Supplementary Material: Appendices for ‘Optimal Universal and Categorical Benefit Provision with Classification Errors and Imperfect Enforcement’

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A Proofs under the fixed budget assumption

A.1 Proof of Proposition 1

Solving the maximisation problem in (8) of the main article yields the following first order conditions (henceforth FOC) characterising $\hat{B}^P$ and $\hat{C}^P$:

$$\begin{aligned} &(B): \quad \theta u_x(\hat{B}^P + \hat{C}^P, 1) + (1-\theta)\int_0^\infty v_M(\omega, \hat{B}^P)dF(\omega) \leq \hat{\lambda}^P \;\; ; \; \hat{B}^P \geq 0 \\ &(C): \quad u_x(\hat{B}^P + \hat{C}^P, 1) \leq \hat{\lambda}^P \;\; ; \; \hat{C}^P \geq 0 \end{aligned} \tag{A.1}$$

The pairs of inequalities hold with complementary slackness and λ denotes the shadow price of public expenditure. Given that the budget constraint must be exhausted (i.e. $B+\theta C = \beta$), we now test the following two hypotheses:

(i) $\hat{B}^P = \beta, \hat{C}^P = 0$ **(Pure universal system)**

If $\hat{B}^P = \beta$ and $\hat{C}^P = 0$ the FOCs in (A.1) become:

$$\theta u_x(\beta, 1) + (1-\theta)\int_0^\infty v_M(\omega, \beta)dF(\omega) = \lambda$$
$$u_x(\beta, 1) \leq \lambda$$

Combining both equations gives the contradictory statement:

$$\int_0^\infty v_M(\omega, \beta)dF(\omega) \geq u_x(\beta, 1)$$

It cannot hold that the average smvi of the able weakly exceeds that of the unable and so the assertion that $\hat{C}^P = 0$ must be false. Instead, we must have $\hat{C}^P > 0$. The shadow price of public expenditure is therefore equal to the smvi for the unable at the optimum.

(ii) $\hat{B}^P = 0, \hat{C}^P = \beta/\theta$ **(Pure targeted system)**

If $\hat{B}^P = 0$ and $\hat{C}^P = \beta/\theta$ the FOCs in (A.1) become:

$$\theta u_x\left(\frac{\beta}{\theta}, 1\right) + (1-\theta)\int_0^\infty v_M(\omega, 0)dF(\omega) \leq \lambda$$
$$u_x\left(\frac{\beta}{\theta}, 1\right) = \lambda$$

Combining both equations gives:

$$\int_0^\infty v_M(\omega, 0)dF(\omega) \leq u_x\left(\frac{\beta}{\theta}, 1\right)$$

The left side is independent of β, whilst the right side is unambiguously decreasing in β. Further, given that $\lim_{x\to 0} u_x(x,l) = +\infty$ it follows from the intermediate value theorem that there is a critical budget level $\bar{\beta}^P$ satisfying:

$$\int_0^\infty v_M(\omega, 0)dF(\omega) \equiv u_x\left(\frac{\bar{\beta}^P}{\theta}, 1\right)$$

■

A.2 Proof of Proposition 2a

Solving the maximisation problem in (12) in the main article yields the following FOCs for the optimal benefits, $\hat{B}^N$ and $\hat{C}^N$:

$$\begin{aligned}(B):\ &\theta(1-p_I)u_x(\hat{B}^N + \hat{C}^N, 1) + (1-\theta)p_{II}\int_0^\infty v_M(\omega, \hat{B}^N + \hat{C}^N)dF(\omega) \\ &+ \theta p_I u_x(\hat{B}^N, 1) + (1-\theta)(1-p_{II})\int_0^\infty v_M(\omega, \hat{B}^N)dF(\omega) \leq \hat{\lambda}^N \quad ;\ \hat{B}^N \geq 0\end{aligned} \tag{A.2}$$

$$\begin{aligned}(C):\ &\theta(1-p_I)u_x(\hat{B}^N + \hat{C}^N, 1) + (1-\theta)p_{II}\int_0^\infty v_M(\omega, \hat{B}^N + \hat{C}^N)dF(\omega) \\ &\leq \hat{\lambda}^N[\theta(1-p_I) + (1-\theta)p_{II}] \quad ;\ \hat{C}^N \geq 0\end{aligned} \tag{A.3}$$

where the pairs of inequalities hold with complementary slackness.

With regard to the above FOCs, we test the following two hypotheses:

(i) $\hat{B}^N = \beta, \hat{C}^N = 0$ (Pure universal system)

If $\hat{C}^N = 0$ then it must hold from the budget constraint that $\hat{B}^N = \beta$. In this case, the FOCs in (A.2) and (A.3) reduce to:

$$\theta u_x(\beta, 1) + (1-\theta)\int_0^\infty v_M(\omega, \beta)dF(\omega) = \lambda$$
$$\frac{\theta(1-p_I)u_x(\beta, 1) + (1-\theta)p_{II}\int_0^\infty v_M(\omega, \beta)dF(\omega)}{[\theta(1-p_I) + (1-\theta)p_{II}]} \leq \lambda$$

Combining both equations gives:

$$\frac{(1-p_I)\left[\lambda-(1-\theta)\int_0^\infty v_M(\omega,\beta)dF(\omega)\right]+(1-\theta)p_{II}\int_0^\infty v_M(\omega,\beta)dF(\omega)}{[\theta(1-p_I)+(1-\theta)p_{II}]}\leq\lambda$$
$$\Leftrightarrow\frac{(1-p_I)(1-\theta)\left[\lambda-\int_0^\infty v_M(\omega,\beta)dF(\omega)\right]}{p_{II}(1-\theta)\left[\lambda-\int_0^\infty v_M(\omega,\beta)dF(\omega)\right]}\leq 1$$
$$\Leftrightarrow 1-p_I\leq p_{II}$$

Given our discriminatory power assumption (i.e. $p_I+p_{II}\leq 1$) this condition can only hold with equality, and thus when $p_I+p_{II}=1$. It follows that $\hat{C}^N=0$ only if the test awarding C has no discriminatory power. Otherwise, $\hat{C}^N>0\ \forall\ p_I+p_{II}<1$.

It immediately follows from (A.3) that $\forall\ p_I+p_{II}<1$:

$$\frac{\theta(1-p_I)u_x(\hat{B}^N+\hat{C}^N,1)+(1-\theta)p_{II}\int_0^\infty v_M(\omega,\hat{B}^N+\hat{C}^N)dF(\omega)}{\theta(1-p_I)+(1-\theta)p_{II}}=\hat{\lambda}^N \tag{A.4}$$

Substituting (A.4) into (A.2) one can readily show that $\forall\ p_I+p_{II}<1$:

$$\frac{\theta p_I u_x(\hat{B}^N,1)+(1-\theta)(1-p_{II})\int_0^\infty v_M(\omega,\hat{B}^N)dF(\omega)}{\theta p_I+(1-\theta)(1-p_{II})}$$
$$\leq\hat{\lambda}^N=\frac{\theta(1-p_I)u_x(\hat{B}^N+\hat{C}^N,1)+(1-\theta)p_{II}\int_0^\infty v_M(\omega,\hat{B}^N+\hat{C}^N)dF(\omega)}{\theta(1-p_I)+(1-\theta)p_{II}}\quad;\ \hat{B}^N\geq 0 \tag{A.5}$$

where the pairs of inequalities hold with complementary slackness. The left side corresponds to $\bar{\varphi}^N_{NR}$ from the main article, whilst the right side is $\bar{\varphi}^N_R$.

(ii) $\hat{B}^N=0,\hat{C}^N=\beta/[\theta(1-p_I)+(\theta)p_{II}]$ **(Pure targeted system)**

First off, it must hold that $\hat{B}^N>0$ whenever $p_I>0$ because $\lim_{x\to 0}u_x(x,l)=+\infty$. Suppose otherwise, then the left side of either (A.5) or (A.2) 'blows up' to infinity whenever some unable individuals have zero income to consume. Next, if $p_I=0$ but $p_{II}\geq 0$ then from (A.5):

$$\int_0^\infty v_M(\omega,0)dF(\omega)\leq\frac{\theta u_x\left(\frac{\beta}{\theta+(1-\theta)p_{II}},1\right)+(1-\theta)p_{II}\int_0^\infty v_M\left(\omega,\frac{\beta}{\theta+(1-\theta)p_{II}}\right)dF(\omega)}{\theta+(1-\theta)p_{II}}$$

The left side is independent of β, whilst the right side is unambiguously decreasing in β. Suppose that $\beta\to 0$, then $\lim_{\beta\to 0}u_x(\beta/[\theta+(1-\theta)p_{II}],1)=+\infty$ such that the right side approaches $+\infty$ and the condition must hold with strict inequality. It once more follows from the intermediate value theorem that there must be a critical budget level $\bar{\beta}^N$ for which the condition holds with equality. Formally, $\bar{\beta}^N$ satisfies:

$$\underbrace{\int_0^\infty v_M(\omega,0)dF(\omega)}_{u_x\left(\frac{\bar{\beta}^P}{\theta},1\right)} \equiv \frac{\theta u_x\left(\frac{\bar{\beta}^N}{\theta+(1-\theta)p_{II}},1\right)+(1-\theta)p_{II}\int_0^\infty v_M\left(\omega,\frac{\bar{\beta}^N}{\theta+(1-\theta)p_{II}}\right)dF(\omega)}{\theta+(1-\theta)p_{II}}$$

■

A.3 Proof of Proposition 2b

To assist us in deriving the welfare effects of classification errors when $p_I + p_{II} < 1$ we use (A.5) to derive an ordering of each average smvi at the optimum. Indeed, from the right side of (A.5) it must hold that:

$$u_x(\hat{B}^N+\hat{C}^N,1) \geq \hat{\lambda}^N > \int_0^\infty v_M(\omega,\hat{B}^N+\hat{C}^N)dF(\omega) \tag{A.6}$$

where $u_x(\hat{B}^N+\hat{C}^N,1)=\hat{\lambda}$ if and only if $p_{II}=0$. By diminishing marginal utility of income we know that $u_x(\hat{B}^N,1) > u_x(\hat{B}^N+\hat{C}^N,1)$ and thus $u_x(\hat{B}^N,1) > \hat{\lambda}^N$. Given this, the left side of (A.5) can only hold if $\int_0^\infty v_M(\omega,\hat{B}^N)dF(\omega) \leq \hat{\lambda}^N$, where $\int_0^\infty v_M(\omega,\hat{B}^N)dF(\omega) < \hat{\lambda}^N$ if $p_I > 0$ (because $\hat{B}^N > 0$). Putting this all together we have:

$$u_x(\hat{B}^N,1) > u_x(\hat{B}^N+\hat{C}^N,1) \geq \hat{\lambda}^N \geq \int_0^\infty v_M(\omega,\hat{B}^N)dF(\omega) > \int_0^\infty v_M(\omega,\hat{B}^N+\hat{C}^N)dF(\omega) \tag{A.7}$$

The immediate implication is that the average smvi of unable individuals will exceed that of able individuals at the optimum whenever classification errors are made.

To proceed, we note the following standard property of concave functions:

$$u_x(B,1)\cdot C > u(B+C,1)-u(B,1) > u_x(B+C,1)\cdot C \tag{A.8}$$

From (A.7) and (A.8) it must therefore hold that:

$$\begin{aligned}\frac{\partial V^N}{\partial p_I} &= \theta\left\{\left[u(\hat{B}^N,1)-u(\hat{B}^N+\hat{C}^N,1)\right]+\hat{\lambda}^N\hat{C}^N\right\} < \theta\hat{C}^N\left[\hat{\lambda}^N-u_x(\hat{B}^N+\hat{C}^N,1)\right] \leq 0\\ \frac{\partial V^N}{\partial p_{II}} &= (1-\theta)\left\{\int_0^\infty\left[v(\omega,\hat{B}^N+\hat{C}^N)-v(\omega,\hat{B}^N)\right]dF(\omega)-\hat{\lambda}^N\hat{C}^N\right\}\\ &< \theta\hat{C}^N\left[\int_0^\infty v_M(\omega,\hat{B}^N)dF(\omega)-\hat{\lambda}^N\right] \leq 0\end{aligned} \tag{A.9}$$

where in deriving $\partial V^N/\partial p_I$ and $\partial V^N/\partial p_{II}$ we have applied the envelope theorem for constrained optimisation. ■

A.4 Proof of Proposition 3a

Solving the optimisation problem in (26) in the main article yields the following FOCs characterising $\hat{B}^F$ and $\hat{C}^F$, respectively:

$$\begin{aligned}(B): \quad & [\theta(1-p_I) + (1-\theta)F(\bar{\bar{\omega}})p_{II}]u_x(\hat{B}^F + \hat{C}^F, 1) + \theta p_I u_x(\hat{B}^F, 1) \\ & + (1-\theta)\left\{(1-p_{II})\int_0^{\bar{\bar{\omega}}} v_M(\omega, \hat{B}^F)dF(\omega) + \int_{\bar{\bar{\omega}}}^{\infty} v_M(\omega, \hat{B}^F)dF(\omega)\right\} \\ & \leq \hat{\lambda}^F\left[1 + (1-\theta)f(\bar{\bar{\omega}})p_{II}\bar{\bar{\omega}}_B\hat{C}^F\right] \quad ; \quad \hat{B}^F \geq 0\end{aligned} \tag{A.10}$$

and

$$(C): \quad u_x(\hat{B}^F + \hat{C}^F, 1) \leq \hat{\lambda}^F\left[1 + \frac{(1-\theta)f(\bar{\bar{\omega}})p_{II}\bar{\bar{\omega}}_C\hat{C}^F}{\theta(1-p_I) + (1-\theta)F(\bar{\bar{\omega}})p_{II}}\right] \quad ; \quad \hat{C}^F \geq 0 \tag{A.11}$$

where the pairs of inequalities hold with complementary slackness and λ is the shadow price of public expenditure. Note that changes in the integral limits of the welfare function cancel (an application of the Leibniz rule using the fact that $v(\bar{\omega}, B) = u(B + C, 1)$).

We proceed to test the following two hypotheses:

(i) $\hat{B}^F = \beta$, $\hat{C}^F = 0$ **(Pure universal system)**:

Suppose that $\hat{C}^F = 0$ - and thus $\hat{B}^F = \beta$ - such that $\bar{\bar{\omega}}(\beta, 0) = \bar{\omega}(\beta)$. It follows $\forall\ \omega \leq \bar{\bar{\omega}}(\beta, 0) = \bar{\omega}(\beta) \ :\ H^*(\omega, \beta) = 0 \Rightarrow v(\omega, \beta) = u(\beta, 1) \Rightarrow v_M(\omega, \beta) = u_x(\beta, 1)$. The FOCs (A.10) and (A.11) therefore become:

$$[\theta + (1-\theta)F(\bar{\omega}(\beta))]u_x(\beta, 1) + (1-\theta)\int_{\bar{\omega}(\beta)}^{\infty} v_M(\omega, \beta)dF(\omega) = \lambda,$$
$$u_x(\beta, 1) \leq \lambda.$$

Combining these equations implies the contradictory statement:

$$\frac{1}{1 - F(\bar{\omega}(\beta))}\int_{\bar{\omega}(\beta)}^{\infty} v_M(\omega, \beta)dF(\omega) \geq u_x(\beta, 1)$$

But the average smvi of those with $\bar{\omega}(\beta) < \omega$ cannot exceed the smvi of the unable when both receive the same unearned income. The assertion that $\hat{C}^F = 0$ must therefore be false. Instead, it must hold that $\hat{C}^F > 0\ \forall\ p_I + p_{II} \leq 1$.

An immediate implication of the result that $\hat{C}^F > 0\ \forall\ p_I + p_{II} \leq 1$ is that (A.11) can be written with equality and thus as:

$$u_x(\hat{B}^F + \hat{C}^F, 1) = \hat{\lambda}^F\left\{1 + \frac{(1-\theta)f(\bar{\bar{\omega}})\bar{\bar{\omega}}_C p_{II}\hat{C}^F}{[\theta(1-p_I) + (1-\theta)F(\bar{\bar{\omega}})p_{II}]}\right\} \geq \hat{\lambda}^F \tag{A.12}$$

Simple rearranging yields:

$$\hat{\lambda}^F = \frac{[\theta(1-p_I)+(1-\theta)F(\bar{\bar{\omega}})p_{II}]u_x(\hat{B}^F+\hat{C}^F,1)}{[\theta(1-p_I)+(1-\theta)F(\bar{\bar{\omega}})p_{II}]+(1-\theta)f(\bar{\bar{\omega}})p_{II}\bar{\bar{\omega}}_C\hat{C}^F} \tag{A.13}$$

Combining (A.12) with (A.10) gives:

$$\begin{aligned} &\theta p_I u_x(\hat{B}^F,1)+(1-\theta)\left\{(1-p_{II})\int_0^{\bar{\bar{\omega}}} v_M(\omega,\hat{B}^F)dF(\omega)+\int_{\bar{\bar{\omega}}}^{\infty} v_M(\omega,\hat{B}^F)dF(\omega)\right\} \\ &\leq [\theta(1-p_I)+(1-\theta)F(\bar{\bar{\omega}})]u_x(\hat{B}^F+\hat{C}^F,1) \\ &\cdot\left\{\frac{[\theta p_I+(1-\theta)(1-F(\bar{\bar{\omega}})p_{II})]+(1-\theta)f(\bar{\bar{\omega}})p_{II}(\bar{\bar{\omega}}_B-\bar{\bar{\omega}}_C)\hat{C}^F}{[\theta(1-p_I)+(1-\theta)F(\bar{\bar{\omega}})p_{II}]+(1-\theta)f(\bar{\bar{\omega}})p_{II}\bar{\bar{\omega}}_C\hat{C}^F}\right\}\ ;\ \hat{B}^F\geq 0 \end{aligned} \tag{A.14}$$

The left side is the aggregate smvi of categorical recipients, φ^F_{NR}. The first term on the right side the aggregate smvi of categorical recipients, φ^F_R, whilst one can readily show that the second term within curly braces corresponds to $-(1+\partial C^F/\partial B)$.

(ii) $\hat{B}^F=0, \hat{C}^F=C^F(0;\beta,\theta,p_I,p_{II})$[1] **(Pure targeted system)**:

If $p_I>0$ then the left side of (A.10)(or A.14) 'blows up' to infinity by (1) in the main article and, consequently, the assertion that $\hat{B}^F=0$ must be false. However, if $p_I=0$ then (A.14) becomes:

$$\begin{aligned} &(1-\theta)\left\{(1-p_{II})\int_0^{\bar{\bar{\omega}}} v_M(\omega,0)dF(\omega)+\int_{\bar{\bar{\omega}}}^{\infty} v_M(\omega,0)dF(\omega)\right\} \\ &\leq [\theta+(1-\theta)F(\bar{\bar{\omega}})p_{II}]u_x(C^F,1)\cdot\left\{\frac{(1-\theta)[1-F(\bar{\bar{\omega}})p_{II}]+(1-\theta)f(\bar{\bar{\omega}})p_{II}(\bar{\bar{\omega}}_B-\bar{\bar{\omega}}_C)C^F}{[\theta+(1-\theta)F(\bar{\bar{\omega}})p_{II}]+(1-\theta)f(\bar{\bar{\omega}})p_{II}\bar{\bar{\omega}}_C C^F}\right\} \end{aligned} \tag{A.15}$$

Whilst the left side is monotonically decreasing in β (because $\bar{\bar{\omega}}$ increases), the right side may be non-monotonically changing in β. Consequently, we can cannot in general define a *unique* critical budget size below which $\hat{B}^F=0$ and above which $\hat{B}^F>0$. This is made clear through the following numerical example.

Example Let $u(x,l)=[\alpha x^{\frac{1-\mathcal{E}}{\mathcal{E}}}+(1-\alpha)l^{\frac{1-\mathcal{E}}{\mathcal{E}}}]$, where $\mathcal{E}$ is the elasticity of substitution between leisure and consumption. Let $\alpha=0.6$ and $\mathcal{E}=0.5$. Also let individual productivities be exponentially distributed with rate parameter $\mu=3$. Figure 1 then illustrates how the left and right sides of (A.14) - i.e. the functions φ^F_{NR} and $\varphi^F_R\cdot-(1+\partial C^F/\partial B)$ from the main text - change with β (we set $\tau=0.3, p_I=0, p_{II}=0.3$).

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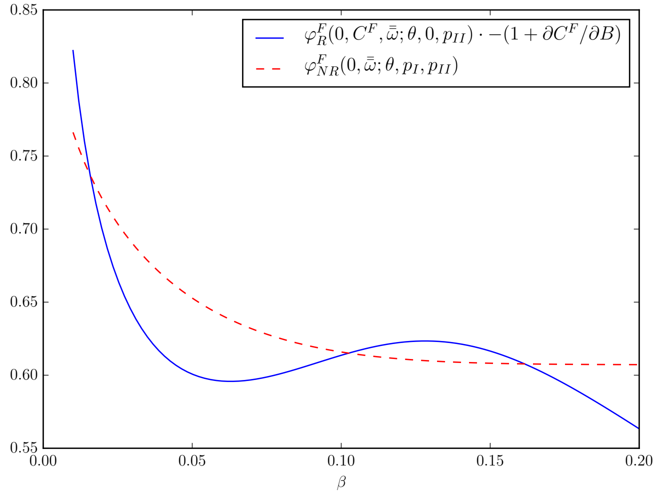
[1] Where from (19) in the main article the function C^F is the level of C that exhausts the budget for any given $B\leq\beta$ (and exogenous parameters).

Figure 1: Variation of $\varphi_R^F \cdot -(1 + \partial C^F/\partial B)$ and φ_{NR}^F with β.



A.5 Proof of Proposition 3b.

To assist in deriving the effect of classification errors on maximum social welfare we note that from (A.12) we can directly ascertain:

$$\frac{1}{F(\bar{\bar{\omega}})} \int_0^{\bar{\bar{\omega}}} v_M(\omega, \hat{B}^F) dF(\omega) > u_x(\hat{B}^F + \hat{C}^F, 1) \geq \hat{\lambda}^F > \frac{1}{1 - F(\bar{\bar{\omega}})} \int_{\bar{\bar{\omega}}}^{\infty} v_M(\omega, \hat{B}^F) dF(\omega) \tag{A.16}$$

where $u_x(\hat{B}^F + \hat{C}^F, 1) = \hat{\lambda}^F$ if only if $p_{II} = 0$.

To establish the effect of an increase in the Type I error propensity on maximum social welfare we use (A.8) - i.e. $u(B + C, 1) - u(B, 1) > u_x(B + C, 1)C$ - and (A.16) to obtain:

$$\begin{aligned} \frac{\partial V^F}{\partial p_I} &= \theta \langle [u(\hat{B}^F, 1) - u(\hat{B}^F + \hat{C}^F, 1)] + \hat{\lambda}^F \hat{C}^F \rangle \\ &= \theta \left\{ \hat{\lambda}^F \hat{C}^F - \left[u(\hat{B}^F + \hat{C}^F, 1) - u(\hat{B}^F, 1) \right] \right\} < \theta \hat{C}^F [\hat{\lambda}^F - u_x(\hat{B}^F + \hat{C}^F, 1)] \leq 0 \end{aligned}$$

Turning to the effect of Type II errors on maximum social welfare, we have:

$$\frac{\partial V^F}{\partial p_{II}} = (1 - \theta) F(\bar{\omega}) J \quad \text{and so} \quad \frac{\partial V^F}{\partial p_{II}} > 0 \text{ if and only if } J > 0 \tag{A.17}$$

where

$$\begin{aligned} J &\equiv \left[u(\hat{B}^F + \hat{C}^F, 1) - \frac{1}{F(\bar{\bar{\omega}})} \int_0^{\bar{\bar{\omega}}} v(\omega, \hat{B}^F) dF(\omega) \right] - \hat{\lambda}^F \hat{C}^F \qquad \text{(A.18)} \\ &= \left[u(\hat{B}^F + \hat{C}^F, 1) - u(\hat{B}^F, 1) - \hat{\lambda}^F \hat{C}^F \right] - \frac{1}{F(\bar{\bar{\omega}})} \int_{\bar{\omega}(B)}^{\bar{\bar{\omega}}} \left[v(\omega, \hat{B}^F) - u(\hat{B}^F, 1) \right] dF(\omega) \end{aligned}$$

To transition from the first line to the second line we (i) add and subtract $u(\hat{B}^F,1)$; and (ii) use the property that $v(\omega,B)=u(B,1)\ \forall\ \omega\leq\bar{\omega}(B)$. But since $v(\omega,B)<v(\bar{\bar{\omega}},B)=u(B+C,1)\ \ \forall\,\omega\in[\bar{\omega}(B),\bar{\bar{\omega}})$ it must hold that:

$$\begin{aligned} J>K &\equiv \left\langle 1-\frac{F(\bar{\bar{\omega}})-F(\bar{\omega}(\hat{B}^F))}{F(\bar{\bar{\omega}})}\right\rangle \Big[u(\hat{B}^F+\hat{C}^F,1)-u(\hat{B}^F,1)\Big]-\hat{\lambda}^F\hat{C}^F \\ &= \frac{F[\bar{\omega}(\hat{B}^F)]}{F(\bar{\bar{\omega}})}\Big[u(\hat{B}^F+\hat{C}^F,1)-u(\hat{B}^F,1)\Big]-\hat{\lambda}^F\hat{C}^F \end{aligned} \tag{A.19}$$

But we know from (A.8) that $u(B+C,1)-u(B,1)>u_x(B+C,1)C$ and so:

$$K>L\equiv\hat{C}^F\left\{\frac{F[\bar{\omega}(\hat{B}^F)]}{F(\bar{\bar{\omega}})}u_x(\hat{B}^F+\hat{C}^F,1)-\hat{\lambda}^F\right\} \tag{A.20}$$

$$=\hat{C}^F\hat{\lambda}^F\left\{\frac{F[\bar{\omega}(\hat{B}^F)]}{F(\bar{\bar{\omega}})}\left\langle 1+\frac{(1-\theta)f(\bar{\bar{\omega}})\bar{\bar{\omega}}_C p_{II}\hat{C}^F}{\theta(1-p_I)+(1-\theta)F(\bar{\bar{\omega}})p_{II}}\right\rangle-1\right\} \tag{A.21}$$

$$=\hat{C}^F\hat{\lambda}^F\left\{\frac{F[\bar{\omega}(\hat{B}^F)]}{F(\bar{\bar{\omega}})}\left\langle \frac{(1-\theta)f(\bar{\bar{\omega}})\bar{\bar{\omega}}_C p_{II}\hat{C}^F}{\theta(1-p_I)+(1-\theta)F(\bar{\bar{\omega}})p_{II}}\right\rangle-\left[1-\frac{F[\bar{\omega}(\hat{B}^F)]}{F(\bar{\bar{\omega}})}\right]\right\} \tag{A.22}$$

Note that the transition from (A.20) to (A.21) uses the definition of $u_x(\hat{B}^F+\hat{C}^F,1)$ from (A.12), whilst the subsequent transition to (A.22) follows simply from the manipulation of terms.

It follows directly from terms within the pair of curly braces in (A.22) that a sufficient condition for maximum social welfare to be increasing in the Type II error propensity is:

$$E>0\Rightarrow\frac{\partial V^F}{\partial p_{II}}>0 \tag{A.23}$$

where:

$$E\equiv\frac{F[\bar{\omega}(\hat{B}^F)]}{\frac{\theta(1-p_I)}{(1-\theta)p_{II}}+F(\bar{\bar{\omega}})}\cdot\frac{\bar{\bar{\omega}}f(\bar{\bar{\omega}})}{F(\bar{\bar{\omega}})}\cdot\frac{\hat{C}^F\bar{\bar{\omega}}_C}{\bar{\bar{\omega}}}-\left[1-\frac{F[\bar{\omega}(\hat{B}^F)]}{F(\bar{\bar{\omega}})}\right] \tag{A.24}$$

If $(1-\theta)p_{II}\approx 0$ then the first term on the right side is approximately zero such that $E<0$. Yet, given that $\partial V^F/\partial p_{II}>(1-\theta)F(\bar{\bar{\omega}})\hat{C}^F\hat{\lambda}^F E$ this is insufficient to sign $\partial V^F/\partial p_{II}$. However, if $\theta(1-p_I)\approx 0$ then:

$$E\approx\frac{F(\bar{\omega}(\hat{B}^F))}{F(\bar{\bar{\omega}})}\cdot\frac{\bar{\bar{\omega}}f(\bar{\bar{\omega}})}{F(\bar{\bar{\omega}})}\cdot\frac{\hat{C}^F\bar{\bar{\omega}}_C}{\bar{\bar{\omega}}}-\left[1-\frac{F(\bar{\omega}(\hat{B}^F))}{F(\bar{\bar{\omega}})}\right] \tag{A.25}$$

If the product of (i) the elasticity of F with respect to ω - evaluated at $\bar{\bar{\omega}}$ - and (ii) the elasticity of $\bar{\bar{\omega}}$ with respect to C - evaluated at $\hat{C}^F$ - are sufficiently high, then we can have $E>0$ and thus $\partial V^F/\partial p_{II}>0$. Since these elasticities depend on as yet unspecified properties of the distribution and utility functions respectively, we have enough degrees of freedom to choose parameters so that $E>0$.

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B Proofs for the Optimal Tax Analysis

B.1 Proof of Proposition 1′

Solving the maximisation problem in (33) of the main article yields the following FOCs characterising the optimal benefits:

$$(B): \quad \theta u_x(\hat{B}^P + \hat{C}^P, 1) + (1-\theta)\int_0^\infty s(n, \hat{\tau}^P, \hat{B}^P, \hat{\lambda}^P)dF(n) \leq \hat{\lambda}^P \ \ ; \ \hat{B}^P \geq 0 \tag{B.1}$$

$$(C): \quad u_x(\hat{B}^P + \hat{C}^P, 1) \leq \hat{\lambda}^P \ \ ; \ \hat{C}^P \geq 0 \tag{B.2}$$

where the pairs of inequalities hold with complementary slackness. We test the following corner solution hypothesis.

$\hat{C}^P = 0, \hat{B}^P > 0$ **(Pure Universal System)**

Setting $\hat{C}^P = 0$ in (B.1) and (B.2) and combining the resulting expressions yields:

$$\int_0^\infty s(n, \tau, B, \lambda)dF(n) \geq u_x(B, 1)$$

Given that (i) $s(n, \tau, B, \lambda) < v_M[\omega(n, \tau), B] \ \forall \ n > \bar{n}$; and (ii) $v_M[\omega(n, \tau), B] < u_x(B, 1) \ \forall \ n > \bar{n}$, this expression must be a contradiction. The hypothesis that $\hat{C}^P = 0$ is thus *false*. We instead have $\hat{C}^P > 0$ and so the optimal benefits are characterised by:

$$\int_0^\infty s(n, \hat{\tau}^P, \hat{B}^P, \hat{\lambda}^P)dF(n) \leq \hat{\lambda}^P = u_x(\hat{B}^P + \hat{C}^P, 1) \ \ ; \ \hat{B}^P \geq 0 \tag{B.3}$$

where the pair of inequalities hold with complementary slackness.

The FOC characterising the optimal tax rate is:

$$(\tau): \quad \int_0^\infty \left\{ y(n, 1-\hat{\tau}^P, \hat{B}^P) - \frac{n v_\omega[\omega(n, \hat{\tau}^P), \hat{B}^P]}{\hat{\lambda}^P} - \hat{\tau}^P \cdot \frac{\partial y(n, 1-\hat{\tau}^P, \hat{B}^P)}{\partial(1-\tau)} \right\} dF(n) = 0 \tag{B.4}$$

Substituting in Roy's identity (i.e. $v_\omega = v_M y/n$) and the Slutsky-Hicks expression (i.e. $\partial y/\partial(1-\tau) = \partial y^c/\partial(1-\tau) + y_M y$ where y^c is compensated earnings) then gives:

$$\hat{\tau}^P = \frac{\int_0^\infty y(n, 1-\hat{\tau}^P, \hat{B}^P)\left[\hat{\lambda}^P - s(n, \hat{\tau}^P, \hat{B}^P, \hat{\lambda}^P)\right]dF(n)}{\hat{\lambda}^P \int_0^\infty \dfrac{\partial y^c(n, 1-\hat{\tau}^P, \hat{B}^P)}{\partial(1-\tau)}dF(n)} \tag{B.5}$$

Letting $\kappa \equiv (\hat{\lambda}^P - \bar{s})$ - where $\bar{s} \equiv \int_0^\infty s dF(n)$ - we can write the numerator of (B.5) as (to save on space we abstract function arguments):

$$\int_0^\infty y(\hat{\lambda}^P - s)dF(n) = \int_0^\infty y\left[(\hat{\lambda}^P - \bar{s}) + \bar{s}\left(1 - \frac{s}{\bar{s}}\right)\right]dF(n) = \kappa\bar{y} - \mathrm{Cov}(y, s) \tag{B.6}$$

Substituting (B.6) into (B.5) and subsequently dividing both sides by $1 - \hat{\tau}^P$ then yields the optimal tax expression in the main article. ■

B.2 Proof of Proposition 2a′

Solving the maximisation problem in (38) in the main article yields the below expressions characterising the optimal benefit levels:

$$\begin{aligned}(B): \quad & \theta(1-p_I)u_x(\hat{B}^N+\hat{C}^N,1)+(1-\theta)p_{II}\int_0^\infty s(n,\hat{\tau}^N,\hat{B}^N+\hat{C}^N,\hat{\lambda}^N)dF(n) \\ & +\theta p_I u_x(\hat{B}^N,1)+(1-\theta)(1-p_{II})\int_0^\infty s(n,\hat{\tau}^N,\hat{B}^N,\hat{\lambda}^N)dF(n) \\ & \leq \hat{\lambda}^N \;\; ; \;\; \hat{B}^N \geq 0\end{aligned} \tag{B.7}$$

$$\begin{aligned}(C): \quad & \theta(1-p_I)u_x(\hat{B}^N+\hat{C}^N,1)+(1-\theta)p_{II}\int_0^\infty s(n,\hat{\tau}^N,\hat{B}^N+\hat{C}^N,\hat{\lambda}^N)dF(n) \\ & \leq \hat{\lambda}^N[\theta(1-p_I)+(1-\theta)p_{II}] \;\; ; \;\; \hat{C}^N \geq 0\end{aligned} \tag{B.8}$$

where the pairs of inequalities hold with complementary slackness.

We now proceed to test the following hypothesis:

$\hat{C}^N = 0, \hat{B}^N > 0$ **(Pure Universal System)**

Setting $\hat{C}^N = 0$ in (B.7) and (B.8) gives:

$$\theta u_x(B,1)+(1-\theta)\int_0^\infty s(n,\tau,B,\lambda)dF(n)=\lambda$$

$$\theta(1-p_I)u_x(B,1)+(1-\theta)p_{II}\int_0^\infty s(n,\tau,B,\lambda)dF(n)\leq\lambda[\theta(1-p_I)+(1-\theta)p_{II}]$$

The first expression implies $u_x(B,1) > \lambda > \int_0^\infty s(n,\tau,B,\lambda)dF(n)$. Combining both expressions so as to eliminate $u_x(B,1)$ then yields:

$$(1-p_I-p_{II})\left\{\lambda-\int_0^\infty s(n,\tau,B,\lambda)dF(n)\right\}\leq 0 \tag{B.9}$$

Given the discriminatory power assumption of $p_I + p_{II} \leq 1$, the above condition can only hold if $p_I+p_{II} = 1$ and thus whenever the test administering C has no discriminatory power. The assertion that $\hat{C}^F = 0$ must therefore be false *whenever* $p_I+p_{II} < 1$. Given that $\hat{C}^F > 0$ whenever $p_I + p_{II} < 1$ the optimal choice of benefits is characterised by:

$$\begin{aligned}&\frac{\theta p_I u_x(\hat{B}^N,1)+(1-\theta)(1-p_{II})\int_0^\infty s(n,\hat{\tau}^N,\hat{B}^N,\hat{\lambda}^N)dF(n)}{\theta p_I+(1-\theta)(1-p_{II})} \\ \leq&\hat{\lambda}^N=\frac{\theta(1-p_I)u_x(\hat{B}^N+\hat{C}^N,1)+(1-\theta)p_{II}\int_0^\infty s(n,\hat{\tau}^N,\hat{B}^N+\hat{C}^N,\hat{\lambda}^N)dF(n)}{\theta(1-p_I)+(1-\theta)p_{II}}\;;\;\hat{B}^N\geq 0\end{aligned} \tag{B.10}$$

This is the optimality condition given in the main text (the left side is $\bar{\varphi}^N_{NR}$, whilst the right side is $\bar{\varphi}^N_R$).

The FOC characterising the optimal tax rate is:

$$(\tau):\int_0^\infty \left\{ \begin{array}{c} -\frac{1}{\hat{\lambda}^N}\left\langle p_{II} n v_\omega[\omega(n,\hat{\tau}^N),\hat{B}^N+\hat{C}^N]+(1-p_{II})nv_\omega[\omega(n,\hat{\tau}^N),\hat{B}^N]\right\rangle \\ +p_{II}\left\langle y(n,1-\hat{\tau}^N,\hat{B}^N+\hat{C}^N)-\hat{\tau}^N\cdot\dfrac{\partial y(n,1-\hat{\tau}^N,\hat{B}^N+\hat{C}^N)}{\partial(1-\tau)}\right\rangle \\ +(1-p_{II})\left\langle y(n,1-\hat{\tau}^N,\hat{B}^N)-\hat{\tau}^N\cdot\frac{\partial y(n,1-\hat{\tau}^N,\hat{B}^N)}{\partial(1-\tau)}\right\rangle \end{array}\right\} dF(n)=0 \tag{B.11}$$

Substituting in Roy's identity and the Slutsky-Hicks equation then gives:

$$\begin{aligned} &\int_0^\infty \left\{ \begin{array}{c} p_{II}\cdot y(n,1-\hat{\tau}^N,\hat{B}^N+\hat{C}^N)\left[\hat{\lambda}^N-s(n,\hat{\tau}^N,\hat{B}^N+\hat{C}^N,\hat{\lambda}^N)\right] \\ +(1-p_{II})\cdot y(n,1-\hat{\tau}^N,\hat{B}^N)\left[\hat{\lambda}^N-s(n,\hat{\tau}^N,\hat{B}^N,\hat{\lambda}^N)\right] \end{array}\right\} dF(n) \\ =&\hat{\tau}^N\hat{\lambda}^N\int_0^\infty \left\{ p_{II}\cdot\frac{\partial y^c(n,1-\hat{\tau}^N,\hat{B}^N+\hat{C}^N)}{\partial(1-\tau)}+(1-p_{II})\cdot\frac{\partial y^c(n,1-\hat{\tau}^N,\hat{B}^N)}{\partial(1-\tau)}\right\} dF(n) \end{aligned} \tag{B.12}$$

From this one can readily establish the optimal tax expression in the main text. ■

B.3 Proof of Proposition 2b′

The immediate implication from the right side of (B.10) is that $\forall\ p_I+p_{II}<1$:

$$u_x(\hat{B}^N+\hat{C}^N,1)\geq\hat{\lambda}^N>\int_0^\infty s(n,\hat{\tau}^N,\hat{B}^N+\hat{C}^N,\hat{\lambda}^N)dF(n) \tag{B.13}$$

where $u_x(\hat{B}^N+\hat{C}^N,1)=\hat{\lambda}^N$ if and only if $p_{II}=0$.

The effect of an increase in the Type I error propensity on maximum social welfare is:

$$\begin{aligned} \frac{\partial V^N}{\partial p_I}&=\theta\left\{\left[u(\hat{B}^N,1)-u(\hat{B}^N+\hat{C}^N,1)\right]+\hat{\lambda}^N\hat{C}^N\right\} \\ &<\theta\hat{C}^N\left\{\hat{\lambda}^N-u_x(\hat{B}^N+\hat{C}^N,1)\right\}<0 \end{aligned} \tag{B.14}$$

The effect of an increase in the Type II error propensity on maximum social welfare is:

$$\frac{\partial V^N}{\partial p_{II}}=(1-\theta)\int_0^\infty \left\{ \begin{array}{c} v[\omega(n,\hat{\tau}^N),\hat{B}^N+\hat{C}^N]+\hat{\lambda}^N\hat{\tau}^N y(n,1-\hat{\tau}^N,\hat{B}^N+\hat{C}^N) \\ -\left\langle v[\omega(n,\hat{\tau}^N),\hat{B}^N]+\hat{\lambda}^N\hat{\tau}^N y(n,1-\hat{\tau}^N,\hat{B}^N)\right\rangle \\ -\hat{\lambda}^N\hat{C}^N \end{array}\right\} dF(n) \tag{B.15}$$

$$=(1-\theta)\int_0^\infty \left\{ \begin{array}{c} \int_0^\infty[s(n,\hat{\tau}^N,\hat{B}^N+\hat{C}^N,\hat{\lambda}^N)-s(n,\hat{\tau}^N,\hat{B}^N,\hat{\lambda}^N]dM \\ -\hat{\lambda}^N\hat{C}^N \end{array}\right\} dF(n) \tag{B.16}$$

$$<(1-\theta)\hat{C}^N\left\{\int_0^\infty s(n,\hat{\tau}^N,\hat{B}^N,\hat{\lambda}^N)dF(n)-\hat{\lambda}^N\right\}\leq 0 \tag{B.17}$$

To transition from (B.15) to (B.16) we imposed $\int_0^{\infty}(v_M + \lambda\tau y_M)dM = v + \lambda\tau y$, whilst to transition from (B.16) to (B.17) we used the concavity property in (A.8) coupled with (B.13). Note that here we make use of the assumption that $\partial s/\partial M < 0$. ■

B.4 Full Enforcement Proofs.

Solving the maximisation problem in (42) in the main article yields the below expressions characterising the optimal benefit levels:

$$\begin{aligned}(B):\quad & [\theta(1-p_I) + (1-\theta)F(\bar{\bar{n}})p_{II}]u_x(\hat{B}^F + \hat{C}^F, 1) + \theta p_I u_x(\hat{B}^F, 1) \\ & + (1-\theta)\left\{\int_0^{\infty} s(n, \hat{\tau}^F, \hat{B}^F, \hat{\lambda}^F)dF(n) - p_{II}\int_0^{\bar{\bar{n}}} s(n, \hat{\tau}^F, \hat{B}^F, \hat{\lambda}^F)dF(n)\right\} \\ & \le \hat{\lambda}^F\left\{1 + (1-\theta)f(\bar{\bar{n}})\bar{\bar{n}}_B p_{II}\left[\hat{C}^F + \hat{\tau}^F y(\bar{\bar{n}}, 1-\hat{\tau}^F, \hat{B}^F)\right]\right\} \quad ;\ \hat{B}^F \ge 0\end{aligned} \tag{B.18}$$

$$(C):\quad u_x(\hat{B}^F + \hat{C}^F, 1) \le \hat{\lambda}^F\left\{1 + \frac{(1-\theta)f(\bar{\bar{n}})p_{II}\bar{\bar{n}}_C\left[\hat{C}^F + \hat{\tau}^F y(\bar{\bar{n}}, 1-\hat{\tau}^F, \hat{B}^F)\right]}{[\theta(1-p_I) + (1-\theta)F(\bar{\bar{n}})p_{II}]}\right\} \quad ;\ \hat{C}^F \ge 0 \tag{B.19}$$

where the pairs of inequalities hold with complementary slackness. Note that in deriving these FOCs changes in the limits of integration cancel.

We now proceed to test the following hypothesis:

$\hat{C}^F = 0, \hat{B}^F > 0$ **(Pure Universal System)**

Setting $\hat{C}^F = 0$ in (B.18) and (B.19) and using the fact that $\bar{\bar{n}}(\tau, B, 0) = \bar{n}(\tau, B)$ yields:

$$\begin{aligned}\theta u_x(B,1) + (1-\theta)\int_0^{\infty} s(n, \tau, B, \lambda)dF(n) &= \lambda \\ u_x(B,1) &\le \lambda\end{aligned}$$

The first condition implies that $u_x(B,1) > \lambda$, but this contradicts the second condition and so the assertion that $\hat{C}^F = 0$ must be false. We thus have $\hat{C}^F > 0\ \forall\ p_I + p_{II} \le 1$. We can combine (B.18) and (B.19) - the latter of which now holds with equality - to obtain:

$$\begin{aligned}&\frac{\theta p_I u(\hat{B}^F, 1) + (1-\theta)\left\{\int_0^{\infty} s(n, \hat{\tau}^F, \hat{B}^F, \hat{\lambda}^F)dF(n) - p_{II}\int_0^{\bar{\bar{n}}} s(n, \hat{\tau}^F, \hat{B}^F, \hat{\lambda}^F)dF(n)\right\}}{\theta p_I + (1-\theta)(1 - F(\bar{\bar{n}})p_{II}} \\ &\le u_x(\hat{B}^F + \hat{C}^F, 1)\cdot\left\{\frac{1 + \dfrac{(1-\theta)f(\bar{\bar{n}})p_{II}[\bar{\bar{n}}_B - \bar{\bar{n}}_C][\hat{C}^F + \hat{\tau}^F y|_{n=\bar{\bar{n}}}]}{[\theta p_I + (1-\theta)(1 - F(\bar{\bar{n}})p_{II})]}}{1 + \dfrac{(1-\theta)f(\bar{\bar{n}})p_{II}\bar{\bar{n}}_C[\hat{C}^F + \hat{\tau}^F y|_{n=\bar{\bar{n}}}]}{[\theta(1-p_I) + (1-\theta)F(\bar{\bar{n}})p_{II}]}}\right\} \quad ;\ \hat{B}^F \ge 0\end{aligned} \tag{B.20}$$

This is the expression given in the main article (the left side is $\bar{\varphi}^F_{NR}$, whilst the first term on the right side is $\bar{\varphi}^F_R$).

Next, the FOC characterising the optimal tax rate is:

$$(\tau) \quad \int_0^\infty \left\{ -nv_\omega[\omega(n,\hat{\tau}^F),\hat{B}^F] + \hat{\lambda}^F \left[y(n,1-\hat{\tau}^F,\hat{B}^F) - \hat{\tau}^F \cdot \frac{\partial y(n,1-\hat{\tau}^F,\hat{B}^F)}{\partial(1-\tau)} \right] \right\} dF(n)$$
$$- p_{II} \int_0^{\bar{\bar{n}}} \left\{ -nv_\omega[\omega(n,\hat{\tau}^F),\hat{B}^F] + \hat{\lambda}^F \left[y(n,1-\hat{\tau}^F,\hat{B}^F) - \hat{\tau}^F \cdot \frac{\partial y(n,1-\hat{\tau}^F,\hat{B}^F)}{\partial(1-\tau)} \right] \right\} dF(n)$$
$$= \hat{\lambda}^F f(\bar{\bar{n}}) p_{II} \frac{\partial \bar{\bar{n}}}{\partial \tau} \left[\hat{C}^F + \hat{\tau}^F y(\bar{\bar{n}},1-\hat{\tau}^F,\hat{B}^F) \right] \tag{B.21}$$

Substituting Roy's identity and the Slutsky-Hicks equation into (B.21) and rearranging then yields:

$$(\tau): \quad \int_0^\infty y(n,1-\hat{\tau}^F,\hat{B}^F) \left[1 - \frac{s(n,\hat{\tau}^F,\hat{B}^F,\hat{\lambda}^F)}{\hat{\lambda}^F} \right] dF(n)$$
$$- p_{II} \left\{ \int_0^{\bar{\bar{n}}} y(n,1-\hat{\tau}^F,\hat{B}^F) \left[1 - \frac{s(n,\hat{\tau}^F,\hat{B}^F,\hat{\lambda}^F)}{\hat{\lambda}^F} \right] dF(n) + f(\bar{\bar{n}}) \frac{\partial \bar{\bar{n}}}{\partial \tau} \hat{C}^F \right\}$$
$$= \hat{\tau}^F \left\{ \int_0^\infty \frac{\partial y^c(n,1-\tau^F,\hat{B}^F)}{\partial(1-\tau)} dF(n) - p_{II} \left[\int_0^{\bar{\bar{n}}} \frac{\partial y^c(n,1-\tau^F,\hat{B}^F)}{\partial(1-\tau)} dF(n) + f(\bar{\bar{n}}) \frac{\partial \bar{\bar{n}}}{\partial \tau} y(\bar{\bar{n}},1-\hat{\tau}^F,\hat{B}^F) \right] \right\} \tag{B.22}$$

Dividing both sides by $1-\hat{\tau}^F$ and rearranging then yields the optimal tax expression in the main text. ■

B.5 Proof of Proposition 3b′

To establish the effect of an increase in the Type I error propensity on maximum social welfare we use the fact that $u_x(\hat{B}^F+\hat{C}^F,1) \geq \hat{\lambda}^F$ from (B.19) in conjunction with (A.8) to obtain:

$$\frac{\partial V^F}{\partial p_I} = \theta \left\{ \left[u(\hat{B}^F,1) - u(\hat{B}^F+\hat{C}^F,1) \right] + \hat{\lambda}^F \hat{C}^F \right\} < \theta \hat{C}^F \left[\hat{\lambda}^F - u_x(\hat{B}^F+\hat{C}^F,1) \right] < 0 \tag{B.23}$$

The effect of an increase in the Type II error propensity on maximum social welfare is:

$$\frac{\partial V^F}{\partial p_{II}} = (1-\theta) F(\bar{\bar{n}}) \cdot J \quad ; \text{where}$$
$$J \equiv u(\hat{B}^F+\hat{C}^F,1) - \frac{1}{F(\bar{\bar{n}})} \int_0^{\bar{\bar{n}}} \left\langle v[\omega(n,\hat{\tau}^F),\hat{B}^F] + \hat{\lambda}^F \hat{\tau}^F y(n,1-\hat{\tau}^F,\hat{B}^F) \right\rangle dF(n) - \hat{\lambda}^F \hat{C}^F \tag{B.24}$$

Adding and subtracting $u(B,1)$ and noting that $v[\omega(n,\tau),B]=u(B,1)\ \forall\ n\leq\bar{n}$ gives:

$$\begin{aligned}J &= [u(\hat{B}^F+\hat{C}^F,1)-u(\hat{B}^F,1)]-\hat{\lambda}^F\hat{C}^F \\ &-\frac{1}{F(\bar{\bar{n}})}\int_{\bar{n}}^{\bar{\bar{n}}}\left\langle v[\omega(n,\hat{\tau}^F),\hat{B}^F]-u(\hat{B}^F,1)+\hat{\lambda}^F\hat{\tau}^F y(n,1-\hat{\tau}^F,\hat{B}^F)\right\rangle dF(n)\end{aligned} \tag{B.25}$$

Using the fact that $v[\omega(n,\tau),B]<u(B+C,1)\ \forall\ n<\bar{\bar{n}}$ it follows immediately that:

$$J>K\equiv\frac{F[\bar{n}(\hat{\tau}^F,\hat{B}^F]}{F(\bar{\bar{n}})}[u(\hat{B}^F+\hat{C}^F,1)-u(\hat{B}^F,1)]-\hat{\lambda}^F\left[\hat{C}^F+\frac{\hat{\tau}^F}{F(\bar{\bar{n}})}\int_{\bar{n}}^{\bar{\bar{n}}}y(n,1-\hat{\tau}^F,\hat{B}^F)dF(n)\right]$$

Using (A.8) we then obtain:

$$K>L\equiv\hat{C}^F\left\{\frac{F[\bar{n}(\hat{\tau}^F,\hat{B}^F)]}{F(\bar{\bar{n}})}u_x(\hat{B}^F+\hat{C}^F,1)-\hat{\lambda}^F\left[1+\frac{\hat{\tau}^F}{\hat{C}^F F(\bar{\bar{n}})}\int_{\bar{n}}^{\bar{\bar{n}}}y(n,1-\hat{\tau}^F,\hat{B}^F)dF(n)\right]\right\}$$

Substituting in (B.19) - which holds with equality because $\hat{C}^F>0$ - yields

$$L=\hat{\lambda}^F\hat{C}^F\left\{\begin{array}{c}\frac{F[\bar{n}(\hat{\tau}^F,\hat{B}^F]}{F(\bar{\bar{n}})}\left\langle\dfrac{(1-\theta)f(\bar{\bar{n}})p_{II}\bar{\bar{n}}_C\left[\hat{C}^F+\hat{\tau}^F y(\bar{\bar{n}},1-\hat{\tau}^F,\hat{B}^F)\right]}{[\theta(1-p_I)+(1-\theta)F(\bar{\bar{n}})p_{II}]}\right\rangle \\ -\left\langle\left[1-\frac{F[\bar{n}(\hat{\tau}^F,\hat{B}^F]}{F(\bar{\bar{n}})}\right]+\frac{1}{F(\bar{\bar{n}})}\frac{\hat{\tau}^F}{\hat{C}^F}\int_{\bar{n}}^{\bar{\bar{n}}}y(n,1-\hat{\tau}^F,\hat{B}^F)dF(n)\right\rangle\end{array}\right\} \tag{B.26}$$

Rearranging the terms in curly braces then yields the sufficient condition for $\partial V^F/\partial p_{II}>0$ provided in the main text. ■