde{p}^i(k)]$, $i \in \{c, f\}$ are negative if the numerators are positive. In the cartel case,

$$(1-\gamma)[2+\gamma(n+k-3)][1+\gamma(n-1)] > 0 \ \forall 2 \leq k \leq n \tag{81}$$

is trivial. In the fringe case,

$$k\gamma(1-\gamma)[1+\gamma(n-1)] > 0 \ \forall 2 \leq k \leq (n-1) \tag{82}$$

is trivial too. Therefore, both direct price effects are strictly negative.

6.2 Proof that cartel price effect is stronger than fringe price effect

The claim is that $\mid -\tau_o \tilde{p}_o^c(k) \mid > \mid -\tau_o \tilde{p}_o^f(k) \mid$. As both $\tilde{p}_o^c(k)$ and $\tilde{p}_o^f(k)$ are positive as shown in the above proof, the equation holds when $\tilde{p}_o^c(k) > \tilde{p}_o^f(k)$. Therefore

$$\begin{aligned} & \tilde{p}_o^c(k) > \tilde{p}_o^f(k) \\ \iff & [2+\gamma(n+k-3)] > k\gamma \\ \iff & 2+\gamma(n-3) > 0 \end{aligned} \tag{83}$$

which always holds. for $n \geq 2$.

6.3 Proof that cartel quantity effect is positive

$$\begin{aligned} \tilde{q}_o^c &= \left(\tilde{p}_o^c \frac{(1+\gamma(n-k-1)}{(1-\gamma)[1+\gamma(n-1)]} - \tilde{p}_o^f \frac{(n-k)\gamma}{(1-\gamma)[1+\gamma(n-1)]}\right) \\ &= \frac{(1-c)[1+\gamma(n-2)]\left(2+\gamma(3n-k-5)+\gamma^2[n^2-4n-k(n-2)+3]\right)}{[2+\gamma(n-3)][1+\gamma(n-1)]\left(2(1+\gamma*(n-k-1))(2+\gamma(n+k-3))-(\gamma^2)*(n-k)k\right)} \end{aligned} \tag{84}$$

which is positive for all $2 \leq k \leq k$ and $\gamma \in (0,1)$, $c \in (0,1)$.

6.4 Proof that fringe quantity effect is negative

The quantity effect $-T\tilde{q}_o^f$ is negative if $\tilde{q}_o^f > 0$.

$$\begin{aligned}\tilde{q}_o^f &= \Big(\frac{k\gamma\tilde{p}_o^c}{(1-\gamma)[1+\gamma(n-1)]} - \frac{[1+\gamma(k-1)]\tilde{p}_o^f}{(1-\gamma)[1+\gamma(n-1)]}\Big) \\ &= \frac{(1-c)\gamma k[1+\gamma(n-2)]^2}{[2+\gamma(n-3)][1+\gamma(n-1)]\Big(2(1+\gamma*(n-k-1))(2+\gamma(n+k-3))-(\gamma^2)*(n-k)k\Big)} \\ &> 0\end{aligned} \tag{85}$$

6.5 Proof that fringe quantity exceeds cartel quantity

$$q_o^f(k) - q_o^c(k) = \frac{(1-c)\Big(\gamma[k-1]-\tau_o[1+\gamma(n-2)]\Big)}{\Big(2(1+\gamma*(n-k-1))(2+\gamma(n+k-3))-(\gamma^2)*(n-k)k\Big)} \tag{86}$$

Where the denominator is positive and hence the equation is positive as long as the numerator is positive. Solving $(1-c)\Big(\gamma[k-1]-\tau_o[1+\gamma(n-2)]\Big) > 0$ for τ_o leads to

$$\tau_o < \frac{\gamma(k-1)}{1+\gamma(n-2)} = \tau_o^{max}. \tag{87}$$

Therefore $q_o^f(k) - q_o^c(k) > 0 \ \forall \ \tau_o < \tau_o^{max}$. For $\tau_o \geq \tau_o^{max}$, the cartel wouldn't form. Hence, for all cases in which a cartel forms, the fringe quantity is above the cartel quantity.

6.6 Proof that cartel profits are decreasing in penalty toughness

The cartel profits are given by

$$\pi_o^c(k) = \pi^c - \tau_o\Big([p^c-p^*]q^* + \tilde{q}_o^c q^c - [p^c-c]\tilde{q}_O^c\Big) + \tau_o^2\tilde{q}_o^c(q^*-\tilde{q}_o^c), \tag{88}$$

which is a U-shaped quadratic function of τ_o because the competitive quantity is larger than the cartel's quantity effect:

$$q^* - \tilde{q}_o^c = \frac{(1-c)[1+\gamma(n-2)][2+\gamma(3n-k-5)]+\gamma^2[(n-k(n+k)-2(n-k)-2n+3]}{[2+\gamma(n-3)][1+\gamma(n-1)]\Big(2[1+\gamma*(n-k-1)][2+\gamma(n+k-3)]-\gamma^2*(n-k)k} \tag{89}$$

where the denominator is positive and the numerator is strictly decreasing in k. For the highest possible cartel size $k = n$, the expression becomes

$$q^* - \tilde{q}_o^c = \frac{(1-c)([1+\gamma(n-2)]}{2[2+\gamma*(n-3)][1+\gamma(n-1)]} > 0. \tag{90}$$

This means that the profit function $\pi_o^c(k)$ is decreasing for low values of τ_o until the function is at a minimum after which it increases again. Evaluating $\frac{\partial \pi_o^c}{\partial \tau_0}$ for $\tau_o = \tau_o^{max}$ shows that the derivative at this point is negative. Therefore, the function is also decreasing $\forall$ $\tau_o < \tau_o^{max}$:

$$\frac{\partial \pi_o^c}{\partial \tau_0}|_{\tau_o=\tau_o^{max}} = \frac{(-1)(1-c)^2(1-\gamma)\gamma k[1+\gamma(n-2)](n-k)}{[2+\gamma(n-3)][1+\gamma(n-1)]\Big(2[1+\gamma*(n-k-1)][2+\gamma(n+k-3)]-\gamma^2*(n-k)k\Big)}, \tag{91}$$

which is clearly smaller than zero $\forall 2 \leq k \leq n$.

7 Appendix C: Revenue Based Penalties

7.1 Proof that profits are decreasing penalty toughness

The revenue based penalties can be written as

$$\pi_r^c = U\frac{(A-\tau_r B)^2}{(1-\tau_r)}, \tag{92}$$

where

$$\begin{aligned} A &= (1-c)(1-\gamma)[2+\gamma(2n-3)] && > 0 \\ B &= \Big((1-\gamma)[2+\gamma(2n-3)] + c\gamma(n-k)[1+\gamma(n-2)]\Big) && \geq A \\ U &= \frac{1+\gamma(n-2)}{(1-\gamma)[1+\gamma(n-1)]\Big(2[1+\gamma(n-k-1)][2+\gamma(n+k-3)]-\gamma^2(n-k)k\Big)^2} && > 0 \end{aligned} \tag{93}$$

where $A \leq B$ follows from $c \in [0,1)$ and $cg(n-k)[1+\gamma(n-2)] \geq 0$. After some rearranging, the first order partial derivative of π_r^c w.r.t. τ_r gives

$$\begin{aligned} \frac{\partial \pi_r^c}{\partial \tau_r} &= U\frac{(A-\tau_r B)^2 - 2B(A-\tau_r B)(1-\tau_r)}{(1-\tau_r)^2} \\ &= U\frac{(A^2-\tau_r^2 B^2) - 2B(A-\tau_r B)}{(1-\tau_r)^2} \end{aligned} \tag{94}$$

where the denominator of the fraction is always positive. This means, the derivative is only negative when $(A^2 - \tau_r^2 B^2) - 2B(A - \tau_r B) < 0$. Taking into account that a cartel only forms if $\tau_r < \tau_r^{max} = A/B$.[34] Therefore $(A - \tau_r B) > 0 \ \forall \tau_r (0, \tau_r^{max})$. From this it follows that

$$\begin{aligned} &(A^2 - \tau_r^2 B^2) < 2B(A - \tau_r B) \\ \iff &(A + \tau_r B) < 2B, \end{aligned} \tag{95}$$

where the left hand side strictly increasing in τ_r. For the highest value of $\tau_r = \tau_r^{max}$ the inequality becomes $2A < 2B$ which holds because $A < B$. Therefore, $\frac{\partial \pi_r^c}{\partial \tau_r} < 0 \ \forall \tau_r \in (0, \tau_r^{max})$.

[34]As defined in Eq. 58, the maximum penalty toughness is $\tau_r^{max} = \frac{(1-\gamma)(1-c)[2+\gamma(2n-3)]}{(1-\gamma)[2+\gamma(2n-3)]+c\gamma(n-k)[1+\gamma(n-2)]} = A/B$.

Collusion, Market Entry and Mergers

Jonas Kalb[1]

PhD Thesis, Chapter 3

Abstract

The majority of traditional models analysing collusive stability take the number of firms in the industry as given. The models which do allow for a change in the number of firms mainly assume that this happens through entry. If free entry is possible a common argument is that collusion is either unstable because any collusive equilibrium would be broken up through additional entry (e.g. Ivaldi et al. (2003)), or that a cartel has to play entry deterring strategies (e.g. Wenders (1971)). This paper contributes to the existing literature by providing a structured analysis of how a cartel's expectation about the competitive dynamic forces of entry, exit, and merger affect the stability of collusive agreements. Contrary to previous literature, it is found that even though entry is free, stable cartels can form without the need for entry deterring actions. Furthermore, the model mimics empirical evidence that the break up of cartels can be followed by mergers and that mergers can lead to collusive equilibria. A further contribution of this paper is a new notion of a long run sustainable market size under merger and entry, in which the entry force is equal to the merger force.

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Contents

1 Introduction

Standard models on the collusive behaviour of firms are set in a static environment in the sense that the number of firms in the market doesn't change. In these industries, firms make decisions about forming a cartel or colluding tacitly, which follow the basic logic of a Prisoner's Dilemma: while coordinated conduct would lead to high profits, each participant has an individual incentive to deviate and secure high profits. As a result, collusion is thought to be inherently unstable in a one-shot game. However, given that interaction between firms is repeated and if firms value future profits high enough, it is possible to sustain a collusive agreement as a Nash Equilibrium. Some examples for literature showing this is Friedman (1971) for infinitely repeated games, and for finitely repeated games Benoit and Krishna (1987) and Harrington (1987). A standard implication from this is that because firms are better off colluding than competing against each other, there will always be some valuation of future profits for which collusion can be sustained for any given number of firms in the industry. However, the question of how the number of firms in the industry is determined has received relatively little attention.

It has been acknowledged that increased industry profits through collusive conduct may trigger entry, which would lessen the firms' incentive or ability to collude. For example, Ivaldi et al. (2003) argue that "the prospect of future entry tends to (...) [limit] the sustainability of collusion"(p.16). Therefore, models directly concerned with entry and collusion often consider the question of how firms in the industry can stop firms from the outside to enter, rather than analysing how a change in the number of firms would alter collusive incentives, e.g. Wenders (1971) or MacLeod and Norman (1987). An exception is given by Brander and Spencer (1985) who argue that partial tacit collusion implies higher profits which may trigger entry of firms which are directly included into the collusive equilibrium. In their model, entry occurs until all firms earn zero profits. A more common way of modelling collusion is then to exclude the possibility of entry altogether by either explicitly or implicitly focussing on industries which are characterised by barriers to entry. This is reflected in Ivaldi et al. (2003) who argue that "collusion cannot be sustained in the absence of entry barriers"(p.19). At the same time, market entry and cartelisation of an industry are not distinct phenomena, as described in Connor (2001) who recount the entry of ADM into the lysine industry and the cartel which formed in it in the early 1990s. Similarly, empirical evidence presented by Hyytinen et al. (2013) suggests that cartels are concerned with entry. The authors analyse legal cartel agreements from a

Finnish data set and shows that considerable parts of some of the contracts are dedicated to the question of how to deal with firms who entered the market.

At the same time, it is an accepted notion that a decrease in the number of firms either through exit or merger can facilitate collusion. There is a variety of arguments for this. For example, Compte and Jehiel (2002) argue that it is harder to agree on a collusive equilibrium when there are more firms to include. Additionally, Bos and Harrington (2015) argue that smaller cartels face a lower probability of prosecution through a competition authority. Finally, Katsoulacos et al. (2015) define the 'intrinsic difficulty of holding the cartel together" (p.71) as a combination of the discount factor and the cartel size. In this model, for a given discount factor the difficulty of sustaining collusion increases in the number of firms. While none of these authors explicitly model a reduction of the number of firms through exit or merger, the idea that few firms are more likely to collude is a one of the main drivers in merger legislation. Therefore, a crucial part of the analysis of a request to merge is to evaluate whether market consolidation may facilitate collusion in the future. This is reflected in the horizontal merger guidelines of many competition authorities. For example, the EU Commission say under section 22(b) that "...horizontal mergers may significantly impede effective competition(...) by changing the nature of competition in such a way that firms that previously were not coordinating their behaviour, are now significantly more likely to coordinate and raise prices or otherwise harm effective competition."(EC (2004)). The DoJ in the USA states similar concerns.[2] If collusion in the future is seen to be a likely result of a merger, it would be blocked, which means that the empirical evidence linking the two phenomena is limited.[3]

An additional recurring topic in the discussion of industry dynamics and collusion is: is a merger the result of failed attempts of collusion? The idea behind this is twofold. On the one hand, both practices can be seen of serving the same purpose: to decrease competition in a market and raise profits. If then firms in a given industry find collusion is too hard to achieve, they might opt for a merger instead, thereby decreasing the competitive pressure on firms and increasing profits. On the other hand, it is possible that firms attempt to integrate one specific firm which broke up a collusive agreement. In this case too, cartel break up would be followed by merger. Whichever explanation holds in a specific

[2]Compare, Department of Justice Horizontal Merger Guidelines 7.1 : "The Agencies are likely to challenge a merger if the (...) market shows signs of vulnerability to coordinated conduct (and) (...) the Agencies have a credible basis on which to conclude that the merger may enhance that vulnerability." DOJ-FTC (2010)

[3]However, Miller and Weinberg (2015) find that a joint venture between two large American brewing companies has lead to pricing behaviour in the beer industry best explained by tacit collusion.

case, there is plenty of empirical evidence demonstrating that the break-up of cartels increases merger activity. As an example, consider the introduction of the Sherman Act of 1890 in the United States (US). While the Sherman Act forbade collusion amongst firms, mergers remained unchallenged until the implementation of the Clayton Act of 1914. Between these two events, up to 50% of the US manufacturing industry took part in mergers (Lamoreaux (1988)). One explanation for this observation is that the introduction of the Sherman Act increased the costs of collusion relative to the costs of merging (Bittlingmayer (1985)). For some firms collusion became unattractive, so in an attempt to decrease the competitive forces in the industry they decided to merge to lower the number of competing firms. Kumar et al. (2015) present evidence from the US pipe industry of the time. They show that a year after being successfully prosecuted for collusion, members of the Addyston[4] cartel merged. Mehra (2008) suggests that merger activity increased in the United Kingdom (UK) after the passing of The Restrictive Trade Practices Act 1956. A more recent empirical study by Hüschelrath and Smuda (2013) shows that cartel breakdown in an industry increases horizontal merger activity by up to 83% in the three years following cartel breakdown. The dataset analysed comprises mergers and cartel cases decided by the European Commission between 2000 and 2011. Additional evidence that the break up of collusion is followed by mergers in Europe is presented by Davies et al. (2015).

Although the empirical evidence suggests a dynamic relationship between collusive break down and mergers, the theoretical literature linking the two practices is limited. One view is that mergers and collusion can be interpreted as substitutes and that firms choose between them. For example, Kumar et al. (2015) as well as Mehra (2008) take this view and show that if mergers don't lead to significant costs synergies, and if collusion is costless, firms strictly prefer collusion over mergers. Both papers don't consider industry dynamics between merger and collusion or include entry into the model.

This gap in the academic literature is what motivates this paper. The model defined aims to nest the analysis of collusive stability into a dynamic framework with competitive forces of entry, exit and mergers. It adds to the existing literature in four ways. Firstly, it provides an analytical framework to explain how the firms' anticipation of long run competitive equilibria influence the incentives to collude. Secondly, it defines market characteristics under which stable, profitable collusion can be sustained in a market with

[4]US pipe makers were convicted of collusion under the US supreme court case *Addyston Pipe and Steel Company et al., Appts., v. United States, 175 U.S. 211*

free entry and without the cartel playing entry deterring strategies. Thirdly, it defines a new notion of a competitive long run stability point under merger and entry. Finally, it provides an analytical framework under which endogenous firm decisions can replicate two observations: a) collusive breakdown is followed by mergers and b) mergers or firm exit can make collusion possible.

The paper is structured as follows. In Section 2 the basic model assumptions are defined. In Section 2.1 the model is solved and analysed for the standard static case in which the number of firms is given exogenously. In Section 2.2 entry and exit dynamics are introduced and the implications of this for competitive and collusive equilibria are discussed. In addition to this, mergers are introduced in Section 2.3. Section 3 provides some discussion and concludes.

2 Model

The model described in this paper follows a three step set up. Firstly, the static base model, in which the number of firms is exogenously given and doesn't change is defined. Secondly, the static model is transferred into a dynamic world with entry and exit forces, which determine the number of firms endogenously. The implications on the industry equilibrium resulting from these dynamics are discussed. Finally, mergers are introduced into the dynamic case and equilibrium dynamics are characterised.

To begin with, the common assumptions of all three models are defined. Firstly, assume an industry in which $n \geq 2$ symmetric, forward-looking firms compete in quantities over homogeneous goods for infinitely repeated periods. Firms face the inverse demand function $P(Q) = a - Q$, where $Q = \sum_{i=1}^{n} q_i$ is the sum of quantities q_i produced by the firms $i = 1, 2, ..., n$. Firms produce at constant marginal cost c and face per period fixed costs f. One unit of profits in the next period has the value $\delta \in (0, 1)$ in the current period. If firms collude, they play a grim trigger strategy against firms that cheat on the collusive agreement. This means that they play collusive strategies initially and then continue doing so as long as no firm deviates from the collusive strategies. If at any point some firm deviates from the collusive strategy, all firms return to playing competitive strategies for all future periods.

2.1 Static Industries

In the static case, the number of firms $n \geq 2$ is given exogenously. It is then straightforward to derive functional forms for the competitive and collusive profits. To begin with, the case in which firms compete against each other is considered. Each firm i maximises individual profits by setting the optimal output q_i, given its competitors output $q_{-i} = Q - q_i$. The maximisation problem is of the form [5]

$$\max_{q_i} q_i \Big[P(Q) - c\Big] - f = \max_{q_i} q_i \Big[a - c - q_i - q_{-i}\Big] - f. \tag{1}$$

From the first order condition, individual best response functions to the competitors output can be determined as

$$q_i^R = \frac{a - c - q_{-i}}{2}. \tag{2}$$

Taking the symmetry of all firms into account and substituting optimal quantities back into the profit functions leads to the competitive per firm profits[6]

$$\pi^*(n) = \left(\frac{a-c}{n+1}\right)^2 - f, \tag{3}$$

which is a strictly decreasing function of n. This implies that for any given level of fixed costs f there is some industry size n^* above which profits in a competitive industry are negative. Solving $\pi^*(n^*) = 0$ for n^* leads to

$$n^* = \frac{a-c}{\sqrt{f}} - 1. \tag{4}$$

For $n < n^*$ firms earn positive competitive profits $\pi^*(n) > 0$, while for $n > n^*$ they occur losses $\pi^*(n) < 0$.

Consider now the case in which firms collude to form a cartel and maximise joint profits. The collusive industry profits are given by $Q[P(Q) - c] - nf$ and hence the maximisation problem for the cartel is

$$\max_{Q} Q\Big[P(Q) - c\Big] - nf = \max_{Q} Q\Big[a - c - Q\Big] - nf. \tag{5}$$

[5] Profits derived step by step in Section 4.1

[6] As all firms are symmetric, the subscript i is omitted in the following

The first order condition leads to the optimal total cartel quantity. After substituting this back into the profit function, the total cartel profits follow.[7] After dividing the total profits by the number of firms inside the cartel, the individual collusive firm profits $\pi^c(n)$ follow as

$$\pi^c(n) = \frac{(a-c)^2}{4n} - f. \tag{6}$$

Similarly to the competitive profits, the collusive profits are strictly decreasing in n and hence, there is some industry size $\overline{n}^c$, for which collusive profits are equal to zero. Solving $\pi^c(\overline{n}^c) = 0$ results in

$$\overline{n}^c = \frac{(a-c)^2}{4f}. \tag{7}$$

From $\frac{\partial \pi^c(n)}{\partial n} < 0$ it then follows immediately that for all $n < \overline{n}^c$ collusive profits are positive, while for $n > \overline{n}^c$ they are negative. Furthermore, from the direct comparison with n^* it follows that the number of firms that can profitably be in a collusive industry is higher than that in a competitive industry: $\overline{n}^c > n^*$.[8]

From the two profit functions, an important standard result of traditional collusive theory follows.

Standard result 1: Relative to competition, it is always profitable to be inside a cartel:

$$\pi^c(n) - \pi^*(n) = \frac{(a-c)^2}{4n} - \left(\frac{a-c}{n+1}\right)^2 = (a-c)^2\left(\frac{(n-1)^2}{4n(n+1)^2}\right) > 0 \;\; \forall n > 1. \tag{8}$$

Hence, it follows that for any given industry size, firms prefer being in a cartel to being in a competitive industry and will therefore always aim to form a cartel.

However, the issue that arises for firms in such a collusive industry is that no binding cartel contracts can be negotiated. Instead, firms merely agree to follow a collusive strategy without any way for the other firms to enforce this action directly. It follows then that any firm could have an incentive to deviate from this agreement and maximise individual profits, given that all other $(n-1)$ firms set the agreed on cartel quantity q^c.

[7] All values are derived step for step in Section 4.1

[8] This inequality holds because $\frac{a-c}{f} > 1$, which is true because $n^* \geq 2$ by assumption.

The defecting firm's maximisation problem is given as

$$\max_{q^d} q^d \Big[P(Q) - c\Big] - f = \max_{q^d} q^d \Big[a - q^d - (n-1)q^c - c\Big] - f. \tag{9}$$

From the first order conditions and the optimal cartel quantities, it follows that

$$q^d(n) = \frac{a - c - (n-1)q^c(n)}{2} = \frac{(n+1)(a-c)}{4n}. \tag{10}$$

After determining the new market price given the deviation, the deviator's profits follow as[9]

$$\pi^d(n) = \Big(\frac{(n+1)(a-c)}{4n}\Big)^2 - f. \tag{11}$$

This shows that collusion is inherently unstable in any one shot game. That is because firms earn more if they deviate than when they collude: $\pi^d(n) > \pi^c(n)$. This follows directly from

$$\pi^d(n) - \pi^c(n) = \frac{(a-c)^2}{4n}\Big[\frac{(n-1)^2}{4n}\Big] > 0 \quad \forall\ n > 1 \tag{12}$$

Thus, although firms would benefit from collusion compared to competition, collusion cannot be upheld because it cannot be supported as a Nash equilibrium. Each firm would have an incentive to deviate from the agreement to secure higher profits.

However, it is assumed in this model that firms play a grim-trigger strategy against defectors and that interaction is infinitely repeated. Collusion can then be stable if the present value of being inside the cartel outweighs the value of earning deviator profits in this period and then earning competitive equilibrium profits afterwards. This implies that collusion is stable as long as

$$\begin{aligned} &\sum_{t=0}^{\infty} \pi^c(n)\delta^t \geq \pi^d(n) + \sum_{t=1}^{\infty} \pi^*(n)\delta^t, \text{ which simplifies to} \\ &\quad \frac{\pi^c(n)}{1-\delta} \geq \pi^d(n) + \delta\frac{\pi^*(n)}{1-\delta}. \end{aligned} \tag{13}$$

This can be rewritten as the following condition

$$\delta \geq \frac{\pi^d(n) - \pi^c(n)}{\pi^d(n) - \pi^*(n)} = \frac{1}{1 + \frac{4n}{(n+1)^2}} := \delta_0^*(n). \tag{14}$$

[9]Profit function as well as price and quantities derived in Section 4.1

Call $\delta_0^*(n)$ the critical discount factor. If the industry's discount factor is above this critical discount factor, firms value the profits of staying in the cartel more than the short term profits of deviation followed by the competitive punishment profits. A cartel only forms when that is given. When $\delta < \delta_0^*(n)$, firms anticipate that each cartel member has an incentive to deviate and the cartel doesn't form in the first place.

$\delta_0^*(n)$ is increasing in n, which means that it is harder to sustain collusion when the industry holds more firms. This notion is commonly referenced in the academic literature.[10]

Furthermore, as it is assumed that $\delta \in (0,1)$, it must be that $\delta^* < 1$ for there to be any chance that $\delta \geq \delta_0^*(n)$. This means

$$\begin{aligned}&\delta_0^*(n) < 1, \text{ which after some rearranging becomes}\\ &\pi^c(n) > \pi^*(n).\end{aligned} \tag{15}$$

As shown above, this holds $\forall n > 1$ and therefore a second important standard result follows.

Standard result 2: For every industry size n, there is some discount factor $\delta \in (0,1)$ for which collusion can be sustained because $\delta \geq \delta_0^*(n)$.

This standard result implies that even unprofitable cartels can be stable for some discount factor. This follows directly from the fact that the number of firms in the industry is expected to be fixed and exogenously given. When firms then make a decision about collusion, they only take into account the relative pay off possible. This means that even when a cartel earns negative profits, the firms know that under competition their losses would be larger.

Overall, the results in this section then imply that it is possible to group market sizes n into three different ranges depending on the profitability of competitive and collusive firms:

1) $n \in [2, n^*]$. In this range, both collusive and competitive firms earn (weakly) positive profits: $\pi^*(n) \geq 0$ and $\pi^c(n) > 0$.

2) $n \in (n^*, \overline{n}^c]$. For these market sizes, firms who compete against each other are unprof-

[10] Compare for example Katsoulacos et al. (2015) who define the *intrinsic difficulty of holding a cartel together* as a combination of the discount factor and the cartel cartel size. In their model, larger cartels increase the difficulty, while higher discount factors decrease it. This implies that a larger cartel requires a higher discount factor to achieve any given level of *intrinsic difficulty.* In this sense, $\delta_0^*(n)$ follows the same logic.

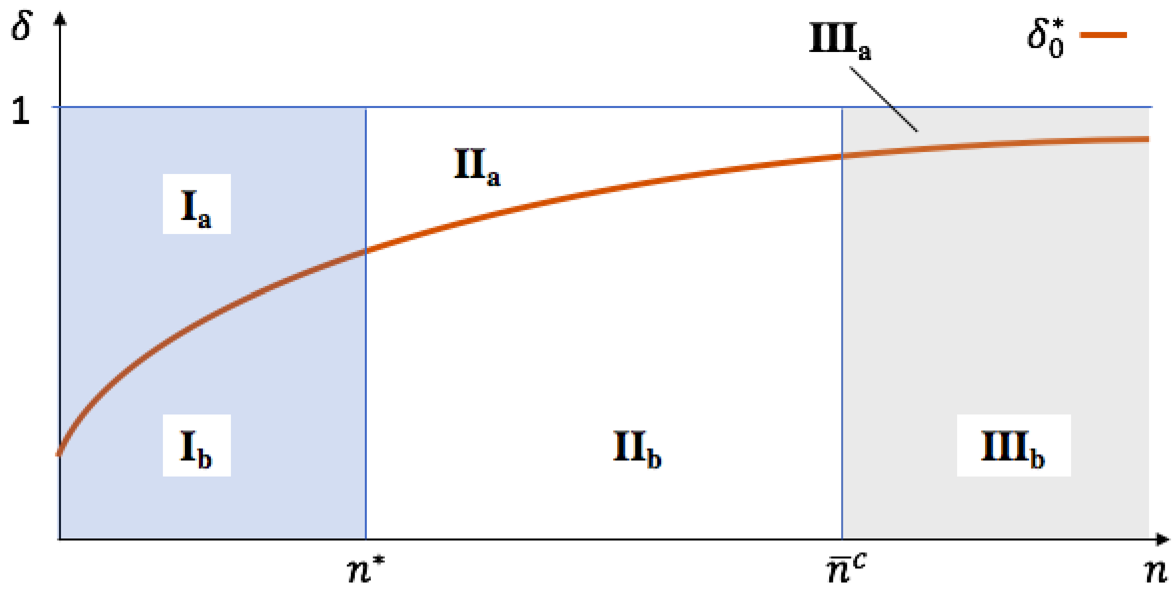

Figure 1: *The graph shows the profitability of firms inside the market and stability of collusive agreements depending on* (n, δ)

itable, while those in collusive agreements earn weakly positive profits: $\pi^*(n) < 0$ and $\pi^c(n) \geq 0$.

3) $n > \overline{n}^c$. In this range, any firm operating in the industry is unprofitable: $\pi^*(n) < 0$ and $\pi^c(n) < 0$.

For each of these industry sizes, firms will collude if their discount factor is above the critical discount factor needed: $\delta \geq \delta_0^*(n)$. This then means that for any $\{n, \delta\}$-tuple, it is possible to determine if a) the industry is competitive or collusive and b) if firms are currently earning positive or negative profits. Fig. 1 gives a graphic example of the possible outcomes. It maps the critical discount factor $\delta_0^*(n)$ as a function of n, and displays the maximum profitable industry sizes n^* and $\overline{n}^c$. This results in 6 possible outcomes that correspond to the outcomes defined above. The subscript a [b] denotes the $\{n, \delta\}$-tuples for which collusion is stable [unstable]. In areas I_i, $i \in \{a, b\}$ both cartel and competitive firms earn positive profits, for II_i, $i \in \{a, b\}$ only collusive firms earn positive profits and finally, for III_i $i \in \{a, b\}$ both cartels and competitive firms are unprofitable.

However, from an economical perspective, some of these outcomes don't seem to be sustainable over time. On the one hand, there are scenarios in which firms are making a loss, which begs the question why they stay inside the industry. On the other hand, some firms can earn big positive profits. This could incentivise firms from the outside to join the industry. In the following section, these two forces are introduced into the framework.

2.2 Dynamic Industries With Entry And Exit

This section aims to introduce some industry dynamics into the above defined framework. This will be done by allowing firms to freely choose to enter or exit the industry. To begin with, the dynamics at work in a competitive industry are considered:

- A competitive industry grows in size through entry. There is a potential group of entrants who will join the industry as long as the profits of doing so are non-negative. Analytically, this implies that an entry force is present as long as

$$\pi^*(n) > 0 \qquad \Longleftrightarrow \qquad n < n^*. \tag{16}$$

- A competitive industry reduces in size through exit. Any firm, which currently earns negative profits have an incentive to leave the industry. Analytically, this implies that there is an exit force present if

$$\pi^*(n) < 0 \qquad \Longleftrightarrow \qquad n > n^*. \tag{17}$$

Combining the entry and the exit force, it is clear that a competitive industry can only be in equilibrium if $n = n^*$ and firms make zero profits. For any industry size $n > n^*$ firms have an incentive to leave as defined in (17), while for any $n < n^*$ there is going to be entry as defined in (16). Therefore, the equilibrium condition for competitive industries is

$$\pi^*(n) = 0 \qquad \Longleftrightarrow \qquad n = n^*. \tag{18}$$

This means that the introduction of entry and exit forces imply a zero profit equilibrium in competitive industries. Independent from the initial industry size, any competitive industry will converge to this equilibrium. When there are too many firms to operate profitably, some of them will leave. If there are only few, but profitable firms, additional entry will occur.

For firms who aim to collude, this has important implications: it means that compared to the static model discussed before, the outside option to collusion is now the zero profit dynamic equilibrium. Therefore, define the realisable profits $\pi_e^r(n)$ as those profits a cartel firm can earn when the collusive agreement with n members breaks up. Assume also that all cartel firms are able to anticipate that the realisable profits are the outside option to colluding. From (18) it follows immediately that $\pi_e^r(n) = \pi_e^r = 0 \ \forall n$ because competitive entry and exit forces will move the market into equilibrium. For now the number of firms in a collusive industry is assumed to be exogenously given. This leads to the first dynamic result.

Dynamic Result 1: When competitive entry and exit dynamics are accounted for, only strictly profitable cartels earn more than competitive firms. Cartels earn strictly positive profits as long as $n < \overline{n}^c$.

This follows immediately from the idea that competitive industries converge to the long run equilibrium profits $\pi^*(n^*) = 0$. Therefore, as long as the $\pi^c(n) > 0 \iff n < \overline{n}^c$, a cartel earns more than under the competitive equilibrium. To analyse how this influences cartel stability, the stability condition is considered again.

A collusive agreement can be called stable when no firm has an incentive to cheat on it, i.e. when the present value of staying inside the cartel outweighs that of deviating and then earning the realisable profits for all future periods:

$$\begin{aligned} &\sum_{t=0}^{\infty} \pi^c(n)\delta^t \geq \pi^d(n) + \sum_{t=1}^{\infty} \pi_e^r(n)\delta^t, \text{ which simplifies to} \\ &\quad \frac{\pi^c(n)}{1-\delta} \geq \pi^d(n) + \delta\frac{\pi_e^r(n)}{1-\delta}, \text{ and therefore} \\ &\quad \frac{\pi^c(n)}{1-\delta} \geq \pi^d(n). \end{aligned} \tag{19}$$

Intuitively, the introduction of the realisable profits concept has three implications. Firstly, for those $n \geq \overline{n}^c$ for which $\pi^c(n) \leq 0$ collusion is expected to be unstable. That is because the cartel's outside option to collusion is the zero profit competitive equilibrium, which is strictly better than negative profits. Secondly, for those industry sizes $n \in [2, n^*)$ for which competitive firms earned positive profits, they will now earn zero profits. This is expected to increase cartel stability as a break up of collusion would result in a more severe punishment. Finally, for industry sizes $n \in (n^*, \overline{n}^c)$ for which cartel firms earn positive profits but competitive firms earn negative profits, collusion is expected to be less stable. That is because for these n, the realisable profits are above the static competitive profits and therefore, the cartel's ability to punish deviators has decreased.

To verify these intuitive expectations, the critical discount factor representation of the Nash equilibrium condition is considered again. It follows from (19) as

$$\delta \geq \frac{\pi^d(n) - \pi^c(n)}{\pi^d(n)} = 1 - \frac{\pi^c(n)}{\pi^d(n)} := \delta_e^*(n), \tag{20}$$

where $\delta_e^*(n)$ is the new resulting critical discount factor above which collusion is stable and the subscript e denotes that it is derived under the free entry and exit conditions. Similarly to before, $\delta_e^*(n)$ is increasing in n.[11]

However, as the intuitive analysis above has already indicated, there are crucial differences to the static case. To begin with, the existence of any stable cartel requires that for some $\delta \in (0,1)$, it holds that $\delta \geq \delta_e^*(n)$. For this, it must be that

$$\begin{aligned} &\delta^*(n) < 1, \text{which implies} \\ &\pi^c(n) > 0, \text{and therefore} \\ &\quad n < \overline{n}^c. \end{aligned} \tag{21}$$

Thus, a stable cartel can only exist as long as it earns positive profits, i.e. $\forall\ n < \overline{n}^c$. One way of interpreting this is that the application of competitive entry and exit forces has a deterrence effect on large cartels. That is because in very large industries with $n > \overline{n}^c$, cartel firms earn negative profits and would therefore rather be part of a competitive zero profit industry or exit the market than to earn permanent losses. This is in contrast to the static case, where the standard result was that there is some discount factor such that collusion is stable for any cartel size.

Dynamic Result 2: When free entry and exit in competitive industries are taken into account, only strictly profitable cartels can be sustained for some discount factor $\delta \geq \delta_e^*(n)$.

Additionally, there is a range of cartel sizes for which collusion is harder to sustain in the dynamic, relative to the static case. That is when $\delta_e^*(n) > \delta_0^*(n)$. Solving this leads to

$$\pi^*(n) < 0 \iff \quad n > n^*, \tag{22}$$

This means that for those industry sizes for which a competitive firm in the static case would make negative profits collusion is harder to sustain when accounting for entry and exit dynamics in competitive industries. In these cases the cartel's ability to punish a deviating firm by returning to competition is lower in the dynamic framework. Instead of being able to threaten negative future profits, their threat is to revert to a zero profit equilibrium.

[11] Proof in Section 5

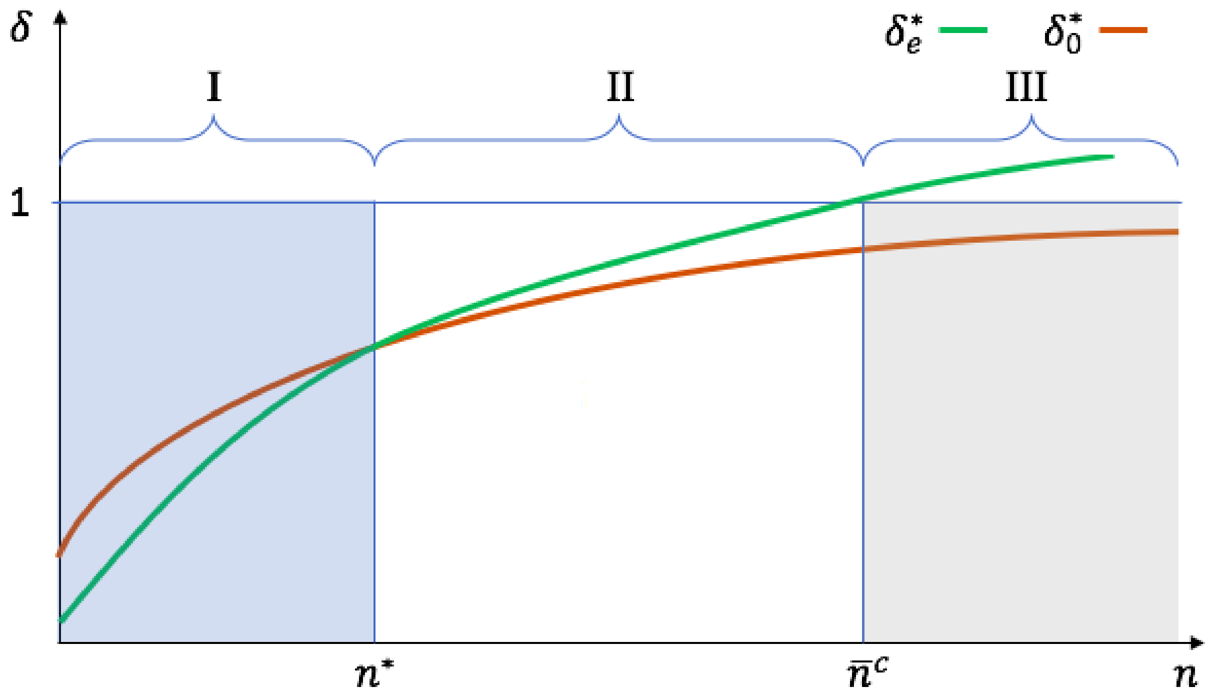

Figure 2: *The graph shows the two critical discount factors $\delta_0^*(n)$(in blue) and $\delta_e^*(n)$ (in red) as a function of the number of firms n*

Similarly, a cartel is easier to sustain in the dynamic case when $\delta_e^*(n) < \delta_0^*(n)$, i.e. when

$$\pi^*(n) > 0 \iff \quad n < n^*. \tag{23}$$

For these industry sizes the competitive profits in the static framework are strictly positive while those under the dynamic approach are equal to zero. Therefore, the cartel's ability to punish a deviator is increased when entry is taken into account. Fig. 2 gives a graphic example of the difference between δ_e^*(in red) and δ_0^* (in blue).

From the definition of the critical discount factor, the first directional results for industry dynamics follow. By definition, when $\delta \geq \delta_e^*(n)$, collusion is sustainable while it isn't for $\delta < \delta_e^*(n)$. Given that for now only competitive industries are dynamic, this divides the (n, δ) plane into two areas: the one above $\delta_e^*(n)$, where firms collude and the one below it where firms compete against each other. Focussing on the latter area, one can identify two opposing industry dynamics. Firstly, when $n < n^*$ firms will enter, according to (16). Secondly, when $n > n^*$ firms earn negative profits and therefore, from (17) it follows that they will leave. This latter case brings up one question: can it be possible that competitive industry forces which decrease the number of firms make collusion possible? As cartels are easier to sustain with less members, there can be cases

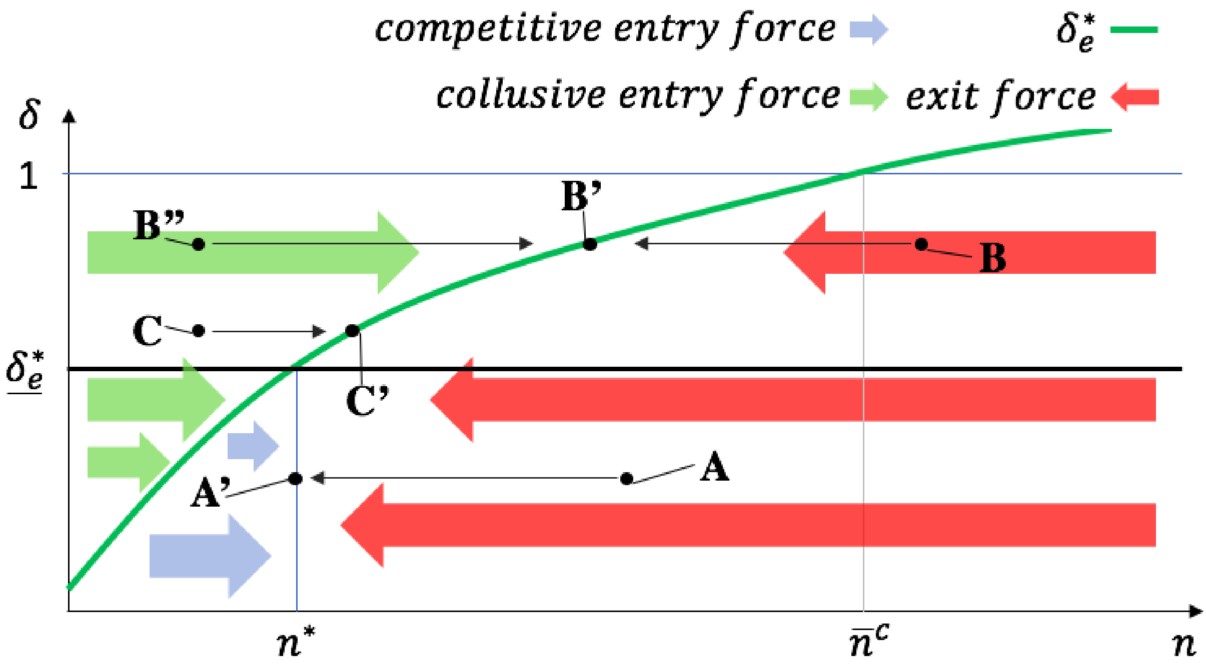

Figure 3: *The graph shows how the industry dynamics depend on the initial industry size n and the industry's discount factor δ.*

in which the industry size decreases enough to enable collusion.

A graphic example of this is given in Fig. 3, which shows the industry dynamics. Consider an industry with an initial (n,δ) like that in point A. For firms in this situation, collusion is not sustainable because $\delta < \delta_e^*(n)$. Furthermore, $n > n^*$ implies that firms earn negative profits. Thus, some firms will leave until $n = n^*$, i.e. when the industry is in point A'. Here, no firm has an incentive to leave or to join, but firms are also not able to collude. This shows that there are some industries which when they start off competitively, will remain so in the long run equilibrium. However, consider now a $\{n,\delta\}$-tuple like that in B. Again, the competitive industry forces will decrease the number of firms. When n is such that the industry arrives at the point B' though, the industry's discount factor is equivalent to the critical discount factor $\delta_e^*(n)$, which means that collusion is now possible. Hence, the process of firms leaving the industry has led to a collusive equilibrium at B'. The difference between the industries A and B is that B's discount factor is high enough to sustain collusion for some industry size $n \geq n^*$. Given this, the firms can return to collusion before the competitive equilibrium size is reached. This difference is represented by the horizontal line which takes its origin on the intersection of n^* and $\delta_e^*(n)$. For all competitive industries which have a discount factor above this line, the outcome will be collusive, while the industries which start off below the line will remain competitive.

Formally, this implies that one can differentiate industries given on how their discount factor relates to the critical discount factor required to sustain collusion at the competitive industry size n^*. Define $\delta_e^*(n^*) := \underline{\delta_e^*}$. Then, assuming that industries start off competitively in the initial period, those with $\delta < \underline{\delta_e^*}$ will remain competitive but will adjust the industry size to $n = n^*$, while those with $\delta \geq \underline{\delta_e^*}$ will be characterized by exit until $\delta = \delta_e^*(n)$ and collusion is stable. What size will these stable cartels have?

Define n_e^c as the maximum stable cartel size for a given discount factor δ. Formally, $\forall$ $\delta \geq \delta_e^*(2)$ $\exists$ $n_e^c \geq 2$, such that $\delta = \delta_e^*(n_e^c)$, where the functional form of n_e^c is given by

$$n_e^c = \frac{(1+\delta)(a-c)^2 + 2(a-c)\sqrt{(a-c)^2\delta - 4\delta(1-\delta)f}}{(1-\delta)(a-c)^2 + 16f\delta}. \tag{24}$$

This is an increasing function of δ, because it the inverse function of the monotonically increasing function $\delta_e^*(n)$. By definition, $\delta \to \underline{\delta_e^*}$ implies $n_e^c \to n^*$ and $\delta \to 1$ implies $n_e^c \to \overline{n}^c$. Furthermore, the formulation $n > n_e^c$ is equivalent to $\delta < \delta_e^*(n)$ and therefore collusion isn't stable for these sizes. Equivalently, $n \leq n_e^c \iff \delta \geq \delta_e^*(n)$, which implies that collusion can be sustained. As $\delta < 1$, it is always the case that $n_e^c < \overline{n}^c$ and hence, it is confirmed again that any stable cartel earns strictly positive profits.

So far, it has been assumed that once firms reach a collusive agreement, the number of firms in the industry is fixed. However, allowing for the number of firms to be endogenously defined during competition but not during collusion is inconsistent. Therefore assume in the following that colluding firms can make the same entry and exit decisions as competing firms. As the analysis is now focussed on sustainable collusive agreements, assume that $n \leq n_e^c$. For any of these industry sizes it it has already been established that $\pi^c(n) > 0$ which directly implies that colluding firms will never exit. Therefore, the only condition to be established is concerned with entry.

Consider for this purpose a firm outside an industry with $n < n_e^c$ firms, which observes that firms inside the industry earn strictly positive profits. There are two possible outcomes for an entrant. On the one hand, if after entry $n \leq n_e^c$, the firm can anticipate that collusion is still stable after it joins and that therefore the profits of entering the industry are strictly positive. Consequently, the firm will enter. On the other hand, if the industry size grew to $n > n_e^c$ through entry, collusion would be unstable and therefore the firm would expect negative competitive profits and would not enter. Thus, entry into the collusive industry occurs as long as the post entry industry size is

$$n \leq n_e^c \qquad \iff \qquad \delta \geq \delta_e^*(n). \tag{25}$$

This means that any $n < n_e^c$ cannot be the equilibrium industry size, because additional entry would increase n. At the same time, sizes of $n > n_e^c$ cannot be a collusive equilibrium because no cartel can form for these n. Hence, the collusive equilibrium industry size is given by

$$n = n_e^c \qquad \Longleftrightarrow \qquad \delta = \delta_e^*(n). \tag{26}$$

For the most part, the dynamics that follow are similar to the ones discussed in the competitive case. To show this, an industry which starts off with an $\{n, \delta\}$-tuple strictly above $\delta_e^*(n)$ is considered. Furthermore, assume that the industry's discount factor is such that $\delta \geq \underline{\delta_e^*}$. Given this initial configuration of parameters, firms in the industry collude and earn strictly positive profits. However, additional entrants anticipate that after entry the industry can still be collusive and profitable. Therefore they will enter until $n = n_e^c$. Fig. 3 gives a graphic example of this. The point C is strictly above $\delta_e^*(n)$ and $\delta > \underline{\delta_e^*}$. Therefore, the industry size grows until the point C' which is on the $\delta_e^*(n)$ line.

The combination of these industry forces for $\delta \geq \underline{\delta_e^*}$ directly imply that in this model, an industry's discount factor determines a unique stable equilibrium size at $n = n_e^c$. For any $n < n_e^c$ additional entry increases n, while for $n > n_e^c$ exit decreases it. This is highlighted in Fig. 3 by the the initial starting points B and B" which both have the same δ but B is competitive to begin with while B" is collusive. Both industries converge towards B' through exit or entry respectively.

The industry dynamics for the case of $\delta < \underline{\delta_e^*}$ and therefore $n^c < n^*$ are analysed now. Those cases which start off competitively have been analysed above and have led to the result that the industries converge towards the competitive equilibrium industry size and remain competitive. When the industry starts off in a cartel, a potential entrant again faces two possible outcomes. Firstly, if after entry the industry can remain collusive, i.e. when $n \leq n_e^c$, any entrant will join because it expects positive collusive profits. Secondly, when after entry $n > n_e^c$, collusion cannot be sustained any longer. In this case any potential entrant expects to earn competitive profits. There are two possible arguments to consider. On the one hand, the entrant could expect to earn the long run realisable profits $\pi_e^r(n)$ if it anticipates that other firms will join the industry too. In this argument firms would not enter. However, strictly speaking this would not be a Nash equilibrium strategy because if all firms refrained from entering, it would pay off for each firm to enter as long after entry $n < n^*$. On the other hand, it is therefore argued that any firm considering to enter only takes into account the direct profit implications of its own actions. This then means, that if after entering the industry $n > n_e^c$ but also $n < n^*$, the entrant expects positive profits and therefore joins the industry. This latter argument implies that no collusive agreement is stable for $\delta < \underline{\delta_e^*}$. From an intuitive point of view, this correlates with the general notion that collusion is more likely when the valuation of future profits is high. Furthermore, it reflects the idea that in some industries cartel's are concerned by potential entry. Therefore, this second interpretation of outcomes is seen preferable and will be used in the rest of this paper.

Concluding, this means that the model defined in this section predicts a unique industry size, depending solely on the industry's discount factor δ. If $\delta < \underline{\delta_e^*}$ the industry will converge towards the competitive equilibrium size $n = n^*$ and will be in a zero profit equilibrium. For $\delta \geq \underline{\delta_e^*}$, the industry converges towards the collusive equilibrium size $n = n_e^c$ and firms earn strictly positive collusive profits. Therefore, stable and profitable cartels can form even in the presence of entry forces. In addition to these dynamics, two major differences to the static result have been identified. Firstly, unprofitable cartels are not sustainable any more. Secondly, collusion is harder to sustain for cartels that do form, as the cartel's ability to punish deviators has decreased. So far, the dynamic forces in the industry have been limited to entry and exit. However, mergers can also decrease the industry size. Therefore, the following section introduces them to the model.

2.3 Dynamic Industries With Entry, Exit And Merger

Assume now that, additionally to the entry and exit dynamics introduced before, firms in a competitive industry can decide to engage in mergers. If two firms merge, the industry size decreases from n to $(n-1)$ and the two firms utilise synergies by stripping out one set of fixed costs.[12] Therefore, a merger pays off when

$$\pi^*(n-1) > 2\pi^*(n), \tag{27}$$

[12] This assumption is consistent with the existing literature on mergers (e.g. Salant et al. (1983), or Davidson and Mukherjee (2007)) and greatly simplifies the following analysis. Nevertheless, the assumption that merged firms do not differ from non-merged firms has been criticized in previous literature. Most notably, Perry and Porter (1985) argue that a merger of "two firms from a symmetric equilibrium of (n + 1) firms should result in an equilibrium with (n -1) old firms and one new firm that is "larger" than the others" (p. 191). In their model, merged firms can increase their production above the level of the other firms in the market. Similarly, Deneckere and Davidson (1985) allow for a distinction of merged firms to other firms by modelling a differentiated goods Bertrand model in which firms require a patent to produce their individual product. Thus, a merged entity owns a larger range of products and can internalize the externalities of their pricing decisions.

which trivially holds $\forall\ n \geq n^*$, as for these values $2\pi^*(n^*) = 0$ by definition, and $\pi^*(n^* - 1) > 0$ from the comparative statics of $\pi^*(n)$. Furthermore, it is intuitively reasonable to expect any merger from $n = 2$ to $n = 1$ to be profitable, because these mergers create a monopoly which leads to the highest producer surplus possible. Substituting the competitive profit function into the the merger condition and rearranging leads to

$$(a-c)^2\Big[\frac{1}{n^2} - \frac{2}{(n+1)^2}\Big] > -f, \tag{28}$$

where the LHS is an asymmetric v-shaped function of n which is positive $\forall\ n < 2.414$, has a minimum at $n = 3.847$ and then converges towards zero as n goes to infinity. This confirms the intuition that mergers from two firms to one firm always pay off. The RHS is strictly decreasing in f, which implies that mergers are more likely to pay off when the fixed costs are high. This makes sense as higher fixed costs imply a larger saving potential through the merger. It is then possible that the fixed costs are high enough to render all mergers profitable. If that was the case, firms would always want to engage in mergers. However, empirical evidence suggests that mergers come in waves, which implies that after some consolidation, further mergers are not attractive any more. To mimic this in the model, it will be assumed that the fixed costs f are small enough that there is some range for which mergers are not profitable. Formally, assume that f is such that

$$\begin{aligned}
&\exists\ n_0^m, n_1^m \text{ where} && 2 < n_0^m < n_1^m < n^* \text{ s.t.}\\
&\forall\ n < n_0^m, && \pi^*(n-1) > 2\pi^*(n)\\
&\forall\ n \in [n_0^m, n_1^m], && \pi^*(n-1) \leq 2\pi^*(n)\\
&\forall\ n > n_1^m, && \pi^*(n-1) > 2\pi^*(n).
\end{aligned} \tag{29}$$

This means that mergers are profitable in small and large markets (more specifically, those with $n < n_0^m$ and $n > n_1^m$), but that there is some range of sizes $n \in [n_0^m, n_1^m]$ for which they are not.

Assume also that there is a CA which forbids mergers to take place in industries with $n \leq n_0^m$. This means that industry dynamics are only affected by mergers when $n > n_1^m$. Equivalently to the entry and exit dynamics, one can then formally define that a merger force is present

$$\forall n > n_0^m \quad \text{s. t.} \quad \pi^*(n-1) > 2\pi^*(n) \quad \iff n > n_1^m \tag{30}$$

Additionally, from the previous section it is known that firms enter as long as the post entry industry size is $n < n^*$ and exit when $n > n^*$. It is also assumed that in their decision to enter an industry, firms don't take into account possible merger dynamics that could have been triggered through their entry.[13] This then allows to define three ranges for the industry size:

- $n \in [1, n_1^m]$, only the entry force is present, when
- $n \in (n_1^m, n^*)$, a merger and an entry force are present,[14] and when
- $n > n^*$, an exit and a merger force are present.

Hence, the introduction of mergers doesn't influence the direction of industry dynamics for small ($n \leq n_1^m$) or large ($n > n^*$) markets. The only difference for these n is that a reduction of firms in the large markets can now have its root in either exit or merger. However, for values of $n \in (n_1^m, n^*)$ the dynamic forces of merger and entry point in opposing directions. Any entry would increase the number of firms, merger would decrease it.

The question that arises for this area is: which force will outweigh the other? From the comparative statics it is known that the net gains of merging increase with the number of firms, while the profits from entry decrease. This implies that those n, for which the incentives to merge are highest, are also those, for which the incentives for entry are lowest, and vice versa. Specifically, consider a market in which n is close to n^*. Any additional entrant would expect to earn profits close to zero after entry, while firms within the market can expect relatively large gains from merging. In the long run, one would then expect more firms to merge than to enter. Contrary, when n is close to n_1^m, the incentives to merge are small while those of entry are large. In this scenario, the long run prediction would be that more firms enter than engage in merger.

Following this intuition, define the long run sustainable number of firms under entry, exit and merger $n^m \in (n_1^m, n^*)$ as the industry size for which the net gains from merging are equal to the profit of entering. A long run stability point is then defined by the

[13]This assumption excludes *entry to merge* strategies in which firms enter a currently unprofitable industry under the assumption that consolidation after entry leaves them a non-zero probability of earning positive profits in the future.

[14]When $n = n^*$, only the merger force is present, as this is a point and not a range though, it is not included in the list. This distinction does not change the equilibrium results or dynamics.

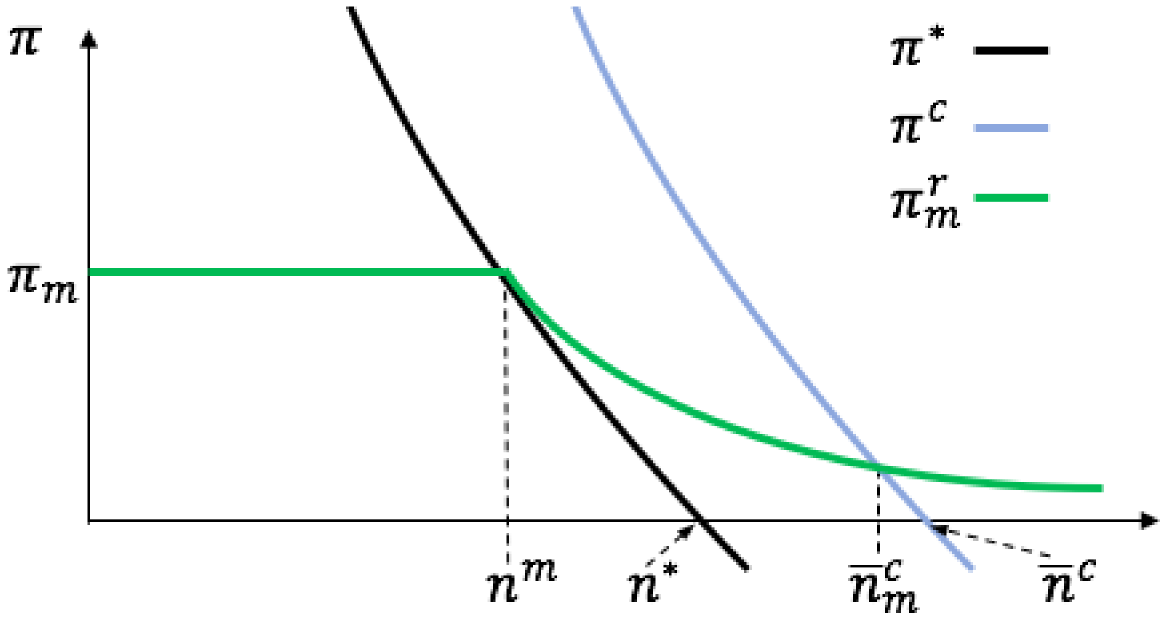

Figure 4: *The graph shows profits of competitive and cartel firms as well as realisable profits for firms inside a cartel after deviation.*

condition

$$\pi^*(n^m-1)-2\pi^*(n^m)=\pi^*(n^m), \tag{31}$$

where the LHS shows the net gains of merging and the RHS those of entry. For any $n<n^m$, the gains of entry outweigh those of merger which is assumed to result in relatively more entries in the long run. Therefore, the industry size n is expected to increase until $n=n^m$. The opposite dynamic argument holds for $n>n^m$. Thus, any strictly competitive industry will converge towards the long run sustainable industry size n^m and earn long run competitive profits of $\pi^*(n^m)$. To simplify notation define $\pi^*(n^m)=\pi_m$. From $n^m\in(n_1^m,n^*)$, it follows directly that $\pi_m>0$.

Similarly to the entry and exit case, define now $\pi_m^r(n)$ as the realisable profits for a firm inside a cartel, after the breakdown of collusion. As long as $n\le n^m$, each firm in the cartel can expect to earn π_m, should the collusive agreement break down. This changes when $n>n^m$, because the firms anticipate that there will be market consolidation under competition, which not all firms are going to survive. As all firms are symmetrical, it is assumed that each firm expects to remain in the industry with probability $\frac{n^m}{n}$. The expected realisable profits $\pi^r(n)$ then take the form

$$\pi^r_m(n) = \begin{cases} \pi_m & \forall n \leq n^m \\ \frac{n^m}{n}\pi_m & \forall n > n^m, \end{cases} \tag{32}$$

where $\pi^r(n) > 0$ holds $\forall\ n \geq 2$. This means that compared to the entry and exit case discussed in the forgone section, the cartel's outside option to collusion is strictly positive profits instead of zero profits. Therefore, it can be expected that collusion is less stable when mergers are taken into account because the cartel's ability to punish deviators is lower. Furthermore, it implies that for some industry size, the realisable profits are just equal to the cartel profits. Formally, $\exists \overline{n}^c_m$, s.t. $\pi^c(\overline{n}^c_m) = \pi^r_m(\overline{n}^c_m)$. The functional form of $\overline{n}^c_m$ follows as

$$\overline{n}^c_m = \frac{(a-c)^2 - \pi_m n^m}{4f} < \frac{(a-c)^2}{4f} = \overline{n}^c. \tag{33}$$

Above this cartel size, firms are better off breaking up collusion and returning to competition and therefore it can be expected that collusion cannot be sustained.

To verify these intuitive results regarding collusive stability, a cartel firm's incentive to deviate is considered again. For any firm to not have an incentive to deviate from the collusive agreement, it must be that earning $\pi^c(n)$ for all periods exceeds earning the deviator profits $\pi^d(n)$ in one period and afterwards reverting to earn the realisable profits $\pi^r_m(n)$. Formally, collusion is stable when

$$\begin{aligned} &\sum_{t=0}^{\infty} \pi^c(n)\delta^t \geq \pi^d(n) + \sum_{t=1}^{\infty} \pi^r_m(n)\delta^t, \text{ which simplifies to} \\ &\quad \frac{\pi^c(n)}{1-\delta} \geq \pi^d(n) + \frac{\delta\pi^r_m(n)}{1-\delta}. \end{aligned} \tag{34}$$

This then leads to the condition for Nash Equilibrium

$$\delta \geq \frac{\pi^d(n) - \pi^c(n)}{\pi^d(n) - \pi^r_m(n)} := \delta^*_m(n), \tag{35}$$

where $\delta^*_m(n)$ is the critical discount factor necessary to sustain collusion for any given industry size n, given that the cartel firms expect to revert to the competitive long run sustainable industry size n^m after deviation. Substituting the functional form of π^r_m into

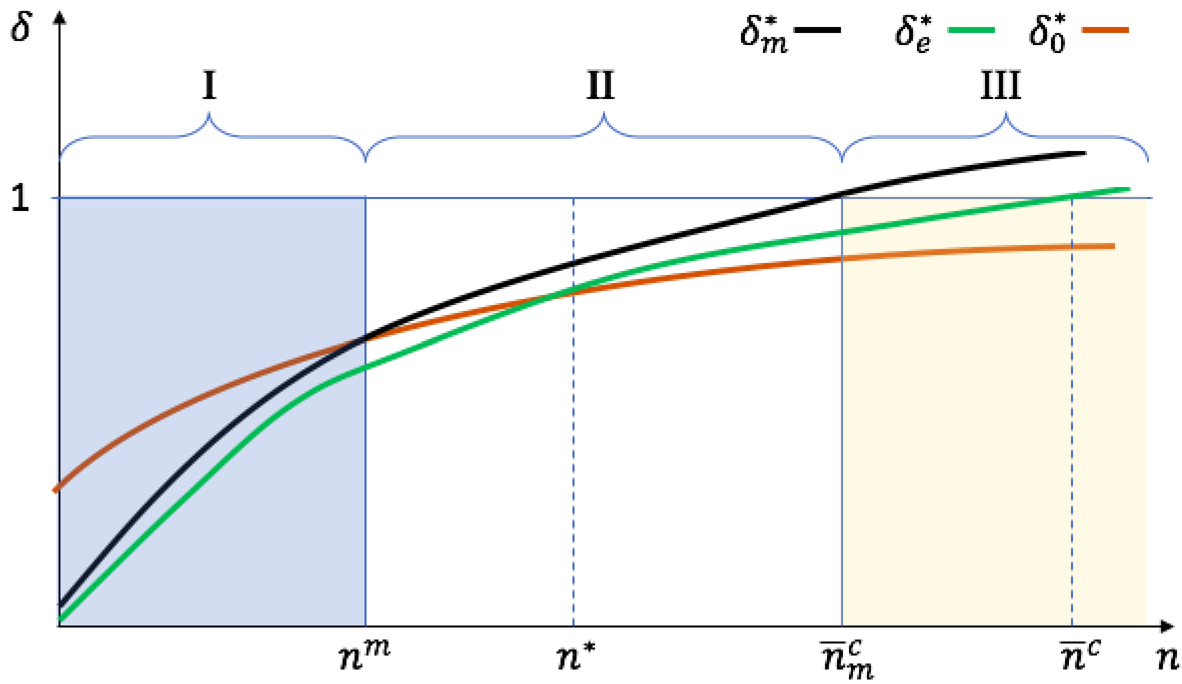

Figure 5: *The graph shows the critical discount factors $\delta_m^*(n)$, $\delta_e^*(n)$, and $\delta_0^*(n)$ as functions of the industry size n.*

this leads to

$$\delta \geq \delta_m^*(n) = \begin{cases} \frac{\pi^d(n)-\pi^c(n)}{\pi^d(n)-\pi_m} & \forall n \leq n^m \\ \frac{\pi^d(n)-\pi^c(n)}{\pi^d(n)-\frac{n^m}{n}\pi_m} & \forall n > n^m. \end{cases} \tag{36}$$

Now, as $\delta \in (0,1)$, the existence of a stable cartel requires that the critical discount factor is below unity. Analytically this means

$$\begin{aligned} &\delta_m^*(n) < 1, \text{ which solves for} \\ &\pi^c(n) > \pi_m^r, \text{ and finally} \\ &\quad n < \overline{n}_m^c. \end{aligned} \tag{37}$$

This confirms that collusion cannot be stable above the market size which equalises cartel and realisable profits.

Dynamic Result 3: When cartel firms anticipate to return to the long run stability point at which entry and merger forces are equivalent, every cartel has to be strictly profitable, though there are some profitable cartel sizes which cannot be sustained for any discount factor $\delta \in (0,1)$.

This result combines two forgone findings. Firstly, for $\delta_m^*(n) < 1$, it must be that $n < \overline{n}_m^c$ and hence $\pi^c(n) > 0$. Secondly, though $\overline{n}_m^c < \overline{n}^c$ implies that there are some cartel sizes $n \in [\overline{n}^c_m, \overline{n}^c)$ for which collusion would be profitable, but it cannot be sustained because the cartel's outside option has a higher expected pay off. Fig. 5 gives a graphic representation of this. One can see that for any $n \geq \overline{n}^c_m$ the critical discount factor $\delta^*_m(n)$ (in black) is unity or greater which implies that collusion isn't stable for these values. This contrasts the results from the entry/merger case discussed in the previous section, where in the range $n \in [\overline{n}^c_m, \overline{n}^c_e)$ collusion was possible.

In terms of magnitude, $\delta^*_m(n)$ can be compared to both the critical discount factor from the static and the dynamic entry and exit case. To begin with, the comparison with $\delta^*_e(n)$ shows clearly that

$$\delta^*_m(n) > \delta^*_e(n) \quad \forall n \geq 2. \tag{38}$$

This follows from $\pi^r_m(n) > \pi^r_e(n) = 0$. Intuitively, this result shows that the cartel's ability to punish deviators is lower when mergers are taken into account because the punishment profits are positive instead of equal to zero. Deviating is then attractive for a larger range of discount factors.

Comparing $\delta^*_m(n)$ to δ^*_0 shows that

$$\begin{aligned}
&\delta^*_m(n) < \delta^*_0(n) \quad && \forall \pi^r_m(n) < \pi^*(n) \quad && \iff \forall n < n^m \\
&\delta^*_m(n) = \delta^*_0(n) \quad && \forall \pi^r_m(n) = \pi^*(n) \quad && \iff \text{for } n = n^m \\
&\delta^*_m(n) > \delta^*_0(n) \quad && \forall \pi^r_m(n) > \pi^*(n) \quad && \iff \forall n > n^m.
\end{aligned} \tag{39}$$

Again, this demonstrates the cartel's altered ability to punish deviators. When $n < n^m$, a cartel in the static case threatens to return to competitive profits of $\pi^*(n)$. Taking the entry and merger dynamics into account then means that firms anticipate additional entry after cartel break up, which decreases the punishment profits expected, compared to the static case: $\pi^r(n) < \pi^*(n)$ $\forall n < n^m$. This makes staying in the collusive agreement more attractive when dynamics are considered. The opposite is true for $n > n^m$: firms in the static case expect lower punishment profits than those anticipating mergers, i.e. $\pi^r(n) > \pi^*(n)$ $\forall n > n^m$. A graphic example of the difference between $\delta^*_m(n)$ and the previously defined discount factors $\delta^*_0(n)$ and $\delta^*_e(n)$ is given in Fig. 5.

With the critical discount defined, it is now possible to determine the industry dynamics taking into account mergers. Similar to the entry/exit section, the competitive dynamics are considered first, assuming that the number of firms in any collusive industry

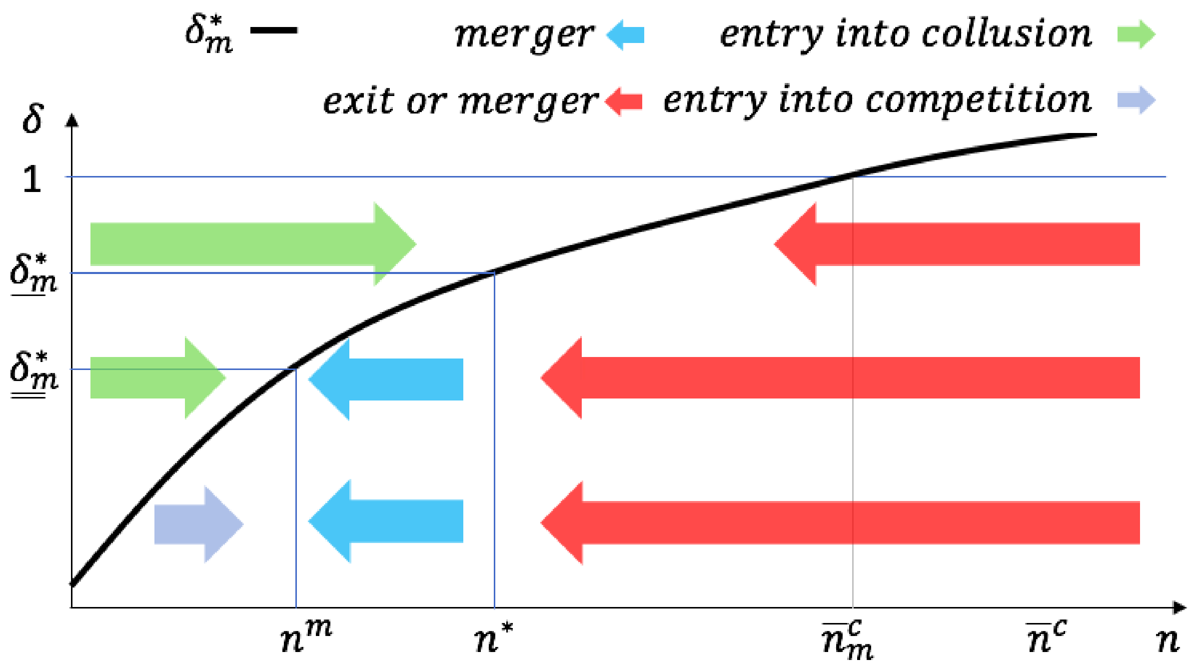

Figure 6: *The graph shows the industry dynamics when mergers are taken into account.*

is fixed. In principle, these dynamics have already been defined. When $n < n^m$ the industry size will grow through entry, while for $n \in (n^m, n^*]$ it will decrease through mergers. Whenever $n > n^*$ either merger or exit will lead to a reduction in n. In a world without collusion, this would always lead to the long run sustainable market size $n = n^m$. However, given that firms can collude and that a cartel is easier to sustain for smaller industries, it is possible that both merger and exit can make collusion stable for some discount factors. To formalise this argument in the same way as in the previous section, two definitions are necessary.

- Firstly, define $\delta^*_m(n^m) := \underline{\underline{\delta^*_m}}$ as the minimum critical discount factor needed to sustain collusion at the long run stability point n^m.
- Secondly, define $\delta^*_m(n^*) := \underline{\delta^*_m}$ as the minimum critical discount factor needed to sustain collusion at the zero profit industry size n^*.

As $n^m < n^*$ and $\delta^*_m(n)$ is increasing in n, it follows immediately that $\underline{\underline{\delta^*_m}} < \underline{\delta^*_m}$. This makes it possible to split the analysis of competitive industry forces and outcomes into three separate cases, depending on the discount factor δ.

1. Low discount factors of $\delta < \underline{\underline{\delta^*_m}}$. For any $n < n^m$, entry forces outweigh merger forces and the industry size will increase until $n = n^m$. For any $n > n^m$, the industry dynamics will decrease n either through entry and exit (when $n > n^*$), or because

merger forces outweigh entry forces (when $n \in (n^m, n^*])$. This implies that when $\delta < \underline{\underline{\delta}}^*_m$, any industry which starts off competitively will remain so in equilibrium. The long run market size is then given by $n = n^m$ and firms earn positive long run profits $\pi^*(n^m)$.

2. High discount factors of $\delta \in [\underline{\delta}^*_m, 1)$. For these discount factors, any competitive industry has at least $n > n^*$ firms in it. To see this, define the maximum number of firms in a stable cartel, accounting for merger dynamics, as n^c_m. For this market size, a cartel can just be sustained, which implies $\delta = \delta^*_m(n^c_m)$ and hence

$$n^c_m = \frac{(1+\delta)(a-c)^2 - 8\delta\pi_m + 2(a-c)\sqrt{\delta[(a-c)^2 - 4(1-\delta)f + (1+\delta)\pi_m]}}{(1-\delta)(a-c)^2 + 16\delta f}. \quad (40)$$

For any $n \leq n^c_m$, collusion can be sustained, while for any $n > n^c_m$, it cannot. As n^c_m is the inverse of the monotonically increasing function $\delta^*_m(n)$, it is increasing in δ. Furthermore, by definition $n^c_m(\underline{\delta}^*_m) = n^*$. In the interval $\delta \in [\underline{n}^*_m, 1)$ this implies that n^* is a strict lower boundary for competitive industry sizes. From this, it then follows that the competitive dynamics on this interval always decrease the number of firms either through merger or exit. After some firms have left and $n = n^c_m$, the remaining firms form a stable cartel and earn strictly positive profits.

3. Medium discount factors in the range $\delta \in [\underline{\underline{\delta}}^*_m, \underline{\delta}^*_m)$. Following the same logic as above, any competitive industry in this range has at least $n > n^m$ firms in it. Therefore, the competitive industry forces reduce the number of firms. As long as the industry is competitive and $n > n^*$, firms will either merge or exit. For all $n \in (n^m, n^*]$ the merger force outweighs the entry force, decreasing n further until at some point $n = n^c_m$. Because $\delta \in [\underline{\underline{\delta}}^*_m, \underline{\delta}^*_m)$ it follows directly that $n^c_m \in [n^m, n^*)$.

To summarise, it was shown that any market which is initially competitive converges towards either the clearly defined collusive industry size n^c_m when $\delta \geq \underline{\underline{\delta}}^*_m$, or towards the competitive long run sustainable size n^m when $\delta < \underline{\underline{\delta}}^*_m$. These results take as a given the assumption that when firms reach a collusive agreement, the number of firms is fixed and further dynamic processes are excluded. It seems to be intuitively fine to assume that firms currently colluding would not want to attract CA attention on their industry by seeking clearance for a merger and it has been established in the forgone section that no firm would want to exit a collusive industry in which it earns strictly positive profits.

However, firms who are currently not in the industry might still have an incentive to enter and participate in the market.

To determine how this affects the dynamics in the model, the three ranges of δ are considered again, but this time it is assumed that industries start off colluding. Discount factors of $\delta < \delta_m^*(2)$ are not considered, as for these values no stable cartel can form by definition.

1. When $\delta \in [\delta_m^*(2), \underline{\underline{\delta_m^*}})$ and the industry is currently collusive, it follows directly that $n \in (2, n^m)$. There are then two possible outcomes for a potential entrant. On the one hand, the post entry size could be such that the industry is still collusive. That is the case when after entry $n \leq n_m^c$. In this case, any entrant can expect to earn positive profits of $\pi^c(n) > 0$ and will therefore enter. On the other hand, it is possible that after a firm has entered, n is too large for the industry to remain collusive. That is the case when after entry $n > n_m^c$. In this case, any entrant can expect to earn competitive profits of $\pi^*(n) > 0$. As $n \in (2, n^m)$ the profits are positive and hence the firm will enter.[15] As a consequence, the collusive agreement dissolves and the competitive industry dynamics as defined above lead to $n = n^m$ in the long run. This shows that for any $\delta < \underline{\underline{\delta_m^*}}$ collusion cannot be sustained in the long run and that any industry with such a discount factor will converge towards the competitive long run stability point n^m.

2. When $\delta \geq \underline{\delta_m^*}$, the maximum stable cartel size is in the range of $n_m^c \in [n^*, \overline{n}_m^c)$. Given that an industry is collusive, there are again two possible outcomes for a potential entrant. If after entry collusion is still sustainable, i.e. when after entry $n \leq n_m^c$, the entrant can expect to join the cartel and earn strictly positive profits of $\pi^c(n)$. However, if entry pushes the industry size above the maximum stable cartel size n_m^c, the firm has to expect competitive profits, which are negative because $n_m^c \geq n^*$. Therefore, entry will not happen. Combining the two arguments then means that any industry with $\delta > \underline{\delta_m^*}$ has entry until the maximum stable cartel size n_m^c and no entry above this. Together with the competitive dynamics, this implies that any industry which values future profits enough to sustain collusion above the competitive zero profit number of firms will converge to some uniquely

[15] The competitive profits in this region are positive because it is assumed that entry is small and the integer problem is assumed away. Therefore it is expected that industry size after entry is in the immediate proximity of the n which solves $\delta = \delta_m^*(n)$. For the highest possible n in the given range, this would be n^m and for $n^m + \epsilon$, $\epsilon > 0$ the competitive profits are positive.

defined industry size n_m^c for which collusion can just be sustained.

3. When $\delta \in [\underline{\underline{\delta_m^*}}, \underline{\delta_m^*})$, the maximum stable cartel size is $n_m^c \in [n^m, n^*)$. Similarly to the other two cases, any firm aiming to join the industry faces two outcomes. Firstly, it is possible that the post entry size is $n \leq n_m^c$, which allows for stable collusion. In this case, any entrant expects to earn $\pi^c(n) > 0$ and will therefore join. Secondly, when the post entry size is such that $n > n_m^c$, entry renders collusion unstable and an entrant has to expect competitive profits. For industry sizes in the direct proximity of $n \in (n^m, n^*)$, these profits are positive $\pi^*(n) > 0$ which means that entry pays off and firms join the market. As a result, the firms revert to competing against each other and thus, the competitive industry dynamics defined above hold. This means that because $n \in (n^m, n^*)$, the merger force outweighs the entry force and therefore it can be expected that the industry size will shrink over time. This can have either of two consequences. On the one hand, it is possible to argue that firms anticipate the instability of collusive agreements and therefore will not attempt to collude again. This would mean that the competitive dynamics lead towards the long run sustainable industry size n^m and firms earn positive profit $\pi^*(n^m) = \pi_m$. On the other hand though, it is possible to argue that after the industry size has decreased, there will be some point at which firms decide to attempt forming a cartel again. If this was the case, additional entry over time would break collusion up again. This would lead to some unstable equilibrium around the maximum stable cartel size n_m^c: for some time, the industry will be collusive until excessive entry renders collusion impossible. This would be followed by a period of consolidation, which could lead to collusion emerging again. Both arguments are characterized by collusive periods which are followed by periods of mergers.

   This is a major novelty of this model specification. By taking into account the competitive dynamics resulting of mergers and entry, endogenous breakdown of collusion is followed by mergers.

Dynamic Result 4: When mergers are taken into account and $\delta \in [\underline{\underline{\delta_m^*}}, \underline{\delta_m^*})$ endogenous breakdown of collusion leads to mergers.

A graphic representation of the collusive and competitive industry forces is given in Fig. 6. The graph plots the critical discount factor $\delta_m^*(n)$ as a function of the market size. Tuples of (n, δ) which are above the critical discount factor show parameter combinations for which collusion is stable, while those tuples below it show competitive markets. Red arrows highlight the areas in which either exit or merger decrease the number of firms. Blue arrows show areas in which the merger force is stronger than the entry force and green arrows areas where firms enter the industry.

To summarize, the dynamic model with entry, exit and merger predict outcomes depending on an industry's discount factor δ. When $\delta \in (0, \underline{\underline{\delta_m^*}})$ the number of firms will converge towards the long run sustainable number of firms in a competitive industry n^m and earn strictly positive profits π_m. When $\delta \in [\underline{\underline{\delta_m^*}}, \underline{\delta_m^*})$, the prediction is unclear. One possible outcome is that firms attempt collusion at the maximum stable cartel size n^m, which fails due to additional entry, after which firms return to competition and n converges towards n^m. The other possible outcome is that the number of firms cycles around the maximum stable cartel size and the industry is characterised by periodical collusion and competition. In both cases, endogenous collusive break down is followed by mergers.

Overall, it has been shown that taking mergers into account has profound implications on the ability of cartels to reach stable agreements which all point into the same direction: they make collusion harder to sustain than in the static model. To begin with, contrary to the purely static case, it was shown that the dynamic entry force can deter collusion for discount factors of $\delta \in [\delta_m^*(2), \underline{\underline{\delta_m^*}})$. For these values of δ small cartels are stable in a static environment. However, when entry is allowed for, collusive agreements cannot be sustained and the industry size increases to the competitive long run sustainable size n^m. Additionally, when $\delta \geq \underline{\underline{\delta_m^*}}$, collusion is less stable in the merger model than in the static model for two reasons. Firstly, firms require higher discount factors to sustain collusion when merger dynamics are included. That is because firms inside the cartel take into account their expected punishment profits in the case of defection are higher. This makes defecting more attractive, which means that less firms can collude sustainably. It also means that the maximum stable cartel size is smaller in the dynamic model. Secondly, for discount factors in the range $\delta \in [\underline{\underline{\delta_m^*}}, \underline{\delta_m^*})$, cartels are either unstable, or the industries cycle between collusive and competitive periods. That is because entry and merger forces are active for those maximum stable cartel sizes which correspond to this range of discount factors. Under competition, merger forces will decrease the number of firms until collusion can be sustained. Under collusion, entry will increase the number of firms until the cartel is rendered unstable.

3 Conclusion

The model discussed in this paper nested the analysis of collusive equilibria into a competitive environment with dynamic forces of entry, exit, and merger, allowing firms to endogenously choose between the practices. Doing so allowed to consider how endogenously determined industry sizes can affect collusion.

In a first step, a static framework was defined and two standard results of collusive theory were replicated. Firstly, that relative to competition, it is always profitable to be inside a cartel. It was shown that from this, it follows secondly, that there is some valuation of future profits (discount factor) above which collusion is stable for each industry size. This implies that overall unprofitable cartels can be sustained. However, standard economic theory suggests that unprofitable firms have an incentive to leave a market in the long run. As this cannot be modelled in the static framework, the paper then adds industry dynamics into the analysis.

In a second step, entry and exit are therefore introduced into the model. The resulting competitive zero profit equilibrium implies that any firm's alternative to collusion are not static competitive profits for a given number of firms, but zero long run profits. Therefore, unprofitable cartels cannot be sustained any longer because the cartel firms anticipate that they would be better off in the competitive equilibrium or by leaving the industry. This leads to the first dynamic result that unprofitable cartels cannot be sustained when competitive free entry and exit are taken into account. It is then shown that entry renders collusion unstable when the industry's discount factor is not high enough to sustain collusion for the competitive equilibrium industry size. However, when the discount factor is above this critical threshold profitable collusion can be sustained. In terms of industry dynamics it is then shown that for each discount factor δ there is some unique equilibrium market size and that the industry converges towards this size. From this, one can identify two kinds of industries: firstly, the ones with relative low discount factors in which collusion can never be sustained due to high competitive pressure through entry. Secondly, the industries in which the valuation of future profits is high enough to sustain stable and profitable collusion. In this latter industry, the size will be such that no firm has an incentive to enter and break up the cartel as this would lead to negative profits.

This is a drastically different result to much of the standard literature, as it implies that strictly profitable cartels can be sustained under the presence of free entry and exit.[16]

In the last step, the possibility of mergers between competitive firms is added into the dynamic entry and exit model. It is argued that a long run stability point is given for the industry size which equalises the gains of entry with those of mergers. This constitutes a new definition of an entry-merger stability point. For this long run sustainable size, firms earn strictly positive profits. A direct implication is that the expected future profits of a firm inside a cartel, which expects the breakdown of the cartel, are strictly positive. Therefore, any sustainable collusive agreement needs to earn profits exceeding a strictly positive outside option. Hence, some profitable cartels, which were stable under the entry/exit specification are not stable when mergers are taken into account because they don't earn profits above the outside option.

In this specification, three separate kinds of industry are identified. Firstly, those with a discount factor below the one necessary to sustain collusion at the long run sustainable industry size will always be competitive, because entrants can break up collusion and still earn positive profits. Secondly, there are the discount factors just above this one, but below the one necessary to sustain collusion at the competitive zero profit industry size. In these industries firms from the outside always have an incentive to join a collusive industry, even if it breaks up collusion. However, once an industry has become competitive, mergers will decrease the number of firms again, thereby making collusion possible again. This means two things: a) endogenous collusive breakdown is followed by mergers, and b) endogenously motivated mergers lead to an industry structure under which collusion is possible. The equilibrium for these industries is not stable, but characterised by cycles of competitive and collusive periods. Finally, those industries with the highest discount factors converge towards stable collusive equilibria at the maximum stable cartel size.

For the last model specification this means that, as long as the discount factor is high, both exit and merger facilitate collusion. Furthermore, there are three ways in which the breakdown of collusion can trigger mergers. Firstly, for the medium defined range of discount factors, this dynamic is part of the endogenous processes as discussed. Secondly, potential exogenous shocks could lead to cartel break up. Some examples of shocks could include decreased discount factors, CA intervention into an industry or if key stakeholders involved in the collusive process switch jobs and therefore cannot ensure adherence to the collusive agreement any longer. Any breakdown of collusion for these industries would

[16] While the model defined here has recurring fixed costs of operating in the industry, entry and exit can be interpreted as there are no one-time, irretrievable investment costs for entering the industry. Compare for this, Baumol and Willig (1981): "(...)fixed costs do not constitute barriers to entry"(p.405).

be followed by either merger or exit. Thirdly, as real world processes aren't as clearly defined as theoretical models, one could argue that firms make mistakes. An example of this could be that firms enter in too large numbers. If wrongly calculated entry broke up collusion, mergers or exit would be the direct consequence.

Overall, the model presented in this paper combines the analysis of collusion with the dynamic forces of entry, exit, and mergers. It is the first model to allow firms to endogenously choose between all four of these options. The results imply that i) profitable collusion can be sustained even under free entry, ii) endogenous mergers can facilitate collusion and iii) endogenous breakdown can lead to mergers in an industry. Additionally, the paper identifies a new definition of a competitive long run stability point under merger and entry.

Further research into this area could involve a more active CA, analyse welfare implications of the firm decisions, and setting out a more general form of the very specific linear cost Cournot model analysed here.

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4 Appendix A

4.1 Derivation of profit functions

There are four profit functions which have to be derived in order for this model to work.

1. Competitive profits $\pi^*(n)$
2. Collusive profits $\pi^c(n)$
3. Deviator profits $\pi^d(n)$
4. Profits of all other firms in the cartel, given that one firm deviated $\pi^{-d}(n)$

Firstly, derive the competitive profits $\pi^*(n)$. Each firm maximises profits, given the output of other firms

$$\max_{q_i} q_i\Big[P(Q)-c\Big]-f=\max_{q_i} q_i\Big[a-c-q_i-q_{-i}\Big]-f \tag{41}$$

From the first order condition, it follows for each firm that the optimal own output q_i, given competitors output q_{-i} is equal to

$$q_i^R=\frac{a-c-q_{-i}}{2}. \tag{42}$$

All firms are symmetrical and hence will set the same output in equilibrium $q_i=q^*(n)$ $\forall i=1,2,...,n$. It follows that

$$\begin{aligned} q^*(n)&=\frac{a-c-(n-1)q^*(n)}{2}\\ &=\frac{a-c}{(n+1)}. \end{aligned} \tag{43}$$

Plugging the equilibrium quantities back into the inverse demand function $P(Q)=a-Q$, where $Q=n*q_i$, the equilibrium price $P^*(n)=P(q^*(n))$ follows as

$$P^*(n)=\frac{a-nc}{n+1}. \tag{44}$$

Finally, equilibrium profits $(P^*(n)-c)q^*(n)-f$ follow as

$$\pi^*(n)=\left(\frac{a-c}{n+1}\right)^2-f \tag{45}$$

Secondly, derive the collusive profits. If there is a cartel, firms maximise joint profits

$$\max_Q Q\Big[P(Q)-c\Big]-nf=\max_Q Q\Big[a-c-Q\Big]-nf. \tag{46}$$

The first order condition leads to the optimal total cartel quantity

$$Q^c=\frac{a-c}{2}. \tag{47}$$

Individual production targets are then $q^c(n)=Q^c/n=\frac{a-c}{2n}$. The market price in a collusive market is $P^c(Q^c)=\frac{a+c}{2}$ which leads to per firms profits of

$$\pi^c=\frac{(a-c)^2}{4n}-f \tag{48}$$

Thirdly, derive deviator profits. A deviator takes as given that all other firms in the cartel set the collusive quantities and responds optimally to this:

$$\max_{q_i} q_i\Big[P(Q)-c\Big]-f=\max_{q_i} q_i\Big[a-c-q_i-(n-1)q^c(n)\Big]-f=\max_{q_i} q_i\Big[\frac{(n+1)(a-c)}{2n}-q_i\Big]-f. \tag{49}$$

From the first order condition, optimal deviator quantities immediately follow as

$$q^d(n)=\frac{(n+1)(a-c)}{4n}. \tag{50}$$

The market price, given deviation is then

$$P^d(n)=\frac{(n+1)(a-c)}{4n}+c, \tag{51}$$

which leads directly to deviator profits of

$$\pi^d(n)=\Big(\frac{(n+1)(a-c)}{4n}\Big)^2-f \tag{52}$$

Finally, the profits of other cartel members, given one firm deviated are derived. These are important to evaluate if cartel members are actually worse off given that one firm deviated. The market price given deviation and the per firm cartel output are defined above. It follows that

$$\pi^{-d}(n) = q^c(P^d - c) - f = q^c(P^c - c)\frac{n+1}{2n} - f < \pi^c(n). \tag{53}$$

5 Appendix B

5.1 Proof Critical Discount Factor Increasing In Number Of Firms

The derivative of the critical discount factor $\delta_e^*(n)$ with respect to n is given by

$$\frac{\partial \delta_e^*(n)}{\partial n} = \frac{4(a-c)^2(n-1)\Big[(n+1)^2(a-c)^2 - 8nf\Big]}{\Big[(n+1)^2(a-c)^2 - 16n^2 f\Big]^2}, \tag{54}$$

where the denominator is always positive. The numerator is positive iff

$$\begin{aligned} &(n+1)^2(a-c)^2 - 8nf > 0 \\ \iff\ &(n+1)^2\frac{(a-c)^2}{4f} > 2n \\ &\iff (n+1)^2\overline{n}^c > 2n, \end{aligned} \tag{55}$$

where the last step follows from the definition of $\overline{n}^c = \frac{(a-c)^2}{4f}$. As $\overline{n}^c > n^*$ which in turn is assumed to be such that $n^* \geq 2$, it will always hold that $(n+1)^2\overline{n}^c > 2n$. Hence, $\frac{\partial \delta_e^*(n)}{\partial n} > 0$.

6 Appendix C

6.1 Discussion of Merger Condition

The merger condition is given by

$$\begin{aligned} &\pi^*(n-1) \geq 2\pi^*(n), \\ \iff\ &\left(\frac{a-c}{n}\right)^2 - 2\left(\frac{a-c}{n+1}\right)^2 \geq -f, \end{aligned} \tag{56}$$

where the RHS is strictly decreasing in f and therefore the function holds more likely for large f. Furthermore, the analysis of the RHS shows two properties. Firstly, as the LHS is only positive $\forall\ n < 2.414$. This follows directly from

$$\begin{aligned}(a-c)^2\Big[1/n^2 - 2/(n+1)^2\Big] &= 0\\ \iff \Big[(n+1)^2 - n^2\Big] &= 0\\ n &= 2.414\end{aligned} \tag{57}$$

This implies a that any merger from 2 to 1 firms also pays off. Secondly, the LHS is decreasing up to a minimum value at $n = 3.847$ after which it is strictly increasing in n. This follows from

$$\frac{\partial\Big(\frac{a-c}{n}\Big)^2 - 2\Big(\frac{a-c}{n+1}\Big)^2}{\partial n} = 2(a-c)^2[2/(n+1)^3 - 1/n^3], \tag{58}$$

which is positive if the value in the brackets is positive, i.e. as long as $2n^3 - (n+1)^3 > 0$ which solves for $n > 3.847$. The second derivative is given by $2(a-c)^2[2/n^4 - 6/(n+1)^4]$ which is positive $\forall\ 0 < n < 5.285$. Hence, for $n = 3.847$ the LHS is at a minimum and the function increases thereafter.

Assuming that the merger condition always holds implies

$$\Big(\frac{a-c}{n}\Big)^2 - 2\Big(\frac{a-c}{n+1}\Big)^2 \geq -f. \tag{59}$$

From the discussion above it follows that the LHS is minimal for $n = 3.847$. Plugging this into the function leads to

$$0.01756(a-c)^2 \leq f. \tag{60}$$

Hence for all $f \geq 0.01756(a-c)^2$ a merger always pays off.

5 Conclusion

This thesis has addressed how endogenous size processes affect the stability and price setting behaviour of cartels. In doing so, it has contributed to research by expanding the existing industrial economics literature along two dimensions. Firstly, in Chapters One and Two, the common assumption that all firms in an industry have to be inside a cartel for it to be stable is relaxed and the consequences of this on stability and price are discussed. Secondly, Chapter Three challenges the assumption that a stable cartel requires the number of firms in an industry to be fixed by presenting a model in which stable collusion can occur in a dynamic industry with entry, exit and merger forces. In combination, the chapters have emphasized the importance of taking into account possible endogenous size variations when evaluating the stability or price setting of cartels.

Specifically, Chapter One discussed how collusive stability depends on two dimensions: the level of horizontal product differentiation and the level of costs associated with collusion. Although there is a wide range of literature analysing both of these dimensions, this is the first model to combine them under the assumption that a cartel can consist of less than all firms in the industry. Three main results are derived. Firstly, when collusion is costless, small cartels tend to become more stable as products become closer substitutes, while large cartels tend to become less stable. Secondly, although smaller collusive agreements are more stable than larger ones when collusion is costless, this result does not apply when costs of collusion are strictly positive. When that is the case, cartel stability needs to be compared on a case by case basis. Finally, it is shown that when the cartel size is determined endogenously, there are cases in which larger costs of collusion can result in more stable cartels. This finding contrasts the standard result that collusion is always harder to sustain when it is more costly. However, the costs modelled in this framework are assumed to be fixed and further insight may be gained by considering more elaborate cost functions. For example, costs of collusion could vary with the degree of product differentiation or be linked to the number of firms in the cartel. Additionally, it is possible to model the costs of collusion as the expected penalties that are imposed on cartels by a CA.

This latter topic is what motivated Chapter Two, which compares the effect that three different penalty regimes have on the price setting of cartels, given that the size of a cartel is determined endogenously. The regimes considered are penalties based on profits, overcharges and revenues. In an infinitely repeated Bertrand competition model over true substitutes, it is shown that the penalty regime influences price setting in two ways. Firstly, they directly affect the price set by a cartel for a given size. Secondly, they affect the endogenous cartel size and thereby indirectly influence price setting. The direct effect for all these penalty regimes has been studied previously by Katsoulacos et al. (2015). However, in their model all firms in an industry have to be part of the cartel and thus the indirect effect of a penalty is not taken into account. At the same time, Bos and Harrington (2015) compare the indirect and direct effect for profits based penalties in a capacity constraint Bertrand model, but do not undertake a comparison of different penalty regimes. The model presented in Chapter Two is thus the first paper to compare the direct and indirect effects for multiple penalty regimes. The results show that the profits based penalties have a weakly negative overall price effect. For overcharge based penalties it is found that imposing a penalty will always decrease the price compared to a situation without a penalty. However, it is also possible that an increase in the penalty rate increases the prices charged in the industry. Revenue based penalties are shown to have an ambiguous overall price effect. When comparing the penalty regimes on the basis that they all deter cartels over the same group of products, it is shown that overcharge based penalties always lead to the lowest prices, followed in the most cases by profits based penalties and then revenue based penalties, though it is possible that in some cases revenue based penalties lead to lower prices than profits based penalties. While this methodology of comparing the penalty regimes provides a first intuition about how the prices under different penalty regimes compare to one another, it is also possible to use further methodologies to gain a deeper understanding. More specifically, a next step in the analysis could be to equalize the penalties such that the expected penalty is equal for firms who create the same harm to the economy.

Chapter Three then analysed the interdependencies between collusion and the dynamic processes of entry, exit and merger, which determine the number of firms in an industry. In much of the literature on cartels these dynamic processes are excluded and the industry size is fixed. Some authors argue that entry into collusive industries would either break up collusion (e.g. Ivaldi et al. (2003)) or that no firm would enter a collusive industry because the incumbent firms play entry deterring strategies. The research presented in this chapter adds to the existing literature by challenging this idea and establishing a framework in which stable collusion can be achieved in an industry in which entry, exit and merger forces are present. It is shown that these dynamic forces are not distinct phenomena to collusion, rather they can all occur sequentially in the same industry, potentially leading it to a collusive equilibrium in which no entry deterring strategies are played. This is in contrast to the previous literature. In arriving at this long run equilibrium, the model also mimics two empirical observations. Firstly, the observation that the breakdown of collusive agreements can be followed by increased merger activity. Secondly, that mergers may make collusion more likely to be sustainable. Finally, the model presents a new notion of a long run competitive stability point under merger and entry forces in which firms earn long run positive profits. A next step for future research into this area could be to add an active competition authority into the framework to analyse in a more elaborate setting how mergers and cartels are connected.

Overall, this PhD thesis has researched how the size of a cartel affects its stability and price setting. While the first two chapters have addressed this question in an industry with a fixed number of firms but endogenous cartel size, the third chapter has focussed on dynamics which altered the number of firms in the industry. It was shown that endogenous cartel size can be an important dimension to consider when making predictions about cartel behaviour.