COGNITIVE DEFICITS UNDERLYING CHILDREN'S MATHEMATICAL DIFFICULTIES

Rebecca Bull

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Cognitive Deficits Underlying Children's Mathematical Difficulties.

Rebecca Bull.

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I, Rebecca Bull, hereby certify that this thesis, which is approximately 71,000 words in length, has been written by me, and that it is the record of work carried out by me and that it has not been submitted in any previous application for a higher degree.

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Dedication.

To my brother Anthony, who is lost from my life, but still lives in my heart.

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Contents.

Abstract.

Many children have difficulties learning mathematics, and the consequences of poor mathematical skills are very far reaching. Studies examining the reasons why children struggle to learn mathematics are scarce, particularly in comparison to studies examining reading difficulties. The studies reported in this thesis attempted to provide insights into the cognitive limitations that may lead some children to have difficulties learning mathematics, re-examining some of the cognitive deficits already thought to be associated with mathematical difficulties, as well as providing the starting point for new lines of enquiry. Five main studies are reported. Four of these studies examined a range of cognitive skills and identify a number of fundamental cognitive mechanisms as playing a role in children's mathematical skills, these being a slowness in the speed of processing information, poor control of executive functioning, evidenced through difficulty switching strategies and poor self-regulation of actions, and a delay in the automatisation of basic arithmetic facts. The final study aimed to investigate the implications of these recognised cognitive difficulties in the teaching of mathematics, and explored the use of two different teaching strategies, rote learning of basic arithmetic facts and a discussion method to allow alternative methods of solution to be learned, both of which attempted to overcome some of these cognitive limitations. Rote learning was found to be an effective device to improve performance in different areas of mathematical skill. The implications of this research and the foundations for future research are also discussed.

Chapter 1.
Introduction.

> *"You cannot evade quantity. You may fly to poetry and music, and quantity and number will face you in your rhythms and octaves"* (Whitehead, 1929, p. 7).

1.1 Mathematics in Everyday Life.

We live in a world filled with numbers. Children need to learn about numbers in order to understand the world around them. Numbers confront us in numerous everyday activities, for example, when children share things with friends, when they are trying to decide how to spend their pocket money, and later in life when we have to purchase goods, listen to the weather forecast, cook, deal with medicine dosages, and try to cope with mortgage and insurance policies. McCloskey and Macaruso (1995) and Nunes and Bryant (1996) provide numerous examples of the importance of numerical knowledge in everyday life, and even how such knowledge is tailored by social and cultural features. As the quote from Whitehead points out, even when we move to the arts, numerical knowledge is still required in its construction and understanding.

Throughout development, children are exposed to physical and social environments rich in mathematical opportunities. Research has shown that children possess a natural understanding of quantity (Antell & Keating, 1983; Starkey, 1992; Wynn, 1996). Studies involving infants have shown that they can discriminate between two rows of dots that differ only in number, provided the number of dots are not too large. This provides the starting point for quantitative judgements, the most reliable form of which is counting. Children often learn number words before attending school, with learning occurring through memorisation of the number sequence. Hughes (1986) showed that pre-school children can carry out simple counting, addition and subtraction, provided that the numbers involved are not too large. These mathematical skills acquired before formal schooling are referred to as informal mathematical concepts.

To be able to count correctly, children must know the sequence of number words, which in turn must be applied to the objects being counted. The child must also be aware that the last number word represents the number of objects that have been counted. This draws on Gelman and Gallistel's (1978) 'how to count' principles of one-to-one correspondence (each counted object must be given only one number tag), stable order (counting proceeds in a set order), cardinality (the last number counted represents the value of the set), abstraction (objects of any kind can be collected together and counted), and order irrelevance (items can be counted in any sequence). An alternative view of counting is that children first learn to count by rote, with no understanding of counting principles, but that gradually children induce the essential and unessential features of counting (Fuson, 1988; Wynn, 1990). Counting provides the foundation for the development of basic arithmetic skills (Kaye, 1986; Resnick, 1989). This is the starting point for arithmetical difficulties shown by some children. Fuson (1988) notes that in the early stages of counting, children make frequent errors, especially as the number of items that must be counted increases.

Following on from the development of counting skills is the acquisition of basic arithmetical skills. Strategies, or algorithms, employed by children in simple arithmetic, were first investigated by Groen and affiliates (Groen & Parkman, 1972; Groen & Resnick, 1977), and has been further elaborated by the Strategy Choice Model of Siegler and Shrager (1984), which will be discussed in greater detail in Section 1.5 The theoretical and practical application of these models have formed the basis for much of the work of David Geary and his colleagues (Geary, 1990; Geary, Bow-Thomas, & Yao, 1992; Geary & Brown, 1991; Geary, Brown, & Samaranayake, 1991, Geary, Widaman, Little, & Cormier, 1987; see Geary, 1993 for a review). The developmental progression of arithmetic skills as depicted in many studies by Geary and Siegler is illustrated in Table 1.1. According to the strategy choice model, children progress through the use of different counting strategies, starting with computational or procedural based-components, characterised by reliance on accessing stored algorithms or rule based strategies for solving arithmetic problems. Eventually, as children become more familiar with arithmetic procedures and number facts, arithmetic problems are more frequently solved through the use of direct memory retrieval of number bonds associated with the presented problem (Widaman, Geary, Cormier, & Little, 1989). What we see therefore is a gradual progression from the use of immature slow counting strategies through to the development of rapid direct memory retrieval of number bonds (see also, Ashcraft, 1982, 1983, 1992; Ashcraft & Fierman, 1982; Campbell & Graham, 1985; Cooney, Swanson, & Ladd, 1988; Siegler, 1988a, 1988b).

Groen and Parkman (1972) have proposed what appears to be the most influential model of counting processes, by suggesting a conceptual model of a mental counter. It is proposed that the counter increments in a unit by unit fashion, with the resulting sum of the increments being read off from a mental display. This counter may also be set to a specified value prior to the start of the incrementing process. The MIN model suggested by Groen and Parkman has received the most empirical support. This model assumes that the individual selects the larger of the two addends to be the value to which the counter is initially set, with the counter then being incremented a number of times equal to the smaller addend. Other models of simple arithmetic solution include the use of rules and heuristics. Baroody (1983, 1985) suggested that strategy choice in arithmetic skill development is a movement from slow procedural processes, such as MIN, to quicker procedural processes, for example, the use of rules (such as n + 0 = n), principles (e.g., commutativity, 3 + 6 is the same as 6 + 3), and heuristics (e.g., reorganisation, n + 9 as [n + 10] - 1), although Baroody also accepted that some arithmetic facts are stored in long term memory (e.g., ties, 3 + 3 =, 4 + 4 =).

Contingent upon the development of these basic arithmetical skills are multi-digit computations. Computations involving single-digit numbers are a necessary component of more complex mathematical problem solving, and therefore, the accuracy and speed with which they can be carried out is of great importance.

TABLE 1.1

The developmental progression of children's counting strategies for solving arithmetic problems.

Strategy	Explanation	Example: 9 + 4 =
Counting fingers.	*Counting all* numbers involved in the problem.	1, 2, 3, 4, 5, 6, 7, 8, 9; 10, 11, 12, 13.
	Counting on from the largest number.	9; 10, 11, 12, 13
Fingers	Fingers are used to represent the integers but are not visibly counted.	
Verbal counting	Children count audibly or implicitly.	
Decomposition	Problem is decomposed into simpler problems, usually involving numbers rounded up to 10, or tie problems	(10 + 4) - 1
Retrieval	No evidence of counting procedures.	

1.2 Aetiology of mathematical difficulties.

Numerous studies have investigated the reading problems of young children and the underlying cognitive mechanisms contributing to their difficulties. However, the occurrence of similar studies to address children's mathematical difficulties is much less common. This is surprising considering the importance placed on mathematics within the school curriculum, and, as noted earlier, because of the frequency with which numbers must be dealt with in many activities in everyday life (Hamann & Ashcraft, 1986; McCloskey & Macaruso, 1995). A number of authors (e.g., Dockrell & McShane, 1993; Hughes, 1986) have reported that children have unexpected difficulties at school with reading and especially with the grasping of mathematical concepts. Reading difficulties certainly hamper educational progress in a wide range of areas, and that the co-occurrence of reading and arithmetical difficulties is very common. Badian (1983) found that of the 6.4% of school age children found to have mathematical disabilities, 43% also showed poor reading achievement. A more recent study by Lewis, Hitch, and Walker (1994) investigating mathematics and reading difficulties in 9- and 10-year old children, reported a lower prevalence, with 1.3% of children showing specific arithmetic problems, and 2.3% showing joint reading and arithmetic problems. Similar information is presented by White, Moffitt, and Silva (1992), who found 10.1% of children to have reading and arithmetic disabilities, but only 4.3% to have specific reading difficulties, and 2.7% to have specific arithmetic difficulties. The prevalence of arithmetical difficulties is considered in greater detail in the methodological note reported in Chapter 2.

Clearly, there are numerous reasons why children may have difficulties learning mathematics. This thesis concentrates on the deficits in cognitive skills which might limit the child's capability to learn arithmetic and mathematical skills. Studies examining children's arithmetic have investigated numerous cognitive factors to try and explain why arithmetical difficulties occur (e.g., Boulton-Lewis, 1993; Boulton-Lewis & Tait, 1994; Fleischner, Garnett & Shepherd, 1982; Geary, 1990, 1993; Geary, et al, 1992; Geary & Brown, 1991; Geary et al, 1987; Hitch & McAuley, 1991; Jordan, Levine, & Huttenlocher, 1995; Kirby & Becker, 1988; Koontz & Berch, 1996; Lehto, 1995; McLean, 1997; Rabinowitz & Wooley, 1995; Rourke & Conway, 1997; Russell & Ginsburg, 1984; Share, Moffitt, & Silva, 1988; Siegel & Linder, 1984; Siegel & Ryan, 1989; Siegler, 1988a; Swanson, 1993). These include links with the problems underlying reading difficulties, problems involving short-term and long-term memory, speed of processing information and processing efficiency, inadequate knowledge of arithmetic procedures, and poor visual-spatial ability. Inevitably, some of these proposed problem areas have overlapping components and theories, especially those which question the involvement of processing efficiency, short-term memory, and long-term memory. In providing some theoretical background to such a wide range of cognitive mechanisms, evidence from three main areas will be reviewed including experimental studies with normal adults and children, developmental studies, and studies in cognitive neuropsychology examining patients with brain damage.

1.3 The Role of Short-term Memory in Arithmetical Skills.

One of the most frequently addressed issues in studies of children's arithmetical difficulties is whether short-term memory plays a specific role in this skill, as short-term memory is thought to have a central role in the acquisition and execution of basic educational skills, including reasoning, language comprehension, long-term learning, reading ability, and reading comprehension (Daneman & Carpenter, 1980, 1983; Daneman & Green, 1986; Gathercole & Baddeley, 1989, 1990a, 1990b 1993; Johnston, Rugg & Scott, 1987; Lehto, 1995; McDougall, Hulme, Ellis, & Monk, 1994; Michas & Henry, 1994; Siegel & Linder, 1984; Snowling, 1980). Research in this area has gained inspiration from the ideas associated with working memory (Leather & Henry, 1994; Swanson, 1993; for a review, see Hulme & Roodenrys, 1995). The working memory model was developed by Baddeley and Hitch (1974; see also Baddeley, 1986, and Baddeley, 1996 for refinements of the working memory model), and is considered as a measure of functional storage capacity, comprising both storage and processing function, measured though the use of tasks such as reading span (Daneman & Carpenter, 1980) and counting span (Case, Kurland, & Goldberg, 1982).

An important feature of working memory is that it has a limited capacity, the theory being that if more demands are being made on the central executive, there will be less processing space and cognitive energy available for the subsystems. Working memory theory suggests that individuals do not differ in a structural fashion, that is, in terms of the number of 'slots' for passively storing items or chunks of information as suggested by traditional short-term memory theory (Miller, 1956). Rather, individuals differ in functional storage capacity, that is, the amount of residual capacity remaining for temporary storage once the processing demands of the task have been met. Therefore, individuals differ in the trade-off between processing and storage functions (Engle, Cantor, & Carullo, 1992).

The core system thought to be responsible for co-ordinating activity within the working memory system is the central executive. This system off-loads some short-term storage functions to slave systems, freeing a portion of its own capacity for performing more complex information processing tasks. These slave subsystems are the articulatory loop and the visual-spatial sketchpad. The articulatory loop is involved in the storage of verbal information, and itself has two subsystems. The first of these is an active subvocal rehearsal process in the form of subvocal articulation. The second is a passive phonologically based store, the contents of which are subject to decay, but may be refreshed through subvocal rehearsal (Baddeley, 1986). Evidence for the existence of rehearsal processes in the articulatory loop comes from observation of findings such as the word length effect and articulatory suppression, with evidence for the phonological store coming from findings associated with phonemic similarity and irrelevant speech effects.

The visual spatial sketch pad serves a temporary storage function for visual and spatial material (Logie, 1986; Quinn, 1994; Quinn & McConnell, 1996). When one considers a complex cognitive task such as mental arithmetic, it is possible to see that it requires the temporary storage of information whilst new information is being processed and other cognitive tasks are being performed, and hence, it is clear why many ideas of children's arithmetical difficulties have appealed to explanations associated with working memory. When carrying out an arithmetical operation, it would be necessary for the central executive to monitor and retrieve information about the operation to be used, for example, addition, while subsidiary systems store specific numbers involved in the calculation (Logie, Gilhooly, & Wynn, 1994). Similarly, a major component of skilled reading is the ability to compute semantic and syntactic relationships among successive words, phrases, and sentences in order to construct a coherent and meaningful representation of the text, that is, there is need for a temporary storage of information whilst new information is being processed, in order for all information to be integrated into a meaningful whole.

Based on the knowledge gained from studies of short-term memory and reading ability, it is possible to start to understand how short-term memory could also play a role in children's mathematical skills. The notion of short-term memory playing a role in skills such as arithmetic and series problems was articulated by Hunter (1957) who described the activity sequence that one has to go through in combining information to reach a solution.

"He completes one stage; he lays the outcome aside; he completes the second stage; and then he recalls the first outcome and combines it with the second. In such an activity sequence, he must retain the first outcome during the time he is arriving at the second outcome. And in general, the more difficult it is for him to complete the second stage, the more likely he is to lose the outcome of the first stage" (p. 297).

Hunter's quotation is illustrated in a study by Hitch (1978), which explored the role of working memory in the performance of mental arithmetic. When solving aurally presented multi-digit problems, subjects reported breaking down the problem into a series of elementary stages. Geary and Lin (in press) also report the strategies used by adults in solving complex subtraction problems, whereby interim solutions are held in working memory. The involvement of each component of working memory in relation to arithmetical skills will be considered in turn.

<u>Articulatory Loop.</u>

The availability of working memory resources is dependent on how much information can be rehearsed in a 2 to 3 second span. Speech rate is considered to be important because of its strong relationship to short-term memory span and various other measures of short-term memory recall (Baddeley, Thomson, & Buchanan, 1975; Chen & Stevenson, 1988; Ellis & Hennelley, 1980; Hitch, Halliday, & Littler, 1989). Evidence to support this relationship comes from both adults and children in the guise of the word length effect, whereby fewer words are remembered which take a long time to articulate (e.g., *elephant*), compared to words which can be articulated quickly (e.g., *cat*). Ellis and Hennelley (1980) showed that arithmetic performance and verbal memory span in Welsh speaking children were poorer than when the same children performed the task in English. This was accounted for by the fact that words spoken in Welsh take longer to pronounce, and hence also take longer to subvocally rehearse. Further evidence to support this comes from Geary, Bow-Thomas, Fan, and Siegler (1993) who attribute some of the arithmetical superiority of Chinese 6 year olds over their American counterparts to their greater digit span. Chinese spoken digits are shorter and hence occupy less memory space. With regard to arithmetic, rehearsal time should be related to counting speed (Baddeley, 1986), such that the faster the counting speed, the longer the short-term memory span for numbers. With a slow counting speed the representation of the integers is more likely to decay before the count is completed.

A study by Siegel and Ryan (1989) compared children with specific reading difficulties to children with specific arithmetical difficulties. Using two working memory tests, one comprising a language component (verbal), and the other a counting component (nonverbal), Siegel and Ryan found that children who had difficulties in reading had lower scores on both the verbal and non-verbal working memory tasks, whereas children reported as having specific arithmetic difficulties were only shown to have a working memory deficit in the task that involved counting (concurrent counting span task).

Work by Dark and Benbow (1990, 1991) with gifted adolescents found that working memory was differentially enhanced depending on whether the adolescents showed verbal or mathematical precocity. Dark and Benbow (1991) examined three core aspects fundamental to successful memory performance, comparing verbal versus mathematical precocity. They investigated the accuracy with which information could be maintained in working memory, the accuracy of manipulating information in working memory, and finally, the speed of activating information from long-term memory into working memory. Span tasks (digits, letters, and locations) were used which assessed ability to encode and recall information, and a paired associate task was also included, as a measure of the effectiveness of central executive processing. In this task, stimulus letters were paired with a digit or a location. The associated digit or location changed continuously, such that subjects had to continually update their representations. Dark and Benbow found that mathematical precocity was best correlated with the ability to manipulate information in working memory, that is they showed better performance on the paired-associate task. Furthermore, subjects in this group were significantly better on tasks involving digits and spatial location stimuli. In contrast, verbal precocity was found to be most highly correlated with enhanced retrieval of representations from long-term memory into working memory, and with speed of encoding information. Verbally precocious subjects were found to have a significant advantage on span tasks involving word stimuli. This finding lends support to the argument that working memory differences are specific to different intellectual domains, that is differences in span tasks involving numerical information and spatial information to differences in mathematics ability, and differences in performance on verbal span tasks to differences in verbal ability. Interestingly, these results also suggest that reading and mathematical skills may be associated with different functional aspects of working memory; reading ability to the ability to encode and recall information, and mathematics ability to the ability to manipulate information. This point will be discussed in detail later in this section. However, Dark and Benbow do not put this result down to differences in memory performance per se. Rather, they suggest that such differences arise due to differences in item identifiability, such that when items are identified more rapidly, this will lead to enhanced memory performance. This supports the work of Case et al (1982). For example, digits may have stronger representations (or more 'compact' in Dark & Benbow's terms) for individuals who are mathematically talented, meaning they are identified more quickly, leading to enhanced memory span for those particular stimuli.

Other researchers (e.g., Hitch & McAuley, 1991; Koontz & Berch, 1996; Swanson, 1993), have argued that working memory does not have a task specific component for arithmetic as suggested by Siegel and Ryan (1989). Swanson (1993) found that children with arithmetical difficulties had a lower working memory span on a wide range of working memory tasks involving language, visual-spatial, and number elements. This is supported by the work of Koontz and Berch (1996) who found that children classified as specific arithmetic disabled had smaller digit and letter spans than their normally achieving peers, arguing against a specific deficit in working memory for numerical information. Further support for such a view comes from Lehto (1995) who found that performance in a sentence word span task (requiring verbal skills) correlated significantly with mathematics skill. Furthermore, an operation span task, relying much less on verbal skills, showed a strong correlation with language skills.

As suggested earlier, the explanation most readily given to account for differences in performance on span tasks, is that some individuals may articulate items more rapidly, and hence will be able to rehearse more information. However, this 'speech rate' explanation of individual differences in short-term memory span may be too simple (see e.g., Dempster, 1981; Ericsson & Kintsch, 1995; Gathercole & Adams, 1994; Henry & Millar, 1991; Hitch & McAuley, 1991; Hulme, Maughan, & Brown, 1991; Roodenrys, Hulme, & Brown, 1993). Performance on short-term memory tasks depends on a number of factors, and it is now accepted that there may also be involvement from long-term memory components. Case et al (1982) proposed that the one important limiting factor on short-term memory is speed of item identification, whereby representations of items are accessed from long-term storage, with developmental increases in memory span being due to extra resources becoming available for storage as a result of a decrease in capacity taken up by item identification. This idea has been supported by findings from recent research. Hitch and McAuley (1991) found that although children with arithmetical difficulties were less fluent in their articulation rate, they were not significantly different from children with normal mathematical ability, despite showing a deficit in digit span. Hitch and McAuley (1991) explain this finding by suggesting that a deficit in measures of short-term memory may be due to slower access to number representations in long-term memory. Work by Henry and Millar (1991) showed that differences in memory span were often still apparent when words were equated for how quickly children of varying ages could articulate them. Furthermore, it was also found that familiarity of words had an effect upon short-term memory span. Such a finding is explained by the fact that familiar words have stronger representations in long-term memory, and hence, are more easily accessible, than unfamiliar words. Hulme et al (1991) also put forward the suggestion that differences in short-term memory span may reflect the operations of long-term memory. Hulme et al (1991) considered memory span for nonwords since these lack phonological representations in long-term memory. With speech rate equated, differences in memory span for words and nonwords were still evident, offering support to the involvement of long-term memory. In a second experiment, Hulme et al also considered words from a foreign language, predicting that memory span would be poor for these words as it had been for nonwords. They further predicted that if subjects were then taught the meanings of these words, then this should lead to the creation of long-term memory representations for them and a corresponding improvement in memory span. This was indeed what was found. Results found convincing evidence for both a long-term memory contribution to short-term memory, and for the operation of a time based articulatory loop, as proposed by Baddeley and Hitch (1974), and Baddeley (1986).

Further support for alternative explanations of individual differences in memory span comes from Cowan (1992). Cowan points out that differences in memory span could be due to differences in covert rehearsal or overt pronunciation. Cowan studied memory span in 4 year old children, as well as the duration of the recall. It was found that duration of recall was not limited to two seconds, but that there was a linear relationship between memory span and duration of recall for span length lists, and that in contrast to previous research, there was no relation between memory span and speech rate. A decay and reactivation account is given for this finding, with Cowan suggesting that individual differences in memory span result from differences in the speed and efficiency with which subjects can retrieve and reactivate items in the pauses between words to be recalled, again providing convincing evidence for the importance of long-term memory in short-term memory recall.

Established findings associated with short-term memory may also provide support for the involvement of a long-term memory element. For example, the phonological similarity effect and the word length effect are often seen as evidence for the existence of speech-based coding in short-term memory. With articulatory suppression (the elimination of speech based coding) both of these effects are abolished, although some serial recall ability remains. It is therefore reasonable to suggest that this remaining recall capacity depends upon activation of information in long-term memory, or may reflect the operation of some other non-phonological information store, such as the central executive.

In summary, short-term memory performance specifically involving the storage and rehearsal capabilities of the articulatory loop may influence the development and use of arithmetical and mathematical skills in a number of ways. Firstly, if information held in the articulatory loop is not rehearsed or rehearsed slowly, then information may be lost before the problem is solved. Furthermore, if information cannot be recognised quickly and activated from long-term memory into short-term memory, then again, not all information will be available when it is needed to solve the problem. The availability of these long-term representations is based on familiarity with the material. However, a number of studies have suggested that differences in performance of mathematics may not be the result of differences in short-term memory capacity per se, but rather the result of differences in other functional characteristics of the working memory system, possibly in the functioning of the central executive and the ability to manipulate information.

Visual-spatial sketch pad.

The involvement of the visual spatial sketch pad in arithmetical performance has been considered in a number of studies (see e.g., Dehaene, 1992; Hayes, 1973; Heathcote, 1994; Hope & Sherill, 1987; Moyer & Landauer, 1967; Restle, 1970; Seron, Pesenti, Noel, Deloche, & Cornet, 1992; Smyth, Morris, Levy, & Ellis, 1987), although many of these studies have been conducted with adult or neuropsychological populations. Therefore, extrapolation of these results to the developing skills of the child must be made with caution. In a series of studies examining the role of the visual spatial sketch pad in the solution of multi-digit arithmetic problems performed by adults, Heathcote (1994) revealed evidence that digits were being stored visually as well as verbally, finding that problems containing visually similar digits resulted in significantly more errors than problems in which the digits were visually dissimilar. This is analogous to the phonological similarity effect observed in studies of the articulatory loop. Furthermore, Heathcote found that visual and spatial interference of the visual-spatial sketch pad produced disruption of mental calculation performance of multi-digit addends. Disruption was greatest on more complex problems involving carrying, suggesting that the carrying process involves spatial working memory. Overall, Heathcote suggests that the articulatory loop and the visual-spatial sketch pad are used in conjunction for solving multi-digit addition problems, the sketch-pad being used as a "mental blackboard or workbench" (p. 237), where visual-spatial material is held whilst operations are performed upon it. The articulatory loop, on the other hand, performs the role of a storage device retaining both the initial problem information and partial results, as well as refreshing images stored in the visual-spatial sketch pad. Hayes (1973) gives a similar interpretation to mental imagery, suggesting that visual imagery acts as a surrogate for external visual cues that would usually be generated using paper and pencil. Other researchers make reference to the use of mental number lines and spatial arrangements which are sometimes used in solving arithmetic problems (Dehaene, 1992; Hayes, 1973; Seron et al, 1992; Smyth et al, 1987).

In an analysis of children's simple addition skills, Geary and Burlingham-Dubree (1989) found that individual differences in the choice of strategies for solving addition problems, that is, counting strategies or direct memory retrieval, were related to spatial ability. Each child was given a score on a strategy choice variable reflecting accuracy of use of both direct retrieval and overt counting strategies. Differences on the strategy choice variable were not related to language ability, but were significantly positively correlated with two measures taken from the Wechsler Preschool and Primary Scale of Intelligence (Geometric Design and Mazes). These tasks require reproduction of simple and complex geometric designs, and spatial scanning. Geary and Burlingham-Dubree suggest that early reliance by children on spatial information and strategies, for example, combining arrays of objects to add and solve arithmetic problems, may contribute to the observed relationship between performance on numerical and spatial ability measures. The importance of visual-spatial skills for basic arithmetic may decline as facts and rules become automated (Fuson, 1988; Hartje, 1987). This would agree with Geary and Burlingham-Dubree's findings that spatial skills were related to strategy choice but not related to speed of fact retrieval.

Davis and Bamford (1995) conducted a study to investigate the use of imagery in young children's arithmetic performance. All children were presented with arithmetic problems that had contextual support, that is, concrete representations for each of the numbers involved in the calculation. Some of the children were also prompted to use an imagery strategy of imagining a picture in their heads of the concrete representations. Davis and Bamford firstly found that contextual support lead to the production of more correct answers. Furthermore, the children prompted in the use of the imagery strategy performed at an even higher level of accuracy. Therefore, it would appear that visual imagery does provide a useful resource in solving simple arithmetical problems.

In an effort to form a taxonomy of arithmetic disability subtypes, Geary (1993) identified three subtypes of arithmetic difficulties, one of which had visual-spatial problems as its basis. Studies of acquired and developmental dyscalculia (e.g., Rourke, 1993; Rourke & Finlayson, 1978, Strang & Rourke, 1985; Temple, 1991) also report difficulties with visual-spatial skills in some cases. Such difficulties lead to problems in the organisation of numerical information, for example, misalignment of numbers in columns, problems with directionality (e.g., taking away the wrong number in subtraction), and misreading of arithmetical signs. Therefore, evidence from developmental studies of children's arithmetical skills, and neuropsychological studies of dyscalculia, suggest that visual-spatial abilities do play a role in the early development and execution of children's mathematical skills, and may also play a role in the solution of more complex mathematical problem solving involving multiple stages.

Central Executive.

Whilst children's arithmetical and reading skills have frequently been assessed in relation to short-term memory span, that is, the functional capacity of the articulatory loop, studies investigating the role of central executive functioning are few and far between. As pointed out by Swanson, Ashbaker, and Lee (1996), performance differences between learning-disabled and non-disabled readers on measures of cognitive function are often attributed to limitations in working memory. However, there is no consensus as to the nature of working memory, or more specifically, the aspect of working memory that is most affected by deficiencies in reading ability. For example, deficits in working memory performance may be due to storage capacity, processing efficiency or both. A similar argument can be levelled at studies of children's arithmetical skills, which have frequently only been concerned with tasks which assess storage and rehearsal capacity of the articulatory loop.

Despite the fact that the central executive is the controller of working memory, and that it is the most important aspect of working memory in terms of its general impact on cognition, very little is know of its functioning, although it is assumed that it co-ordinates the operations of the articulatory loop and the visual spatial sketch pad. The view held by researchers as to the nature of the central executive is also subject to considerable variation, with some researchers viewing the executive as a unitary system that may form the basis of a general factor of intelligence (Duncan, Williams, Nimmo-Smith, & Brown, 1991; Kyllonen & Chrystal, 1990), whilst others suggest the executive comprises a range of relatively independent sub-process (Baddeley, 1996; Duncan, 1993).

The concept of the central executive was strongly influenced by the Supervisory Activating System (SAS) model of attentional control proposed by Norman and Shallice (see Shallice, 1982). This model involves two sources of action control, one dealing with well-learned habitual patterns, with the other acting as an attentional controller, capable of overriding habitual response patterns when it is necessary to initiate new behaviour. Other functions have been proposed as being under the control of executive functioning. These include planning and goal direction (Duncan, 1986), an inhibitory function where dominant action tendencies are suppressed in favour of more goal-appropriate behaviour (Diamond, 1989), dual-task performance, (Baddeley, Bressi, Della Sala, Logie, & Spinnler, 1991; Baddeley & Wilson, 1988), selective attention and capacity to switch attention, and retrieval of information from long-term memory (see Baddeley 1996 for a recent overview of the functions served by the central executive). Cantor and Engle (1993) suggested that working memory span may reflect the temporary activation of areas of long-term memory, with high span subjects being able to activate more extensive regions of long-term memory due to higher levels of activation being spread between different items of information

The development of such executive control processes have long been thought to underlie developmental improvements in a wide variety of cognitive contexts. This can be thought of in terms of Piaget's developmental stages. During the pre-operational phase the child's thinking is inflexible and captured by external salient stimuli. Mental representations do not contain all the relevant task information, abstract sequences of actions, or imagined goals or outcomes. As the child progresses through the concrete operations and formal operations stages of development, the child becomes able to reflect on abstract notions such as his or her own knowledge base, and efficacy of behaviour, thereby facilitating executive functions such as planning and self-monitoring. Self-monitoring is necessary for gauging the effectiveness of behaviours currently in operation, so that it may be maintained and flexibly adapted. For example, in the case of arithmetic, the child who begins to solve the question 9 + 5 by counting both numbers using their fingers as concrete representations has to monitor the situation, and discover that the 'counting all' method is not going to be sufficient to solve this problem, because the child will run out of fingers before the count is completed. The child will also need to decide whether they know of another counting procedure that will enable them to solve this problem, for example, counting on or decomposition.

Studies in developmental neuropsychology have also noted that the central executive is a complex structure, identifying similar functions performed by the frontal lobes, thought to be the neurological location of executive functioning (Welsh, Pennington, & Groisser, 1991). Welsh et al (1991) examined neuropsychological and cognitive tests of executive functioning, and the performance of children of differing ages on these tasks. Factor analysis was conducted to see if all tasks would load on one executive function measure. However, three measures were revealed, the first being referred to as Fluid and Speeded Response (verbal fluency, visual search and motor sequencing). The second factor was referred to as Hypothesis Testing and Impulse Control (Wisconsin Card Sorting Test and the Matching Familiar Figures Test, which is a cognitive measure designed to capture individual differences in cognitive style). The Tower of Hanoi loaded on the final factor. Therefore, in contrast to earlier work which viewed the central executive as a unitary store, more recent evidence suggests that this store is more like a cluster of relatively autonomous control processes.

Given the size and mass of connections with other parts of the nervous system, the frontal lobes have been implicated in a variety of behaviours, including motor skills, disrupted organisational and planning skills, generalised memory deficits, difficulties with mental flexibility, and poor task initiation, as well as behavioural difficulties such as distractibility and problems with sustained attention during complex information processing (Stuss & Benson, 1984; Wolfe, 1996).

Frontal lobe functioning in children shows its main developmental increase from the ages of 7 to 10, and around 10 years of age children are reaching adult performance levels of measures of executive functioning, such as the Wisconsin Card Sorting Test (Case, 1992; Chelune & Baer, 1986; Levin, Culhane, Hartmann, Evankovich, Mattson, Harward, Ringholz, Ewing-Cobbs, & Fletcher, 1991; Passler, Isaac, & Hynd, 1985; Welsh et al, 1991). Children acquire executive skills at different rates. It is reasonable to suggest therefore that at seven years of age, some children may be more developmentally advanced in their frontal lobe development than other children of the same age, that is, there will be individual differences, as is the case with virtually all cognitive skills. If frontal lobe executive functioning does underlie working memory skills, which is in turn the proposed limiting function for the development of other cognitive skills, then we may presume that some children will be more advanced than others simply due to their neurophysiological development. This is one issue rarely addressed in any studies of children's learning difficulties, but must be a point of consideration. One exception to this is Denkla who has carried out a number of electrophysiological studies of dyslexic and control groups. These studies indicate that there are differences in EEG rhythms in the frontal regions between the control and dyslexic groups, when they were engaged in reading and other language-related tasks (see e.g., Denkla, 1983). Obviously problems arise in trying to measure the degree of frontal lobe functioning, and without the use of direct neurological measures we are left to deduce the extent of frontal lobe or executive functioning from cognitive measures. Fortunately, studies in developmental neuropsychology are attempting to identify measures used in neuropsychological studies of adults, that might also be readily adapted and applied to developmental studies of children (e.g., Gnys and Willis, 1991).

Wolfe (1996) examined cognitive functioning attributed to the frontal lobes in children aged 7 to 12 years with learning disorders. This study incorporated the use of a number of measures, these being a freedom from distractibility measure derived from factor analytic studies of the Wechsler Intelligence Scale for Children - Revised (WISC-R, Wechsler, 1977), showing sensitivity to behavioural and cognitive abilities, including attention and concentration, and sequential processing. The Wisconsin Card Sorting Test was also administered, along with the Trail Making test, considered to be a measure of conceptual flexibility, visuo-motor tracking, the ability to mentally follow a sequence, and to deal with more than one stimulus at a time. Results showed that children with learning disabilities performed more poorly on the freedom from distractibility measure, indicating deficits in sequencing and related executive functions. These children also performed poorly on the trail making task, suggesting problems with shifting cognitive set in the alternating of sequential plans. Wolfe concluded by saying that learning disabled children experience more difficulty on tasks requiring a high demand for selective attention, inhibiting interference, shifting of conceptual sets, sequential reasoning, and the organisation and integration of new information.

One study which has examined the role of the articulatory loop and the central executive in relation to academic achievement is that of Lehto (1995), who studied Finnish children aged 15 to 16 years old. Lehto used simple span tasks (digit span and word span) to measure the functioning of the articulatory loop. Complex span tasks were also used which simultaneously tax the storage and processing functions of working memory, in a similar manner to the reading span task of Daneman and Carpenter (1980). These tasks are thought to probe the central executive rather than specifically the articulatory loop. A further memory updating task was administered which is suggested to measure the central executive more specifically (Morris & Jones, 1990). In this task, subjects are required to recall the last four or six items of a string of unrelated consonants. However, the length of the presented lists varied unexpectedly, and may have been two, four, or six items longer than the subjects were asked to recall. Therefore, the task for the subjects is to drop earlier items of the list in order to recall the last items, that is, it requires a shift in rehearsal frame from the first item to a later item. Performance on these memory tasks was considered in conjunction with four school subjects; Finnish, foreign language, mathematics, and geography. Performance on the memory updating task correlated significantly with performance on all four subjects. Further correlational analysis showed that when performance in simple span tasks was partialled out, correlations between complex span and the memory updating task and school performance remained significant. Focusing on mathematics performance, the highest correlation came from the memory updating task, a central executive measure. When complex span and the memory updating task were partialled out, simple span only correlated with foreign language performance. From these results, Lehto suggested that the articulatory loop, measured through simple span measures, is related to foreign language success. In contrast, general school achievement, including mathematics, is more strongly related to the functioning of the central executive.

Logie et al (1994) considered the role of all functional aspects of working memory and their relative contribution to mental arithmetic performance in adults. Logie et al used a dual-task procedure to examine the extent to which disrupting each component of working memory would affect mental arithmetic performance. Articulatory suppression (repeating 'the' throughout presentation of the arithmetic items) was used to disrupt functioning of the articulatory loop. To disrupt the functioning of the visual-spatial sketch pad, hand movements were made and irrelevant pictures were shown. Finally, random letter generation was used to disrupt functioning of the central executive. Results showed that disruption of the central executive through random letter generation produced the most errors in mental addition performance, with articulatory suppression producing a smaller decrement in performance. Mental addition remained largely unimpaired by hand movements or irrelevant pictures. These results suggest that mental addition relies on central executive resources.

Similar results were found by Lemaire, Abdi, and Fayol (1996) who also used a dual task methodology to look at arithmetic performance when the articulatory loop was disrupted through articulatory suppression, and when the central executive was disrupted by the necessity to randomly generate letters. Results showed that disrupting the central executive led to the greatest slowing of response time, and to increased errors. This was particularly the case with more difficult problems which were more difficult to retrieve from long-term memory. Lemaire et al go on to suggest that the central executive is the critical component of working memory involved in simple mental arithmetic.

Evidence published after the research for this thesis was started now leads to speculation that there may not be a definitive link between arithmetic and short-term memory. Butterworth, Cipolotti, and Warrington (1996) report the case of a patient (MRF) who showed poor performance on short-term memory tests but whose arithmetic skills were intact. MRF was found to have a poor short-term memory span on tasks requiring spoken recall, or a written or pointing response, with his span for digits and letters being between only two and three items. The Brown-Peterson task was also administered to assess the rate of forgetting of short-term information. On this task MRF showed abnormally fast forgetting for two letters and even for a single item. However, MRF performed at normal levels on the calculation tasks administered, with his ability ranging from normal to superior on all tests. The Paced Auditory Serial Addition task was also administered specifically to investigate the relationship between short-term memory and arithmetic calculation. This task requires the subject to add a new digit to the previously presented digit, ignoring the intervening response (e.g., presented with the digits 2, 1, respond 3, presented with the digit 6, respond 7). Hence, successful performance on this task depends on selecting the correct item from many in short-term memory and calculating a sum. MRF's performance on this task was reliably good. Clearly, this finding does not correspond to previous results from cognitive studies investigating short-term memory and arithmetic performance. Butterworth et al go as far as to propose that the whole concept of working memory should be subject to careful consideration.

Therefore, the evidence relating short-term memory to arithmetical difficulties is somewhat inconclusive despite a wealth of studies. There are discrepancies between studies over the extent and nature of the relationship between short-term memory and arithmetic, and when short-term memory difficulties are found, the explanation as to why those difficulties occur is often inconclusive. Furthermore, studies have focused almost exclusively on the articulatory loop by using span tasks which assess the storage capabilities of short-term memory. The theoretical understanding of the visual-spatial sketch pad, and even more so, the central executive, is far from fully developed, and because of this, many studies have been reluctant to measure these aspects of memory and to assess their roles in the learning of academic skills. However, those studies that have dabbled their feet in the relative unknown have shown that visual-spatial and central executive skills may have a crucial role to play in the solution of arithmetical and more general mathematical problems.

1.4 The Role of Processing Speed and Efficiency in Arithmetical Skills.

Consideration of short-term memory leads directly onto related issues concerning processing speed and efficiency. One reason why differences in processing speed are potentially important is that tests reporting to measure processing speed are consistently found to be related to performance of other cognitive skills, and to measures of higher-order cognitive processes, for example, short-term memory (Fry & Hale, 1996; Hale, 1990; Kail, 1991a, 1991b, 1992, 1997; Kail & Hall, 1994; Kail & Park, 1994; Kail & Salthouse, 1994; Rabbitt & Goward, 1994). It is also assumed that general limitations such as the speed with which we can process information impose constraints on many types of processing, which in turn impacts on the performance of many cognitive tasks (Salthouse, 1996). In understanding changes in processing speed, many studies have considered the decline in speed of executing cognitive operations that accompanies ageing in adults, although the basic mechanism is relevant across the entire life span (see Salthouse, 1991, 1994). Salthouse (1992a) provides an example of this where he investigated the effects of processing speed on adult age differences in working memory. Working memory performance is shown to deteriorate with age, and Salthouse showed that age-related variance in composite measures of working memory was reduced by 70% after statistically controlling for a composite measure of perceptual comparison speed.

Salthouse (1996) discusses the results of many of his studies and those showing a general slowing of processing speed accompanying ageing in terms of a limited time mechanism, and a simultaneity mechanism. The limited time mechanism suggests that when the early cognitive operations of a task are executed slowly, there will be a greatly limited time available to complete the processing required for later stages of the task. Therefore, all of the necessary operations required by the task will not be completed if the processing is too slow. The metaphor provided by Salthouse for the limited time mechanism is:

"...an assembly line because if all relevant processing operations are not successfully completed within a particular temporal window, then the quality of the final product is likely to be impaired because later processing operations would either be less effective or only partially completed. " (Salthouse, 1996, p. 404).

The simultaneity mechanism is subtly different. This mechanism is based on the idea that the products of early processing may be lost by the time later processing is completed, if the operations required by the task are executed slowly. Therefore, not all relevant information would be available when it was needed. This would result in a high degree of errors or time consuming repetition of operations. Again, Salthouse provides a useful metaphor in terms of juggling for understanding this mechanism:

"...the fundamental principle is that many complex activities require synchronisation of the constituent tasks, and synchronisation is easier when the relevant processing operations can be executed rapidly." (Salthouse, 1996, p. 404).

Much of the research on processing speed has considered age related changes, such that with increasing age, the speed of processing information becomes progressively more rapid. Two distinctive views have been put forward to account for such changes. One view emphasises experiences that lead to changes in the speed of processing in specific domains, (*local trend hypothesis*; Hale, 1990). For example, differences in processing speed may reflect more efficient strategies for task solution, or the fact that knowledge in specified domains becomes more elaborate, leading to more rapid access of information. These changes in knowledge elaboration come about through experience in a specific domain, and make items of information required to deal with a particular problem more accessible, more richly interrelated, and co-ordinated into larger chunks (Anderson, 1988; Chi, 1978; Chi & Ceci, 1988; Logan, 1988; Roth, 1983). This view is illustrated by Logan's (1988) 'instance theory', where each time a task is performed, the stimulus and response are stored in memory. Repeated task performance increases the strength of these representations, so that the answer can eventually be retrieved directly from memory rather than being computed using an algorithm. Therefore, speed of information processing in that specific domain increases.

An alternative view of processing speed emphasises more global changes (e.g., *global trend hypothesis*; Hale, 1990; Pascual-Leone, 1970). Such a view suggests that all information processing components develop at the same rate, due to actual changes in processing capacity. Increased processing space could result in more processing occurring in parallel, or that fewer results of processing must be exchanged between the computing space and long-term memory, therefore leading to more rapid processing of information. Such changes in processing speed would result in differences in performance on most speeded tasks, rather than those specific to certain domains. To illustrate this an analogy is made to computer hardware (Ridderinkhof & van der Molen, 1997; Salthouse & Kail, 1983). Processing speed is compared to the clock speed of a computer, where as the young child's clock speed increases, the speed of processing all cognitive processes increases until an adult level is reached. The relation between age and processing speed has been observed on a number of tasks such as simple classification (Akhtar & Enns, 1989), mental rotation (Kail, 1986), reading (Kail & Hall, 1994), visual-spatial skill (Kail, 1997), picture and name matching (Bisanz, Danner, & Resnick, 1979), simple response time, and perceptual motor tasks (Kail, 1991a, 1991b). Hale (1990) also provides evidence to support this view. She tested 10, 12, 15, and 19 year olds on four measures of speeded performance (letter matching, choice reaction time, abstract matching, and mental rotation). Results showed that processing speed increased across all tasks at a constant rate until the age of 15. Findings such as these provide strong evidence for the view that there is a general mechanism responsible for age related changes in processing speed. However, whether this is due to a change in processing *capacity*, as would be suggested by Pascual-Leone, or a global change in processing *efficiency* remains to be determined.

As a combination of these two alternative explanations, Kail (1991a) further proposes that initial performance on a task is likely to be based on a globally speeded mechanism, with later performance on processes tailored to a particular task. Whenever execution of an operation requires cognitive resources, processing speed will be limited by the availability of those resources. Greater task experience means that performance is more likely to reflect retrieval of a stored response, meaning that speed is no longer constrained by the available resources. Further to this, Geary (1995; see also Geary & Lin, in press) makes the distinction between biologically-primary and biologically-secondary abilities, suggesting that the usual decline in speed of processing which accompanies the ageing process may only be for biologically-primary abilities, and not necessarily those which are seen as secondary. Biologically-primary abilities are those that emerge through natural activities in childhood, and as such, are to be found in schooled and unschooled populations. Examples of primary mathematical abilities include enumeration, magnitude comparison, and simple arithmetic abilities involving small numbers. As noted in Section 1.1, there is evidence that infants are able to quantify small groups of objects of 3 or 4 items (Antell & Keating, 1983; Starkey, 1992; Wynn, 1996), suggesting that this is a primary numerical ability. Magnitude comparisons thought to be primary abilities are limited to quantities associated with 1 to 3, with a similar limit being placed on primary arithmetical abilities. Secondary abilities are defined as cognitive competencies that emerge in unnatural contexts, such as school. For example, skills learned in school, such as reading and complex arithmetic, do not emerge in unschooled populations. This includes counting of larger value numbers, magnitude comparisons of larger value numbers, and more complex arithmetical skills learned in school. Geary and Lin (in press) found that speed of executing primary processes did decline with age, but that secondary abilities showed no disadvantage for the older adults, for example, enumeration of larger values, magnitude comparison of large numbers, and arithmetic fact retrieval.

Processing speed and changes in processing efficiency, may be observed through the various forms of counting strategies that children use to solve arithmetical problems. On the basis of processing load involved, the most inefficient way to perform addition is to use analogues to support limited knowledge of place value, for example, using fingers to represent integers as in the counting-all and counting-on strategies identified by Geary (1990). The normal developmental progression of addition strategies is from counting fingers using the counting-all and counting-on procedures, through verbal counting, and finally to direct retrieval. In terms of skill development, the preferred strategy choice is direct memory retrieval because this strategy requires less time to execute, and places fewer demands on working memory resources.

Studies concerning learning difficulties in arithmetic, such as those by Geary & colleagues (see also Fleischner et al, 1982), suggest that performance characteristics of children with arithmetic learning difficulties include the frequent use of inefficient problem solving strategies, such as the use of analogues (physical representations) for counting, strategies often employed by younger, academically normal children (Geary, 1990: Geary & Brown, 1991) Use of inefficient problem solving strategies may be in part due to an immature or abnormal development of long-term memory representations of basic addition facts. Accompanying this characteristic are long solution times to arithmetic problems, and frequent computational and memory related errors. Computational or memory related errors may be related to the availability of working memory resources, which in turn may be influenced by counting speed, attentional allocation or rate of decay. Where processing requirements extend over a period of time, as in the case of solving arithmetical problems, it may be possible that the products of early processing begin to decay before later requirements of the processing have been completed. This would correspond to Salthouse's notion of the simultaneity mechanism.

Speed of processing information has not been considered in relation to mathematics ability directly, only in terms of the speed of retrieving arithmetic facts from long-term memory. Clearly, it has been shown in these studies that children who are poorer at arithmetic are slower to retrieve known facts from long-term memory. This may be accounted for by a very specific difficulty related to the ability to process numerical information, or could be the result of a general slowness in processing information, that will impact on all aspects of mathematical skill. Furthermore, whilst most studies of children's mathematical difficulties explain these difficulties at the level of short-term memory, it may be the case that differences in the speed of processing information is fundamental to short-term memory, and that we should be looking at this lower level for an explanation of differences in mathematical ability.

1.5 Long-term Memory Organisation and Retrieval of Arithmetic Facts and Procedures.

Many ideas concerning long-term memory pull from theories and research considering processing speed and efficiency, with an emphasis on the importance of the automatisation of arithmetic facts. Long-term memory is considered as a vast, relatively permanent store of information. Ashcraft (1987, 1992) proposes that arithmetic knowledge is represented and accessed in the same fashion as other forms of long-term memory knowledge. Geary and Brown (1991) suggest that long-term memory representations in children with arithmetical difficulties may be immaturely or perhaps abnormally organised. This explanation is consistent with studies that have considered arithmetical counting strategies (e.g., Geary, 1990). Children with mathematical difficulties rely on memory retrieval of number facts far less than children who show good mathematical ability. When memory retrieval is used, retrieval times have been shown to be highly unsystematic, that is, not related in any way to the difficulty of the problem being solved, and frequent retrieval errors are made. This again provides a substantial link to short-term memory problems, because a failure to solve answers correctly in the first instance may lead to inaccurate or slow to form representations of number bonds in long-term memory.

This line of reasoning becomes clear when one considers theories of number representation in long-term memory (see McCloskey & Macaruso, 1995, for a review of the representation of numerical information). A number of studies offer support for the role of long-term memory involvement and automatic fact retrieval. In a study by Ashcraft and Battaglia (1978), it was found that performance on verification questions (deciding whether a presented answer is true or false), was slower when that problem would have been correct if a difference operation were being used (e.g., 6 + 1 = 5). They suggested that this was due to a relatedness effect among arithmetic problems and operations (see also Zbrodoff & Logan, 1986). In a more recent study, Lemaire, Fayol, and Abdi (1991) reported that children aged 9 and 10 years exhibited associative confusion effects in an arithmetic verification task. Children were slower to reject questions as being false when the question would have been correct if a different operation were being performed, suggesting that sums and products were being automatically retrieved from long-term memory. Similar results were found by Lemaire, Barrett, Fayol, and Abdi (1994). Association between a number pair and its sum or product were of sufficient strength during primary school years to produce interference effects. The size of integers presented was a critical determinant of whether interference effects were found in a particular age group, with the range of problems affected by interference increasing throughout the school years.

Studies have also considered the types of retrieval errors made when answering arithmetic questions. Campbell and Graham (1985) tested multiplication performance and tabulated the types of errors that were made. For adults, more than 90% of errors were 'table errors', for example, when presented with the question '6 x 4', a likely error would be 32, because this is a multiple of four. Similar results were also found for children who had only recently been introduced to multiplication. Priming effects have also been observed in arithmetic. Campbell & Clark (1989) demonstrated that after processing an arithmetic question, such as 4 x 6, then the probability of incorrectly responding "24" to 3 x 7 is increased. These priming effects were found to last for approximately one minute, suggesting that stored arithmetic facts behave like logogens with a fluctuating threshold of activation.

Such patterns of errors and relatedness effects (comparable to frequency effects in lexical access) provide information as to how arithmetic facts may be represented and organised in long-term memory. Two classes of memory retrieval model have been proposed, these being tabular and nontabular. The three main models to consider the representation of arithmetic facts are the network retrieval model (Ashcraft, 1987), the distribution of associations model (Siegler & Shrager, 1984), and the network interference model (Campbell, 1987). All three models share a number of common assumptions. Firstly, performance on simple arithmetic questions depends on retrieval from long-term memory. In general, memory network retrieval reflects some form of search through a stored network of answers for the correct answer to a given problem and then retrieval of the correct answer once it is located. Secondly, memory representations are organised and structured in terms of the strength of individual connections, reflecting differing degrees of relatedness among the elements. Finally, the strength with which the elements are stored, and hence the probability or speed of retrieving information, depends on experience.

An early tabular model of arithmetic fact organisation was proposed by Ashcraft and Battaglia (1978). This model is conceptualised as a square, symmetrical two-way table with nodal values ranging from 0 to 9. These values correspond to addends of a simple addition question. The correct answer to an addition question is stored at the intersection of the two nodal values in the problem. Retrieval time is thought to be related to the distance from the origin of the table (0, 0) to the intersection of the nodal values for the given arithmetic problem. The basic assumption of the network retrieval model is that arithmetic number facts are represented in an interconnected network in long-term memory, being accessed and retrieved through a process of spreading activation. This spread of activation is triggered by three sources: the problem addends, the answer stated in the problem, and nodes in the network that had been activated during retrieval. Activation spreads through the network in parallel, with nodes accumulating various levels of activation depending on their strength and relatedness, the answer selected being the most highly activated node. Ashcraft states that the strength with which nodes are stored and interconnected is a function of the frequency of occurrence and practice, particularly in early education. This retrieval from long-term memory, according to Ashcraft, represents declarative knowledge. Where such declarative knowledge is not present, children will rely more heavily on procedural knowledge of arithmetic, this largely entailing the mental process of counting. Hamann and Ashcraft (1985) compared performance on simple and complex arithmetic questions over various ages of children. They found that declarative knowledge, that is, the memory network, increased with age, with procedural knowledge becoming more elaborated such that the procedures required for answering more complex arithmetic questions could be applied. A speed advantage for older children was also found, which Hamann and Ashcraft explained as being due to an increase in the automaticity of responding.

Two shortcomings of this model have been proposed. Firstly, the model only takes account of direct memory retrieval and fails to consider alternative strategies for reaching the correct answer to an arithmetic question. Secondly, the model fails to account completely for errors in arithmetic fact retrieval. Campbell's network interference model draws on many of the assumptions made by the network retrieval model. Campbell's unique contribution comes from a focus on interference as an unavoidable part of the retrieval process, and his elaboration on the kinds of stored connections. Campbell suggests that there are not only associations from individual numbers to answers, but also associations from whole problems to answers. In this model, each act of retrieval activates a large number of associations and potential responses, such that interference occurs from other activated nodes, with the most strongly activated value being retrieved.

An example of a nontabular memory network retrieval model is that proposed by Siegler and Shrager (1984), this being the distribution of associations model. This is the model most frequently employed in recent studies to explain patterns of arithmetic development and retrieval errors. Nontabular models presuppose that number facts are stored in memory in networks resembling those proposed for semantic facts. Associations between problems and answers are formed each time a particular arithmetic problem is encountered, regardless of the correctness of the answer. The distribution of associative strengths of alternative potential answers for each problem varies from very peaked to very flat as a function of problem size. A problem with a more peaked associative strength requires less time to verify as less effort is required to distinguish between the correct sum and alternative incorrect sums. The associative strength is governed by the frequency of correct and incorrect answers associated with the problem. Where the correct answer has frequently been associated with a particular problem the distribution of associations will be peaked, with the correct answer very clearly having the highest strength of association. Retrieval errors will occur if an incorrect association is retrieved from memory, resulting from an incorrect answer frequently being associated with a particular problem, or a flat distribution of associations due to lack of practice with a particular problem. Alternatively, errors will occur if an alternative backup counting strategy has not been correctly executed. The probability of an answer being correctly retrieved is also determined by a confidence criterion and a search time parameter. If a retrieved answer does not reach the confidence criterion due to a flat distribution of associations, or if the search time parameter (number of searches) has been exceeded, then an alternative strategy will be employed, for example, verbal or finger counting. Therefore, according to this model, children begin with an initial distribution of associations of varying strengths between each simple arithmetic problem and possible answers. With experience, the structure of associations changes. Connections between increasing larger integers and their sums and products are strengthened, and activation of these associations becomes automatic. Therefore, accuracy is a very important factor because if the child makes many computational errors, particularly within the early stages of learning, the probability of retrieving the incorrect answer during subsequent attempts to solve the problem will be increased.

Geary (1990) compared normal and mathematically-disabled children on the strategies used and time taken to solve simple addition questions. After one year of remedial instruction, the mathematics-disabled group were split into an improved group and a no-change group. The normal and improved groups showed no significant differences in strategy choice, errors, or in the rate of information processing. Performance of the no-change group in comparison to the other ability groups included frequent verbal counting and memory related errors, frequent use of immature counting strategies, a variable rate of executing the verbal counting strategy, and solution times for memory retrieval that were unsystematic. Such performance was interpreted as indicating a lenient confidence criterion (hence the large number of errors), and flat distributions of associations of arithmetic facts (and therefore leading to infrequent use of direct memory retrieval). Results from this study also suggested that the use of immature problem solving strategies may have been related to poor working memory resources. Therefore, these results are consistent with conceptual models of the maturational development of arithmetic skills.

A follow-up study of the children examined by Geary (1990) was conducted by Geary et al (1991) in which the pattern of skill development in arithmetic was evaluated and measures of working memory performance were administered. Results from this study showed that for the normal ability group, arithmetic skill development reflected increased reliance on memory retrieval, a decrease in the frequency of errors, and decreased reliance on immature counting strategies. The mathematics-disabled group showed no change in their use of strategies, although they were more skilled counters compared to performance in Geary (1990). Verbal counting and memory retrieval were the most frequently used strategies by both ability groups. Results from working memory tests were consistent with the claim that mathematically-disabled children have poor working memory resources (Geary, 1990).

A number of studies have investigated the relationship of individual differences in strategy execution with measures of achievement or ability. One such investigation carried out by Widaman, Little, Geary, & Cormier (1992) aimed to determine which factors were related to arithmetic efficiency. They found that individual differences in addition efficiency were strongly related to individual differences in mathematics achievement, verbal comprehension and reading skills. A speediness variable comprising encoding and intercept speed was found not to be related to any of the three latent variables of mathematics ability, verbal comprehension and reading skills. They did however find that counting speed explained significant amounts of variance in each of three mathematics achievement subtests (computations, concepts, and applications), and three subtests related to reading skills (comprehension, reading and word study skills, and spelling). In summary, this suggests that the more highly automated the counting process or the retrieval process, the higher the level of achievement as shown in the mathematics and reading latent variables. However, speediness both for digital counting (computation) and for memory retrieval were not significantly related to measures of achievement. Geary and Widaman (1992) also added a working memory latent variable. For college students, working memory was found to be related to a general reasoning factor, but not with numerical facility or arithmetic efficiency. From this they concluded that working memory capacity influences certain types of performance such as reasoning, but does not influence the numerical facility directly. Little and Widaman (1995) carried out a similar study but with a wider range of participants ranging from school children to college students. Results for the college students supported those of Geary and Widaman (1992), but interestingly found that working memory was related to numerical facility and perceptual speed for the school age children. This lead Little and Widaman to suggest that relations among processing components change during development, with working memory capacity having an influence on ability performance at early ages, but not at later ages when performance is more highly automated.

Results from studies that have considered children's arithmetical difficulties are consistent with these models of long-term memory representations. Patterns of change in the use of different counting strategies suggests that long-term memory organisation of basic arithmetic facts may be one of the major factors contributing to mathematical achievement. Children's arithmetical difficulties may to some extent be explained by a delayed or abnormal development of these long-term representations (Geary, 1990; Geary & Brown, 1991; Geary et al, 1991). Such problems may arise due to initial working memory difficulties, particularly associated with speech rate. With a slow counting speed, the representations of the numbers are more likely to be forgotten before the count is completed. Even if the child reaches the correct answer using a computational strategy such as counting, the answer might not become strongly associated with the problem. Therefore, the development of long-term memory representations of basic arithmetic facts is likely to be related to the speed of executing computational strategies, as well as to computational accuracy, which also appears to be related to memory span (Geary et al, 1991) Incorrectly answering arithmetic questions will lead to incorrect associations being formed in long-term memory, resulting in a flat distribution of associations. Future attempts to solve the problem may lead to the incorrect association being retrieved from long-term memory (Geary, 1990). Only when the representation is strengthened, that is, when the distribution of associations is more peaked, will the shift from the use of counting strategy to accurate direct retrieval of arithmetic facts take place (Geary & Burlingham-Dubree, 1989; Lemaire et al, 1991; Lemaire et al, 1994; Lemaire & Siegler, 1995).

Difficulties in the early stages of arithmetic with basic numbers operation such as counting, addition, and subtraction, leads to later difficulties in that the child has a poor grasp of the basic skills on which the development of more complex skills rests. The child may also lack motivation due to difficulties they are faced with, which may in turn lead to the avoidance of activities with numbers. Without sufficient practice and experience with number and number combinations, representations will be slow to form in long-term memory (Ashcraft & Faust, 1992; Strang & Rourke, 1983). This returns to the ideas raised by Hitch and McAuley (1991) that deficits in short-term memory span showed by children with arithmetical difficulties may be in fact due to slow access of representations from long-term memory. Low familiarity with numbers may account for slow access to number representations in long-term memory, and in turn explain slow counting and low digit span. Hitch and McAuley (1991) found that children with arithmetical difficulties did understand the basic principles of counting and how to apply them (evidenced though the accurate counting of dots on cards). However, these children were much less fluent than children of normal arithmetic ability.

Work carried out by Kirby and Becker (1988) considered whether children's arithmetical difficulties were due to deficits in the encoding of numbers, the efficiency of executing operations, or the use of differing strategies for carrying out arithmetical calculations. Findings indicated that arithmetical difficulties were characterised by very slow execution of operations. Kirby and Becker noted that this was not the result of lack of familiarity with numbers as children had received formal instruction in mathematics for a number of years. Rather, they suggested that such a finding was apparent because the children had failed to automate the basic operations of arithmetic.

To summarise, children with mathematical difficulties show quite specific performance characteristics on measures of simple arithmetic skill. These children do not retrieve as many facts directly from long-term memory, and are slow in retrieval and execution of counting strategies. Evidence from models of long-term arithmetic fact organisation helps to interpret these results, for example in terms of lack of familiarity leading to weak associations in long-term memory, or early errors leading to similar associative strengths to a number of answers.

1.6 Neuropsychological studies of acquired and developmental dyscalculia.

Neuropsychological studies put forward a cognitive architecture that is modular in structure, based on patterns of selective impairment shown by brain damaged patients. Such modularity has been found in studies of dyscalculia, with models being proposed to account for the fractionation of arithmetic skills (McCloskey & Aliminosa, 1991; McCloskey, Aliminosa, & Macaruso, 1991; McCloskey, Caramazza, & Basili, 1985; Temple, 1991, 1994; Warrington, 1982). The definition of developmental dyscalculia has varied over the years, the most simple definition being a "difficulty with counting", through to the more complex definition of the disorder given by Kosc (1974):

> *"a structural disorder of mathematical abilities which has it's origins in a genetic or congenital disorder of those parts of the brain that are the direct anatomico-physiological substrate of the maturation of mathematical abilities adequate to age, without a simultaneous disorder of general mental functions"* (p. 165).

Hinshelwood (1917; cited in Temple, 1994), observed selective disorders in arithmetic skills, and emphasised that processes involved in numerical processing were distinct from those involved in reading. Consideration of acquired and developmental dyscalculia, that is, impaired number comprehension, number production, or calculation resulting from brain damage, offers insight into the normal cognitive processes underlying arithmetical skills. McCloskey and Aliminosa (1991) examined the cognitive mechanisms underlying performance in arithmetic by considering single case studies of patients. This type of analysis shows how cognitive number processing and calculation mechanisms may be fractionated, and using such approaches it has been possible to classify dyscalculia into three distinct disorders. The first of these is alexia or agraphia for numbers. This deficit involves difficulties in the reading and writing of numbers with intact skills in other areas of arithmetical processing (McCloskey et al, 1985). The second classified deficit is spatial dyscalculia, characterised by difficulties in the spatial representation of numerical information, leading to a lack of understanding of the rules placing digits in the proper order, for example, misalignment of numbers, problems with directionality (e.g., taking away the wrong number in subtraction), number rotation, difficulties with regard to visual detail such as misreading arithmetic signs, and difficulties with place value and decimals, but with intact number reading and writing skills and basic arithmetic fact retrieval (see e.g., Rourke, 1993; Rourke & Conway, 1997; Rourke & Finlayson, 1978; Strang & Rourke, 1985). Such an association between visual-spatial skills and arithmetic performance was referred to earlier in discussing the visual-spatial sketch pad. The final disorder is anarithmetria, which is a difficulty in the retrieval of basic arithmetic facts from long-term memory. There may also be some confusion of arithmetical operations, as well as difficulty in operations requiring number sequencing.

A number of studies provide examples of dissociations between these skills. Warrington (1982) reports patient DRC, who was found to have intact number comprehension and production, but who showed impairment in the retrieval of arithmetic facts. McCloskey et al (1985) reported patient MW, who again showed impaired retrieval of arithmetic facts in the presence of intact execution of calculational procedures. Patients have also been presented showing opposing patterns of results, offering support for the fractionation of arithmetic skills.

Whilst the studies carried out for this thesis do not include individuals with clinically diagnosed dyscalculia, it is clear that the subtyping of mathematical difficulties in this population resembles the subtyping described by Geary (1993), who was able to classify mathematical disabilities into three subtypes, one revolving around a deficit in visual-spatial skill, another with difficulties in retrieving information from long-term memory, and the third with procedural difficulties. As such, these studies are of value in providing a more detailed understanding of the types of mathematical and cognitive difficulties identified, and the extent of generalisation that can be made between studies of extreme mathematical difficulties and those examining more common mathematical difficulties.

1.7 The Relationship Between Reading and Arithmetical Skills.

> *"the number faculty largely emerges through the interaction of central features of the language faculty with other cognitive capacities relating to the recognition and manipulation of concrete objects and collections....It is therefore not necessary to postulate an autonomous 'faculty of number' as a separate module of mind."* (Hurford, 1987, p.3).

Earlier sections of this introductory chapter, and indeed a number of previous studies, provide evidence of a possible connection between children's reading and mathematical skills (Ackerman, Anhalt, & Dykman, 1986; Ackerman & Dykman, 1995; Chansky, Czernik, Duffy, & Finnell, 1980; Geary, 1993; Kulak, 1993, Light & DeFries, 1995; Pearn, 1996; Rourke & Finlayson, 1978; Share et al, 1988). Davis, Merrifield, Pearn, Price, and Smith (1996) considered the connection between counting and reading and found that around 73% of variance in children's arithmetic performance could be accounted for by variation in phonological awareness and attention. A child's readiness for reading appears to be a function of the child's ability to be aware of phonemic units, and to operate flexibly with them. Children's counting difficulties are also linked to their developing ability to operate flexibly with composite numerical units. Davis et al (1996) conclude by proposing that a common feature in the observed connection between reading and arithmetical difficulties may be in a common "unitising" feature of the brain. Geary (1993) suggests that reading and arithmetical difficulties often co-occur because of a common underlying neuropsychological deficit which manifests itself as difficulties in the representation and retrieval of information from long-term memory. He also points to the findings of Rourke (1989) that children with phonological and auditory memory deficits often have a deficit in arithmetic fact retrieval, whereas children with orthographic reading difficulties perform at normal levels on arithmetical problems.

Rasanen and Ahonen (1995) also considered arithmetic error types in relation to specific arithmetic and joint arithmetic and reading difficulties. This study set out to discover whether there was a relation between reading performance (speed and accuracy) and arithmetic errors, which we would expect given the findings of Rourke (1989). Results showed that the number of fact retrieval errors made was significantly correlated with reading speed and accuracy in both the arithmetic disabled group and a control group. Children with specific difficulties made more errors of all types compared to children normally achieving in arithmetic. Children with comorbid reading and arithmetic difficulties exhibited more difficulties in retrieving arithmetic facts. In a number of studies (e.g., Chansky et al, 1980; Perry, Guidubaldi, & Kehle, 1979), it has been recognised that abilities such as letter recognition and dot counting in pre-school children are both equally good predictors of later reading and arithmetical achievement. Indeed, some investigators (e.g., Muth, 1984) believe that arithmetical difficulties may be wholly attributable to reading skills. Muth (1984) found that when the computational and reading demands of arithmetical problems were manipulated, 14% of the variance could be uniquely attributed to reading skills, 8% to mathematical skills, with 32% of the explained variance being due to joint variance between both reading and arithmetical abilities. Rourke and Strang (1983) argued that the difficulties children have memorising tables and procedural steps are reflections of verbal impairments. A study by Geary et al (1991) examined strategy use and working memory in the ability to solve simple addition questions. Results revealed that strategy choice and memory span indices were moderate predictors of mathematical achievement. In addition to this, Geary et al also noted that the same regression equation was also moderately predictive of reading achievement. They suggest that the finding that reading achievement was predicted from the frequency of long-term memory retrieval of addition facts does not necessarily indicate that knowledge of arithmetic facts contributes to reading skills. A more likely explanation is that a failure to represent addition facts in long-term memory parallels phonological processing deficits associated with some forms of reading disorders, such as difficulties in learning the verbal labels for words leading to problems in retrieving information from long-term memory. Errors made during verbal counting (computational skills) which were not found to be predictive of reading performance, appear to be domain specific. Therefore, long-term memory deficits may extend to other semantic networks, influencing not only arithmetic fact retrieval, but also language skills such as reading.

A recent study by Leather and Henry (1994) compared a number of short-term and working memory tasks (involving verbal and numerical components), and phonological awareness to discover which would be the best predictor of reading and mathematics ability, assuming that if each of these were specific skills, reading ability should be predicted by phonological awareness and memory measures involving verbal components, whereas arithmetical ability would be best predicted by memory measures incorporating a numerical component. Phonological awareness tasks were found to be the best predictors of reading accuracy, reading comprehension, and arithmetic. Leather and Henry interpreted this by the fact that phonological awareness tasks involve 'taking away' (deleting final and initial phonemes), and 'adding' (blending phonemes). These findings support the results of Siegel and Linder (1984), who found that children with arithmetical and reading disabilities showed a lack of sensitivity to phonological similarity.

These studies provide just a few illustrations of how investigations of children's mathematical difficulties have been guided by studies that have considered children's reading problems. For example, theories of short-term memory and working memory have often been used to offer explanations for children's reading difficulties (e.g., Daneman & Carpenter, 1980), and similar ideas resulting from this research are being used to try and explain children's arithmetical difficulties (e.g., Logie et al, 1994; Swanson, 1993). Linked to this are notions of processing capacity and efficiency which have been two of the major contributors to explanations of children's arithmetical difficulties. Furthermore, models of the representations of arithmetic facts in long-term memory posit similar principles of organisation and retrieval as semantic memory. If this is so, then factors such as semantic relatedness and accessibility would be expected to have similar effects in the domain of arithmetic as well as verbal knowledge. For example, word frequency effects have an influence on the ability and speed of retrieving words, and a similar idea is put forward for the frequency of exposure to particular number combinations, such that arithmetic problems vary in difficulty due to differential experience (Ashcraft, 1992).

A further similarity between mathematics and reading is in the sequence of skill development. For beginning readers, one of the first components of reading they must develop is the ability to decode words into their constituent parts, that is, phonemes. With age and experience the child will move from this sound-letter correspondence, to direct visual processes for recognition of familiar high frequency words. However, low frequency words are still decoded using phonological information (Bruck, 1988; Frith, 1985; Marsh, Friedman, Welch, & Desberg, 1981). The development of arithmetical skills parallels that of reading development. The primary task in the development of arithmetic is the acquisition of basic arithmetic facts. This acquisition is characterised by a transition from reconstructive procedural strategies, such as counting, to automatic retrieval of arithmetic facts. However, procedural strategies will still be used for obtaining the answer to low frequency arithmetic facts. Clearly, the development of reading and arithmetic skills are conceptually similar. Learning in both domains begins with effortful procedural strategies, phonemic analysis or counting. These procedures are, with experience, replaced with automatic processes requiring few attentional resources.

Therefore, some evidence suggests that certain cognitive abilities contribute to both reading and arithmetic skills. This is an issue that has often been overlooked in previous studies, in favour of studying reading and arithmetic as distinct, academic domains. Research investigating mathematical ability may benefit from the theoretical and methodological knowledge in what is inevitably the more advanced area of research that has examined the development of reading skills. Furthermore, the identification of converging trends across the domains of mathematics and reading may lead to the development of a more coherent theory of learning disability as a whole.

1.8 Summary.

There are a number of possible problem areas for children with arithmetical and mathematical difficulties. The simplest of these is in the development of basic counting skills. Without knowledge of the correct sequence of numbers, arithmetical skills will not develop accurately. The next level of problems concerns the learning of arithmetical skills, including knowledge of the written symbolism used in mathematics, and the use of correct procedures for solving arithmetic questions. Accompanying this are differences in the efficiency of strategies used by the child, and a general slowness in speed of processing, possibly leading to short-term memory difficulties. Following on from the types of counting strategies used is the establishment of number representations and number facts in long-term memory. Until automatisation of simple arithmetic knowledge is nearing completion, arithmetic skills will not be fluent or rapid, and the development of complex arithmetical abilities will be slow to form. The research carried out for this thesis concentrates on the development of basic arithmetic skills and simple mathematical knowledge.

By considering both cognitive and neuropsychological studies of children and adults, and by drawing on the large body of evidence investigating children's reading skills, it has been possible to show how certain cognitive limitations may impact on the development of mathematical skills. The first study, reported in Chapter 3, aims to pinpoint what is potentially the fundamental cognitive limitation for children with mathematical difficulties, and Studies 2, 3, and 4 provide a follow up to Study 1. The final study, reported in Chapter 7, attempts to pull together the cognitive deficits identified in the previous chapters, in order to investigate the efficacy of two approaches to teaching mathematics in the primary school.

Chapter 2.
Methodological Note.

In the introduction it was reported that there have been many studies examining children's mathematical skills. Some of these studies claim to be examining specific mathematical difficulties, whilst others simply refer to learning difficulties. The different selection criteria used for classifying children as having mathematical difficulties in these studies raises a number of methodological concerns, the most obvious being the problem of comparing results across studies. Table 2.1 shows some examples of the selection criteria used for classifying children as having mathematical difficulties. The majority of these studies use a 'cut-off' selection criteria where the child has to be beyond certain cut-off points to be classified as having a learning difficulty (Hitch & McAuley, 1991; Koontz & Berch, 1996; Lewis et al, 1994; Rasanen & Ahonen, 1995; Share et al, 1988; Siegel & Linder, 1984; Siegel & Ryan, 1989). The mathematics criteria set for cut-off in these studies are variable with some studies choosing mathematics performance below the 30th or 25th percentile as the cut-off. This means that to be classified as having mathematical difficulties the child has be in the bottom 30% of the sample. Other studies classify children who score below one standard deviation from the mean as having mathematical difficulties, that is, those children in the bottom 16%. There is also variability in the cut-off for reading ability when trying to identify specific mathematical difficulties. Suggested cut-offs for reading ability are at least 4 points above mathematics (Hitch & McAuley, 1991), above the 25th percentile (Lewis et al, 1994), above the 30th percentile (Siegel & Ryan, 1989), above the 35th percentile (Share et al, 1988), or within one standard deviation of the mean, that is, above the 16th percentile (Rasanen & Ahonen, 1995). Other studies make no specific reference to measures of reading ability (Geary et al, 1992; Geary & Brown, 1991; Geary et al, 1991). These studies require the child to be scoring below the 46th percentile in mathematics, with no selection cut-off being stated for reading performance. These studies do, however, state that many of the children classified with mathematical difficulties were also receiving remedial help for reading.

Another methodological issue raised by some of these studies, for example, Swanson (1993), Rourke and associates, and Geary and his colleagues, is the exclusive use of children from referred populations, that is, children clinically referred by physicians or paediatricians, or referred for remedial help by schools. Clearly, such children represent the extreme in learning difficulties and are likely to be atypical of the general population from which they are drawn. Therefore, it may be difficult to generalise these results to children in the school classroom who also show difficulties learning mathematics.

A number of studies have considered the prevalence of mathematical difficulties in children. Lewis et al (1994) carried out a study to assess the prevalence of arithmetic difficulties in 9 and 10 year old children. Standardised tests of arithmetic, reading and nonverbal reasoning ability were administered to a sample of 1056 children. Children were classified as having a specific arithmetic deficit if their reading and intelligence scores were above the 25th percentile, and if their arithmetic scores were below one standard deviation from the mean. Similar cut-off points were set for classifying children as having a specific reading deficit, with children falling into this category if they obtained arithmetic and nonverbal intelligence scores above the 25th percentile, and a reading score below one standard deviation. Children were classified as having arithmetic and reading difficulties if their nonverbal intelligence score was above the 25th percentile and their arithmetic and reading scores were below one standard deviation from the mean. Results showed that 1.3% of children had specific arithmetic difficulties, 3.9% had specific reading difficulties, and 2.3% had reading and arithmetic difficulties. Rasanen and Ahonen (1995) carried out a study looking at the errors made by children with arithmetical difficulties and those with comorbid reading difficulties, and found that of 2,673 children, only 3% fulfilled the requirement for being arithmetic disabled, whether this was accompanied by reading disabilities or not. A study by Siegel and Linder (1984) looking at reading and arithmetical difficulties failed to identify any children below the age of 8 years with specific arithmetical difficulties, and few below the age of 10 years. Finally, Ackerman & Dykman (1995) point out that in their 25 years of research with learning disabled populations, those showing specific arithmetical difficulties have been few and far between, and present results from such subjects with caution.

Therefore, the main concern that arises from the issues discussed above is that studies claiming to be examining mathematical difficulties may not in fact be dealing with a specific disorder. Given the variability of the criteria set for selection as having mathematical difficulties, it is difficult to ascertain whether such studies are actually considering mathematical difficulties per se, or whether in fact they are examining children of generally low or average academic ability (similar concerns are also pointed out by Ginsburg, 1997). This can be illustrated with the following example. Child A may achieve an arithmetic score of 80 and a reading score of 105. In this case, there is clearly some kind of deficit specific to arithmetic. Child B achieves an arithmetic score of 84 and a reading score of 91. In this case the specificity of the arithmetic deficit is much less apparent. Both of the examples meet the classification criteria as set by Lewis et al (1994) for having a specific arithmetic deficit. However, the specificity of the second case is much less clear than the first. Therefore, the use of such selection criteria may be misleading.

It was planned that children with specific arithmetic deficits would be identified for the studies in this thesis, as clearly this would be the best method for studying the cognitive deficits underlying specific mathematical difficulties. Therefore, prior to any experimental work, large scale screening was initiated, testing children on reading and mathematics ability, to identify children who may have a specific deficit in mathematics. The results from this screening are shown in Figure 2.1, and are as reported at the beginning of the subjects section in chapter 3. The criteria set for inclusion as having specific arithmetic difficulties were that a child achieve a standard age score below the 25th percentile (standard age score of 90 or below) in mathematics, but score above the test mean for reading (standard age score of 100 or above). This ensured that reading scores would be at least 10 points above mathematics scores, and would add credence to the specificity of the deficit. As can be seen from Figure 2.1, this process only identified one child has having specific mathematical difficulties (upper left quadrant). This, of course, agrees with the low percentage of children identified in previous studies as having specific mathematical learning difficulties. This raises major difficulties for any experimental studies. In order to achieve a sample of 20 children, which is still a relatively small sample in experimental terms, this would entail screening 1,860 children. Access to schools does not allow such large scale screening. Hitch (personal communication) has suggested that studies with older children, around 10 or 11 years of age, would allow for the identification of more children showing specific mathematical difficulties, as the difference between reading and mathematical skills becomes more pronounced as the child progresses through school. However, cognitive developmental work such as that by Little and Widaman (1995) suggests that change in addition processing skills develops rapidly during the first three or four years of primary school, after which improvements in such skills is much less pronounced. The reason for so much improvement at this early stage is not clearly defined. It may be that children at this age have a particular readiness to learn. This clearly suggests that educational influences have their strongest impact during the first few years in school, and that identification of children with arithmetical difficulties at this stage would be the most beneficial and successful stage for giving assistance.

Given that the numbers of children with specific mathematical difficulties is so low, it is reasonable to suggest that time would be better spent studying the majority of children who are not at these extremes, but rather those children who show below average mathematical abilities, whether or not these difficulties are accompanied by reading difficulties. Therefore, the strategy adopted in all of the studies reported in this thesis involves including all children for whom access was permitted, and not choosing children according to the specificity of their mathematical or reading difficulties. Because the major focus of this research concerns cognitive skills involved in mathematics, statistical techniques (analyses of covariance and partial correlations) have been used to control for individual differences in reading ability. By doing this, it can be inferred that any significant results found can only be explained by individual differences in mathematical ability. Furthermore, as an adjunct to these findings, this statistical technique is also reversed to control for individual differences in mathematics ability, to discover whether the same cognitive skills can also be used to explain differences in reading ability. These results are not reported in as much detail, but it is theoretically important to carry out this additional analysis, in order to decide whether it is possible to study reading and mathematical difficulties in conjunction as they may share the same underlying cognitive mechanisms.

TABLE 2.1

Examples of selection criteria used for classification of children with arithmetic disabilities.

Study	Mathematics ability	Reading ability or IQ
Hitch & McAuley (1991)	< 25th percentile	> 4 points above mathematics
Strang & Rourke (1985)		WRAT reading score 2 years higher than arithmetic score
Siegel & Ryan (1989)	< 25th percentile	> 30th percentile
McLean (1997)		> 25th percentile, < 75th percentile
Share, Moffit, & Silver (1988)	< 30th percentile	> 35th percentile
Koontz & Berch (1996)		
Kirby & Becker (1988)	< -.7 SD	> 1 SD discrepancy to mathematics score
Siegel & Linder (1984)	< 21st percentile	> 34th percentile
Lewis, Hitch, & Walker (1994)	< -1 SD	> 25th percentile
Rasanen & Ahonen (1995)	< -1 SD (sample mean)	> - 1SD (test mean)
Russell & Ginsburg (1984)	< 1 year below grade level	Above average
Cawley & Millar (1989)	< 2 SD's	Within 1SD of mean
Geary, Brown, & Samaranayake (1991)	< 46th percentile	Many children receiving remedial help.
Geary and Brown (1991)		
Geary, Bow-Thomas, & Yao (1992)		Not specified
Geary (1991)	Remedial help	Remedial help
Garnett & Fleischner (1983)		

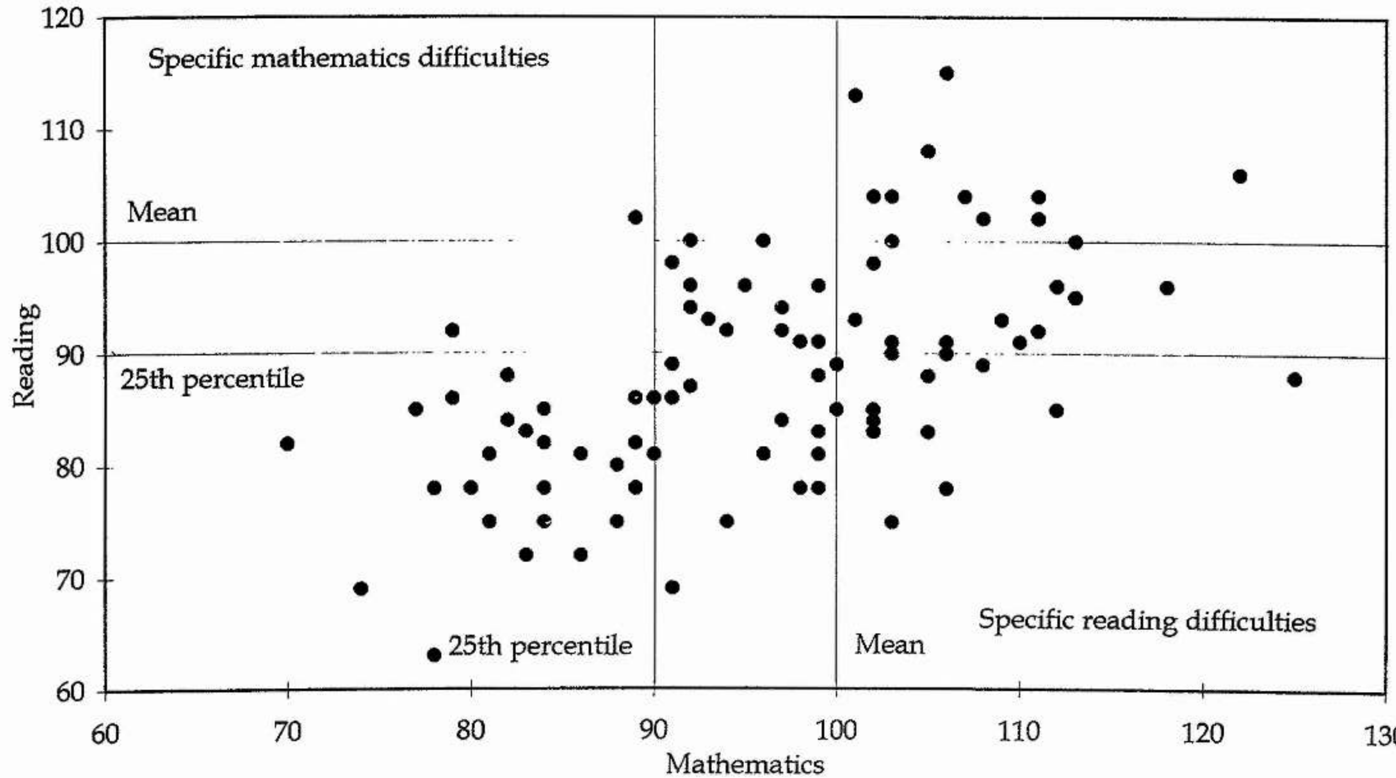

FIGURE 2.1.

Criteria set for selection as having specific mathematical difficulties, that is, a score below 90 (25th percentile) for mathematics performance, and above mean, 100, for reading.

Chapter 3.

Identifying the Fundamental Cognitive Deficits Underlying Mathematical Difficulties.

3.1 Theoretical Background.

Many studies examining the cognitive mechanisms involved in mathematics have concentrated almost exclusively on the role of short-term memory, with a number of studies demonstrating that poor mathematical skills are accompanied by deficits of short-term memory (Geary, 1990; Hitch & McAuley, 1991; Siegel & Linder, 1984; Siegel & Ryan, 1989; Swanson, 1993). Many of these studies have measured short-term memory span, a task thought to tax the storage and rehearsal properties of the articulatory loop. Differences in memory span are usually explained in terms of speed of articulation rate, with differences in rate of articulation, and hence, differences in rate of subvocal rehearsal, influencing how much information can be rehearsed and recalled from short-term memory. However, many studies have ignored the possibility that individual differences in articulation rate may reflect a more global developmental difference in general processing speed (Hale, 1990; Kail, 1991a, 1991b, 1992, 1997; Kail & Hall, 1994; Kail & Park, 1994; Kail & Salthouse, 1994; Rabbitt & Goward, 1994). For example, Kail suggests that whilst differences in articulation rate can account for differences in short-term memory span, these differences in articulation rate are actually secondary to a more fundamental difference in general processing speed, and that these differences in speed of processing information account for both individual differences and age related changes in a range of cognitive skills. Differences in processing speed will also influence memory more directly, for example, the initial encoding of items to be remembered may be more rapid, leading to greater recall.

Many researchers have now departed from an explanation of short-term memory differences in terms of articulation rate, and have recognised the importance of the influence of long-term information. Factors such as the familiarity of the material, the strength of long-term memory representations, speed of identification of items, and the fluency which with sequences of operations can be performed will influence the efficiency of short-term memory and, therefore, how much information we are able to hold and recall from short-term memory (Case et al, 1982; Ericcson & Kintsch, 1995; Gathercole & Adams, 1994; Henry & Millar, 1991; Hitch & McAuley, 1991; Hulme et al, 1991; Roodenrys et al, 1993).

One line of enquiry that has departed from the usual explorations of short-term memory, considers the role of visual-spatial skills in mathematics performance. Whilst few cognitive studies have documented direct associations between mathematics and visual-spatial skills, neuropsychological studies of acquired and developmental dyscalculia, and studies with learning disabled children, have identified a subset of subjects whose mathematical difficulties do appear to be the result of poor visual-spatial skills (Geary, 1993; Rourke, 1993; Rourke & Finlayson, 1978; Strang & Rourke, 1985).

Given all the possibilities raised by the literature as to the cognitive deficits that may underlie poor mathematical skills, the first study set out to be very exploratory and to examine a range of cognitive mechanisms, in an attempt to identify what could be fundamental cognitive difficulties for children with poor mathematical skills. Mechanisms investigated include short-term memory span and associated factors concerning articulation rate, speed of item identification, processing speed, visual-spatial ability, the ability to sequence information, and retrieval from long-term memory. This study also included a battery of reading measures to assess how these cognitive skills were in turn related to reading ability. This helps to identify which cognitive skills are common to both mathematical and reading skills, and those cognitive mechanisms which may be more specific to one or the other.

3.2 *Study 1a.*

3.2 *(i)* Method.

Subjects.

103 children (mean age 7 years, 7 months, *SD* = 4 months) were initially screened prior to experimental testing. Children attended three different schools in Dundee, all of which were in working class areas. 10 children had to be excluded from further analysis due to absence on one or more of the screening tests, leaving a total of 93 children who participated in all three screening tests. No children were excluded due to any intellectual or behavioural difficulties. Children were initially screened for mathematics and reading ability using the Group Mathematics Test (Young, 1970), the Primary Reading Test (France, 1981), and the English Picture Vocabulary Test (EPVT, Brimer & Dunn, 1968), the latter being used as an additional assessment of general intellectual ability. All were group administered tests carried out in the classroom. The Group Mathematics Test consists of word problems (read by the experimenter) which correspond to an associated picture on the

page. The test also requires single- and multi-digit addition and subtraction. The Primary Reading Test involves both word recognition (matching words to pictures) and reading comprehension (choosing the correct word to allow a sentence to make sense). The English Picture Vocabulary Test requires the child to circle the picture corresponding to the word spoken by the experimenter.

As discussed in the methodological note in Chapter 2, large scale screening was carried out in an attempt to identify a sample of children with specific mathematical difficulties. The selection criteria set for classification as having a specific arithmetic deficit were that the child achieved a mathematics standard age score below the 25th percentile (standard age score of 90 or below), and a reading standard age score above 100 (above the test mean). There was a strong correlation between reading and mathematics ability, and reading performance was fairly low, so only one child fulfilled the criteria for having specific mathematical difficulties. Following from this, it was decided to study children from two classes in the same school, thus reducing the possibility of results being due to differences in teaching style and content between the schools. From these classes, 24 children (mean age 7 years, 7 months, *SD* = 4 months) were chosen who were representative of the total sample, this smaller group having the same mean and variability of performance on the screeing tests as the larger screening group.. Standard age scores were calculated using the test norms from the Group Mathematics Test, Primary Reading Test, and English Picture Vocabulary Test (for all tests, mean = 100, *SD* = 15). The children's standard age scores for mathematics ranged from 70 to 122 (mean = 98.25, *SD* = 14.71). Reading standard age scores ranged from 72 to 106 (mean = 88.67, *SD* = 9.22), with vocabulary standard age scores ranging from 79 to 101 (mean = 89.92, *SD* = 6.48). The generally low performance on the Primary Reading Test occurred because of difficulties with the comprehension section of the test. Therefore, an additional measure of reading ability was administered, this being the British Ability Scales (BAS) Word Reading Test (Elliott, Murray, & Pearson, 1979). This is a test purely of single word reading ability, requiring no reading comprehension. Children are simply required to read words which become progressively more difficult, with testing being discontinued when the child makes ten consecutive reading errors. This test has a mean of 50, and a standard deviation of 10, and the children showed performance very similar to this standard (mean = 49.42, *SD* = 12.65, scores ranging from 31 to 73). A strong correlation was found between mathematics ability and this measure of word reading ability, $r\ (22) = .74, p < .001$.

Children were placed into one of two groups according to their mathematics standard age score. Mean performance on the Group Mathematics Test for the sample was 98.25 (SD = 14.71). Those children scoring above the mean performance for this sample of children (a standard age score above 98) were placed into the high ability mathematics group (n = 14, mean = 109.14, SD = 6.85). Those children performing below the mean (standard age score 98 or below) were placed into the low ability mathematics group (n = 10, mean = 83.00, SD = 6.51). One way analyses of variance (ANOVA) were conducted comparing groups (low versus high mathematics ability) on measures of mathematics, reading, and vocabulary. This revealed there to be a significant difference between the groups in mathematics ability, F (1, 22) = 88.44, $p < .0001$, and in reading ability, F (1, 22) = 4.66, $p < .05$. There was no significant difference between the groups in performance on the English Picture Vocabulary Test, F (1, 22) = .15, $p > .05$, and no difference in chronological age, F (1, 22) = .28, $p > .05$, (see Table 3.1 for performance and age characteristics of each ability group).

To establish that differences in mathematics ability were still apparent once differences between the groups in reading ability had been controlled for, analyses of covariance (ANCOVA) were carried out, controlling for reading differences using performance on the BAS Word Reading Test. Whilst ANOVA revealed there to be no group differences in IQ (as measured by the EPVT), children in the higher ability mathematics group did show a general tendency to score slightly higher on this test. For the reason, the precaution was taken to include EPVT performance as a second covariate in all statistical analyses. ANCOVA controlling for reading ability and IQ, revealed that while there were still significant differences between the high and low ability mathematics groups in mathematics ability, F (1, 20) = 70.86, $p < .0001$, differences between the two groups in performance on the Primary Reading Test were removed, F (1, 20) = 0.13, $p > .05$ (see Table 3.1 for adjusted means after controlling for differences in reading ability and IQ).

TABLE 3.1.

Performance of each ability group on mathematics, reading, and vocabulary screening tests, and adjusted means controlling for differences in reading ability and IQ (*SD*).

Measure	High ability *M*	High ability Adjusted *M*	Low ability *M*	Low ability Adjusted *M*
Group Mathematics Test	109.14 (6.85)	106.34	83.00 (6.51)	85.80
Primary Reading Test	91.86 (8.50)	88.47	84.20 (8.65)	87.58
EPVT	90.36 (6.37)		89.30 (6.93)	
Chronological age (years, months)	7,7 (0,4)		7,8 (0,4)	

Materials and Procedure.

Experimental tests: Experimental testing started approximately one month after the initial screening. Children were tested individually in a quiet room in the school, over a total of six sessions, each session lasting approximately 20 minutes. Testing consisted of a number of cognitive measures, including short-term and working memory, processing speed, speed of identifying numbers and letters, sequencing ability, and visual-spatial skills. Children were further tested on their ability to solve simple and complex arithmetic questions. Various measures of reading ability were also administered. Due to the large number of experimental tasks undertaken and their differing procedures, each will be described separately.

Measures of reading ability.

Regularity task: Stimuli for this test consisted of 14 low frequency regular words (mean frequency 25.9, *SD* 21.6) and 14 low frequency irregular words (mean frequency 28.1, *SD* 23.3, Carroll, Davies, & Richman, 1971). Stimuli were presented at the centre of a computer screen. Responses were made by way of a voice key, with the stimulus presentation being terminated when the child responded by speaking into the microphone.

Accuracy of word reading was recorded for both regular and irregular words. Use of the voice response key also allowed for the collection of response times for the reading of words.

Nonword reading: Stimuli consisted of 20 short and 20 long pronounceable nonwords which were presented in the same way as stimuli for the regularity task. Again, accuracy and speed of reading was recorded.

Phoneme deletion: This task was constructed by Duncan (1991). Stimuli were presented auditorily by the experimenter. Words were read to the child, with the child's task being to say what the word would be with either the initial or final phoneme deleted. 24 words and 24 nonwords were presented, 12 in each category having the initial phoneme deleted, and 12 in each category having the final phoneme deleted. The order in which words were presented was counterbalanced for both words and nonwords, and for initial and final phoneme deletion. Testing was tape recorded for later analysis, with the accuracy of each response being recorded (Stimuli for all reading tests are presented in Appendix I).

Visual-spatial ability.

Block Design and Object Assembly. Children were tested using the Block Design and the Object Assembly tests from the Wechsler Intelligence Scale for Children (WISC-R, Wechsler, 1977). The block design task assesses the ability to perceive and analyse forms, and involves visual organisation and visual-motor co-ordination, along with the ability to reason about spatial relationships. The child is required to copy patterns from a booklet, using either 4 or 9 blocks. Testing was discontinued after the child had made two successive failures. Object Assembly requires the child to complete pictures which have been separated to form simple jig-saws. The time taken for the child to complete each trial in each task was recorded, as well as the accuracy of the completed designs. In both tests, accurate performance within a fast time was rewarded with bonus points, in accordance with the standard procedure.

Processing speed.

Visual number matching and cross-out tasks: These tests were based on the Woodcock-Johnson tests of Cognitive Ability, but with stimuli derived by the experimenter. The visual number matching task comprised 30 rows of 6 digits, with two digits in each row being identical (for example, 7 1 2 6 1 6). The child was instructed to circle the identical digits in each row, and to work as quickly and as accurately as possible. The performance measure in this case was the time taken to complete all 30 rows of digits. The cross-out task consisted of 12 rows of a geometric figure at the left hand side of the row and 19 similar figures to the right. An example of this may be a triangle containing a dot. The 19 alternative figures are all triangles but contain various internal objects, for example, a square, an addition sign, a diamond shape. The child's task was to cross out the five figures of the 19 that were identical to the target stimulus presented on the left hand side of the page. Again, performance was measured by the time taken to complete all 12 rows. The visual number matching and cross out tasks were chosen as measures of processing speed because they were initially devised to assess the processing speed factor in the theory of fluid and crystallised intelligence (Cattell, 1963), and have been shown to be valid measures of processing speed in previous studies (e.g., Kail & Hall, 1994).

Perceptual motor speed: This was measured by the use of a peg board, with children being required to transfer 10 pegs from one side of the board to 10 holes in the other side of the board as quickly as possible. The time taken to complete this task with both the right and left hand was recorded.

Short-term memory span.

Memory span: Auditory memory span for digits and one syllable words was measured to assess short-term storage capacity, along with counting span as an additional measure of storage and processing efficiency. Digit span was tested using the WISC-R sub-test (Wechsler, 1977). One syllable words were used for the word span test with a mean age of acquisition of 3.1 years (Carroll & White, 1973). The words used were *king, rope, horse, leaf, tree, knife, tent,* and *snake*. Administration of the digit and word span tasks followed the same procedure. After an initial practice session, the digits or words were presented auditorily to the child at a rate of approximately one item per second, starting from a span length of two. Each child was required to recall the words in the order in which they had been presented. If the items were recalled in the correct serial order, the span length was increased by one. If the child recalled the digits or words incorrectly, the span length was repeated with a different set of items. If the child failed on the second attempt of any particular span length, then testing was discontinued.

Stimuli for the counting span test consisted of plain white cards with between 1 and 9 green spots and 1 and 9 red spots on each card, each spot being 8 mm in radius. Children were instructed to count the number of green spots on each card presented, and to try and remember the number of spots counted on each card. After an initial practice session, children were presented with two cards faced down on the table. The experimenter then turned the first card over and after the child had counted the spots, this card was turned face down again, and the next card turned up. This card was then turned face down and the experimenter pointed to the first card and then the second, asking the child to recall the number of spots counted on each card. The child was presented with 2 sets of cards at each span length. If the child passed on one or both of the trials, the child was then presented with cards of the next span length, i.e., 3 cards, up to a maximum of 8 cards. Testing was discontinued when the child failed both trials at any one span length. Digit span, word span and counting span were determined to be the length of the longest correct serial recall.

Articulation rate: This was measured by asking the child to repeat two 1 syllable words (*king* and *tree*). Children were instructed to carry on repeating the word pair until told to stop. This was recorded on audio tape for later analysis. If the child made an error or stopped for any other reason, the child was stopped and instructed to begin again. The time taken by the child to repeat the stimulus pair five times was measured using a stopwatch, with timing being started at the beginning of the second repetition of the words and terminated at the end of the sixth repetition. The same procedure was administered for three syllable words (*banana* and *elephant*). The number of words articulated per second was calculated.

Speed of counting was measured using cards previously used for the counting span test. The time taken for children to count the number of red spots on five cards was recorded, and any errors of counting were noted. The number of dots counted per second was calculated.

Long-term memory.

Speed of number and letter identification: The child was presented with a number or letter in the centre of the computer screen, with the child being instructed to name the letter or number as quickly as possible. This was achieved through the use of a voice response key attached to the computer. Timing for identification of the stimulus began when it was presented on the screen and was stopped when the child responded with the answer by means of the voice response key. The numbers used were 1 to 20 presented in Arial font. All the letters of the alphabet were used, presented in Arial font, apart from the letter 'a' which was presented in Century Gothic ('ɑ'). Letters and numbers appeared in the same random order for each child. For the purposes of statistical analysis, any incorrect responses were later excluded.

Sequence knowledge: The child was asked to complete a number of sequences which would provide a comparison to the sequence task of number counting. As well as being asked to count from one to twenty, children were also asked to complete other well known sequences, for example, saying the letters of the alphabet, days of the week, and months of the year. Accuracy of the sequence was taken to be the number of items remembered in the correct sequence order before the first mistake was made. No credit was given for any items placed into the correct sequence after one mistake had already been made.

Additional measures of mathematical skill and knowledge.

Single digit addition. In order to measure the child's ability to solve simple arithmetical questions, single-digit arithmetic problems were presented to each child, with stimuli being presented in the centre of the computer screen (all stimuli were presented in Arial font). Addition questions from 2 +3 = through to 9 + 8 = , were presented, excluding tie questions such as 2 + 2 =, 3 + 3 =, and '+ 1' questions. These questions were excluded as previous studies have suggested that the answers to such questions are usually well learned for children of all abilities. This left a total of 56 questions which were administered in two blocks, each block containing 28 trials, with blocks being counterbalanced to account for any practice effects or teaching that may have occurred between each time of testing. Children were instructed to say the answer as soon as they knew it, into a microphone held by the experimenter. The microphone was linked to a voice response key which terminated the presentation of the stimulus presentation. Both time taken to reach the answer, accuracy, and strategy use were recorded for every arithmetical question. The child was instructed to use any strategy for reaching the answer that they wished (for example, counting using physical objects (children were provided with counting blocks to use if they so wished), counting on fingers, verbal counting, direct memory retrieval, or any other feasible strategy). If the strategy use was not immediately obvious, the experimenter recorded the most likely strategy and then questioned the child as to how they had solved the problem. If the experimenter's and the child's opinions on the strategy used were different, the strategy described by the child was taken to be correct. This procedure is given credence by a number of studies which have demonstrated that children can accurately describe problem solving strategies in arithmetic, if they are asked immediately after the problem has been solved (Siegler, 1987; Siegler & Shrager, 1984).

Single-digit subtraction. Arithmetic problems used were all combinations involving the values of 1 to 9, resulting in a positive result. This left a total of 36 problems which were presented in one block. Stimuli characteristics and administration procedure were as described for the single-digit addition problems. Accuracy, strategy use, and solution times were recorded.

3.2 *(ii)* Results.

Differences between the groups on the various measures were first analysed by way of ANOVA. To be certain that any differences found between the groups were due to mathematics ability alone, ANCOVA was also carried out, controlling for differences between the groups in word reading ability and IQ. In all cases the covariates were performance on the BAS Word Reading Test and EPVT.

Measures of reading ability.

Mean performance for each group on each measure of reading ability is reported in Table 3.2. Results from each reading task will be described in turn.

Regularity task: For each child, the percentage correct performance on the reading of regular and irregular words was calculated. A 2 x 2 repeated measures ANOVA with one between subjects factor, group (low versus high ability mathematicians), and one within subjects factor, regularity (regular versus irregular words), showed there to be a significant main effect of group, $F\ (1, 22) = 11.95$, $p < .01$, with there being no significant main effect of regularity, and no significant interaction between group and regularity. Newman Keuls analysis revealed that children in the high ability mathematics group read significantly more regular and irregular words correctly than children in the low ability mathematics group (p's $< .01$). However, statistically controlling for differences between the groups in reading ability and IQ through ANCOVA removed the significant main effect of group, $F\ (1, 20) = 2.57$, $p > .05$.

Time taken to correctly read regular and irregular words was also recorded and submitted to repeated measures ANOVA comparing the two ability groups on both regularity conditions. This revealed a significant main effect of group, $F\ (1, 20) = 6.44$, $p < .05$, with no other main effect or interaction being significant. Newman Keuls analysis revealed that children in the low ability mathematics group were significantly slower at correctly reading both regular ($p < .05$) and irregular words ($p < .01$). Controlling for differences in reading ability and IQ through ANCOVA removed the significant main effect of group, $F\ (1, 18) = 1.47, p > .05$.

Nonword Reading: The percentage of short and long nonwords read correctly was calculated. A 2 x 2 repeated measures ANOVA with one between subjects factor, group (low versus high ability mathematicians), and one within subjects factor, word length (short versus long nonwords), revealed a significant main effects of group, $F\ (1, 22) = 13.91, p < .01$, and word length, $F\ (1, 22) = 21.12, p < .001$, along with a significant interaction between group and word length, $F\ (1, 22) = 5.23, p < .05$. ANCOVA controlling for differences in reading ability and IQ did not remove the significant main effect of group, $F\ (1, 20) = 4{,}77, p < .05$. Newman Keuls analysis revealed that high ability mathematicians read significantly more short and long nonwords correctly (p's $< .01$). Children in the low ability mathematics group read significantly more short than long nonwords correctly ($p < .01$), with high ability mathematicians showing no significant difference in the percentage of short and long nonwords read correctly.

Time taken to read short and long nonwords correctly was also calculated. A 2 x 2 repeated measures ANOVA comparing the two ability groups on both word length conditions, revealed a significant main effect of word length, $F\ (1, 18) = 26.64, p < .001$, with there being no significant difference between the two ability mathematics groups, and no significant interaction between group and lexicality. Both ability groups correctly read short nonwords significantly faster than long nonwords.

Phoneme Deletion: The percentage correct performance for words and nonwords, and for initial and final phoneme deletion were calculated. A three way repeated measures ANOVA was conducted, with one between subjects factor, group (high versus low ability mathematicians), and two within subjects factors, lexicality (words versus nonwords), and position (initial or final phoneme deletion e.g., m/ind, l/ext, des/k, nus/t). A main effect of group was found, $F\ (1, 22) = 12.86, p < .01$, together with significant interactions between lexicality and position, $F\ (1, 22) = 25.57, p < .001$, and group, lexicality, and position, $F\ (1, 22) = 4.38, p < .05$. Newman Keuls analyses revealed that irrespective of lexicality (words or nonwords), or deletion position (initial or final phoneme), children in the high ability mathematics group performed significantly better than children in the low ability group (p's $< .01$) Children in the low ability mathematics group were better at initial phoneme rather than final phoneme deletion in words, with this pattern of results being reversed for nonwords (*p*'s < .01). Children in the high ability mathematics group showed no differences in performance of either words or nonwords, or initial or final phoneme deletion. However, ANCOVA controlling for differences in reading ability and IQ between the groups removed the significant main effect of group, F (1, 20) = 3.25, p = .087.

In summary, ANOVA revealed that children in the high ability mathematics group were significantly more accurate and significantly faster at reading both regular and irregular words, and were more accurate at reading both short and long nonwords. Children in the high ability mathematics group also showed significantly better performance on the phoneme deletion task. However, controlling for differences between the groups in reading ability and IQ removed virtually all of these significant differences between the two mathematics ability groups, the only significant difference remaining being in accuracy of nonword reading. This supports the use of BAS Word Reading performance as the covariate in controlling for reading ability, as it was clearly able to account for many of the group differences on a wide variety of measures assessing different aspects of reading related ability.

TABLE 3.2.

Mean performance on measures of reading ability (Regularity task, Nonword Reading, and Phoneme Deletion) for each ability group, and adjusted means controlling for differences in reading ability and IQ (*SD*).

	High ability		Low ability	
Measure	*M*	Adjusted *M*	*M*	Adjusted *M*
REGULARITY TASK				
Regular words (% correct)	77.04 (20.43)	64.60	42.86 (32.82)	55.29
Irregular words (% correct)	71.92 (19.90)	60.22	37.86 (28.94)	49.55
Regular words (time)	1.95 (0.89)	2.70	4.80 (3.37)	4.05
Irregular words (time)	2.42 (1.12)	2.95	3.64 (2.30)	3.12
NONWORD READING				
Short nonwords (% correct)	87.86 (12.51)	79.74	58.50 (29.91)	66.62
Long nonwords (% correct)	79.64 (24.30)	65.59	34.00 (35.65)	48.05
Short nonwords (time)	2.18 (1.17)	2.69	3.44 (1.82)	2.92
Long nonwords (time)	3.64 (2.06)	4.40	5.95 (4.53)	5.19
PHONEME DELETION				
Total correct (%)	79.17 (16.26)	69.31	44.38 (31.06)	54.24
Words (% correct)	82.74 (14.88)	73.46	49.58 (32.95)	58.86
Nonwords (% correct)	75.60 (18.62)	65.15	39.17 (29.47)	49.62
Initial phoneme (% correct)	78.57 (19.05)	67.65	39.17 (33.86)	50.09
Final phoneme (% correct)	80.06 (15.43)	71.22	49.58 (29.03)	58.42

Cognitive tasks.

All means and adjusted means (controlling for differences in reading ability) from the cognitive tasks are reported in Table 3.3.

Visual-spatial ability.

Block design test: Each child's raw score on the block design test was transformed into a scaled score taking in account chronological age using tables in the WISC-R manual. A one way ANOVA comparing groups (low versus high ability mathematicians) revealed there to be a significant difference, $F\ (1, 22) = 9.84$, $p < .01$, with children in the high ability mathematics groups performing significantly better than children in the low ability group. ANCOVA controlling for differences between the groups in reading ability and IQ removed the significant difference between the groups in performance on the Block Design task, $F\ (1, 20) = 2.21$, $p > .05$.

Object Assembly: Raw scores were again transformed into scaled scores taking into account chronological age. A one way ANOVA comparing the two ability groups showed there to be no difference in performance, $F\ (1, 22) = .86$, $p > .05$. ANCOVA also showed there to be no significant differences between the groups, $F\ (1, 20) = .03$, $p > .05$.

Processing speed.

Visual number matching: The total time taken by each child to complete 30 items was recorded. A one way ANOVA was carried out comparing the high and low ability mathematics groups, revealing there to be a significant difference, $F\ (1, 22) = 5.84$, $p < .05$. Children in the high ability mathematics group were significantly faster to finish the visual number matching task than children in the low ability mathematics group. Controlling for differences in reading ability and IQ reduced this difference, although the difference between the group still approached significance, $F\ (1, 20) = 4.25$, $p = .052$.

Cross-out task: The total time taken to complete 12 items was recorded. A one way ANOVA comparing groups (low versus high ability mathematicians) showed there to be a significant difference between the groups, $F\ (1, 22) = 6.59$, $p < .05$. ANCOVA controlling for differences in word reading ability and IQ did not remove this result, with significant differences still being present between the groups, $F\ (1, 20) = 4.43$, $p < .05$. Observation of the mean times to complete the task show that the children in the high ability mathematics group completed the task significantly more quickly than children in the low ability mathematics group.

General speed of motor functioning: For the purposes of analysis the time taken to complete the peg board task with the preferred hand was calculated (preferred hand; right = 23, left = 1). A one way ANOVA comparing groups (low versus high ability mathematicians) revealed there to be no significant difference between the groups, F (1, 22) = 3.57, p = .07. However, the difference was approaching significance, with the children in the high ability mathematics groups tending to show faster perceptual motor speed than children in the low ability mathematics group. ANCOVA controlling for differences in reading ability and IQ also showed there to be no significant differences between the groups, F (1, 20) = 0.76, p > .05.

Short-term memory.

Digit Span: Digit span was taken to be the longest span length at which the child recalled all numbers in the correct serial position. A one way ANOVA comparing the two ability mathematics groups showed there to be a significant difference in digit span between the groups, F (1, 22) = 6.19, p < .05. Observation of the mean digit span for each group revealed that children in the high ability mathematics group had a significantly longer digit span. However, ANCOVA controlling for differences in reading ability and IQ removed this significant difference in digit span between the groups, F (1, 20) = .78, p > .05.

One syllable word span: Word span was taken to be the longest span length at which words were recalled by the child in the correct serial position. A one way ANOVA comparing the high and low ability mathematics groups showed there to be a significant difference in word span, F (1, 22) = 17.92, p < .001. ANCOVA controlling for differences in reading ability and IQ did not remove this significant difference between the groups, F (1, 20) = 9.66, p < .01, with children in the high ability mathematics group achieving a significantly longer word span than children in the low ability mathematics groups.

Counting span: Counting span was taken to be the longest span length where stimuli were recalled in the correct serial position order. A one way ANOVA comparing the low and high ability mathematics groups revealed no significant difference in counting span between the groups, F (1, 22) = 1.99, p > .05. ANCOVA controlling for differences in reading ability and IQ again revealed no significant differences between the groups, F (1, 20) = .13, p > .05.

Speech rate: Speech rate for one and three syllable words was calculated as the time taken for five repetitions of the word pairs. From this the number of words articulated per second was calculated. A two way repeated measures ANOVA, with one between subjects factor, groups (low versus high ability mathematicians), and one within subjects factor, word length (one versus three syllables), revealed significant main effects of group, $F(1, 22) = 8.24$, $p < .01$, and word length, $F(1, 22) = 173.22$, $p < .001$, with there being no significant interaction between group and word length. Newman Keuls analysis revealed that children in the high ability mathematics group had significantly faster articulation rates for both one and three syllable words, compared to the children in the low ability mathematics group ($p < .05$ and $p < .01$ respectively). Both groups showed the normal word length effect, articulating significantly more one syllable words per second than three syllable words (p's $< .01$). However, ANCOVA, controlling for differences in word reading ability and IQ removed the significant difference between the groups in speech rate, $F(1, 20) = 3.06$, $p > .05$.

Counting rate: Counting rate was calculated as the number of dots counted per second. A one way ANOVA comparing groups (low versus high ability mathematicians), showed there to be no significant differences between the groups, $F(1, 22) = .00$, $p > .05$. ANCOVA controlling for differences in reading ability and IQ also revealed there to be no significant difference between the groups, $F(1, 20) = .04$, $p > .05$.

Speed of retrieval from long-term memory.

Speed of letter identification: Mean letter identification time for correct identification only was calculated. A one way ANOVA comparing low and high ability mathematics groups showed a significant difference, $F(1, 22) = 12.64$, $p < .01$. ANCOVA controlling for group differences in reading ability and IQ did not remove this significant difference between the groups in speed of letter identification, $F(1, 20) = 5.97$, $p < .05$. Examination of the mean times revealed that children in the high ability mathematics group were significantly faster at identifying letters than children in the low ability mathematics group.

Speed of number identification: All children identified the numbers correctly, and the mean identification time was calculated. A one way ANOVA comparing low and high ability mathematics groups revealed there to be a significant difference between the groups, $F(1, 22) = 16.97$, $p < .001$. ANCOVA controlling for differences in word reading ability and IQ also showed there to be a significant difference between the groups, $F(1, 20) = 7.23$, $p < .05$. In a similar manner to speed of letter identification, children in the high ability mathematics groups were significantly faster at identifying numbers, compared to children in the low ability group.

Sequence knowledge: Knowledge of well known sequences was examined, these being counting from 1 to 20, the letters of the alphabet, days of the week, and months of the year. Any particular sequence was marked as being correct up to the point where the first sequencing error occurred. This value was taken to be the raw score. All children correctly counted from 1 to 20 and so this sequence item was excluded from further analysis. All raw scores were then converted to percentages to enable comparisons across different sequences. Three one way ANOVA's were conducted to compare the high and low ability mathematics groups on each sequence type. This revealed there to be significant differences between the groups in the ability to recall both the alphabet and months of the year in sequence; alphabet, $F(1, 22) = 7.71, p < .05$; months, $F(1, 22) = 8.01, p < .01$. In both cases, children in the high ability mathematics group were able to correctly produce more of the sequence than children in the low ability mathematics groups. There was no significant difference between the groups in their ability to produce the days in sequence, $F(1, 22) = 3.58, p = .072$. However, ANCOVA controlling for differences between the groups in reading ability and IQ changed this pattern of results, and removed the significant differences between the groups in alphabet sequencing, $F(1, 20) = 3.20, p > .05$, and in the sequencing of months, $F(1, 20) = 2.65, p > .05$. This shows that the initial difference found between the groups in sequencing ability can be accounted for by differences between the groups in reading ability and IQ, not mathematics ability.

In summary, the initial analyses of variance comparing the two ability mathematics groups revealed that children in the high ability group performed significantly better on the block design test (measuring visual-spatial ability), and were significantly faster to complete the visual number matching task and the cross out task with geometric shapes. Children in the high ability mathematics group were also found to have a significantly longer short-term memory span for both digits and one-syllable words, and significantly faster articulation rate for both one- and three-syllable words. These children were also significantly faster to identify numbers and letters, and were more accurate at sequencing letters of the alphabet and months of the year. However, at this point it was not possible to say whether these results are due to the differences in mathematics ability between the groups, or the co-occurring differences in reading ability.

Analyses of covariance controlling for differences in reading ability and IQ removed many of these significant results. However, there were still significant differences in performance on the cross-out task (matching geometric shapes), and also differences in one-syllable word span. Significant differences in performance were also still apparent in speed of number and letter identification. Marginally significant results were also found for the visual number matching task. Therefore, there are differences between the groups in speed of information processing, speed of item identification, and in one measure of short-term memory span, which can only be ascribed to difference between the groups in mathematics ability, given that differences between the groups in reading ability and IQ had been controlled for.

TABLE 3.3.

Mean performance on all cognitive tasks by high and low ability mathematics groups, and adjusted mean performance controlling for group differences in reading ability and IQ (*SD*).

	High ability		Low ability	
Measure	*M*	Adjusted *M*	*M*	Adjusted *M*
VISUAL-SPATIAL ABILITY (SCALED SCORE)				
Block Design	10.57 (2.68)	9.70	7.30 (2.26)	8.17
Object Assembly	9.36 (2.21)	8.84	8.50 (2.27)	9.01
PROCESSING SPEED (TOTAL TIME, SECS)				
Visual matching	156.64 (45.34)	157.73	198.59 (36.41)	197.50
Cross out task	147.27 (53.93)	147.49	198.96 (39.80)	198.68
Peg Board	9.63 (2.16)	9.98	11.18 (1.69)	10.83
SHORT-TERM MEMORY SPAN AND ARTICULATION RATE (ITEMS PER SECOND)				
Counting span	4.21 (1.12)	3.77	3.50 (1.35)	3.94
Digit span	5.57 (1.22)	5.17	4.30 (1.25)	4.70
Word span	4.57 (0.76)	4.46	3.40 (0.52)	3.51
Counting rate	2.45 (0.54)	2.44	2.44 (0.41)	2.45
1 syll speech rate	1.47 (0.27)	1.42	1.24 (0.15)	1.29
3 syll speech rate	0.82 (0.15)	0.80	0.65 (0.19)	0.66
SPEED OF ITEM IDENTIFICATION (MSECS)				
Letter identification	1048 (152)	1075	1274 (156)	1246
Number identification	744 (112)	772	967 (154)	939
SEQUENCE KNOWLEDGE (% CORRECT SEQUENCING)				
Alphabet	72.80 (38.33)	68.39	33.46 (27.20)	37.88
Days	100.00 (0.00)	96.57	82.86 (34.21)	86.98
Months	72.02 (36.36)	61.63	27.50 (39.88)	37.89

Single-digit addition.

Frequency of strategy use: For each addition question presented, the strategy used by the child to answer the question was recorded at the time of testing. For the purposes of analysis, strategy use was split into two classifications. The first of these was overt counting strategies, encompassing counting using concrete representations, including fingers, and verbal counting, evidenced audibly or through movement of the lips and head. The second strategy classification was direct memory retrieval of arithmetic facts, where the child showed no evidence of using overt counting strategies. A one way ANOVA was conducted to compare the high and low ability mathematics groups on the frequency of correctly using overt counting strategies for the solution of simple addition problems. This revealed there to be no significant difference between the groups, F (1, 22) = 2.46, $p > .05$. ANCOVA controlling for differences between the groups in reading ability and IQ also showed there to be no significant difference between the groups, F (1, 20) = 1.11, $p > .05$. ANOVA comparing the groups on the frequency of correctly retrieving answers directly from long-term memory revealed there to be a significant difference between the groups, F (1, 22) = 11.17, $p < .05$, with children in the high ability mathematics group retrieving significantly more answers directly from long-term memory compared to children in the low ability mathematics group. Controlling for reading ability and IQ through ANCOVA removed this significant difference between the groups, F (1, 20) = 3.95, $p = .061$.

The total number of addition problems answered correctly using any strategy was also analysed. ANOVA comparing the two ability mathematics groups revealed there to be a significant difference between the groups, F (1, 22) = 16.80, $p < .001$. This result remained significant after controlling for reading ability and IQ, F (1, 20) = 7.10, $p < .05$. Children in the high ability mathematics group answered significantly more additions problems correctly than children in the low ability mathematics group.

Strategy times: For each strategy, the mean time taken to produce correct responses to arithmetical questions using that particular strategy was calculated. Analysis of solution time using overt counting strategies revealed there to be a significant difference between the high and low ability groups, F (1, 22) = 11.80, $p < .01$. Children in the high ability mathematics group were significantly faster to reach correct solutions using overt counting strategies. ANCOVA controlling for reading ability and IQ did not remove this significant difference in solution time between the groups, F (1, 20) = 7.49, $p < .05$. It was not possible to consider retrieval times for analysis, as only one out of ten of the children in the low ability mathematics group retrieved arithmetic facts directly from memory, compared to thirteen out of fourteen of the children in the high ability mathematics group. This in itself provides some evidence to suggest that children who are poor at mathematics are delayed in their automatisation of basic arithmetic facts.

TABLE 3.4.

Performance of high and low ability mathematicians on measures of simple arithmetic (*SD*).

	High ability		Low ability	
Measure	*M*	Adjusted *M*	*M*	Adjusted *M*
Overt counting (frequency)	43.79 (8.06)	43.17	36.70 (14.11)	37.32
Direct retrieval (frequency)	9.43 (8.55)	7.99	0.30 (0.95)	1.74
Total correct (max = 56)	53.21 (3.56)	51.16	37.00 (14.31)	39.06
Overt counting solution time	4.54 (1.97)	4.68	12.49 (8.43)	12.33
Direct retrieval solution time	1.94 (0.57)	1.95	3.92 (0.00)	3.91

Note. Solution time is in seconds.

Single digit subtraction.

Frequency of strategy use: As for the addition problems, the percentage of trials using each solution strategy (overt counting and direct memory retrieval), resulting in correct answers was calculated. One way ANOVA comparing the low and high ability mathematics groups in frequency of use of overt counting strategies revealed there to be no significant difference, $F\ (1, 22) = 3.42\ p = .078$. ANCOVA controlling for differences in reading ability and IQ also showed there to be no significant differences between the groups, $F\ (1, 20) = 2.00, p > .05$. Analysis of the frequency of direct memory retrieval revealed a significant difference between the groups, $F\ (1, 22) = 11.29, p < .01$, with children in the high ability mathematics group retrieving significantly more correct answers directly from long-term memory than children in the low ability mathematics group. Controlling for differences in reading ability and IQ through ANCOVA did not remove this significant difference between the groups, $F\ (1, 20) = 9.55, p < .01$.

A one way ANOVA was also conducted to compare overall accuracy between the groups. This revealed there to be a significant difference between the groups, $F(1, 22) = 7.66$, $p < .05$, with children in the high ability mathematics group answering more subtraction problems correctly compared to children in the low ability mathematics group. However, ANCOVA removed this significant difference between the groups in overall accuracy, $F(1, 20) = 3.77$, $p = .067$ (see Table 3.5).

Strategy times. A one way ANOVA was conducted to compare the two ability mathematics groups in the time taken to correctly solve subtraction problems using overt counting strategies. Analysis of solution times revealed there to be a significant difference between the groups, $F(1, 22) = 11.29$, $p < .01$. ANCOVA controlling for differences in reading ability and IQ did not remove this significant difference between the groups, $F(1, 20) = 5.84$, $p < .05$. Children in the high ability mathematics groups correctly solved problems using overt counting strategies more quickly than children in the low ability mathematics group. As for addition problems, analysis of solution times for direct memory retrieval was not possible as only one child out of ten in the low ability mathematics group showed direct retrieval of subtraction facts, compared to thirteen out of fourteen children in the high ability mathematics group (see Table 3.5).

TABLE 3.5.

Performance of low and high ability mathematicians on single-digit subtraction (*SD*).

	High ability		Low ability	
Measure	*M*	Adjusted *M*	*M*	Adjusted *M*
Overt counting (frequency)	23.79 (7.50)	24.12	23.90 (6.77)	28.99
Direct retrieval (frequency)	11.07 (8.25)	10.37	0.40 (1.26)	1.10
Total correct (max = 36)	34.86 (1.29)	34.48	29.70 (6.86)	30.07
Overt counting solution time	4.59 (3.02)	4.89	8.59 (2.64)	8.29
Direct retrieval solution time	1.87 (0.41)	2.02	2.72 (0.00)	2.57

Note. Solution time is in seconds.

Correlational analyses.

Initially, factor analysis was performed to try and form composite variables of mathematics and reading ability from the measures used. For mathematics ability, the variables entered into factor analysis were performance on the Group Mathematics test, counting errors in addition and subtraction, and frequency of memory retrieval of arithmetic facts for addition and subtraction. Statistics for this factor analysis are shown in Table 3.6. The determinant of the correlation matrix shows that the data do not suffer from multicollinearity or singularity, and the measures of sampling adequacy and sphericity reveal that the data was sufficient for satisfactory factor analysis to proceed. The measures entered into this factor analysis all converged on one variable, and so it was decided to use the measure that had the highest correlation to the overall latent variable, which was performance on the Group Mathematics Test, $r = .90$.

TABLE 3.6

Statistics from factor analysis for measures of mathematics ability.

Measures entered into factor analysis	Correlation to latent variable
Group Mathematics Test	.90
Counting errors in addition	-.79
Counting errors in subtraction	-.80
Memory retrieval of addition facts	.82
Memory retrieval of subtraction facts	.85

Determinant of correlation matrix = .0309

Kaiser-Meyer-Olkin measure of sampling adequacy = .79805

Bartlett test of sphericity = 71.242, sig = .00000

Eigenvalue of variable extracted = 3.50, accounting for 70.1 % of variance

The same procedure was carried out for reading ability, with the following measures being entered into the factor analysis; performance on the Primary Reading Test and BAS Word Reading test, regular and irregular word reading, short and long nonword reading, and performance on the phoneme deletion measure. Statistics from the factor analysis are shown in Table 3.7. The determinant of the correlation matrix shows that the data do not suffer from multicollinearity or singularity, and the measures of sampling adequacy and sphericity reveal that the data was sufficient for satisfactory factor analysis to proceed. Again, all of these measures converged on the same variable, and as for mathematics ability, the measure was used that showed the highest correlation to the latent variable identified in factor analysis, this being BAS Word Reading, $r = .95$.

TABLE 3.7

Statistics from factor analysis for measures of reading ability.

Measures entered into factor analysis	Correlation to latent variable
BAS word reading test	.95
Primary Reading Test	.80
Regular word reading	.94
Irregular word reading	.92
Short nonword reading	.90
Long nonword reading	.94
Phoneme deletion	.89

Determinant of correlation matrix = .0000509

Kaiser-Meyer-Olkin measure of sampling adequacy = .85911

Bartlett test of sphericity = 196.074, sig = .00000

Eigenvalue of variable extracted = 5.74, accounting for 82.1 % of variance

For the purposes of correlational analysis, a number of composite measures were formed. These were *speed of item identification* (mean letter identification time, r (22) = -.71, and number identification times, r (22) = -.69), *short-term memory span* (mean counting span, r (22) = .49, digit span, r (22) = .57 and one-syllable word span, r (22) = .65), *articulation rate* (one syllable speech rate, r (22) = .53, and three syllable speech rate, r (22) = .47), *sequencing ability* (mean percentage of alphabet, r (22) = .59, days, r (22) = .53, and months, r (22) = .64, sequenced correctly), *visual spatial ability* (block design, r (22) = .68, and object assembly, r (22) = .31), and *processing speed* (visual matching (time), r (22) = -.48, cross out (time), r (22) = -.48, and peg board, r (22) = -.33). All correlation coefficients reported above are those between mathematics ability and the reported variables. As can be seen, all variables that were combined together to form a new composite variable had similar correlations with mathematics ability, and therefore no spurious correlations were formed by combining these variables. This was further supported by the transformation of r to Fisher's z, showing there to be no significant differences between variables which went to form the composite variables in the strength of their relationships to mathematics ability. Correlational analysis was first carried out to investigate the relationships between the composite variables and mathematics ability, followed by partial correlation analysis controlling for differences in reading ability according to performance on the BAS Word Reading Test. These results are reported in Table 3.8.

Initially, all composite variables were found to be significantly correlated with mathematics ability; *articulation rate*, r (22) = .58, $p < .01$; *item identification speed*, r (22) = -.76, $p < .001$; *sequencing ability*, r (22) = .74, $p < .001$; *processing speed*, r (22) = -.52, $p < .01$; *short-term memory span*, r (22) = .67, $p < .001$; *visual-spatial ability*, r (22) = .61, $p < .01$. However, once differences in reading ability and IQ had been controlled for through partial correlation (controlling for performance on the BAS Word Reading and EPVT), this pattern of results changed. Previously significant correlations to mathematics ability from articulation rate, short-term memory, and visual-spatial ability were eliminated to leave the following significant correlations to mathematics ability; *speed of item identification*, pr (20) = -.60, $p < .01$; *sequencing ability*, pr (20) = .55, $p < .01$; *processing speed*, pr (20) = -.52, $p < .05$.

Further correlational analyses were carried out to see which variables were correlated with reading ability (see Table 3.9). All composite variables with the exception of speed of processing were found to be significantly correlated with reading ability; *articulation rate*, r (22) = .52, $p < .05$; *speed of item identification*, r (22) = -.60, $p < .01$; *sequencing ability*, r (22) = .63, $p < .01$; *short-term memory span*, r (22) = .73, $p < .001$; *visual-spatial ability*, r

(22) = .66, $p < .001$. Partial correlation analysis was also conducted to see which are these variables remained specifically correlated to reading ability once differences in mathematics ability and IQ had been controlled for. This revealed only short-term memory span to be significantly correlated with reading ability, *pr* (20) = .48, $p < .05$, with all other previously significant correlations to reading ability being eliminated by controlling for individual differences in mathematics ability and IQ.

TABLE 3.8.

Mathematics ability: Correlations coefficients to experimental measures (below principal diagonal) and partial correlation coefficients controlling for BAS Word Reading and EPVT (above principal diagonal).

Measure	1	2	3	4	5	6	7
1. Group Mathematics Test	—	*ns*	*ns*	-.60**	-.52*	.55**	*ns*
2. Short-term memory span	.67***	—	*ns*	*ns*	*ns*	*ns*	*ns*
3. Articulation rate	.58**	.59**	—	*ns*	*ns*	.46*	*ns*
4. Item identification (speed)	-.76***	-.45*	-.48*	—	.46*	ns	*ns*
5. Processing speed	-.52**	*ns*	*ns*	.49*	—	*ns*	*ns*
6. Sequencing ability	.74***	.60**	.66***	-.66**	*ns*	—	*ns*
7. Visual-spatial ability	.61**	.58**	*ns*	*ns*	-.46*	.51*	—

Note: For correlations, $df = 22$, for partial correlations, $df = 20$.

* $p < .05$

** $p < .01$

*** $p < .001$

TABLE 3.9.

Reading ability: Correlation coefficients to experimental measures (below principal diagonal) and partial correlation coefficients controlling for mathematics ability and EPVT (above principal diagonal).

Measure	1	2	3	4	5	6	7
1. BAS Word Reading	—	.48*	*ns*	*ns*	*ns*	*ns*	*ns*
2. Short-term memory span	.73***	—	*ns*	*ns*	*ns*	*ns*	*ns*
3. Articulation rate	.52*	.59**	—	*ns*	*ns*	*ns*	*ns*
4. Item identification (speed)	-.60**	-.45*	-.48*	—	*ns*	*ns*	*ns*
5. Processing speed	*ns*	*ns*	*ns*	.49*	—	*ns*	*ns*
6. Sequencing ability	.63**	.60**	.66***	-.66***	*ns*	—	*ns*
7. Visual-spatial ability	.66***	.58**	*ns*	*ns*	-.46*	.51*	—

Note: For correlations, $df = 22$, for partial correlations, $df = 20$.

* $p < .05$

** $p < .01$

*** $p < .001$

3.2 *(iii)* Summary of results.

These results suggest that certain cognitive skills may be more closely tied to either mathematics or reading ability. Mathematics ability appeared to be more closely associated with speeded performance measures such as speed of identifying numbers and letters, speed of matching shapes and numbers, and perceptual motor speed as well as a general ability to be able to recall sequences of information. This suggests that one potential difficulty for children who are underachieving in mathematics may stem from a difficulty in retrieving information from long-term memory, which may result from a fundamental deficit in underlying speed or fluency of processing information. This is shown in this study by the fact that children who were poor at mathematics were slower to retrieve the names of known numbers and letters from long-term memory. Further to this, these children were also found to have a low frequency of direct arithmetic fact retrieval from long-term memory.

On the other hand, correlation analyses revealed reading ability to be strongly associated with measures of short-term memory span, and not to measures of speeded performance. In many studies examining reading skills, short-term memory has been found to play a role in children's reading ability (Daneman & Carpenter, 1980, 1983; Johnston et al, 1987; Lehto, 1995; McDougall et al, 1994). Children's poor reading skills when coupled with such short-term memory difficulties are often interpreted in terms of a phonological coding deficit, particularly in young readers, whereby poor or young readers have poorer access to phonetic codes and representations, or are unable to rehearse the items presented (Johnston, 1982; Mann, Lieberman, & Shankweiler, 1980; Shankweiler, Liberman, Mark, Fowler, & Fisher, 1979; Siegel & Linder, 1984). Differences in short-term memory span are also explained in terms of differences in articulation rate (Henry & Millar, 1991), and speed of item identification (Case et al, 1982; Hitch et al, 1989). All of these issues are discussed in some detail in the introductory chapter.

One fundamental problem with the first study which must not be overlooked is the small sample size, which leads to lack of statistical power especially with regard to correlation analyses, and rules out the possibility of performing regression analyses on the grounds of the criteria that must be fulfilled to perform such analyses, namely, a subjects to variables ratio of at least 5:1. Such a lack of statistical power also leads to the possibility that these results will not be replicated. To counteract these problems, it was decided to test a further sample of children on some of the measures that had produced the most interesting results with the initial sample of 24 children. The following subject descriptions and results are for a total sample of 68 children, that is, it includes the 24 children reported in Study 1a, with a narrower range of tasks being administered.

3.3 Study 1b.

3.3 *(i)* Method.

Subjects.

69 children (mean age 7 years, 5 months, *SD* = 4 months, 39 males and 30 females) were initially screened prior to experimental testing; all children attended two different urban schools in Dundee. One child was later excluded from further analysis due to absence on a number of the experimental testing sessions, leaving 68 children who participated in later experimental testing. No child was excluded due to any intellectual or behavioural difficulties. Children were initially screened for mathematics and reading ability using the Group Mathematics Test (Young, 1970), and the BAS Word Reading Test (Elliott et al 1979). Standard age scores for mathematics and reading ability were calculated using the test norms from the Group Mathematics Test (mean = 100, *SD* = 15), and from the BAS Word Reading Test (mean = 50, SD = 10). Results from these tests revealed a range of reading performance from 31 to 73 (mean = 49.84, *SD* = 10.80). Mathematics standard age scores ranged from 70 to 122 (mean = 96.37, *SD* = 6.34). A significant correlation was found between mathematics and reading ability, r (66) = .67, $p < .001$. IQ was also estimated using measures of vocabulary knowledge (EPVT or WISC-R). As the initial sample of 24 children had been administered the EPVT as a measure of vocabulary, whilst the additional 44 children had completed the WISC-R vocabulary subtest, scores for both samples of children were converted to standardised scores according to the test means (for EPVT, mean = 100, *SD* = 15; for WISC-R vocabulary, mean = 10, *SD* = 3), and then converted into standardised scores with a mean of 0 and a standard deviation of 1.

Children were placed into one of two groups according to their mathematics standard age score, forming a high ability group whose mathematics standard age scores were above the mean score for the entire sample, and a low ability group whose standard age scores fell below the group mean of 96. Thirty-six children (21 males and 15 females) were placed into the high ability mathematics group due to their standard age scores falling above 96, and 32 children (17 males and 15 females) were placed into the low ability mathematics group, having standard age scores falling below 96. A one way ANOVA was conducted comparing groups (high and low mathematicians), which showed there to be no significant difference between the groups in chronological age, F (1, 66) = .61, $p > .05$. Two way analyses of variance were conducted examining for differences between the groups and for sex differences on the Group Mathematics Test and the BAS Word Reading Test. Results showed there to be a significant difference between the groups on mathematics ability, $F\ (1, 64) = 149.77$, $p < .0001$, and on reading ability, $F\ (1, 64)\ 30.17$, $p < .0001$. There were no significant sex differences in either mathematics ability, $F\ (1, 64) = .008$, $p > .05$, or reading ability, $F\ (1, 64) = .440$, $p > .05$. ANOVA comparing the groups on vocabulary performance revealed the difference between the high and low ability groups to be approaching significance, $F\ (1, 66) = 3.24$, $p < .063$, with children in the high ability mathematics group having a higher standardised IQ score than children in the low ability group (see Table 3.10 for mean mathematics and reading standard age scores and estimated IQ for each group).

To establish that differences in mathematics ability were still apparent once reading ability had been controlled for, ANCOVA was carried out, controlling for reading differences using performance on the BAS Word Reading Test. Because the difference between the groups in estimated IQ was also approaching significance, vocabulary knowledge was included as a second covariate in all statistical analyses. This analysis revealed that there were still significant differences between the groups in mathematics ability, $F\ (1, 64) = 87.09$, $p < .0001$. Performance on the BAS Word Reading Test and vocabulary measures were used to control for differences in reading ability and IQ in all the statistical analyses reported below.

Materials and Procedure.

Experimental testing began approximately one month after the screening tests had been administered. Children were tested individually over six short sessions in a quiet classroom in the school, with tasks being given to each child in the same order. Experimental tasks are as reported for the initial study of 24 children, unless specifically stated here. Some of the initial experimental tasks were not administered in this study. These included all the measures of reading ability with the exception of BAS Word Reading, and the measures of visual-spatial skill. Given that the results from study 1a showed there to be similar performance characteristics by the children on addition and subtraction measures, only addition questions were examined in this study. The tasks administered were as follows.

Short-term memory span.

Measures of memory span and articulation rate administered were as previously stated. These measures were digit span, one syllable word span, counting span, one and three syllable speech rate, and counting rate.

Processing speed.

Measures of processing speed are as previously stated. These measures are visual number matching, cross out task, and perceptual motor speed measured by the peg board.

Long-term memory.

Again, these measures were kept as previously stated. These measures were speed of identification of numbers and letters.

Sequencing ability: Some of the sequencing tasks were altered slightly. This was to overcome problems associated with simple repetition of overlearned sequences. Again, the child was asked to count from one to twenty, but was also asked to count higher numbers which are not so well learned (31 to 50), and asked to say the letters of the alphabet in sequence. To be certain that performance on this task was not simply a measure of repetition ability, the children were also asked to put a range of numbers in numerical order, and also words in alphabetical order. For example, the child was presented with six cards each displaying a different number and the child was asked to put them in number order. In the alphabetical ordering task, the child was again presented with six cards, each displaying a different word, which the child was asked to put in alphabetical order, with alphabetical order being described as the order with which the first letter of each word appears in the alphabet. To make this more obvious to the child, the first letter of each word was written in a different colour. Familiar words were used which could be easily read by the child. Each child was asked to complete three trials of six items for both the numerical and alphabetical ordering tasks. Accuracy of the counting and alphabet sequences was taken to be the number of items remembered in the correct sequence order before the first mistake was made. No credit was given for any items placed into the correct sequence after one mistake had already been made. Accuracy of the numerical and alphabetical order tasks was measured as the number of items placed in the correct alphabetical or numerical order. For purposes of comparison between the tests, all scores were converted to percentages. Data for only the new sample of 44 children will be discussed subsequently.

Single-digit addition.

Apparatus, stimuli, and administration procedure were as described in Study 1a. Measures of strategy use were taken, with the distinction being made between direct memory retrieval of arithmetic facts, and the use of overt counting strategies. Accuracy was also noted for each question, with the computer also recording solution time for each question.

3.3 (ii) Results.

The results from this additional study were combined with the results from the initial study of 24 children. Therefore, all of the analyses reported are for a total of 68 children, with the exception of measures of sequencing ability where only the results from the additional 44 children from this study will be reported.

Differences between the groups on the various measures were first analysed by way of 2 x 2 analysis of variance, testing for significant differences between the high and low ability mathematics groups and for sex differences. This was thought necessary as some studies have shown sex differences not only in mathematical skill but also in some cognitive skills. Sex differences were not found on any of the experimental measures, and so only the one way ANOVA's comparing high and low ability mathematics groups will be reported. To be certain that any differences found between the two mathematics ability groups were due to mathematics ability alone, ANCOVA was also carried out for each task, controlling for differences between the groups in reading ability and IQ. In all cases the covariates were performance on the BAS Word Reading Test and vocabulary knowledge. All results from these analyses are reported in Table 3.10.

To summarise, analyses of variance showed there to be significant differences between the groups on virtually all cognitive measures, with the exceptions of counting span, three-syllable speech rate, and counting rate. Analyses of covariance controlling for differences between the groups in reading ability and IQ reduced the number of significant differences. The low ability mathematics group showed significantly poorer performance on measures of speed of number identification, processing speed (i.e., visual number matching and perceptual motor speed), one syllable speech rate, and sequencing ability (alphabet and numerical ordering), even when differences in reading ability and IQ between the groups had been controlled for. No differences were found between the low and high ability mathematics groups on any of the short-term memory span measures once differences in reading had been controlled for.

TABLE 3.10

Performance characteristics of low and high ability mathematicians on screening tests and experimental measures (*SD*)

Measure	Low ability		High ability		ANOVA			ANCOVA		
	M	Adjusted *M*	*M*	Adjusted *M*	*df*	*F*	*p*	*df*	*F*	*p*
Group Mathematics	86.1 (6.1)	87.94	105.5 (6.5)	103.6	1, 66	149.77	< .001	1, 64	87.09	< .001
BAS Word Reading	43.5 (8.4)		55.5 (9.5)		1, 66	30.17	< .001			
Estimated IQ	-0.24 (1.00)		.21 (0.95)		1, 66	3.59	= .06			
SHORT-TERM MEMORY SPAN AND ARTICULATION RATE (ITEMS PER SECOND)										
Counting Span	3.3 (1.1)	3.5	3.6 (1.2)	3.3	1, 66	.72	*ns*	1, 64	.35	*ns*
Digit Span	4.3 (0.9)	4.6	5.2 (1.2)	4.9	1, 66	10.75	< .01	1, 64	1.08	*ns*
Word Span	3.7 (0.6)	3.9	4.3 (0.7)	4.2	1, 66	16.15	< .001	1, 64	3.07	*ns*
1 Syll Speech Rate	2.3 (0.4)	2.4	2.9 (0.6)	2.8	1, 66	25.36	< .001	1, 64	12.09	< .01
3 Syll Speech Rate	1.6 (0.4)	1.6	1.8 (0.3)	1.8	1, 66	3.21	*ns*	1, 64	1.43	*ns*
Counting rate	2.5 (0.5)	2.5	2.9 (1.0)	2.9	1, 66	3.77	*ns*	1, 64	2.58	*ns*
PROCESSING SPEED (TOTAL TIME IN SECONDS)										
Cross Out task	185.4 (40.1)	180.7	151.4 (45.0)	155.9	1, 66	10.73	< .01	1, 64	3.88	= .053
Visual Matching	210.7 (39.3)	208.2	171.9 (41.7)	174.4	1, 66	15.41	< .001	1, 64	7.84	< .01
Peg Board	10.0 (1.4)	9.9	9.3 (1.5)	9.4	1, 66	4.65	< .05	1, 64	4.54	< .05
SPEED OF ITEM IDENTIFICATION (MEAN TIME IN MSEC)										
Letter ID	1138 (211)	1101	983 (169)	1019	1, 66	11.26	< .01	1, 64	2.32	*ns*
Number ID	894 (171)	860	740 (114)	774	1, 66	19.65	< .001	1, 64	4.61	< .05
SEQUENCING ABILITY (% CORRECT)										
Alphabet	40.2 (17.7)	46.3	80.3 (26.3)	74.2	1, 42	24.23	< .001	1, 40	7.88	< .01
Numerical Order	65.9 (29.3)	66.3	88.1 (20.9)	87.7	1, 42	8.39	< .01	1, 40	4.61	< .05
Alphabetical Order	47.0 (29.1)	55.0	70.0 (24.7)	60.8	1, 42	7.96	< .01	1, 40	.28	*ns*

Single-digit addition.

Strategy use: For each addition question presented, the strategy used by the child to answer the question was recorded at the time of testing. These strategies were split into two main strategies, counting (finger and verbal counting) and direct retrieval from long-term memory. One way ANOVA was carried out comparing groups (low versus high ability mathematicians) on their frequency of use of direct memory retrieval in solving arithmetic problems. Significant differences were found between high and low ability mathematicians, F (1, 66) = 38.92, $p < .001$. ANCOVA did not remove this result with significant differences still being present between the groups once differences in word reading ability and IQ had been controlled for, F (1, 64) = 19.51, $p < .001$ (see Table 3.11). Children in the high ability mathematics group used direct memory retrieval to solve addition problems significantly more frequently than children in the low ability mathematics group.

Error frequency: The number of errors made by each group when using counting strategies only was calculated. Only this strategy was considered because very few errors were made by any child when using memory retrieval. A one way ANOVA comparing groups, (low versus high ability mathematicians) revealed significant differences between the groups, F (1, 66) = 18.78, $p < .0001$. ANCOVA did not remove this result, with differences between the groups in frequency of counting errors made still being apparent once differences between the groups in word reading ability and IQ had been controlled for, F (1, 64) = 4.72, $p < .05$. Observation of the mean number of errors made shows that low ability mathematicians made significantly more counting errors than high ability mathematicians (see Table 3.11).

Strategy times: For each strategy, the mean time taken to produce correct responses to arithmetical questions using that particular strategy was calculated. A two way repeated measures ANOVA was carried out with one within subjects factor, strategy (counting versus direct memory retrieval), and one between subjects factor, group (low versus high mathematicians). This revealed significant main effects of group, F (1, 46) = 15.90, $p < .0001$, and strategy, F (1, 46) = 185.56, $p < .0001$, along with a significant interaction between group and strategy, F (1, 46) = 15.77, $p < .001$. This result remained significant after differences in word reading ability and IQ had been controlled for through ANCOVA, with main effects of group, F (1, 44) = 14.55, $p < .001$, and strategy, F (1, 46) = 185.56, $p < .001$, being found, along with an interaction between group and strategy, F (1, 46) = 11.47, $p < .01$. Newman Keuls analyses revealed that although high ability mathematicians were faster at reaching the correct answer using both counting and memory retrieval, they were only significantly faster when using counting strategies, ($p < .01$), with the difference between the groups in memory retrieval time proving not to be reliable. Both ability groups were significantly faster at using direct memory retrieval than counting to reach correct answers, (p's $< .01$) (see Table 3.11). Differences in retrieval time may not have been found due to a phenomena known as the 'problem size effect'. This is where the answers to more difficult arithmetic problems take longer to retrieve directly from memory. Evidence from the frequency of memory retrieval of arithmetic facts shows that children in the high ability mathematics groups are retrieving more answers directly from long-term memory, and these are answers to more difficult problems. Therefore, if the difficulty of the problem for which the answer was being retrieved was taken in consideration, the differences between the groups in retrieval time would probably be increased.

TABLE 3.11.

Mean frequency of strategy use, mean frequency of counting errors, and mean time taken to answer simple addition questions by low and high ability mathematicians (*SD*).

		Counting		Memory retrieval	
Group		*M*	Adjusted *M*	*M*	Adjusted *M*
Low	Frequency	95.42 (7.04)	94.07	4.57 (7.04)	6.01
High	(%)	74.48 (17.73)	75.83	25.50 (17.76)	24.06
Low	% errors made	30.51 (25.22)	26.21		
High		10.22 (11.67)	14.52		
Low	Time (correct	9.60 (5.66)	9.30	3.22 (0.69)	3.21
High	answers only)	5.21 (1.82)	5.50	2.46 (0.64)	2.47

Correlational analyses.

For the purposes of correlational analysis, all raw scores on the experimental tests were converted to standard scores with a mean of zero and a standard deviation of one. A number of composite measures were formed by combining together experimental variables theoretically thought to be measuring the same substrate and taking the average standard score of the measures, with reliability analysis being performed to confirm these theoretical beliefs. All the correlation coefficients reported below are those between mathematics ability and the reported variables, with the reliability coefficient being that of all the variables which were combined to form the composite variable. These were *speed of item identification* (mean letter identification time, r (66) = -.44, and number identification times, r (66) = -.54, standardised alpha = .74), *short-term memory* (mean counting span, r (66) = .28, digit span, r (66) = .50 and word span, r (66) = .51, standardised alpha = .78), *sequencing* (mean percentage of alphabet, r (42) = .48, numerical order, r (42) = .36, and alphabetical order, r (42) = .39, sequenced correctly, standardised alpha = .65), *processing speed* (cross out task, r (66) = -.42, visual number matching, r (66) = -.49, and peg board, r (66) = -.23, standardised alpha = .71), and *articulation rate* (counting rate, r (66) = .14, one syllable speech rate, r (66) = .51, and three syllable speech rate, r (66) = .22, standardised alpha = .66).

Correlation coefficients revealed a number of composite variables which correlated significantly with mathematics ability (see Table 3.12). Indeed, all the experimental measures of item identification, sequencing ability, articulation rate, speed of processing, and short-term memory span were significantly correlated with mathematics ability. However, once differences in word reading ability and IQ had been controlled for through partial correlation (controlling for performance on the BAS Word Reading Test and vocabulary measures), sequencing ability and articulation rate were no longer significantly correlated to mathematics ability. Performance variables found to be significantly correlated with mathematics once differences in word reading ability had been controlled for were *speed of item identification* (speed of identifying numbers and letters), pr (64) = -.32, $p < .01$, *processing speed* (time), pr (64) = -.39, $p < .01$, and *short-term memory*, pr (64) = .25, $p < .05$ (see Table 3.12).

In order to gain insight into how these cognitive processes may affect the component mechanisms underlying simple arithmetic, data from the arithmetic questions were also entered into the correlational analysis, these being the frequency of memory retrieval to arithmetic problems, the time taken to reach correct answers both in counting and direct memory retrieval, and the number of counting errors made. Partial correlations controlling for performance on the BAS Word Reading Test and IQ revealed that frequency of memory retrieval was significantly correlated with a number of the performance variables, these being *articulation rate*, pr (64) = .34, $p < .01$, *speed of item identification*, pr (64) = -.30, $p < .05$, *processing speed*, pr (64) = -.24, $p < .05$, and *sequencing ability*, pr (40) = .32, $p < .05$. The time taken to correctly solve arithmetic questions using counting strategies was found to be significantly correlated with *speed of item identification*, pr (64) = .31, $p < .05$, with the time taken to retrieve correct answers from long-term memory being significantly correlated with *processing speed*, pr (44) = .50, $p < .01$. The number of counting errors made was significantly correlated with *short-term memory span*, pr (64) = -.30, $p < .05$, and *processing speed*, pr (64) = .26, $p < .05$ (see Table 3.12).

Fixed order multiple regression analyses.

Specific links between predictor variables and mathematics ability were further tested in fixed order multiple regression analyses, assessing whether the variables speed of item identification, short-term memory, and processing speed accounted for further significant unique variance in mathematics ability, once the variance associated with word reading ability had been removed (BAS Word Reading was always entered on the first step of each regression equation). Initially, vocabulary was also entered into the regression equation after BAS Word Reading, but made negligible difference, and so was excluded from the analysis (R^2 change = .00001, F change = .00136, p = .9707). Excluding this extra variable also keeps the statistical power of the regression analysis as high as possible. The outcomes of the regression analyses are summarised in Table 3.13. The first thing to note is the substantial amount of variance accounted for by word reading ability (45%). A significant unique relationship between mathematics ability and processing speed was found however, with processing speed accounting for a further 8% ($p < .01$) of variance in mathematics ability after the variance associated with reading ability had been accounted for. No variables entered on later stages of the regression equation after processing speed were found to add a significant amount of unique variance, suggesting that processing speed accounts for all the variance associated with short-term memory and speed of item identification. Indeed, even when processing speed was entered on the final step of the regression equation, it was still found to add a significant amount of unique variance which was not accounted for by other variables in the regression equation. Short-term memory appears to be the least predictive of mathematics ability, compared to processing speed and speed of item identification. All of the performance variables entered in the regression equation after short-term memory were found to be contributing a significant amount of unique variance in predicting mathematics ability, with the exception of speed of item identification which was not adding a significant amount of unique variance when entered on the final step of the regression equation. Therefore, speed of processing makes a contribution towards predicting differences in mathematics ability that is independent of the contribution made by speed of item identification and short-term memory.

TABLE 3.12.

Correlation coefficients between test measures (below principle diagonal) and partial correlation coefficients controlling for reading ability and IQ (performance on the BAS Word Reading Test and vocabulary measures), above principle diagonal.

	1	2	3	4	5	6	7	8	9	10
1. Short-term memory span	–	*ns*	*ns*	*ns*	*ns*	.25*	*ns*	*ns*	*ns*	-.30*
2. Speech rate	.29*	–	-.28*	*ns*	*ns*	*ns*	.34**	*ns*	*ns*	*ns*
3. Item identification (time)	-.24*	-.38**	–	.34**	*ns*	-.32**	-.30*	.31*	*ns*	*ns*
4. Processing speed (time)	-.33**	*ns*	.46**	–	-.34*	-.39**	-.24*	*ns*	.50**	.26*
5. Sequencing ability.	*ns*	*ns*	*ns*	-.47**	–	*ns*	.32*	*ns*	*ns*	*ns*
6. Group Mathematics Test	.50**	.34**	-.54**	-.51**	.54**	–	.51**	-.46**	-.69**	-.43**
7. Memory retrieval (freq)	.31**	.42**	-.47**	-.36**	.50**	.63**	–	-.27*	*ns*	*ns*
8. Counting (time)	*ns*	-.26*	.42**	*ns*	-.42**	-.56**	-.38**	–	.78**	*ns*
9. Memory retrieval (time)	-.29*	*ns*	*ns*	.54**	*ns*	-.68**	-.32*	.80**	–	*ns*
10. Counting errors	-.48**	-.27*	.28*	.39**	-.41**	-.62**	-.33**	.38**	.39**	–

Note. Correlation coefficients: $df = 66$ in all cases apart from sequencing ability where $df = 42$, and memory retrieval (time) where $df = 46$. Partial correlation coefficients: $df = 64$ in all cases apart from sequencing ability where $df = 40$, and memory retrieval (time) where $df = 44$. Where sequencing ability and memory retrieval (time) correlate, $df = 32$ for correlation and $df = 30$ for partial correlation.

* $p < .05$

** $p < .01$

TABLE 3.13.

Summary of multiple regression analyses (outcome measure: mathematics ability as measured by the Group Mathematics Test).

Order of entry in regression equation	R^2	R^2 change	*df*	*F*	*p*
1 BAS Word Reading	.45	.45	1, 66	54.45	< .0001
CONTRIBUTION OF ITEM IDENTIFICATION AND PROCESSING SPEED, CONTROLLING FOR BAS READING AND STM.					
2 STM	.48	.03	2, 65	4.39	< .05
3 Item identification	.54	.06	3, 64	7.93	< .01
4 Processing speed	.58	.04	4, 63	5.23	< .05
3 Processing speed	.55	.07	3, 64	9.64	< .01
4 Item identification	.58	.03	4, 64	3.65	*ns*
CONTRIBUTION OF ITEM IDENTIFICATION AND STM, CONTROLLING FOR BAS READING AND PROCESSING SPEED.					
2 Processing speed	.53	.08	2, 65	11.69	< .01
3 STM	.55	.02	3, 64	2.62	*ns*
4 Item identification	.58	.03	4, 63	3.65	*ns*
3 Item identification	.56	.03	3, 64	3.00	*ns*
4 STM	.58	.02	4, 63	3.29	*ns*
CONTRIBUTION OF PROCESSING SPEED AND STM, CONTROLLING FOR BAS READING AND ITEM IDENTIFICATION.					
2 Item identification	.51	.06	2, 65	7.28	< .01
3 Processing speed	.56	.05	3, 64	7.09	< .01
4 STM	.58	.02	4, 63	3.29	*ns*
3 STM	.54	.03	3, 64	5.05	< .05
4 Processing speed	.58	.04	4, 63	5.23	< .05

Note. Item identification = speed of number and letter identification; processing speed = mean time to complete cross-out, visual number matching and peg board tasks; STM = short-term memory span.

3.4 Discussion.

The results clearly show that children with arithmetical difficulties have particular problems on a number of tasks designed to assess cognitive processes. Initially, analyses of variance revealed that children in the low ability mathematics group performed significantly worse than children in the high ability mathematics group on measures assessing short-term memory span, speed of item identification, speech rate for one syllable words, processing speed, and sequencing ability. However, to get a true account of the cognitive processes specifically underlying mathematics ability it was necessary to control for differences between the groups in word reading ability and estimated IQ. Controlling for reading ability and IQ differences through analyses of covariance eliminated previously found differences between the groups on a number of the performance variables, including all of the short-term memory measures, the cross-out task (matching geometric shapes), and speed of letter identification. However, significant differences were still apparent between the groups in speed of number identification, processing speed (visual number matching and perceptual motor speed), one syllable speech rate, and sequencing ability. Partial correlation coefficients, also controlling for differences in word reading ability and IQ, revealed a number of the composite measures to be related to mathematics ability. These included processing speed, speed of item identification, and short-term memory. Fixed order multiple regression analyses however, revealed speed of processing to be the best predictor of mathematics ability, subsuming all the variance that speed of item identification and short-term memory accounted for in mathematics ability.

One of the main differences found between groups, once reading ability had been controlled for, was the speed of identifying numbers, which constitute representations which must be retrieved from long-term memory. Measures of strategy usage for the arithmetical problems presented in this study also showed that children in the low ability mathematics ability group were less likely to use direct memory retrieval to solve arithmetical questions. Furthermore, when this strategy was used, lower ability mathematicians were significantly slower than higher ability mathematicians in retrieving the answer. It has been proposed in previous studies that factors associated with long-term memory, such as familiarity of the stimuli and the strength of representations, have an influence on short-term memory, such that items which can be identified and retrieved from long-term memory more rapidly will aid short-term recall (Gathercole & Adams, 1994; Henry & Millar, 1991; Hulme et al, 1991). Additionally, studies investigating children's counting strategies point out that children with arithmetical difficulties have poor working memory resources leading to poor or incomplete representations of numbers and number facts in long-term memory (Geary, 1990; Geary & Brown, 1991; Hitch & McAuley, 1991; Siegler & Shrager, 1984). This study is supportive of this view as it was found that speed of item identification, involving retrieval of information from long-term memory, was significantly correlated with short-term memory span. Children with arithmetical difficulties were indeed significantly poorer than children in the other ability group on measures of short-term memory, but not after differences in reading skill and IQ had been controlled for.

Another factor found to be strongly correlated with mathematics ability was processing speed. It was further found that low ability mathematicians tended to be slower to complete the cross-out task (comparing geometric shapes) than the other ability group, although this difference was not significant after differences between the groups in reading ability and IQ had been controlled for. Low ability mathematicians were significantly slower at completing the visual number matching and peg board tasks than the high ability group. Correlational analyses showed a composite measure of processing speed to be significantly related to mathematics ability once differences in reading ability and IQ had been partialled out, that is, as mathematics ability increased, time taken to complete the processing speed tasks decreased. Furthermore, multiple regression analyses revealed processing speed to be the best predictor of mathematics ability.

Salthouse's (1996) assumptions regarding a limited time mechanism and a simultaneity mechanism are useful in understanding the link between the speed of processing information and mathematics ability. According to the limited time mechanism, when the early cognitive operations of a task are executed slowly, there is a greatly restricted portion of time left to complete all later processing that may be necessary, and this will have a particularly strong impact when the cognitive task to be performed is complex, such as when there are concurrent demands. We know that multi-digit addition and subtraction, and the solving of word problems, requires intermediate information to be held in memory whilst further processing is taking place to reach a final solution, unless of course, one is an expert at mathematics and is able to rely almost independently on direct retrieval from long-term memory. Therefore, those children who are slow to complete the early stages of the mathematical solution process, will have a restricted portion of time available to execute later stages of the solution process, hence resulting in errors, or very slow solution times, with fewer problems being completed. The simultaneity mechanism would account for a link between processing speed and performance on a cognitive task by the idea that early processing may be lost by the time later processing is completed, such that relevant information is either lost, or in a unusable form, when it is needed. Therefore, it is not possible to integrate all of the relevant information. Again, this may impact on mathematics performance because if the results from earlier lower level processing are lost, then not all relevant information will be available for solving a mathematical problem.

Garnett and Fleischner (1983) and Geary (1993) proposed that the major problem for children with arithmetical difficulties is associated with the slow execution of operations, particularly with regard to long-term memory access. This can be interpreted in several ways. On the one hand, children with mathematical difficulties may simply be slower in general information processing, in the way that other children may be slow at running. Alternatively, these children may have specifically failed to automate basic arithmetical operations. If this is so, performance on more complex mathematical tasks is also likely to suffer, as performance on such tasks is contingent upon the fluency of carrying out the simple operations underlying them. Lack of automaticity also provides a link to the finding that children who were poor at mathematics were slow to identify numbers, and that they used direct memory retrieval of number bonds much less frequently than children in the other ability groups. Low ability mathematicians may be slow to automate numbers and number bonds, which may be due to lack of experience and familiarity with the subject area (Hitch & McAuley, 1991). Frequency of use of direct memory retrieval was found to be partially correlated with numerous performance measures, including speech rate, speed of item identification, processing speed, and sequencing ability. It would therefore appear that the development of mature, efficient strategies for solving simple arithmetic questions, such as direct memory retrieval, is dependent on the establishment of a firm basis of intellectual development, whereby performance becomes automated and information can be processed efficiently. Time taken to retrieve arithmetic answers from long-term memory was also correlated with processing speed, again suggesting this variable measure represents a fundamental deficit in children's arithmetical difficulties.

If lack of automaticity is simply due to lack of practice, then the prospects for remediation are good. If there is no underlying cognitive deficit for children with arithmetical difficulties, then frequent practice of the basic skills, such as counting and simple arithmetic, may provide a simple but highly effective approach for teaching such children, which should enable children to quickly catch up with their peers. Geary and Brown (1991) noted that as children had more experience with numbers and number bonds, fewer computational errors were made and the child relied more frequently on direct memory retrieval of arithmetic facts. Therefore, such difficulties may simply represent a developmental delay. However, Howell, Sidorenko, and Jurica (1987) showed that drill and practice of arithmetic facts, whilst having some short-term benefits on mathematical ability, did not have any long lasting effects, and failed to lead to any improvements in strategy development. They found that teacher intervention was much more beneficial, as the specific difficulties of each child could be addressed. Memory retrieval deficits may represent a more serious difficulty. Geary and Brown noted that retrieval deficits, such as slowness, do not disappear, and that this may represent a much more fundamental deficit. In the study reported here, mathematics ability was significantly correlated with various measures of processing speed, some of which did not contain number components, such as speed of identifying letters, speed of matching shapes (cross-out task), and speed of motor functioning. Given these findings, it may be that children with arithmetical difficulties do indeed have a more centrally limiting cognitive deficit in the speed of executing operations, rather than simply a developmental delay in the automatisation of numbers and number facts. However, the major differences between the two ability groups appeared to be on tasks containing number components such as speed of number identification, and visual number matching. Therefore, the possibility of a very specific deficit relating to the automatisation of numbers and arithmetic facts in long-term memory cannot be ruled out.

Sequencing ability has not been extensively examined in studies of children's arithmetic, with most studies simply ensuring the child is able to count from one to twenty. In this study it was found that children with arithmetical difficulties were able to count accurately from one to twenty, and were also able to count higher value numbers. However, they tended to be less accurate at sequencing letters of the alphabet, and were significantly less accurate at placing items in numerical order. Partial correlation coefficients showed that sequencing ability was not significantly correlated with mathematics ability. One reason for children to be able to successfully count in sequence from one to twenty may be that this is a such a well learned sequence, whereas the numerical ordering task requires the child to order numbers out of context where repetition ability cannot be relied upon. If the child relies on counting to solve arithmetic problems, then this sequence of numbers will be used each time an arithmetic question is encountered. It may be that children with arithmetical difficulties are less fluent in their counting, which may impede their speed of arithmetical functioning, although in the present study low ability mathematicians were not impaired in speed of counting. It may be necessary in future studies to concentrate more specifically on assessing the child's counting knowledge in a similar fashion as reported in Geary et al (1992). Whilst children may be able to count in sequence, more needs to be known about the child's understanding of counting principles.

Findings associated with the speech rate measures are inconclusive. There were significant differences between the high and low ability mathematics groups on the measure of one syllable speech rate, even after differences in reading ability and IQ had been controlled for. However, one drawback of both the one and three syllable speech rate tasks was that familiar words were used, which may involve retrieval from long-term memory. Indeed, partial correlation coefficients did reveal that speech rate was significantly correlated with speed of item identification, a measure concerned with retrieving known representations from long-term memory. One consideration for future studies would be the use of nonwords to measure speech rate, which would reduce the confounding effect of long-term memory processes. However, the lack of significant differences between the two ability mathematics groups on the counting rate task cannot be overlooked. It has been proposed in previous studies that children with arithmetical difficulties may have particular problems in the fluency of number counting. This study found that children with lower mathematical ability were not significantly different in their speed of counting dots, suggesting that lack of fluency in counting is not a problem. One important finding concerning counting is with regard to the number of counting errors made when solving arithmetic problems. Low ability mathematicians made significantly more counting errors than high ability mathematicians, suggesting that there is some underlying problem that leads these children to make mistakes whilst counting. Clearly, this is not a problem involving the speed of counting. Partial correlation coefficients revealed that the number of counting errors made was correlated with short-term memory span and processing speed. Therefore, one explanation for these increased counting errors may be that children with arithmetical difficulties are slow at processing information, meaning their efficiency of manipulating information in short-term memory may be reduced. This supports the argument of Kail (1992) who proposes that speed of processing is an important variable in performance and intellectual development, and the earlier arguments relating to Salthouses's (1996) limited time and simultaneity mechanisms.

Given the abundance of studies showing children with arithmetical difficulties to have some form of short-term memory deficit, it can be questioned why no such clear cut deficit was found in this study, independently of reading skill and IQ. Once differences in reading ability and IQ had been controlled for, there were no differences between the two ability groups on any of the short-term memory measures. An explanation of this result can be found by considering a recent study conducted by Swanson et al (1996), who suggest that where a group of children show poor reading and mathematical skills, there will be a general depression in working memory performance, based on the assumption that verbal working memory correlated best with reading, and visual-spatial working memory correlated best with mathematics. Clearly, in the studies reported in this chapter, there was a general working memory deficit when there were differences between the groups in reading and mathematical skills. However, this deficit on verbal working memory measures was not evident once differences in reading ability had been controlled for. It was also found that controlling for mathematics ability did not remove the correlation between reading ability and measures of verbal working memory. Therefore, in this respect the current results do agree with recent research, and suggests that if a measure of visual-spatial working memory had been included, then the distinction of reading ability to verbal working memory, and mathematics to visual-spatial working memory may have been apparent.

Another issue to consider in relation to this result is the fact that children at this age very rarely use mnemonic strategies such as rehearsal, and this may be one reason for there being no clear cut differences in short-term memory span, particularly on measures taxing the functioning of the articulatory loop. Such differences may become more apparent as the children get older. However, Hitch et al (1989) suggest that developmental difference in memory span may reflect speed of item identification when the effects of rehearsal are minimised. In this study, whilst there were differences between the groups in speed of identifying items, there was no corresponding differences in measures of memory span.

Hulme and Roodenrys (1995) suggest that associations between cognitive impairments and deficits in short-term memory skills should be interpreted cautiously. They point out that there has been little evidence for direct causal links between limitations of short-term memory and other impairments of cognitive development, and suggest that weaknesses of short-term memory should be considered in conjunction with other cognitive weaknesses when trying to interpret problems associated with cognitive skills such as reading and arithmetic. Furthermore, as pointed out in the methodological note in Chapter 2, previous studies which have claimed to examine *specific* arithmetical difficulties have used lenient selection criteria in classifying children as having a specific deficit, and in many cases poor arithmetical skills may have been accompanied by relatively poor reading skills (e.g., Geary et al, 1991; Hitch & McAuley, 1991; Siegel & Ryan, 1989). Therefore, it is difficult to ascertain for certain whether previously found results are in fact the difficulties associated with specific arithmetic difficulties, or whether they are the outcome of generally poor academic skills, incorporating both reading and arithmetic difficulties. As found in Study 1a, short-term memory span was significantly correlated with reading ability, and so previous results claiming an association between mathematics and short-term memory span may have actually been an artefact of associated difficulties in reading that had not been fully accounted for. Furthermore, these studies have concentrated on the functioning of the articulatory loop, when in fact, memory problems may be isolated in another region of working memory, such as the central executive or visual-spatial sketch pad.

As shown in this study, an investigation purely of short-term memory skills would not have revealed the true nature of the deficits underlying children's arithmetical difficulties. Whilst there clearly is involvement from short-term memory in children's arithmetical difficulties, as shown by the significant correlations between mathematics ability and short-term memory after controlling for word reading ability and IQ, this needs to be considered in conjunction with other factors, such as the ability to identify numbers, which is dependent upon retrieving information from long-term memory. Furthermore, speed of processing obviously represents an element of major importance in the explanation of children's mathematical difficulties. This study found that children with mathematical difficulties were slow in the speed of executing operations, such as the speed of identifying numbers, speed of matching numbers and shapes, speed of perceptual-motor performance and the speed of executing arithmetical procedures. This finding could be linked to lack of familiarity with the subject area. On the one hand, this could simply represent a developmental delay, particularly in the automatisation of basic arithmetic facts. Alternatively, and more seriously, it could represent a more fundamental speed of processing deficit, which may be specific to arithmetic, or a general deficit which may also underlie often associated reading difficulties. There is clearly a need in studies of children's arithmetical difficulties to address the issues raised by this research, rather than simply relying on an explanation in terms of a short-term memory deficit.

Studies 2 and 3, reported in chapters 4 and 5 respectively, set out to consider the issues regarding working memory and processing speed in a subset of children reported in this first study. Study 2 pays close attention to working memory, but moves away from the use of short-term memory tasks, to concentrate in more detail on the roles of central executive functioning and visual-spatial sketch-pad in the development of mathematical and arithmetical skills. Study 3 considers in more detail the processing speed deficit identified in this first study, and aims to discover whether this processing speed deficit is specific for information with numerical content, or whether it represents a global deficit observed over a wide variety of tasks. Chapter 5 ends with a synthesis of all the results presented in Studies 1 to 3, exploring the inter-correlations between processing speed and working memory measures to measures of general mathematical ability and simple arithmetical skills.

Chapter 4.

Working Memory Mechanisms: Central Executive Functioning and Visual-Spatial Skills.

The results reported in Chapter 3 opened the door to another course of investigation. As is very clear, short-term memory has been the focus of much work in children's arithmetical skills. However, the results from study 1b found that when differences in reading skills had been controlled for, short-term memory, specifically the functioning of the articulatory loop, did not represent a fundamental deficit for children of low mathematical ability. The study reported in this chapter examined the role of other working memory mechanisms in mathematical and arithmetical skills, namely the central executive and the visual-spatial sketchpad. A detailed theoretical background for the central executive and visual-spatial sketch pad was discussed in the introduction, and will be briefly referred to here to provide the theoretical underpinning for the study reported in this chapter.

4.1 Theoretical Background.

Recent evidence suggests that poor arithmetical skills might not be so easily explained by a deficit in short-term memory span. Butterworth et al (1996) report the dissociation of short-term memory span and arithmetic performance in the case of a patient (MRF), who showed poor performance on short-term memory tests but whose arithmetic skills were intact. Further evidence comes from the results of Studies 1a and 1b (now reported in Bull and Johnston, 1997), where it was found that children with mathematical difficulties did not differ significantly from high ability mathematicians in word span, digit span, or counting span, once differences in reading ability had been controlled for. Whilst short-term memory did correlate with mathematics ability after controlling for differences in reading ability, it actually accounted for the least amount of variance when entered into a regression equation along with speed of item identification and general speed of information processing, and accounted for no unique variance above and beyond these measures. One reason for a short-term memory span deficit not being detected may be that we are not measuring the part of working memory that is deficient in these children. The possible influence of executive functioning and visual-spatial skill in mathematics has frequently been left open to question in previous studies.

The involvement of visual spatial skills in arithmetical performance has been noted in a number of studies (see e.g., Dehaene, 1992; Hayes, 1973; Heathcote, 1994; Hope & Sherill, 1987; Moyer & Landauer, 1967; Restle, 1970; Seron et al, 1992; Smyth et al, 1987), which report both children and adults making use of visual-spatial skills in arithmetic. This has been evidenced through the use of tasks which disrupt the functioning of the visual-spatial sketch pad (Heathcote, 1994), through reports of the use of mental number lines and spatial arrangements to aid counting and solution of arithmetic problems (Dehaene, 1992; Hayes, 1973; Seron et al, 1992, Smyth et al, 1987), and through studies with young children which have shown visual-spatial skills and visual-imagery to be related to arithmetic accuracy (Davis & Bamford, 1995; Geary & Burlingham-Dubree, 1989). There is also evidence from neuropsychological studies and studies of learning disabilities that there is a subtype of mathematical difficulties which have a deficit in visual-spatial skills as its basis (Rourke, 1993, Rourke & Conway, 1997; Rourke & Finlayson, 1978; Strang & Rourke, 1985; Temple, 1991).

There has been very little research, particularly with children, to ascertain the role of the central executive in children's mathematical skills. This is despite the fact that the central executive is probably the most important component of working memory in terms of its general impact of cognition. One exception to this comes from the work of Lehto (1995) who found a significant correlation between performance on a central executive task to mathematics ability in Finnish children aged 15 to 16 years old. However, cognitive studies do report findings with adults linking central executive performance with mathematical skill (Logie et al, 1994). Further theoretical understanding and research findings also come from the developmental neuropsychological literature. These studies provide evidence for the main developmental increases in executive functioning as linked to frontal lobe development, with one of the most important developmental phases thought to be between the ages of 7 to 10 years old (Case, 1992; Chelune & Baer, 1986; Levin et al, 1991; Passler et al, 1985; Welsh et al, 1991). These studies also provide evidence for the kind of cognitive and behavioural skills thought to be linked to executive functioning, such as planning and goal direction, inhibition of habitual responses, dual-task performance, attention, and retrieval of information from long-term memory.

The study reported here represents a move away from traditional cognitive memory span tasks which focus specifically on the functioning of the articulatory loop, to instead focus on the largely neglected involvement of the central executive and the visual-spatial sketch pad in children's mathematical and simple arithmetical skills. It is predicted that as the findings of Studies 1a and 1b showed there to be no differences between low and high ability mathematicians in short-term memory span, once differences in reading ability and IQ had been controlled for, that mathematics ability would instead be related to other mechanisms of working memory, that is, the central executive and visual-spatial sketch pad, in a subset of these children.

4.2 Study 2.

4.2 *(i)* Method.

Subjects

Children taking part in this study were a subset of those reported in Chapter 3, Study 1b (N = 44). Children were screened for mathematics ability and reading ability using the Group Mathematics Test (Young, 1970) and the BAS Word Reading Test (Elliott et al, 1979). Intelligence quotients were also estimated using the vocabulary subtest of the WISC-R (Wechsler, 1977). A significant correlation was found between mathematics and reading ability, r (42) = .63, p <.001. The mean chronological age was 7 years, 3 months (*SD* = 3 months); mean mathematics standard age score was 95.34 (*SD* = 9.50); mean reading standard age score was 50.07 (*SD* = 9.80), and mean estimated IQ was using WISC-R vocabulary was 10.97 (*SD* = 2.94). The Group Mathematics test has a mean of 100 (*SD* = 15); for BAS Word Reading, a mean of 50 (*SD* = 10); for WISC-R vocabulary, a mean of 10 (*SD* = 3).

Children were split into two groups according to their performance on the Group Mathematics Test. Children achieving a standard age score of 96 or above (i.e., above the sample mean) were allocated to the high ability mathematics group (N = 24, 9 girls, 15 boys). Those children with a standard age score of 95 or less were allocated to the low ability mathematics group (N = 20, 9 girls, 11 boys). One way ANOVA's comparing the two ability mathematics groups revealed there to be significant differences between the groups in mathematics ability, F (1, 42) = 100.32, p < .0001, word reading ability, F (1, 42) = 23.60, p < .0001, and in IQ, F (1, 42) = 5.34 p < .05 (see Table 4.1).

ANCOVA was also conducted to ensure there were still differences in mathematics ability between the groups once differences in word reading ability and IQ had been controlled for (controlling for performance on the BAS Word Reading test and WISC-R vocabulary). This revealed that there were still significant differences between the groups in mathematics ability, independent of reading ability and IQ, F (1, 40) = 49.63, p < .001.

Test Procedures.

Experimental testing began approximately one week after screening tests had been completed, with all children completing the tests in the same order. Children were tested individually over a total of three sessions each lasting approximately 20 to 30 minutes, in a quiet area outside the classroom. The tasks were administered in the order Corsi Blocks, Card Sorting, Addition 1 and 2 (addition blocks 1 and 2 counterbalanced). Each experimental task will be described in turn.

Visual-Spatial Sketch Pad.

Corsi Blocks Task: This apparatus consisted of nine blocks nailed to a board (to assure the blocks remain in the same position for each child), all of which were painted black so that it was not possible to discriminate in any way between the blocks, other than by their spatial position in relation to the other blocks on the board. The blocks were in no definable pattern. The experimenter pointed to a sequence of blocks starting from a span of two, and instructed the child to point to the same blocks in the same order. If the child succeeded, the experimenter progressed to three blocks, and so on. If the child failed, a second trial was administered at the same span length. Testing was discontinued if the child failed both trials at any particular span length. Span length was taken as being the maximum number of blocks pointed to in the correct serial order.

Central Executive functioning.

Wisconsin Card Sorting Test (WCST): This test was chosen because it has frequently been found to be a good measure of frontal lobe functioning, and is now successfully being applied and normalised for the study of children (WCST revised and expanded, Heaton, Chelune, Talley, Kay, & Curtiss, 1993). This test has also been used with adults as a measure of central executive functioning. Furthermore, Shute and Huertas (1990) found WCST performance to load on the same factor as cognitive measures (Trail Making and Category Test). In addition, a Piagetian test of formal operational reasoning loaded on the same factor, suggesting that it is appropriate to draw such links between neuropsychological and developmental measures. In the WCST, three dimensions are used for the classification of a series of cards (colour, shape, and number). Four key cards are placed in front of the child, each with a different shape (triangle, circle, square or star), different numbers of the shapes (one, two, three, or four), and each in a different colour (red, green, blue, or yellow). These three distinctions were made very clear to the child.

The child was then instructed to pick up the first card and asked to match it to one of the four key cards by one of the sorting dimensions, i.e., by colour, shape, or number. If the child matched the to correct category, the experimenter said "correct" and the child went onto the next card. If the categorisation response was incorrect, the child was required to match the next card on one of the other sorting dimensions. When the child had maintained the correct sorting classification for 10 consecutive trials, the experimenter changed the sorting criteria without telling the child. It is the child's task to use the feedback given by the experimenter to determine that a previous sorting criteria that was correct is now incorrect, and a different sorting criteria needs to be applied. This procedure continued until the child had completed six category changes, or until the child had run out of cards (total of 128 trials). A number of results from the WCST were used in the analyses: percentage of errors made (perseverative and non-perseverative); failure to maintain set (when the child makes five or more correct responses and then makes an error); number of categories completed; percentage of perseverative responses (perseverative errors and correct perseverative responses); total number of trials administered.

Counting strategies in simple addition.

This followed the same procedure as reported in Study 1a. For the purposes of analysis, the number of correct answers retrieved directly from long-term memory was calculated, as well as the frequency of using overt counting strategies (counting using fingers or verbal counting). The time taken to retrieve the answers directly from memory and the time taken to reach correct solutions using overt counting procedures was also calculated.

4.2 *(ii)* Results.

To examine for differences between high and low ability mathematicians, between groups analyses of variance (ANOVA) were conducted. Analyses of covariance (ANCOVA) were also carried out to control for differences between the groups in word reading ability and IQ (as measured by performance on the BAS Word Reading Test and WISC-R vocabulary). All main effects of group (high versus low ability mathematicians) for both ANOVA and ANCOVA are reported in Table 4.1.

In summary, ANOVA revealed there to be significant differences between the high and low ability groups on a number of measures. Children in the high ability mathematics group showed better performance on the Wisconsin Card Sorting Test, with high ability mathematicians making significantly fewer errors, fewer perseverative errors and perseverative responses, completing more category changes, and taking fewer trials to complete the task. There were no significant differences between the groups in visual-spatial span as measured by the Corsi Blocks.

When differences between the groups in word reading ability and IQ were controlled for through ANCOVA, differences between the groups remained on various measures of the WCST, which is thought to assess central executive functioning. Groups differed significantly in the percentage of errors made, both perseverative and non-perseverative, with children in the low ability mathematics group making more errors. Low ability mathematicians also made significantly more perseverative responses. There was also a significant difference in the number of trials administered, with children in the low ability mathematics group requiring more trials to complete the test. These differences in performance on the WCST can only be ascribed to differences between the groups in mathematics ability, as differences between the groups in reading ability and IQ had been statistically controlled for.

TABLE 4.1.

Performance characteristics of high and low ability mathematicians on screening tests and experimental measures (*SD*).

Measure	High ability		Low ability		ANOVA			ANCOVA		
	M	Adjusted *M*	*M*	Adjusted *M*	*df*	*F*	*p*	*df*	*F*	*p*
Group Mathematics	102.54 (5.38)	101.53	86.70 (5.03)	87.71	1, 42	100.32	< .001	1, 40	49.63	< .001
BAS Reading	55.38 (8.86)		43.70 (6.65)		1, 42	23.60	< .001			
Estimated IQ	10.96 (2.63)		9.00 (2.99)		1, 42	12.11	< .01			
Corsi Blocks	4.21 (0.83)	4.16	4.05 (1.10)	4.10	1, 42	0.30	*ns*	1, 40	0.54	*ns*
% errors	16.96 (4.48)	16.56	26.50 (7.34)	26.90	1, 42	28.11	< .001	1, 40	20.07	< .001
% perseverative. errors	9.63 (2.68)	9.99	14.3 (5.04)	13.93	1, 42	15.46	< .001	1, 40	13.36	< .01
% perseverative. responses	10.88 (4.01)	11.22	15.65 (5.95)	15.30	1, 42	10.01	< .01	1, 40	8.52	< .01
% nonperseverative errors	7.42 (3.60)	7.45	12.25 (3.86)	12.22	1, 42	18.40	< .001	1, 40	11.98	< .01
Failure to maintain set	0.92 (1.06)	0.93	1.20 (1.06)	1.19	1, 42	0.78	*ns*	1, 40	0.52	*ns*
Categories completed	5.96 (0.20)	5.95	5.60 (0.68)	5.61	1, 42	6.03	< .05	1, 40	3.98	= .053
No. of trials administered	90.75 (16.59)	90.79	107.70 (18.13)	107.66	1, 42	10.46	< .01	1, 40	6.26	< .05

Performance on simple addition questions.

Frequency of use of direct retrieval and overt strategies. The total number of arithmetic facts retrieved directly from memory was calculated, as well as the number of problems correctly solved through the use of overt counting procedures. ANOVA with one between subjects factor, group (low versus high ability mathematicians), and one within subjects factor, strategy (retrieval versus overt counting), revealed significant main effects of group, F (1, 42) = 14.13, $p < .01$, and strategy type, F (1, 42) = 101.94, $p < .001$, and a significant interaction between group and strategy type, F (1, 42) = 10.74, $p < .01$. ANCOVA controlling for differences in reading ability and IQ did not remove the significant main effect of group, F (1, 40) = 4.23, $p < .05$. Newman Keuls tests examining the significant interaction group by strategy revealed that high ability mathematicians retrieved more correct answers directly from long-term memory than low ability mathematicians ($p < .01$). There was no significant difference between the two ability groups in the frequency of use of overt counting strategies for reaching correct answers. The total number of correct answers using any strategy was also calculated. ANOVA revealed there to be significant differences between the groups, F (1, 42) = 14.24, $p < .001$, with high ability mathematicians showing higher overall accuracy than low ability mathematicians. ANCOVA controlling for differences in word reading ability and IQ did not remove this significant difference between the groups in overall accuracy, F (1, 40) = 4.23, $p < .05$ (see Table 4.2).

Solution times using direct memory retrieval and overt counting strategies. Mean solution times for both correct retrieval and for problems solved using overt counting strategies were calculated. ANOVA with one between subjects factor, group (low versus high ability mathematicians), and one within subjects factor, strategy (retrieval versus overt counting), revealed significant main effects of group, F (1, 32) = 5.90, $p < .05$, strategy, F (1, 32) = 167.55, $p < .001$, and a significant interaction between group and strategy, F (1, 32) = 5.22, $p < .05$. Note that the degrees of freedom are lower in this analysis because not all children used direct memory retrieval, and hence had no associated time. ANCOVA controlling for differences in reading ability and IQ removed the significant main effect of group, F (1, 30) = 4.11, $p = .052$. Newman Keuls test examining the significant interaction of group by strategy revealed that children in the high ability mathematics group were significantly faster in reaching correct solutions using overt counting strategies than children in the low ability mathematics group ($p < .01$). There were no significant differences between the groups in time taken for memory retrieval of correct answers, although the difference did approach significance ($p = .053$), with high ability mathematicians being faster to retrieve answers. Furthermore, this difference may be an underestimation of the actual difference in speed of retrieval. Children in the high ability mathematics group were retrieving more answers from memory, these being answers to more difficult problems. As discussed in Study 1b, it is a well established finding that answers to more difficult problems take longer to retrieve from memory (the problem size effect, see e.g., Hamann & Ashcraft, 1986; LeFevre, Sadesky, & Bisanz, 1996). Therefore, if the difficulty of the problem for which answers were being retrieved were controlled for, a larger difference in retrieval time between the high and low ability groups would be apparent.

TABLE 4.2.

Performance on simple addition questions and adjusted means controlling for differences in word reading ability and IQ, for low and high ability mathematicians (*SD*).

	High ability		Low ability	
Measure	*M*	Adjusted *M*	*M*	Adjusted *M*
Frequency of direct retrieval	16.87 (9.57)	16.87	2.83 (3.54)	2.83
Frequency of overt counting	33.87 (9.32)	31.94	36.17 (13.32)	38.10
Solution time (direct retrieval)	2.77 (0.48)	2.77	3.19 (0.70)	3.19
Solution time (overt counting)	5.89 (1.48)	6.25	8.39 (3.45)	8.03
Total correct (max = 56)	80.75 (5.05)	48.82	39.00 (14.31)	40.93

Correlational analyses.

Correlation coefficients were computed to discover whether either of the experimental measures were associated with measures of mathematical performance (Group Mathematics test and arithmetic strategy use and solution times). Performance measures from the WCST were entered into principal components analysis to identify any latent variables. This analysis revealed two latent variables, one being a general measure of performance incorporating failure to maintain set, nonperseverative errors, total number correct, and number of trials administered. The second measure included measures of perseveration. Therefore, three experimental measures were entered into the correlation analyses. These were WCST general performance, WCST perseveration, and Corsi Blocks. All scores were converted to standardised scores with a mean of 0 and standard deviation of 1. IQ was found to be significantly correlated with mathematics ability, as was WCST perseveration, and, as would be expected, performance measures from the addition tasks (see Table 4.3). Frequency of direct memory retrieval of arithmetic facts was also significantly correlated with WCST perseveration. There were no significant correlations between any measures of mathematics ability and Corsi block performance or WCST general performance.

Partial correlation coefficients were examined to determine if any of the significantly correlated variables would still be correlated specifically with mathematics ability once differences between the groups in word reading ability and IQ had been controlled for (performance on the BAS Word Reading Test and WISC-R vocabulary). Performance on the Group Mathematics Test was found to be significantly correlated only with the simple addition measures, with the previously significant correlation to WCST perseveration being eliminated. However, amount of perseveration on the Wisconsin Card Sorting Test was found to be significantly correlated with frequency of memory retrieval of arithmetic facts, $pr\ (41) = -.32$, $p < .05$, indicating that a higher level of automatisation of arithmetic facts was correlated with a lower level of perseveration. The frequency of use of direct retrieval of arithmetic facts had the highest correlation with performance on the Group Mathematics Test, $pr = .72$, $p < .001$, suggesting this to be a very important component of general mathematical skill. No significant partial correlations were found to the other measures of arithmetic performance (speed of direct retrieval or overall accuracy).

TABLE 4.3.

Correlation coefficients between measures (below principal diagonal) and partial correlation coefficients controlling for word reading ability and IQ (BAS Word Reading and WISC-R vocabulary), above principal diagonal.

	1	2	3	4	5	6	7	8
1. Group Mathematics Test	--	.72***	*ns*	-.40*	-.42**	*ns*	*ns*	*ns*
2. Frequency of direct retrieval	.77***	--	-.52***	-.37*	*ns*	*ns*	*ns*	-.32*
3. Frequency of overt counting	*ns*	-.42**	--	*ns*	-.39*	*ns*	*ns*	*ns*
4. Solution time for direct retrieval	-.47**	-.43*	*ns*	--	.80***	*ns*	*ns*	*ns*
5. Solution time for overt counting	-.54***	-.42**	-.38*	.79***	--	*ns*	*ns*	*ns*
6. Corsi Blocks	*ns*	*ns*	*ns*	*ns*	*ns*	--	*ns*	-.45**
7. WCST general performance	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	--	.50**
8. WCST perseveration	-.35*	-.39**	*ns*	*ns*	*ns*	-.44**	.50**	--

Note. For correlations, $df = 42$, apart from solution time for direct retrieval where $df = 32$; Partial correlations: $df = 40$, apart from solution time for direct retrieval where $df = 30$.

* $p < .05$, ** $p < .01$, *** $p < .001$

4.3 Discussion.

This study set out to examine the roles of the central executive and visual-spatial sketch pad in children's mathematical skills. The most interesting results came from the WCST, used to measure central executive skill. Children in the lower ability mathematics group made a higher percentage of errors, both perseverative and non-perseverative, and made more perseverative responses (errors and correct responses) even when reading and IQ differences had been controlled for. Shallice (1988) explains perseveration as arising when the control of action is captured at a low level by a single powerful schema that continues to inhibit all other schema. This is due to the fact that the SAS is impaired and unable to interrupt an established schema that has become highly activated. Perseverative errors indicate that the child is failing to change psychological set (in this case, sorting criterion), perseverating on a sorting criteria that was once correct but is subsequently incorrect. Lower ability mathematicians also required more trials to make the required six category changes. Partial correlation coefficients revealed a significant negative correlation between frequency of arithmetic fact retrieval and perseveration, but no significant correlation with overall performance on the Wisconsin Card Sorting Test. This result supports preliminary evidence discussed by Welsh and Pennington (1988) who found that individual differences in children's cognitive performance were significantly correlated with perseveration scores on the WCST. Support is also given to the work of Lehto (1995), who found significant correlations between mathematics performance and measures of central executive performance, even when other short-term memory factors such as digit span and word span were partialled out. There is clearly some mechanism involved in skilled arithmetic performance which depends on mental flexibility and the ability to readily shift psychological set.

These results can be explained in terms of Norman and Shallice's (1980) SAS model of action control, and in terms of Baddeley's (1996) ideas regarding the multi-purpose nature of the central executive. High levels of perseveration by children in the low ability mathematics group show that they found it difficult to switch from habitual responding to one classification system to a new classification system. This explanation fits with the SAS model in that deficient functioning of the central executive would cause problems in trying to override such habitual responding. One method by which this could be examined would be to look for what have been termed 'procedural bugs' in complex addition and subtraction (Ginsburg, 1997; VanLehn, 1990, 1996). This is where the child uses procedures that are correct for some problems, but are incorrectly applied to other problems. For example, the procedure of always taking away the smallest number from the largest in subtraction does not always hold true, for instance, given the problem 14 - 6 =, and particularly when presented in columnar format, many children will apply this rule, subtracting 4 from 6, to achieve an incorrect answer of 12. This may be seen as perseveration to an habitual strategy, with the child failing to monitor their own actions and knowledge.

A lack of mental flexibility and an inability to use feedback to change response behaviour, indicated by high level of perseverative responses, may impact on arithmetic performance because the child may be unable to adapt existing strategies of known procedures to new situations, and they may be reluctant to drop strategies which are useful when the child is first learning arithmetic, but which subsequently become inefficient procedures. Lemaire and Siegler (1995) point out that one measure of improvement in arithmetic is change in strategy use, including acquisition of new strategies and abandonment of old ones. Another measure of arithmetic knowledge is deciding which strategy is best to use in different situations. Observation of the strategies used in this study revealed that some of the children in the low ability mathematics group were using procedures for certain questions which almost inevitably resulted in an incorrect answer being reached, for example, using a counting-all strategy (using fingers) for solving problems that have a sum greater than 10. Even though the children realise the difficulties of using such a strategy when they run out of fingers to count, many still persist in using the counting-all strategy again to see if it will work on a second attempt. One boy was also observed counting imaginary fingers when the sum was greater than 10, which, for the majority of occasions, resulted in under- or over-counting. However, if prompted to use an alternative strategy such as counting-on rather than counting-all, he could frequently reach the correct answer. This shows an inability by the children to regulate their own actions in favour of alternative, more appropriate counting strategies.

Logie et al (1994) suggested that the role of the phonological loop was to keep track of running totals and to maintain accuracy in calculation, whereas the role of the central executive was to select and implement appropriate heuristics when the solution to a problem was not directly available through retrieval from long-term memory. As tasks become more routine and automated, their demands on the central executive are assumed to decrease. As has been shown in the present study and in previous studies investigating children's counting strategies (Geary, 1990; Geary et al, 1992; Geary & Brown, 1991; Geary et al, 1991), children of low mathematics ability do not retrieve as many arithmetic facts from long-term memory, and therefore the use of the central executive may play a distinctive role in their solutions to arithmetic problems, as the central executive is used to implement an appropriate solution heuristic. Teaching children to a level of mastery such that they have automatic retrieval of arithmetic facts may remedy any disadvantage children of lower mathematics ability may have from poor central executive skills. Therefore, the necessity of central executive resources in arithmetic may be very specific to levels of arithmetic development. At early stages where the child is using concrete representations to aid counting, visual-spatial skills may play a greater role in acquiring mathematical knowledge (Geary & Burlingham-Dubree, 1989; Luria, 1966). As children become skilled with arithmetic procedures and facts, more of this arithmetic knowledge will become automated in long-term memory, and the central executive, along with general working memory limitations may have less of a role to play in the solution of arithmetical problems (see e.g., Geary & Widaman, 1992; Little & Widaman, 1995). The central executive may be most relevant to the intermediate stages of arithmetic development, where a choice has to be made between many alternative solution strategies, or where older children are asked to solve more complex mathematical problems, where some parts of the problem require a solution heuristic to be implemented. Studies investigating executive functioning in relation to arithmetic skills at different ages and abilities will provide more substantiating evidence to support this claim.

This finding could also be interpreted in terms of a generalised memory deficit. The central executive is thought to be the controller of working memory, co-ordinating the actions of the articulatory and visual-spatial sketch pad. There is the possibility that the articulatory loop and the visual-spatial sketch pad on their own, function perfectly well, and it is only when this information has to be co-ordinated and integrated that problems arise, a problem which would manifest itself as a problem with the central executive's co-ordinating properties. The central executive is also thought to be involved in the retrieval of information from long-term memory. We know from the results of Studies 1a and 1b that children who are poor at mathematics are slower in retrieving known information from long-term memory, for example, in identifying numbers and letters. This difficulty in retrieving long-term information may impact on the temporal limitations of working memory, again adding to the finding of a generalised memory deficit.

An alternative neurological explanation can be offered for this pattern of results. Welsh and Pennington (1988) note that the prefrontal cortex may be one underlying substrate for a limiting mechanism such as working memory, and a number of studies have shown developmental increases in performance on tests of frontal lobe functioning from the ages of 7 to 10 years. The possibility exists therefore, that some children may be more developmentally advanced in their frontal lobe development than other children, and will hence show better performance on tasks such as the Wisconsin Card Sorting Task, which purports to measure frontal lobe functioning. Of course, this is a very tentative conclusion, and no firm conclusions can be made without neurological assessment. However, Case (1992) found similarities between cycles of EEG coherence and cycles of cognitive growth, and he found that the growth of attentional span depends to some extent on the functioning of the frontal lobes. It may be interesting to examine whether the role of central executive functioning in arithmetic performance differs at various ages. If frontal lobe development is more complete at the ages of 10 to 12 years, then we may find that individual differences in executive functioning are less apparent if the children are tested at an older age, and arithmetic performance is therefore not influenced by possible differences in neurophysiological development.

The results of the present study suggest that the visual spatial sketch pad does not play a major role in simple arithmetic performance, shown by the lack of significant difference between low and high ability mathematicians in performance on the Corsi Blocks task, and the fact that there was no significant correlation between performance on this task with mathematical ability. This does not support the initial hypothesis that visual-spatial skills would play a role in children's mathematical performance. This result does, however, support the finding of Logie et al (1994) who found that disrupting the functioning of the visual-spatial sketch pad had minimal effect on adults arithmetic performance. Those studies involving children which have found a connection between visual-spatial skills and arithmetic performance have used very general measures of visual-spatial skill that do not involve any aspect of memory retrieval, unlike the measure used in this study which had the requirement of serial recall.

An interesting discovery was made when questioning children about the strategies they were using for solving simple arithmetic questions. A number of children in both ability groups reported using an imagery procedure of counting dots on the numbers (6 out of 44 children, 2 in the low ability group, and 4 in the high ability group). For example, when the number 4 was presented, they reported seeing four dots at certain locations on the number. An alternative procedure described was the use of specific spatial arrangements of dots. For example, presented with the number 6, one child reported visualising two rows of three dots, and when presented with the number 9, three rows of three dots. A similar procedure is described by Hayes (1973, p 186, Figure 2), whereby each digit has its own unique set of counting points related to the form of the number being counted. Various methods for visualising numbers are also described by Seron et al (1992), who suggest that one of the common characteristics of such visual number forms is their emergence during infancy with no evident relationship to any number or calculation teaching methods at school. This suggests that imagery may be a successful back up strategy used by children to solve arithmetic problems. The role of the visual-spatial sketch pad in arithmetic may become more pronounced when solution of multi-digit problems or carrying problems is required, as found by Heathcote (1994). Furthermore, the use of a visual-spatial *span* task in this study may have negated any differences between the groups, given that the findings from Study 1b showed no differences on span measures taxing the performance of the articulatory loop. A simple task of visual-spatial skills or visual imagery, for example, mental rotation, that does not have a memory span requirement may prove to be more revealing of any differences associated with mathematical ability.

The case for investigating cognitive mechanisms such as executive functioning and visual-spatial skill, which may be related to cognitive skills such as reading and mathematics, is of great importance, particularly in light of recent findings from Butterworth et al (1996), Bull and Johnston (1997), and the points made in the theoretical review of Hulme and Roodenrys (1995), all of which have failed to find a direct association between short-term memory span and arithmetical skills. This study moved away from the traditional use of verbal memory span tasks in investigating the role of memory in relation to children's cognitive skills, and instead focused on the much less understood areas of central executive and visual-spatial functioning. This was achieved through the use of neuropsychological tasks which have been applied to the study of normally developing children, and by considering both cognitive and neuropsychological explanations for these results. Clearly, an investigation of executive functioning and its role in the developing mathematical skills of children has proved to be fruitful. The WCST can only be regarded as a very general measure of executive functioning, and from this measure, and given the exploratory nature of this study, the only conclusion that can be drawn is that central executive functioning plays some role in mathematical skill. Further research is needed to establish these findings across a wider battery of tests designed to measure the central executive and visual-spatial sketch pad. At present, as pointed out by Parkin (in press), the functions of the central executive are typically defined in terms of performance that cannot be readily attributed to the articulatory loop or the visual-spatial sketch pad. That is, functions of the central executive are defined by default rather than by direct theoretical findings. Given the recently renewed interest in delineating the organisation and function of the central executive (Baddeley, 1996; Lehto, 1996; Phillips, in press), it will become possible to concentrate on finer grained details as to central executive functioning, and use tasks which tax these specific central executive skills. This will help to give a more precise description of the types of executive processes involved in the development of reading and arithmetic skills.

Chapter 5.

Processing Speed and Mathematics Ability: A Specific or General Deficit?

Study 1b showed that children with lower mathematical skills exhibited a general slowness in completing a number of tasks. Such children were slower at identifying numbers and letters, were slower at matching numbers and shapes, and were slower on a perceptual motor task, the peg board. These children were also slower to retrieve known arithmetic facts directly from long-term memory. The study reported in this chapter considers alternative explanations of how differences in processing speed emerge, with the aim of discovering whether children who are poor at mathematics have a specific processing speed deficit just for numerical information, or whether they have a general deficit extending over a range of tasks.

5.1 Theoretical Background.

Two alternative explanations have been put forward to account for individual differences in processing speed. One view emphasises global changes (Pascual-Leone, 1970), whereby all information processing components develop at the same rate. This is suggested to be the outcome of changes in actual processing capacity which allows more processing resources to be allocated to a task at any one time. A supporter of this global mechanism view is Kail who says that children often show a consistent pattern of age related differences in many tasks assessing cognitive development, suggesting that there is a global limiting mechanism, limiting the speed with which children can process any type of information

The alternative view of processing speed emphasises experiences that lead to changes in speed of processing in specific domains. For example, differences in processing speed may indicate that more efficient strategies for task solution are being used, or that knowledge in specified domains becomes more elaborate, leading to more rapid access of information. These changes in knowledge elaboration come about through experience in a specific domain, and make items of information required to deal with a particular problem more accessible, more richly interrelated, and co-ordinated into larger chunks (Anderson, 1988; Chi, 1978; Chi & Ceci, 1988; Logan, 1988; Roth, 1983).

Combinations of these two alternative explanations have also been explicated. Kail (1991a) proposes that initial performance on a task is likely to be based on a globally speeded mechanism, with later performance on processes tailored to a particular task. Whenever execution of a process requires processing resources, processing speed will be limited by the availability of those resources. Greater task experience means that performance is more likely to reflect retrieval of a stored response, meaning that speed is no longer constrained by the available resources. Further to this, Geary (1995; see also Geary & Lin, in press) makes the distinction between biologically-primary and biologically-secondary abilities, suggesting that the usual decline in speed of processing which accompanies the ageing processes may only be for biologically-primary abilities, and not necessarily those which are seen as secondary. All of these theories of changes in processing speed and how they impact on performance of cognitive skills are discussed in more detail in Section 1.4 of Chapter 1.

There appear to be no studies, apart from the attempts in Studies 1a and 1b, which have directly examined the relationship between speed of information processing and mathematics ability. In order to address this issue, Study 3 looked in more detail at the information processing speed deficit found in the first study, and aimed to discover if this finding could be interpreted in terms of a general processing speed limitation, or a slowness in processing speed for a specific domain, in this case, numerical information processing. If this is a general processing speed limitation, then we would expect to see children who are poorer at mathematics exhibiting slower performance on a range of processing speed measures, regardless of the type of information that is to be processed. On the other hand, a specific processing speed deficit for numerical information would mean that children who are poorer at mathematics would only show slower performance on tasks that require some element of numerical processing, that is, they have less task-relevant knowledge pertaining to numerical information, and show less rapid access to that knowledge.

5.2 Study 3.

5.2 (i) Method.

Subjects.

Children taking part in this study were as described in Study 2, reported in chapter 4, with the same split into high and low ability groups. Because of the differences found between the two groups in reading ability and IQ, ANCOVA was also carried out with two covariates, reading ability and IQ (as measured by the BAS Word Reading test and WISC-R vocabulary.) This way, any significant differences that are still apparent between the groups can only be the result of differences in mathematics ability. One way ANCOVA controlling for differences between the high and low ability groups in reading ability and IQ revealed that there were still significant differences between the groups in mathematics ability, F (1, 40) = 49.63, $p < .001$, with adjusted means for the high and low ability groups being 101.53 and 87.71 respectively.

TABLE 5.1.

Performance on mathematics and reading tests, and estimated full scale IQ for high and low ability mathematics group (*SD*).

Group	Mathematics	Reading	Estimated IQ
High (N = 24)	102.54 (5.38)	55.38 (8.86)	10.96 (2.63)
Low (N = 20)	86.70 (5.03)	43.70 (6.65)	9.00 (2.99)
Total (N = 44)	95.34 (9.50)	50.07 (9.80)	10.07 (2.94)

Test Procedures.

Experimental testing began approximately one week after screening tests had been completed, with all children completing the tests in the same order (apart from those where counterbalancing was necessary). Children were tested individually over a total of five sessions each lasting approximately 20 minutes, in a quiet area outside the classroom. Only the additional speed of processing tasks were administered. For those tasks which were as in Study 1b, the results were re-analysed for the smaller sample of 44 children. Each processing speed measure will be described in turn. Unless otherwise described, administration procedures for each task are as described in Studies 1a and 1b.

Motor tasks.

Peg Board. Time taken to move the pegs from one side of the board to the other with the preferred hand was recorded.

Finger Tapping: For this task, the use of a response key was incorporated which was attached to the computer. The child was instructed to tap the response key as fast as possible with their preferred hand. This was first demonstrated by the experimenter. The child was told that when they saw a circle appear on the screen they should start tapping the key as fast as possible and that they should not stop until they see a star appear on the screen. Timing began when the child made the first key press and the star appeared after a four second time period. The computer recorded the number of taps performed in each 4 second period. A total of ten trials were administered, and for the purposes of analysis the mean number of taps per second across all trials was calculated.

'Simple' Processing Speed.

Number and Letter Identification. Speed of correctly identifying numbers and letters was recorded.

Visual Number Matching. Total time to complete thirty items on the visual-number matching task was recorded.

Cross out task. Total time to complete twelve items on the test was recorded.

Simple Reaction Time: In this task, the child was initially shown a letter and asked to identify it. They were then instructed that they were going to see lots of letters on the computer screen, and that every time they saw the stimulus letter, they had to press the response key as fast as possible. One letter was shown on each trial. A total of 50 items were shown, 10 of which were stimulus items which the child had to respond to. The mean reaction time to the stimulus letter was recorded. An identical procedure was carried out with numbers. Administration of these tasks was counterbalanced to overcome practice and order effects.

WISC-R Coding: This test involves the timed substitution of number codes for a list of abstract symbols and is normally taken to be an indicator of processing speed. This task has also been described as a 'practical' skill, with contributions from speed, visual and motor co-ordination, along with sustained effort. The test was carried out in accordance with procedures stated in the WISC-R manual. The child was given the test sheet and was asked to look at the key, which consisted of shapes such as a star, a triangle, and a circle. It was made clear to the child that each shape contained different things, for example, one of the shapes contains a line that goes up and down, another two lines that go across, and so on. The child's task was to then look at the test shapes, compare them to the key, and draw in the correct number and orientation of lines, as shown in the key. The child was instructed to work as quickly as possible without making mistakes. At the end of two minutes the child was told to stop, and the total number of correct responses was recorded.

If the child finished before the time limit the time was recorded, again the number of correct responses was calculated. If the child had perfect performance within the time limit, they were awarded bonus points in accordance with the standard procedure.

'Complex Processing Speed.'

Choice Reaction Time: The child was told that they would see either a triangle or a circle on the computer screen. Two response keys were attached to the computer, one with a picture of a triangle on it, and the other with a picture of a circle. The child was instructed that they had to press the key that was the same as the shape that was on the computer screen, and that they had to try and make this choice as quickly as possible. A '+' sign appeared on the screen to indicate to this child that they could proceed to the next trial. The experimenter pressed a key to present the stimulus on the computer screen (either a circle or a triangle), with timing starting with the onset of the presentation. The child had to press the key which corresponded to the shape being shown. Timing was terminated when the child pressed the key. The experimenter recorded the accuracy of the choice response, and for the purposes of analysis, only correct choice response times were included.

Counting strategies in simple addition.

This followed the same procedure as reported in Study 1a. For the purposes of analysis, the number of correct answers retrieved directly from long-term memory was calculated, as well as the frequency of using overt counting strategies (counting using fingers or verbal counting, including decomposition). The time taken to retrieve the answers directly from memory and the time taken to reach correct solutions using overt counting procedures was also calculated.

5.2 (ii) Results.

Results from the between groups analyses will be briefly summarised. All reported ANOVA's are one way comparing the low and high ability groups. All reported ANCOVA's have two covariates, reading ability and IQ, and again are one way, comparing low and high ability mathematics groups. Means, adjusted means, and significance of ANOVA and ANCOVA results are reported in Table 5.2.

ANOVA's of the motor speed tasks revealed there to be a significant difference between the groups in speed of finger tapping, with the children in the high ability mathematics group tapping more times per second than the children of lower mathematical ability. There was no significant difference in performance on the peg board task.

The next tasks considered were those classified as measures of simple processing speed. Children in the high ability mathematics group were found to be significantly faster at identifying both numbers and letters. The higher ability mathematics group were also found to complete the cross-out task (matching geometric shapes), and the visual number matching task faster than children in the low ability mathematics group. These children also achieved a significantly higher standard score on the WISC-R Coding, indicating that children in the high mathematics group were completing more of the task in the time limit than children of lower mathematical ability. There were no significant differences between the groups in simple reaction times to numbers or letters.

Finally, analysis of the more complex measure of processing speed, choice reaction time, revealed there to be no significant differences between the two ability mathematics groups on this task.

However, controlling for differences between the groups in reading ability and IQ changed these results dramatically. ANCOVA revealed there only to be a significant difference between the groups on the visual number matching task, F (1, 40) = 4.11, $p < .05$, with children in the high ability mathematics group being faster to complete the task. The only other difference to be approaching significance is speed of number identification, F (1, 40) = 4.06, $p = .051$, with the children in the high ability group identifying numbers faster than children in the low ability mathematics group.

Observation of the adjusted mean times shows that for all tasks, with the exception of choice reaction time, children in the high ability mathematics group were faster than children in the low ability mathematics group. Therefore, despite the fact that most of these differences were not reaching significance, there is a general tendency for higher ability mathematicians to show faster processing for various types of information.

TABLE 5.2

Means and adjusted means for ANOVA and ANCOVA (controlling for reading ability and IQ), for low and high ability mathematics groups (*SD*).

	Low ability		High ability		ANOVA			ANCOVA		
Measure	*M*	Adjusted *M*	*M*	Adjusted *M*	*df*	*F*	*p*	*df*	*F*	*p*
MOTOR SPEED										
Finger Tapping[a]	3.75 (0.40)	3.85	4.03 (0.44)	3.93	1, 42	4.95	< .05	1, 40	.30	*ns*
Peg Board[b]	9.51 (0.83)	9.46	9.05 (0.87)	9.09	1, 42	3.18	*ns*	1, 40	1.26	*ns*
SIMPLE PROCESSING SPEED										
Coding[c]	9.65 (2.11)	9.87	11.29 (2.07)	11.08	1, 42	6.73	< .05	1, 40	2.20	*ns*
Cross-out[b]	182.11 (38.83)	174.77	153.61 (39.03)	160.95	1, 42	5.84	< .05	1, 40	0.89	*ns*
Visual matching[b]	220.67 (38.44)	215.29	180.78 (36.26)	186.17	1, 42	12.50	< .01	1, 40	4.11	< .05
Letter ID (ms)	1096 (200)	1058	935 (170)	973	1, 42	8.28	< .01	1, 40	1.83	*ns*
Number ID (ms)	888 (160)	859	725 (120)	754	1, 42	15.19	< .001	1, 40	4.06	= .051
RT letters (ms)	680 (100)	675	660 (100)	664	1, 42	0.40	*ns*	1, 40	.08	*ns*
RT numbers (ms)	721 (140)	715	641 (90)	647	1, 42	5.77	< .05	1, 40	2.64	*ns*
COMPLEX PROCESSING SPEED										
Choice RT (ms)	865 (140)	853	858 (170)	870	1, 42	0.02	*ns*	1, 40	.07	*ns*

Note. [a] taps per second; [b] total time (secs); [c] standardised score.

Correlational analyses.

a) Correlations to general mathematics ability (Group Mathematics Test).

Initially, each independent measure of processing speed was entered into the correlation analysis, along with mathematics and reading ability. This revealed many of the variables to be significantly correlated with mathematics and reading, with the exception of peg board, simple reaction times to numbers and letters, and choice reaction time to shapes. The correlations to mathematics and reading ability are reported in Table 5.3. As can be seen, correlations between the measures of processing speed to mathematics and reading proved to be very similar in the majority of cases.

The second correlational analysis to be carried out used IQ as a covariate. Controlling for differences in IQ reduced the number of variables that were still significantly correlating with mathematics and reading ability. Measures found still to be significantly correlating with mathematics, after controlling for IQ, included Coding, cross-out task, visual number matching, and speed of number identification. Fewer measures were found to be correlating with reading ability, these being cross-out task, visual number matching, and speed of number identification. The correlation between tapping rate and reading ability approached significance, pr (41) = .30, p = .051.

Finally, specific correlations to mathematics ability were examined by controlling for IQ and reading ability. This revealed only two variables which remained significantly correlated with mathematics ability, these being WISC-R Coding, pr (40) = .33, $p < .05$, and visual number matching, pr (40) = -.33, $p < .05$. A similar analysis to investigate correlations specific to reading ability was also conducted, controlling for IQ and mathematics ability. None of the measures of processing speed were found to be significantly correlated with reading ability once IQ and mathematics ability had been partialled out of the correlation.

TABLE 5.3.

Correlation and partial correlation coefficients (pr^1 controlling for IQ, pr^2 controlling for IQ and reading ability, pr^3 controlling for IQ and mathematics ability), for each measure of processing speed in relation to mathematics and reading ability.

	Mathematics ability			Reading ability		
	r	pr^1	pr^2	r	pr^1	pr^3
Finger Tapping (taps per second)	.30*	.23	.06	.39**	.30	.21
Peg board (total time)	-.23	-.21	-.14	-.21	-.18	-.07
Coding Task (scaled score)	.45**	.42**	.33*	.32*	.27	.03
Cross-out Task (total time)	-.37*	-.36*	-.16	-.42**	-.42**	-.27
Visual Number Matching (total time)	-.49**	-.45**	-.33*	-.40**	-.34*	-.10
Letter Identification (mean time, ms)	-.38*	-.28	-.21	-.35*	-.20	-.04
Number Identification (mean time, ms)	-.47**	-.42**	-.25	-.47**	-.40**	-.21
Letter RT (mean time, ms)	-.06	-.01	-.01	-.06	-.01	.00
Number RT (mean time, ms)	-.27	-.18	-.17	-.21	-.08	.03
Choice RT (mean time, ms)	-.04	-.03	.02	-.09	-.08	-.08

Note. For correlations, $df = 42$, for pr^1, $df = 41$, for pr^2 and pr^3, $df = 40$.

* $p < .05$, ** $p < .01$.

b) Correlations to simple arithmetic ability.

Correlational analyses were also conducted on the data from the simple arithmetic problems. Measures taken were frequency of using overt counting strategies and direct memory retrieval (for correct answers only), retrieval times when using both strategies, and overall accuracy. As for the Group Mathematics test, correlation coefficients were compared to partial correlation coefficients, after controlling for reading ability and IQ. These results are shown in Table 5.4. None of the processing speed measures were significantly correlated to frequency of using overt counting strategies. The pattern of results was slightly different for frequency of direct retrieval, with the Coding task, visual-number matching, and speed of number identification all significantly correlating with retrieval frequency. These correlations reveal that faster or better performance on these tasks is related to a higher frequency of direct arithmetic fact retrieval. However, partial correlation coefficients controlling for reading ability and IQ eliminated these significant correlations.

Time taken to correctly solve problems when using overt counting strategies was significantly correlated with finger tapping, Coding, cross-out task, visual-number matching, and speed of number identification. However, partial correlation coefficients removed most of these significant correlations, to leave just performance on the Coding task to be correlating with solution time, $pr\ (40) = -.31, p < .05$. A similar outcome was also found for solution times when the problems were answered by direct memory retrieval of an arithmetic fact, $pr\ (30) = -.50, p < .01$. These results indicate that better performance on the Coding task is related to faster solution times for using both overt counting strategies and direct retrieval of arithmetic facts. Overall accuracy was not significantly correlated with any of the measures of processing speed, once reading ability and IQ had been controlled for.

Summary.

Numerous measures of processing speed were found to be significantly correlated with mathematics ability and measures of simple arithmetic ability, with the strongest correlations being to the Coding task, visual-number matching, and speed of identifying numbers. Controlling for differences in reading ability and IQ revealed that only the Coding Task remained significantly correlated with both mathematics and simple arithmetic measures, with the visual-number matching task remaining significantly correlated only to mathematics ability. Examining the correlations of processing speed to reading ability again revealed a wide range of significant correlations, although once differences in mathematics ability and IQ had been taken into account, none of the correlations remained significant.

TABLE 5.4.

Correlation and partial correlation coefficients between simple addition performance and measures of processing speed.

	Counting frequency		Retrieval frequency		Counting time		Retrieval time		Overall accuracy	
	r	*pr*	*r*	*pr*	*r*	*pr*	*r*	*pr*	*r*	*pr*
Finger tapping (taps per second)	.16	.16	.21	.03	-.31*	-.16	-.03	.06	.33*	.20
Peg Board (total time)	.13	.17	-.24	-.17	.12	.04	.08	.03	-.07	.03
Coding task (scaled score)	-.01	-.05	.33*	.23	-.40**	-.31*	-.53**	-.50**	.28	.15
Cross-out task (total time)	-.15	-.08	-.23	-.08	.36*	.25	.32	.22	-.34*	-.16
Visual Number Matching (total time)	.03	.07	-.39**	-.25	.42**	.30	.39*	.34	-.31*	-.15
Letter identification (mean time)	.17	.17	-.25	-.08	.19	.17	.13	.10	-.05	.12
Number identification (mean time)	.06	.12	-.36*	-.19	.36*	.12	.25	.16	-.26	-.04
Letter RT (mean time)	.24	.23	-.23	-.20	-.06	.23	-.05	-.06	.04	.07
Number RT (mean time)	.03	.01	-.29	-.20	.12	.01	-.15	-.19	-.22	-.18
Choice RT (mean time)	.19	.21	-.00	.04	-.07	-.11	.21	.20	.18	.26

Note. For correlations, $df = 42$, and for partial correlations, $df = 40$, with the exception of correlations to retrieval time, where $df = 32$ for correlations, and $df = 30$ for partial correlation coefficients.

* $p < .05$, ** $p < .01$.

5.3 Exploring the interrelations between processing speed, working memory, and mathematical and arithmetical skills.

Studies 1, 2, and 3 have identified working memory mechanisms and measures of processing speed that may have a role to play in the development of children's mathematical skills. This final section of chapter 5 aims to draw together the findings of these studies for the 44 children who took part in all three studies, in order to gain some insight as to the relative importance of working memory mechanisms and processing speed in relation to mathematical and simple arithmetic skills. This is achieved through a series of partial correlational analyses controlling for working memory or processing speed measures to see how they impact on correlations to mathematics and arithmetical skills.

A number of composite variables were formed. These were measures associated with the functioning of the articulatory loop, short-term memory span (digit span, one-syllable word span, and counting span), and articulation rate (one and three-syllable speech rate, and counting rate). Measures of central executive functioning and visual-spatial functioning from Study 2 were also included, these being WCST overall performance, WCST amount of perseveration, and Corsi Blocks. Finally, a composite measure of general processing speed was also formed from the results of Study 3. All scores of the processing speed measures were converted to standardised scores with a mean of 0 and a standard deviation of 1. Z scores for the Coding task and for finger tapping were reversed such that a negative score on these tasks indicates better performance, and hence has the same directionality as the other measures of processing speed. Scores from the finger tapping task, Coding task, Cross-out task, visual-number matching, and letter and number identification were added together, and a mean Z score was calculated.

Correlation coefficients between these composite variables to mathematics and arithmetic ability are shown in Table 5.5. This table also reports partial correlation coefficients between all measures after controlling for reading ability and IQ. Frequency of using overt counting strategies was not significantly correlated with any of the working memory or processing speed composite variables and so this variable was excluded from all further analyses. In line with the results reported in the previous studies, mathematics ability was found to be significantly correlated with short-term memory span, articulation rate, WCST amount of perseveration, and processing speed. Processing speed is the only measure to remain significantly correlated to mathematics ability after controlling for reading ability. However, the measures from simple arithmetic performance reveal a slightly different story. Frequency of retrieving arithmetic facts directly from long-term memory was found to be significantly correlated with short-term memory span, articulation rate, WCST perseveration, and processing speed. However, only measures of memory span, articulation rate, and WCST perseveration remained significantly correlated with retrieval frequency after controlling for reading ability and IQ, indicating that there may be a general working memory deficit for children who are poor at mathematics, when they are required to solve simple arithmetic problems. Short-term memory span is also found to be significantly partially correlated with overall arithmetic accuracy.

These correlation analyses were then taken one stage further. Three separate analyses were conducted, adding an additional covariate to reading ability and IQ. In the first of these analyses, processing speed was added as the third covariate to see if controlling for processing speed would reduce any of the correlations to non-significant levels (see Table 5.6). Short-term memory span and WCST perseveration were still significantly correlated with frequency of direct memory retrieval of arithmetic facts, but the significant correlation between short-term memory span and overall arithmetic accuracy was removed. Secondly, short-term memory span was added as the third covariate in the correlation analysis (see Table 5.7). Controlling for short-term memory span did not remove the significant correlation between processing speed and performance on the Group Mathematics test, or the significant correlation between WCST perseveration frequency of direct memory retrieval. Finally, WCST perseveration was entered as a third covariate in the analysis (see Table 5.8). Again, this failed to account for the significant correlation between short-term memory span and retrieval frequency, and for the significant correlation between processing speed and general mathematics ability.

TABLE 5.5

Correlation coefficients between composite measures and mathematics and arithmetical ability (below principle diagonal), and partial correlation coefficients controlling for reading ability and IQ (above principle diagonal).

	1	2	3	4	5	6	7	8	9	10	11	12
1. Group Mathematics test	—	*ns*	.72***	-.42**	-.40*	.47**	*ns*	*ns*	*ns*	*ns*	*ns*	-.32*
2. Counting frequency	*ns*	—	-.52***	-.39*	*ns*	.62***	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*
3. Retrieval frequency	.77***	-.41**	—	*ns*	-.37*	.34*	.35*	.31*	*ns*	*ns*	-.35*	*ns*
4. Counting time	-.54***	-.38*	-.42**	—	.80***	-.71***	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*
5. Retrieval time	-.47**	*ns*	-.43*	.79***	—	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*
6. Overall accuracy	.62***	.61**	.47**	-.73***	-.35*	—	.31*	*ns*	*ns*	*ns*	*ns*	*ns*
7. STM span	.36*	*ns*	.46**	-.31*	*ns*	.40**	—	*ns*	*ns*	*ns*	*ns*	*ns*
8. Articulation rate	.40**	*ns*	.39**	*ns*	*ns*	.30*	.30*	—	*ns*	*ns*	*ns*	*ns*
9. Corsi Blocks	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	—	*ns*	-.45**	-.37*
10. WCST	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	—	.50**	*ns*
11. WCST perseveration	-.35*	*ns*	-.39*	*ns*	*ns*	*ns*	*ns*	-.32*	-.44**	.50**	—	*ns*
12. Processing speed	-.54***	*ns*	-.41**	.44**	.35*	-.33*	-.32*	-.38*	*ns*	*ns*	*ns*	—

Note. For correlations, $df = 42$, with the exception of correlations to retrieval time, where $df = 32$. For partial correlation, $df = 40$, with the exception of correlations to retrieval time, where $df = 30$.

* $p < .05$, ** $p < .01$, *** $p < .001$.

TABLE 5.6.

Partial correlation coefficients showing correlations of mathematics and arithmetic ability to experimental measures, after controlling for reading ability, IQ, and processing speed.

	1	2	3	4	5	6	7	8	9	10
1. Group Mathematics test	—									
2. Retrieval frequency	.70***	—								
3. Counting time	-.36*	*ns*	—							
4. Retrieval time	*ns*	*ns*	.78***	—						
5. Overall accuracy	.45**	.33*	-.71***	*ns*	—					
6. STM span	*ns*	.34*	*ns*	*ns*	*ns*	—				
7. Articulation rate	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	—			
8. Corsi Blocks	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	—		
9. WCST	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	—	
10. WCST perseveration	*ns*	-.33*	*ns*	*ns*	*ns*	*ns*	*ns*	-.43**	.52***	—

Note. $df = 39$ in all cases, with the exception of correlations to retrieval time, where $df = 29$.

* $p < .05$, ** $p < .01$, *** $p < .001$.

TABLE 5.7.

Partial correlation coefficients showing correlations of mathematics and arithmetic ability to experimental measures, after controlling for reading ability, IQ, and short-term memory span..

	1	2	3	4	5	6	7	8	9	10
1. Group Mathematics test	—									
2. Retrieval frequency	.71***	—								
3. Counting time	-.40*	ns	—							
4. Retrieval time	-.41*	-.40*	.81***	—						
5. Overall accuracy	.44**	*ns*	-.70***	*ns*	—					
6. Processing speed	-.31*	*ns*	*ns*	*ns*	*ns*	—				
7. Articulation rate	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	—			
8. Corsi Blocks	*ns*	*ns*	*ns*	*ns*	*ns*	-.36*	*ns*	—		
9. WCST	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	—	
10. WCST perseveration	*ns*	-.32*	*ns*	*ns*	*ns*	*ns*	*ns*	-.43**	.49***	—

Note. $df = 39$ in all cases, with the exception of correlations to retrieval time, where $df = 29$.

* $p < .05$, ** $p < .01$, *** $p < .001$.

TABLE 5.8.

Partial correlation coefficients showing correlations of mathematics and arithmetic ability to experimental measures, after controlling for reading ability, IQ, and WCST amount of perseveration (central executive performance).

	1	2	3	4	5	6	7	8	9	10
1. Group Mathematics test	—									
2. Retrieval frequency	.69***	—								
3. Counting time	-.47**	-.38*	—							
4. Retrieval time	-.41*	-.38*	.82***	—						
5. Overall accuracy	.48**	.36*	-.72***	*ns*	—					
6. Processing speed	-.31*	*ns*	*ns*	*ns*	*ns*	—				
7. Articulation rate	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	—			
8. Corsi Blocks	*ns*	*ns*	*ns*	*ns*	*ns*	-.35*	*ns*	—		
9. WCST	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*	—	
10. STM span	*ns*	.32**	*ns*	*ns*	.31*	*ns*	*ns*	*ns*	*ns*	—

Note. $df = 39$ in all cases, with the exception of correlations to retrieval time, where $df = 29$.

* $p < .05$, ** $p < .01$, *** $p < .001$.

5.4 Discussion.

Analyses of between group differences found that children in the low ability mathematics group performed significantly more slowly on a wide range of tasks involving the processing of different forms of information. However, once differences between the groups in reading ability and IQ had been controlled for, the only differences between the groups were on measures that had some element of numerical processing, these being the visual number matching task, and speed of identifying numbers. Clearly, there is a connection between these two tasks, in that children who are slower to identify numbers will be slower to match them. So, what there appears to be is a general slowness of processing information which is related to general academic performance and IQ, with this slowness for poor mathematicians being even more pronounced for the processing of numerical information. This pattern of results could be explained in terms of Kail's attempt to draw together the two contrasting theories of global and specific differences in processing speed. Kail (1991a) suggested that initial performance on a task is likely to reflect a globally speeded mechanism, and with more experience and familiarity, resources become tailored specifically to a particular task. Speed of processing will then be limited by the particular resources. Greater task experience means that performance is more likely to reflect retrieval of a stored response, such that speed of processing information is no longer limited by the available resources. Results show that children in the high ability mathematics group retrieve more arithmetic knowledge directly from memory, and therefore, their performance in mathematics will be less restricted by the available resources, and will hence show faster processing, particularly of numerical information. Children in the low ability mathematics group show less reliance on retrieving numerical information from long-term memory, therefore meaning that their speed of processing numerical information will be slowed because of the extra demands being made on the available resources. This difference may come about simply because children who are poor at mathematics may have had less experience and are less familiar with numerical information. However, the underlying slowness in general speed of information processing may have impacted on the child's ability to learn basic numbers and arithmetic operations in the first instance, meaning that they are delayed in automating information in memory.

Examination of simple arithmetic skills revealed that, after controlling for differences in reading ability and IQ, the only significant correlations were with performance on the Coding task, which was significantly negatively correlated with solution times using both overt counting strategies and direct memory retrieval, that is, faster solution times were correlated with better performance on the Coding task. Reference to studies of the development of automaticity suggest that automaticity, that is, rapid, effortless processing, is a necessary component of skill development, and that higher-level aspects of skill will not be acquired until lower level aspects have become automated. In the case of mathematics, the lower-level processing would be the recognition of numbers, and the retrieval of basic numerical facts and procedures. Therefore, what the results from this study and studies 1a and 1b show are that performance on basic number tasks is dependent on general speed of processing, shown by slow retrieval of known representations from memory, and by the significant correlation in this study between the Coding task and performance on the simple addition problems. On the other hand, performance on general, higher-level mathematical knowledge is also affected by this general slowness, but is even more affected by the more specific slowness associated with the lower-level attributes of mathematical knowledge relating to simple arithmetic and numerical processing. Therefore, the general or specific nature of the processing speed deficit will be related to the type of outcome measure used, that is, whether general mathematics ability or simple arithmetic skills are the focus of attention.

Drawing together the results of the first three studies and examining the correlations to mathematics and arithmetical skills has revealed a very striking pattern of results. Mathematics ability is found to be significantly correlated to processing speed after controlling for reading ability and IQ, and remains significantly correlated after further controlling for short-term memory span and central executive performance. Processing speed is not significantly correlated with any of the measures of simple arithmetic performance after controlling for reading ability and IQ. On the other hand, and in complete contrast, short-term memory span and executive functioning are not significantly correlated with performance on the Group Mathematics test after controlling for reading ability and IQ, but are significantly correlated to measures of simple arithmetic ability, most notably, the frequency of retrieving answers to arithmetic problems directly from memory. This remains the case even after controlling for processing speed. This indicates that the measure used for assessing mathematics ability reveals different cognitive performance deficits, presumably indicative of the different types of processing that need to take place on different tasks. Indeed, if we think of the distinction between tests of word recognition and reading comprehension, this helps in the understanding of how different these measures of mathematical performance are. Performance on the Group Mathematics test is analogous to the more complex reading comprehension test. It requires the solving of single-digit arithmetic problems and more complex multi-digit addition and subtraction problems. The test also has a requirement to answer problems read by the tester in relation to a picture shown on the test sheet. Therefore, this test requires a higher degree of conceptual understanding. Performance on simple arithmetic problems, on the other hand, is the equivalent of a simpler, more pure measure of word reading ability. It requires the application of procedural knowledge rather than conceptual knowledge, which may well be affected by general working memory problems. If the child does have a working memory deficit, then there may be problems in keeping the associated answers to arithmetic problems in short-term memory long enough to be transferred to a long-term memory representation. If the strength of the representation is particularly weak, this will also lead to problems of retrieving items from long-term memory.

This synthesis of the results may provide a neurological explanation for the central executive and processing speed deficits identified. As pointed out in Chapter 4, one of the main developmental phases of frontal lobe development is from the ages of 7 to 10 years old. Developmental changes in processing speed may have a maturational basis, such as the increasing myelinization of the integrative systems of the brain (Bjorklund & Harnishfeger, 1990; Eaton & Ritchot, 1995). Myelinization allows for the more rapid transmission of neural information, "thus enhancing the ability to hold a number of concepts in mind simultaneously, to integrate non-specific or abstract concepts from a variety of modalities, and to engage in complex abstract thought" (Hallett & Proctor, 1996, p. 11). We also know that performance on central executive tasks is linked to development of the frontal lobes, and therefore, it is not unreasonable to suggest that processing speed, central executive performance, and other cognitive deficits would have similar biological aetiologies.

It is also possible to draw a theoretical comparison between the central executive and processing speed deficits. Skilled performance, as in skilled mathematical performance, is dependent on rapid, automatic processing to some extent, but as pointed out by Logan (1985), skilled performance also involves other forms of knowledge, such as knowledge about one's own capabilities and strategic options (see also Garafalo & Lester, 1985). This metacognitive knowledge allows the more skilled performer to make better use of their automatic procedures. Therefore, the finding of a central executive deficit and a processing speed deficit may provide a justifiable account of children's mathematical difficulties, in that skilled performance in mathematics requires the ability to carry out basic numerical procedures rapidly, and preferably automatically, and the child should be able to think strategically about the problem they are required to solve, and to be flexible in their adaptation of strategies for solution.

These results may also provide some indication as to the structure of working memory. At present, working memory is seen as a multi-component structure, with the central executive co-ordinating the actions of the articulatory loop and the visual-spatial sketch pad. The results from this analysis revealed that the central executive and articulatory loop were each contributing independently to mathematical skills, with short-term memory span not being able to account for the correlation of mathematics ability to central executive processing, and vice versa. This suggests that the central executive and the articulatory loop may indeed be reasonably independent of each other, showing different functional characteristics. However, performance on the Corsi Blocks, used to test the functioning of the visual-spatial sketch pad, also reveals an interesting pattern of results. Performance on this task was not correlated with short-term memory span, but was significantly correlated with central executive performance. This remained so after controlling for processing speed, and short-term memory span. Furthermore, these partial correlations are fairly strong (p's $< .01$), indicating that it is probably not just a chance occurrence. Therefore, the functioning of the central executive and visual-spatial sketch pad may not be as distinct as portrayed by the working memory model. This connection has been alluded to in previous studies although there is no direct evidence reported showing this to be the case.

To conclude, these results appear to correspond most closely to a theory of processing speed that combines both a general and specific explanation. Early development of basic number skills is influenced by a general speed of processing factor, such that children who are slower to process information use more effortful processes for solving arithmetic problems. Combining the results from the first three studies also reveals that simple arithmetic performance is related to general memory difficulties involving both the articulatory loop and the central executive. When the child is then required to use this basic arithmetic knowledge in higher-level mathematical problem solving, the child is restricted not only by the underlying processing speed deficit, but also by the more specific slowness of processing numerical information.

Chapter 6.
Organisation and Retrieval of Arithmetic Facts from Long-Term Memory.

The final cognitive deficit to be examined in detail is a very pervasive one, and has proved to be a very robust finding in virtually all studies investigating children's arithmetical skills, including all experimental studies reported here. Performance characteristics of children with poor arithmetical skills include the infrequent use of direct memory retrieval of arithmetic facts, and slow retrieval times when direct retrieval is used. One explanation of this may be that arithmetic facts in children of low arithmetical ability are poorly or abnormally organised in long-term memory (e.g., Geary & Brown, 1991). The study reported in this chapter aimed to investigate the development of memory retrieval over the course of one school year. Part of the study also used a priming procedure to probe possible differences in the organisation of arithmetic facts in long-term memory. This will give some indication as to whether the performance characteristics of children who are poor at mathematics represent a simple developmental delay, or a more substantive developmental difference.

6.1 Theoretical Background.

The development of children's arithmetic skills is characterised by an early reliance on strategies involving counting objects which act as physical representations of the numbers. There is a gradual transition through progressively more efficient strategies such as verbal counting and decomposition, until the eventual outcome of automatisation of arithmetic facts, whereby answers to arithmetic problems can be retrieved directly from long-term memory. This is achieved through exposure with, and strengthened familiarity to, arithmetical procedures and facts. This is thought to be the most efficient strategy for solving arithmetic problems, as it requires minimal involvement from limited cognitive resources.

Studies investigating learning difficulties in arithmetic show that children who are poor at arithmetic show performance attributes that would be characteristic of younger children. They will frequently use less efficient counting strategies such as counting on fingers, resulting in long solution times, frequent errors, and difficulties in establishing long-term representations of arithmetic facts in memory. With a slow counting speed, the representations of the numbers are more likely to decay or be forgotten before the count is completed. Where errors occur in answering arithmetic questions, incorrect associations may be formed in long-term memory, meaning that future attempts to solve the problem may lead to the incorrect association being retrieved from long-term memory.

The model most frequently referred to when discussing children's use of counting strategies is the Distribution of Associations Model (Siegler & Shrager, 1984), an example of a nontabular model. According to nontabular models, number facts are stored in long-term memory in networks that resemble those for semantic facts. In such models, solution times for arithmetic problems are typically viewed as being a function of the associative strength between the integers comprising a problem and their correct sum, as well as competing associations among incorrect sums and related integers. The stronger the association between the problem and its correct sum, the greater is its activation strength and the quicker the solution is retrieved. In the distribution of associations model, memory representation of arithmetic facts contain both correct and incorrect answers. Associations between problems and answers are formed each time the child encounters an arithmetical problem, regardless of the correctness of the answer, so mistaken associations may be formed. Strategies differ in the probability of producing the correct answer, in the duration of the problem solving process, and in the demands placed on limited working memory resources (Siegler, 1987). The specific strategy chosen is governed by the peakedness of the distribution of associations between the problem and all possible answers. Siegler (1988a, 1988b) reports that when associative strength is concentrated on a single answer, then the distribution will be peaked. Alternatively, if the associative strength is distributed among many potential answers then the distribution will be flat. This peakedness of distribution determines when direct memory retrieval will be used. However, the choice of strategy is further governed by a confidence criterion, which gauges confidence as to the correctness of the retrieved answer, the rigour of which varies from child to child (Siegler, 1988a). For example, because problems with two large value integers (e.g., 7 + 9), tend to be presented less frequently than problems with smaller value integers (Hamann & Ashcraft, 1986, Siegler & Shrager, 1984), they probably have a flat distribution, and therefore, the associated strength of the retrieved answer is not likely to meet the confidence criterion. In this case, the child would resort to using a backup strategy such as counting with the aid of concrete representations.

Models of arithmetic fact organisation in long-term memory show a number of parallels to the organisation of semantic and lexical knowledge in long-term memory. With tabular models, activation time reflects the distance or area in memory that is searched to retrieve a problem's sum. With nontabular models, activation reflects the associative strengths between integers and their correct sum as well as competing associations among incorrect sums and related integers, due to errors made during acquisition. Furthermore, the developmental progression of arithmetical and reading skills show many similarities. Therefore, one way to examine arithmetic fact retrieval would be to draw on studies of the development of reading skills and the retrieval of lexical and semantic information from long-term memory, and apply the methods used in these studies to investigations of arithmetic fact retrieval. One example of such a method is priming, which has proved to be an extremely useful tool for the examination of cognitive functioning, and has often been used to investigate the structure and organisation of lexical and semantic information in both normal and brain-damaged readers, (e.g., Besner, Smith, & MacLeod, 1990). The usual method used with priming is to show a word preceding a target word which is either semantically related (e.g., *table - chair*) or unrelated (e.g., *cow - chair*) to the target word, the result being faster identification and greater accuracy of the target word when it is preceded by a semantically related word. Priming effects have been explained by the concept of spreading activation (Anderson, 1976, 1983; Collins & Loftus, 1975; see McNamara, 1994 for a review), similar to the notion of spreading activation applied to models of long-term storage of number bonds described earlier. The fundamental assumption underlying spreading activation is that an item is retrieved by activating its internal representation, with this activation then spreading to associated concepts which facilitates their retrieval. Whilst such a priming technique has been successfully applied to the study of associative and semantic memory, there have been few studies which have used a number prime to assess the impact on arithmetic fact retrieval.

It can be seen from studies investigating children's arithmetic that the ability to retrieve numerical information directly from long-term memory plays a major role in skilled mathematical performance, and is undoubtedly necessary for the solving of more complex mathematical problems. Studies reported in previous chapters have shown that the frequency of retrieving arithmetic facts from memory has a very strong correlation to general mathematics ability. Performance on simple addition questions provides invaluable information as to strategy development and speed of information processing, and provides detailed information as to the underlying component skills associated with mathematics. A focus on very basic tasks, such as mental addition rather than problems in word form, enables a more precise assessment of the fundamental cognitive deficits of children with mathematical difficulties. It is also hoped that results from the study will provide insights as to the organisation of arithmetic facts in long-term memory, achieved through the use of an arithmetic priming procedure and by examining the development of direct retrieval of arithmetic facts. If it is the case that children who are poor at arithmetic have an abnormally organised representation of arithmetic facts in long-term memory, then we should find that they would be less susceptible to a prime, showing no improvement in either accuracy or speed of responding compared to non-primed arithmetic questions.

6.2 Studies 4a and 4b.

Subjects.

Again, a subset of those children reported in Study 1b took part in this study. Forty-four Primary 3 children from one school in Dundee took part in the first stage of the study. Three children left the school before the second part of the study took place at the end of the school year, leaving a total of 41 children who completed both stages of the study. Only data from these 41 children will be used. At the first stage of testing (October) children were screened for mathematics and reading ability using the Group Mathematics Test (Young, 1970), and the BAS Word Reading Test (Elliott et al, 1977). Intelligence was also estimated using the vocabulary subtest from the Wechsler Intelligence Scale for Children (WISC-R). The Group Mathematics Test and BAS Word Reading were administered again at the beginning of the second stage of the study (May) to examine developmental changes in performance. Mean chronological age, mathematics and reading standard age scores, and mean estimated IQ are shown in Table 6.1 (for the Group Mathematics test, mean = 100, SD = 15, for BAS Word Reading, mean = 50, SD = 10; for WISC-R vocabulary, mean = 10, SD = 3).

To assess whether there were significant differences on the simple arithmetic measures children were split into two groups, according to their scores at the first time of testing. Seventeen children who fell below the mean mathematics performance of the sample (less than or equal to a standard age score of 95) were placed into the low ability mathematics group. Twenty-four children who achieved mathematics standard age scores above the sample mean (greater than or equal to 96) formed the high ability mathematics group. One way analyses of variance comparing the two ability mathematics groups revealed there to be significant differences between the groups in mathematics ability, F (1, 39) = 86.96, $p < .0001$, reading ability, F (1, 39) = 22.34, $p < .0001$, and estimated IQ, F (1, 39)

= 5.47, $p < .05$. There was no significant difference between the groups in chronological age.

TABLE 6.1.

Overall performance characteristics on screening tests at times 1 and 2, and performance of each ability group (*SD*).

	CA (years:months)	Mathematics SAS	BAS Word Reading	Estimated full scale IQ
Time 1	7:3 (0:3)	96.14 (9.26)	50.34 (10.03)	10.07 (3.04)
Time 2	7:10 (0:3)	99.83 (10.70)	51.12 (8.86)	
		ABILITY GROUPS		
Low	Time 1	87.12 (4.97)	43.23 (6.87)	8.82 (3.21)
	Time 2	90.29 (7.32)	43.29 (6.47)	
High	Time 1	102.54 (5.38)	55.38 (8.86)	10.96 (2.63)
	Time 2	106.58 (6.85)	56.67 (7.91)	

Procedures.

This study was split into two parts, the first investigating the use of a priming procedure in simple arithmetic performance, and the second investigating the development of arithmetic fact retrieval across time.

6.2 (i) Study 4a.

Priming study.

Method and procedure.

This study was carried out at the first time of testing in October, 1995. Children were visually presented with simple addition questions which they were simply instructed to answer as quickly and accurately as possible. Arithmetic questions used ranged from 2 + 3 to 9 + 8, with tie questions (e.g., 2 + 2 =, 3 + 3 =) and questions requiring '+1' being removed. This left a total of 56 questions which were administered in two separate trials, each trial containing 28 questions. Children saw each question under a primed condition, and non-primed condition. In the primed condition, each arithmetic question was preceded with a prime, which was one of the numbers in the arithmetic problem to be presented after the prime. For example, a number was presented on the screen (e.g., 3) for the amount of time necessary for the child to identify the number. On identification of the number, the experimenter pressed a response key which started the presentation of the arithmetic problem (e.g., 6 + 3 =). Half of the primes were the addend and half the augend of the immediately following arithmetic problem. This priming task was carried out in the hope of finding out whether children with arithmetical difficulties were less susceptible to the prime, perhaps indicating poor or idiosyncratic organisation or poor representation of arithmetic facts in long-term memory. In the non-primed condition, children simply answered the arithmetic questions, without the additional task of identifying a number. This provided a total of four arithmetic trials with two conditions, primed and non-primed. These conditions were counterbalanced to overcome any practice effects or teaching that may have occurred between each trial. Each arithmetic question was presented in the centre of the computer screen. Time to solve the question began with the onset of the question, and was terminated by use of a response key when they child reported the answer. The experimenter recorded the strategy used by the child to solve each addition question, in this case overt counting strategies (finger counting and verbal counting), or direct memory retrieval. In most cases, it was possible to observe counting taking place through movement of the fingers, lips or heads, or through audible counting of the numbers. In cases where the strategy use was not clear, the child was questioned on the strategy they had used. Where the opinion of the child and the experimenter differed, the strategy reported by the child was recorded. Accuracy was also recorded by the experimenter.

6.2 (ii) Results.

For all parts of the study, results were analysed first by ANOVA and then by ANCOVA controlling for differences between the high and low ability mathematics groups in reading ability and IQ, according to performance on the BAS Word Reading test and WISC-R vocabulary.

Frequency of strategy use.

The frequency of using different solution strategies for reaching correct answers only was calculated. For the purposes of analysis, the strategies used were split into two types, direct memory retrieval of arithmetic facts and overt counting strategies (counting using fingers, verbal counting, counting in head). Each repeated measure ANOVA performed had one between subjects factor, group (low versus high ability mathematicians), and one within subjects factor, condition (primed and non-primed). Analysis of the frequency of memory retrieval under both conditions revealed there to be significant differences between the groups, $F(1, 39) = 28.10$, $p < .001$, with no other main effect or interaction proving to be significant. Repeated measures ANCOVA controlling for reading ability and IQ did not remove the significant main effect of group, $F(1, 37) = 16.07$, $p < .001$. Newman Keuls analysis revealed that high ability mathematicians recalled significantly more addition facts directly from long-term memory than low ability mathematicians, under both primed and non-primed conditions (p's $< .01$).

Analysis of the frequency of use of overt counting strategies revealed no significant main effects of group or condition, and no significant interaction (see Figure 6.1). Therefore, priming of the arithmetic questions did not lead to any significant changes in the accuracy of using overt counting strategies for either the low or high ability mathematics groups.

<u>Solution times.</u>

A repeated measures ANOVA was conducted comparing the low and high ability mathematics groups under primed and non-primed conditions were conducted to examine solution time using both direct retrieval of arithmetic facts and overt counting strategies. Analysis of retrieval times for correct answers retrieved directly from memory revealed significant main effects of group, $F(1, 28) = 11.67$, $p < .01$, and condition, $F(1, 28) = 15.20$, $p < .01$, with the interaction between group and condition proving not to be significant. ANCOVA controlling for differences in reading ability and IQ did not remove the significant main effect of group, $F(1, 26) = 7.65$, $p < .05$. Newman Keuls analysis revealed that children in the high ability mathematics group were faster to retrieve answers directly from long-term memory than low ability mathematicians under both conditions (p's $< .01$). Furthermore, both ability groups were significantly faster at retrieving answers under the primed condition than the non-primed condition (for high and low ability groups, $p < .05$ and $p < .01$ respectively). This suggests that both the low and high ability groups were equally susceptible to the prime.

Analysis of the correct solution times using overt counting strategies revealed significant main effects of group, $F(1, 39) = 18.92$, $p < .001$, and condition, $F(1, 39) = 12.54$, $p < .01$, with there being no significant interaction between group and condition. ANCOVA also showed there to be a significant difference between the groups, $F(1, 37) = 7.78$, $p < .01$.

Newman Keuls analysis revealed that children in the high ability mathematics group showed faster solution times under both primed and non-primed conditions, than children in the low ability group (p's <.01). However, only the low ability group showed significantly faster solution times under primed compared to non-primed conditions ($p <$ .01) (see Figure 6.2).

Overall accuracy.

Finally, analysis of total accuracy when using both overt counting strategies and direct memory retrieval revealed significant main effects of group, F (1, 39) = 11.81, $p < .01$, and condition, F (1, 39) = 5.10, $p < .05$, with there being no significant interaction between group and condition, F (1, 39) = 3.11, $p = .086$. High ability mathematicians had higher overall accuracy in answering the addition questions under both primed and non-primed conditions (p's < .01). Both groups correctly answered more questions under the primed condition than the non-primed condition, but only the low ability mathematics group showed a significant improvement under the primed condition as compared to the non-primed condition ($p < .01$). This might be expected, as the accuracy level of the high ability mathematics group was already very high (mean accuracy = 50.75 out of a total of 56), and clearly the low ability mathematics groups had more to gain through use of the prime. However, when differences between the groups in reading ability and IQ were controlled for through ANCOVA, the initially significant difference between the groups in overall accuracy was reduced to a non-significant level, F (1, 37) = 3.38, $p = .074$ (see Figure 6.3).

These results, showing both groups to improve in both retrieval time and counting time under the primed condition, can perhaps go some way to ruling out the possibility of a poorly or abnormally organised memory network of arithmetic facts for low ability mathematicians. An alternative priming procedure that might be considered in future studies could use a number related to the target answer as the prime rather than the augend or the addend that were to be shown in the arithmetic question. This procedure may have led to a different pattern of results emerging, and would certainly tell us more about how the closeness of the relationship between the prime and the target influences the speed and accuracy of responding to the arithmetic question presented. However, it appears to be the case, as would be predicted by previous research, that low ability mathematicians have failed to automate arithmetic facts to the extent of high ability mathematicians, and hence show a lower frequency of direct memory retrieval.

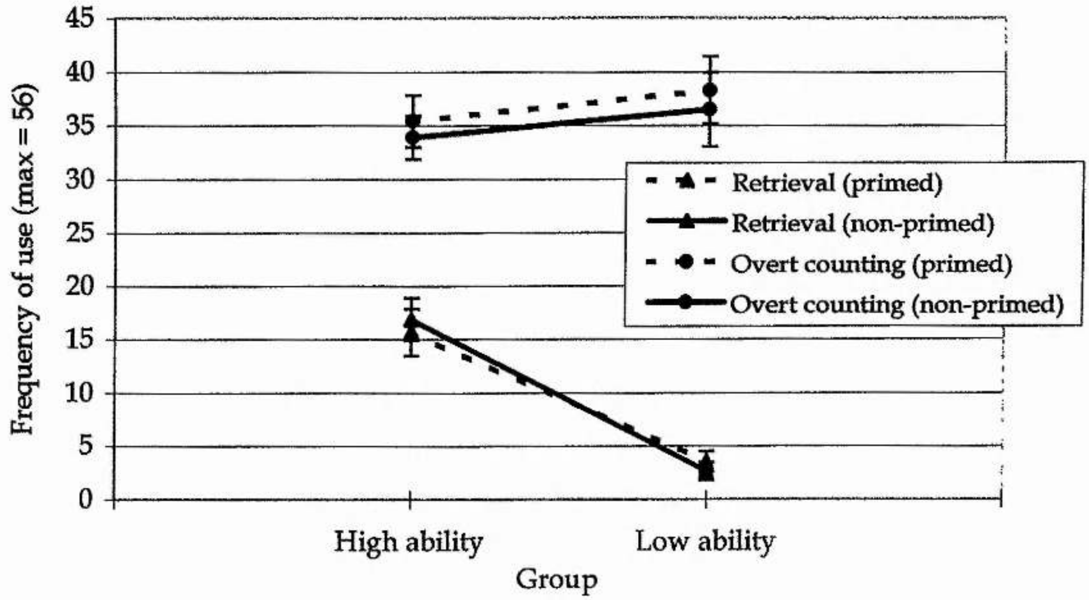

FIGURE 6.1

Frequency of strategy use (direct retrieval and overt counting strategies) under primed and non-primed conditions, for high and low ability mathematics groups (bars show standard error).

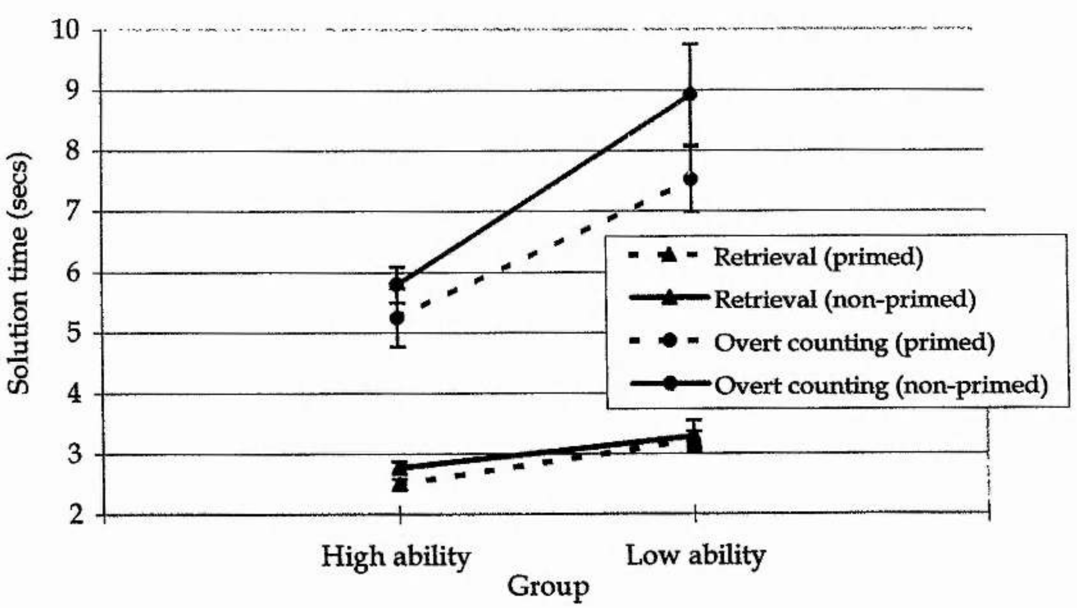

FIGURE 6.2

Solution times for correct answers only, using direct memory retrieval and overt counting strategies under primed and non-primed conditions, for high and low ability mathematics groups (bars show standard error).

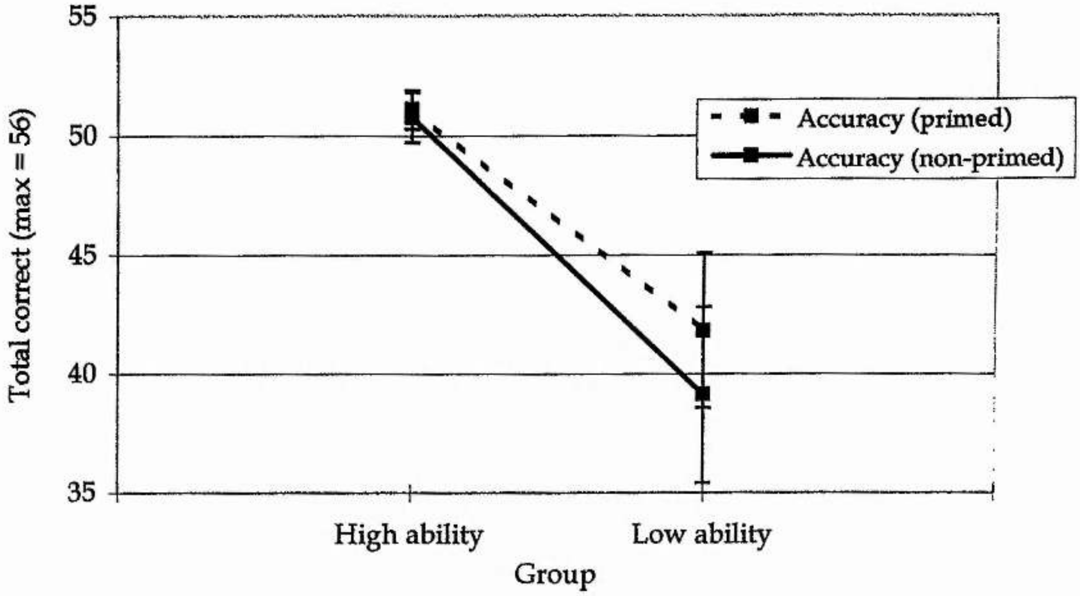

FIGURE 6.3

Total accuracy (direct retrieval + overt counting strategies) under primed and non-primed conditions, for low and high ability mathematicians (bars show standard error).

6.3 (i) Study 4b.

The development of arithmetic fact retrieval.

Method and procedure.

Another way of determining if children are showing an idiosyncratic development of arithmetic skills is to compare the development of their arithmetic skills to those of higher ability mathematicians across time. The data from the non-primed condition completed in October was compared with performance seven months later (May, 1996) under identical non-primed testing conditions. Differences in the frequency of using overt counting strategies and direct retrieval, solution times, and improvements in overall accuracy were examined across time.

Tie questions and + 1 questions (time 2).

Performance on these questions was examined in this analysis as many studies report that all children, whatever their mathematical ability, frequently solve such problems through direct retrieval of an arithmetic fact from long-term memory. The procedure for presentation of the questions and response were as for simple addition questions under non-primed conditions in the priming study. The questions in this block consisted of those described as tie problems (e.g., 3 + 3 =), with all combinations from 1 + 1 = to 9 + 9 = being used. Questions requiring + 1 were also included, with all combinations using the numbers from 1 to 9 inclusive being presented, that is with 1 as the addend in eight questions (1 + 2 = through to 1 + 9 =), and 1 as the augend in eight questions (2 + 1 = through to 9 + 1 =). This left a total of 25 questions that were presented in one block. Strategy use, solution times, and accuracy were recorded.

6.3 (ii) Results.

The purpose of this short longitudinal examination was to assess if any substantial improvements had been made by the children during the course of the academic year, and if so, whether both groups appeared to be progressing at the same rate. Performance measures considered in this instance were frequency of strategy use (direct memory retrieval versus overt counting strategies), solution times, and total accuracy of using overt counting and direct memory retrieval. All of these performance measures were investigated through repeated measures ANOVA's, each with one between subjects factor, group (low versus high ability mathematicians), and one within subjects factor, time (performance at time 1 versus performance at time 2). Repeated measures ANCOVA's were also carried out controlling for differences between the groups in reading ability and IQ.

Frequency of strategy use.

Analysis of the frequency of direct memory retrieval of addition facts revealed a significant main effect of group, F (1, 39) = 29.58, $p < .001$, with no other main effect or interaction proving to be significant. ANCOVA did not remove this significant difference between the groups, F (1, 37) = 13.64, $p < .01$. Newman Keuls analysis revealed that high ability mathematicians used direct memory retrieval of addition facts significantly more frequently at both times of testing (p's < .01). Whilst both groups used direct memory retrieval more frequently at the second time of testing, this increase was not significant. Analysis of the frequency of use of overt counting strategies revealed no significant main effects or interactions (see Figure 6.4).

Solution times.

Analysis of the solution times for correct answers retrieved directly from long-term memory revealed significant main effects of group, F (1, 29) = 10.11, $p < .01$, and time, F (1, 29) = 8.99, $p < .01$, with the interaction between group and time proving not to be significant. ANCOVA controlling for differences in reading ability and IQ did not remove this significant difference between the groups, F (1, 27) = 7.64, $p < .05$. Newman Keuls analysis revealed that high ability mathematicians were significantly faster than low ability mathematicians at retrieving answers directly from memory at both time of testing (p's < .01). Whilst both ability groups were faster at retrieving answers directly from memory at the second time of testing, only the low ability group showed significantly faster retrieval at time 2 compared to time 1 ($p < .05$) (See Figure 6.5). Analysis of solution times using overt counting strategies revealed a significant main effect of group, F (1, 39) = 16.47, $p < .001$, with no other main effect or interaction being significant. Again, controlling for differences between the groups in reading ability and IQ through ANCOVA did not remove the significant main effect of group, F (1, 37) = 4.96, $p < .05$. Newman Keuls analysis revealed that children in the high ability group showed significantly faster solution times at both times of testing (p's < .01).

Overall accuracy.

Analysis of overall accuracy using both overt counting strategies and direct memory retrieval revealed significant main effects of group, F (1, 39) = 14.26, $p < .01$, time, F (1, 39) = 18.46, $p < .001$, and a significant interaction between group and time, F (1, 39) = 4.50, $p < .05$. The significant main effect of group remained after controlling for differences in reading ability and IQ through ANCOVA, $F(1, 37) = 4.43$, $p < .05$. Newman Keuls analyses revealed that overall accuracy of the high ability group was higher at both times of testing compared to the low ability group (p's $< .01$). Both mathematics ability groups were significantly more accurate at time 2 compared to performance at time 1. However, only the low ability mathematics groups showed a significant improvement in accuracy from time 1 to time 2 ($p < .01$) (see Figure 6.6). This finding that the high ability mathematics group did not show a significant improvement in accuracy across time may be the result of performance being almost at ceiling for this group (overall accuracy at time 1 = 50.75, time 2 = 53.04, maximum = 56).

Again, these results are encouraging for children in the low ability mathematics group. With regard to retrieval time for answers retrieved directly from memory, the low ability mathematicians, despite still being slower than their higher ability peers, have shown faster retrieval times at the second time of testing, matching the developmental improvements of the high ability mathematics group. Furthermore, low ability mathematicians showed a significant improvement in their overall accuracy at the second time of testing, suggesting their understanding and performance on simple arithmetic questions is gradually catching up with that of the high ability mathematicians. This would suggest a developmental delay between the low and high ability mathematics groups rather than an idiosyncratic developmental difference.

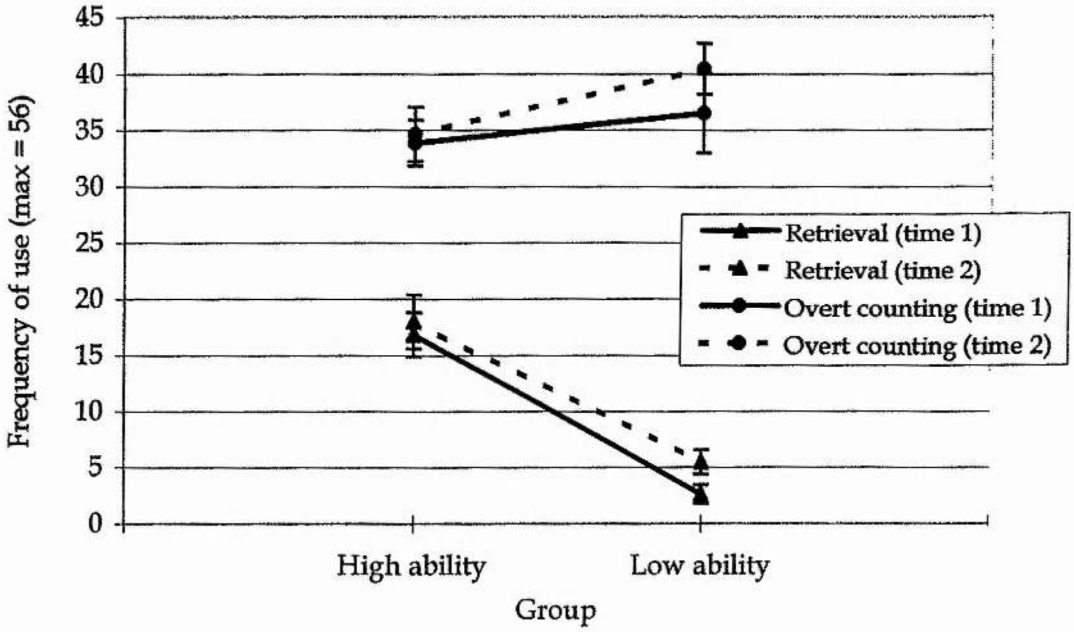

FIGURE 6.4.

Frequency of strategy use (direct retrieval and overt counting) at time 1 and time 2, for high and low ability mathematicians(bars show standard error).

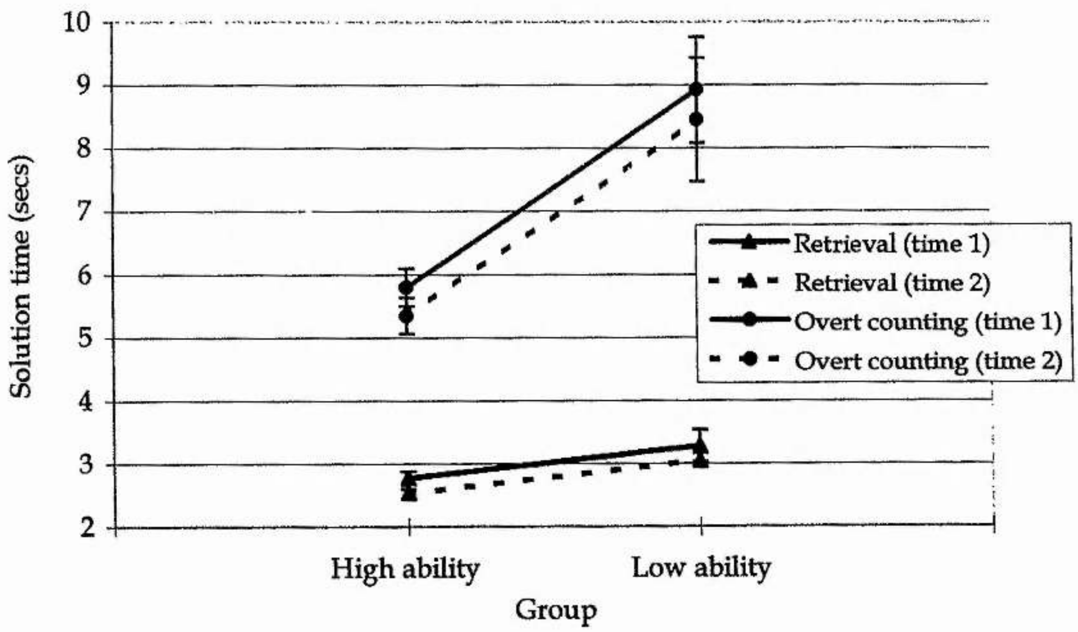

FIGURE 6.5.

Solution times for correct answers only, using direct memory retrieval and overt counting strategies at time 1 and time 2, for high and low ability mathematics groups (bars show standard error).

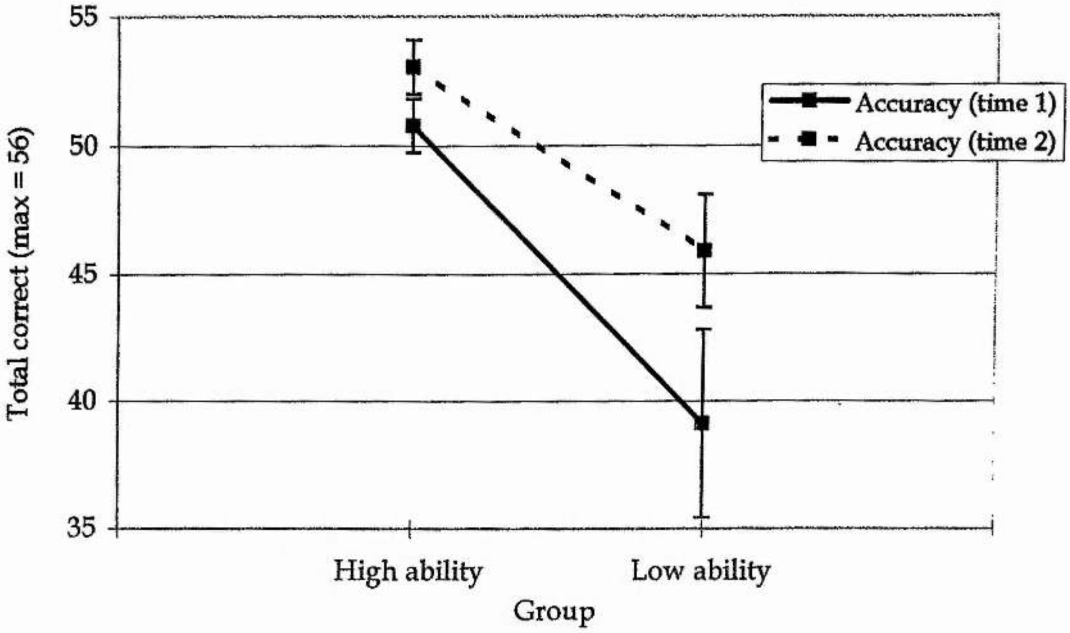

FIGURE 6.6.

Total accuracy (direct retrieval + overt counting strategies) at time 1 and time 2, for high and low ability mathematicians (bars show standard error).

Analysis of tie questions and + 1 questions at time 2.

The main reason for examining performance on these questions was that they are regarded as the questions that are most well learned by children, and hence easiest to solve. Therefore, the possibility remained that children of low mathematical ability would be more on a par with their higher ability peers on these questions. Performance measures included looking at the frequency of use of computational procedures and direct memory retrieval, computational and retrieval times for correct answers, and associated error rates. All analyses conducted were one-way ANOVA's with one between subjects factor, group (low versus high ability mathematicians), and one way ANCOVA's controlling for differences in reading ability and IQ.

Examination of the frequency of use of direct memory retrieval revealed there to be a significant difference between the groups in the frequency of use of direct memory retrieval, $F\ (1, 39) = 11.62, p < .01$. High ability mathematicians were found to retrieve significantly more answers directly from long-term memory than low ability mathematicians. However, controlling for differences between the groups in reading ability and IQ through ANCOVA removed the significant difference between the groups, $F\ (1, 37) = 1.55, p > .05$. Analysis of the number of incorrect responses retrieved from memory revealed there to be no significant differences between the groups, $F\ (1, 39) = 0.51, p > .05$. Analysis of the time taken to retrieve correct answers directly from memory revealed there to be significant differences between the groups, $F\ (1, 39) = 17.18, p < .001$, with high ability mathematicians retrieving answers significantly more quickly than low ability mathematicians. Again, ANCOVA controlling for reading ability and IQ removed the significant difference between the groups in speed of memory retrieval, $F\ (1, 37) = 3.89, p = .056$.

With regard to the use of computational procedures, analysis of the error rates revealed a significant difference between the groups, $F\ (1, 39) = 4.25, p < .05$, with low ability mathematicians making significantly more computational errors than high ability mathematicians. ANCOVA controlling for reading ability and IQ removed the significant difference between the groups in error frequency, $F\ (1, 37) = 0.06, p > .05$. There was no significant difference between the groups in the time taken to compute correct answers, $F\ (1, 26) = 0.01, p > .05$ (see Table 6.2 for performance measures). It must be noted that not all children used computational procedures on these questions, with eleven children in the high mathematics group, and two children in the low mathematics group retrieving all answers directly from long-term memory.

To summarise, initial findings from ANOVA revealed that children in the high ability mathematics group retrieved significantly more arithmetic facts directly from long-term memory, and that they were faster to retrieve answers directly from memory. Results also showed that children in the low ability mathematics group made significantly more errors when using overt counting strategies than did children in the high ability group. However, this pattern of results changed completely once differences between the groups in reading ability and IQ had been controlled for through ANCOVA. All previously significant results were removed, leaving no significant differences between the groups on any of the measures. Therefore, when the only difference between the group is in mathematics ability, there are no differences in performance on tie questions and + 1 questions. This is in agreement with previous studies showing that such questions are well learned in children of all abilities, with there being an almost complete reliance on retrieval of facts from long-term memory.

TABLE 6.2.

Performance on tie and + 1 questions by low and high ability mathematicians (*SD*).

	High ability		Low ability	
	M	Adjusted *M*	*M*	Adjusted *M*
Direct retrieval (frequency)	23.00 (1.96)	21.99	19.41 (4.62)	20.42
Overt counting (frequency)	2.00 (1.96)	3.01	5.59 (4.62)	4.58
Direct retrieval (no. of errors)	0.21 (0.66)	0.24	0.35 (0.61)	0.32
Overt counting (no. of errors)	0.29 (0.62)	0.67	1.18 (1.98)	0.80
Direct retrieval (solution time)	1.83 (0.32)	1.92	2.25 (0.33)	2.17
Overt counting (solution time)	6.42 (3.06)	6.68	6.52 (2.10)	6.26

Note. Time in seconds.

6.4 Discussion.

The use of counting strategies in this study reflects the pattern of results found in previous studies (e.g., Geary, 1990; Geary et al, 1992; Geary & Brown, 1991; Geary et al, 1991). Children in the low ability mathematics group relied on inefficient counting strategies such as verbal and finger counting, whereas high ability mathematicians relied more heavily on direct memory retrieval of arithmetic facts. Furthermore, in a similar manner to previous studies, low ability mathematicians were slower to retrieve correct arithmetic answers from long-term memory, but did not however, show a high error rate for direct retrieval. Both groups of children had a very low error rate for direct memory retrieval. Also, in agreement with previous studies, there was no difference in performance between the groups on well learned arithmetic problems, such as tie questions.

One explanation offered for this pattern of results in previous studies is that children with poor mathematical skills may have either a delayed or abnormal development of arithmetic facts in long-term memory. To explore this further, a priming procedure was used, which has been used in studies of semantic memory. Previous research shows that when a target word is primed with a semantically related word, time to identify a target word is decreased. If children of low mathematics ability do in some way differ in their long-term arithmetic fact organisation from their higher ability peers, it was hypothesised that these children would show a differing effect when an arithmetic question was primed with a related number. For example, they might show less facilitation to the prime if their network of arithmetic facts is poorly organised. Contrary to this notion, it was found that all children were equally susceptible to the prime, with both ability groups showing faster retrieval times for arithmetic facts compared to the non-primed condition.

The development of memory retrieval of arithmetic facts was also investigated over the course of the academic year. The pattern of strategy use at the end of the academic year revealed that all children were showing increased reliance on direct memory retrieval, although high ability mathematicians were still using direct memory retrieval significantly more frequently than low ability mathematicians. These results do not support the findings of Geary et al (1991), who reported that a mathematics disabled group showed no change in their strategy use, whilst more able mathematicians showed an increased reliance on direct memory retrieval.

Results from the priming and strategy development aspects of this study allow us to draw a tentative conclusion, in that children with poor mathematical skills in this study did not show idiosyncratic patterns of performance, allowing us to suggest that these children simply have a delayed development of arithmetic fact automatisation, as opposed to an abnormal fact organisation. Differences in the speed of arithmetic fact retrieval may simply be accounted for by the fact that children of higher mathematics ability may be more familiar with the numerical material, with information being activated to a high level more rapidly, resulting in faster retrieval times. This is very encouraging as, with extended practice and greater familiarity with numerical information, it may be possible for low ability mathematicians to reach the level of performance of their higher ability peers. Evidence to support this comes from performance on well learned arithmetic problems, such as tie questions and + 1 questions, where children in the low ability mathematics groups showed no difference in performance to their higher ability peers, once differences between the groups in reading ability and IQ had been accounted for.

These differences in performance between the two groups can be interpreted by considering tabular and nontabular models of arithmetic fact organisation and retrieval. For brevity, only the distribution of associations model proposed by Siegler and Shrager (1984) will be considered, as this is the most widely acknowledged model of arithmetic fact organisation, and overcomes problems associated with earlier models. Associations between problems and answers are formed each time an arithmetic problem is encountered, regardless of the correctness of the answer. Therefore, retrieval errors occur due to accessing an incorrect association in memory, or due to an alternative counting strategy not being correctly executed. The probability of correctly retrieving an answer depends on the peakedness of associations between the problem and an answer, compared to other possible answers, on the confidence criterion, and on the search time parameter. If the retrieved answer does not reach the confidence criterion due to the association between the problem and the retrieved answer being low, and if the search time parameter (number of searches) has been exceeded, then an alternative back up counting strategy will be employed. The distribution of associations changes with experience. Clearly, accuracy is important in early arithmetic learning. Producing a variety of correct and incorrect answers to a problem will result in a problem becoming associated with many potential answers, and the distributions of associations will be flat. Therefore, retrieved answers will not reach the confidence criterion, and the child will revert to using counting strategies to solve the problem. These are known to be much less efficient and more prone to errors, meaning the process of automating arithmetic facts in long-term memory is going to take much longer. This may be the problem for children who show poor arithmetic performance, which may be overcome by increased exposure to basic arithmetic problems at an early stage in mathematics learning.

Whilst strategies used by the children were categorised as computation or direct retrieval, it was clear that some of the computational strategies were much more efficient and frequently produced more correct answers than other computational strategies. These strategies were realised by hearing children verbally count, or by the children's self-report of the strategy they were using. One of these strategies involved a method of decomposition, whereby an addition question which could not be solved by direct retrieval, was broken down into smaller units to which the answers were able to be retrieved directly from memory. For example, given 6 + 9 =, the child would convert this into 5 + 10, such that the child is performing the following (6 - 1) + (9 + 1). This decomposition frequently involved rounding numbers in the question up or down to units of 5 or 10, or working for an answer to a known tie question. To provide a few examples; 9 + 8 becomes (9 + 9) - 1; 5 + 7 becomes (5 + 5) + 2; 8 + 7 becomes (5 + 5) + (3 + 2). Initially, this appeared to be a fairly complex method for solving simple addition questions, requiring a number of computational steps. However, this proved to be a very successful method, presumably because it involves a stage of direct memory retrieval of known answers and therefore requires very little counting. What the use of this strategy does show is a good understanding of simple arithmetic procedures, and a flexibility of this understanding to know that each addition question could be broken down into sub-components. This flexibility of problem solving was further shown by the switch between addition and subtraction in the solving of some of the questions by decomposition. Given that this method was so successful, it may be possible to use this strategy to teach children who are poor at mathematics how to use the long-term knowledge they have in order to learn and eventually automate other arithmetic facts. This may be particularly successful if arithmetic problems can be decomposed into the facts that are well learned, such as tie problems, where the children of low mathematics ability show equivalent levels of automatisation of arithmetic facts as their higher ability peers.

One point of consideration is what we actually take to be a measure of improvement in the use of strategies. Some researchers may suggest the acquisition of new, more effective approaches is the key to understanding strategic change in arithmetic. However, as indicated by Lemaire and Siegler (1995), the more efficient use of existing strategies can also be seen as a measure of improvement, for example, improvements in speed and accuracy, both of which were evidenced in this study.

Clearly, before firm conclusions can be drawn, more research needs to take place. This is so particularly with regard to the priming procedure. For example, this study only looked at a primed versus a non-primed condition, but a condition which also used false primes (e.g., prime is 3, arithmetic problem is 5 + 2 =), would have revealed more detailed results as to the possible organisation of arithmetic facts in long-term memory. For example, we would expect a false prime to have a deleterious effect on the problem solving process, as we are essentially activating an incorrect internal representation, and setting the child off down the wrong number path towards solving the arithmetic problem. Also, further studies need to be conducted which examine extended practice of basic number facts, and which assess the development of arithmetic skills over a longer time period. This would give a more accurate depiction of whether a developmental delay in arithmetic fact automation could be easily overcome by extended practice.

Chapter 7.

Teaching Strategies in Mathematics: Views from Educational Philosophy and Psychology.

The final study to be described is somewhat different to those reported in previous chapters. This study aimed to take the cognitive deficits identified as playing a role in poor mathematical skills, and to incorporate them into teaching strategies that would hopefully alleviate some of these difficulties. This chapter includes a theoretical background relating to the teaching of mathematics in schools, with views on mathematics teaching from both educational philosophy and psychology.

7.1 Theoretical Background.

National testing has revealed that many children are underachieving in mathematics. Recent results from the testing of 11 year old children in schools in England point out the poor standards of many children in mathematics and English (Scott-Clark reporting in The Sunday Times, 25th August 1996). The Sunday Times on November 24th 1996 reported that Scottish children were trailing near the bottom of the world mathematics league table. More recently, an article by MacLeod in the Guardian (June 11th, 1997) reported that children in primary schools in England are falling further behind children in other countries in mathematics. This article even suggested that teachers foster a positive self-image of mathematical skill in children, when in fact they are accepting lower standards than they should. This is supported by the finding that 90% of English and Scottish nine year olds thought they did well in mathematics, compared to 74% in Japan and 77% in Singapore. Documents from the National Council of Teachers of Mathematics (1989) have emphasised the need for mathematical literacy. Martin Hughes (1986) in his book 'Children and Number' provides some insight into the problems faced by children in mathematics, and discusses the intense public debate and concern about the teaching of mathematics in schools. A number of claims are made. Firstly, that standards of numeracy are generally inadequate. Secondly, progressive teaching methods in the primary school have failed. Thirdly, teachers themselves do not know enough mathematics, and finally, employers are having to provide remedial training in basic mathematical skills to their young employees. Although many students with mathematics deficiencies exhibit characteristics that predispose them to such difficulties, for example, problems in memory, language, reading, reasoning, and metacognition, their learning difficulties are often accompanied by ineffective instruction. This study focuses on two instructional techniques that have been used to teach mathematics, and aims to assess the effectiveness of each for teaching mathematics in the early years of primary school. These instructional techniques provide a link to the cognitive difficulties identified in earlier studies. One of these methods of instruction is currently very influential in curriculum development, that is the ideas associated with constructivism. The second technique arises from experimental psychology, but would not now be accepted by many educators, that is, rote learning. The ideas surrounding constructivism and its application to education will be discussed first.

Constructivism and its application to education.

During the 1950's and 1960's, the idea of progressivism was introduced into primary schools, along with curriculum innovations which placed an emphasis on practical activities and discovery methods (for example, the Nuffield Primary Mathematics Project). These teaching methods adopt the views of psychologists such as Piaget and Vygotsky. Drawing from the work of Piaget and Vygotsky, constructivists view the child as active learners who must construct mathematical knowledge for themselves. Mathematical learning is seen as a social enterprise, with disagreements about the meanings of mathematical concepts providing the emphasis to change and accommodate current mathematical understanding. Griffin and Case (cited in Murray, 1997), propose that children construct mathematics knowledge, using what they already know about numbers to add new knowledge. Again, this fits Piaget's ideas of assimilation and accommodation. The ultimate goal of progressivism was the replacement of methods based on drill and rote-learning with methods based on discovery and understanding. A quote taken from an early document of the progressive movement sums up this goal.

> *"understanding must precede drill or formal exercise intended to develop memory, mechanical accuracy or speed.... We are convinced that the important thing is to help children to understand mathematical ideas and to recognise the kind of computation or other thought processes which a problem situation demands."*

Recent reform recommendations place an emphasis on applications and connections of mathematics to the real world. This involves a reduction in practice exercises of written computation procedures and mechanical drill, and an increase in meaningful problem situations for which a solution procedure has not already been memorised. Such reforms argue that when mathematics is acquired in these realistic situations, students see them as being more useful, rather than simply being an acquisition of knowledge that is isolated from real situations (Hiebert, Carpenter, Fennema, Fuson, Human, Olivier, & Wearne, 1996, Murray, 1997). Much of this research is being conducted in schools in the United States, where children's low achievement in mathematics is a primary concern. One such teaching method is described by Hiebert et al (1996) as 'problematising', whereby children discover methods for solution of problems (usually word problems), through discussion with other children. This is often referred to as a learner-centred approach by American mathematics educators, which allows students to shape their own learning, and which allows them to engage in mathematics related activities that make them think. This allows the children to see alternative methods of solving problems, rather than simply using techniques prescribed to them by the teacher.

These ideas draw on Dewey's (1933) notion of 'reflective inquiry'. Dewey suggests that much of school instruction is plagued by a push for quick answers, which inhibits the search for alternative methods of solution. Whilst this may result in correct answers, it also results in mechanically executed procedures and away from a search for higher quality methods. Dewey maintains that it is the quality of mental processes, not the production of rapid, correct answers, that is the measure of educative growth. One of the fundamental features of Dewey's notion of 'reflective inquiry' is that problems should be studied through active engagement, which helps the problems to be understood more fully. This is achieved by searching related information to the problem, formulating hypotheses, and interacting with the problem. This allows conclusions to be reached as the problems are solved. By reaching conclusions, new relationships between information are uncovered and understood (see Hiebert et al, 1996, for more details on 'reflective inquiry'), rather than the task simply being a recall of facts or rules. Ackerman and Dykman (1995) suggest that one major cognitive problem for both reading disabled, and reading and arithmetic disabled children, is in their lack of generalisation, and inferential reasoning, that is, they have difficulty appreciating patterns and going beyond the information that has been given to them. Examples of this include knowledge of commutativity (knowing that 3 x 6 = 18, and that 6 x 3 = 18), reciprocation of multiplication and division (6 x 3 = 18, 18 divide by 3 = 6), and reciprocation of addition and subtraction. Further support for this view comes from Hiebert et al (1996) who suggest that functional understanding in mathematics relies on analysing and understanding the adequacy of current methods, thus building the social and intellectual community of the classroom. This is achieved by sharing the results of enquiries with others and justifying methods. Most importantly, Hiebert et al (1996) say that procedural and conceptual understanding should go hand in hand. To be able to simply memorise and retrieve facts is not enough without understanding.

Another key determinant in children's success in mathematics is their motivation and interest in the subject. This aspect has not been studied in great detail, particularly with children, although the literature has suggested methods of improving interest in mathematics. Lave, Smith, & Butler (1988) suggest that problematising mathematics provides individuals with the opportunity to recognise the inventiveness of their own practice, and to see mathematics as an intellectual activity in which they can participate. Ackerman and Dykman (1995) also comment on the passivity on learning disabled students, including giving up on problems in both reading and arithmetic, and not getting as much practice as needed to automate facts and procedures.

These types of teaching strategies require the child to partake in higher level thinking about different strategies they can use for solution, and so can be linked to studies investigating metacognitive skills. Some researchers have pointed out that there is a positive relationship between a well developed cognitive monitoring system and the effective use of learning strategies, with some suggesting that metacognitive strategies could be incorporated into the study of mathematics learning (e.g., Burton, 1984; Garafalo & Lester, 1985). Flavell (1976) refers to metacognition as knowledge of one's own cognitive processes, and to the active monitoring and regulation of these processes. The best example of this is a control system that oversees the mental flow of information. Metacognition has also been described as 'executive control' (Gagne, 1983), and reflective intelligence (Skemp, 1980). The results from Study 2, reported in chapter 4, showed that executive performance was closely associated with mathematical skill, and that children who were poor at mathematics showed a high degree of perseveration. This means they found it difficult to switch between strategies for the solution of a problem, and they were poor at self-monitoring their own knowledge and actions. Therefore, given the difficulties found in higher level thinking, one of the teaching strategies adopted in this study aimed to draw together the recognised deficit with the problematising teaching strategy. It was hoped that this would extend the children's knowledge of different solution strategies and would go some way towards overcoming problems associated with lack of mental flexibility. So, by asking the children to think about and justify what they are doing, and by sharing their strategies for solution, children are engaging in higher metacognitive thinking, and this may bypass some of the difficulties previously identified related to poor executive functioning.

Mechanistic methods.

The alternative teaching strategy to be discussed in this study is based on psychological research which has been largely ignored by educational researchers. Since the 1970's, mathematics education has been criticised for fostering rote learning without understanding. However, much of the psychological literature suggests that rote learning and the automatisation of basic arithmetic facts must be the first thing which the child develops before going on to more complex problem solving (Fleischner et al, 1982; Kirby & Becker, 1988). The National Council of Supervisors in Mathematics (1988) suggested that gaining proficiency in using basic arithmetic operations is one component of effective mathematics teaching. This report states that knowledge of basic arithmetic facts is essential, and mental arithmetic is important. It suggests teaching to a level of mastery, with rate of responding being a good measure of mastery. By teaching to a level of mastery, the individual can concentrate on using, rather than remembering, the skill. The Third International Mathematical and Science Study also points to the low standard of mathematics in primary and secondary schools in England, compared to other Western European and Pacific rim countries. Arithmetic is confirmed as the branch of mathematics with the greatest shortfall in children's attainment in both primary and secondary school. A statement in an article by Prais in the Times (Friday 18th July, 1997) illustrates the importance of arithmetic knowledge.

> *"It is easy to discount failing in arithmetic as being only a small part of the broader canvas of mathematics needed by the aspiring modern mathematical or technological specialist. But to do so would overlook the role of arithmetical competence as a pedagogical foundation stone for progress in other branches of mathematics and science."*

Other researchers have pointed out that unfamiliarity with basic number facts plays a major role in mathematical difficulties (DeCorte & Verschaffel, 1981; Russell & Ginsburg, 1984). However, Geary & Brown (1991) make it clear that whilst rote learning may have short-term benefits, the long-term advantages are much more in question. Despite this, David Geary and his colleagues have presented evidence showing that the inadequacy of basic number fact knowledge of American school children is fundamental to their mastery of more complex mathematical procedures (Geary, 1993; Geary, Bow-Thomas, Liu, & Siegler, 1996; Geary, Fan, & Bow-Thomas, 1992; Geary & Widaman, 1992). These studies suggest that if the mastery of basic skills underlies the acquisition of more complex mathematical procedures and concepts, then Chinese children, who practice basic academic skills to a higher extent, will have a consistent 3 to 4 year advantage in their development of mathematical cognition. Goldman (1989) states that readily accessible knowledge of basic single-digit addition, subtraction, multiplication, and division facts is a critical enabling condition for the application of strategies required to solve problems.

Practice may be particularly important for children with learning difficulties, because it is often assumed that such children have memory and attentional difficulties, limiting the effectiveness of any form of instruction. Increases in the efficiency of cognitive processes involved in accessing and using the knowledge base of arithmetic facts, brought about by practice, may mitigate the impact of such constraints.

Constructivists fail to recognise the potential benefits of teaching based on drill and practice. Their view of such a mechanistic approach sees the child as a passive learner receiving information from the environment, that is, the teacher. Constructivists would also suggest that any overt changes shown by the child, for example, improvements in the number of arithmetic problems solved, do not represent a fundamental change in conceptual mathematical understanding. Therefore, such techniques may in fact be viewed as detrimental to the child's development of mathematical skills. However, procedural learning does require extended practice so that the child knows which mathematical procedure to use, and when (see Geary, 1995). Procedures should be practised until they can be carried out automatically. Once this is achieved, very little conscious effort is required to apply the procedure, which frees attentional and working memory resources for use on other important features of the problem (Geary & Widaman, 1992).

Russell and Ginsburg (1984) carried out a cognitive analysis of children's mathematical difficulties, looking in particular at possible differences or deficiencies shown by children who were poor at mathematics in informal (non-schooled) concepts of arithmetic. Several measures of mathematical knowledge were considered, these being informal concepts and calculation skills (e.g., which number is more, which number is closer to a target number, mental addition, and estimation), base ten concepts and related enumeration skills (enumeration by 10's, counting large numbers, multiples of large numbers, and place value), error strategies and other calculational procedures (accuracy and bugs in written addition and subtraction, monitoring errors), knowledge of number facts (retrieval of basic arithmetic facts directly from long-term memory), and finally, problem solving skills (use of principles, e.g., commutativity and reciprocity of addition and subtraction, and story problems). Results showed that mathematics disabled children were not deficient in knowledge of basic mathematical concepts and non-algorithmic procedures, such as those measured by informal concepts. However, mathematics disabled children did show some difficulties in the execution of procedures for mental addition problems, and more substantial difficulties in the application of these basic concepts to problems involving larger numbers. Many studies report that difficulties with base ten concepts underlie mathematical problems, and a similar hypothesis was proposed by Russell and Ginsburg. However, it was found that mathematics disabled children did possess elementary knowledge of base ten concepts, so they were able to use positional information to determine the magnitude of large numbers, and were able to distinguish between tens and units in place value. They were also able to count by tens. Problems arose with large numbers and Russell and Ginsburg concluded that the mathematics disabled children lacked computational skills with large numbers. One of the most severe difficulties displayed by mathematics disabled children was in their knowledge of basic addition facts, with their mastery of addition facts being significantly lower than their peers. This finding was clearly later supported by the work of Geary and his affiliates. Russell and Ginsburg concluded by saying that these children display relatively normal but immature and inefficient mathematical knowledge, so for example, they show problems in calculation that would normally be attributed to younger children. They also show inattention, poor execution of strategies, and lack of facility for dealing with large numbers. However, these results also suggest that mathematics disabled children also have a number of strengths which could be worked on to improve their weaknesses in other areas. These children showed simple understanding of base ten notions, insightful problem solving, and the ability to interpret elementary story problems. Since mathematics is a cumulative subject where early knowledge forms the building blocks of later knowledge, then mathematics disabled children, if not helped at an early age, may eventually lose touch with the subject and may exhibit more serious forms of mathematical difficulties.

The aim of this study was to compare two different teaching methods for the learning of arithmetic facts, these being problematising (metacognitive) and rote learning (mechanistic), and to assess the short-term benefits of each.

7.2 Study 5.

7.2 (i) Method.

Subjects.

59 children (mean age 8 years, 1 month) from two classes in one school in Dundee took part in this study. Children were screened before the start of the training period for mathematics ability, reading ability, and estimated IQ, using the Group Mathematics Test (Young, 1970), the BAS Word Reading Test (Elliott et al, 1977), and the Vocabulary subtest from the WISC-R (Wechsler, 1977). Children were formed into mathematics ability groups by the teachers, and for the purposes of the training period, children were taught in these mathematics groups. Three groups were classified as high ability, having a mean mathematics standard age score for the group falling above 96, and three low ability groups, having mathematics standard age scores falling below 96. One high and one low ability group were assigned to the problematising training strategy (referred to as the discussion group from here onwards), one high and one low ability group to the rote learning strategy, and one high and one low ability group were assigned as the control group. This group did work with the experimenter in similar manner to the training groups, but received no specific training strategy. The performance characteristics and number of children in each group are shown in Table 1. One way ANOVA's were conducted to compare the three high and three low ability group in their performance on the Group Mathematics Test. This revealed there to be no significant difference between the three low ability groups in mathematics performance, F (1, 30) = 1.09, $p > .05$, although there was a small significant difference in mathematics performance between the three high ability groups, with children in the high discussion group achieving a mean higher standard age score than children in the high rote and high control groups. All high ability groups were achieving significantly higher standard age scores than all low ability groups. One way ANOVA comparing the 6 groups on reading ability revealed there to be significant differences, F (5, 53) = 4.83, $p < .01$. Newman Keuls analyses revealed that all high ability groups had significantly higher reading ability than all low mathematics ability groups (p's < .05). There were no significant differences in reading ability between the three high ability mathematics groups, and no significant differences between the three low ability groups. ANOVA comparing the three high ability groups on estimated IQ revealed there to be no difference between the three groups, F (2, 23) = 1.66, $p > .05$. A similar result was found comparing the three low ability groups in estimated IQ, with there being no

significant differences between the groups, $F(2, 30) = 2.51, p > .05$. There was no significant difference between the groups in chronological age, $F(5, 53) = 1.95, p > .05$.

TABLE 7.1

Mathematics, reading, and estimated IQ for each training group (*SD*).

Group	N	Chronological age (years:months)	Mathematics SAS	Reading SAS	Estimated IQ
High Rote (HR)	12	7, 11 (0, 3)	106.42 (6.20)	52.25 (6.86)	9.83 (2.62)
High Discussion (HD)	6	8, 2 (0, 4)	115.50 (12.13)	58.33 (10.58)	12.00 (2.67)
High Control (HC)	8	8, 1 (0, 3)	107.12 (2.64)	59.50 (8.04)	11.13 (2.17)
Low Rote (LR)	10	8, 2 (0, 3)	89.20 (4.59)	49.30 (8.17)	10.90 (2.13)
Low Discussion (LD)	14	8, 3 (0, 3)	85.36 (7.78)	45.36 (6.99)	9.00 (3.14)
Low Control (LC)	9	8, 0 (0, 3)	86.00 (6.04)	48.78 (9.19)	8.33 (2.24)

Training Strategies.

Discussion (problematising): The training in this group consisted of discussion between the children of their methods for solving arithmetic problems. The focus in this training strategy was not on automating as many arithmetic facts as possible, but rather on understanding relationships between numbers, such that unknown arithmetic facts could be derived through known arithmetic facts. This included showing the children number patterns so that the children were able to handle higher value numbers by applying their knowledge of procedures for lower value numbers. Arithmetical principles such as commutativity, reciprocation of addition and subtraction, reciprocation of multiplication and division, and the N + 1, N - 1, and N x 10 principles, were also made apparent to the children. Examples of these principles are as follows:

1. Commutativity - if 7 + 5 = 12, then 5 + 7 must also equal 12.
2. Reciprocation of addition and subtraction - if 46 + 27 = 73, then 73 - 27 must be 46.

3. Reciprocation of multiplication and division - if 3 x 6 = 18, then 18 divide by 6 must be 3.
4. N + 1 - if 21 + 15 = 36, then 21 + 16 must be 37.
5. N - 1 - if 9 + 8 = 17, then 9 + 7 must be 17 - 1 or 16.
6. N x 10 - if 21 + 15 = 36, then 210 + 150 must be 360.

Rote: In this case, the children were simply given extended and frequent practice of basic arithmetic questions, with the aim of increasing automaticity and speed of responding for basic number facts. Children were required to complete worksheets of basic arithmetic problems, and arithmetic facts were practised by asking children to recite and quickly answer basic arithmetic problems. No discussion of procedures for solving the problems was allowed, and no patterns or arithmetic principles were made apparent to the children.

Control: Children in the control group simply received their normal classroom teaching. During the training period, all groups were seen for one hour per week, for a total of eight weeks.

Screening and outcome measures.

All children were screened using the Group Mathematics Test (Young, 1970) to assess their mathematics ability. This was done in the first session with each group. A further measure of mathematical skills was constructed by the experimenter, the test materials taking a similar format to that described by Russell and Ginsburg (1984). The material for this adapted test is provided in the Appendix. This format was chosen because the type of questions asked covered the majority of the topics covered in the mathematics curriculum in the previous school year, and so would therefore provide a good measure of the child's mathematical learning. The format of the test was as follows.

1. The first tasks were designed to measure *informal concepts of mathematics.* Informal mathematical knowledge is the outcome of the child's encounters with quantity in the social environment, before entrance to school. The child will typically develop notions such as more and less, adding and taking away, and relative frequency. Therefore, the measures included in the test examined the concept of 'more' and knowledge of the relative magnitude of numbers. For example, questions were asked such as "Which number is more?", and "Which number is closer to X?" Children were also required to answer addition questions without writing them down, and their ability to estimate answers was also assessed by asking whether answers given to questions were close to or far away from the real answer.

2. The second group of tasks were designed to measure the child's understanding of *base ten concepts and related enumeration skills.* For example, tasks were given which require enumeration by tens (counting dots), counting large numbers (in the context of money), multiples of large numbers (how many 10's are in 100?), making magnitude judgements of large written numbers (requires knowledge that the left-most number is most important in determining value), and representation of place value (what does the 2 stand for in 327?).

3. Two tasks were given to investigate children's *strategies for solving written addition and subtraction* problems. In the first of these the children were asked to write down and solve a number of addition and subtraction problems, some of which involved no alignment or renaming (carrying or borrowing) difficulties, some of which involved alignment difficulties, and some of which involved renaming difficulties. A number of points were taken into consideration, including correct alignment of numbers, whether the correct operation being carried out, and overall calculation accuracy. The second task required the child to look at addition and subtraction problems which had already been solved and to say whether they had been solved correctly or incorrectly. Some of the questions had been answered correctly, but other questions contained common errors often made by children, for example, calculation errors, alignment errors, and renaming errors. The child was required to explain why these questions were incorrect.

4. The ability to solve story word problems was taken as a measure of *problem solving* ability. Each child was asked to solve thirteen story problems involving addition, subtraction, multiplication, and division.

5. Knowledge of *basic number facts* was assessed by presenting the children with a random selection of 15 addition, 15 subtraction, and 15 multiplication questions on a computer screen, one problem at a time. The child was simply told to work out the answer as quickly and as accurately as possible. The experimenter recorded the strategy used for solution of the problem, that is, whether the answer was retrieved directly from memory, or whether a counting algorithm was used, and also accuracy. The computer recorded solution times, with timing being started with the onset of presentation of the arithmetical problem, and terminated when the child gave an answer, and the response key was pressed by the

experimenter. Additional analysis of the data included an analysis of the types of errors made.

These screening tests were administered again to each group after the eight weeks of training, to assess for any improvements made from pre- to post-training.

7.2 *(ii)* Results.

Results for each measure will first be reported by means of 6 x 2 repeated measures ANOVA's, with one between subjects factor, group (HR, HD, HC, LR, LD, LC), and one within subjects factor, time of testing (pre- and post-training). It should be noted that results for children in the high ability mathematics group are very variable, as they were reaching ceiling levels of performance on many parts of the screening measures. The mean performance on each measure at both times of testing, and the significance of all post hoc Newman Keuls comparisons are summarised in Table 7.2. Newman Keuls comparisons reported in the tables are only those comparing the change in performance within each individual group from pre- to post-training. Groups were not matched on some of the tasks at pre-training, and to take this into account improvement scores were calculated by subtracting the pre-training score from the post-training score. This way, comparisons are being made of absolute improvement, and is irrespective of performance differences at pre-training. Improvement scores for the six groups were submitted to one way ANOVA, and Newman Keuls analyses reported in the text are for comparisons between the groups on the level of improvement made.

Group Mathematics Test.

Repeated measures ANOVA comparing the 6 groups in performance on the Group Mathematics Test from pre- to post-training revealed a significant main effect of group, F (5, 53) = 32.48, $p < .001$, and a significant interaction between group and time, F (5, 53) = 2.76, $p < .05$. Newman Keuls tests analysing the change in mean performance for each group revealed that only the LR group showed a significant improvement from pre- to post-training ($p < .01$, see table 7.2 for mean performance). For the improvement scores, one way ANOVA revealed there to be significant differences between the groups, F (5, 53) = 2.76, $p < .05$. Newman Keuls test revealed that the LR group showed significantly greater improvement compared to the HD group ($p < .05$). No other comparisons were found to be significant. These results are illustrated in Figure 7.1. Observation of mean standard age scores for the HR and HD groups indicates that performance has deteriorated from pre- to post-training. This result may come about because at post-training, when children were 4 months older, they are required to answer more questions correctly to achieve an equivalent standard age score to pre-training. This is very difficult when the children are scoring so highly at pre-training.

To analyse the Group Mathematics Test further, scores on the two sections of the test were considered separately, that is, scores on the oral word questions, and scores on the addition and subtraction calculation questions. A 6 x 2 repeated measures ANOVA comparing groups at pre- and post-training on the oral questions revealed a significant main effect of group, F (5, 53) = 21.34, $p < .001$, with all high ability groups scoring higher than all low ability groups. Whilst most of the groups showed some improvement in performance from pre- to post-training, none of these changes were significant (see Table 7.2). A one way ANOVA showed that there were no significant differences between the groups in the level of improvement made, F (5, 53) = .99, $p > .05$.

Performance on the calculation questions of the Group Mathematics Test revealed significant main effects of group, F (5, 53) = 29.32, $p < .001$, and time, F (1, 53) = 18.16, $p < .001$, along with a significant interaction between group and time, F (5, 53) = 3.13, $p < .05$. Newman Keuls tests analysing the performance for each group at pre- and post-training, revealed that only the LR and LD groups showed significant improvement from pre-to post training ($p < .01$, and $p < .05$ respectively; see Table 7.2). One way ANOVA comparing the levels of improvement made by each group revealed there to be significant differences between the groups, F (5, 53) = 3.13, $p < .05$. Newman Keuls analysis showed the LR group to have made significantly more improvement than the LC, HR, HD, and HC groups, but did not make significantly more improvement than the LD group (see Figure 7.3).

TABLE 7.2.

Performance on the Group Mathematics Test at pre- and post-training, and summary of Newman Keuls comparisons of performance within each group (*SD*).

	Pre-training	Post-training	Newman Keuls
GROUP MATHEMATICS TEST (standard age scores).			
HR	106.42 (6.20)	104.50 (5.79)	*ns*
HD	115.50 (12.13)	112.00 (10.20)	*ns*
HC	107.13 (2.64)	108.13 (4.05)	*ns*
LR	89.20 (4.59)	95.10 (4.91)	$p < .01$
LD	85.35 (7.78)	87.86 (6.27)	*ns*
LC	86.00 (6.04)	86.11 (10.30)	*ns*
GROUP MATHEMATICS TEST: ORAL WORD PROBLEMS (max = 28)			
HR	21.92 (2.57)	21.67 (1.50)	*ns*
HD	23.83 (3.13)	23.67 (2.34)	*ns*
HC	22.38 (1.77)	24.25 (1.83)	*ns*
LR	18.70 (2.00)	19.80 (2.15)	*ns*
LD	14.64 (4.03)	16.36 (2.59)	*ns*
LC	15.00 (3.43)	15.44 (5.05)	*ns*
GROUP MATHEMATICS TEST: CALCULATION (max = 30).			
HR	23.58 (3.77)	24.92 (3.77)	*ns*
HD	27.50 (3.15)	27.83 (2.64)	*ns*
HC	24.88 (3.31)	25.75 (2.61)	*ns*
LR	13.20 (1.69)	20.30 (4.69)	$p < .01$
LD	12.21 (5.07)	16.79 (5.09)	$p < .05$
LC	12.11 (4.08)	13.78 (5.89)	*ns*

Note. HR = High Rote, HD = High Discussion, HC = High Control
LR = Low Rote, LD = Low Discussion, LC = Low Control

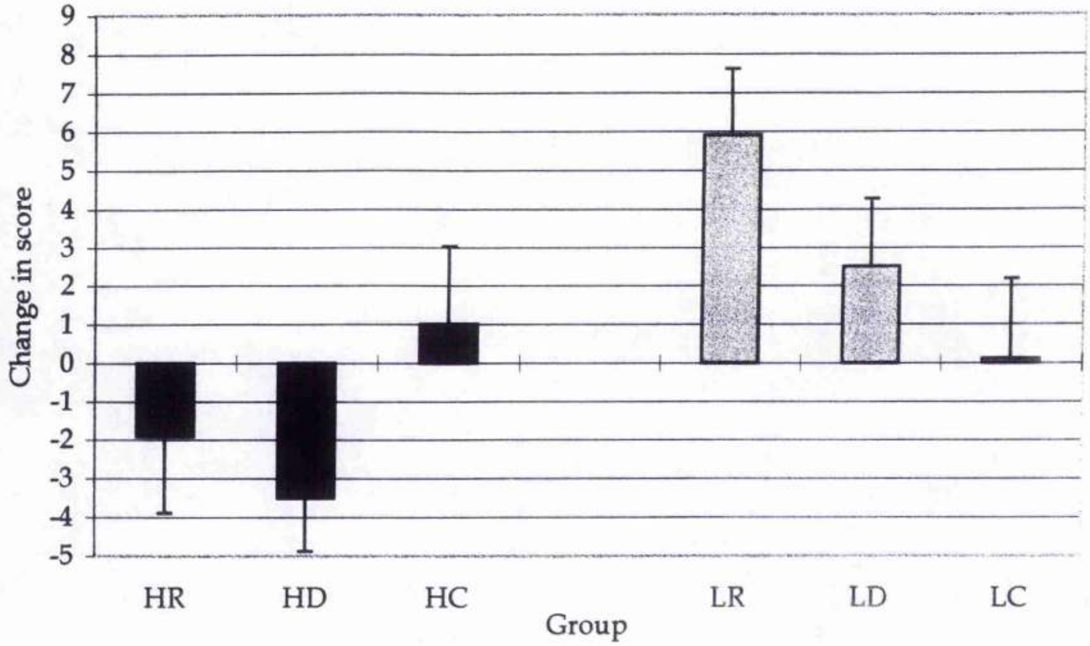

FIGURE 7.1.
Change in performance (age standardised scores) on the Group Mathematics Test (bars show standard error).

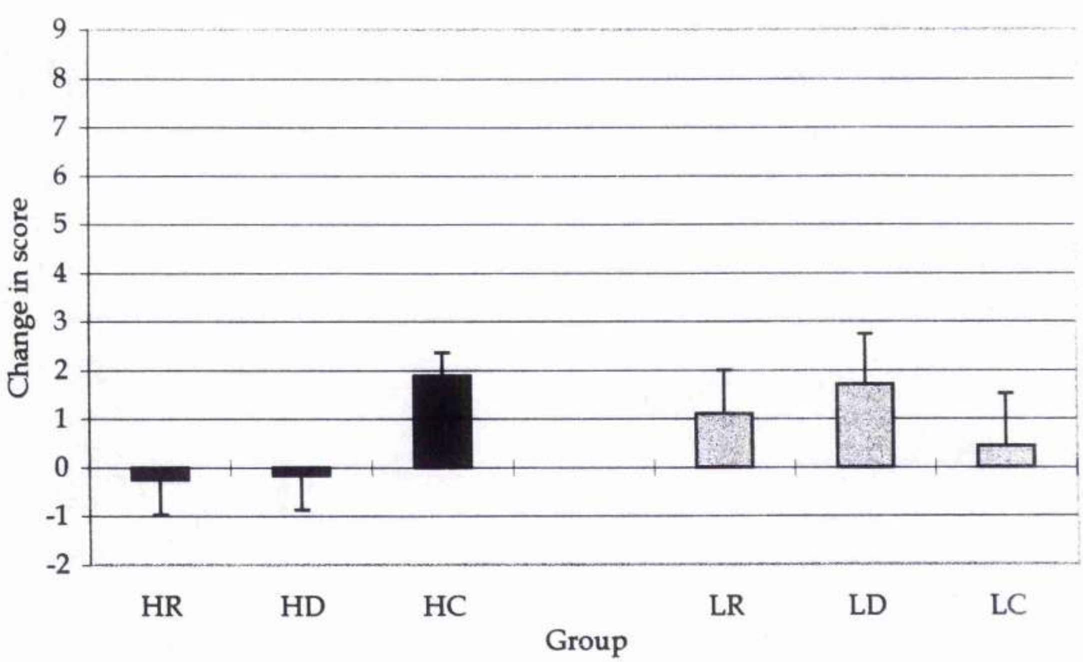

FIGURE 7.2.
Change in performance on oral word problems from the Group Mathematics Test (bars show standard error).

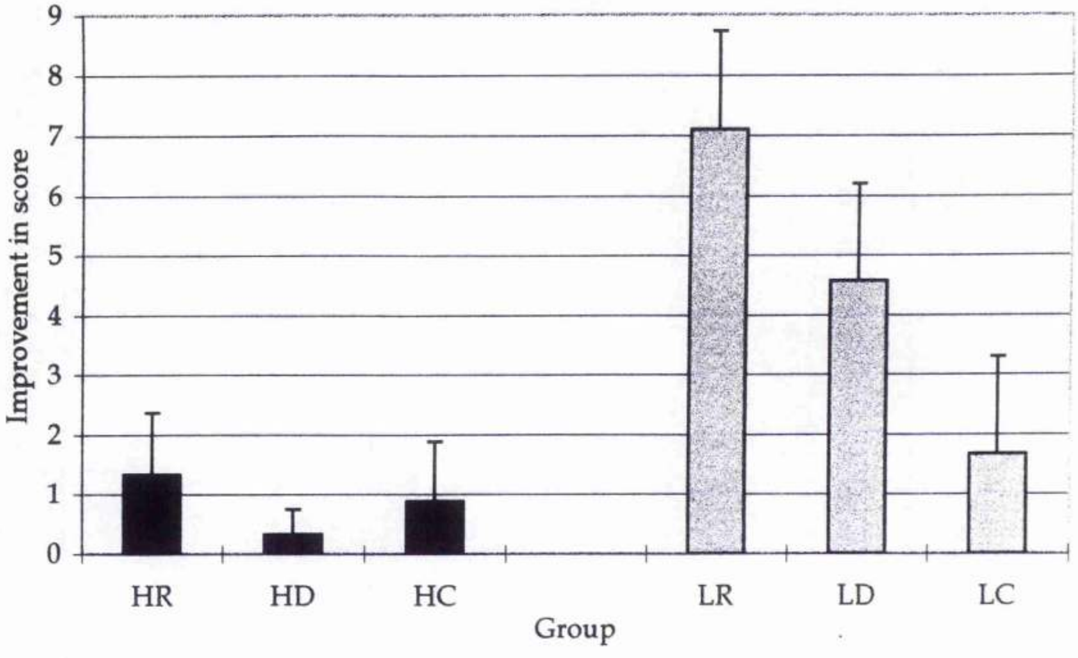

FIGURE 7.3.
Improvement in performance on addition and subtraction calculation questions from the Group Mathematics Test (bars show standard error).

Adapted Russell and Ginsburg test results.

Informal concepts and calculation skills. Performance scores from questions 1 to 4 were added together to give a total score on this mathematical skill (maximum score = 24), at pre- and post-training. ANOVA comparing the 6 groups from pre- to post-training revealed significant main effects of group, F (5, 53) = 10.43, $p < .001$, and time, F (1, 53) = 7.04, $p < .05$, with the interaction between group and time proving not to be significant. Observation of mean accuracy, as reported in Table 7.3, shows that all groups showed some degree of improvement from pre- to post-training. Newman Keuls analysis revealed that whilst all groups did show improvement, no individual group showed significant levels of improvement. There were no significant differences between the groups in the amount of improvement made, F (5, 53) = .465, $p > .05$ (see Figure 7.4).

Base ten concepts and related enumeration skills. Scores from questions 5, 6, 7, 8, and 9 were added together to give a total score on this mathematical skill (maximum score = 32). A 6 x 2 repeated measures ANOVA comparing the six groups at pre- and post-training, revealed significant main effects of group, F (5, 53) = 13.84, $p < .001$, and time, F (1, 53) = 24.57, $p < .001$, with there being no significant interaction between group and time. Again, observation of the mean accuracy rates reported in Table 7.3, indicates than all groups showed some level of improvement from pre- to post-training. Newman Keuls analysis examining within group improvement across time revealed that only the HR and LR groups showed significant levels of improvement across time ($p < .01$, and $p < .05$ respectively). One way ANOVA comparing improvement scores made by each group revealed there to be no significant differences in the levels of improvement made, F (5, 53) = .558, $p > .05$ (see Figure 7.5).

Accuracy and bugs in calculation procedures. Scores on questions 10 and 11 were added together to give a total score for this skill (maximum score = 29). A 6 x 2 repeated measures ANOVA comparing the six groups at pre- and post-training, revealed significant main effects of group, F (5, 53) = 10.91, $p < .001$, and time, F (1, 53) = 5.31, $p < .05$, along with a significant interaction between group and time, F (5, 53) = 3.12, $p < .05$. Newman Keuls tests analysing the change in performance within each group revealed that only the LC group made a significant improvement ($p < .01$; see Table 7.3). This may be the result of very low performance at pre-training compared to all the other groups, such that the LC group has ample room for improvement compared to the other groups. One way ANOVA comparing the levels of improvement revealed there to be significant differences, F (5, 53) = 3.13, $p < .05$. Newman Keuls analyses revealed that the LC group showed significantly higher improvement than the LD group ($p < .05$), with none of the other comparisons between the groups reaching significance (see Figure 7.6).

Problem solving. The maximum score on this skill was 13. A 6 x 2 repeated measures ANOVA comparing the six groups at pre- and post-training, revealed significant main effects of group, F (5, 53) = 12.65, $p < .001$, and time, F (1, 53) = 12.65, $p < .001$, with there being no significant interaction between group and time. Again, all groups showed some level of improvement from pre- to post-training. Newman Keuls analysis of within group improvement revealed that both the HR and LR groups made significant improvement from pre- to post-training (p's $< .05$; see Table 7.3). One way ANOVA comparing the levels of improvement made revealed there to be no significant differences between the groups, F (5, 53) = 1.41, $p > .05$ (See Figure 7.7).

TABLE 7.3.

Performance on the adapted Russell and Ginsburg Test at pre- and post-training, and summary of Newman Keuls comparisons of performance within each group (*SD*).

	Pre-training	Post-training	Newman Keuls
INFORMAL MATHEMATICAL CONCEPTS (max = 24).			
HR	17.92 (2.61)	18.08 (1.93)	*ns*
HD	20.83 (1.72)	21.83 (2.31)	*ns*
HC	18.75 (3.06)	19.75 (2.49)	*ns*
LR	14.20 (3.82)	15.30 (2.26)	*ns*
LD	13.29 (5.15)	15.21 (3.75)	*ns*
LC	10.56 (6.00)	12.44 (4.07)	*ns*
BASE TEN CONCEPTS AND RELATED ENUMERATION SKILLS (max = 32).			
HR	22.67 (5.60)	27.00 (3.84)	$p < .01$
HD	28.17 (3.37)	29.67 (0.52)	*ns*
HC	25.13 (4.42)	28.63 (2.26)	*ns*
LR	18.30 (4.55)	21.40 (4.40)	$p < .05$
LD	15.50 (7.22)	17.43 (5.89)	*ns*
LC	12.78 (5.91)	16.22 (6.38)	*ns*
ACCURACY AND BUGS IN WRITTEN CALCULATION (max = 29).			
HR	21.75 (2.70)	24.00 (3.86)	*ns*
HD	26.67 (1.37)	27.00 (0.52)	*ns*
HC	23.50 (3.67)	25.13 (1.64)	*ns*
LR	20.30 (3.53)	21.20 (3.97)	*ns*
LD	19.07 (5.59)	17.07 (5.14)	*ns*
LC	12.56 (5.64)	17.33 (6.75)	$p < .01$
PROBLEM SOLVING (max = 13).			
HR	6.08 (1.98)	8.33 (2.46)	$p < .05$
HD	10.00 (2.19)	10.50 (2.07)	*ns*
HC	6.75 (1.58)	8.63 (2.26)	*ns*
LR	3.30 (1.49)	6.00 (1.83)	$p < .05$
LD	3.00 (2.96)	4.50 (2.44)	*ns*
LC	4.44 (2.40)	4.67 (3.39)	*ns*

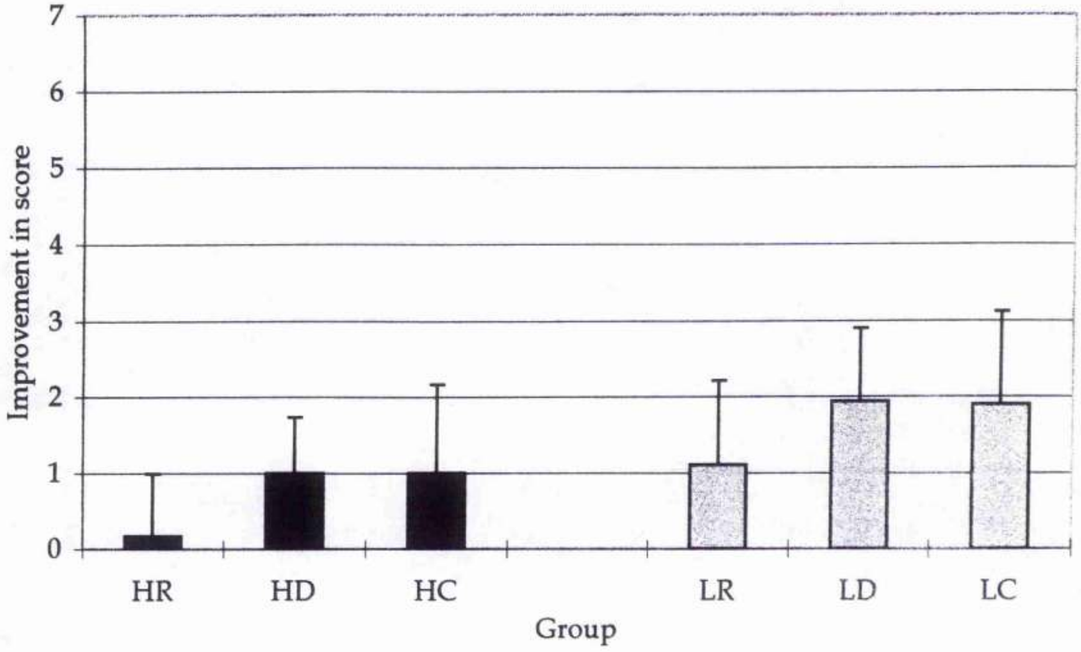

FIGURE 7.4.
Improvement in performance on questions measuring knowledge of informal mathematical concepts (bars show standard error).

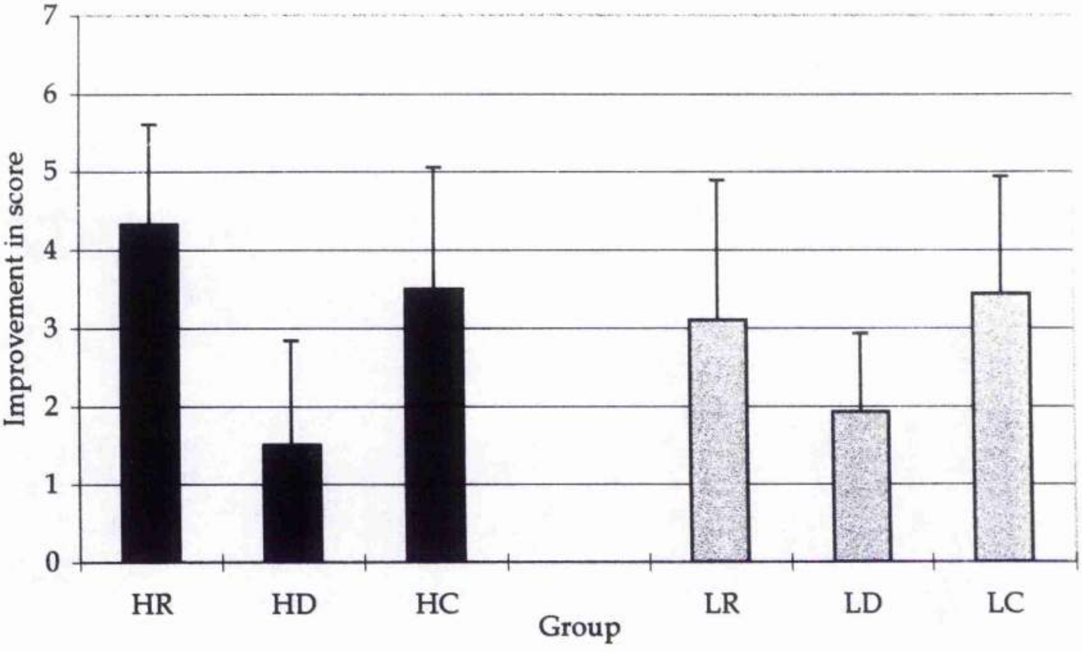

FIGURE 7.5.
Improvement in performance on questions measuring knowledge of base ten concepts and related enumeration skills (bars show standard error).

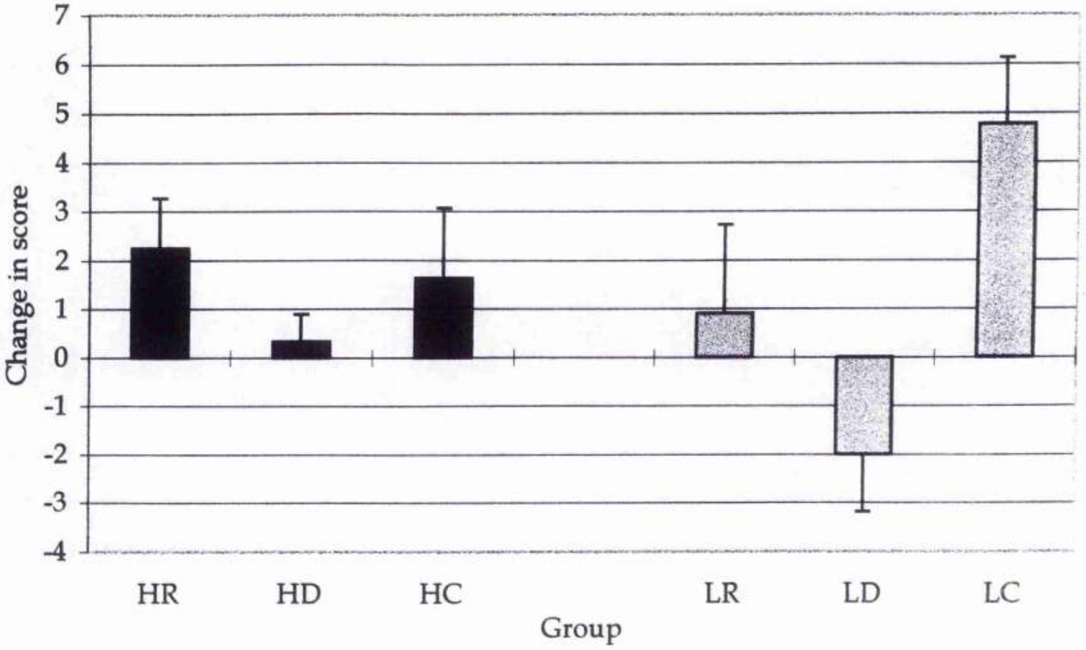

FIGURE 7.6.
Change in performance on questions measuring accuracy and bugs in written addition and subtraction (bars show standard error).

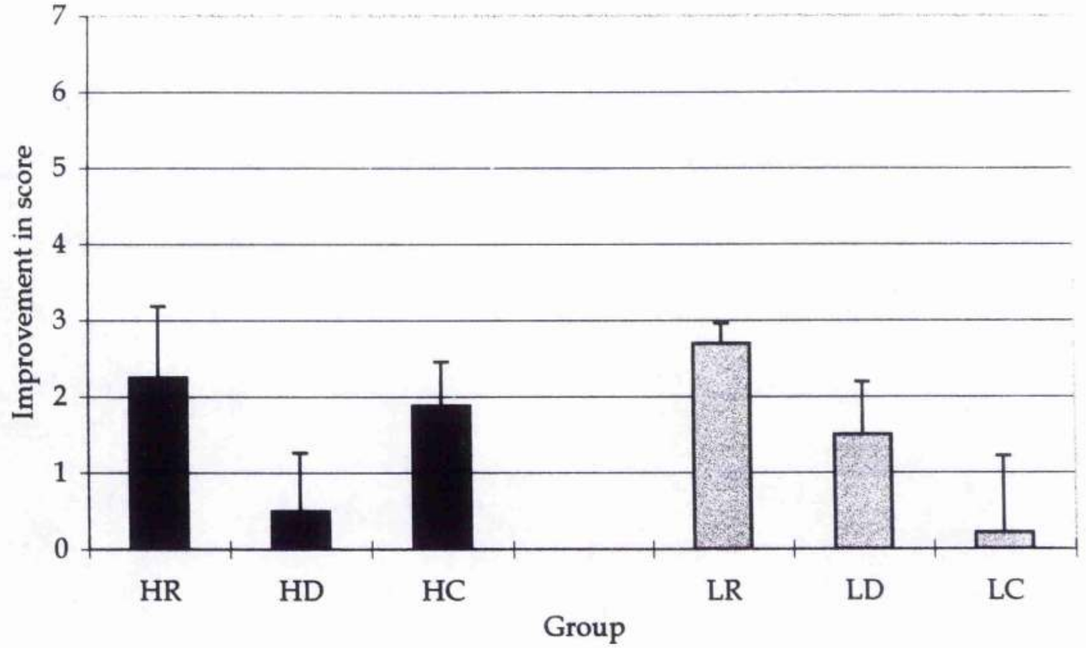

FIGURE 7.7.
Improvement in performance on questions measuring problem solving skills (bars show standard error).

Arithmetic problems (addition, subtraction, and multiplication).

In the analyses of arithmetic problems the three problem types, addition, subtraction, and multiplication, were considered together due to small cell sizes which would have resulted in lack of statistical power.

Strategy use.

The total number of problems solved correctly by direct memory retrieval and overt counting strategies were calculated (addition, subtraction, and multiplication questions), as well as the total number of correct answers, using either strategy. Overt counting strategies in addition involved counting using fingers, verbal counting, and decomposition. Subtraction counting strategies included counting down and counting up, whilst multiplication counting strategies frequently involved repeated addition.

6 x 2 repeated measures ANOVA's were conducted to examine the frequency of use of direct memory retrieval and overt counting strategies at pre- and post-training, by the six different training groups. Analysis of the frequency of direct memory retrieval of arithmetic facts revealed significant main effects of group, F (5, 53) = 6.35, $p < .001$, and time, F (1, 53) = 6.22, $p < .05$, with there being no significant interaction between group and time. Newman Keuls analysis revealed that at pre-training, the HD and HC groups retrieved significantly more answers directly from long-term memory than all low ability groups (p's < .01, with the exception of the comparison of HC to LR where $p < .05$). The HR group retrieved significantly fewer answers directly from memory than the HD group ($p < .05$), but retrieved significantly more answers compared to the LC group ($p < .01$). The HR group showed no significant difference in frequency of memory retrieval when compared with the LR and LD groups. At post-training, all high groups retrieved significantly more answers directly from long-term memory than all low group (p's < .01). The LR and LD groups retrieved significantly more answers direct from memory than the LC group at pre- and post-training (p's < .05). None of the groups showed significant improvements from pre- to post-training.

Analysis of the frequency of use of overt counting strategies at pre- and post-training revealed a significant main effect of time, F (1, 53) = 4.63, $p < .05$, with no other main effect or interaction being significant. All groups showed an increase in the use of counting strategies from pre- to post-training, with the exception of the HD group. The LD group showed the highest degree of increase in the frequency of use of overt counting strategies. The only significant difference at pre-training came from the comparison of the HD group to the LD group, showing the HD group to be using overt counting strategies significantly more frequently than the LD group ($p < .05$). At post-training there were no significant differences between the groups in the frequency of use of overt counting strategies.

Overall accuracy.

The total number of problems answered correctly using any strategy was also analysed. A 6 x 2 repeated measures ANOVA comparing the six ability groups at pre- and post-training revealed significant main effects of group, $F\ (5, 53) = 10{,}70$, $p < .001$, and time, $F\ (1, 53) = 18.32$, $p < .001$, with the interaction group by time not being significant. Newman Keuls analysis revealed that at pre-training the HD group were significantly more accurate than all low ability groups (p's $< .01$), with the HR and HC group being significantly more accurate than the LD and LC groups (p's $< .01$), but showing no difference to the HR group. The HD group were also more accurate compared to the HR group ($p < .01$) and HC group ($p < .05$) at pre-training. At post-training, all high ability groups were significantly more accurate than all low ability groups, with the exception of the comparison between HR and LR groups which remained non-significant (all p's $< .01$, apart from the comparison between HR and LD where $p < .05$). At pre-training, the LR group achieved more correct answers than the LD and LC groups (p's $< .01$), with there being no difference between the LD and LC groups. All groups improved from pre- to post-training, with the LD group being the only group to show significant improvement. At post training, both the LR and LD groups achieved significantly more correct answers than the LC group (p's $< .01$).

Solution times.

Overall mean solution times to arithmetic questions (using both computation and direct memory retrieval) were calculated. Again, 6 x 2 repeated measures ANOVA's were conducted to examine for differences between the groups at pre- and post-training. Analysis of the solution times when using direct memory retrieval revealed a significant effect of time, $F\ (1, 47) = 19.65$, $p < .001$, with there being no significant main effect of group and no significant interaction. All groups showed a decrease in solution times from pre- to post-training.

Analysis of the time taken to solve problems using overt counting strategies revealed a significant main effect of group, $F\ (5, 53) = 6.12$, $p < .001$, and a main effect of time which was approaching significance, $F\ (1, 53) = 3.89$, $p = .054$. There was no significant interaction between group and time. At pre-training, the HD group were significantly faster to use overt counting strategies than all low groups (p's $< .05$). The HR group were significantly faster than the LD and LC groups (p's $< .05$), but not the LR group ($p = .053$).

the HC group showed no significant difference to any of the low groups, although the differences were all approaching significance (p = .090, p = .056, and p = .055, for the LR, LD, and LC groups respectively). The LR group showed the highest decrease in solution time from pre- to post-training, such that at post-training, the LR group showed no significant difference in solution times to the high groups. All other comparisons remained as for pre-training.

Error analysis of arithmetic questions.

Errors were classified into four subtypes according the error analysis made by Rasanen and Ahonen (1995). These were wrong operation (for example, performing a subtraction instead of addition), computation errors (obvious errors made in counting), fact errors (table or closely associated errors in multiplication problems), and random errors (errors that were totally incongruent with the arithmetic problem presented, and don't know responses). A 6 x 4 x 2 mixed design ANOVA was conducted comparing the six training groups on each of the four error types at pre- and post-training. This revealed significant main effects of group, F (5, 53) = 10.20, $p < .001$, time, F (1, 53) = 18.88, $p < .001$, and error type, F (3, 159) = 15.48, $p < .001$, along with significant two-way interactions between group and error type, F (15, 159) = 4.79, $p < .001$, and error type and time, F (3, 159) = 2.73, $p < .05$. There was no significant interaction between group and time, and no significant three way interaction between group, error type, and time. Newman Keuls analysis revealed that there were no differences between the groups in the frequency of wrong operation errors or fact errors. The LC group made significantly more random errors than all other groups (all p's < .01). The largest differences between the groups were found in the frequency of computation errors. All high ability groups made significantly fewer computation errors than the LD group (for HR and HD, p's < .01; for HC, $p < .05$). The HR and HD groups also made significantly fewer computation errors than the LC group (p's < .05). There were no significant differences between the three low ability groups in the number of computation errors made (see Table 7.4).

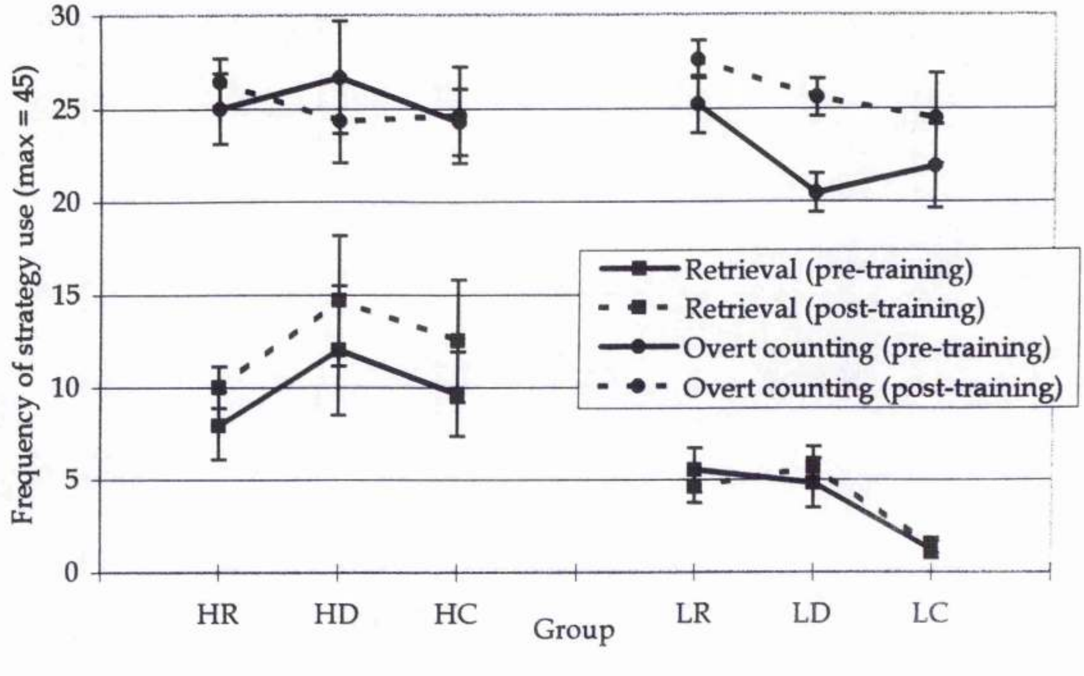

FIGURE 7.8.

Frequency of strategy use (direct memory retrieval or overt counting) for each training group, at pre- and post-training (bars show standard error).

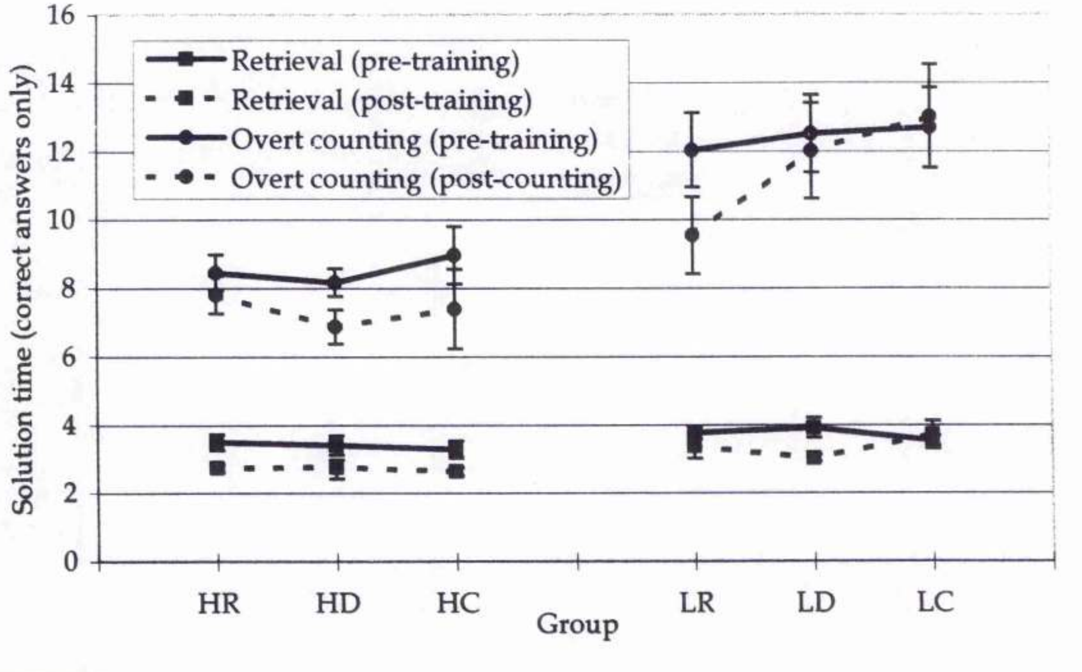

FIGURE 7.9

Retrieval times for direct memory retrieval and solution times for overt counting strategies, for each group, at pre- and post-training (bars show standard error).

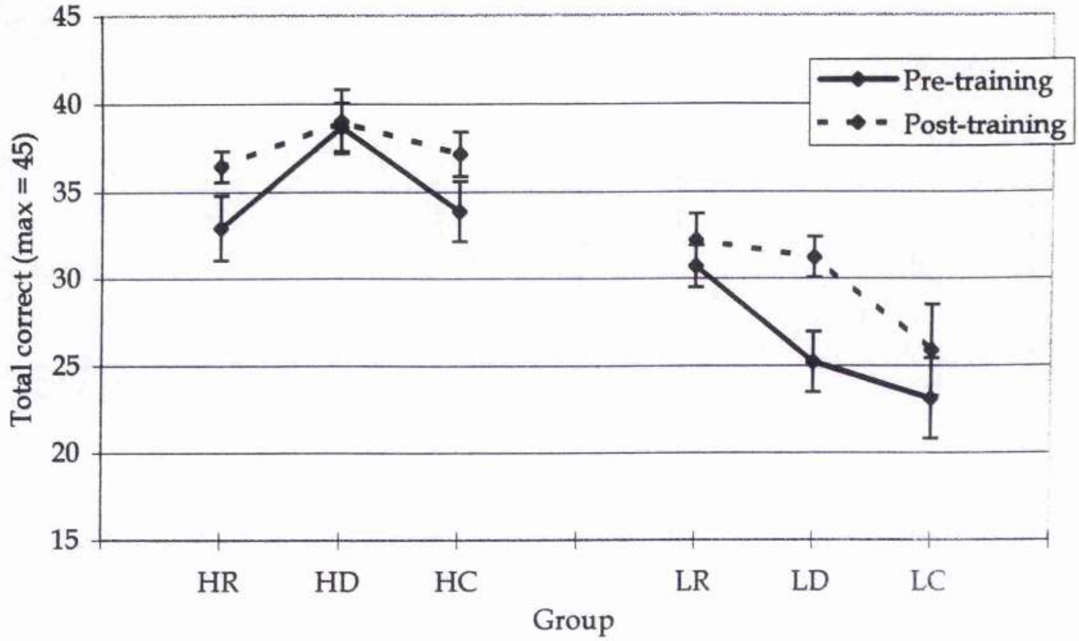

FIGURE 7.10.

Total number of arithmetic problems answered correctly by each group, at pre- and post-training (bars show standard error).

TABLE 7.4

Frequency of errors made by each group at pre- and post training (*SD*).

	Pre-training						Post-training					
Error type	HR	HD	HC	LR	LD	LC	HR	HD	HC	LR	LD	LC
Wrong operation	5.17	1.17	1.25	0.90	1.00	1.89	2.08	0.33	2.25	1.00	0.64	2.22
	(6.18)	(1.17)	(1.58)	(1.29)	(1.71)	(3.10)	(1.38)	(0.52)	(2.87)	(1.25)	(1.22)	(2.54)
Computation errors	1.42	0.67	3.25	3.80	7.14	6.00	1.83	2.17	1.50	3.70	5.50	5.00
	(1.38)	(0.82)	(2.05)	(2.35)	(4.37)	(3.91)	(1.85)	(2.79)	(1.41)	(2.21)	(2.71)	(2.96)
Fact errors	1.50	1.00	3.38	3.90	4.64	3.11	2.25	2.33	2.88	3.70	3.14	2.89
	(1.24)	(1.26)	(3.29)	(2.88)	(3.97)	(3.55)	(0.86)	(1.97)	(1.55)	(1.95)	(1.70)	(2.09)
Random errors	4.0	3.50	2.88	5.60	6.79	11.00	2.50	1.33	1.50	4.30	4.36	9.00
	(2.83)	(3.99)	(1.46)	(3.24)	(4.64)	(5.02)	(1.73)	(1.97)	(2.07)	(1.95)	(3.86)	(6.02)

Analysis of improvement frequency.

Because of the wide deviations within each group which meant that some of the differences between the groups were not significant, it was decided to look at the numbers of children in each group showing improvement, or a decline in performance, from pre- to post-training. For the Group Mathematics Test, those children achieving the same standard score at both pre- and post-training were said to have improved, because the child would need to answer more questions correctly at post-training to achieve the same standard age score as at pre-training. Chi-squared tests for independent samples were conducted to examine frequency of improvement and decline in each training group. Analysis of performance on the Group Mathematics test revealed a significant result, χ^2 (5 df) = 11.44, $p < .05$. Analysis of the residuals, following the procedure described by Siegel and Castellan (1988), revealed that the significant effect was apparent for the HD group, who were found to have significantly more children showing a decline in performance than would be expected ($p < .01$). The only other result proving to be marginally significant was for the LR group, who were found to have more children showing improvement than would be expected ($p < .10$). Observed and expected frequencies, and standardised residuals are shown in Table 7.5.

Chi-squared analysis was also conducted to investigate the frequency of improvement on the sub-sections of the adapted Russell and Ginsburg test. No significant results were found; Informal mathematical concepts, χ^2 (5 df) = 6.66, $p > .05$; Base ten concepts and related enumeration skill, χ^2 (5 df) = 6.08, $p > .05$; Accuracy and bugs in written addition and subtraction, χ^2 (5 df) = 5.94, $p > .05$; Problem solving, χ^2 (5 df) = 9.37, $p = .095$. This suggests that the frequency of improvement or decline made was not outside the frequency that would normally be expected.

TABLE 7.5.

Chi-squared statistics for frequency of improvement on the Group Mathematics Test, showing observed and expected frequencies, and standardised residuals.

		HR	HD	HC	LR	LD	LC
	Observed	6	1	4	9	11	5
Improvement	Expected	7.3	3.7	4.9	6.1	8.5	5.5
	Residual	-0.68	-2.14*	-0.61	*1.71*	1.20	-0.31
	Observed	6	5	4	1	3	4
No improvement	Expected	4.7	2.3	3.1	3.9	5.5	3.5
	Residual	0.78	2.62**	0.61	*-1.75*	-1.18	0.32

Note. For values shown in italics, $p < .10$.

* $p < .05$

** $p < .01$

Summary of results.

Analysis of the improvements made from pre- to post-training on the various measures of mathematical ability reveals a somewhat varied pattern of results. Firstly, focusing on performance of the three low ability groups, the rote learning group showed most improvements. The rote learning group was the only low ability group to show a significant improvement in standard age score on the Group Mathematics Test, and at post-training, the mean standard age score of the group approached the cut-off of 96 which had been used for classifying children as low or high ability mathematicians. Neither the discussion group nor the control group showed a significant improvement, with their mean standard age scores remaining below 90. None of the three low ability groups showed significant improvement on the oral questions of the Group Mathematics Test, but for the calculation questions, both the rote learning and discussion groups showed improvements across time, with the rote group answering around eight more questions correctly and the discussion group answering around seven more questions correctly. The control group showed no improvement in performance across time.

Measures of informal concepts and calculation skills from the adapted Russell and Ginsburg test revealed that whilst all of the low ability groups improved from pre- to post-training, none of the changes were significant. However, the measure of base ten concepts and related enumeration skills revealed that whilst again all groups showed some improvement at post-training, the rote learning group was the only group to show a significant level of improvement. For the measure of accuracy and bugs in calculation procedures, the only significant improvement shown was by the control group. This was probably the result of very poor performance at pre-training, leaving much greater scope for improvement for this group. At post-training, the performance of the control group was much closer to that of the other low ability groups. Finally, for the measure of problem solving, all three groups showed an improvement in performance across time, although this improvement was only significant for the rote learning group.

The measures of simple arithmetic knowledge showed that there were no significant changes in the frequencies of using direct memory retrieval and overt counting strategies. However, the discussion group was the only low ability group to show a significant increase in overall accuracy from pre- to post-training. Observation of solution times revealed that for direct retrieval, all groups were faster at post-training, although none of the changes were significant. Solution times for the use of counting strategies revealed that the rote learning group showed the most improvement in solution time, although this was not significant, and at post-training, the solution times of the rote group were not significantly different from any of the high ability groups.

Turning attention to the high ability mathematics groups, performance on the Group Mathematics Test revealed that none of the groups showed a significant improvement from pre- to post-training. This is due to the fact that the scope for improvement is limited, given their high performance scores at pre-training. Performance on the adapted Russell and Ginsburg test showed that the only significant improvements in performance from pre- to post-training were made by the rote learning group on measures of base ten concepts and related enumeration skills, and on the measure of problem solving. None of the groups showed significant changes in performance on the simple arithmetic measures.

7.3 Discussion.

This study aimed to assess the effectiveness of two strategies for mathematics teaching that were derived from the results of the previous experimental chapters which identified two cognitive difficulties shown by children who were below average at mathematics (these results are also reported in Bull & Johnston, 1997, and Bull, Johnston, & Roy, in press). One of these strategies aimed to overcome the pervasive problem of poor arithmetic fact retrieval shown by children of lower mathematical ability. The other teaching strategies revolved around the finding that children who are poorer at mathematics have difficulty in their flexibility of thinking and in higher metacognitive thought. This study revealed that rote teaching of arithmetic facts may provide a simple but effective teaching strategy in early primary school mathematics. Children in the rote group showed significantly more improvement in standardised mathematics test performance, and in problem solving.

This result can be explained very simply by previous results from experimental psychology. One of the major problems previously identified for children who are poor at arithmetic, has been in the retrieval of arithmetic facts accurately and rapidly from long-term memory. Children who are poor at mathematics show very low levels of automated fact retrieval, instead relying on inefficient and immature back-up counting strategies such as counting on fingers. These strategies place extensive demands on limited cognitive resources, and this leads to errors being made, and less work being completed. Again, if we refer back to Salthouse's ideas regarding the limited time mechanism and the simultaneity mechanism, then it is easy to see how a slowness to complete the very basic of arithmetic operations will impede on the child's ability to complete the processing of more complex mathematical problems. This will be either the result of insufficient cognitive resources being available to complete later operations required by the task (as predicted by the limited time mechanism), or the result of loss of initial information that is required for completion of the task (as predicted by the simultaneity mechanism). Teaching children arithmetic to a level of automaticity through rote learning means that answers to arithmetic problems are retrieved automatically, placing little, if any, demands on limited cognitive resources. This not only benefits the solution of basic arithmetic problems, but also helps in the solution of more complex problems, for example, in the solution of multi-digit arithmetic problems and in problem solving, as more cognitive resources are available for solution of other parts of the mathematical problem, rather than all of the cognitive resources being taken up simply for solving the easiest parts of the problem.

Results from the simple arithmetic measures revealed that the eight weeks of training was not sufficient to lead to a significant change in the frequency of directly retrieving arithmetic facts from long-term memory for the low rote learning group. However, these children did show the greatest improvement in solution times for using overt counting strategies, which may form the basis of their observed improvements on various parts of the Group Mathematics Test, and the Russell and Ginsburg test. Whilst automaticity does not appear to have improved per se, more efficient strategies may be being used by the children, or there could be increased reliance on a form of decomposition to known number facts, as discussed in chapter 6.

Of course, the discussion, or problematising, approach used in this study also has its benefits. This technique may be more advantageous for older children who are learning more complicated mathematical procedures, and where knowledge of multiple routes to solution would be of benefit to the child. As Geary (1995) points out, a method such as this which reveals multiple routes to solution is likely to foster conceptual understanding of the problem, but not necessarily procedural understanding, which is obviously very important when learning mathematics at this young age. One of the reasons for such a metacognitive method not being as effective in this study may be that the attention span of children at this age may be insufficient to support the discussion necessary to gain full benefit from such an approach. Children frequently lost trains of thought and diverted to social talking with other children in the group (a point also raised in other studies investigating collaborative learning, e.g., Tudge, Winterhoff, & Hogan, 1996). It has been noted in a number of studies that children who have difficulty learning are very passive in their approach to learning. Also, the expressive language abilities of these children, that is, their ability to express their ideas and strategies for solution of mathematical problems may not be well enough developed at this age to gain full benefit from what is clearly a knowledge rich technique for learning mathematics. Furthermore, our previous research (Bull, Johnston, & Roy, in press) has found that children who are poor at mathematics have poor executive skills, that is, lack of higher level thinking, poor task initiation, and poor attentional skills. If we have a curriculum based on metacognition and higher level thinking, that requires children to be flexible in their thinking for applying the most appropriate solution heuristics, then clearly children who may have a cognitive difficulty with executive functioning will find it difficult to cope with such a curriculum, and may benefit more from teaching procedures based on drill and practice. Furthermore, for those children who find it difficult to maintain attention, an effective approach to teaching might be to strip away all irrelevant details of a problem so that the child is not distracted from the solution of the specific problem. Once the child is successful with such problems, it should be possible to introduce more difficult problem solving formats.

It has been pointed out in a number of studies that identifying specific arithmetic disabilities in a sample of young children, such as those in the present study, is very difficult, and very few children at this age will show a specific deficit. However, as Ackerman and Dykman (1995) report, the older the child becomes, the more severe the arithmetic retardation becomes. Clearly, if children with arithmetical difficulties can be identified at an early age and given help, then this will reduce the degree of arithmetic retardation in later years when more complex mathematical problem solving will be occurring.

There were a number of methodological and logistic difficulties associated with this study. The formation of groups was restricted because of the necessity to take children out of the classroom in the mathematics groups that had already been established by the teachers. The initial methodological design incorporated mixed ability teaching groups, so that children who were poor at mathematics would benefit from the ideas of their higher ability peers. If we refer to Vygotsky's ideas about how children learn through social interaction with others, we find that collaboration with adults or higher level peers is of critical importance to the development of mental operations. This may have particularly impacted on the discussion technique where children of higher ability would have been able to contribute more to the discussion of methods for solution of mathematical problems. Recent research has also noted the importance of peer collaboration in learning skills, and research is being undertaken to identify the most effective forms of peer collaboration and group learning, for example, what skills are necessary to be a good teacher and at what age it is possible to engage in this type of instruction (see, e.g., Flynn, Ding, & O'Malley, 1997; Smith, Howe, & Noble, 1997; Tudge et al, 1996). Developments in the understanding of peer collaboration and learning will obviously have a bearing on the direction of future educational research such as that reported in this chapter.

Clearly, the results from this study are not as clean and clear cut as we might have wanted, and certainly do not allow us to say that rote teaching of arithmetic facts is the best method for teaching early mathematics. However, children who did undergo rote learning did show some significant improvements in specific aspects of mathematical skills. The logistical and methodological problems must also be taken into consideration. This was a study where the training only lasted for eight weeks, and where circumstances were far from ideal. Despite this, some improvements were still found. This suggests that a larger scale study over a longer period of teaching and under conditions where the teaching being carried out in the classroom could be controlled, might result in much more clear cut results, and would identify effective techniques for teaching mathematics to children.

In summary, I have no doubt that constructivism is an effective teaching method, but only for some children, who have the intellectual and social capacity to deal with such a teaching method. Furthermore, this technique may take much longer to implement and for there to be any signs of improvement by the children. Children might find this very discouraging. Those children who find it difficult to learn may not be able to benefit fully from such a teaching method, with the only method succeeding to 'get the information in', being through extended practice through drill and rote learning. Whilst short-term benefits have been found with such teaching methods, the longer term effectiveness is still open to question. However, rapid improvements in arithmetic skills may lead the child to feel more confident about their own abilities and to engage in more mathematical activities. Therefore it is essential to bear in mind when assessing the effectiveness of teaching strategies that not all children will learn in the same way, and teaching strategies may need to be adapted to fit the intellectual, cognitive, and social capabilities of the child.

Chapter 8.
General Discussion.

8.1 General Summary of Findings.

The research presented in this thesis was carried out with the intention of identifying cognitive difficulties which may impact on the child's ability to learn simple arithmetical and mathematical skills. To this end, four studies were conducted examining a range of cognitive skills, and whether they were in some way connected with numerical skills. Study 1 revealed that mathematical and arithmetical skills were associated with cognitive mechanisms such as short-term memory and speed of retrieving information from long-term memory. However, regression analyses revealed that there was something more fundamental to these cognitive deficits; a deficit in the speed of processing information which had an influence on skills relating to the storage of information in short-term memory and the retrieval of information from long-term memory. Examining this processing speed deficit further in Study 3 revealed that whilst there may be a general slowness in processing information which accounts for poor mathematical skills, this is further influenced by a more specific and more obvious slowness to process basic arithmetical information.

The finding of no direct relationship between short-term memory and mathematical skills in Study 1, which goes against the majority of previous research, led to more careful consideration in Study 2 of the functional mechanisms of working memory. This suggested that one possible link to poor mathematical skills may come from poor executive processing, for example, in the ability to flexibly switch retrieval strategies, to self-monitor one's own actions, and to plan ahead.

Given the finding showing poor mathematicians to be slow processors of numerical information, and based on numerous previous studies showing children of poor mathematical ability to use immature, slow solution strategies for arithmetic problems, Study 4 used two methods to try and discover whether this could be attributed to a simple developmental delay, or a more significant developmental difference. Using a priming procedure, and taking a short longitudinal measure of the development of automated arithmetic fact retrieval, it was found that children of both high and low mathematics ability were equally susceptible to a prime, indicating similar long-term organisational attributes of arithmetic facts. Further, children of low mathematics ability were progressing in their automatisation of arithmetic facts at a similar rate to their higher ability peers, only at a lower level of performance, indicating a developmental delay rather than some idiosyncratic difference. Armed with the knowledge that children of low mathematics ability are slow to execute basic numerical operations, that they are delayed in learning basic arithmetic facts, and that they show difficulties in switching strategies for solution and monitoring their own actions, the final study, Study 5, aimed to compare two different approaches to teaching mathematics. Rote learning and metacognitive strategies were used. Performance measures revealed that for children of lower mathematical ability, rote learning of basic arithmetic facts may be a necessary pre-requisite for the learning of more complex mathematical skills.

8.2 General Discussion.

The recent concerns over numeracy standards in schools makes this research particularly timely. This is a very under-researched area, and in some respects a number of the studies reported in this thesis suggest new lines of enquiry for trying to understand the cognitive difficulties underlying children's poor mathematical skills. The results from the studies reported in this thesis have uncovered a number of cognitive limitations not previously recognised in studies of children's mathematics, for example, the speed of information processing deficit, and poor executive functioning. Of course, to gain a full understanding of children's mathematical difficulties it will be necessary to consider these cognitive limitations in conjunction with educational research which tries to find the best ways of teaching school subjects to children. In order to extrapolate these results to a wider range of children, it will also be necessary to consider social, cultural, and motivational issues relating to both learning and academic performance in the school, and interest shown in the home environment. Only then will a full picture emerge as to the reasons why some children have greater difficulties than others in learning mathematics.

One of the fundamental difficulties identified in this research, and one which has for the most part been completely overlooked in previous studies, was that children who were poor at mathematics were slower to process information of various types. There is indeed a growing body of evidence to suggest that differences in the speed of processing information underlie individual and age related changes in performance on a variety of cognitive skills. Salthouse (1996) even suggests that "every theory examining cognitive processes will need to take into account factors relating to basic processing efficiency, or else they run the risk of focusing on merely another symptom of what could be a broader and more fundamental phenomenon" (p.425). The fundamental nature of this deficit in processing speed can be illustrated by considering briefly all the main findings reported in this thesis. Some children are slower to process information than others; Where speed of processing information is slow, and excessive demands are placed on working memory, errors will be made, or less work will be completed. This is a line of reasoning that follows from Salthouse's (1996) descriptions of the simultaneity and limited time mechanisms. Both errors and a lower frequency of exposure will lead to either incorrect or particularly weak associations being made in long-term memory. This will mean that information will be slower to be automated and less efficient strategies for the solution of basic arithmetic problems will be used. This is indicative of the classic performance characteristics shown by children who are poor at arithmetic. Finally, performance on more complex mathematics and problem solving is dependent upon the efficiency of executing the more basic, lower level arithmetic operations. Therefore, the fundamental theme is that those children who are slow to process information will be less efficient at using a range of cognitive mechanisms, and will find it more difficult to learn cognitive skills such as mathematics.

Another of the main aims of this research was to discover whether the mathematical difficulties shown by children at this age can be thought of in terms of a developmental delay, or, perhaps more seriously, as an idiosyncratic developmental difference. Of course, the most appropriate methods for studying developmental delay would be to use matched control conditions where a group of lower ability mathematicians are matched on chronological age to a group of higher ability peers, and to the mathematics age of a normally performing younger group of children. Only by using such methods can we make definitive statements about developmental delay or difference. Unfortunately, this method was not possible given relatively small samples of children, but should perhaps be incorporated into the design of future experiments. However, by considering the performance characteristics discussed in studies of younger children, we know that the children of lower mathematics ability in these studies were showing very similar performance characteristics to those we would expect in a sample of younger children. The results from this research would tend to suggest that most children are simply developmentally delayed in their automatisation of arithmetic facts, resulting from limitations in cognitive functioning, and it is this difficulty in using basic information which manifests itself in more complex mathematical problem solving. Of course, for those people at the extreme of the mathematical difficulties spectrum, their difficulties may be more idiosyncratic rather than just being a developmental delay. Therefore, it is difficult to extrapolate many of the results from the studies reported in this thesis to populations who have more extreme difficulties. However, by studying children in the classroom, who do not represent the extremes in learning disabilities, the results reported here are much more easily generalisable to the relatively large number of children who have difficulties learning mathematics in school, but who are not referred for remedial help. Studying other populations of children, such as those clinically diagnosed with dyscalculia and severe forms of mathematical difficulties, may show a variety of different cognitive deficits which account for their problems. Obviously, a comparison of results across these broad abilities would be helpful in pinpointing the cognitive limitations that may have the greatest impact on serious mathematical difficulties.

A factor only touched upon by these studies is the importance of the outcome measure of mathematics ability used. The results reported at the end of Chapter 5 revealed that different cognitive limitations were associated with different measures of mathematical ability, working memory being associated with arithmetical skills, and processing speed being associated with a more general measure of mathematics ability. Other studies have shown that mathematical skills such as geometry may rely heavily on visual-spatial skills. Needless to say, that if the adapted Russell and Ginsburg test used in Study 5 had been considered in conjunction with these cognitive mechanisms, the results may have become even more confusing. This need for a more specific analysis is also made apparent if we consider how age related changes in mathematical performance may be related to the need to use different cognitive skills to a greater or lesser extent. This issue was briefly referred to in the discussion of Study 2, where it was noted that visual-spatial skills may be particularly important for younger children who are using physical representations to aid counting, with more complex mathematical procedures performed by older children perhaps becoming more reliant on both short-term and long-term memory skills, and the ability to rapidly process information. Both of these issues lead to one important consideration for future research, that is, to focus more closely on specific aspects of mathematics ability within different age groups, and to examine the cognitive mechanisms related to skilled performance in that specific ability. By doing this, the specific mathematical difficulties of a child at any particular age can be taken into consideration, and the associated cognitive limitations associated with that skill can be assessed.

8.3 Future Research.

The burning question is what direction should this research now take? Obviously, the need for effective teaching techniques, that may have to be adapted to meet the cognitive limitations of some children, is of critical importance. Study 5 showed that one effective teaching technique might be rote learning of basic arithmetic facts, and possibly, the greatest benefit from such a teaching approach may be found by applying it to very young children, either before formal schooling or during the first year of schooling. This way, the basic facts can be built upon during later years of mathematical learning. Learning complex mathematical skills will be difficult if the bare bones are not fully developed. However, children may find such teaching methods rather dull and uninteresting, so it would seem essential to present such methods in a more interesting format. One format currently being developed by myself and a colleague is a computer game involving frequent exposure to and solution of basic arithmetic facts, such that children think they are playing the game, when in fact there is also incidental learning of basic arithmetic facts taking place. We hope to pilot this programme in the near future with children in the first few years of school, and possibly pre-school children.

Studies also need to be conducted to provide insights into how the developmental sequence of counting strategies might be abbreviated. For example, it may be possible to teach a visual-spatial counting strategy that may be more efficient and effective than strategies involving using physical representations to aid counting, or to introduce decomposition strategies at an earlier age. Clearly, the sooner basic mathematical facts and procedures can become efficiently used, the lesser the impact on limited cognitive mechanisms.

In continuing with research examining cognitive deficits in mathematical skills, there are two very interesting lines of enquiry to follow. The first of these revolves around the observed deficit in processing speed shown by children with poor mathematical skills. As referred to earlier, Salthouse suggests that virtually every theory examining cognitive processes will need to take into consideration changes in processing speed and efficiency. Salthouse also points out that one important issue that must be considered is how these changes in processing speed come about, with particular emphasis on the importance of the neurophysiological basis of changes in speed of processing. These include changes in the extent of myelinization, or in the propagation of the connections between neural units, for example, by changes in the functioning of neurotransmitters (see Salthouse, 1992b). Such changes would account for more rapid processing of information by older (or more able) children, as well as the slowing of information processing in older adults. Differences in brain development at an early age may be the result of a number of factors. Perhaps most importantly for this research, the experiences in the early years are critical, as those children who do not have enriched, stimulating environments may be delayed in, or fail to develop, the neural connections and pathways that will facilitate later learning. Of course, this is pure conjecture, and is well beyond the scope of the research conducted for this thesis.

However, one possible explanation which incorporates the findings of the experimental studies in this thesis is that some children may show a developmental delay in neurological maturation. This may lead to two problems. Firstly, delayed neurological maturation shown by the extent of myelinization of nerve fibres may lead to a general slowness to process information. Secondly, we know that with the development of the frontal lobes comes skilled executive functioning, so another difficulty shown by children with delayed neurological maturation may be a deficit on tasks requiring executive skills. A deficit in processing speed and in executive functioning, especially when occurring in conjunction, will lead to general short-term memory deficits, which will further impact on the child's ability to learn cognitive skills such as mathematics and reading. Tentative suggestions have been made that there may be a neurophysiological explanation for this deficit, and one way to investigate this directly would be through the use of brain scanning techniques that would allow observation of blood flow and electrical activity in the brain. Of course, this is difficult to administer with children, but may be one of the few methods of determining for certain whether differences in processing speed do have a neurophysiological basis. The second line of enquiry to follow would be further investigations of the role of central executive skills. However, before practical applications of central executive skills can be understood in detail, research needs to be conducted to strengthen our theoretical understanding. Currently, tasks are used and functions are ascribed to the central executive that are not fully understood. Researchers need to address this issue rather than keep conducting research that may not be theoretically sound or fully reasoned through.

Some of the studies reported in this thesis did touch on the possible connection between reading and mathematical skills, and identified some cognitive limitations that may be applicable to both, and others that may be more specific to either mathematical or reading difficulties. The initial larger scale screening did reveal that many children with mathematical difficulties also had corresponding reading difficulties, and this was further supported in all the reported studies which showed there to be very strong correlations between mathematics and reading ability. Therefore, it would seem appropriate that future studies might consider in more detail why such a co-occurence of reading and mathematical difficulties is so prevalent and what are the cognitive limitations underlying such difficulties. This may in fact prove to be a more fruitful line of research than trying to conduct experimental studies with small groups of children who have very specific difficulties. In studying children with specific deficits, the best way forward will be to conduct detailed case studies to compare their performance with that of children who show general academic difficulties.

8.4 Concluding Remarks.

The study of children's mathematical difficulties is akin to a book that has just been opened; the outline and the plot have been set, but new plots and characters will be incorporated into the story as it progresses, there are still numerous possible endings to the book, and there is still a long way to go. The possibilities for research in the field of children's mathematical difficulties are still very much open to new and exciting lines of enquiry. Recent research has set the groundwork and the basic ideas are established, but new lines of thought are being introduced all of the time. Certainly, there are still numerous possible endings to what will inevitably be a very long story, but hopefully, the research carried out for this thesis, whilst also investigating recognised deficits, has set the scene for original lines of reasoning about why some children have difficulties in learning mathematical skills.

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Stimuli for Phoneme Deletion Task.

Words		Nonwords	
Initial phoneme	Final phoneme	Initial phoneme	Final phoneme
h/ard	lear/n	f/ard	pur/m
b/lood	sca/le	k/lud	spa/le
m/ind	stoo/d	g/ind	spoo/t
f/lat	sma/ll	s/mab	sno/l
f/loor	des/k	f/rosh	bes/k
w/ild	tur/n	j/ild	fer/m
w/ork	sal/t	d/urk	nol/p
b/rown	mus/t	t/rown	nus/t
g/rass	brea/th	p/rass	prea/th
s/tep	slee/p	s/kep	smee/p
c/ost	mos/t	n/ost	kos/p
n/ext	cla/ss	l/ext	bla/ss

Stimuli for Regularity Task.

Regular words	Irregular words
stuck	touch
dust	deaf
pest	pint
luck	lose
wake	aunt
gang	wool
treat	bush
base	doll
rub	sew
mile	soul
spade	steak
spear	prove
slate	glove
dive	broad

Stimuli for Nonword Reading Task.

Short nonwords	Long nonwords
hig	gantok
nal	muntal
kug	renbok
bis	sanlud
gok	minlan
dep	ritney
foy	yomter
kun	nurdal
ged	daspog
lar	ludpon
jek	bosdin
lan	culgin
mip	fambey
pos	kesdal
ruk	libnol
dal	bantik
ped	lemfid
fik	mitson
lom	goklup
sul	puklon

Adapted Russell and Ginsburg Test used in Study 5.

Please write your name ______________________________

1. You are going to see two numbers and you have to say which number is more. Here's some practice ones

Which is more, 5 or 10? __________
Which is more, 45 or 19? __________

Now lets try some bigger numbers.

Which is more - 8000 or 3200? __________
Which is more - 372 or 701? __________
Which is more - 1800 or 3000? __________
Which is more - 603 or 552? __________

2. If I asked you which number is closer to 6, the 4 or the 9, the answer would be 4, because 4 is only 2 away from 6, but 9 is 3 away from 6? Here's a practice.

Which number is closer to the 7, 2 or 10? __________

Now lets do the same with some bigger numbers.

Which number is closest to 200. 99 or 400? __________
Which number is closest to 6000. 2000 or 8000? __________
Which number is closest to 700. 300 or 900? __________
Which number is closest to 5000. 2000 or 9000? __________

3. Now I'm going to give you some adding and take away problems. I want you to try and work out the answers by counting in your head or using your fingers.

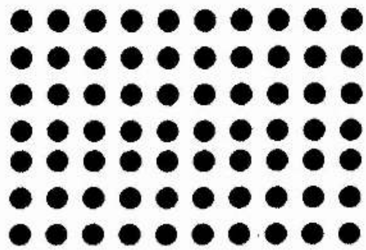
17 + 5 ____________
12 - 4 ____________
57 + 23 ____________
16 + 19 ____________
42 - 7 ____________
40 - 16 ____________
220 + 110 ____________
38 - 18 ____________
114 + 178 ____________
73 - 21 ____________

4. I'm going to show you some problems and some answers. Some of the answers are close to the real answers and some are far away. Like 2 + 2 = 5. That's not the right answer, but it is close to the right answer because 2 + 2 = 4. How about 2 + 2 = 100? That's far away from the real answer. I'm going to give you some more bigger problems, and I don't want you to work out the answer, I just want you to write down whether the answer is close to or far away from the real answer. If you think the answer is close put a tick, if you thing it is far away put a cross.

31 + 42 = 70 ____________
92 + 23 = 50 ____________
58 + 32 = 926 ____________
353 + 520 = 900 ____________
210 + 560 = 800 ____________
460 + 570 = 8000 ____________

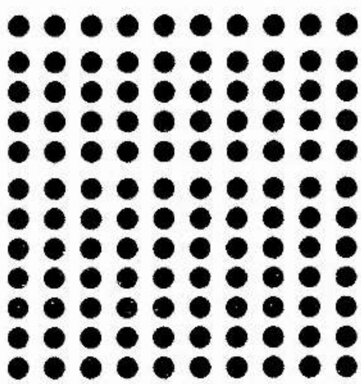
5a. Write down how many dots there are.

Answer ______________



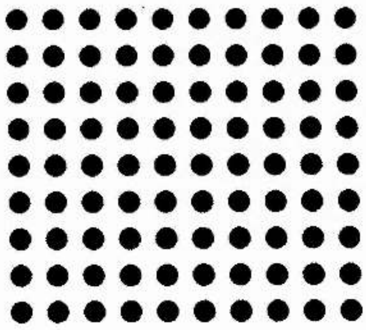
5b. Write down how many dots there are.

Answer ______________



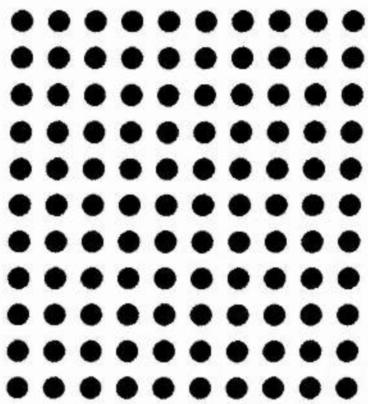
5c. Write down how many dots there are.

Answer ____________



5d. Write down how many dots there are.

Answer _________



6. Ask me for the piles of play money. Write down how much money you think there is in each pile.

1. ____

2. ____

3. ____

4. ____

7. If I ask you how many 2's you must add together to make 4, you would say 2, because two 2's make 4. How many 2's would you need to add together to make 8? How many three's do you need to add together to make 9? Now try and do the same with some bigger numbers.

1. How many 10's are there in 100? ______________________

2. How many 2's are there in 10? ______________________

3. How many 100's are there in 1000? ______________________

4. How many 50's are there in 100? ______________________

5. How many 20's are there in 100? ______________________

6. How many 500's are there in 1000? ______________________

8. Now I'm going to show you some numbers and I want you to put a circle around the number which you think is the biggest.

822 288	577 775	3999 6222	5444 4555	799999 811111	522222 288888
833333 177777	944444 499999				

9. Now we are going to do some work with hundreds, ten and units. Here's a practice. If you think of the number 25, the number 2 stands for 2 tens (twenty), and the number 5 stands for 5 units.

T	U
2	5

What does the number:

2 stand for in 297 ____________________
9 stand for in 893 ____________________
7 stand for in 372 ____________________
9 stand for in 129 ____________________
0 stand for in 230 ____________________
6 stand for in 632 ____________________

10. I'm going to give you some adding and take away problems, and I want you to write down the sum, and then work it out. I have written the first one out to show you what to do.

21 + 37	21
28 - 7	+ 37
49 - 32	____
12 + 6	____
57 + 25	
185 + 72	
64 - 28	
234 + 43	
179 + 153	
252 - 198	

12. To finish with we are going to solve some story problems.

Charlie had 11 pennies. When he was walking to school he found seven more pennies. How many pennies did he have altogether?

Mary liked to eat sweets. She had 14 sweets and ate seven of them. How many sweets did she have left?

Larry collected football stickers. On Monday, he collected 7 stickers, on Tuesday ten stickers, and on Wednesday six stickers. How many stickers did he have altogether?

There are 23 children in the dinner hall. Seven are boys and the rest are girls. How many girls are there in the dinner hall?

Mary and John were playing cards. After seven turns, Mary had 2 points and John had eight points. How many points was John ahead after seven turns?

For three days Jack did housework to make money. He got £5 for washing dishes, nine pounds for cleaning the car, and seven pounds for tidying his room. How much money did Jack get for three days work?

At Joe's garage there were 10 cars. Each car had three flat tyres. How many tyres did Joe have to fix?

Mary had 20 pieces of banana bubble gum. She wanted to give the same number of pieces of bubble gum to four friends. How many pieces should each friend get?

Sarah gave 7 sweets to her friend and she still had nine left. How many sweets did she have to start with?

Ben had 34 pence and he went to the shop to buy a drink. When he left the shop he had seven pence left. How much money did he spend in the shop?

Sally was given marbles by the teacher for doing well in her work. She was given 23 marbles on Monday and Tuesday and Wednesday. If the teacher gave her eight marbles on Monday, and four marbles on Tuesday, how many marbles did the teacher give Sally on Wednesday?

John had 4 bags of bricks, and each bag had 6 bricks in it? How many bricks did John have altogether?

Steven had 18 sweets and three friends. If Steven gives the sweets to his friends, how many sweets would each friend get?