MACROECONOMETRIC FORECASTING IN DEVELOPING COUNTRIES WITH SPECIAL REFERENCE TO FISCAL POLICY : A CASE STUDY OF INDIA

Dinesh Kumar Srivastava

A Thesis Submitted for the Degree of PhD
at the
University of St Andrews

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MACROECONOMETRIC FORECASTING IN DEVELOPING COUNTRIES

WITH SPECIAL REFERENCE TO FISCAL POLICY:

A CASE STUDY OF INDIA

DINESH KUMAR SRIVASTAVA

Dissertation submitted to the University of St. Andrews in partial fulfilment of the requirements for the degree of *Doctor of Philosophy*

St. Salvator's College
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Abstract

This work was undertaken with a view to construct a macroeconometric model for the Indian economy for purposes of forecasting and policy simulation. As a prelude to this exercise, we have surveyed available forecasting techniques, techniques of evaluation of forecasts and forecasting models and the issues concerning the use of macroeconometric models in the context of policy analysis. We have also considered specific issues and considerations relevant in the context of developing countries. As a second step towards providing a proper perspective to our model, and to derive useful guidelines, we have surveyed and reviewed existing macroeconometric models of the Indian economy. This survey concerns the models built by Narasimham, Choudhry, Krishnamurty, Krishnamurty-Choudhry, Marwah, Mammen, Agarwala, Pandit, Gupta, Bhattacharya and UNCTAD. We notice that an interesting variety of sectoral emphasis is offered in these models although in general they are all based on the IS-LM framework.

As a part of the review of the existing models, we have re-estimated three models, viz., models by Choudhry, Marwah and Bhattacharya under common sample conditions and estimation techniques and have compared their forecasting performance against alternative autoregressive benchmark models. In general, the benchmark models do better but the performance of Bhattacharya and Marwah models, in their adapted versions, seems to be satisfactory.

A common shortcoming of all these models is an underexploration of the fiscal sector of the economy. Generally, the government budget restraint has been ignored, the treatment of tax functions is highly aggregated and all government expenditure variables are treated as exogenous. Furthermore, the estimates in these models have become dated because of major data revisions. On these grounds and also as a part of continuing efforts towards building macroeconometric models for the Indian economy, we have specified, estimated and analysed a new model containing thirty four equations out of which eighteen are stochastic.

Its special features are an endogenous treatment of government consumption expenditure, a disaggregated treatment of tax-revenue functions, an endogenous money-supply function and a distinction between the agricultural and non-agricultural sectors in terms of prices, outputs and investments. The model is estimated by mixed estimation procedures. In particular, two stage least squares with subsets of predetermined variables in the first stage and with first order autoregressive corrections in a few cases have been used.

The model is used for forecasting and policy simulation. Its forecasting performance, within the sample period, and in a 'pseudo' forecast period is found to be satisfactory compared against 'naive' and 'not-so-naive' extrapolative benchmark models. Various policy simulations have been done and subsequently the model is used for conditional forecasting. We find that increases in government consumption expenditure have detrimental effects on real output, that changes in tax-rates and discount rates have very marginal impact on the system and that important policy changes relate to expenditure variables and government deficit financing.

DECLARATION

I hereby declare that the research reported in this thesis was carried out by me and that the thesis is my own composition. No part of this work has previously been submitted for a higher degree.

The research was conducted under the supervision of Dr G.K.Shaw, Department of Economics, University of St. Andrews.

D. K. SRIVASTAVA
(candidate)

CERTIFICATE

I hereby certify that the aforesaid candidate has fulfilled the conditions of General Ordinance No.12 (Resolution of the University Court No.1, 1967) and that he is qualified to submit the accompanying thesis for the Degree of *Doctor of Philosophy*.

DR. G.K.SHAW
(supervisor)

Statement of Academic Qualifications

I hold the degrees of M.A. (University of Allahabad, India) and M.Litt. (University of St. Andrews, U.K.). I am currently a Stanley Smith Fellow at the University of St. Andrews. I have worked as a Lecturer in Economics, University of Allahabad and as a Senior Economist at the National Institute of Public Finance and Policy, New Delhi.

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The typing of my original manuscript was an arduous job. I heartily thank Mrs J.V.Reed both for the quality of her typing and her patience and the good humour with which she has completed it within a given time limit.

I would also like to put on record my appreciation for my wife, Veena and my daughter, Mrinmayee for gracefully permitting, during the currency of this work, persistent inroads into many hours which rightfully belonged to them.

In the end, may I say, anticipating a phrase much used in the text, that while I have benefitted from the help of all these people, the errors that may yet remain would not, by any means, significantly (!) regress on them.

PART I

MACROECONOMETRIC FORECASTING: TECHNIQUES AND ISSUES

Chapter 1

Introduction

Speculation about the future is an age-old phenomenon. Traditionally, forecasting has been treated as an art dependent on the subjective judgement of the forecaster and the use of implicit or informal models where although the predictions may uncannily come true, the working mechanism inside the 'crystal ball' remains obscure. Thus, Huxley's The Brave New World or Orwell's Nineteen Eighty Four may be cited as examples of informal forecasting. In contrast to this, the scientific approach to forecasting calls for a more explicit treatment of the forecasting mechanism.

Within the context of a macroeconomy, the scientific approach to forecasting would entail the proposition of 'formal' forecasting models, which are intended to serve as simplified 'images' of the complex system of economic relationships. Although many rough approximations and abstractions are necessarily involved in such models, they have the distinct advantage of clearly outlining the interrelations and hypotheses that are considered and those that are ignored. The explicit treatment has made the subject objective and has taken much of the mystique out of 'crystal ball' approach to prediction although even in the modern approach many subjective elements may be involved. In the last twenty-five years or so, considerable activity has taken place in the field of forecasting, and it is appropriate to begin by examining why one is interested in predicting economic phenomena.

1.1 The Case for Prediction: General Observations

There are sound practical reasons for economic forecasting, but the case for prediction may well be initiated by asserting that prediction is interesting in its own right. As Klein (1971) says, 'It is an interesting intellectual challenge to see if prediction is possible.'

In more practical terms, however, since forecasting reduces uncertainty, it should be useful wherever decision-making is involved. Knowledge about the future is an important input in all decision-making processes. Prediction of uncertainty makes decisions better and where multi-level decision-making is involved, it attempts to make them consistent.

Forecasting has a role in each of the present-day economies that constitute the rich cross-spectra from a highly centralised socialist developed economy to a highly decentralised capitalist developed economy and all kinds of combinations of centralisation and decentralisation, development and under-development in between. This is so because forecasting can serve as a means of regulating spontaneous socio-economic processes that originate within the government and the private sectors of an economy as also those that originate between them. It can also serve as a means of linking medium- and short-term economic processes and long- and medium-term economic processes. Forecasting, thus, characterises the decision-making space over time and space.

In a capitalist society, both the government and the businesses can work out their respective roles in the future given a consistent forecast of most of the economic indicators regarding the future. Current plans for expansion or otherwise can suitably be adjusted in relation to these forecasts. For the individual decision-maker, the macro-economic forecasts provide a consistent frame of reference for a study of changes in major economic variables as also the planned role of the government regarding these variables. For the government, prediction makes a scientific exploration of the future possible; it also provides a framework for the study of alternative policy configurations in solving future problems wherever they may be spotted.

Short-term economic forecasts in these economies help to smooth out short-term economic fluctuations and irregularities that characterise them. Medium-term forecasts help the governments to formulate economic plans. The problem in these economies is, however, to link the medium-term equilibrium trends to the succession of related short-term equilibria. But since medium-term projections are based on mechanisms that operate over a longer time and override the short-term processes, this is a difficult task. Techniques for making 'revolving projections' and 'sequential modeling' that may provide a dynamic linkage between the medium- and the short-term are currently being experimented with in some of the Western economies.

In the socialist economies also, forecasting is an important stage in pre-planning work. The centralised economies, however, are not so much troubled by short-term disequilibria. Wherever these occur, they have techniques other than forecasting, such as 'control' and 'programming' to by-pass them. Only recently interest has been shown in short-term forecasting in these economies. The primary concern in the socialist economy has been to incorporate within the conventional five-year plans, the effects of processes which depict their cyclical behaviour over a longer time-horizon. Once long-term projections are available, suitable adjustments in the five-year plans can be made to incorporate their implications and thus to avoid frequent under- and over-estimation of demand and supply resulting in miscalculations and disproportions.

The case for econometric forecasting in developing countries is specifically examined in a later section. Since they have a lot to gain from the experience of macro-econometric forecasting in modern industrialized nations, the main trends in this are examined first.

1.2 Macro-Econometric Model-Building in Developed Countries

Efforts in developed countries both in the market economies and in the centrally planned economies have produced a number of macro-econometric models which have been used both for forecasting and for policy-simulations. The hectic work in these countries is well gauged by the proliferation of surveys* which were necessitated to keep track of the model-building activity. Some of the more well-known and currently operational models include BEA, Brookings, DHL III, DRI 71, Fair, FRB St.Louis, Wharton-Mark III, Wharton (anticipations), MPS, Hickman-Coen, Liu-Hwa models of the U.S. economy, 'Stone', Par and Treasury Models of the U.K. economy, CANDIDE, RDX models of the Canadian economy, the FIFI model of the French economy and the EPA model of the Japanese economy.

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* Reference can be made to the following surveys and reviews which refer mainly to models of the U.S. economy; Christ (1956), Christ (1975), Ball, ed. (1973): models of Project LINK - various countries), Fromm and Klein (1973), Hilton and Heathfield eds. (1970: models of U.K.), Nerlove (1965: models of U.K.), Nerlove (1966), Hickman ed. (1972), Hickman (1973: various countries), Shapiro and Halabuk (1976: including models of socialist countries), Zarnowitz (1967), Liggins (1972: models of the French economy), Wynn and Holden (1974), Waelbroeck (1973: short-run model-building outside U.S.).

There have been many forerunners to these models. Not all of these continue to be operational in the sense that the models and their forecasts are not continually updated. The forerunners have dropped out of the race, as it were, after having served their purpose in providing valuable insights for the currently used models. It is also indicative of the highly costly nature of the exercise that model-building activity in the developed countries has moved, by and large, from a single or multi-author level (like Klein, Klein-Goldberger models) to an institutional (like BEA, Brookings, Wharton models) and inter-institutional (like Fed- MIT, LINK models) level.

The variety, coverage and structural detail of models built in the developed countries are highly impressive. An idea as to the variety of sizes may be obtained from the following table adapted from Fromm (1972) for ten models of the U.S. economy.

Table 1.1*

Models of the U.S. Economy

Model	Number of: Stochastic Eqn.	Identities	Endogenous Var.	Exog. Var.	Policy Instruments
BEA 70	58	41	98	83	14
Brookings (condensed)	81	86	167	117	19
DHL III (1972)	24	20	47	26	7
DRI (1970)	200	168	368	120	29
Fair	14	6	19	16	3
FRB St.Louis	5	4	9	4	2
MPS (Fed-MIT)	64	113	177	119	29
Wharton (old)	47	52	99	39	7
Wharton (Mark III)	67	134	201	104	25
Liu-Hwa	42	59	101	14	12

* Total number of equations and identities may be greater than the number of endogenous variables due to disequilibrium and alternative specifications.

Different models built for the same economy differ in their estimation techniques, in the choice of sample-periods and in their internal emphasis and purpose. However, the main thrust of model-building in the Western market economies has been towards short-term forecasting and policy-analysis. Shapiro and Halabuk (1976) trace this short-term emphasis to two sources: (i) the time-unit of analysis (quarterly, monthly) and (ii) the dominant role of aggregate demand expenditures in determining current output.

There are, of course, important exceptions to this dominant trend. Among the earlier studies with a long-term perspective mention may be made of Valvanis (U.S.), Caves and Hilton (Canada), Tinbergen (U.K.), Ueno (Japan) and among the more recent works, Almon (1975, U.S.), Hickman-Coen (1975, U.S.), Preston (1972, U.S.), and McCracken (1973, Canada). Another exception to the predominant trend in terms of hypotheses involved are the so-called 'monetarist' models. These models are generally smaller in size(1 to 8 equations) and based on the assumption of a stable demand for money. They try to highlight the effect of monetary expansion on the aggregate level of economic activities in value terms. Also, there are many models which incorporate 'supply-side' influences in terms of using elaborate input-output blocks (e.g. Preston, Almon, Candide-models).

On the whole, however, it can be said that the primary emphasis in model-building in the developed market economies has been on the short to medium term perspective.

In short-term forecasting, at least, the model-building activity in Western countries could be said to have achieved some maturity. In a review of models for the U.S. economy, Christ (1975) writes:

> "Econometric models of the U.S. economy have been developed to the point where forecasters who use them can forecast real and nominal GNP two or three quarters ahead with root-mean-square errors of less than 1% and six quarters ahead with RMS errors of one to two per cent. The best of them now usually do better than forecasters who do not use such models."

The emphasis, on the other hand, in macro-economic forecasting exercises in the centrally planned economies, has been on the long-run perspective. This is so because many processes that exert significant influence on the structure and rate of development but which do not fit within the framework of the conventional five-year plan may be forecast and taken account of.

The need for forecasting in the Soviet Union and in other East European countries has assumed significance in the wake of greater autonomy for the productive enterprises introduced through the economic reforms carried out in the latter part of the sixties. An effective combination of centralization with autonomy for enterprises requires the use of sophisticated economico-mathematical models. To achieve this end, in the U.S.S.R. and in other East European countries, conventionally the method of 'intersectoral balances' has been used for extrapolation of established structures of intersectoral relations; its counterpart in the West is the input-output method. Fedorenko (1969) reports the current use of intersectoral balances for four main purposes: (i) long-term planning, (ii) current planning, (iii) preliminary stage of planning, and (iv) terminal stage of planning.

Apart from projections based on intersectoral balances, considerable interest has now been evinced in constructing macro-econometric models. In Shapiro and Halabuk (1970) a number of such models and their distinguishing features are reviewed. Mention may be made of models of the Polish economy developed at the Katowice College of Economics and a series of KP-models (KP-1, -2) for the Polish economy by the Commission of Planning, a series of M-models for the Hungarian economy (M-1, -2, -3, -4) prepared by the Hungarian Central Statistical Office, a VVS-series of models for Czechoslovakia (VVS-1, -2, -3) constructed at the Computing Research Center UNDP at Bratislava, and a series of UKR-models (UKR-1, -2) for the Ukraine region in the U.S.S.R. Other models include works of Adamec and Fundarek et al (1971) for Czechoslovakia, Yemelyanov and Kushnirskij (1974) and Adirim et al (1975) for the U.S.S.R. and Anders and Franken et al (1971) for the German Democratic Republic.

The basic characteristics of models built in these countries has been (i) a medium- to long-term focus, (ii) consideration of variables in volume rather than value terms, (iii) successive disaggregation of variables and enlargement of model sizes, and (iv) an important role for sectoral production functions.

More recently, however, interest in the study of short-term perspective by macro-econometric models is being shown in the centrally planned economies on the one hand, and on the other, an incorporation of flow-of-funds analysis, longer-term perspective and supply-side considerations are being advocated (e.g. Klein (1978) for model-builders in the developed market economies.

Developing countries stand to gain much from this rich experience in econometric forecasting already at hand.

1.3 The Case for Prediction in a Developing Economy

The large underdeveloped world operating primarily under a mixed economy framework is the last to show interest in consistent forecasting techniques, but given its reliance both on planning and on the market, it stands to benefit most from short- and medium-term, as also from long-term forecasting.

Interest in forecasting economic magnitudes of various types in developing countries arises not only within the country but also outside it. Short-term lending agencies such as the IMF and medium- and long-term lending agencies such as the World Bank and the IDA have a natural interest in the viability of their customers and in the direction and pattern of development that might take place in these countries. International agencies such as UNCTAD which initiate or execute actions in the developing countries have also to plan beforehand for an action they might want taken in future.

The major use for forecasting in a developing country remains in consistent planning for economic development. A plan which may remain consistent over time can only be formulated with the support of a comprehensive forecasting framework. The current practices in plan preparation in a developing economy are seriously lacking in this respect generally.

Currently, four main ingredients seem to characterise the prototype of a plan document in a developing country*, viz., (i) a historical over-all or sectoral diagnosis of the causes of economic backwardness in the country; (ii) a set of projections for the plan period; (iii) a comprehensive listing of planned investment projects with information about their financing; and (iv) a brief explanation of the main economic policies and economic reforms stipulated in the plan.

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* See Foxley and Garcia (1972) for a discussion.

The resultant efforts in these directions is a weighty cumbersome document comprising an incoherent set of projects, programmes and policies. As such its usefulness as a guide to public or private decision-making is limited. Its structure and method of construction is such that it cannot easily be revised once the set of basic data change. It does not offer obvious 'entry points' for information so that obsolete information may easily be deleted and replaced by new information.

In contrast, reliance on an econometric model framework† can provide a solid and coherent guide for public and private decision-making. It also provides a framework in which changes in data preferences, or policies may be easily and consistently incorporated. A change of current practice in this direction should be able to produce a consistent set of forecasts for the guidance of public and private sectors of the economy during the plan period. With the policy options and their projected effects clearly stated, it would also be a much more reliable guide so as to genuinely induce action from the private enterprise in the desired directions.

A medium-term forecasting system can be complemented by and linked to a short-term forecasting system. This would help formulate annual operating plans and define the nature of relationship between short-term and medium-term objectives of planning and make the whole plan much more operational.

In addition, a long-term projection system may also be employed so that the decision-makers may have a better time base for planned action. This would enable them to consider a wider range of options than is possible in a medium-term plan as also to accommodate within the medium-term plan, the implications of long-term objectives. The

† Some people take the extreme position that in view of various non-economic and non-quantifiable influences which it may not be possible to incorporate in a macroeconometric framework, such exercises are futile. We take note of this view but do not concur with it.

primary long-term objective in a developing country is to bring about structural change and this can systematically be studied only in a long-term forecasting framework. Research in this field is currently being carried out in developed countries, and the outcome should have important effects on developing countries as well.

The role of forecasting in a developing country, and the warranted emphasis on the type of forecasting, viz., short-, medium- or long-term, depends to some extent on the degree to which an economy is centralised or decentralised. For a highly decentralised economy short-term forecasting is important, as in the case of developed economies with 'indicative planning' for bringing about coherence in the complex of private and public decisions. Medium- and long-term forecasting will also have a role to play for planning and for bringing about structural changes. For a highly centralised economy, short-term forecasting may not be so important. For smoothing over short-term disequilibria, control and programming techniques may have a greater role to play. In these economies medium- and long-term forecasting may assume a greater emphasis.

In general, one can say that developing economies need, in varying degrees of importance, a central nucleus of macro-economic projections based on an econometric model, a set of detailed short-term projections based on an econometric or input-output model, and a set of stipulated long-term projections based on a study of structural changes.

Care, however, must be taken not to effect a wholesale transfer of the blueprint provided by the developed world but to make adjustments for the specific requirements of a developing economy.

1.4 Plan of the Present Work

The present work concerns itself with the consideration of issues to be faced in forecasting aggregate economic magnitudes in developing countries in general terms. It then adapts the context of the Indian economy as a case study. The entire study is divided in three parts. In Part I, apart from this introductory chapter, there are chapters on techniques of macro-econometric forecasting and related issues, specific issues in constructing macro-econometric models for developing countries, evaluation of forecasts and forecasting models, and on the use of prediction as a guide to policy evaluation. As such, Part I is devoted to a consideration of available techniques both for generating forecasts and evaluation as also for using forecasts in policy analysis with a view to applying them to developing countries.

In Part II, a review of macro-econometric models of the Indian economy is undertaken. Main features of various models are examined and their shortcomings are noted. Drawing upon the theoretical background provided in Part I and the empirical background provided in Part II, an attempt is made to develop a forecasting model for the Indian economy using more recent data and longer time series. The model is used for policy analysis, especially for the consideration of fiscal policy options. Its differences with other models are noted and, its properties and forecasting performance are examined in detail.

Chapter 2

Techniques of Macro-econometric Forecasting and Related Issues

If forecasting were a purely mechanistic phenomenon, it would not be necessary to distinguish between stages of economic development. However, forecasting is not just mechanistic: its practice is loaded with empiricism. A number of options are to be exercised at various stages before a final forecast is produced. In the exercise of these options, the forecaster has to be fully alive to the context he is analysing. It is, therefore, useful to consider these options in terms of available forecasting and estimation techniques and other related issues.

2.1 Methods of Forecasting

Although the focus here is going to be primarily on multi-equation forecasting systems, two options to these may be noted at the outset. These are single-equation models and extrapolative techniques.

Single-equation econometric models postulate a one-way relationship between two or more variables. The variable to be predicted is the dependent variable. The current value of this variable is seen as related to current and lagged values of other variables which are considered 'exogenous' or 'independent'. In addition, lagged values of the dependent variable may also be used. In general, such models could be written as

$$y_t = f(x_1 x_2, \ldots, x_j, y_{t-i}) + u_t \qquad (2.1)$$

where y is the dependent variable, x's are the independent variables and u is a stochastic term. The subscript t-i indicates lagged values. A lagged exogenous variable can be treated as a separate variable.

The forecaster has to specify the functional form and the lag-structure. He can then estimate the parameters of the relationship using sample data and some standard estimation technique. Given the estimated model, forecasts could be generated by feeding in independently obtained future values of the independent variables and by using either independently obtained or model-generated values of the lagged dependent variables. An appropriate assumption about the stochastic term will also have to be made.

The viability of this method depends on the exogeneity of the explanatory variables. If these variables happen to be jointly dependent with y, i.e. there is a feedback from y to these variables in the current period, the appropriate framework for forecasting would be a system of simultaneous equations. The nature of economic phenomenon is such that joint dependence of variables would generally be the case. However, if there is reason to believe that variables on the right-hand-side are 'near-exogenous' i.e. the feedback from the dependent variable is negligible or not strong enough to justify the additional costs of building a large multi-equation system, he would be justified in proceeding with the single-equation framework.

A second option to multi-equation systems is provided by extra-polative techniques. These are also single equation models. However, in order to forecast a future value of any variable, they use only the past history of this variable. This is basically a statistician's world. Here, a knowledge of established economic relations is not always a prerequisite. A mathematical relation between current and past values is fitted using relevant statistical criteria and forecasts are successively generated using lagged values. If sufficient information about the past history of a series is available, the purely statistical techniques could indeed produce very good results. Experience in DCs shows that these techniques do at least as well if not better as complex econometric structures in terms of forecasting performance. Extrapolative techniques could range from simple moving average, autoregressive or exponential smoothing schemes to fairly sophisticated models like integrated moving average autoregressive (ARIMA) procedures. It should be noted that the more sophisticated techniques here require a fairly large sample of past history of a series. This requirement may not always be met in LDCs for many macro-economic aggregates as one would generally be constrained to work with annual observations. It is also possible to extend the univariate forecasting framework to bivariate and multivariate models.

Forecasts generated with statistical techniques substantially exploit information on the past history of a series and they may be used in conjunction with forecasts generated by multi-equation models either (i) competitively, in which case the statistical forecasts can serve as a mirror in which to judge the relative performance of the econometric forecast or (ii) complementarily, in which case, one can combine alternative forecasts and generate composite forecasts making a better use of available information. In view of this dually useful nature of statistical forecasting techniques, a small sub-section is added at the end of this chapter outlining the relevant forecasting procedures.

However, our main concern is with multi-equation models.

2.2 Multi-Equation Models

Multi-equation models may be useful even if, in certain cases, they are outperformed *vis a vis* their predictive accuracy by single-equation econometric or statistical models. For one thing, their forecasts are 'consistent', e.g. forecasted expenditure components would add up to income etc. This is done by introducing appropriate restrictions in the model. This would not be so in the case of independently produced forecasts of these variables from the single-equation techniques. Secondly, these models would be an aid to policy analysis in a way which cannot be accommodated in other techniques. In particular, these models would be able to predict the simultaneous impact on a number of variables of interest following changes in some policy variables or configurations of these.

Two types of variables occur in a multi-equation econometric model - endogenous and exogenous. A variable is endogenous if it is generated within the model; it is exogenous if it is given to model and not affected by its endogenous variables. Where causes are found to produce their effects after a delay, lagged endogenous variables are used. These act much like the exogenous variables in that they affect current endogenous variables but are not affected by them. However, over a sequence of time, they are determined within the system. There may also be lagged exogenous variables but these could be treated as separate exogenous variables. It is convenient to group lagged endogenous and exogenous variables into a common class called predetermined variables.

A multi-equation model is a system of equations and identities in the endogenous and the predetermined variables. The identities are established using *a priori* knowledge of relations between economic aggregates. The equations are also specified using knowledge of the structural relations in the economy.

2.3 Structural-, Reduced- and Final Forms

A multi-equation model could be represented in a number of ways. When it is written in the structural form, endogenous variables are related to or expressed as a function of other endogenous variables, predetermined variables and disturbances. In the reduced-form of the model, endogenous variables are expressed as a function only of of the predetermined variables and disturbances. In the final form, each endogenous variable is expressed as a function of its own lag structure and a lag function of exogenous variables and errors.

For a linear system of equations these forms could be symbolically* represented as follows. Let there be n endogenous variables $y_{j,t-i}$ where j varies from 1 to n and i refers to time and varies from 0 to p. Then the vector of endogenous variables for period t-i could be written as

$$Y_{t-i} = (y_{1,t-i}, y_{2,t-i}, \ldots, y_{n,t-i})$$

Let there be n exogenous variables x_t (lagged exogenous variables treated as separate variables) so that the vector of exogenous variables could be written as

$$X_t = (x_{1,t}, x_{2,t}, x_{3,t} \ldots, x_{m,t})$$

Let the vector of error be denoted by

$$U_t = (u_{1,t}, u_{2,t}, \ldots u_{n,t})$$

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* The presentation here closely follows Klein (1971) which in turn is based on Howrey (1967).

Then the structural form of the model is written as

$$\sum_{i=o}^{p} A_i Y_{t-i} + BX_t = U_t \qquad \ldots \quad (2.2)$$

where A_i and X are matrices of structural coefficients. The reduced-form of the model is derived as follows, write (1) as

$$A_o Y_t + \sum_{i=1}^{p} A_2 Y_{t-i} + BX_t = U_t$$

or

$$Y_t = -A_o^{-1} \sum_{i=1}^{p} A_i Y_{t-i} - A_o^{-1} BX_t + A_o^{-1} U_t \qquad \ldots \quad (2.3)$$

In order to derive the final form of the model, a lag operator L is defined as

$$L^r Y_t = Y_{t-r} \qquad r = o, 1, 2, \ldots p$$

Let A(L) be a polynomial expression in L :

$$A(L) + A_o + A_1 L + A_2 L^2 + \ldots + A_p L^p$$

Then the system given by (1) can be written as

$$A(L) Y_t = -BX_t + U_t \qquad \ldots \quad (2.4)$$

Let $\alpha(L)$ be the adjoint of A(L) and $\Delta(L) = |A(L)|$ denote the determinant polynomial of A(L). Premultiplying both sides of (2.4) by $\alpha(L)$, the final form of the system is written as

$$||\Delta(L)|| \; Y_t = -\alpha(L) BX_t + \alpha(L) U_t \qquad \ldots \quad (2.5)$$

where $||\Delta(L)||$ is a matrix with $\Delta(L)$ on the main diagonal and zeros everywhere else. A particular solution of (2.5) can be written as

$$Y_t = -\frac{\alpha(L)}{\Delta(L)} BX_t + \frac{\alpha(L)}{\Delta(L)} U_t \qquad \ldots \quad (2.6)$$

The general solution is given by

$$Y_t = K\lambda^t - \frac{\alpha(L)}{\Delta(L)} BX_t + \frac{\alpha(L)}{\Delta(L)} U_t \qquad \ldots \quad (2.7)$$

where K is a (n x np) matrix of constants determined by the initial conditions of the system, and λ is an np - element vector of roots of the characteristic polynomial associated with the autoregressive structure.

The significance of the difference between different forms of expressing the same set of linear equation may be noted. The coefficients in the structural form usually have direct economic interpretation in terms of marginal propensities, marginal productivities, elasticities, reaction coefficients etc. The reduced- forms are used in estimation theory. They link the first and second stages of two (or three) stage least squares and are important in considerations of 'identification' properties of the model. They also provide short-term or impact multipliers and form the basis of single-period ahead predictions.

The final form shows the dynamic or autoregressive structure of the system. This form is convenient for (1) making multi-periods-ahead forecasts, (2) showing the implications of the recursive solution of the reduced-form equations for several successive periods, and for (3) showing the distinctive roles of (i) the internal dynamics from the elements of λ, (ii) the time paths of independent variables, and (iii) the accumulation of stochastic errors.

Whereas, it is not difficult to derive the solution and properties of linear systems, once appropriate conditions of identification etc., are satisfied, the same is not true of non-linear systems. A symbolic representation of a non-linear system could be as follows:

Assume Y_t, a n-vector of endogenous variables, X_t, a K-vector of exogenous variables, U, a n-vector of disturbance terms and G a column vector of functions G^j, j = 1, 2, ...n. A non-linear system may be written as

$$Y_t = G\,(Y_t,\ Y_{t-i},\ X_t) + U_t \tag{2.8}$$

where i may vary from 1 to p. The functions G^j may be non-linear in both endogenous and predetermined variables. The reduced-form, if it exists, could be written as

$$Y_t = H\,(Y_{t-i},\ X_t,\ U_t) \tag{2.9}$$

In general, H would also be non-linear in the stochastic term.

There are various ways for deriving solutions to, obtaining properties of, and generating forecasts from non-linear systems. Some of these will be taken up later in this chapter.

2.4 Recursive, Block-Recursive and Simultaneous Equation Systems

An important property of multi-equation models lies in their recursive, block-recursive or simultaneous nature. This has implications both for estimation of their parameters and their solutions.

A recursive multi-equation model is really an ordered set of single equation models. The ordering is such that each equation is independent of endogenous variables being determined in subsequent equations. Thus, the first equation may contain only predetermined variables. The second may have the first endogenous variable and predetermined variables. The third may have the first two endogenous variables apart from predetermined variables and so on. The random variables are assumed to be independent. For example, a system of n equations may be written as

$$
\begin{aligned}
y_1 &= f(x_1,x_2,\ldots, x_k; U_1) \\
y_2 &= f(x_1,x_2,\ldots, x_k; y_1; U_2) \\
y_3 &= f(x_1,x_2,\ldots, x_k; y_1,y_2; U_3) \\
y_n &= f(x_1,x_2,\ldots, x_k; Y_1,Y_2,\ldots Y_{n-1}; U_n)
\end{aligned}
\tag{2.10}
$$

where $y_1,\ldots y_n$ are endogenous variables; $x_1,\ldots x_k$ are exogenous variables; U_1, $U_2,\ldots U_n$ are random variables. Lagged endogenous variables may also occur in any of the equations but any ordering condition is not required.

Recursive systems are also called triangular systems because the coefficients of the endogenous variables form a triangular array: the main diagonal contains units and zeros appear above the main diagonal.

A fully simultaneous equation system will have equations with one or more endogenous variables on the right-hand side so that they cannot be ordered in the manner described above.

Frequently, one would encounter block-recursive systems. Here, each block of equations would be a subset of simultaneous equations. Successive blocks may also have endogenous variable on the right-hand side determined in a previous block but equations in any one block will not contain endogenous variables being determined in a succeeding block as explanatory variables.

2.5 Stages in Forecasting

Having considered some of the basic features of multi-equation models in the previous two sections we can now go on to see how these may be used in order to produce forecasts. In preparing a forecast, the forecaster has to go through a number of stages. These are listed below:

1. Specification of equations and identities
2. Considerations of identification
3. Estimation of parameters
4. Obtaining conditions on or values of exogenous variables for the forecast period
5. Preparation and presentation of forecast

Once sufficient information about the forecasting performance and properties of his model become available, he can undertake a process of revision going through the same stages again. Various considerations arising in each of these stages are examined below.

2.5.1 Specification of Models

Specification refers to the structural form of the model. As a first step, he has to consider or choose those macroeconomic aggregates in which he is primarily interested. He has then to specify equations and identities explaining these endogenous variables. It is here that the economists' knowledge of the structure of the economy, behavioural patterns and historical insights are incorporated into a macro model.

The structural relations are generally behavioural (like consumption, investment and money-demand functions) or technological (like production functions) or simple price relations. Considerable theoretical groundwork† would normally be available to the forecaster to choose the likely explanatory variables and sometimes functional forms for the endogenous variables in question. Once a choice regarding explanatory variables *vis a vis* the first set of endogenous variables are made, the analyst would generally be faced with a situation where some of the explanatory variables (other than the original endogenous variables) would seem to be jointly dependent and also dependent on the original list of endogenous variables. It would then be necessary to have additional endogenous variables and specification of equations which may determine these. Ultimately, the forecaster should have a list of exogenous variables so that he is reasonably sure that none of these is significantly affected by any of the endogenous variables.

The specification stage may be divided into three sub-stages (e.g. see, Brown, 1970): primary hypothesis, primary extended hypothesis and testing. Primary hypothesis involves forming opinions about the basic economic mechanism for each sector and market. The next stage is to deduce the consequences of these hypotheses at the level of <u>observable</u> behaviour. This is primary extended hypothesis. It means relating observable consequences to observable causes. These relationships could be tested by observing the behaviour of the micro consuming and producing units of the economy. Based on these tests, certain hypothesis may be rejected or reformulated. Once this is done, the specification stage can be finalised by developing the extended hypothesis at the macro-level. This will imply a decision as to which variables

† e.g. approaches to models within the monetarists, Keynesian, international monetarists, Kaldorian income-distribution type of frameworks. We consider this issue also in Chapter 3.

belong to which side of the equation as also to the form of the equation. A forecaster would generally be in a situation where he can draw upon related studies in partial equilibrium or micro contexts and a judicious selection procedure could save him much labour and costs.

Once the structural form of the equations has been identified coupled with necessary identities, its identification properties could be checked and estimates of parameters could be obtained. Frequently, it may be necessary to revise the original specification in view of the considerations of identification and the results of estimation.

The role of identities should not be under emphasised. The suppression of identities would generally lead to misleading results. Again, established knowledge of economics and accounting would be an aid in the formulation of the identities.

2.5.2 Considerations of Identification

Once a model has been hypothesised, it becomes necessary to see whether its equations can be statistically estimated. This leads us to the considerations of identification which arise both for individual equations (other than identities) of the model and also for the model as a whole. The problem of identification has implications for the estimation procedures to be adopted and the appraisal of a model although basically it is a problem of model formulation. Because of the necessity of statistical estimation, the identification problem in econometric model-building is more than the mathematical conditions required for the solution of a simultaneous equation system, although the latter are clearly included. Mathematically, an equation-system is exactly determined and solved if it has the same number† of (unique) equations as unknowns (here, endogenous variables). If the number of equations is less, there will not be a unique solution, if more, there may be no solution at all.

Econometric estimation of equations in forecasting models, however, calls for a consideration of identification for each equation. The problem arises because of the fact that if in a system two (or more) equations have the same statistical form, there is no unique way of knowing as to parameters of which equation will be estimated. For example, suppose we have data on X, Y and Z, and we formulate a model

$$X = a + bY + cZ + u \qquad \dots \quad (2.11)$$

and

$$Y = \alpha + \beta X + \gamma Z + v \qquad \dots \quad (2.12)$$

then an infinite number of equations of the same form can be generated. Premultiplying the equations with arbitrary constants λ and μ and by adding and rearranging the terms, we will have

$$X = \frac{1}{\lambda - \beta\mu}\left[(a\lambda + \alpha\mu) + (b\lambda - \mu)Y + (C\lambda + \gamma\mu)Z + \lambda\mu + \mu v\right] \qquad \dots \quad (2.13)$$

If we try to estimate parameters of equation (2.11) by using data on X, Y and Z we cannot be sure whether or not we have estimated the parameters of equation 2 or a set of 'mongrel' parameters from any of the infinite possibilities in (2.13) depending on different values of λ and μ. Such a problem would not have arisen, however, if, for example,

† Only a necessary condition.

equation (2.12) had an additional variable not contained in equation (2.11), because then no combination of the two equations will give rise to the same statistical form as equation (2.11). This equation will then be identified. Similarly (2.12) will also be identified if it did not contain some variable included in equation (2.11).

Thus, the identification of an equation depends on variables excluded from it, while being included in other equations of the model. In other words, a system is identified if it has a unique statistical form, i.e. no other equation in the system or combinations of other equations would contain exactly the same variables as the equation in question. The identification problem does not arise for identities in the system because these will not have parameters to be estimated statistically.

A system as a whole would be underidentified if one or more of its equations are underidentified. Further considerations will show that when an equation is identified it may either be <u>exactly</u> (or just) identified or <u>over</u>identified. A system would be identified as long as all the equations in it are identified (either just identified or overidentified).

There are two <u>formal</u> conditions of identification, viz., the order condition and the rank condition. These could be examined either with reference to the structural form of the model or its reduced form.

The order condition as applicable both to the structural and reduced-forms of the model could be written as follows:

> An equation is identified if the total number of variables excluded from it and included in other equations is higher than or equal to the number of equations minus one.

Thus, if,

G = total number of equations (or endogenous variables);

K = number of all variables in the model, endogenous and predetermined,

and M = number of variables, endogenous and predetermined, included in a given equation, then this equation is identified if

$$K - M \geqslant G - 1 \qquad \ldots (2.14)$$

The order condition, although necessary for identification is not a sufficient condition. It must be supplemented by the rank condition which can be stated with reference to the structural form of the model as follows:

> In a system of G equations, any given equation is identified if and only if at least one non-zero determinant of order (G - 1) can be formed from the coefficients of variables excluded from this equation but included in other equations of the model.

The same condition with reference to the reduced-form of the model could be stated as follows:

> An equation with G* endogenous variables is identified if and only if at least one non-zero determinant of order G* - 1 can be formed from the reduced-form coefficients of the predetermined variables excluded from that equation.

If the rank condition is satisfied, then an equation would be exactly identified or over-identified, depending on, with reference to the order condition, whether $(K - M) = G - 1$ or $K - M > G - 1$ in relationship (2.14).

In practice, the construction of appropriate determinants for checking the rank condition of each equation is likely to be a costly and time-consuming affair - especially for large-scale models. Fortunately, the order condition, generally applicable to linear systems, is much easier to test and does prove an adequate guide to identification especially in large models although special kinds of restrictions still need to be guarded against. It is considered that models containing more than 3 to 4 equations will not usually suffer from identification problems (Wynn & Holden, 1974, p.115) except in special cases.

There may be cases where the investigator is either unable to specify the entire model or he is interested in only a few of the equations of the model and therefore does not want to specify the full model. In such cases, the formal rules of identification cannot be tested for the equations in question as other equations have not been specified. It might still be possible to establish identification by using certain identifying restrictions on the values of parameters, on the relative variance of random variables, and on the mathematical form of the equations.

Parameter value restrictions generally are 'zero-restrictions' or 'equality restrictions'. If the investigator thinks on the basis of _a priori_ knowledge that certain variable are likely to appear in other equations but are not important for an equation in question, he can restrict its coefficient to zero. The variable does not appear in this equation but is expected to have non-zero coefficients in other equations. This would help identification as identification criteria are based on inclusion and exclusion of relevant variables. Sometimes, _a priori_ knowledge, also helps to formulate equality restrictions, like $b_1 + b_2 = 1$ in $X = b_o L^{b_1} K^{b_2}$ if constant returns to scale are expected in a production function. Usually, it would be possible to statistically test the validity of the restrictions. Sometimes extraneously known values are used for unidentified parameters.

2.5.3 Estimation Techniques

Once a model has been hypothesised and one is reasonably sure that there are no identification problems, one can proceed to estimate the stochastic equations of the system. A variety of estimation techniques exist and it is crucial that an appropriate technique is chosen.

From the viewpoint of estimation of structural parameters, multi-equation systems are best considered as falling into three categories: (i) recursive models; (ii) simultaneous-equation models with exactly identified equations, and (iii) simultaneous-equation models with over-identified equations. The last category of models is the type one most frequently encounters in practice.

The main reason why single-equation model estimation techniques (like OLS) cannot always be extended for multi-equation systems is the phenomenon called simultaneous equation bias. This arises from the dependence of some explanatory variables in any equation on the random variable. Consider, for example, the system

$$M = a + bY + u \qquad \ldots \quad (2.15)$$

and

$$Y = \alpha + \beta M + \gamma I + v \qquad \ldots \quad (2.16)$$

where M = money-supply, Y = Income, I = Investment and u and v are random terms.

Since Y can be written as

$$Y = \alpha + \beta(a + bY + u) + \gamma I + v \qquad \ldots \quad (2.17)$$

it becomes dependent on the random variable u, which implies that an OLS estimate of equation (2.15) is not appropriate as one of its assumptions, viz., the independence of explanatory variables from the error-terms is violated. Such violation results in estimates which are both biased and inconsistent.

However, this problem is bypassed for recursive multi-equation models. Consider the system,

$$\begin{aligned} Y_1 &= f(X_1, X_2, \ldots, X_K; U_1) \\ Y_2 &= f(X_1, X_2, \ldots, X_K, Y_1; U_2) \\ Y_3 &= f(X_1, X_2, \ldots, X_K, Y_1, Y_2; U_3) \\ Y_G &= f(X_1, X_2, \ldots, X_K, Y_1, Y_2, \ldots Y_{G-1}, U_G) \end{aligned} \qquad (2.18)$$

Here, if OLS is applied step-by-step, the problem of bias would not arise. The procedure would be to estimate $\hat{Y}_1$ using values of $X_1, X_2 \ldots, X_K$ in equation (1); use the generated $\hat{Y}_1$ which are independent of the stochastic term in equation (2), and thus generate $\hat{Y}_2$ and use these in the estimation of $\hat{Y}_3$ etc.

The procedure cannot be followed for systems which have simultaneous relations. If the equations in such a system are exactly identified, the estimation technique called indirect least squares (ILS) can be adopted. In this procedure, OLS is applied to the reduced-form of each equation. Since, in this form only exogenous variables enter as explanatory variables, the interdependence of these with random terms is avoided. The parameters of the reduced-form can be expressed as functions of the parameters of the structural-form and the latter can be obtained from the estimates of the former as long as the equations in the system are exactly identified. ILS may be applied in such a case provided the usual assumptions of OLS are satisfied for the reduced-form of each equation.

The estimation of structural parameters obtained in this manner are considered to be biased for small samples, but they are consistent, i.e. their bias tends to zero as the sample size increases and their distribution collapses on the true parameters values. For the property of consistency, ILS procedures are usually preferred to OLS which are biased as well as inconsistent in this context.

However this method cannot be extended to models with over-identified equations because in such a case the reduced-form estimates cannot be uniquely solved for obtaining the structural-form parameters. A variety of estimation techniques exist in order to meet this situation. These techniques can be divided into two categories: an equation-by-equation approach and a systems approach. In the first case, estimation is done one-equation after another while in the latter case, ultimately, all equations are considered together. The more popular of the techniques are listed below.

Table 2.1

Estimation Techniques for Simultaneous Equation Models

Approach	Methods
Equation-by-equation approach	OLS, ILS, IV, 2SLS, LIML, RR, 3-Pass, k-class, Double k-class, Mixed estimation techniques
Systems approach	FIML, 3SLS

In the instrumental variable (IV) technique, endogenous variables on the right-hand side of the structural equations are replaced by appropriately chosen exogenous variables of the system. In choosing the exogenous variable it is borne in mind that this variable is (i) strongly correlated with the dependent variable in the given equation; (ii) strongly correlated with endogenous variable it is replacing; (iii) least correlated with other exogenous variables in the equation, and that it is (iv) truly exogenous, and, therefore, uncorrelated with the random term of the equation. If more than one instrumental variable is used in one equation, they should be least correlated with each other. The least correlation criterion is used to avoid multi-collinearity in explanatory variables. Once endogenous variables are replaced by their instruments, the structural equation is multiplied throughout by each instrument (as well as exogenous variables, being their own instruments) and the sum of the terms overall sample observations is obtained. This procedure provides as many linear equations as there are unknown parameters. The equations are solved after setting summation terms with random variables to zero.

The IV method is based on all the usual assumptions of least squares and it yields estimates which are biased for small samples but consistent for large samples.

The two-stage least squares(2SLS) method is appropriately viewed as the application of OLS in two stages. In the first stage, OLS is applied to the reduced-form equations in order to obtain estimated values of endogenous variables. These estimates are then substituted for endogenous variables occurring as explanatory variables in the structural-form equations and OLS is again applied to estimate the structural parameters. In the first stage, an estimate of the exact and random components of an endogenous variable is obtained and in the second stage the random component is dropped in an attempt to remove the simultaneous equation bias.

It can be seen that this method is an extension of IV and ILS methods. It is generally preferred to the IV method as it uses information on all predetermined variables for separating the random component from endogenous variables whereas the IV method uses information only of a subset of predetermined variables.

2SLS estimates are biased for small samples but they are asymptotically unbiased for large samples, are consistent and asymptotically efficient under certain conditions. Usual stochastic assumptions must be satisfied in both stages of estimation.

The k-class estimatorsare a generalisation of 2SLS estimator. In the first stage OLS is applied to the reduced-form equations, generating estimates of endogenous variables. Thus,

$$y_i = \hat{y}_i + u_i \,, \qquad (i=1,2,..\ G) \quad \ldots \quad (2.19)$$

$\hat{y}_i$ is obtained by regressing y_i on all predetermined variables $\hat{y}_i = \hat{\Pi}_{i_1}x_1 + \hat{\Pi}_{i_2}x_2 + \ldots + {}_{iK}x_K$ where $(x_1, x_2 \ldots . x_K)$ are the predetermined variables. In the second stage a composite variable

$$\bar{y}_i = (1-k)y_i + k\hat{y}_i = y_i + k(\hat{y}_i - y_i) \ (i=1,2,\ldots G)$$

is constructed and used to replace the endogenous variables which occur as explanatory variables in the structural-form of equations. It is easily seen that when k = 0, OLS estimates would result and when k = 1, 2SLS estimates would result. The value of k can be set arbitrarily or according to some predefined rules and using sample data. The k-class estimates change considerably if the value of k greatly exceeds unity. If k does not exceed unity substantially, the estimators do not vary appreciably.

The limited information maximum-likelihood method (LIML) also called the least variance method (LVR) is also an equation-by-equation approach to the estimation of a simultenous equation system. Consider, that one has to estimate

$$y_i = ay_2 + b_1x_1 + b_2x_2 + u_1 \qquad \ldots \quad (2.20)$$

This may be written as

$$y_i^* = y_1 - ay_2 = b_1x_1 + b_2x_2 + u_1$$

For each 'a', a y_i^* can be constructed. In each regression of y_i^* on x_1 and x_2, a residual sum of squares will be yielded. Call these RSS_1. Similarly, if y_i^* was regressed on all the predetermined variables of the system another residual sum of squares would be obtained. Call this RSS_2. What is expected is that the extra reduction in the residual sum of squares is minimal when additional (other than those included in the structural form) exogenous variables are used. So that LIML/LVR method determines a value of 'a' such that $(RSS_1 - RSS_2)/RSS_1$ or RSS_1/RSS_2 is minimised. Once 'a' is determined, using the constructed y_i^* and running a simple regression, b_1 and b_2 can be determined. The same procedure is adopted for all equations and it is easily extended for additional endogenous variables on the right-hand side.

The LIML estimates are biased for small samples but they are consistent. If the disturbances of the structural model are normally distributed, the estimates would be asymptotically efficient. The LIML also belongs to the k-class estimators. It is similar to 2SLS in many ways but it is computationally more difficult and in small samples 2SLS may have an advantage over LIML (see Nagar, 1960, Cragg, 1967).

In the repeated reduced form (RR) method[1], first consistent estimates (by applying 2SLS or some other consistent method) are obtained for the structural parameters of the model. The system is then solved to generate estimates of endogenous variables for given exogenous variables and initial values of lagged endogenous variables. The new estimates of endogenous variables are then used equation-by-equation in the structural equations. For a linear system this means that the estimates of endogenous variables to be used in the second step are derived from the restricted reduced-form. Howrey et al (1974) have shown that for non-linear systems, the method may not yield consistent estimates of parameters.

Various mixed estimation techniques result from the use of *a priori* information or prior restrictions on parameters, use of cross-section and time-series information together, autoregressive corrections of first and higher-orders, and the use of principal components of predetermined variables in the first stage (of 2SLS or 3SLS) estimation. The last mentioned technique is especially useful when sample sizes are limited, specifically when the number of predetermined variables exceed the number of observations. The CS-TS combination can also be usefully employed where there are problems of identification or multicollinearity.

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1 Attributed to Jorgenson. Analysed in Klein (1971) and Howrey, Klein and McCarthy (1974). Variants of the method have been analysed by Houthakkar, Theil, and Nagar.

In the single-equation methods of estimation like the 2SLS or LIML, knowledge of the structural specification of the entire system is not required for the estimation of any one particular equation as long as one knows all the predetermined variables occurring in the system. However, in systems-methods of estimation like 3SLS or FIML, the structural specification of the entire system must be known. These methods yield the estimates of the structural parameters of all the equations in the model at the same time.

FIML is a systems-extension of the maximum likelihood method for single equation models which allows for the possibility that the random variables of each equation (assumed normally distributed with zero means and constant variance) may be intercorrelated. FIML estimates are biased for small samples but for large samples they are consistent and efficient. The method involves estimating equations which are non-linear in parameters except when the model is recursive. As such the computational costs are considerable.

3SLS involves the application of least squares in three stages, the first two stages being coincident to 2SLS. In the third stage generalised least squares are applied to a set of transformed structural equations, the transformation being dependent on the residuals obtained in the second stage. The basic difference between 2SLS and 3SLS estimation is that in the latter procedure inter-equation contemporaneous correlation between error terms is allowed. The error terms are still independent of lagged values in its own equation and those in other equations. The 3SLS estimates are also biased but consistent. They are more efficient than 2SLS for overidentified systems. In exactly identified systems 3SLS yields estimates which are asymptotically identical to 2SLS estimates.

The above merely refers to the more popular of estimation techniques. There are others and indeed mixed estimation can give rise to a wide range of techniques. The choice as to one technique *vis a vis* another depends both on the context one is analysing and the purpose for which the analysis is being done. In both cases, properties of the estimators are a good guide towards exercising a choice and since in an LDC context the general situation is likely to be one of small samples, it is the small sample properties which may be relevant. The small sample properties cannot be theoretically derived but they have been widely studied with Monte Carlo type of simulations.

From the viewpoint of ranking, it is useful to distinguish whether one is interested in the (i) reduced-form estimates and/or (ii) structural-form estimates and whether specification is correct or one or a combination of errors are present. Estimators are variously ranked according to bias, variance, mean square error (or root mean square error) and proportion of incorrect references. A summary of ranking generated by various studies on small sample properties of estimators[1] is given in following tables.

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1 Based on studies by Bassman (1958), Nagar (1960), Summers (1965), and Cragg (1966, 1967) as presented in Koutsoyiannis (1974) & Christ (1966).

Table 2.2

Structural Parameters: Ranking of Estimators Based on Small Sample Properties

	Criteria			
	Bias $(\bar{\hat{b}}-b)$	Variance $\frac{\Sigma(\hat{b}-\bar{\hat{b}})^2}{n}$	Root Mean Square Error	Proportion of Incorrect Inferences
Correct Specification of Model	1. FIML≃LIML 2. 2SLS 3. OLS	1. OLS 2. FIML 2SLS LIML	FIML 2SLS LIML OLS	2SLS LIML OLS
Type of Error	**Incorrect Specification**			
a. Omission of variable	2SLS≃LIML OLS FIML	OLS FIML 2SLS LIML	2SLS≃LIML ≃OLS FIML	
b. Multi-collinearity	2SLS LIML FIML OLS	OLS FIML 2SLS LIML	2SLS≃FIML OLS LIML	
c. a & b	2SLS FIML OLS LIML	OLS FIML≃2SLS ≃LIML	2SLS OLS FIML LIML	
d. Auto-correlation				2SLS≃LIML ≃OLS
e. d & b				2SLS OLS LIML
f. Errors of Measure-ment				LIML 2SLS OLS
g. f & b/ f & b & d/ f & d				LIML 2SLS OLS

Notes: The ≃ sign denotes that the methods 'linked' with it yield almost identical results b = true parameter value, $\hat{b}$ = estimate of parameter. Proportion of incorrect inference based on selection with reference to t-values.

If the interest lies primarily in the reduced-form parameters which may be so if one is considering only policy analysis or forecasting, the following table may provide useful information.

Table 2.3

Reduced-Form Parameters: Ranking of Estimators Based on RMSE Criterion

a.	No Specification Error	2SLS, LIML, LSNR, OLS
b.	Multicollinearity	FIML, LIML, 2SLS, OLS
c.	Serial Correlation and b	OLS, 2SLS, LSNR, LIML
d.	Measurement Error & b/ Measurement Error & c	2SLS LIML, LSNR, OLS

Note: LSNR = least squares - with no restrictions

These rankings do not provide definitive statements on the properties of the estimator and are sensitive to the original specifications provided in the respective Monte Carlo studies.

On the whole it seems that in general OLS is inferior to consistent methods where there are no specification errors; in the presence of errors, however, this may be queried. In the presence of errors 2SLS seems to be the best method among consistent estimators. It is also preferred on the basis of its simplicity of computations. Not much small sample evidence on 3SLS is available. It is well known however, that 3SLS and FIML perform badly in the presence of specification errors because such errors in any one equation are transmitted to the entire system. Their data requirements are also large.

2.5.4 Preparation of Forecasts

Once an estimated model is available, the procedure for construction of forecasts boils down to solving the system for endogenous variables for one or more sets of values for the predetermined variables. The assumption in such forecasting is that the source of changes in endogenous variable are changes in the predetermined variables while the parameters of the system (structural- and reduced-form coefficients) are held constant. In the special cases of variable parameter and random coefficients models, the coefficients are also allowed to vary but in a predefined way.

Because of the fact that a path for endogenous variables can be generated for a given path of predetermined variables, various possibilities arise. The purest form of forecasting is <u>ex ante forecasting</u> where future values of both endogenous and exogenous variables are unknown. The forecaster uses his <u>best</u> judgement as to the future values of exogenous variables. By feeding these period-by-period into the model he can generate future values of lagged endogenous variables. By using independently obtained values of exogenous variables and model forecasted lagged endogenous variables, he gets the requisite set of predetermined variables for forecasting.

At this stage forecasting needs to be distinguished from simulation. Whereas, in a forecast, the forecaster's best judgement as to the future values of exogenous variables is required, he can also solve the system for various other combinations of hypothetical values (even if unlikely) of exogenous variables. These solutions are generally called simulations and these may be very useful for policy analysis.

Both these possibilities can be distinguished from ex post forecasting. When the original forecast period has elapsed in historical time and actual path of both exogenous variables and lagged endogenous variables has been observed, this new information can be used within the framework of the old model in order primarily to judge the forecasting performance of the model. If actual values of both exogenous and lagged endogenous variables are used we get ex post forecasts; if only actual values of exogenous variables are used (and initial values of lagged endogenous variables, where needed), but all subsequent lagged endogenous variables are generated within the model, the forecasts are called ex post simulation forecasts, or dynamic simulation forecasts. All ex post forecasts have to be 'beyond sample'. They may be with reference to a period either preceding, succeeding or intervening the observation period, as long as the observations in question have not been used for the estimation of the model.

As such ex post forecasts also need to be distinguished from model-estimates of endogenous variables. These are estimates obtained by the model for the observation period by feeding in the actual values of predetermined variables. Again, if only actual values of exogenous variables and initial values of lagged endogenous variables are fed in, we get dynamic simulation estimates. These possibilities are presented in a tabular form below.

Table 2.4

Types of Forecasts and Estimates

	Predetermined Variables	
	Exogenous	lagged Endogenous
Forecast (Beyond Sample)		
ex-ante forecast	Independently obtained, forecaster's best judgement	Model-generated except initial values
ex-ante simulation	Hypothetical values	"
ex-post forecast	actual	actual
ex-post dynamic (simulation) forecast	"	Model-generated except initial values
Estimates (Within Sample)		
(Model-) Estimates	actual	actual
Dynamic (Simulation) Estimates	"	Model-generated except initial values

These terms have not always been used with a uniform meaning in the literature and the user has to be cautious as to exactly what is meant where.†

† We should also recognise that model specification involves the specification both of the deterministic and stochastic parts of the equation. There are no a priori rules as to how the error terms should be treated in a forecast period. Various types of stochastic simulations can be done. It introduces an element of subjectivity in the forecasts.

Forecasts can be presented both as point forecasts, and under certain assumption, interval forecasts. In the latter case a confidence interval is built round a point estimate, depending on the variance of the forecasted variable, which is assumed to be a random variable.

Assuming, values of the predetermined variable are given for the forecast period, the solution of a linear system is straight-forward. Calling arrarys of endogenous and predetermined variables in time t, as Y_t and Z_t and their respective matrices of structural coefficients as A and B, the structural system can be written as

$$AY_t + BZ_t = U_t \qquad (2.21)$$

From which we have

$$Y_t = -A^{-1}BZ_t + A^{-1}U \qquad \ldots \quad (2.22)$$

If $-A^{-1}B$ is written as Π, where Π is the matrix of reduced form coefficients, we have

$$Y_t = \Pi Z_t + A^{-1}U \qquad \ldots \quad (2.23)$$

Assuming $\hat{U} = o$, we can obtain non-stochastic predictions $\hat{Y}_t$ for given Z_t as

$$\hat{Y}_t = \Pi Z_t \qquad \ldots \quad (2.24)$$

It should be noted that estimates of parameters in the Π matrix, i.e. reduced-form parameters can be obtained either from the structural parameters (by inversion of A and multiplication by B, as above), or by directly regressing endogenous variables on all predetermined variables (i.e. the first-stage of 2SLS). For those equations which are exactly identified the two results would be the same. For over-identified equations they would be different inasmuch as the derivation of the reduced-form from structural parameters takes account of parameter restrictions imposed on the specification of the model. Sometimes, the validity of the overidentifying restrictions is tested by a comparison of estimates of endogenous variables generated by the two types of reduced-form parameter estimates. Direct estimates of reduced-form may also be useful when the entire structure of the model is not completely known but a knowledge about the likely predetermined variables of the system is available.

For non-linear systems, the solutions for endogenous variables is not straightforward and one can proceed in a number of ways. One alternative is to linearise* the non-linear system around some observation (or a combination of these, like the mean level etc.).

For a system of G equations,

$$f_i\,(Y_1, Y_2, \ldots Y_G, Z_1, Z_2, \ldots\, Z_K) = 0 \qquad i = 1,2,\ldots G \qquad (2.25)$$

one can obtain the total differentials

$$df_i = \sum_{g=1}^{G} (\partial f_i / \partial Y_g)\, dY_g + \sum_{k=1}^{K} (\partial f_i / \partial Z_K)\, dZ_K = 0 \qquad (2.26)$$

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* For a discussion, see Wynn & Holden (1974).

The partial derivatives can be evaluated at a given point. If at this point, the relevant values are, for the array of endogenous and predetermined variables respectively as

$$Y = Y_o$$
$$Z = Z_o$$

then evaluations of

$$\partial f_i / \partial Y_g \quad \text{and} \quad \gamma f_i / \gamma Z_K$$

at the points $Y_{o1}, Y_{o2}, \ldots Y_{oG}$ and $Z_{o1}, Z_{o2}, \ldots Z_{oK}$ would provide two coefficients matrices A_o and B_o respectively of order GxG and GxK,and the system given by equation (2.25), can be written as

$$A_o dY = -B_o dZ$$

or

$$dY = -A_o^{-1} B_o dZ$$

$$dY = \Pi dZ \qquad (\Pi = -A_o^{-1} B_o) \quad \ldots \quad (2.27)$$

so that for changes,

$$\dot{Y} = \Pi \dot{Z} \quad \ldots \quad (2.28)$$

The usefulness of linearising non-linear models is limited to small changes around a particular set of values (Y_o, Z_o) and it may therefore be used only for short-term forecasting.

There are however, other ways of solving non-linear simultaneous equation models such as Newton's method where each of the non-linear equation is expanded in a Taylor's series around an initial set of trial values of the endogenous variables with predetermined variables set at the forecast level. Successive approximations of the endogenous variables are generated until a predefined level of accuracy is reached. Another alternative is the Gauss-Seidel method where iterations are done without resort to linearising approximations. Equations are successively solved by assuming values of other endogenous variables, and the solutions are fed in successive equations until one round of solution for all endogenous variables is available. Successive iterations are done until convergence is achieved. There are various precedures for expediting the attainment of convergence.

2.6 Time-Series Models

The preceding discussion has been related to the problems and procedures arising in forecasting in the context of multi-equation models. However, as mentioned in section 2.1, an alternative forecasting framework is provided by time-series models. Here, rather than predicting future values of a variable with the help of explanatory variables (in single-equation models) or predetermined variables (multi-equation models), predictions are generated by using only the past values of the variable in question.

These models may be useful (i) when it is difficult to construct an appropriate structural model for lack of data or knowledge about causal relations, (ii) when it is desired to combine forecasts using different methods, and (iii) when an alternative forecast is needed to evaluate a given forecast (from a model, say). A basic advantage in these models is that information regarding future paths of explanatory or predetermined variables is not needed. The limitation is that apart from the simplest type of time-series models, a large sample is required in the construction of these models. This limits their usefulness in LDCs especially if only annual observations are available. However, even in this case, useful autoregressive models can be constructed. Furthermore, although a large sample may not be available for all the variables used in an alternative procedure like a simultaneous equation model, there may be many variables for which observations are available at shorter time intervals (like money-supply, prices, interest-rates) and an adequate sample can be constructed to permit the use of the more sophisticated time-series models.

A further difficulty arises because of the fact that most of these models have been built in the context of stationary series which is taken to mean roughly a series that fluctuates round a given mean level. Most economic time-series, especially in LDCs where a strong trend would be observable in series, are non-stationary. It is possible, however, by successive differencing to bring a series towards stationarity and at least for these series - called homogenous non-stationary - it should be possible to use time-series models with benefit.

It is useful to note that such models built on the basis of purely statistical procedures without recourse to an underlying economic theory have performed very well in terms of predictive accuracy. Based on evidence provided by studies like Bray (1971) and Cooper (1969), Granger and Newbold (1975) advocate the case of these techniques succinctly,

> '... forecasts derived from entirely statistical procedures provide opponents with which econometricians can spar ..... the available evidence .... suggests that so far the sparring partner is consistently outpointing the potential champion.'

Statistical time-series models range from naive to highly sophisticated procedures and they can be divided into two categories: deterministic and stochastic. In the latter type of models, it is assumed that the time series is generated by a stochastic process, i.e. each value in the series is drawn randomly from a probability distribution.

Deterministic models of time series have long been in use for extrapolation. Examples of these are, linear trend models ($y_t = c_1 + c_2 t$), exponential growth model ($\log y_t = c_1 + c_2 t$), autoregressive models ($y_t = c_1 + c_2 y_{t-1} + c_3 y_{t-2} + \ldots$), moving-average models (like, $y_t = \frac{1}{4}(y_{t-1} + y_{t-2} + y_{t-3} + y_{t-4})$, exponentially weighted moving average models (like, $y_t = \alpha\sum_{\tau=0}^{\infty}(1-\alpha)^{\tau} y_{t-\tau}$). In the examples above, t = time, and other symbols have obvious meanings.

Interest now, however, centres round stochastic models of the time series. The simplest example of these is a random walk:

$$y_t = y_{t-1} + \varepsilon_t \tag{2.29}$$

where $E(\varepsilon_t) = 0$, $E(\varepsilon_t \varepsilon_s) = 0$ for $t \neq s$

More sophisticated procedures include autoregressive models, moving average models, autoregressive-moving average models and integrated autoregressive-moving average models.

A moving average process of order q, denoted as MA(q), is generated by a weighted average of random disturbances going back q periods. Its equation is

$$y_t = \mu + \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-2} - \ldots \theta_q\varepsilon_{t-q} \quad \ldots \quad (2.30)$$

θ's may be positive or negative. The random disturbances are assumed to be generated by a white noise process, i.e. each ε-term is assumed to be a normal random variable with zero mean, and variance = σ^2_ε . and covariance $\gamma_k = o$, for $k \neq o$. μ is the mean of the process and is independent of time.

An autoregressive process of order p, denoted AR(p) is generated by a weighted average of past observations and a current random disturbance. An AR(p) can be written as

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi y_{t-} + \delta + \varepsilon_t \quad \ldots \quad (2.31)$$

An autoregressive-moving average process of order (p,q), denoted as ARMA(p,q) is given by

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \delta + \varepsilon_t - \theta_1 \ldots - \theta_q \varepsilon_{t-q} \quad (2.32)$$

The autoregressive-integrated-moving average processes (ARIMA) developed by Box and Jenkins, differ from ARMA models in that successive differencing is adopted for a homogenous non-stationary series in order to reduce it to stationarity before an ARMA model is written. ARIMA models have three parameters (p, d and q) where p and q refer respectively to the AR and MA processes as before and d refers to the order of the differencing to be adopted. Thus, once a process y_t is differenced d times to yield w_t, $w_t = \Delta^d y_t$, ARMA (p,q) process is applied to w_t.

In practice, forecasts using any of these methods have to be preceded by specification, that is the choice of p in AR-models, q in MA-models, p and q in ARMA-models and p, d and q in ARIMA-models and by estimation, that is estimation of ϕ's, θ's, μ and δ as required from sample observations. The specification of parameters is helped by examining the autocorrelation function of the time-series of concern. Box and Jenkins propose that specification of the model should be considered as an iterative process in three steps: identification, fitting and diagnostic checks, at least for the more complicated of these models. After an initial identification and estimation, diagnostic checks are performed to find out model-inadequacies and the model is revised in the light of these until it begins to perform satisfactorily. In this sense, even purely statistical models do not remain a mechanistic phenomenon but require forecaster's judgement at each stage.

In practice, economic time-series are found to be non-stationary but they are reduced to stationarity by a low order differencing (first or second order) or quasi-differencing (a weighted average of first and higher order differences). Once a series is properly differenced, any of the above procedures can be applied provided the sample is adequate.

One time-series procedure which might be useful in practice and applicable even when the sample is not very large is a step-wise auto-regressive procedure. The model is specified as

$$w_t = a + \sum_{j=1}^{k} b_j w_{t-j} + u_t \qquad (2.33)$$

where w_t is a properly differenced series and k is some integer.

Here a direct application of OLS in order to estimate a and b's would yield highly inefficient estimates even if k is very small. It has been suggested (see Granger and Newbold,1975), that a step-wise procedure may prove very useful. First, those lagged values w_{t-j} are introduced which most explain the variation in w_t; and then that lagged value of w_{t-j} is introduced which best improves the fit and so on.

If differencing has been adopted, the model would forecast differences. Forecasts of original levels can then be obtained by successive integration.

2.7 Combination of Forecasts

Different forecasting procedures utilise information contained in the sample in different ways. It is possible that in any one forecast all information is not fully utilised. It is useful in these cases to generate composite predictions by combining forecasts generated by different methods. It may be particularly useful to combine a statistical extrapolative forecast with an econometric forecast as the former utilises extrapolative information without using autonomous information (i.e. information contained in the relationship of different variables) while the latter utilises autonomous information, without always fully utilising the extrapolative information.

Work already done in this respect like Granger and Newbold (1974), Bates and Granger (1969) and Nelson (1972), indicates that combining forecasts can indeed improve results.

A linear composite prediction (C) of two forecasts for period t ($F_{1,t}$ $F_{2,t}$) can be written as

$$C_t = \beta_1 F_{1,t} + \beta_2 F_{2,t} \tag{2.34}$$

The combination can include any number of forecasts if their inclusion improves prediction performance under some chósen criteria.

If the forecasts F_1 and F_2 are individually unbiased, a weighted average of these like

$$c_t = KF_{1\ t} + (1-K)F_{2\ t} \qquad 0 \leqslant K \leqslant 1$$

would also be unbiased. The problem then is of choosing the value of K_t. A useful criterian for this purpose is to take that value of K which minimises the variance of the combined forecast error. It has been shown that such minimisation is obtained when

$$K_t = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 \quad \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

where σ_1^2 = var ($e_{1,t}$); σ_2^2 = var ($e_{2,t}$); ρ = cov ($e_{1,t}, e_{2,t}$) and $e_{j,t} = X_t - F_{j,t}$ for j=1, 2,. X is the realization of which F_1 and F_2 are forecasts.

K_t would be zero if $\rho\sigma_1\sigma_2 = \sigma_2^2$ and unity if $\rho\sigma_1\sigma_2 = \sigma_1^2$. Except for these special cases K_t would lie between 0 and unity and by a suitable choice of weights one can find a composite forecast which would outperform the individual forecasts on the minimum variance criterion.

In practice, however, one does not know the values σ_1^2, σ_2^2 etc. Bates and Granger (1969) have put forward some practical suggestions based on two principles: (i) most weight should be given to that forecast which has performed best in recent past, and (ii) the weight function should adapt itself to allow for the possibility of a non-stationary relationship over time between individual forecasting procedure. In particular, the following five weighting schemes may be used:

1. $$K_t = \sum_{t=T-v}^{T-1} e_{2,t}^2 \Big/ \sum_{t=T-v}^{T-1} (e_{1,t}^2 + e_{2,t}^2) \qquad 0 \leqslant K_t \leqslant 1$$

2. $$K_t = \sum_{t=T-v}^{T-1} (e_{2,t}^2 - e_{1,t}\, e_{2,t}) \Big/ \sum_{t=T-v}^{T-1} (e_{1,t}^2 + e_{2,t}^2 - 2e_{1,t}\, e_{2,t}) \quad 0 \leqslant K_t \leqslant 1$$

3. $$K_t = \alpha K_{T-1} + (1-\alpha) \left[\sum_{t=T-v}^{T-1} (e_{2,t}^2) \Big/ \sum_{t=T-v}^{T-1} (e_{1,t}^2 + e_{2,t}^2) \right] \qquad 0 < \alpha < 1$$

4. $$K = \sum_{t=1}^{T-1} w^t e_{2,t}^2 \Big/ \sum_{t=1}^{T-1} w^t (e_{1,t}^2 + e_{2,t}^2); \qquad w \geqslant 1,\ 0 \leqslant K_t \leqslant 1$$

5. $$K = \sum_{t=1}^{T-1} w^t (e_{2,t}^2 - e_{1,t}\, e_{2,t}) \Big/ \sum_{t=1}^{T-1} w^t (e_{1,t} + e_{2,t} - 2e_{1,t}\, e_{2,t})$$

$$w \geqslant 1,\ 0 \leqslant K_t \leqslant 1$$

2.8 Summary

In this chapter we have surveyed the techniques of forecasting which may be relevant in a macroeconomic context. Although the techniques basically relate to the construction and estimation of multi-equation models, the construction of single equation statistical and econometric models is also considered. These may serve both a competitive and a complementary role. Issues relating to the specification, estimation and preparation of forecasts are successively considered. Finally, techniques of combining forecasts generated by alternative methods and sources have been discussed.

In the next chapter we propose to discuss issues relating to macroeconometric forecasting which may be especially relevant for developing economies in the light of the experience of existing exercises of this nature.

Chapter 3

Macroeconometric Models for Developing Countries: Specific Issues

The amount of structural detail needed in building an appropriate macroeconometric model for any economy whether in the developed or in the developing category is so large that ultimately each case has to be treated as unique and distinct in itself. Still there are certain general features and issues which need to be borne in mind when modelling a developing economy. These arise both from *a priori* considerations and from existing experience in modelling developing economies.

Model building activity in and for developing economies is not new. Traditionally, such models were of the 'programming' and 'optimization' type* rather than econometric models for forecasting and simulation. However, even though this latter variety is relatively a late-comer, especially for use in economic planning, a considerable amount of activity has already taken place in this field. Efforts have been due both to institutions like UNCTAD (1968, 1972), ECAFE (1968) and the project LINK (1973) and to individual researchers.

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* Examples and sources include Blitzer et al (1974), Adelman and Thorbecke (1966), Cabejon (1969), Chenery and Strout (1966), Clark and Foxley (1970, 73), Eckaus and Parikh (1968), Fei and Ranis (1964), Manne (1974).

Many of the early models do not continue to be operational in not having been revised, updated and used as continuing instruments of forecasting. Nor, in general, a rigorous process of testing and evaluating their forecasting performance has been undertaken. These models were built in the spirit of experimentation and they do shed valuable light on the nature of problems involved.* Some of these issues are considered below.

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* Some of the models built for developing countries are listed below:

Behrman (1971, Chile), Albertelli (1967, Mexico), Barraza-Allande and Solis (1974, Mexico), Beltran del Rio (1973, Mexico), Cerboni (1975, Venezuala), Davila (1960, Ecuador), Dutta and Su (1969, Argentina), Hansen (1973, Afghanistan), Kelso (1973, Peru), Khan (1974,Venezuela), Lawrsen (1967, Columbia), Liu and De Vries (1969, Brazil), Manhertz (1971, Jamaica), Marwah (1969, Columbia), Molina and Mellor (1974, Chile), Navanjo-Villa Pobos (1974, Honduras), Pujol (1963, Argentina), Stahl (1965, Puerto Rico), Stavron and Arboleda (1975, Panama), Steed (1969, Argentina), Wong (1974, Singapore), Abbey and Clark (1974, Ghana), Nava and Sabater (1974, Philippines), Otani and Park (1976, Korea), Persad (1975, Trinidad and Tobago); UNCTAD (1968, 1972), Ball (ed.) (1973: project LINK); ECAFE (1968); models for the Indian economy are extensively reviewed in Part II of this work.

3.1 High Degree of Aggregation

Available models for the developing countries are generally characterised by a high degree of aggregation. For example, of the eighteen models given in UNCTAD (1968), with a total of 370 equations, 281 equations have, apart from the constant term, only one regressor; 67 are with two regressors, and 21 with three regressors. Dependent variables are also highly aggregative. Although there are practical limitations of obtaining appropriately disaggregated data, aggregation involves certain assumptions which may not always be warranted.

Thus, if both parameters and variables change between different micro-economic functions then they will be different for the macro-economic function. In the case of a linear function, if only the variable changes between micro-units but not its parameter, the same parameter will apply to the corresponding macro-economic function. Similarly, if the explanatory variable is the same for all micro units, then the same variable in a macro function will have a parameter which is the sum of parameters in the micro-functions. These special circumstances are rarely met. In practice an aggregation error has to be allowed for to some extent. However, care should be taken to disaggregate a variable as far as possible into relatively homogenous groups.

In the context of a developed country, one experiment to study the effects of disaggregation on forecasting performance was done with the Brookings Quarterly Model of the U.S. economy. Two versions of the model, a 'condensed' version and a highly disaggregated 'large' version of the model were produced. Various interim-sample and post-sample simulation experiments with the two versions (see e.g. Fromm and Schink 1973, Fromm 1972) led to the conclusion that disaggregation leads to a significant improvement in the overall model simulation and forecasting performance. In general, however, since errors would tend to accumulate relatively more in large models, increasing the size of the model may involve a sacrifice of predictive accuracy.

3.2 Limitations of Data

Models have generally been estimated with annual observations. The sample periods have been small* and the quality of data doubtful. It has been said that the data base for the elaborate superstructure of large econometric models has been more of the nature of 'shifting sand' rather than 'solid rock', that available statistics are such an opaque glass through which to view experience and, that 'econometricians too often put the cart (intricate analysis) way out in front of the horse (reliable data)'. Specific problems arising from data limitations may be identified as below.

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* For example, the limited number of observations used in the UNCTAD (1968, 72) and ECAFE (1968) models may be studied in the following tables. These are adapted respectively from Shourie (1972) and Sastri (1975).

No. of observations	5-7	8-10	11-13	14-16
	No. of countries			
UNCTAD 68 (14 models)	2	7	3	2
ECAFE 68 (8 models)	4	1	1	2

Four other models in the UNCTAD (1968) study use varying number of observations. The situation is somewhat improved in the latter study.

No. of observations	11-14	15-18	19-21
	No. of countries		
UNCTAD 72 (55 countries)	11	4	40

Since only annual observations are available it is not possible to do any shorter term forecasting nor is it possible to study finer lags in the stochastic equations and related dynamic properties of the model unless various interpolations are done.

Secondly, when the sample size is small it is not feasible to use some of the more sophisticated simultaneous-equation estimation procedures like FIML and 3SLS. Even in the application of 2SLS and LIML there are problems if the number of predetermined variables in the system exceed the number of observations unless some mixed estimation procedure involving principal components etc. in the first stage is adopted.

Thirdly, the models soon become 'dated' as national accounts in most of the developing countries are constantly undergoing revisions. The revisions may not only affect individual coefficients but also the overall conclusion that derive from a model.

Even if a longer uniform time-series are available the analyst may be forced to choose a smaller sample because of structural changes involved in the very nature of development.

For all these reasons data problems do look formidable. Vernon (1966) has used this as a major argument in proposing that model building in developing countries for planning purposes is not a very useful exercise. Although these problems are real, such a blanket pessimism is not warranted. With the amount of resources and effort now devoted both at the national and international level towards building an appropriate data-base, and a number of years already past after the initiation of such efforts, one can confidently expect that the data problem is continuously receding in importance.

It is also useful to distinguish between developing countries where national accounts are of a better quality and those where they are not. There is a constant interaction between the sophistication of techniques and the data-base. The data-base improves upon itself not in a vacuum but in response to the demand-pattern which promises to use it fruitfully. Experience of developed countries itself substantiates this. For example, Grove (1969) writes that most of the economic time-series in the U.S. 'now considered to be well-established, got their start in just that way, and are not now, for their earlier years, nearly as accurate as is commonly supposed'.

3.3 The Demand- and the Supply-Sides

Barring exceptions, econometric models of developing countries have borrowed from models of the western developed countries an over-statement of the demand aspects of the macro-economy. The usual practice has been to construct a model within the Keynesian framework with national income being divided into a number of expenditure components and then estimating separate demand functions for these components. The greater emphasis on demand aspects and a primary concern with policies of demand management may be justified in developed economies where supply constraints are not present or are of minor importance. However, in developing economies supply constraints and bottlenecks are important and should not be underplayed.

In a macro economic context 'demand' and 'supply' aspects are to be interpreted carefully. For example, when one includes a demand for investment function, the capital-stock which increases from such demand has implications for the supply of goods and services. Further, if separate functions for factor demands are introduced, these functions will be basically supply-side considerations going beyond the demand for capital formations which enters into analysis as demand for final goods in GNP. Klein (1978) writes:

> 'The demand for labor, like the demand for capital, is supply side analysis .... Behind the IS-LM diagram or other simplified rendition of the aggregate demand model lie many supply-side relationships.'

A desirable model framework is where both demand (expenditure) and supply (output) are considerably disaggregated so that an explanation of sectoral prices by line of production can be offered. If sectoral production functions are introduced, gross sectoral output are appropriately treated as functions of intermediate inputs and labour and capital inputs. In this framework the role of intermediate imports can also be highlighted.

However, even if demand aspects are substantially disaggregated there is no absolute necessity for supply aspects to be equally disaggregated if data or other limitations do not permit this. Aggregate production functions may be usefully employed as long as a flow of income equal to the flow of expenditure is generated. Although in this framework separate price-relatives arising from detailed demand flows can only be related to the average price level arising from the aggregated treatment of output. Further, in an aggregate production function which explains only value-added as a function of primary inputs, the role of intermediate inputs especially intermediate imports cannot be analysed. For this purpose, ideally input-output framework needs to be integrated with the overall model.

As a matter of fact the importance of supply-side considerations which Klein (1965) had emphasised earlier for model-building in developing economies, he has more recently (Klein 1978) advocated for models of western market economies as well. It is argued that demand-related policies, deriving from the Keynesian system which itself originated in the context of a pervasive deficiency of demand, has served the western economies well until now but it must now be supplanted by supply-side considerations 'in order to develop policies that will be able to deal with the world's contemporary economic problems.'

3.4 Theoretical Foundations of the Model

To a large extent both the degree of disaggregation and the relative emphasis on supply and demand aspects depends on the theoretical foundations of the overall model of income determination which would govern its causal structure. In broad terms, three basic approaches to the overall framework may be distinguished: simple quantity theory, the income-expenditure theory and the monetary theory of nominal income. It is useful to distinguish between thse approaches at a highly simplified level although various refinements, adjustments and combinations would clearly be possible. The simplified framework provided by Friedman (1971, 72) has been adopted here for this purpose.

For representing a macro-economy, the following six equations may be considered as common to the three approaches.

Common Equations

$$M^d = P.l(Y/P, r) \quad (3.1)$$

$$M^s = h(r) \quad (3.2)$$

$$M^d = M^s \quad (3.3)$$

$$C/P = f(V/P, r) \quad (3.4)$$

$$I/P = g(r) \quad (3.5)$$

$$Y/P = C/P + I/P \quad (3.6)$$

where,

M^d = quantity of money demanded, M^s = quantity of money supplied, Y = nominal income, P = price-level, r = interest rate, C = consumption, I = investment. As the system stands there are seven endogenous variables with equations (1-3) relating to the money sector and equations (4-6) relating to the real (savings-investment) sector. In order to make the system determinate additional information is needed since there are seven endogenous variables in six equations. It is the way in which this information is provided that one approach differs from the other.

In the simple quantity theory, the following equation is added.

$$\frac{Y}{P} = y_o \qquad (3.7a)$$

This enables equations (4-6) to be solved for the interest rate. From equation (1-3), an equation relating the price-level to the nominal quantity of money is obtained.

In the simple income-expenditure approach, the equation

$$P = P_o \qquad (3.7b)$$

is added. Now equations (1-3) are used to define one relation between the interest rate and real income (the LM curve) and equations (4-6) to define a second such relation (the IS curve) which together give interest rate and real income.

In the third approach, i.e. monetary theory of nominal income, the system is rewritten in the following way:

Demand for Money $\quad M^d = Y.1(r) \qquad (3.1a)$

The assumption here is that the elasticity of demand for money with respect to real income is unity. The next two equations are retained:

$$M^s = h(r) \tag{3.2a}$$

$$M^d = M^s \tag{3.3a}$$

Other equations of the system are replaced by a new equation

$$r = (P^* - g^*) + (\frac{1}{Y}\frac{dY}{dt})^* \tag{3.7d}$$

where,

P^* = permanent real rate of interest $\left[P^* = r-(\frac{1}{P}\frac{dP}{dt})^*\right]$

g^* = permanent rate of growth of real income $\left[g^* = (\frac{1}{y}\frac{dy}{dt})^*\right]$

where y is real income and an asterisk indicates its permanent version,

and $(\frac{1}{Y}\frac{dY}{dt})^*$ = permanent rate of growth of nominal income.

The assumption is made that $(p^* - g^*) = k_o$ is determined outside the system. This implies either (i) that over a time-interval relevant for the analysis of short-period fluctuations, p^* and g^* are separately regarded as constant, or (ii) that they both move together so that their difference is a constant.

Furthermore, at any point of time $(\frac{1}{Y}\frac{dY}{dt})^*$ is taken to be a pre-determined variable assumed to depend partly on past experience and partly on considerations outside the model. The resultant system of four equations in four unknowns - (M^d, M^s, Y and r) is thus determinate.

In a system like this, prices and quantities are not separable in the nominal income. Thus the way in which real and nominal variables can be related has not been outlined and Friedman regards 'the saving-investment sector as unfinished business.'

The question as to whether one model is suitable or another as far as the determination of national income is concerned is an empirical one and must be resolved for individual economies separately. However, each of these formulations should be treated only as broad frameworks within which necessary extensions and details can be appropriately added. The extensions may relate to incorporating additional variables in the existing functions, disaggregating existing variables, and introducing additional functions.

One way in which such disaggregation may be initiated is via equations (3.6) and (3.7). Equation (3.6) is the income-expenditure identity and terms on the right-hand side may be disaggregated and subsequently separate demand functions for the expenditure items may be introduced.

An important distinction here is between private and government (or public) expenditures.† The role of the public sector is important both in view of the fact that its expenditure propensities would be different from those of the private sector and that in almost all developing countries its share in total expenditure has been increasing. The public sector has the avowed role of initiating economic development and a separation of private and public sector expenditures would be very useful for policy analysis. Another necessary disaggregation would, of course, be in terms of treating an 'open' economy and introducing various import and export components. The supply-side

† The framework given in this section would be modified in view of the developments in the 70s emphasising the interdependence between real and monetary sectors through public sector identities and wealth effects.

influences come from equation (7a) which can be replaced by one or more production functions relating to sectoral outputs. Here, at least, a distinction between agricultural and non-agricultural output is necessary so that the much debated 'dualistic' aspects of a developing economy can be accommodated.

3.5 Nominal or Real Variables?

After the overall approach is chosen, the question of specifying individual functions, within limits permitted by the overall approach, arises.

In specifying individual functions, often one has to choose whether to express variables in nominal or in real terms. This has implications for estimation and interpretation of coefficients, for the hypothesis being advanced and for linearity and non-linearity of equations.

As far as technological equations like production functions are concerned there is little choice. If data is available, one should use only quantities in volume terms or an appropriately deflated series at constant prices.

However, for demand equations, the choice between series measured at current prices and those measured at constant prices is meaningful and important. Relevant considerations here are (i) whether money illusion is hypothesised or not, (ii) whether price relations in the forecast period are expected to continue as in the sample period, and (iii) whether one wants information regarding a change in a nominal variable being decomposed into changes due to variations in the corresponding real variable and in prices. It may not, of course, always be possible to get this information as in the case of the monetarist theory of determination of nominal income.†

From the statistical point of view it is relevant to note that for countries which have experienced a high rate of inflation, the use of nominal variables on both sides of equations will give a very good fit (in terms of R^2 etc.) which may be misleading. It may also add to the problem of multicollinearity among regressor measured at current prices. Further, if the appropriate relationship is in real terms, but it is estimated in nominal terms, the estimates will in general be biased.

For example, if the true relationship between two variables x and y is given by

$$\log x = a + b \log y + u \tag{3.8}$$

so that in a least squares estimate,

$$\hat{b} = \frac{\text{cov.}(\log x, \log y)}{\text{var.}(\log y)}$$

but the estimates are actually obtained from

$$\log X = a + b \log Y + u \tag{3.9}$$

where X (=xp) and Y (=yp) are corresponding nominal variables with p as the price-level, we will have

$$\hat{b} = \frac{\text{cov.}(\log X, \log Y)}{\text{var.}(\log Y)}$$

where cov.(log X, log Y) = cov (log x, log y) + cov (log x, log p)

+ cov (log

and var (log Y) = var (log y) + 2 cov.(log y, log p) + var (log p)

† Also if the entire model is specified in real terms, it may be very tricky to move to nominal magnitudes using exogenous information on prices.

The extent of bias between the two estimates of $\hat{b}$ is given by the additional terms in the numerator and denominator in the latter case. In the simplest case, where nominal variables are not correlated with real variables, cov (log x, log p) = cov (log y, log p) = 0, we will have

$$\hat{b} = \frac{\text{cov.}(\log x, \log y) + \text{var}(\log p)}{\text{var}(\log y) + \text{var}(\log p)}$$

which is biased towards unity.

The implication of possible bias in estimates call for additional care in interpreting these. However, if the interest is not in the individual valuesof coefficients but only in predictions one may decide to use biased coefficients as long as the relationship between prices, and real variables in the sample period may be expected to replicate itself in the forecast period.

In the specification of demand-functions, money-illusion is an important consideration. If it is present for example, in demand for money (M^d), the appropriate dependent variable is M^d in nominal terms; however, if the hypothesis is about the absence of money illusion, the appropriate dependent variable is M^d/P. It is difficult to know _a priori_ whether in any given context money illusion is present or not. In many cases, it may be possible to test for its presence.

For example, if a consumption-function is defined in log-linear terms

$$\frac{C}{P} = \alpha^1 \frac{Y}{P}^{\beta_1} \frac{W}{P}^{\beta_2} \tag{3.10}$$

so that real consumption expenditure is a function of real income and real wealth, then defining $\frac{C}{P} = c$, $\frac{Y}{P} = y$ and $\frac{W}{P} = w$, we can estimate the equation

$$\log c = \alpha + \beta_1 \log y + \beta_2 \log w + \beta_3 \log P + u \tag{3.11}$$

and test whether β_3 is significantly different from zero.

The test could be extended to accommodate various lag-adjustment hypotheses.* If it is established that variations in price-levels have an independent influence on real demands, either price-levels should be introduced as additional explanatory variables, or, one may justify the use of variables defined at current prices.

3.6 Testing for Structural Changes in the Sample

The process of development involves structural changes by definition and unless proper care is taken that the structure in the forecast period is not widely different from the structure in the sample period, the model would show poor prediction performance. The consideration of structural changes has relevance for the choice of the sample period and this is likely to compound the difficulty of the availability of small samples.

It is reasonable to expect that the more recent the experience, the more likely it is to correspond with the forecast period. Hence, formulations of tests for structural changes should start by constructing a sample of observations for relatively recent years and finding out whether the extension of sample backwards provides evidence of structural shifts. If there is *a priori* evidence of unusual changes in a year or two even if recent which are not likely to continue, these years may be dropped from the sample. In individual functions some once-for-all changes in the intercept and slope coefficients can be accommodated by the use of appropriate dummy variables which thus permit the use of a larger set of information than would otherwise be possible.

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* For a further discussion, see Branson and Klevorick (1969, 72) and Cukierman (1972). It has been shown that the test is sensitive to changes in lags and adjustment hypotheses.

However, it is not only the structural coefficients of individual functions but also the structure of the model as a whole, that is relevant. For testing the validity of the model as a whole, revelant tests are formulated only in terms of prediction performance of the model. Considerations relevant for this purpose are discussed in chapter 4. However, even at the stage of constructing the model, many mistakes can be avoided by direct testing for the stability of parameters in individual functions within the chosen sample period. As a result, sample periods may be revised or dummy variables and other modifications in the model may be introduced.

If the available sample is large enough, the 'chow' test can be applied by dividing the sample in two parts of sizes, n_1 and n_2, and observing whether there are significant differences between estimated parameters in the earlier part and the more recent part. For example, if for the first set, the estimates are

$$Y = \hat{\beta}_o + \hat{\beta}_1 X_1 + \ldots + \hat{\beta}_2 X_k + e_1 \tag{3.12}$$

and for the second set,

$$Y = \hat{\beta}_o + \hat{\beta}_1 X_1 + \ldots + \hat{\beta}_K X_k + e_2 \tag{3.13}$$

with symbols having usual meanings, one has to test whether

$$\hat{b}_i \neq \hat{\beta}_i \quad (i=1, \ldots, K)$$

The test statistic is defined as

$$F^* = \frac{\left[\Sigma e_p^2 - (\Sigma e_1^2 + \Sigma e_2^2)\right] \Big/ K}{(\Sigma e_1^2 + \Sigma e_2^2) \Big/ (n_1 + n_2 - 2K)}$$

where e's refer to the residuals and subscripts p, 1 and 2, respectively to the 'pooled' series, the 'first part' and the 'second part'. K is the number of parameters being estimated. One needs to compare the computed F* ratio with the theoretical value of F for $(K, n_1 + n_2 - 2K)$ degress of freedom for an appropriate level of significance.

Samples may need to be truncated if significant differences are observed. A useful procedure would be to move backwards from the beginning of the second-part of the sample in steps by adding a few observations at a time and testing for structural stability in the enlarged sample. For this purpose the following F -ratio would need to be computed,

$$F^* = \frac{(\Sigma e^2 - \Sigma e_1^2) / n_2}{\Sigma e_1^2 / n_1 - K}$$

where e refers to the residuals of the enlarged sample, e_1 to those of the original sample. n_1 is the number of observations in the original sample and n_2 is the number of additional observations. F* is to be compared to the theoretical value of F for $(n_2, n_1 - K)$ degrees of freedom. The same procedure rather than the chow test may be applied if the sample cannot be divided into two sets large enough to permit the estimate of K parameters.

It must be remembered that such testing would only be preliminary and notional because of the presence of simultaneous-equation bias in the estimates of coefficients in the context of OLS being used in simultaneous-equation models.

In some cases it may be possible to replace estimates of endogenous variables occurring on the right-hand side by regressing such variables on all the predetermined variables of the system, if these are known. One can then go through all the steps using explanatory variables from which the influence of errors has possibly been filtered. But this procedure calls for a relatively large sample to begin with so that for each sub part of the sample the number of observations are more than the number of predetermined variables in the system.

3.7 Choice of Exogenous Variables

Along with the choice of the sample period, the choice of exogenous variables in the system is also important. It would affect the quality of forecasts as also the estimated parameters.

In deciding as to which of the explanatory variables entering the right-hand side of various equations can be treated as exogenous, a priori information would be very useful. If there is reason to believe that an explanatory variable is affected significantly by one or more of the endogenous variables of the system, such an explanatory variable should be made endogenous by adding an equation for the purpose. The process is iterative in the sense that now the existing list of exogenous variables should be checked again to see if the new endogenous variable does not affect any of these significantly.

There are a number of variables like time-trends and policy instruments (in general) where such a choice is not difficult. But in many cases the choice may not be easy.

Often any direct testing of the 'exogeneity' of a variable by regressing it on a list of endogenous variables would not be useful. This is because direct regression may yield significant coefficients because of 'spurious' correlations.

There are some procedures developed by Sims (1972) and others following a theoretical treatment of the subject by Granger (1969) towards a direct testing of exogeneity of variables in models of income-determination. These tests have been applied primarily to the question of exogeneity of money in the money-income relationship in advanced economy contexts.

Where possible, similar tests can be applied in the context of developing economies also. In a model of income-determination, there are some important choices to be made, like should variables like population, foreign-aid, exports and money-supply be given an endogenous treatment or they are appropriately treated as exogenous.

The empirical counterpart of Granger's propositions as developed for a two-variable case involves the following practical tests.*

> If X causes Y, a regression of Y on past, current, and future values of X should exhibit significant coefficients for past and current values but insignificant coefficients on future values of Y.
>
> Also, if X causes Y, a regression of X on past, current, and future values of Y should exhibit significant coefficients on future values of Y and may, or may not, exhibit significant coefficients on present and past values.

If tests are carried with respect only to the future coefficients in the above regressions, the test will indicate the existence of what has been called 'proper' causality - where the causal effect takes at least one-time period to manifest itself. Contemporaneous causality can be tested by including the present term with future lags when carrying out the tests.†

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* See, for example, William, Goodhart and Gowland (1975), Barth and Bennet (1974), Sharpe and Miller (1975).

† See, for example, William, Goodhart and Gowland (1975).

The regression tests are based on the assumption that the X and Y series are jointly covariance stationary (Sims, 1972) each with zero mean. Since economic processes are generally non-stationary, the non stationary behaviour should either be modelled by a time-trend, if appropriate, or it should be eliminated by an appropriate degree of differencing. Many economic time series can be 'filtered' to obtain residuals that are approximately 'white-noise'. A filter of the type $(1-kB)^p$ with appropriate values of k and p, where B is difference operator have been generally used.

The tests proposed above can be used for preliminary testing of some of the specifications in the model and for finding out whether or not a particular variable is truly exogenous to the model.

There are certain cases where, however, the test may be misleading. These are:

(i) if there is a third variable, say Z, which affects X quicker than Y. In this case the future lags of Y will be significant when explaining X even though X and Y themselves may not be causally related.

(ii) if factors affecting one variable, say Y, include the expected future values of the other variable X and yet there is no causal relationship from Y to X. In the event of expectations being accurate, the future values of X may be significant when explaining Y because of the autocorrelation of X.

(iii) where contemporaneous causality is involved the tests will fail to detect feedback of a certain type which occurs within the period of analysis (quarter, year etc.). The 'innovation' in the stochastic-process X is that part of X which cannot be predicted from X's own past (i.e. the residual in a regression of X on its own past). If X and Y are connected by two causal relations - one from X to Y involving a distributed lag and the other from Y to X with only the current innovation in Y on the right-hand side - then the tests will not detect the Y to X feedback.

If serial correlation is negligible, this failing of the test becomes important. In this case Y and X are their own innovations and causal relations may be purely contemporaneous. In the general case, with serially correlated data, this failing is not likely to be important (e.g. see, Sims, 1972).

This last item makes it necessary that data with finer disaggregation in time be available. If only annual data is available a relationship with up to 3 quarterly lags would be treated only as contemporaneous. This limits the applicability of tests in cases where only annual data may be available. However, there would be many cases where even annual data may be fruitfully used. For example, one can test whether population is affected by income or aid is affected by income. In these cases annual lags are meaningful.

3.8 Prediction of Exogenous Variables

Once an appropriate list of exogenous variables is available, the forecaster will have to obtain predictions for future values of these for forecasting endogenous variables of the model.

Apart from a correct specification of the model, the quality of forecasts depends vitally on the accuracy of independently obtained forecasts of the exogenous variables.

In economies that are more advanced in the art and practice of macroeconomic forecasting, a variety of forecasts would normally be available for most of the exogenous variables one might be using. These forecasts would stem from various formal econometric models being used by forecasting institutions and agencies as also from administrative bodies which may base their forecasts on expert opinions and less formal models. In any case, the forecaster is likely to be in a position where he can choose from a number of alternative forecasts regarding his exogenous variables. The situation would not be so promising in a developing economy. Independent forecasting institutions undertaking macroeconomic forecasts based on formal and scientific methods would be few and far between. Forecasts made by government administrative bodies may be biased for obvious political reasons and any adjustments in them would be difficult as the forecasting structure behind them, generally being informal, would not be available.

In practice, the forecaster himself may have to produce additional forecasting models which are independent of the main model for predicting future values of the exogenous variables. The type of models needed and relevant for the purpose depends to a certain extent on the type of exogenous variable in question. There may be certain variables like population, share of rural or urban population and such other demographic magnitudes where a simple time-trend model or an exponential smoothing procedure might be appropriate. There may be other variables where a more rigorous extrapolative model can be built. However, problems arise for variables which are exogenous to the national economy but would be endogenous to an international economy like world income, world trade prices etc. For these variables either some subjective forecasts will have to be produced dependent on an understanding of the anticipated changes in the production, trade and marketing conditions in the international economy or forecasts by international agencies like the World Bank and UNCTAD on an informal basis, or on a formal basis like those originating from a project like the LINK project will have to be used.

Another set of exogenous variables may relate to government policy *vis a vis* expansion of money-supply, levels and distribution of public expenditures and revenues. For the immediate future, usually some information is provided in the budget documents and for the longer-term there may be useful information in the relevant plan documents. However, it does seem that in dealing with day-to-day crises, unforeseen events on the international scene and some built-in bias for practical or other reasons, the actual levels of variables would deviate widely from the estimates in budgets and plan documents in the developing countries. The forecaster may therefore need to make his own adjustments. There may be cases where a study of the historical data about budget-estimates and corresponding actuals may highlight some systematic bias. In such cases 'formal' models may be introduced for making the adjustments.

On the whole, the more 'formal' the models that are used for obtaining predictions of the exogenous variables, the easier it would be to trace the sources of prediction errors and introduce necessary corrections later on.

3.9 The Use of Preliminary and Revised Data

With reference both to endogenous variables and lagged exogenous variables, one may be foreed to use a considerable amount of preliminary data and data in various stages of revision for a period immediately preceding the forecast period. This is a kind of period of 'flux', where although observations are available in historical past they are still being collected and processed but a set of preliminary and revised estimates are available. In developed countries where components of national income and other economic data are published quarterly, this period of flux may be small. For example, in the U.S., national accounts data are published in the 'Survey of Current Business'. A preliminary estimate is published one month after a quarter has ended with a revision in the following month. In July each year, the figures for the first quarter of the current year as well as the previous three years are revised. These latter revisions differ much more from the preliminary data than the first revisions. Thus, there is a lag of at least one quarter and at most four quarters before revised data are available.

In developing countries, however, the period of flux is much bigger. Preliminary estimates are normally published with a gap of one year, revised estimates with a gap of two years and final estimates with a gap of three years. The quality of forecasts is therefore substantially affected unless careful use of preliminary and revised data is made.

Studies in developed countries like Cole (1969a, b) Denton and Kuiper (1965), Stekler (1967) and Howrey (1978), indicate that the use of preliminary rather than revised data can lead as far as to double the forecasting errors.

It is possible however, to build forecasting models which make an optimal use of preliminary data, or more generally, data infested with measurement errors where optimality is defined such that forecasts minimise an expected loss function under certain assumptions.

The information that is used in constructing such optimal predictions is a vector of relationship between observed values and their preliminary or revised estimates. This has been called the 'observation' system. If the rth observation on x_t is denoted by $y_{r,t + l(r)}$ to designate that it is available with a lag of l(r) periods, the observation system may be written as,

$$y_{rt} = \theta_r x_{t-l(r)} + v_{rt} \tag{3.14}$$

where θ is a matrix of coefficients to be estimated from past values of y's and x's which are vectors of endogenous variables of the forecasting model. The information on θ is then used for constructing optimal forecasts. For a derivation of such predictors and their properties reference can be made to Astrom (1970), Denton and Kuiper (1965) and Howrey (1978).

3.10 Some Methodological Problems

A review of certain methodological problems that arise in standard statistical estimation of various functions is now undertaken. Routine application of least squares estimation procedures to a system of equations meant for use in forecasting and policy analysis needs to be carefully interpreted. There are standard textbook type of warnings against multi-collinearity, autocorrelation in residuals etc. which are too often ignored in practice. Test-statistics like t-ratios, R^2 and D-W statistics are mechanically reported and liberally interpreted. For building appropriate forecasting frameworks there are at least some methodological problems that must be emphasised here.

3.10.1 Multicollinearity

In least squares estimates, the problem of multicollinearity arises because of linear or near-linear relationships among explanatory variables. In a multiple regression of the type

$$Y = b X_1 + b_2 X_2 + \ldots + b_n X_n + u \qquad (3.15)$$

the parameters b 's cannot be estimated if the X's are perfectly collinear, i.e. correlation coefficients between these are unity. On the other hand, if the X's are orthogonal i.e. correlation coefficients between these are zero, there is no need to perform a multiple regression. The parameters b's can be obtained by separately regressing Y on different X's. In practice, the usual situation lies between these two extremes. If the situation is such that explanatory variables are highly correlated, parameter estimates become very sensitive to measurement errors and sample coverage etc. and become unreliable. The typical fruits of multicollinearity are statistically insignificant coefficients, wrong signs and inaccurate magnitudes of parameters.

The situation encountered in the context of a developing country, especially if highly aggregated regressors are being used and the sample period is limited is one where a high degree of collinearity is present among the regressors because of strong trend components.

An indication of how frequently might high correlation coefficients be encountered if highly aggregated regressors are used in the context of developing countries is provided by Shourie (1972). In the UNCTAD (1968) study about 168 equations use more than one regressor. Correlations between regressors in 118 of these were calculated involving in all 168 correlation coefficients. The distribution of these according to magnitude is as follows:

Table 3.1

Collinearity Among Regressors in UNCTAD (1968) Models

Value of the corr. coeff.	0.0- 0.6	0.6- 0.65	0.65- 0.70	0.70- 0.75	0.75- 0.80	0.80- 0.85	0.85- 0.90	0.90- 0.95	0.95 1.00
Number	12	6	6	12	6	4	6	13	103

Note: The columns should be read as, e.g. >0.6 to ⩽0.65, i.e. values that are equal to 0.6 are included in the previous column.

Thus, about 60 per cent of the correlations were higher than 0.95 and about 70 per cent, higher than 0.90. The incidence of multicollinearity in developing countries with many highly aggregated, annual and trend-ridden series is likely to be high.

It is necessary, therefore, that before selecting structural equations and their specifications on the basis of least squares estimates and usual test-statistics like t, F and R^2, some tests are made for the presence or otherwise of multicollinearity among the regressors. This is especially so when unexpected signs and magnitudes of parameters are encountered.

Some idea as to the presence of multicollinearity can be obtained by examining the partial correlation coefficients between explanatory variables. However, a number of more extensive test procedures exist such as those proposed by Farrar and Glauber (1967) and Silvey (1969). There is also a method based on Frisch's confluence analysis and another based on the method of principal components.

Farrar and Glauber have suggested a series of three tests in order to (i) detect the existence and severity of multicollinearity, (ii) for locating variables which are multicollinear and for (iii) finding out the pattern of multicollinearity. The tests are based respectively on a χ^2-test, an F-test and a t-test.

There may be cases when the multicollinearity problem may solved by appropriate data transformations and by application of generalised least squares or restricted least squares. However, in general, the solution may need additional information in terms either of increasing the sample size or pooling time-series data with additional information from cross-section samples.

The multicollinearity problem may sometimes be ignored if the purpose of analysis is only forecasting and there is no particular interest in magnitudes of individual parameters. This can be done as long as one expects that the relationship between the collinear regressors would not change in the forecast period as compared to the sample period.

3.10.2 Specification Errors

Another set of related difficulties arises from whether the specification of the regression equations is correct or not. Both deficient data and insufficient knowledge as to the correct form of the relationship between dependent and explanatory variables lead to specification errors. Deficient data may arise through errors of measurement in explanatory and/or dependent variables. If there has been aggregation or grouping of data this can also lead to errors.

An incorrect specification may take the following forms:

(1) Omission of a relevant explanatory variable

(2) Inclusion of an irrelevant explanatory variable

(3) Mis-specified relationship between variables

Specification errors may also be due to the way the disturbance terms enters the regression equation or when qualitative changes in regressors are ignored. The main source of specification errors however concerns the question as to what variables should be included as regressors in a particular equation. The estimates will be biased under least squares estimates if certain variables are left out from the equations for reasons of multicollinearity, lack of data, etc., where, in fact, they should have been included in the model.

In general, suppose a vector Y of dependent variables is to be explained. Let X and Z be matrices and b, β, θ, U and W be vectors. Assume that the model used for estimation is of the type

$$Y = X\beta + U \qquad \ldots \quad (3.16)$$

There are different implications, if the correct model is

$$\text{(i)} \quad Y = X\beta + U, \qquad \ldots \quad (3.17)$$

and if it is

$$\text{(ii)} \quad Y = X\beta + Z\theta + W \qquad \ldots \quad (3.18)$$

It can be shown, with reference to least-squares estimates of parameters, that if Z is omitted in the estimation procedure, whereas a correct specification requires that they should not be excluded, not only estimates of coefficients linking Z to Y will not be available but also the coefficient linking X to Y will be biased. If Z is included where in fact Y is independent of them ($\theta = 0$); or if Z is left out, when although affecting Y, they are independent of X ($A = 0$), then the least-squares estimates of β would remain unbiased.

The implication of this type of specification error is more important when the model is to be used for policy analysis in addition to pure forecasting. If the purpose is forecasting alone, one may let X work for Z as well if the relationship between X and Z in the forecasting period is not expected to change from that in the sample period. For policy prescription, however, the estimates of coefficients linking Y and X may be too misleading when Z is left out. In this case, and if the reason for omitting Z was collinearily between Z and X, a search for parent variables that explain Z and X themselves is recommended.

Another form of specification error might arise due to the type of functional form chosen for the equations. There are definite economic assumptions behind each functional form and as such the choice of an appropriate form is important. Some of the forms usually used are listed below.

Table 3.2

Some Functional Forms

Form	Type
$y = a + bx$	linear with an intecept
$y = bx$	linear through the origin
$\log Y = \log a + b \log x$	log linear
$y = a + b \log x$	semi-log
$\log Y = a - b\ x$	log-inverse
$y = a + bx + cx^2$	quadratic
$y = a_o + a_1x + a_2x^2 + \ldots + a_px^p$	polynominal

In a linear relation with an intercept the marginal rate ($dy/dx = b$) remains unchanged; if there is no constant term, the average rate ($y/a = b$) also remains unchanged. In the log linear relationship the elasticity of y with respect to x $\left[(x/y)(dy/dx) = b\right]$ remains unchanged although the marginal and average rates vary. In the semi-log relationship, the elasticity falls as x increases. In the quadratic relation, the rate of change in the marginal rate ($d^2y/dx^2 = 2C$) remains constant. But in this, as in the general polynominal case, there are no restrictions as to the marginal and average rates and the elasticity of y with respect to x.

Except in special cases, however, economic theory does not tell which particular form to use in which instance. There are no *a priori* rules in choosing the functional form. 'Best fit' criteria are not necessarily adequate. In particular the aims of the model should be borne in mind, i.e. whether it is for pure forecasting or also for policy analysis; whether it is for short-term forecasting or for long-term.. Furthermore, the increased computational costs when using non-linear specifications in parameters and variables in multi-equation models may need to be balanced against permitting prediction errors in using only linear specifications.

3.10.3 <u>Limits to Reliance on Statistical Tests</u>

Statistical tests such as the coefficients of determination ($\bar{R}^2$), t-ratios and Durbin-Watson statistics are designed to warn against inadequacies of the model, so that if important explanatory variables have been left out, estimators are not significant, or if strong auto-correlation exists in the residuals, it would automatically be pointed out. As such the analyst may feel comfortable if these tests show highly optimistic results for his model.

However, a number of situations are frequently encountered where mechanical reliance on the coefficient of determination may easily be misleading, i.e. we may get a high value for $\bar{R}^2$ but the use of the equations as if they were adequately specified may not be warranted because of spurious correlations. This arises when two series, although causally uncorrelated, may have strong trends or cyclical components. We will get a high value for $\bar{R}^2$ but it will not be warranted to rely on the model for forecasting or policy purposes unless there is reason to believe that the same relation would continue to exist in future. In the absence of evidence for the existence of a causal relationship, one can rarely be sure that two series will move together in future if they have done so in the past.

Often, therefore, models use transformed data such as the first differences so that auto-correlation in a series may be avoided. Such transformation is not yet very popular in models for developing countries. Nor may it be desirable in every circumstance. In particular, care must be taken that the data transformation itself does not produce serially corrleated errors.

Similarly, there are difficulties in using the Durbin-Watson statistic which is designed to highlight existence of auto-correlation in the residuals. The assumption of randomness in the error-term is critical to regression and econometric model-analysis. If there is evidence of auto-correlation in the residuals, it may imply that a systematic element has been ignored in the explanation. It also means that the last squares estimates, although still unbiased, are no more efficient, and the usual significance tests are not directly applicable.

It is important to note the limitations of the Durbin-Watson statistics in this context. First, it is designed to test only first-order serial correlation, i.e., correlation between an error term and with its immediately preceding error term. It throws no light on either the higher order serial correlation or non-linear forms of relationship between residuals. Secondly, it is not appropriate to use this statistic for measuring auto-correlation if there are lagged endogenous variables among the explanatory variables. Thirdly, there is a range of values of this statistic over which the proposed test is inconclusive. Fourthly, the sample size should be at least thirteen to fifteen observations before tabulated values of the statistic may be used.

In the UNCTAD (1968) and ECAFE (1968) studies there are actually many models where the sample size is smaller. This last problem is however receding in importance as more data is becoming available. The h-statistic, which is a companion to the Durbin-Watson, for cases containing lagged dependent variables may not be used for small samples for little is known about its small sample properties.

Where the Durbin-Watson and h statistics are not relevant, alternative tests should be carried out to test for auto-correlation. The simplest procedure is to test with various forms of autoregressive structures once estimates of regression residuals, from OLS, 2SLS etc., techniques as applied to a preliminary specification, are available. Then one can test for separate orders and forms of serial correlation by using t- and F-Tests in regressions like

$$e_t = pe_{t-1} + u_t$$

$$e_t = pe^2_{t-1} + u_t$$

$$e_t = p_1 e_{t-1} + p_2 e_{t-2} + u_t \quad \text{etc.}$$

If auto-correlation is noted, this problem should be solved by adding additional explanatory variables, changing functional forms or by appropriate transformation of original data.

The t-statistic is omnipresent in econometric exercises. However, an injudicious use of this test should be guarded against. Frequently, the estimated t-ratio is compared with a critical value of 2. There is some justification for this procedure if the degree of freedom in question are more than 8 because then the change in tabulated values of t for different levels of significance becomes considerably smaller.

However, where sample sizes are smaller and the number of regressions large, the test should be more rigorously conducted. Similarly, one should also make sure whether the test being performed is one-tailed or two-tailed i.e., whether the estimated parameter is expected to be positive or negative or merely different from zero.

Further caution is needed when simultaneous equation estimation procedures are adopted. In the case of 2SLS and 3SLS estimators, the ratio of the estimated coefficient to its estimated standard error does not follow the t- distribution (see Christ, pp.515-16, Kmenta, p.584). In these cases the standard t- test is not strictly applicable. The t-ratio should then be used only as a notional or a qualitative guide to the relative importance of individual regressors.

3.11 Some Remarks on the Specification of Individual Functions

Basically, the specification of individual demand, supply and other functions remains a subject of empirical testing in a given context.

For many of the commonly employed functions like consumption functions, demand-for-money function, demand-for-import function, and various production functions, the model builder will have to choose from various available hypotheses and formulations in the theoretical literature. He can also usefully draw upon the relevant empirical literature in both developed and developing countries.

For example, if a consumption function is being specified, there are various hypotheses to choose from like those of habit-persistence, stock-adjustment, permanent income, life cycle, income and wealth, and income and liquid assets etc. In a developing country, among other things, the relevant considerations may be a distinction between subsistence and non-subsistence consumption. For subsistence consumption the m.p.c. is expected to be 1. On the whole, a high marginal propensity to consume would be expected in developing countries. Among other things, the model builder may like to distinguish between agricultural and non-agricultural or rural and urban incomes.

Similarly, in specifying investment functions the commonly used hypotheses are the 'acceleration' principle (Samuelson, 1939, Tinbergen, 1938) based on stock-adjustment hypothesis, 'permanent income' hypothesis (Eisner, 1967, Almon, 1968), the neoclassical approach (Jorgenson, 1963), and the Keynesian approach. Again, depending on the context, a synthesis of elements of more than one theory may be employed. Klein (1965) has suggested that the stock-adjustment form of investment function is not suitable for most developing economies where many opportunities of investment exist. In these countries, it is suggested that 'existence of unused capacity may promote new investment rather than hinder it.' Klein (1965) also suggests that the rate of interest may not be an important variable when 'there are so many worthwhile ventures all economically sound, that close calculation by systematic pattern is unnecessary.' It seems useful to distinguish between private and government investment functions in developing countries as the latter is likely to be a substantial proportion of the total investment and considerations affecting government investment decisions would be quite different from those affecting private decisions.

In establishing the demand for money function there are various variants of permanent income, non-human wealth, non-human wealth and income approaches apart from the basic Keynesian approach where current income and the rate of interest are the relevant variables.

In specifying import-demand functions it is necessary to bring in influences of changes in exchange-rate variations, the influence of the foreign exchange constraint and the variation in domestic prices relative to world prices. An interesting formulation of the import-demand function is provided in UNCTAD (1968) in, for example, the model for the Indian economy. A distinction is made between years where the foreign exchange constraint is operative and where it is not by introducing two functions for non-food imports (M_{of})

$$M_{of} = \text{Minimum } (M^1_{of}, M^2_{of})$$

where

$$M^1_{of} = \frac{f(Y, X\ P_X\ R)}{P_m}$$

$$M^2_{of} = f(Y)$$

where Y is income and X = exports, R = index of official exchange rate, P_X/P_m = price of exports relative to the price of imports.

3.12 <u>Models for Developing Countries: An Arche-type</u>

Structural details of individual countries necessitate considerable and important differences in models of economies in the developing world. An attempt to formulate an arch-type is therefore not very rewarding except inasmuch as it may provide a basic framework which makes the distinctions between developing countries and industrial nations. One such framework has been constructed and used by the UNCTAD secretariat for the LINK project.

The model is specified as follows:

I. Supply of Outputs

1. Agricultural output $Y_a = a_o + a_1 t$

2. Non-agricultural capcity output $Y_{na} = b_o + b_1 K_{-1}$

3a. Capacity utilization ratio $Y^P_{na}/Y_a = c_o + c_1 M + c_2$

II. Demand for Output

3b. Demand for non-agricultural output $Y = d_o + d_1 C + d_2 I + d_3 X$

4. Per capital consumption $\frac{C}{N} = e_o + e_1 \frac{YN}{N} + e_2 \frac{C}{N}_{-1}$

5. Investment in Fixed Assets $IF = f_o + f_1 YD + f_2 YD_{-1}$

6. Exports $X = g_o + g_1 TW + g_2 \, p_y/p_w$

7. Imports $M = h_o + h_1 YD + h_2 (R/p_m)_{-1} + h_3 \, p_m/p_z$

III. Price Equations

8. Agricultural prices $p_a = k_o + k_1 Y_a + k_2 Y_{a-1} + k_3 Y_{na}$

9a. Price of non-agricultural goods $p_{na} = l_o + l_1 Y + l_2 \frac{L}{YD} + l_3 p_a + l_4 p_m$

9b. $p_{na} = m_o + m_1 Y_{na} + m_2 Y_{na-1} + m_3 \frac{L}{YD}$

10. GDP deflator $p = n_o + n_1 p_a + n_2 p_{na}$

11. Export unit value index in domestic currency $p_{xd} = r_o + r_1 p + r_2 X + r_3 p_{xd-1}$

IV. Others

12.	Net factor income payments	$YF^* = a_o + a_1 CDF^*_{-1} + a_2 Xp_x$
13.	Capital stock	$K = \sum_1^t I_t$
14.	Cumulative external deficit	$CDF^* = \sum_1^t (Mp_m + YF^* - X.p_x)$
15.	Real net factor income payments	$YF = \frac{YF^*}{p_m}$
16.	Gross domestic product	$YD = Y_a + Y_{na}$
17.	Gross national product	$YN = YD - YF$
18.	Inventory Investment	$IS = YD + M - C - I - X$
19.	Foreign exchange reserves	$R^* = R^*_{-1} + F^* + Xp_x - Mp_m - YF^*$
20.	Export prices in dollars	$p_x = p_{xd}.Z$

The exogenous variables of the system are

TW = index of world trade; p_m = import unit value index in dollars; F^* = net foreign capital inflow; L = index of cash-balances with the public; Z = index of exchange rates expressed as dollars per unit of domestic currency; t = time trend; N = population.

Symbols marked with an asterisk (*) represent variables expressed in current prices. Equation subscripts as a, b, etc. represent possible options in the specification.

The special feature of the model is that it provides a useful role to the supply side considerations and is able to capture the basic elements of duality in the economy. The role of foreign exchange reserves is explicitly brought about in the import demand function. Also, inventories are treated as a residual.

For any individual country study various modifications in the function will need to be introduced. Some of simplistic explanations like agricultural output being a linear function of time can be easily modified by bringing into variables like agricultural inputs, acreage etc.

Since the model is basically formulated for purposes of projecting trade flows, it is not very competent for domestic policy analysis. The model would need to be extended to bring in a tax sector and a monetary sector. However, as a basic frame of reference, this model does provide a useful starting point.

3.13 Summary

In this chapter we consider some of the problems one may frequently expect to encounter in macroeconometric forecasting in developing countries such as data limitations, a high degree of aggregation, possible understatement of supply side considerations, and the problems created by structural changes. We take note of the options regarding the basic theoretical foundations of a model, the choice of exogenous variables and the implications of specifying income-expenditure variables in nominal and in real terms. We also take note of some of the standard methodological problems like multicollinearity, specifications errors etc., the incidence of which is likely to be high in a developing economy context. Finally, we discuss some relevant considerations in the specification of a few mainstream functions and also an archetype model for developing economies considered as a class.

In the next chapter we propose to discuss available techniques for evaluating forecasts and forecasting models.

Chapter 4

Evaluation of Forecasts and Forecasting Models

A model needs to be evaluated both during the process of its construction as also subsequent to its release. The more reliable a model turns out to be, the more useful it will be in decision-making situations. Although various considerations arise in model evaluation, the most relevant of these is its forecasting performance, i.e., its ability to guide in situations outside its sample. Many available test procedures relate to single-variable forecasts which are compared against corresponding realizations and/or alternative forecasts. Since a multi-equation model predicts a number of variables at the same time, the usual practice of evaluating its performance variable-by-variable needs to be supplemented by additional considerations. In the following discussion, we shall first consider test procedures relating to single variables, and then the additional question of evaluating the model as a whole.

4.1 Some General Considerations

In evaluating a forecast two general considerations need to be borne in mind; first, the costs associated with prediction errors, and second, the costs associated with producing a forecast. For incorporating the former consideration into analysis, some kind of cost function is introduced. If a prediction error of size x is made, then a function g(x) may represent the cost associated with it. Normally, g(x) is taken to be a quadratic function, $g(x) = ax^2$, which implies that the costs rise in proportion to the square of the error. The function is symmetric in treating negative and positive errors and it has attractive mathematical properties. However, there may be many practical situations where a non-quadratic and non-symmetric error function may be relevant. Many such situations are listed in Granger (1969). However, in practice, it is difficult to find the correct form and estimate of a cost function.

It has been argued by Granger and Newbold (1973) that when only ordinal judgements are required, the choice of the function is not too critical. As long as the cost functions monotonically increase from a zero to higher levels of errors, predictors will be ranked in the same way whether one applies one criterion or another. However, if one wants not merely a ranking but also the extent by which one forecast outperforms another, the choice of the cost function is important.

The second consideration, viz., the cost of producing a forecast is sometimes brought into analysis by considering the amount of information used in its construction. Purely extrapolative forecasts, for example, are based on the past history of the predicted series. In a multi-equation econometric model, the information on the past history of many other series is also needed as well as information as to the anticipated values of exogenous variables. When forecasts generated from alternative models are compared, they can be properly ranked only if each of the model uses the same information set. In practice, again this qualification is rarely met or followed.

4.2 Evlauation Procedures: Single-Variable Forecasts

Once a forecast series P_t and a series of realizations A_t for t = 1,2,...n become available there are various ways in which how closely the predictions emulate the realizations can be described. This can be done both with respect to levels and changes in levels of variables. It is useful to consider this distinction first.

4.2.1 Levels and Changes in Levels

Many of the descriptive measures of forecast accuracy have evolved with reference to changes in variables even if the model forecasts level. Although the measures can easily be adapted to refer to levels, sometimes their interpretation would change. It is useful therefore, to bear this distinction in mind.

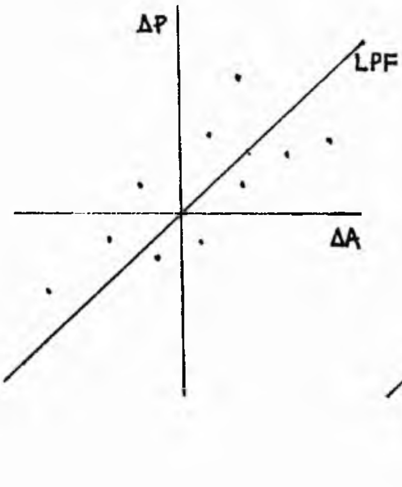
When a model predicts levels, one can calculate 'changes' in one of two ways: (i) by successive differences between predicted levels ($\Delta P_t = p_t - p_{t-1}$), and (ii) by taking the difference between predicted level of a period and the actual level of the previous period ($\Delta P_t = P_t - A_{t-1}$). In the latter case, a comparison between predicted and actual changes is equivalent to a similar comparison between levels, i.e.

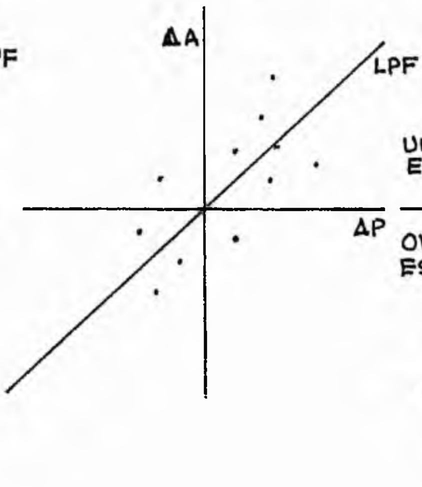
$$\Delta P_t - \Delta A_t = (P_t - A_{t-1}) - (A_t - A_{t-1}) = P_t - A_t$$

It is not always necessary to cast the evaluation framework in terms of changes as long as the measures used are correctly interpreted.

4.2.2. Prediction-Realization Diagrams

A visual description of the forecasting performance can be obtained by plotting predicted and realized changes on the Cartesian axes. In the following diagrams, predicted changes have been alternatively plotted on x- and y- axes. In diagram 4.1, where predicted changes are on the y-axis, one can observe how the predicted changes are distributed given the realized changes. In diagram 4.2, realized changes are on the y- axis and one observes how these are distributed given the predicted changes. The 45° line is the line of perfect forecasts (LPF). Rotating diagram 4.1 until the LPF coincides with the horizontal axis, we obtain the prediction-realization diagram (4.3).





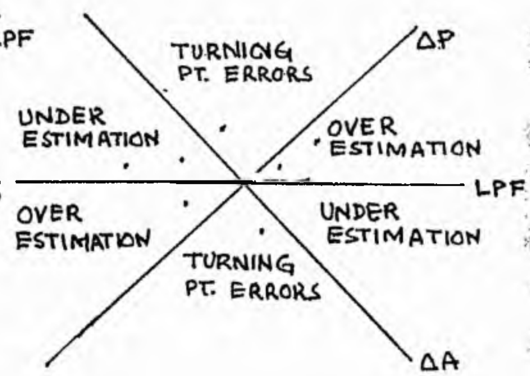


Fig.(4.1) Fig.(4.2) Fig.(4.3)

Prediction Realization Diagrams

Observations about over- and under-estimation of changes and turning point errors can be made in all the diagrams. Relevant areas are indicated in diagram 4.3.

Although prediction-realization diagrams can also be drawn for levels of predictions and realizations rather than changes, the latter procedure seems more useful. It has been contended that (Granger, 1966) the typical time series of economic levels is a near random walk. For such series a graph of predictions against realizations would reveal a close fit, even a 'no-change' or 'constant-change' predictor would look good in this light. Box and Newbold (1971) have demonstrated that by this graphical criterion one random walk can appear to closely predict another random walk. It is therefore more useful to plot the changes. Although the performance would look worse than in the case of levels, it can provide more useful information. The diagrams are also sometimes drawn for relative changes, i.e. $\Delta P_t/P_{t-1}$ and $\Delta A_t/A_{t-1}$

Although a diagrammatic representation is a help, usually the evaluation exercise involves a number of variables and alternative forecasts, and 'summary' statistics are needed to rank forecasts.

4.2.3 Some Summary Statistics of Forecast Accuracy

A number of these have been considered in the literature. The procedure is to describe the accuracy of forecasts over the entire forecast period by a single statistic. Some of these are considered below.

1. Correlation Coefficient between Predictions and Realizations

The correlation coefficient between two series is designed to indicate how closely the two move together. It, therefore, automatically suggests itself for use when it is desired to measure how closely P_t's follow A_t's. The coefficient is defined as

$$\frac{\Sigma(P - \overline{P})\ (A - \overline{A})}{\sigma_P\ \sigma_A} \tag{4.1}$$

Symbols have usual meanings. The properties of this measure, viz., that it is independent of the origin and unit of measurement, which would be useful in a different context, render it somewhat inappropriate in the present context. These properties imply that if all the forecasts were multiplied by a constant or a constant was added to these, the measure would not be able to pick it up. The implicit scale of the measure from a minimum of -1 to a maximum of 1 remains a useful property.

2. Average Absolute Error

Average absolute error is defined as

$$\sum_{t=1}^{n} |P_t - A_t| = \sum_{t=1}^{n} |u_t| \tag{4.2}$$

It has a minimum value of zero when all $P_t = A_t$. It has no maximum value and it is not able to distinguish between turning point errors and ordinary errors.

3. Mean Square and Root Mean Square Errors

These measure are respectively defined as

$$M = \frac{1}{n} \sum_{t=1}^{n} (P_t - A_t)^2 \qquad (4.3)$$

and

$$RMS = \sqrt{\frac{1}{n} \sum_{t=1}^{n} (P_t - A_t)^2} \qquad (4.4)$$

They also have a minimum value of zero in the case of perfect forecasts. There is no upper limit. Their inadequacy lies in not having a proper unit of measurement. They give the same weight to a deviation whether a variable is measured in dollars, or billion dollars or percentages. They however, have interesting mathematical and statistical properties and lend themselves to useful decompositions. As such, they have become very popular.

4. Inequality Coefficients

The inequality coefficients due to Theil (1961, 1966) which are defined below are based on the mean square error. But, in addition, they also represent a search for a proper unit of measurement. They may be defined both with respect to changes and levels. In each case they are to be carefully interpreted.

(a) Changes in Variables

With respect to changes, two inequality coefficients may be defined as follows:

$$U_{C1} = \frac{\left[\Sigma(\Delta P_t - \Delta A_t)^2\right]^{1/2}}{(\Sigma \Delta P_t^{\,2})^{1/2} + (\Sigma \Delta A_t^2)^{1/2}} \qquad (4.5)$$

and, $$U_{C2} = \left[\frac{\Sigma(\Delta P_t - \Delta A_t)^2}{\Sigma \Delta A_t^2}\right]^{\frac{1}{2}} \qquad (4.6)$$

U_{C1} has the advantage that it varies within a finite range from 0 to 1. If $\Delta P_t = \Delta A_t$, the coefficient is zero. If, $\Delta P_t = 0$, or if $\Delta P_t = -b\Delta A_t$ (b>0), the coefficient is unity. But the measure cannot distinguish between different values of 'b', i.e. when predicted changes differ from realized changes by a higher or a lower proportion. The limitation of the measure arises because its unit of measurement (the denominator) is dependent on predictions themselves.

U_{C2} has the advantage that its unit of measurement is independent of the predictions. However, it does not have a finite range of variation. It varies from zero to infinity. But the coefficient does have some useful 'benchmark' values. For example, for 'no change' extrapolation $U_{C2} = 1$. In this case when $\Delta P_t = -b\Delta A_t$(b>0), the measure is able to pick up the impact of different 'b' values. Here,

$$U_{C2} = (1 + b)\frac{(\Sigma A^2)^{\frac{1}{2}}}{(\Sigma A^2)^{\frac{1}{2}}} = (1 + b)$$

(b) Levels of Variables

The same coefficients can be defined with respect to levels.

$$U_{L2} = \frac{\left[\Sigma(P_t - A_t)^2\right]^{\frac{1}{2}}}{(\Sigma P_t^2)^{\frac{1}{2}} + (\Sigma A_t^2)^{\frac{1}{2}}} \quad \text{and} \quad U_{L2} = \left[\frac{\Sigma(P_t - A_t)^2}{\Sigma A_t^2}\right]^{\frac{1}{2}} \qquad (4.6 \ \& \ 7)$$

The remarks applicable to U_{C1} above are also applicable to U_{L1}.

U_{L1} will also have a value of unity when $P_t = -bA_t$, however it is no longer synonymous to a no-change extrapolation. Similarly, the benchmark value of unity, in the case of U_{L2} also does not arise from a 'no-change' extrapolation but when all levels are predicted to be equal to zero. A different benchmark value in this case is 2 which occurs when predictions are of the same magnitude but opposite signs compared to realizations.

If the choice is between U_{C1} and U_{C2} or between U_{L1} and U_{L2}, it must be noted that respectively U_{C1} and U_{L1} would put the forecasts in more favourable light as they are measured with respect to predictions themselves.

5. Somermeijer's Coefficient

Another measure, similar to the inequality coefficients (U_{C1} and U_{L1}) has been called Somermeijer's coefficient. It is defined below.

$$S = 1 - \frac{\Sigma(P_t - A_t)^2}{\Sigma P_t^2 + \Sigma A_t^2} \qquad (4.8)$$

It differs from the inequality coefficients inasmuch as squared terms are considered here and the coefficient is measured as a deviation from unity. In the case of perfect forecasts ($P_t = A_t$) the coefficient is unity. If the variables are defined as 'changes', S would be zero for 'no-change' extrapolation. In general, when $P_t = bA_t$, $b>0$, $S = 0$. It is thus not able to distinguish between higher and lower values of 'b'. Also, since it is measured with respect to predictions themselves, it puts the forecasts in a relatively better light than would otherwise be the case.

Most of the statistics defined above have an implicit quadratic cost of error function and lead to a least squares ranking criterion. This has attractive properties. However, if least squares ranking is adopted care must be taken that it is so defined as to be a monotonically increasing function of the sum of squared errors. If this is not so, the criterion can choose sub-optimal predictors as producing the best forecasts.

From this point of view, inequality coefficients defined as U_{C1} and U_{L1} and Somermeijer coefficient are functions which do not increase monotonically with the sum of squared errors. As such they may choose sub-optimal predictors. Granger and Newbold (1973) illustrate this point by taking the case of a process where realizations are generated by a first order auto-regressive process:

$$A_t = \alpha A_{t-1} + \varepsilon_t \qquad 0<\alpha<1 \qquad (4.9)$$

where ε_t is a zero mean white noise process. If forecasts are generated by the model

$$P_t = \beta X_{t-1} \qquad 0<\beta<1 \qquad (4.10)$$

then var (P) = β^2 var (A), and for $n \to \infty$, we have for parameters referring to the population values,

$$\frac{1}{N\to\infty} \Sigma(P_t - A_t)^2 = (1 - \alpha^2) + (\beta - \alpha)^2 \text{ var } A$$

From this it can be derived that

$$\frac{\Sigma(P_t - A_t)^2}{\left[\Sigma(P_t)^2\right]^{1/2} + \left[\Sigma A_t^2\right]^{1/2}} = 1 - \frac{2\beta(1 + \alpha)}{(1 + \beta)^2} \qquad \ldots \quad (4.11)$$

Thus, the squared inequality coefficient defined here is not a minimum for the optimal forecast when $\beta = \alpha$, but rather when $\beta = 1$. The same criticism applies to the S-coefficient.

6. Janus Quotient

All the statistics defined above use information for forecasts and realizations beyond the sample period. The model which predicts for the future can also be used to generate values of endogenous variables within the sample period. The coefficient defined above can also be used to find out how the model performs within the sample period.

One coefficient, however, has been defined which uses information on both within-sample and beyond-sample predictions. Let the sample period be $t = 1,2, \ldots n$ and the forecast period, $t = n + 1, n + 2, \ldots n + m$, then the Janus Quotient* is defined as

$$J = \left[\frac{\sum_{t=n+1}^{n+m} (P_t - A_t)^2 / m}{\sum_{t=1}^{n} (P_t - A_t)^2 / n}\right]^{1/2} \qquad \ldots \quad (4.12)$$

* See Gadd and Wold (1964).

J varies between zero and infinity. If there is perfect forecast beyond the sample, its value is zero. If the structure of the model has not changed in the forecast period as compared to the sample period, it is expected that the quotient will tend towards unity. Higher values indicate changes in the structure under certain conditions.

The coefficient as defined above is sensitive to magnitudes of variables. If they differ widely between sample and beyond-sample situations, the coefficient may be misleading. For example, if there is a strong trend in the variable, magnitudes in the forecast period may be considerably larger than in the sample period. The quotient will take a relatively high value because of this. It is useful therefore to use this either only for stationary series, or to redefine it in the following manner

$$J^* = \left[\frac{\sum_{t=n+1}^{n+m} (P_t - A_t)^2 \Big/ \sum_{t=n+1}^{n+m} A_t^2}{\sum_{t=1}^{n} (P_t - A_t)^2 \Big/ \sum_{t=1}^{n} A_t^2} \right]^{1/2} \qquad (4.13)$$

Alternatively, the coefficient may be defined in relative terms, i.e. for P'_t and A'_t where

$$P'_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad \text{and} \quad A'_t = \frac{A_t - A_{t-1}}{A_{t-1}}$$

Sometimes the square of J has also been used.

7. Tracking Measures

Sometimes, there is greater interest in turning points predictions rather than all predictions, especially if the model is used for predicting critical fluctuations. In this case forecasts may also be ranked according to the scores they get for (i) the number of incorrectly predicted turning points and (ii) the number of turning points missed.

4.2.4. Limitations of Comparisons between Predictions and Realizations

The intuitive basis of all the measures defined above is the belief that the more closely predictions follow realizations, the better they are. This must however be qualified by the consideration that for all stochastic processes forecasts will be made with errors even if all the information in the universe is used (Granger, 1972). In such a case, optimal predictors are not necessarily those where the variances of predictions are equal to the variance of realizations. The point has been illustrated by decomposing the expected squared forecast error in the following manner

$$S = E(P-A)^2 = (\mu_P-\mu_A)^2 + \sigma_P^2 + \sigma_A^2 - 2\rho\sigma_P\sigma_A \qquad (4.14)$$

where μ_P, μ and σ_P and σ_A are respectively the population means and variances of predictions and realizations and ρ is the correlation coefficient. Assuming S to be a function of μ_P, σ_P and ρ, the following necessary conditions for minimising S can be obtained

$$\frac{\partial S}{\partial \mu_P} = -2\ (\mu_P - \mu_A)$$

$$\frac{\partial S}{\partial \sigma_P} = 2\ (\sigma_P - \rho\sigma_A) \tag{4.15}$$

$$\frac{\partial S}{\partial} = -2\ \sigma_P\sigma_A$$

Thus S is minimised by taking ρ as large as possible with $\mu_P = \mu_A$ and $\sigma_P = \rho\sigma_A$. Whereas the mean of the two series should coincide, the variances need not be equal.

4.2.5. Inter-temporal Weighting of the Prediction Errors

Typically, forecasts are made for several periods ahead. In all the evaluation criteria considered above, the weights assigned to errors for different periods are the same. It can be argued that depending on the purpose of forecasts, i.e. short-, medium- or long-term, these weights should be changed. Thus, for a short-period forecast, errors further ahead from the point of forecast should be given less weight while for long-period forecasts errors nearer to the point of forecast should be given less weight. This means the adoption of a weighted quadratic loss function where forecasts with different time perspectives are being compared. Again, in practice, it may be difficult to obtain the relevant weights and some arbitrary procedure may need to be adopted.

4.2.6. Evaluation Criteria and Statistical Inference

The criteria discussed above are merely descriptive. In particular no statistical inference is made as whether the statistic in question is significantly different from zero (or other benchmark values). Although not widely used in practice some tests have been developed in this context.

For example, for a single-equation linear model with a single explanatory variable, $Y = b_o + b_1X + u$, if a one period ahead forecast, $\hat{Y}_F = b_o + b_1X_F + u_F$ is produced, it can be shown that the statistic

$$t^* = (Y_A - \hat{Y}_F) \Big/ \hat{\sigma}_u \left[1 + \frac{1}{n} + \frac{(X_F - \bar{X})^2}{\Sigma(X - \bar{X})^2}\right]^{1/2} \qquad (4.16)$$

has a t-distribution with n-2 degrees of freedom where symbols have obvious meanings. Thus, a comparison between t^* and the theoretical value of t for the relevant degrees of freedom can be used to construct a test so that, if $t^* < t$, we conclude there is no significant difference between Y_A and $\hat{Y}_F$ at an appropriate level of confidence. The test can also be used for finding if there is a significant change in the structural relationship provided there are no abnormal conditions in the forecasts period.

However, single-period forecasts from single-equation linear models are not of great attraction. What one needs is a finite sample test for simultaneous equation models for multi-period ahead forecasts. Dhrymes et al (1972) have developed a test for a situation where the reduced-form of a linear simultaneous equation system, estimated with ordinary least squares is used to predict new observations on m endogenous variables under the assumption that predetermined variables are non-stochastic, the reduced-form error terms are normally distributed, serially independent but contemporaneously dependent.* The assumption of non-stochastic predetermined variables may also be relaxed in certain ways. The test is based on constructing an expression for the mean squared error of the forecasts. Similarly, if one is using inequality coefficients (U_{C2} and U_{L2}), a test based on Gamma distribution developed by Nagar (1962) may be used for single variable forecasts.

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* The test involves the construction of a statistic

$$(e_o^G)' \left[\Sigma^{-1} \otimes (I_m + X_o S^{-1} X_o')^{-1} \right] (e_o^G) \frac{T-K-G+1}{mG\ (T-K)}$$

where Σ is an estimate of contemporaneous covariance matrix of reduced-form error terms, I_m is an mxm identity matrix, T is the sample size, K is the rank of the matrix of reduced form coefficients, $S = X'X$, where X is the observation matrix, X_o is the mxk observation matrix to be used for predictions, and e_o^G is the matrix of prediction errors. $\otimes$ represents the Kronecker product. It has been shown that the above regression is distributed as (central) F with mG and (T-K-G+1) degrees of freedom if the structure is unchanged.

4.3 Diagnostic Checks on Forecasts

Apart from ranking forecasts, a comparison of predictions and realizations may also be used for diagnostic checks on the forecasting procedures with a view to modify them.

4.3.1 Systematic Errors of Bias and Slope

Some insight into the nature of prediction errors is obtained by regressing realizations on predictions or vice versa. The two relationships can be written as

$$A_t = \alpha + \beta P_t + u_t \tag{4.17}$$

$$P_t = a + bA_t + v_t \tag{4.18}$$

Preference has been shown for the former regression (Mincer and Zarnowitz, 1969) for two reasons: first, P_t's are by definition the predictors, and secondly they become available before realizations. This type of line is drawn in diagram 4.4.

A value of a or α equal to zero means that the regression line passes through the origin, and a value of b or β equal to one means that it coincides with the line of perfect forecasts. In the case of unit correlation between P_t and A_t ($u_t = 0$, $v_t = 0$) we will have $b, \beta = 1$. Thus, the non-zero values of a and α and non-unity values of b and β have been interpreted as 'systematic' errors in the forecast.

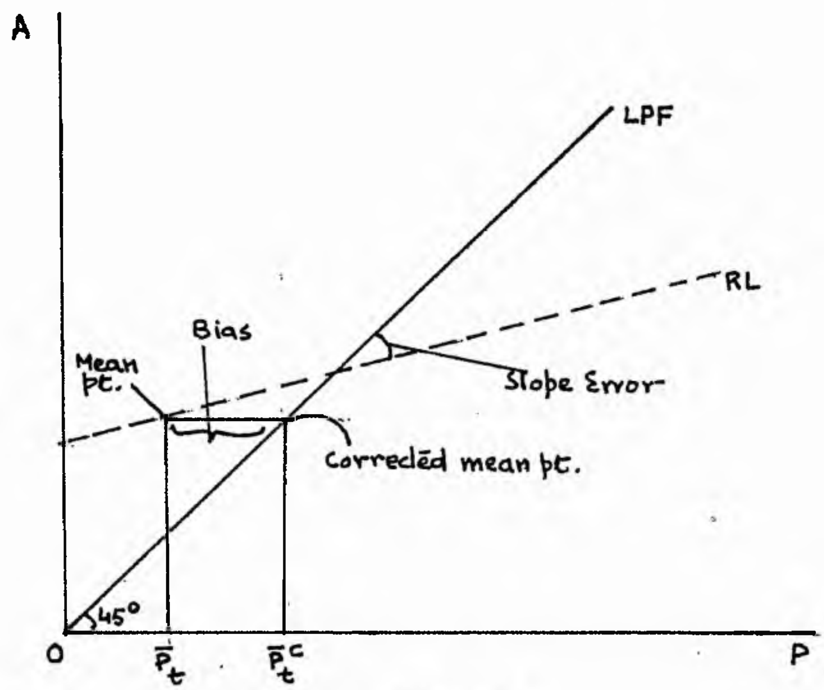

Fig. 4.4

Errors of Bias and Slope

We observe that the mean point $(\overline{P}_t, \overline{A}_t)$ does not lie on the LPF. This is a source of systematic bias and can be removed by shifting the regression line until the mean point lies on the LPF. As it is desirable for the mean point to be on the LPF, so also it is intuitively desirable that the whole regression line coincides with the LPF. If this is so, the forecast is called efficient (Mincer and Zarnotwitz, 1969). When it is not so, such efficiency can be obtained by rotating the shifted regression line such that it coincides with the LPF.

In practice the two changes are obtained by setting the least-squares value of

$$\alpha = \bar{A}_t - \beta\bar{P}_t$$

equal to zero, and that of

$$\beta = \frac{\Sigma(P_t-\bar{P}_t)(A_t-\bar{A}_t)}{\Sigma(P_t-\bar{P}_t)^2} \qquad (t=1,\ldots,n).$$

equal to 1. Once each forecast is multiplied by β and added to a constant α, the new set of forecast has undergone what has been called an 'optimal linear correction' (Theil, 1961) of forecasts.

We can interpret the two changes with the help of mean square prediction error. A decomposition of M_p is given by

$$M_P = (\bar{P}_t-\bar{A}_t)^2 + \sigma^2(U_t) \tag{4.19}$$

When the forecast is corrected for bias, $\bar{P}_t=\bar{A}_t$. This will yield a mean square error for the corrected forecast

$$M_P' = \sigma^2(U_t)$$

when the forecasts are corrected for slope, i.e. $\beta = 1$ in

$$A_t = \alpha + \beta P_t + U_t$$

This will imply that the forecast error U_t is uncorrelated with the forecast values P_t. In this case the variance of the residuals $\sigma^2(V_t)$ is equal to the variance of the forecast error $\sigma^2(U_t)$. Otherwise, $\sigma^2(U_t)>\sigma^2(V_t)$. Based on this, Mincer and Zarnotwitz (1969) define 'efficiency' of forecasts from the condition $\sigma^2(U_t) = \sigma^2(V_t)$

Thus, an unbiased and efficient forecast will have $\alpha=0$, and

$$M_p = \sigma^2(V_t) = \sigma^2(U_t) \tag{4.20}$$

In general, $M_p > \sigma^2(U_t) > \sigma^2(V_t)$ (4.21)

In the case of corrected forecasts, the new mean square error is

$$M'_p = \sigma^2(V_t) < \sigma^2(U_t) < M_p \tag{4.22}$$

Granger and Newbold (1973) find the definition of efficiency in the Mincer and Zarnowitz sense ($\beta=1$) unacceptable. In particular, they point out the case of a process where the actual realizations are generated by a random walk model,

$$A_t = A_{t-1} + \varepsilon_t \tag{4.23}$$

where ε_t is zero mean, finite variance, white noise. Then the Mincer and Zarnowitz criterion of efficiency would find that all the one step predictors of A_t, defined below

$$P_t^{(j)} = A_{t-j} \;;\; j=1, 2, 3, \ldots. \tag{4.24}$$

are efficient. In theory a regression of A_t or $P_t^{(j)}$ for any j, will have a zero intercept and unit slope. The efficiency criterion is thus not able to distinguish between different predictors for different values of j in the above example.

Granger and Newbold (1973) have suggested that as a criterion for 'efficiency' the following definition may be used

$$E = \frac{\text{MSE for optimum predictor using information set } A_n}{\text{MSE for any predictor } P_n}$$

In practice, however, the optimum predictor for any given information set A_n will not be known in general. The concept of 'relative efficiency' defined subsequently may be more useful in practice.

4.3.2 Relative Efficiency and Combination of Forecasts

The concept of relative efficiency is defined by Mincer and Zarnowitz as

$$RE = \frac{\text{MSE of } P_1}{\text{MSE of } P_2}$$

where P_1 is the predictor in question and P_2 is an alternative 'benchmark' forecast. The criterion has been suggested as a 'rate of return' index inasmuch as it takes the return to be inversely proportional to the mean square error of P_1 and the cost to be inversely proportional to the mean square error of the 'benchmark' prediction. The size of RE indicates whether the prediction is superior to benchmark or not. If RE>1, the forecast is prima facie inferior. If RE<1, the forecast is superior to the benchmark. Granger and Newbold (1973) argue the size of RE cannot indicate what the improvement in forecast performance would be if greater effort or a larger information set was used in constructing the forecasts.

In particular, they suggest that useful insights in forecast evaluation can be obtained by combining forecasts. For two forecasts, P_1 and P_2, if they are both unbiased, and if the forecast errors, e_1 and e_2 are bivariate stationary, combinations of the following form are appropriate.

$$P_c = KP_1 + (1-K)P_2 \tag{4.25}$$

It can be established that the sample variance of the error of composite forecasts (S_c^2) is minimised by taking

$$K = \frac{S_2^2 - rS_1S_2}{S_1^2 + S_2^2 - 2rS_1S_2}$$

where S_1^2 and S_2^2 are sample variances of respective forecast errors of predictors P_1 and P_2. If K is so chosen, then

$$S_c^2 \leqslant \min (S_1^2 , S_2^2)$$

If the condition 0<K<1 is met 'K' itself can be taken as a measure of relative efficiency. If this condition is not met, or otherwise, the following criterion of 'conditional efficiency' of P_1 with respect to P_2 may be considered

$$CE (P_1/P_2) = S_c^2/S_1^2 \quad \text{if} \quad (S_1^2 \leqslant S_2^2)$$

If P_1 is the optimum forecast for a given information set, then CE = 1. The conditional efficiency is taken to be a measure of extra information contained in the pair of predictors rather than if just one is used.

The usefulness of this concept of conditional efficiency is limited by the assumption of stationarity. To some extent non-stationarity can be removed by differencing. Altneratively, K may be allowed to vary over time.

The question as to how 'benchmark' forecasts should be constructed is also important. The convention has been to use naive models like 'no-change' or 'constant-change' extrapolation. More recently, sophisticated autoregressive and/or moving average models have also been used. In practice, if one decides to use extrapolative models, it does seem that parameters of the model should be so chosen as to best utilise the available information contained in the past history of the predicted series.

If forecasts based on different information sets are compared, the problem arises as to how the results should be interpreted. Usually, a multi-equation model forecast will use the past history of many variables and an extrapolative forecast, the past history of a single variable. Granger and Newbold suggest that appropriate models for comparison are constructed by using the same set of information as in the original model. This calls for building multi-variate time-series models even if the specifications do not conform to economic theory. If benchmarks are constructed in this manner, the criterion of conditional or relative efficiency then boils down to the condition that a forecast is adequate if it cannot be significantly improved by combining with the benchmark forecasts.

4.3.3. Decompositions of Mean Square Error

Theil (1958) has suggested that the mean square error

$$M_p = \frac{1}{N} \Sigma (P_t - A_t)^2$$

can be decomposed in two ways, viz.,

$$M_p = (\overline{P} - \overline{A})^2 + (S_P - S_A)^2 + 2(1-r)S_P S_A \quad (4.26)$$

and

$$M_p = (\overline{P} - \overline{A})^2 + (S_P - rS_A)^2 + (1-r^2)S_A{}^2 \quad (4.27)$$

where $\overline{P}$ and $\overline{A}$ are the sample means of predictions and realization, S_P and S_A are their standard deviations and r is the correlation coefficient between them. The division of the terms on the right-hand side by the mean square error gives rise to the following quantities which have been called 'inequality proportions'

U^M	$= (\overline{P} - \overline{A})/M_p$	mean proportion
U^S	$= (S_P - S_A)^2/M_p$	variance proportion
U^C	$= 2(1-r)S_P S_A/M_p$	covariance proportion
U^R	$= (S_P - rS_A)^2/M_p$	slope proportion
U^D	$= (1-r^2)S_A/M_p$	disturbance proportion

We have,

$$U^M + U^S + U^C = 1$$

and

$$U^M + U^R + U^D = 1$$

The terms thus provide information on the relative importance of one source of error rather than another.

The mean proportion has a positive value if $\bar{P} \neq \bar{A}$. This is due therefore to 'bias'. The variance proportion is zero when the two standard deviations are equal. It is therefore interpreted as error due to incomplete variation and would arise because of a forecaster's neglect of causes of fluctuations in the two series. The covariance term is zero when r=1 or when $S_A\ S_P.r = S_A S_P$, i.e. when the covariance of predictions and realizations takes its maximum value which is the product of the two standard deviations. Any positive value for this term is, therefore, due to incomplete variation. Errors of this type stand less chance of correction.

Similarly, the terms in the second decomposition are interpreted respectively as those due to bias, slope, and a residual or a disturbance component. Granger and Newbold observe that this decomposition is not so useful as the first one.

As an example, they consider the case where A is generated by a first order autoregressive process,

$$A_t = \alpha A_{t-1} + \varepsilon_t \qquad 0<\alpha<1 \tag{4.28}$$

where ε_t is a zero-mean white noise process. The optimal one-step ahead predictor based on the information set (A_{t-j}, $j \geqslant 1$) is given by

$$P_t = \alpha A_{t-1} \tag{4.29}$$

For this prediction, as $N \rightarrow \infty$,

$$U^M = 0, \quad U^S = \frac{1-\alpha}{1+\alpha}, \quad U^C = \frac{2\alpha}{1+\alpha}$$

If one varies α from 0 to 1, U^S and U^C can take any values apart from the restrictions $0 \leq U^S, U^C \leq 1, U^S + U^C = 1$. This makes the interpretation of these proportions impossible. The difficulty arises because for an optimal predictor one does not expect S_P and S_A to be equal.

In the first decomposition, however, for the same example of first order autoregressive process, U^M and U^R tend to zero for the optimum predictor and so U^D tends to unity.

More useful information can be obtained by an examination of forecast errors. Errors should be tested for randomness and their autocorrelation properties should be explored. First order autocorrelation can be tested by the von Neumann ratio and higher order and other forms of serial correlation can be tested by direct regressions of current errors on lagged errors.

4.4 Evaluation of Multi-equation Econometric Models

The procedures outlined above relate to single variable forecasts produced in any manner. We now consider the requirements for evaluating the forecasting performance of a multi-equation model. This can be done by a comparison with realizations against predicted values, a comparison with other 'benchmark' methods like autoregressive or single equation models, and a comparison with other multi-equation models producing forecasts for similar variables.

4.4.1 Variable-by-variable comparisons

The established practice, although not entirely satisfactory, is to evaluate the performance of a model by applying techniques outlined in previous sections variable-by-variable for all or a subset of the endogenous variables of the model. Thus, indicators like the mean square error and the inequality coefficients are constructed for each variable. Such a procedure can indicate variables for which the model does better and those for which the model is not satisfactory. However, except in cases where one model is overwhelmingly superior to another, this procedure cannot be used to rank the prediction performance of different models. If one model gets a few less or more scores, it cannot be established categorically that it is inferior or superior to another model.

4.4.2 Construction of Composite Performance Indices

One way of ranking models according to prediction performance is to construct composite indices which can refer to all variables together. There are, however, a number of difficulties involved. If one takes an average of an indicator like the mean square error, one has to contend with at least three problems: (i) the units of variables may be different, some being measured in percentages, some in nominal currency units etc., (ii) the weight which should be given to one variable rather than another, and (iii) how many variables should be included in the composite index. This last problem is important, as by a judicous decomposition of variables in the identities any number of variables can be constructed and the prediction performance can be made to look better.

The problem of units of measurement can be resolved, to some extent, by either defining the mean square error in terms of percentages, or choosing an index like Theil's second inequality index where the problem of units has been taken care of. Thus if we have predictions for n variables for m periods P_{it}, $(i=1, \ldots, n,\ t=1, \ldots, m)$, and corresponding realizations A_{it}, then the mean square error may be defined with respect to percentage changes P'_{it} and A'_{it} where

$$P'_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} \quad \text{and} \quad A'_{i,t} = \frac{A_{i,t} - A_{i,t-1}}{A_{i,t-1}}$$

A composite standardised mean square error can then be written as

$$\text{CSMSQ} = \frac{1}{mn} \sum_{i=1}^{n} \sum_{t=1}^{m} (P'_{i,t} - A'_{i,t})^2$$

The problem of weights is however more intractable. What relative weights should be given to prediction error of one variable *vis a vis* another calls for a cost function in errors of all variables in question. In practice it will be difficult to obtain such a function. If it is decided to give arbitrary weights to the errors, some guidelines can be obtained from the purpose of the forecasts, viz., whether the model is designed to forecast primarily the national income components or prices etc. Similarly, errors can be inter-temporally weighted depending on whether the forecasts are in a short-, medium- or long term perspective.

4.4.3 Information Measures of Predictive Accuracy

Information measures of predictive accuracy (suggested by Theil (1961, 1966) are applicable in the case where positive fractions adding to unity are predicted and where it is assumed that the cost function is of the log-linear type. The measure is useful in input-output models and models which forecast national income components. If both national income and its components are forecasted, one can define variables such that all components are positive and calculate predicted income-share in each case. Assuming that P_i, $(i=1,...n)$ are n forecasted shares and A_i are corresponding realized shares, for any given period of forecast, then the information measure

$$I(A : P) = \sum_{i=1}^{n} A_i \log \frac{A_i}{P_i}$$

can indicate the power of the model to forecast components of income or output jointly. The lower the value of $I(A : P)$, the better is the forecasting power.

4.4.4 Comparisons with Other Models

Multi-equation model forecasts are usually compared with time-series models and forecasts generated from other multi-equation models. Requirements of using a similar information set have been considered in section 4.3.2. It is generally observed that adequately built time-series models, either univariate or multi-variate, perform well in terms of predictions at least for the short-run. In this context, a multi-equation model may be considered quite satisfactory if it performs at least as well as the competing time-series model. However, even if it performs worse, the extra information it may contain for purposes of policy analysis in its multi-variate specification, may render it useful.

Comparisons among different multi-equation models are also beset with problems. The problems of weighting and the number of variables to be used for comparisons have already been noted. Proper comparisons also cannot be made when the size, estimation techniques and sample-periods of different models differ.

One way round this problem is to re-estimate competing models under similar conditions of information and estimation as far as possible. Thus, models may be re-estimated for a common sample period and with a common estimation technique. An important study of this kind for some U.S. models is by Cooper (1972).

However, apart from being a costly precedure, there are other problems in this exercise. Some estimation techniques are more appropriate for one set of data and specification, and some for others, especially if non-linearities are involved. Re-estimation with a common technique may, therefore, discriminate against some models. Also, a mechanical application of the same set of rules violates against the essence of model-building where the analyst reacts by testing and retesting his hypothesis with a given set of data. If data were to change, it is expected that his reactions would also have changed. Howrey et al (1974) have called for a 'tender loving care' (TLC) in the re-estimation of models. Frequently, very small changes in the original model when re-estimated with new data can substantially improve its performance. In particular, since economic theory does not tell much about precise lag-structures and functional forms which are a matter of sample experimentation, the re-estimated model may deviate from the original model in these matters in the light of revised information. Howrey et al (1974) suggest that in view of the (i) different requirements of degrees of freedom and (ii) different characteristics of collinearity among variables in different models, the requirements of a homogenous sample period and estimation techniques may be compromised.

4.4.5 Pre-release Period and Pseudo-forecasts

Before models are released, a standard practice is to 'save' some observations in the available sample, estimate the model with the remaining observations, and use the 'saved' observations for a test on forecasting performance. Such forecasts have been termed 'pseudo-forecasts'. This may be a useful and unavoidable procedure in the model-building stage but a proper evaluation of the performance can only be done after the model has been released. Available data in the pre-release period are expected to affect the model-builder and he would normally make sure that for these the model provides a good fit. Christ (1975) has argued that if one is

> 'to discriminate between the (inferior) models that have chosen to fit the random or non-enduring features of the economy, and the (superior) models that have been chosen to fit primarily the systematic and enduring features of the economy'

one should use a test period beyond the model-building period.

4.4.6 Diagnostic Checks: Role of Exogenous Variables

In the post-release period, an interesting diagnostic check is to allocate the prediction error between structural mispecification and incorrectly predicted exogenous variables. One such decomposition has been suggested by Theil (1961).

Theoretically, one can think of the role of the use of incorrect exogenous values in the following way. Suppose the predictions are made by a linear system of equation such that a vector Y^P of predictions is generated by using a vector Z^P of already predicted exogenous variables. If Π_e is the estimated coefficient matrix of the equation system, the reduced-form of the system may be written an

$$Y^P = \Pi_e Z^P + U^P \qquad \ldots \quad (4.30)$$

where U^P are the reduced-form predicted errors. If Z is the corresponding vector of 'true' values of the exogenous variables, and Π the 'true' coefficient matrix, assuming that the true system is also linear, we can write the true vector Y of predicted variables as

$$Y = \Pi Z + U \qquad \ldots \quad (4.31)$$

where U is the vector of 'trne' reduced-form errors. The prediction error is then given by

$$Y^P - Y = \Pi_e Z^P - \Pi Z + U^P - U \qquad \ldots \quad (4.32)$$

or

$$Y^P - Y = (\Pi_e - \Pi)Z + (Z^P - Z)\Pi + (\Pi_e - \Pi)(Z^P - Z) + (U^P - U) \qquad \ldots \quad (4.33)$$

The first term in (4.33) arises because of the discrepancy between Π_e and Π and is, therefore, due to an incorrectly specified model. The second term arises because of the deviation of Z^P from Z and is, therefore, due to incorrectly predicted exogenous variables. The third term is a mixture of the two but is of second order of importance, i.e., if the first two terms are small, the third term will be negligible.

It is difficult to imagine that these three components of the forecast errors could be computed because although Z will be known ex-post, we will rarely know the 'true' equation system Π. What we will know is the vector of the true values of the predicted variables, i.e., Y. Hence it may be better to rewrite (4.32) in the following form:

$$Y^P - Y = (\Pi_e Z - Y) + \Pi_e(Z^P - Z) + (U^P - U) \qquad \ldots. \quad (3.34)$$

In this, the second and third terms of (4.33) are combined. As such the term $\Pi_e (Z^P - Z)$ will indicate the prediction error due to exogenous variables given the estimated model and not given the true model as in $\Pi (Z^P - Z)$. But under normal circumstances it will be possible to calculate only $\Pi_e(Z^P - Z)$.

4.4.7 Diagnostic Checks: Analysis of Prediction Errors

Much insight into the nature of errors and adequacy of specification can be obtained by a direct study of inter-temporal and inter-equation properties of the predictions errors rather than a simple comparison of predictions and realizations. Prediction errors for each of the endogenous variables may be tested for normality. They may also be tested for first and higher order serial correlation and/inter-equation correlation. One method is to directly regress errors on past values, contemporaneous errors in other equations and past errors in other equations and test whether the regression coefficients are significantly different from zero by the usual t- and F- tests. If serial correlation is observed, it might suggest that some new variable needs to be introduced in the specification of equations; if inter-contemporaneous inter-equation correlation among the errors is observed, it might suggest that some simultaneous equation estimation techniques like 3SLS or FIML is required.

Another interesting study in this context is to see whether errors in the forecast period are systematically related to errors in the sample period. Since information on sample period error is already available, any systematic relationship with prediction errors implies that such information was not fully utilized. If 't' is the point of forecast, the regression

$$e_{t+n} = \beta e_{t-n} + u_{t+n} \qquad (n=1, 2, \ldots k)$$

for k-period forecasts may be used to test the hypothesis that $\beta=0$. McNees (1978) has used this as a partial test of 'rationality' of forecasts suggesting that it is irrational to leave information in the sample period, if by using it forecasts could have been improved.

4.5 Summary

The discussion in this Chapter is divided in two main parts. First, we consider techniques relevant for the evaluation of forecasts of individual variables. These range from prediction-realization diagrams to the construction of various indices summarizing the forecasting performance. We also consider measures for locating the nature and strength of different sources and types of errors. The need for having proper cost or gain functions and for the consideration of the costs of producing a forecast have also been discussed. Finally, we discuss relevant considerations for comparing and evaluating forecasting models as a whole which may be needed to supplement the information provided by a variable-by-variable comparison of the forecasting performance.

In the next Chapter we propose to discuss the role of macro-econometric models as an aid to policy analysis.

Chapter 5

Prediction as a Guide to Policy Evaluation

Uncertainty and, therefore, prediction are inextricably mixed with policy-making. In addition to forecasting, therefore, a macroeconometric model is a useful framework for policy analysis provided its structural specification adequately provides for the interrelations between instruments of policy and its targets. Since such a model is a consistent framework of analysis, it is clearly superior to single relation, *ceteris paribus*, partial equilibrium type of policy evaluative exercises.

5.1 Preliminary Considerations

For purposes of using a macroeconometric model for policy analysis, one needs to subdivide the list of endogenous variables into 'target' variables and 'irrelevant' variables, and the list of exogenous variables into 'controlled' variables or 'instruments' and 'non-controlled' variables, sometimes called 'data'. Target variables are those in which the policy maker is interested. The 'irrelevant' variables are determined within the model but there is no interest in the values they take. Controlled variables are those for which arbitrary values subject to predefined limitations can be assigned. The values of non-controlled variables have to be obtained independently.
Difficult problems of estimation and interpretation may arise when instrument variables are not strictly exogenous. Sometimes, these are called 'semi-endogenous', i.e., there is some degree to which the government can assign any chosen value, but they also respond systematically to changes in the other endogenous variables of the model.

In addition to an econometric model which basically describes the working of an economy and thus is in the nature of an operating constraint, a 'preference' function is also needed for ranking alternative policy configuration. The preference function can be incorporated into the analysis in a number of ways and on this basis, and in a historical context, three approaches to policy analysis can be distinguished. These may be referred to as a constrained maximisation approach due to Theil (1961, 1966), an aspiration levels approach due to Tinbergen, and the policy simulation approach. More generally, these should be viewed as variants of the 'synoptic' approach to policy formulation compared to the 'piecemeal' or 'incremental' approach.*

5.2 Constrained Maximisation Approach

In this approach, the basic tenets of the classical problem of utility maximisation under a budget constraint are used. For this purpose it is convenient to write the reduced-form of the system after omitting equations relating to irrelevant endogenous variables and by lumping together the constant terms, non-controlled variables and stochastic terms. Suppose these are n relevant endogenous variables (y's) and m controlled exogenous variables (Z's). Then a linear reduced-form system can be written as

$$\begin{aligned} y_1 &= a_{11} Z_1 + \; \ldots . + a_{1m} Z_m + u_1 \\ \\ y_n &= a_{n1} Z_1 + \; \ldots . + a_{nm} Z_m + u_n \end{aligned} \tag{5.1}$$

where u's are the lumped terms described above.

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* See Popper (1963).

In a matrix form, this system can be written as,

$$Y = AZ + U \tag{5.2}$$

where Y, Z and U are appropriately sized vectors respectively of targets, instruments and a sum of lagged values, non-controlled exogenous values and disturbances. A is a (nxm) matrix of reduced-form coefficients.

Assuming that a preference function of the type

$$W = W(Y,Z) \tag{5.3}$$

which can describe the ordering of alternative outcomes according to increasing preference is available. The policy maker's problem is then one of constrained maximisation. It can be written as.

$$\begin{array}{ll} \text{Maximise} & W = W(Y,Z) \\ \text{Subject to} & Y = AZ + U \\ \text{or more generally,} & Y = F(Z) + U \end{array} \tag{5.4}$$

where F is an arbitrary vector of functions $f_1, f_2, \ldots f_n$.

There are many difficulties with this approach. In real world situations it is difficult to know both the parameters and the functional form of the preference function. Policy makers do not provide explicit quantitative information about their utility functions. They have to be estimated implicitly from observed data in the hope that the preferences have been 'revealed' through the actions of the policy makers in historical time. There are difficult problems of estimation here. There is also the problem of changing preferences where policy makers continuously keep adjusting the policy and target-weights to suit the needs, whims and moods of their electorate.

5.3 Aspiration Levels Approach

Here, rather than using a preference function, specified numerical values for target variables are used. The conditions for the solution of the system require that the number of instruments be as many as there are targets. Let the structural form of the system be written as,

$$\phi_i \, (y,z,x,u) = 0 \qquad . \; i=1,2,\ldots p \qquad (5.5)$$

where y's are n target variables, z's are m instrument variables and x's are k non-controlled endogenous variables and u's are disturbances. When (5.5) is taken as a prediction problem, values for y and x are predicted for given values of z and u. The total number of unknown is n+k and hence the number of equations p should be equal to this sum.

The problem is converted into a policy problem by assigning target values to y's. Now z's become the unknown. The total number of unknowns now is m+k. Since available equations are p(= n+k), for the system to remain consistent and uniquely soluble, we must have $m+k = n+k$ or $m=n$.

This implies that the conversion of prediction problem into a policy problem depends on the equality of the number of targets and instruments. In such a case the problem would be soluble except in special cases. The solution will be unique if the equations are linear and independent. There will be infinite solutions if the equations become dependent and no solutions if they become incompatible. Such dependence or incompatibility may arise even when the original system is not dependent or incompatible. This depends on the numerical values assigned to the targets and the coefficients with which the instruments, which are the new unknowns, appear.

In practice, there is no reason to assume that the number of instruments and targets will be equal. If instruments are more than targets (m>n), an infinite number of solutions would be possible. If arbitrary values to (m-n) instruments are assigned, the system can be made consistent. If, n>m, (n-m) targets will have to be dropped.

Compared to the Theil approach, this method does not call for a preference function or rather/it uses a special type of preference function where targets are constrained to take fixed values depending on levels of aspirations of the planners. In this sense, a lesser degree of information is asked for in this approach. It is doubtful whether even this information would always be available. There is also the difficulty of finding target priorities when instruments are insufficient and targets need to be dropped. This cannot be done unless trade-offs between targets are known which necessitates finding a preference function.

5.4 Uncertainty and the Theil-Tinbergen Approaches

There are two types of uncertainties involved in a policy approach of the type Max. W (Z,Y) subject to Y = A Z + U, where the symbols have meanings as set out in section 5.2. The first is about the specification of A. The policy-maker is uncertain about the response of Y to any given policy action Z. Secondly, he is uncertain about the impact of the exogenous and stochastic terms contained in U.

The first choice for the policy-maker is to work as if he were in a world of certainty. He can then take his sample estimate of $A = \bar{A}$ as representing the true population value of A and make conditional predictions like when $Z = Z_1$, $Y = Y_1$; when $Z = Z_2$, $Y = Y_2$, etc. This also assumes that he believes that he has correctly predicted the exogenous variables and replaced the disturbance term by its mean value. Given the predictions of the above type he will choose a Z such that that W (Z,Y) is maximised.

However, if he does not assume to be in a world of certainty, it would no longer be possible to uniquely predict the value of Y for given values of Z. The impact on Y will depend on the vector of the policy-response coefficient A and the disturbances that are determined by a chance mechanism. Assuming discrete probability distributions, the kind of information which the analyst can still produce would be of the type: if $Z = Z_1$, $Y = Y_{11}$ with probability p_{11}; if $Z = Z_2$, $Y = Y_{21}$, with probability p_{21} and so on. It would now be desirable to maximise the expected or mean value of the welfare function, viz., E(W (Z,Y)).*

If however, the analyst is prepared to neglect the uncertainty about Y by replacing the disturbance vector U by its mean value, he would choose to maximise W (Z,E(Y)).

In general, unless the utility function is linear, the maximisations of E(W (Z,Y)) and W (Z,E(Y)) will lead to a different choice of the policy vector Z.

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* Provided W (Z,Y) satisfies the Von Neumann-Morgenstern assumptions (Theil, 1961).

However, Theil has been able to show that under the two conditions stated below, one of which relates to the constraint, and the other to the utility function, the two maximisations will lead to the same choice of instruments. These conditions are as follows:

(1) The vector of non-controlled variables Y contains only real-valued variables; they are connected with Z by an equation system Y = F (Z) + U (linear case, Y = A Z + U), where F is a column vector of functions f_1, f_2,....; the mean value of disturbance vector U is zero, i.e. E (U) = 0 for any Z; and, its covariance matrix E (U U') is finite and independent of Z. Both F and U are independent of Z.

(2) All vectors Z,Y are completely ordered in such a way as to allow representation by means of a real-valued welfare function such as the following:

$$W (Z,Y) = \alpha(Z) + \sum_{i=1}^{n} \alpha_i (Z)\alpha y_i + \tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} y_i y_j \qquad (5.6)$$

where (Z) and α_i (Z) are arbitrary functions of the vector Z and are independent of Y. If some of the arguments are stochastic in the welfare function, the policy-maker values according to the mean-value of (5.6).

It should be noticed that the welfare function given by (5.6) is quadratic in non-controlled variables and contains interaction terms. It is thus more general than a linear function but quadratic terms in the controlled variables are not permitted.

The satisfaction of these two conditions defines a world of 'certainty-equivalence' and leads to the same choice for Z whether we maximise E (E (Z,Y)) or W (Z,E(Y)) provided such a maximum exists.* The upshot of this conclusion is that under these conditions any information about the probability distribution of Y beyond the mean value is superfluous for the policy-decision.

In practice, however, the actual world does not leave the policy-maker in the envious positions either of certainty or of certainty-equivalence. Brainard (1967) has been able to show that in a world characterised by uncertainty, it is useful to employ the knowledge not only of the mean or expected value but of higher moments of the distribution of Y. This is easily illustrated in the case of one target-variable (y) and one policy-instrument (z). If the impact of all other variables is contained in the disturbance-terms (u), the relation between y and z can be written in a linear form:

$$y = a z + u \qquad (5.7)$$

Let the policy-maker's task be to minimise a quadratic loss function**

$$l = (y - y^*)^2 \qquad (5.8)$$

where y^* is the target value.

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* For proof see Theil (1961), chapter VIII.

** Alternatively, one can work with the maximisation of a utility function,

$$U = - (y - y^*)^2.$$

In a world of uncertainty, the policy-maker may believe that the response-coefficient, a in (5.7), is a random variable depending on some unobserved variables, and that it is correlated with u. Compared to the case of certainty-equivalence, the difference lies in the constraint - in that a and u are not independent - rather than in the utility function. In this case then, y is a random variable, and its variance is given by,

$$\sigma_y^2 = \sigma_a^2 z^2 + \sigma_u^2 + 2\rho\sigma_a \sigma_u z \text{*} \tag{5.9}$$

Here, σ_a^2 and σ_u^2 are the variances of a and u and ρ is the correlation coefficient between u and a.

Assuming then that y is a random variable, the expected disutility associated with a given policy will be given by

$$E(1) = (\bar{y} - y^*)^2 + \sigma_y^2$$

or

$$E(1) = (\bar{a} z + \bar{u} - y^*) + \sigma_a^2 z^2 + \sigma_u^2 + 2\rho\sigma_a\sigma_u z \tag{5.10}$$

Here, $\bar{a}$ and $\bar{u}$ are the expected values of a and u.

Differentiating (5.10) with respect to z, we have the condition

$$(\bar{a} z + \bar{u} - y^*)\, \bar{a} + \sigma_a^2 z + \rho\sigma_a \sigma_u = 0,$$

which yields the optimum value of z given by

$$z^* = \frac{\bar{a}(y^* - \bar{u}) - \rho\sigma_a \sigma_u}{\bar{a}^2 + \sigma^2} \tag{5.11}$$

---

* The policy-maker may also work with the assumption that a is a random variable but it is not correlated with u. In this case $\rho = 0$, and if z is measured as a deviation from its value in the previous period z, say $\bar{z}$, the variance of y will be given by $\sigma_y^2 = \sigma_a^2 (z - \bar{z})^2 + \sigma_u^2$.

Thus, in the determination of optimal policy, the analyst can fruitfully employ not only the knowledge of expected values $\bar{a}$ and $\bar{u}$, but also that of their variances σ_a^2 and σ_u^2, and correlation ρ.

This analysis has implications for Tinbergen's approach as well. Tinbergen's approach works in a world of certainty as compared to certainty-equivalence or uncertainty. The implication of Tinbergen's analysis was that whenever the instruments exceed targets, the excess instruments can be freely discarded. This conclusion is dependent on the assumption of certainty. In an uncertain world, it can be established (e.g. Brainard, 1967) that it is more useful to use all available instruments in the pursuit of a single goal.

5.5 Optimal Use of Forecasts of Non-controlled Exogenous Variables

At a time when, in a predictive exercise, decisions about instruments are taken, values of some or all non-controlled exogenous variables are not known but some predictions regarding these are available. Hersoug and Johansen (1975) have outlined a procedure by which those predictions can be used optimally in policy decisions. In this procedure, a linear model and a quadratic utility function have been assumed.

Let there be n target variable, m instrument variables ($m<n$) and q exogenous non-controlled variables given respectively by vectors X, T, and Z and related by the reduced-form model

$$X = A T + B Z \tag{5.12}$$

with A and Z being appropriately sized coefficient matrices.

The preference function

$$\phi = \sum_{i=1}^{n} w_i(x_i - \bar{x}_i)^2 = (X - \bar{X})' W (X - \bar{X}) \tag{5.13}$$

where $\bar{X} = (\bar{x}_1, \bar{x}_2, \ldots \bar{x}_n)'$ represents most desirable values of targets and w_i's represent weights attached to the squared deviation of actual from desired levels of targets. An optimal policy decision consists in choosing values of instruments T, so that ϕ is minimised.

Suppose there are existing or 'raw' forecasts of non-controlled variables given by $Z^* = (Z_1^*, Z_2^*, \ldots Z_q^*)$ and that the true values become known only later. It is assumed that the instrument values are related to the information contained in forecasts Z^* by a set of relations

$$T = K Z^* + K_0 \tag{5.14}$$

where K is an mxq matrix and K_0 is an m- vector respectively of coefficients and constants. The problem is to determine values of K and K_0 such that the policy leads to the minimal value of E (ϕ).

It is established in Johansen (1972) that this procedure, under the assumption of linearity of equations in (5.14), leads to a certainty equivalence policy with the implication that optimal values of instruments T are to be calculated just as under certainty, only with the values of non-controlled variables Z being replaced by their conditional expectations $\hat{Z} = (\hat{z}_1, \hat{z}_2, \ldots \hat{z}_q)$ where $\hat{z}_i$ are estimates of z_i from the regression of z_i^* on all forecasts $z_1^*, z_2^*, \ldots z_q^*$. The operational procedure thus involves the calculation of regressions of each z_i^* and all z_i^* and use $\hat{z}_i$ generated from these regressions in the policy decision model.

5.6 The Policy Simulation Approach

The policy simulation approach does not require an integrated treatment of a social welfare function and a macroeconometric model. Rather, the problem is tackled in two distinct stages. First, for alternative combinations of instrument values, the effects on endogenous variables are obtained from the model. These effects are then used with a welfare function for generating rankings of policy configurations. As a result, this approach does not produce an 'optimum' policy configuration but merely a set policy rankings.

For a system written in the reduced-form,

$$Y_t = f(X_t, Y_{t-j}, Z_t ; U_t) \tag{5.15}$$

where Y is a vector of relevant endogenous variables, X and Z, vectors of non-controlled and controlled exogenous variable, Y_{t-j} a vector of lagged endogenous variables, and U_t, a vector of reduced-form disturbances, it is possible to generate the time path of Y_t for independently predicted values of X, model-generated values of Y_{t-j} and a specified set of values for the policy instruments. The stochastic term may either be suppressed or taken from a probability distribution. The process can be repeated for alternative sets of policy instruments and the results could be compared to deduce the effectiveness of alternative combinations of instrument values.

Since a preference function is needed only at this stage, it is easily possible, if desired, to use several (hypothetical) functions alternatively and to compare how rankings change when the functions are changed.

Apart from pure policy evaluation, the simulation framework offers other interesting possibilities for prediction *per se*. For example, stochastic disturbances in the experiment can be given assumed serial correlation and covariance properties. Changes in the initial conditions may be introduced. Selected parameters may be changed. Shocks can be imposed on a set of endogenous or exogenous variables. These are system simulations in contrast to pure policy simulation.

The task of designing a policy-simulation experiment has several aspects. First, it is necessary to choose appropriate policy-configurations. This is a problem of <u>factor design.</u> When each factor (policy variable) is assigned one specific value, a <u>design point</u> is generated. A <u>full factorial design</u> will consist of a very large set of such design points. The number of design points in a full factorial design will be given by the product of the number of levels each policy-variable can take. In practice, therefore, it might be desirable to concentrate on a limited number of design points.

The second problem is about the inclusion of <u>stochastic disturbance</u> in the simulation experiment. Such randomization may be indispensible when (i) long-term properties of the model are being investigated, (ii) there are non-linearities in the endogenous variables, and when (iii) it is desired to make statistical inference and test hypotheses from data generated by the replication of the experiment. The long-term properties of the model are not adequately depicted by the characteristics of the non-stochastic model. Also, when non-linearities are contained in the endogenous variables, non-stochastic simulation procedures yield results that are not consistent with the properties of the reduced-form of the model (Howrey and Klein, 1972).

The question of the number of replications or the sample size in a simulation experiment is also important. Frequently, this is chosen arbitrarily. However, there must be an optimal sample size dependent on, as Naylor (1970) points out, how large a shift in population parameters one wishes to detect; how much variability is present in the population; what size risks one is willing to take; and how much would the computing costs be. Similarly, the choice of the length of the simulation run is important.

A number of analytical and statistical techniques are employed to decipher the effectiveness of different policies from the profuse output data produced by simulation experiments. Among the analytical techniques, multiplier and utility analyses are employed. Among the statistical techniques for analysing data, the F- test, multiple comparison methods and multiple ranking criteria have been used.

5.6.1 Multiplier Analysis

There are two main types of multipliers used for comparison of policies, viz., impact or dynamic multipliers, and static or equilibrium multipliers in a time dimensionless sense.

The impact multipliers show the ultimate one-period response in a current endogenous variable following an initial unit change in a policy variable, or some other exogenous variable. In a linear model, the impact multipliers are simply read in the coefficient-matrices of the reduced-form of the system.

The values of impact multipliers depend, among other things, upon whether the model in question is highly recursive or highly simultaneous. Characteristically, lower impact multipliers will be produced in a highly recursive model. In such a model, the high degree of recursiveness tends to dampen the reaction of the system to shocks during the initial period.

Also, these multipliers will not be independent of the policy-parameter changes. Hence, whenever policy-configurations are changes such that the coefficient parameters are altered, these will have to be recalculated.

Further, when the system is non-linear the size of the multipliers will depend on the size of the multiplicands. If a linear approximation is calculated for a non-linear model, these approximations will have to be changed every time there is an alternative value for an exogenous variable.

Impact multipliers, however, provide information about one-period response only. When they are calculated period-by-period, they provide a series of <u>dynamic multipliers</u>.

When a stream of dynamic multipliers is available, it may be desired to integrate them over the entire period. One method for such integration is discounting. The 'present value' of a <u>discounted multiplier</u> stream will be given by

$$\frac{1}{n} \sum_{t=1}^{n} \frac{K\,(i,k)\,t}{(1+r)^t}$$

where K (i,k) t refers to the multiplier related to the ith endogenous variable and the kth policy-parameter in period t. 'r' is the social rate of time preference and n the total number of periods from the start of the simulation over which the integration is done. These may be called discounted multipliers.

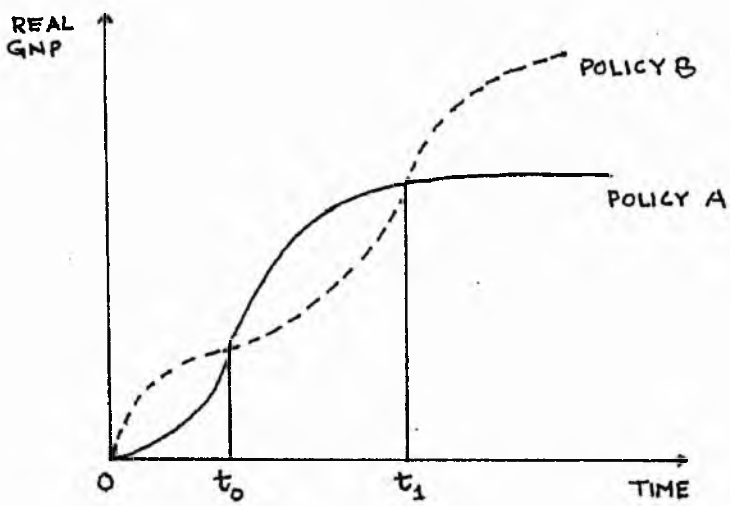
In contrast to impact or dynamic multipliers, there are also 'static' or 'equilibrium' multipliers. These measure the increase in an endogenous variable following a unit change in a policy or other exogenous variable after all transitional effects have subsided. Apart from the fact that at times there may be policies for which comparable static multipliers cannot be calculated there are other problems in defining these multipliers. In particular, they have to be defined separately with respect to each policy variable.

Both the static and the dynamic multipliers can be defined with respect to a unit change in a policy-input measured at constant or current prices. When the increase in a real endogenous variable is measured with respect to a constant unit of additional resource input, we have real input multipliers.

When the increase in a real endogenous variable is measured with respect to a current unit of additional resource input, we have current input multipliers or fiscal multipliers. As these multipliers are dependent on the current price level, they are not fixed in magnitude even when the economic structure is unchanged. Hence they are relevant for policy comparisons only at a given point in time or over a short span of time. The current input multipliers may be of greater interest to legislative and administrative policy-makers whose interest centres on what change in an objective variable can be brought about by a change in the nominal unit of a policy variable no matter what the purchasing power of the nominal unit may be in real terms.

The current input multipliers are less stable than real input multipliers, in general. If relative or absolute prices change over time the same amount of expenditure, say, will buy less goods in future and will yield a smaller multiplier, with respect to, GNP, say. The use of real values in the denominator will generally yield more stable results. However, the use of real input multipliers is limited for fiscal policy in current nominal units, the effects of which are not revealed by them. It is useful, therefore, to use both real input and current input multipliers.

Because of the fact that different policies have different time response functions, the knowledge of equilibrium or long-run multipliers is not sufficient for a decision on the relative merits of the policies. If, for example, an endogenous variable, say, real GNP, increases in a manner shown below, in response to two policies with equal resource inputs, it would be desirable to bring in <u>response-path</u> considerations into the analysis. For example, for the response-paths described in Fig. 5.1,



<u>Fig. 5.1</u>

Response Path of Alternative Policies

Policy B is preferable before t_0 and beyond t_1, whereas policy A is preferable between them. If the two policies be exclusive, so that either A or B must be chosen, the policy with a higher discounted GNP will need to be chosen. It is here that the use of discounted dynamic multipliers using a social time preference rate is useful.

5.6.2 Utility Evaluation

Policies can be ranked with the help of a preference function of the policy-maker or the politician which reflects his subjective trade-offs between different variables. Such an analysis can be conducted with a utility function or a loss function. Since it is difficult to estimate the exact shape of the preference function unless a vast quantity of appropriate information is available, it might be desirable at times to generate policy rankings with hypothetical utility functions with assumed values for parameters.

The arguments (say, x_{it}) in these functions may refer to both the target and the instrument variables. Linear ($u_t = [\Sigma\, \beta_i\, \beta_i\, x_{it}]$), quadratic ($u_t = [\Sigma\, \beta_i\, x_{it}]^2$), Cobb-Douglas ($u_t = \Pi\, x_{it}^{\beta_i}$) and CES ($u = [\Sigma_i \beta_i\, x_{it}^{\delta}]^{1/\delta}$) type of functions seem to be attractice for their analytically tractable properties.

Since the response of the economy to policy change is dynamic, i.e. the effects of policy changes are distributed over time with each policy having a different lag-response function, it is not possible to choose the best policy for period 1, and then for period 2, and so on, separately. It is necessary to find a method to aggregate utility over the entire time-path.

One such method is to use a discount rate, so that the aggregate utility may be given by,

$$u_0 = \sum_{t=1}^{m} \frac{u_t}{(1 + r)^t} \tag{5.16}$$

This is a sum of discounted utilities. This permits the use of different arguments for different periods because for each period the utility is to be calculated separately. Also, the discount rate need not be constant.

It is also desirable, especially in the case of calculation of utility over a cyclical path, to consider the disutility of increased variability of the arguments. One possibility is to use the reciprocal of the variables of the arguments over the sample period, to indicate that the utility increases as variance decreases. Thus, e.g. in the Cobb-Douglas case,

$$U_v = \left(\frac{1}{v_1}\right)^{\beta_1} \left(\frac{1}{v_2}\right)^{\beta_2} \cdots\cdot \left(\frac{1}{v_n}\right)^{\beta_n} \qquad (5.17)$$

The β's here may not be the same as in the original function. Once the utility related to the variability (U_v) is calculated, total aggregate utility may be taken as a linear or dependent combination of U_0 and U_v, e.g.

$$U_T = \gamma_0 U_0 + \gamma_v U_v,$$

or, (5.18)

$$U_T = U_0^{\gamma_0} \cdot U_v^{\gamma_v}$$

One last step in utility evaluation is the consideration as to how the arguments should be defined. All policies start from the same base, variously called as the 'original', the 'control', or the 'neutral' solution. Interest lies in seeing how the policies change the path of the economy from this original solution. It may, therefore, be desirable to express the X's in a deviation form or as a ratio of the original solution values.

Such scaling is also necessary in a variance function written in a multiplicative form. Here, zero variance for any argument will indicate infinite utility. Thus, instead of using v_i in the variance function, if one uses a new argument

$$C'_{x_i} = \frac{s_i + \bar{x}_i}{\bar{x}_i}$$

where $s_i = \sqrt{v_i}$ and x_i is the mean, the value of C'_{x_i} will be 1 whenever $v_i = 0$. Hence, the variance function (5.17) may be redefined as

$$U_v = \left(\frac{1}{C'_{x_1}}\right)^{\beta_1} \left(\frac{1}{C'_{x_2}}\right)^{\beta_2} \cdots \left(\frac{1}{C'_{x_n}}\right)^{\beta_n} \qquad (5.19)$$

It should be possible, in this framework, to rank policies according to the utility they generate.

The use of a disutility function rather than a utility function has been proposed by Theil. Here, the arguments are expressed as deviations from desired targets. The objective is to minimise this function rather than maximising the utility function with arguments expressed as deviations from zero. If target values are denoted by x_i^*, then a linear and a quadratic loss function can be defined as follows:

$$l = \sum_{i=1}^{n} \beta_i \, (x_i - x_i^*) \; ; \qquad (5.20)$$

$$l = \sum_{i=1}^{n} \beta_i \, (x_i - x_i^*)^2 \qquad (5.21)$$

Normally, $\beta_i > 0$.

Expressed in these forms, the marginal rates of substitution are the same in the two cases. When the quadratic function is expressed in a more general form, e.g., in the two variable case,

$$1 = \beta_1' (x_1 - x_1^*)^2 + \beta_2' (x_2 - x_2^*)^2 + \beta_3' (x_1 - x_1^*)(x_2 - x_2^*) \quad \ldots \quad (5.22)$$

the marginal rate of substitution will be equal to that in the linear form only when $\beta_3' = 2\,\beta_1'\,\beta_2'$. In general, they will not be equal.

The quantification of disutility takes place in two steps. For a one-instrument/one-target case it has been shown in Chapter 2. First, disutility is to be minimised subject to the constraint under which the economy is operating, viz., the relation which governs the variables. This yields the target values for the arguments. Secondly, these targets and the loss function are used to quantify the disutilities attached to different policies. A ranking may now be generated with the help of these.

5.6.3 Analysis of Simulated Data

When simulation experiments contain stochastic disturbance terms, they produce a large amount of data depending on how many times the experiment is replicated for each policy. Such replication may be achieved by using different starting values for the pseudorandom number generator used to generate the stochastic term. For each policy, then, a set of sample parameters would be available corresponding to each replication. Appropriate statistical techniques need to be employed to see how far these sample parameters would guide us towards their corresponding population parameters.

The purpose of this exercise would be to tell, first, whether different policies differ in their impact on the response variable(s) as shown by the population parameters corresponding to each policy. If they do, it may be worthwhile building confidence intervals for the difference between population parameters for different policies. This would indicate the nature and strength with which the different policies differ in their impact. Ultimately, one would like to rank the policies.

Several statistical techniques are available to meet these ends. It is suggested for example, that the F-test may be used to find whether the policies differ in affecting the response variable. If we have a sample parameter (e.g. mean or variance of a response-variable) X, such that its expected value is E (X), we can test the null hypothesis

$$H_0 : E(X_1) = E(X_2) = \ldots = E(X_k)$$

where k is the number of policies. The use of the F- statistic will provide us with the decision-rule to reject the hypothesis if

$$F > F_{\alpha,\ k-1,\ k(n-1)}$$

where α is the significance level, and n is the number of replications. If the hypothesis is not rejected, one would tentatively conclude that the sample differences between policies are attributable to random fluctuations and not to actual differences in their population values.

The value of F will be computed by the following formula, assuming that replications are not treated as a factor so that a single-factor design is appropriate.

$$F = M\,S_p \,/\, M\,S_e, \tag{5.23}$$

where

$$M\,S_p = n \sum_{j=1}^{k} (\overline{X}_j - \overline{X})^2 \,/\, k - 1$$

and

$$M\,S_e = \sum_{i=1}^{n} \sum_{j=1}^{k} (X_{ij} - \overline{X}_j)^2 \,/ k\,(n - 1)$$

The symbols have the following meanings:

X_{ij} - value of the parameter for the ith replication and jth policy.

$\overline{X}_j$ - average of X_{ij} for policy j over all replications (i = 1, 2, ..., n).

$\overline{X}$ - average of $\overline{X}_j$ over all policies (j = 1, 2, ..., k).

When it is the case that policies do differ, this knowledge as revealed by the F-test can be supplemented by the use of multiple comparisons method (5.18). This would help us build confidence intervals for the difference between population parameters relating to the response variable for different policies and thus indicate how far the policies differ.

The multiple comparisons method involves the use of a modified form of t-statistic. When pairs of policies j, J ($j = 1, 2, \ldots, k$, $j \neq J$) are compared, the confidence interval would be given by

$$E\,X_j - E\,X_J = (\overline{X}_j - \overline{X}_J) \pm q_{k,v}\,\sqrt{M\,S_e\,/\,n} \qquad (5.24)$$

qk,v is the modified t-statistic provided by Tukey (1953) and is called 'Studentized Range Statistic', k is the number of sample means and v is the number of degrees of freedom for $M\,S_e$, i.e. $k\,(n - 1)$. It will provide confidence intervals which are <u>simultaneously</u> true for all policies which would not have been the case if the familiar t-statistic were used.

If rather than comparing pairs of policies, it is desired to compare all policies with a control solution, Dunnett's d-statistic may be used. The confidence interval would then be given by

$$(E\,X_j - E\,X_C) = (\overline{X}_j - \overline{X}_C) \pm d\,\sqrt{2\,M\,S_e\,/\,n}$$
$$(j = 1, 2, \ldots, n, \neq C) \qquad (5.25)$$

Here subscript C refers to the control solution parameters and d is the percentile of Dunnett's d-statistic with k (n - 1) degrees of freedom.

It would be possible to say whenever the difference between the two sample means exceeds the computed confidence allowance ($d\,\sqrt{2\,M\,S_e/n}$ or $q_{k,v}\,\sqrt{M\,S_e/\,n}$) that the difference between the corresponding population means is 'statistically significant', if this kind of analysis is desired.

Ultimately, one would need to rank the policies. One method of ranking is simply to use sample means associated with alternative policies. However, since sample means may not truly reflect population means, such a ranking may not be reliable. Multiple ranking procedures are recommended at this stage. These help us to associate a probability with which we may believe that a ranking by samples means truly reflects a ranking by population means.

5.7 Simulations for Past Policy Evaluation

A spin-off of a forecasting model is to use it to analyse past policy actions. Here, rather than using the model for future predictions, one simulates the model for the historical past with given values of all exogenous variables except the policy-instruments. The model is used to estimate the effects on relevant endogenous variables by changing the values of policy-instruments within the empirically relevant range and by comparing it with the actual values and/or model-estimated values of these endogenous variables when policy-instruments take their actual values. Ranking techniques similar to those described earlier can then be used to compare alternative policy configurations. Suggestions can then be made as to under what conditions the performance of the economy could have reasonably been improved if a different set of policy options were followed rather than the ones actually used.

This kind of exercise needs to be tempered by the considerations (i) that recommended policy options be politically practicable, (ii) and that the kind of information now available was not available at the time of policy-making. There is no way in which perfect hindsight today can be transplanted as perfect foresight in the past.

Sometimes a currently operational model may need to be partly respecified in order to better predict the past.

5.8 Estimation of Preference Functions

A preference (or loss) function is needed at some stage or another in all approaches to economic policy. Early attempts towards estimating parameters of such a function, e.g., Johnson and Dewald (1963), Wood (1967), Reuber (1964), Havrilesky (1967) and Christian (1968), were cast in terms of estimating a 'reaction function' in the context of a single policy. Given an appropriate statistical indicator of a policy variable and certain 'objective' or target variables such as employment, price-level, economic growth and balance of payments, the operational procedure was to regress the policy variable on the objective variables. Single equation estimation techniques were used and functional forms were decided either on analytical or statistical grounds. The estimated multiple regression coefficients are interpreted as 'reaction coefficients' of the authorities indicating the 'implicit' trade-offs which they had in mind between one objective and another *vis a vis* a given policy.

Such an approach, however, can at best be useful in the context of one policy-instrument at a time. In a general equilibrium framework, where a multiple of policy-instruments and target-variables are involved and interacting, it needs to be generalized. One of the first attempts towards such generalization was made by Friedlander (1973). Her illustration specifically uses a qualified quadratic utility function and a linear constraint. It can be extended to a general class of utility functions, especially if they are less complicated. It should be noted that when the instruments are endogenously determined, estimation becomes much more difficult.

Given a utility function and an econometric model which expresses the operative structural constraint, it is possible to express each policy-instrument as a reaction function in target variables. The coefficients of these reaction functions are shown to be combinations of the policy-weights and reduced-form coefficients.

The technique may be outlined as follows. Let the utility function be given by

$$U = \alpha' Z + \beta' Y - \tfrac{1}{2} Z' AZ + Y' BY \qquad (5.26)$$

where Z and α are (mxl) vectors of policy-instruments and weights attached to these, Y and β are (nxl) vectors of target variables and their weights, A is a (mxm) diagonal matrix of weights attached to the squared values of policy-instruments and B is a (nxn) symmetric matrix of weights attached to the squared values of target variables. Thus, a_{ii} in the diagonal of A is interpreted as the weight attached to the squared value of target i, given the value of target j. The primes refer to the transpose of vectors and matrices.

It is normal to express the values of policy-instruments and target variables as deviations from desired values (as in Friedlander, 1973) rather than as deviations from zero as expressed here. It does not, however, affect the procedure for the derivation of policy weights.

The formulation of the utility function as above, permits interactions between target variables but not between policy-instruments. This is due to the fact that A is taken as a diagonal matrix so that the weight attached to any interaction term in instruments is zero. The symmetric matrix B permits positive weights for interaction terms in target variables. The implication is that the policy-maker is concerned about the relative magnitude of the target variable but is indifferent to the policy-mix. The utility function also does not permit interactions between targets and instruments which implies that the policy-maker is indifferent as to how the targets are achieved. As such the preference function is of limited scope.

The next step is to specify the constraint under which the economy is operating. Let a linear model describing the economy be:

$$Y = R\,Z + S\,U \tag{5.27}$$

where,

R = (n x m) matrix of reduced-form coefficients relating the targets to the instruments;

S = (n x p) matrix of reduced-form coefficients relating the targets to lagged endogenous and exogenous variables; and,

U = (p x 1) vector of lagged endogenous and exogenous variables.

The policy problem is one of maximising (5.26) under the constraint (5.27). Substituting (5.27) in (5.26) and differentiating with respect to Z, we have the first order condition

$$(\alpha + R'\beta) - A\,Z - R'BY = 0$$

or

$$Z = A^{-1}(\alpha + R'\beta) - A^{-1}(R'\beta)\,Y \qquad (5.28)$$

Thus, any element z (j = 1,...,m) of the vector Z has two parts: (i) a constant term, and (ii) a linear combination of all the y's. Since all the policy-instruments and the target variables are observable over time, it is possible to estimate regressions of the form:

$$z_i = \gamma_j + \sum_{i=1}^{n} \gamma_{ij}\, y_i + u_j \qquad (j = 1,...m) \qquad (5.29)$$

It can be seen that the coefficients γ_j and γ_{ij} (j = 1,...,m; i = 1,...,n) are combinations of various policy weights and reduced-form coefficients.

Since A is a diagonal matrix, we will have from (5.29)

$$\gamma_j = 1/a_j\,(\alpha_j + \sum_{i=1}^{n} r_{ij}\,\beta_i) \qquad (5.30)$$

and,

$$\gamma_{ij} = -\,1/a_j\,(\sum_{h=1}^{n} r_{hj}\,b_{hi}) \qquad (i = 1,...,n) \qquad (5.31)$$

The number of equations defined by the set (5.30) and (5.31) is m + (m x n). γ_j and γ_{ij} (i = 1,...,n; j = 1,...,m) are estimated regression coefficients and are known. r_{ij} (and r_{hj}) are reduced-form coefficients and, therefore, known. The unknowns are a_j, β_i, α_j and b_{hi}.

Since B is a symmetric matrix, the total number of unknowns is

$$2m + n + \sum_{i=0}^{n} (n - i).$$

out of which (m + n) are linear weights and (m + n(n + 1)/2) are quadratic weights. Since it is not possible to find absolute weights but only relative weights, in each case one of the weights will have to be used as a numeraire i.e. set equal to 1 so that ultimate results will be relative weights. The number of equations will generally be greater than the number of unknowns and solutions will be possible only in special cases.

The reliability of policy weights derived in this manner depends on the (i) correct choice of the welfare function (quadratic or otherwise and with what interaction terms), (ii) correct choice of the model representing the operational constraint of the economy, and (iii) correct identification of the model. As Friedlander admits, this is an exercise in third-order theorizing. The analysis is sensitive to the specific form of the welfare function, specific targets and instruments included in the welfare function, the assumed time horizon or the forecasting period, and the form of the operating constraint. Important problems of specification and estimation arise when many policy-instruments are not strictly exogenous, i.e. they systematically react to the path taken by endogenous variables.

5.9 Summary

The use of macroeconometric models in policy analysis implies a 'synoptic' approach to policy which permits interdependences and interactions between target and policy variables. In this Chapter we consider three main ways in which a macroeconometric model can be used as a guide to policy evaluation. These are the constrained maximisation approach, the aspiration-levels approach and the policy-simulation approach. In all these variants preference functions are needed at one stage or another for ranking alternative policy configurations. We consider the problems and possibilities of estimating preference functions and note how qualified and tentative such estimates may be.

We have abstracted from a discussion of control theory which is another way in which the policy problem may be approached. We also abstract from the extreme view that various rigidities, imperfect information and other political and psychological factors warrant that only an 'incremental' approach to policy would be realistic.

PART II

MODELS OF THE INDIAN ECONOMY: A SURVEY

Chapter 6

Macroeconometric Models of the Indian Economy: An Overview

The construction of economy-wide models for the Indian economy has a history of twenty-five years or so. Although beset by a number of technical, statistical and conceptual problems, this activity has nevertheless yielded a rich crop in terms of the number and variety of models. Most of the early forecasting models do not continue to be active instruments of forecasting in not having been revised and updated. Still, a review of these models is likely to prove a useful guide to future model-building. First, however, two distinct categories of models built for the Indian economy, viz., programming models and forecasting models, may be distinguished.

6.1 Programming vs. Forecasting Models

Programming models, sometimes also called consistency models, consist of an optimization function or a set of numerically specified targets and a set of constraints in the form of equations and inequalities. The solution of the system provides an 'optimum' and 'consistent' path for instruments in order to satisfy the targets in a terminal year without violating the constraints. Although the constraints may be econometrically estimated equations, the usual practice for the Indian models* has been to use 'likely' values for parameters defined by the analyst and estimated parameters in input-output blocks. Until now this variety of models has been in the background of Indian Plans.

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* Pioneered by Mahalnobis , a number of such plan models have been built, e.g., Palvia (1953), Manne-Rudra (1965), Srinivasan Saluja-Sabherwal (1965), Bergsman-Manne (1966), Chakravarty-Lefeber (1965), Frisch (1960), Sandee (1960), Weisskopf (1967, 71), Manne-Weisskopf (1970). Useful reviews of these models are available in Rudra (1975), and Tendulkar (1974).

In the present study, however, our concern is primarily with the other category of models, viz., macroeconometric forecasting models. An objective function is not an integral part of these models. Their main concern is to describe the structural relationships in the economy with the help of econometrically estimated relations with a view to forecasting and/or policy simulation. Historically, the consistency type models have focussed more on technical constraints and input-output relations whereas the forecasting models have concentrated more on demand side relations. There is no reason, however, that the econometric models cannot contain input-output blocks and other technical relations. They can also be used in conjunction with an objective function and offer a greater degree of flexibility of approach.

Although consistency models have been rather popular with the Indian planners, the main difficulty with these is that once actual achievements begin to deviate from the charted time-paths for the target variables, the entire solution is rendered useless as numerical values for different variables are integrally linked with each other. Forecasting models, on the other hand, provide a more flexible framework. Being independent of an objective function, various simulated paths are generated for the target variables, and results are easily modified when additional information becomes available or when extraneous changes occur. A case in favour of the use of this variety of models in Indian planning can therefore be made both for short-period and for medium-to-long term planning.

6.2 Macroeconometric Models of the Indian Economy: A Listing

An attempt is made here to survey the contributions of following authors towards building macro-models of the Indian economy.

1. Narasimham, N.V.A. (1956).
2. Choudhry, N.K. (1963); four versions to be respectively called Choudhry I, II, III and Revised-III.
3. Krishnamurty, K. (1964); four versions to be respectively called Krishnamurty I, Revised-I, II and III.
4. Marwah, K. (1964, 1972); three models, to be called, Marwah I, alternate-I and II.
5. Mammen T. (1967); separate models of the monetary and real sectors and a joint model, to be called, respectively, Mammen M, -R and -J.
6. Krishnamurty, K and N.K. Choudhry (1968)
7. UNCTAD (1968, 1973); the latter model is contained in a report on the LINK project (Ball, R.J. ed., 1973).
   The models may respectively be called, UNCTAD -I and UNCTAD -LINK.
8. Agarwala, R. (1970)
9. Pandit, V.N. (1973)
10. Gupta, G.S. (1973)
11. Bhattacharya, B.B. (1975)*

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* These models cover the main model-building activity for the Indian economy. There are, however, other models built in the past and in the pipeline. Some of the models not included here like Diwan (1967) and Pani (1971) have many common features with the models covered. For example, UNCTAD (1968) is very similar to the Diwan model, and UNCTAD (LINK) borrows heavily from the Pani model.

Many of these models remain unpublished. Barring a few, most of these have arisen out of Ph.D. level dissertations or subsequent revisions by the respective authors, and have been built outside India. An imprint of Western model-building is clearly visible on them. In an earlier survey of some of these models, Desai (1973) has put this succinctly, "If models were to be classified as thorough-breds, these are clearly from the Tinbergen-Klein stables."

Since a continuous process of forecasting, updating and revisions has not been kept up by their builders, they are best viewed as pioneering work and as providing a valuable background to future model-building in India. Many of the authors have shown the experimental nature of their work by providing more than one version of the same basic model framework. The costly nature of work in keeping a model updated and for providing forecasts at regular time intervals, implies that ultimately most of these exercises will have to be taken up by institutions rather than individual authors.

6.3 Size and other Basic Features of the Models

Due, no doubt, to the limitations of data, these models in general are fairly small relative to some of the vast macroeconometric structures built for the Western economies. The variety of sizes and structural detail as judged by the number of endogenous and predetermined variables, stochastic equations and identities can be read from Table 6:1.

Ranging from the smaller models containing less than 10 equations to larger models containing upwards of 70 equations, these models display a variety of sizes. There is no unique way of knowing what is the right size of a model. It depends to some extent on the purpose of the model, and to some extent on the size of the sample of observations. A

Table 6.1

Size and Other Basic Features of Models†

Model	No. of Equations		No. of variables		
	stochastic	definitions	Endo genous	Exo- genous	lagged endogenous
Narasimham	11	7	18	16	8
Choudhry -I	7	3	10	10	4
-II	13	7	20	19	4
-III	14	8	22	18	5
Revised-III	12	8	20	19	5
Krishnamurty-I	15	6	21	16	6
Revised -I	14	5	19	17	6
-II	6	3	9	10	3
-III	8	4	12	12	3
Marwah-I	14	3	17	14	8(+1)*
Alternate-I	15	6	21	17	8(+1)*
-II	39	9	48	26	19
Mammen-Monetary	10	2	12	7	1
-Real	8	4	12	7	2
-Joint	18	8	26	12	3
Krishnamurty-Choudhry	20	10	30	16	10
UNCTAD-I **	17	15	31	11	8
-LINK	51	20	71	18	3
Agarwala	17	7	24	13	7
Pandit	11	5	17	15	2
Gupta	17	25	42	31	13
Bhattacharya	6	3	9	7	2

* An additional lagged endogenous variable occurs for a respecified equation for a sub-sample.

† The number of variables listed here may occasionally differ from those listed in the original models because variable occurring in identities which do not have a differential effect on the system have sometimes been integrated.

** The number of equations exceeds the number of endogenous variables to permit disequilibrium.

relatively larger model has the advantage of providing extensively disaggregated information but built with smaller samples, estimation techniques have to be compromised. In particular, when the sample-size is smaller than the number of predetermined variables, models need to be estimated by OLS or by the use of principal components or subsets of variables in 2SLS. It becomes difficult to use more sophisticated techniques of simultaneous equation estimation. In countries where current practices in national accounting produce only annual data on national income and its components, the size of available samples with uniform structural characteristics is bound to be small. Consequently, the sizes of models, which are judged more appropriately by the number of stochastic equations since any number of equations can be produced from the identities, will need to be modest in this context. This is borne out, for example, by a survey of models of different countries in Project LINK (Waelbroeck, ed., 1976). For country models using annual data, the number of stochastic equations and the estimation technique used is given in Table 6.2.

Table 6.2

Size and Estimation Techniques for Models using Annual Data in Project LINK

Country	No. of stochastic equations	Estimation Technique	Country	No. of stochastic equations	Estimation Technique
Austria	54	OLS	Nether-lands	13	LIML/2SLS
Belgium	19	OLS	Sweden	75	OLS
Canada	44	OLS	Developing America	11	OLS
France	19	OLS	South & East Asia	13	OLS
W.Germany	51	FIML	Middle East + Libya	9	OLS
Italy	53	OLS	Africa minus Libya	10	OLS

It is seen that, with the exception of a few, these models are moderately sized and estimated by OLS. Details of estimation techniques used in models for the Indian economy are studied in the following section.

6.4 Sample Periods and Estimation Techniques

By and large, all the models have used annual observations in a time-series. In some cases, however, a pooling of time-series and cross-section data has been done for some equations (e.g. Marwah II) or cross-sections estimates have been used for comparisons (e.g. Agarwala). In most cases, the number of observations fall short of the predetermined variables and the model builders were constrained to modify simultaneous equation estimation procedures. Thus, some times principal components or subsets of predetermined variables were used in the first stage of 2SLS. In general uniform samples have been used. But in some cases (e.g. Marwah I and II) the sample size has been different for different equations and estimation procedures had to be modified. Where models were estimated by 2SLS/LIML, OLS estimates have been provided for comparison. Systems approaches to estimation like 3SLS and FIML have not been used. In Table 6.3, the nature of data, i.e. whether time-series or cross-section, the sample periods and the estimation techniques for these models have been surveyed.

It would be observed that there are some models which use data extending into the pre-independence period, e.g., Narasimham, Choudhry, Krishnamurty (for demographic equations only) and Marwah. Reliable forecasting from any econometric model depends on the crucial assumption about the stability of parameters both within the sample period and between the sample and the prediction periods. It is therefore useful to question whether degrees of freedom should be bought by using the pre-independence period. All available information indicates substantial structural shifts in the Indian economy in view of planned efforts towards economic development. The partition of the country on the eve of Independence also necessitates an artifical splitting up of data in

Table 6.3

Sample Periods and Estimation Techniques

Model	Nature of Data	Main Sample	Sub-Samples	Estimation Techniques
Narasimham	TS	1919-52	-	OLS
Choudhry I,II,III and R-III	TS	1930-55	-	OLS
Krishnamurty I, R-I,II & III	TS	1948-61	48-59,49-61, 22-61	OLS/2SLS-S
Marwah I and A-I	TS	1939-60	47-60,44-62	OLS/2SLS-M
Marwah II	TS,CS-TS	1939-65	52-64,51-65	OLS/2SLS-M
Mammen-M & R	TS	1948/9-63/4	-	OLS/2SLS
Mammen-J	TS	1948/9-63/4	-	2SLS-PC
Krishnamurty-Choudhry	TS	1948-61	-	OLS
UNCTAD-I	TS	1950/1-64/5	50/1-63/4, 50/1-62/3 50/1-61/2	OLS
UNCTAD-LINK	TS	1950/1-68/9	-	OLS
Agarwala	TS	1948/9-60/1	-	OLS,2SLS/LIML*
Pandit	TS	1950/1-65/6	-	2SLS
Gupta	TS	1948/9-67/8	-	2SLS-S
Bhattacharya	TS	1949/50-67/8	-	OLS/2SLS

Notes: (i) in 2SLS-S estimation, a subset of predetermined variables were used in the first stage.

(ii) 2SLS-M arises in Marwah's models because of non-uniformity of sample period. In the first stage, only those endogenous variables were estimated, equations for which could be estimated for the full-sample period. These estimates were used in the second stage irrespective of sample size.

(iii) 2SLS-PC refers to the use of a set of principal components of the predetermined variables in the first stage.

order to get comparable series. Further, the quality,collection and coverage of data have also changed. Although the model-builders in yesteryears were induced to minimise the importance of splitting up of pre-independence data in order to make up for the smaller post-independence sample, model-builders now can afford to start at the 1947 watershed or even from 1950 which heralds the planning era.

It is arguable, however, whether even the post-1950 years represent a homogenous period. In particular, the growing role of government, appearance of food grain surplusses in the Seventies compared to the Sixties, a comfortable foreign exchange position in recent years may represent basic changes in the economy. A test for structural stability of coefficients in major stochastic relations in these models has been carried out in Chapter 10.

But even supposing that the post-1947 years represent a homogenous period, the usefulness of models built with post-independence data has become limited because of data revisions. With the exception of a few (e.g. Pandit), most models surveyed here have been built on what is known as the 'conventional' series of national income and its components. The Central Statistical Organisation (CSO) subsequently revised these series and parameters estimated by the old series might be affected not only because of structural shifts but also because of changes in weights, coverage and methodology used in the construction of the two series. Apart from this, any tests on the prediction performance of these models cannot be done directly. Whereas model predictions would be in terms of the old series, corresponding realizations would be in revised terms. Even if one circumvents this problem by comparing rates of growth rather than levels of variables, assuming that the revision of data would not affect the former, it is doubtful whether, in many cases, predictions could be generated at all. These are the cases where for ex-post predictions the actual values of such exogenous variables are needed which are themselves available only in revised terms.

6.5 The Basic Approaches

Whereas specific equations are important in themselves, the overall approach to the model is of crucial importance. This would govern the way in which individual equations are interlinked. Although the overall picture would differ from model to model depending on the degree of disaggregation, it could be deciphered as to whether a model adopts the aggregate demand approach in the IS-LM tradition, or the monetarist approach to the determination of nominal income, and whether elements of aggregate supply especially the dualistic nature of the economy have been incorporated. The theoretical foundations of these approaches have been considered in Chapter 3.

It would be recalled that the IS-LM framework is useful for short-term determination of income, when productivity, technology, etc. can be assumed to be constant. The longer the time horizon, the more difficult it becomes to ignore the supply side considerations. The mainstream models of the Indian economy have been built within the IS-LM framework. This includes models by Narasimham, Choudhry, Krishnamurty, Marwah -I, Mammen, UNCTAD-I, Pandit and Bhattacharya. This, no doubt, has been due to the overriding influence of the model-building tradition in the Western market economies. The main exception to this trend is Agarwala's model which is built primarily on considerations of aggregate supply and incorporates the dualistic nature of the economy by a distinction between 'agriculture' and 'non-agriculture' at all stages of analysis. Marwah II and UNCTAD-LINK also give primary importance to aggregate supply considerations. Other models incorporating one or more production functions are Choudhry, Krishnamurty, Krishnamurty-Choudhry, UNCTAD-I and Gupta. Although basically within the IS-LM framework, many models do contain elements of other approaches and as such they are best viewed as of a hybrid variety.

It is not possible to contend that the aggregate demand approach should be completely ignored in India. There is evidence (e.g. Pandit, 1973) that unutilised capacity exists in many industries including those producing essential consumer goods.

6.6 Sectoral Emphasis

Models differ from one another in terms of emphasis on different sectors. Thus, the monetary sector has been emphasized in Choudhry, Mammen, Bhattacharya and Gupta. Mammen's model was developed by first separately specifying a model for the monetary sector and another for the real sector and then combining these to produce a joint model. Gupta's model is the most disaggregated as far as the monetary sector is concerned. It contains a four-fold division of monetary activities in the economy by distinguishing between (i) government including the central bank, (ii) commercial banks, (iii) State co-operative banks and (iv) the private non-banking sector. The model is built with a view to study monetary policy, and the real sector, in a very aggregated form, is introduced only because it contains some of the target variables. In Bhattacharya's model, a separate money supply function has been introduced for endogenous determination, and 'monetized' income and its components are used for estimation.

The foreign trade sector has received an extensive treatment in the UNCTAD models. These models were prepared primarily with a view to study resources requirements, especially foreign exchange requirements, and the linkage of the economy with the rest of the world via the trade equations. In these models imports are divided between food and non-food imports. Exports are divided between tea, jute and others in UNCTAD-I and between tea, jute, engineering goods and others in UNCTAD-LINK. All these categories of imports and exports receive an endogenous treatment in these models. The only other models where a substantial disaggregation of imports is offered is Marwah II. Here imports are considered in various S.I.T.C. categories. Most other models contain one or more equations for components of income. A few of the models, like Pandit's model, also treat exports as endogenous.

The fiscal sector is relatively underemphasized in almost all models. Very few models contain any disaggregated treatment of government revenues and expenditures. Aggregate tax equations are contained in Krishnamurty-Choudhry, Bhattacharya, UNCTAD-I and Gupta. In UNCTAD-LINK, there is a greater disaggregation of government revenues. A government budget constraint has been incorporated only in Gupta and UNCTAD-LINK models.

A disaggregated treatment of prices is offered in Marwah-II and UNCTAD-LINK. More aggregate price equations are available in Pandit, UNCTAD-I and Gupta. Most other models do not contain price equations. Aggregate supply considerations have been used sometimes via the price equations and sometimes directly via production functions. There are aggregate production functions in Choudhry, Marwah-II, and UNCTAD-I. Separate production functions for agricultural and non-agricultural outputs have been introduced in Krishnamurty and Agarwala models. In Krishnamurty-Choudhry, there is a production function for agriculture and one for output in mining, manufacturing and trade. In Gupta's model there is a production function for non-agricultural output.

In Krishnamurty and Krishnamurty-Choudhry models, an endogenous explanation of population has been offered. These are the only models where population has been explained as more than a fixed function of time as in Gupta's model, if not altogether exogenous. Thus Krishnamurty-Choudhry is the only model containing a separate demographic sector.

Similarly, an endogenous explanation of money supply has been incorporated in only a few models. These are Krishnamurty-Choudhry, Agarwala, Bhattacharya, Pandit, Gupta and UNCTAD-LINK. Whereas in the former four models, the treatment is highly aggregative inasmuch as one equation for supply of money or demand deposits has been introduced in each of these models. In the latter two models, the treatment of money supply is somewhat disaggregated.

6.7 Nominal and Real Variables and Other Special Features

It is interesting to note whether in these models variables apart from price and volume indices, and ratios like interest rates, and more specifically, income expenditure components and money supply etc. have been defined in real* or nominal terms. Specification of demand equations in nominal terms may imply money-illusion or a non-homogenous impact of price changes and it also has implications for the quality of estimation.

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* The terms 'real' and 'nominal' are used respectively for variables measured at constant prices or deflated by price indices and variables measured at current prices.

In Narasimham's model the income-expenditure components are defined in nominal terms. Further, all variables are measured as deviations from 9-yearly moving averages. In models by Choudhry and Bhattacharya also, equations are specified in variables measured in current prices. In Bhattacharya's model, moreover, only 'monetized' levels of relevant variables are used. Variables are in nominal terms for the monetary sector equations in Mammen's and Gupta's models. In the UNCTAD models, both real and nominal counterparts of most variables are introduced. Generally the stochastic equations in these models are in real terms and identities and definitions contain both real and nominal variables. These include definitions relating real variables to nominal variables by appropriate price deflators. In other models, most relations are defined in real variables. However, implications of non-linearities arising out of definitions of variables as products and ratios have largely been ignored.

The lag-structures in these models have generally been very simple. Since only annual observations are used, one year lags and, in a few cases, two year lags have been introduced. In models by Narasimham, Marwah and Choudhry, sometimes half year lags are introduced by constructing variables as averages of current and previous year's values. Distributed lags have normally been handled by Koyck transformations resulting in a lagged dependent variable on the right-hand side of the respective equations.

6.8 Introduction of Policy Instruments

Apart from forecasting, an explicit purpose of these models has been policy analysis. Considerable attention has, therefore, been paid to the specification of individual equations especially those incorporating policy instruments. There are, however, salient differences in the number, types and ways in which these policy instruments have entered into different models and it is interesting and useful to survey differences in variables appearing as instruments in these models.

Fiscal policy variables appear in nearly all the models. Salient differences in the definitions of these variables and the way they have been disaggregated both on the expenditure and the revenue side are noted in Table 6.4.

With the exception of a few like Gupta and UNCTAD-LINK models, these models do not provide explicitly for an ex-post matching of government revenues and expenditures. This absence of a budget constraint implies the assumption that the government can freely fix the values of all components of its expenditure and revenues.

On the expenditure side the main difference is in the degree of aggregation involved in various models. In some models a catchall remainder term is used which captures all non-private expenditures (like Marwah) and in some cases it covers parts of private expenditures as well (as in Bhattacharya and Pandit). In some models a distinction between aggregate government consumption and investment expenditure has been made and, in some, investment expenditure is further disaggregated. An aggregate expenditure variable implies that a unit increase in one type of expenditure rather than another is expected to have a similar impact on the endogenous variable. Conversely, disaggregation is useful if a differential impact of different expenditure categories is envisaged.

Table 6.4a

Fiscal Policy Instruments

Narasimham	E	Govt. consumption exp. (U^g), investment exp. (V^g)
	R	Direct taxes on non-corporate incomes (T^{NC}), on farm incomes (T^f); indirect taxes (T)
Choudhry I	E	Govt. exp. net of indirect taxes ($G-T^{ind}$), Transfer payments (T^r);
	R	Direct taxes (T^y)
Choudhry II	E	Govt. real investment (I_g); others as in Choudhry I.
Krishnamurty	E	Govt. investment - in stocks (I_g^s), in machinery and equipment, construction and exp. on maintenance (I_g^{me} + I_g^c + G'); govt. welfare expenditure (G^w); govt. imports (M_g), exp. on repair and maintenance (I_g^{rp}, in some versions); transfer payments (T)
	R	Direct taxes (T_d)
Krishnamurty-Choudhry	E	I_g^{me}, I_g^s, I_g^c, G^w, G' (meanings as in Krishnamurty)
Marwah I	E	total govt. expenditure (H)
Marwah II	E	H as in Marwah I
	R	rate of import duties (D)
Mammen	E	Govt. real consumption expenditure (C_g/P_i)
UNCTAD I/		

Table 6.4 (continued)

UNCTAD I		No explicit fiscal policy instruments
UNCTAD-LINK	E	Govt. investment - in departmental undertakings, (IGD*), other than in departmental undertakings (IGO*); deficit exp. on current a/c (GD*)
	R	average rate of - , direct taxes expressed as % of national income (RDT); indirect taxes other than customs duties expressed as % of national income (RIT1); of customs duties expressed as % of value of imports and exports.
Pandit	E	Real govt. investment and consumption exp. including all investment in inventories, depreciation and a statistical discrepancy term (H); Deficit exp. in the previous year D_{-1}.
	R	Direct taxes (T^d)
Gupta	E	Govt. consumption exp. (CO^g); investment exp. (I^g)
	R	Average rate of , direct taxes (t_d); indirect taxes (t_i); net 'other' sources of govt. finance (OS^g)
Bhattacharya	E	Govt. expenditure plus imports minus exports (E_n)

Note: E and R respectively refer to expenditure and revenue variables.

On the revenue side, most models are not able to distinguish between automatic and discretionary changes in tax revenues. Either tax revenues are themselves taken as policy instruements, in which case they do not have an automatic component or they are made a function of income and other tax-bases, in which case they do not have a discretionary component. In particular, tax rates have not been explicitly introduced except in Gupta and UNCTAD (LINK) models. Even in these cases, the tax rates are average effective tax rates which are obtained by dividing tax revenues by national income or other tax-bases. As such they are not able to tell how actual tax rates are translated into effective tax rate changes.

In some models (e.g. Bhattacharya) total tax revenues are taken as one aggregate category while in some others (e.g. Choudhry, Krishnamurty) a distinction between direct and indirect taxes have been made. This distinction is extended in Narasimham for direct taxes which are divided between those on corporate and non-corporate incomes and in UNCTAD (LINK) for indirect taxes which are divided between customs duties and 'other than' customs duties. In some models no tax variable is used (e.g. Marwah I) and no distinction is made between total and disposable incomes.

Very few of the models introduce any kind of government borrowing as policy instruments. One variable which is often used in the theoretical literature in explaining inflation in the Indian context is deficit financing. Two of the models which introduce a variable in this spirit are Pandit and UNCTAD (LINK) which have an explicit variable defined as government deficit on current account.

The second main category of instruments relates to monetary policy. Monetary policy instruments occurring in different models are surveyed in Table 6.5.

Table 6.4b

Monetary Policy Instruments

Krishnamurty	Ratio of excess to required reserves (ER/RR); bank rate (BR)
Krishnamurty-Choudhry	Reserves of scheduled banks with RBI (RES), bank rate (i_B)
Marwah I & II	Short term rate of interest (i_s)
Mammen	Supply of -, currency (C); of govt.interest bearing securities (G^s); rate of borrowing by scheduled banks from RBI (BR)
Bhattacharya	Unborrowed money reserves (U); discount rate (r^d); net liquidity ratio (l)
UNCTAD I	Supply of currency plus demand plus time deposits (L)
UNCTAD-LINK	Discount rate (RB), currency with public (C*)
Gupta	Bank rate (i_B), bank rate on borrowing by scheduled commercial banks from RBI (weighted average) (i_{Bl}); private non-bank liabilities held by RBI (LI^g); dummy variable for bill market scheme (d_B); total stock of securities held outside the govt. sector (G^s); reserve requirements ratio against -, demand liabilities of commercial banks (r_d^{cb}), demand liabilities of State co-operative banks (r_d^{sb}), time liabilities of commercial banks (r_t^{cb}), time liabilities of State co-operative banks (r_t^{sb})

Thus, in most models some monetary policy variable occurs and in many cases, money supply either as an aggregate, or broken into currency, demand and time deposits is itself taken as exogenous. However, in a few of the models like Mammen, Bhattacharya and Gupta, money supply has been treated as endogenous and accorded a disaggregated treatment.

The main instruments in the models surveyed here relate to domestic fiscal and monetary policy. In a few of the models there are other scattered instruments. Among these, of interest are a coefficient of import liberalization defined by Choudhry and a target real income level defined by Mammen. In only one model (UNCTAD I) an exchange-rate variable has been used. Until 1969-70, the official exchange-rate was fixed having undergone a devaluation in 1966, but in later years it has been made quasi-flexible in the sense that it has been linked with a basket of currencies and changes in it are announced periodically.

In some cases, a few policy changes have been introduced by using dummy variables (e.g. Marwah and Choudhry models) and in some cases policy instruments under the control of the rest of the world like foreign aid and capital inflow have been incorporated, as in Krishnamurty-Choudhry, UNCTAD I and UNCTAD-LINK.

6.8 A Survey of Main Stochastic Functions

Although any equation in a model is best viewed in the context of the whole model, there are certain core relations like consumption, investment and money-demand functions which normally occur in all models and a survey of these would indicate the variety of hypotheses that have been tried in the Indian context and the quality of statistical fits obtained. This is likely to serve as a guide for these mainstream equations in any future model-building. Whereas individual coefficients in estimated relationships of similar equations are not always directly comparable because of differences in variable definitions and units, this survey is undertaken with a view to highlight differences in specification in terms of explanatory variables, functional forms and lags and statistical properties of fits as revealed by quantities like R^2, d-W, V-N and t-ratios.

The functions surveyed here relate to (i) consumption; (ii) investment; (iii) demand for money; (iv) imports and (v) production. Selected estimated equations relating to these are given in Tables 6.4 and 6.8. Only selected equations are given in order to keep the number of equations within manageable limits. Care is taken that the few equations that were dropped are similar to some that were included.

There are a number of differences in the statistical presentation of equations in various models. Thus, multiple correlation properties have been variously indicated by R, R^2, $\bar{R}^2$; first-order autocorrelation in error terms, by d-W or VN statistics; statistical significance of individual coefficients by t-ratio or standard error of estimate (SEE). Sometimes the t-ratio or SEE is given for the constant term and sometimes it is not. Rather than presenting individual t-ratios, we have indicated[†] whether it is greater than one by (*) and greater than two by (**). Except for constant terms, if no asterisk appears, it means that the t-ratio is less than one. For constant terms, sometimes there is no asterisk because no t-statistic or SEE has been reported. Variables defined in current prices appear with a prime.

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† Following the practice in an earlier survey by Desai (1973).

Variable definitions common to Tables 6.5 through 6.9 are given below:

Consumption expenditure, total (C), private (C_p), government (C_g)

Investment expenditure, total (I), in agriculture (I_a), in industry (I_i), of government (I_g), private (I_p); private investment in machinery and equipment (I_p^{me}), in construction (I_p^c), in inventories (I_p^s); depreciation (Dep); investment in fixed assets (ICF); investment in inventories (H); govt. investment in machinery and equipment (I_g^{me}), in construction (I_g^c).

Money-supply (M), currency (CR), demand-deposits (D), time-deposits (T); total bank credit (LI).

Rate of interest, short-term (R), long-term (R_l), loan-rate (i_L: defined as average of call loan rate of RBI to commercial banks and 'hundi' rate), bank-rate (R_b), yield on government bonds (i_g); by commercial banks on 12-month deposits (R_{12}); time-deposit rate (i_r); yield on long-term govt. bonds (R_{gi}).

Imports, total (IM), food (IM_f), non-food (IM_{of}), of capital goods (IM_k), of consumer goods (IM_c), of raw materials (IM_R), of machinery (IM_m); private imports (IM_p).

Price-indices, of imports (P_m), of agricultural output (P_a), of non-agricultural output (P_i), of investment goods (P_{inv}), of industrial securities (P^s), of exports (P_x); of food (P_f); of food-imports (P_{mf}); of non-food imports (P_{nmf}); whole-sale price index (P_w); price deflator of GNP (P); of foreign prices (P_f); of world prices (P_w)

Gross national product (Y), gross domestic product (GDP), agricultural output (Y_a), non-agricultural output (Y_{na}), private disposable income (Y^d), population (N), agricultural employment (N_a), non-agricultural employment (N_i), foodgrains output (F_a); industrial profits (Π); net worth of the private sector (NW).

Government deficit on current a/c (GD), Balance of payments deficit on current a/c (BPD).

Capital stock, total (K), private (K_p), government (K_g), in agriculture (K_a), in non-agriculture (K_{na}), inflow of foreign capital (IFK)

Foreign aid (A), time-trend (t), foreign exchange reserves (FE) external finance of corporate sector (EF); utilisation of capacity in non-agricultural sector (U); taxes, in agriculture (T_a), in industry (T_i).

Gross acreage, sown (L), irrigated (L_i); % deviation from 'ideal' rainfall (R_1); utilisation of capacity in non-agricultural sector (U_1).

Table 6.5

Selected Consumption Functions

Choudhry I

$$C'_p = 13.41 + \underset{(**)}{0.92Y^{d'}} + \underset{(*)}{0.074M'} - \underset{(**)}{0.14N} + 0.024C'_{-1} \qquad R^2 = 0.99$$

II

$$C'_p = \underset{(*)}{-0.94} + \underset{(**)}{0.88Y^{d'}} + \underset{(*)}{0.17M'} + \underset{(*)}{0.05}C_o \qquad R^2 = .99,\ VN = 1.02$$

Krishnamurty and Krishnamurty-Choudhry

$$C_p/N = 17.54 + \underset{(**)}{0.81}\ Y^d/N + \underset{(*)}{0.54}\ (Y_a/Y_{na}) - 1.70t$$

Marwah I

$$C_p = 17.21 + \underset{(**)}{0.87Y} + \underset{(**)}{0.23}(M'_{-2})/P + \underset{(*)}{0.046N} \qquad \bar{R}^2 = .98$$

II

$$C_p = -10.63 + \underset{(**)}{0.84Y} + \underset{(*)}{0.27}(M'_{-2})/P + 31.8N \qquad \bar{R}^2 = .98,\ d = 1.42$$

Bhattacharya

$$C'_p = .428 + \underset{(**)}{.848Y^{d'}} \qquad \bar{R}^2 = .99,\ d = 2.243$$

Pandit

$$C_p = -9.85 + \underset{(**)}{0.73Y^d} + \underset{(*)}{0.18}Y_a/Y_{na} \qquad \bar{R}^2 = .96,\ d = 1.16$$

Gupta

$$C'_p = \underset{(**)}{3105.12} + \underset{(**)}{0.9094Y^{d'}} \qquad R^2 = .998,\ d = 1.195$$

Notes: (i) C_o = highest previous consumption;

(ii) in Bhattacharya's equation variables relate to 'monetized' levels.

Except for Agarwala's model, all models have a consumption or a saving function in one form or another. In Agarwala's model, demand for consumption goods has not been considered a constraint.

In UNCTAD models, the determination of consumption expenditures was approached from the savings side. The data on savings in India estimated on the basis of net-worth approach is considered to be quite reliable. It also permits a disaggregation of savings in terms private, corporate and government savings. As such the approach from the savings side is quite viable and useful.

In functions surveyed above (Table 6.5) the consumption variable has been considered in real, nominal or monetized terms. Either aggregate private consumption or per capita private consumption were taken as the dependent variable. Among the explanatory variables, income or disposable income appears. A distinction between different propensities to consume in rural and urban sectors is sometimes captured by a ratio of agricultural to non-agricultural output. Population, real money balances and a time-trend have occasionally appeared as explanatory variables. These consumption functions have closely followed the Keynesian absolute income hypothesis. Indeed, income alone seems to explain more than 99% of variation in private consumption expenditure (see e.g. Mammen, Bhattacharya). Only Choudhry seems to utilise a lagged consumption variable on the right-hand side in one formulation and previous highest consumption in another. These were used respectively to capture the permanent and the relative income hypotheses.

A greater disaggregation and variety in specification can be observed in investment functions. These are surveyed in Table 6.6. Except UNCTAD-I model, all other models have a demand-for-investment function of one kind or another. In the UNCTAD-I model, this demand is indirectly determined in a production function which provides investment requirements for a given level of output.

Alternative schemes for disaggregating investment expenditure have been adopted in different models. In the main, it has been disaggregated between (i) private and government investment, (ii) investment in agriculture and industry, and (iii) investment in fixed assets, and inventories. An income variable occurs in various forms: disposable income, some index of profits or gross output. The long-term rate of interest was used as an explanatory variable in Marwah, Bhattacharya, Choudhry, Krishnamurty, Krishnamurty-Choudhry and Gupta. Only in Agarwala's model, an attempt is made to incorporate the influence of price expectations and government control. In this model, government control is introduced by defining a variable, GC, which is estimated as a percentage of capital issues sanctioned by the government against total amount for which applications were made. Capital stock appears directly into analysis in Choudhry and Gupta and indirectly via a lagged endogenous variable on the right-hand side in some formulations of Marwah, and in Mammen, Bhattacharya and Pandit. In Mammen's model, the influence of inflow of foreign capital is explicitly introduced. Mammen's specification of government investment demand takes account of a 'target' income variable. Thus, a policy parameter enters into specification in his and Agarwala's analysis mentioned above.

Table 6.6

Selected Investment Functions

Choudhry I

$$I' = \underset{(*)}{-3.05} + \underset{(*)}{.028}Y^{d'} + \underset{(*)}{0.02}Y' + 0.14R_{-1} + \underset{(**)}{0.13}M' + \underset{(**)}{0.098}K'_{-1} \qquad R^2 = .94$$

II

$$I' = 0.27 + \underset{(**)}{.024}Y^{d'} - \underset{(**)}{.15}M_{-1} + \underset{(*)}{.016}Y'_{-1} - .16R_{-1} + \underset{(**)}{.05}K'_{-1} \qquad R^2 = .94,\ VN = 1.13$$

III

$$I' = \underset{(**)}{-3.46} + 1.18\frac{(\Pi' + \Pi'_{-1})}{2} + .0001\frac{\Delta P}{P_{-1}} + \underset{(**)}{1.39}\frac{K'_{-1}}{Y'_{-1}} + .039M'_{-1} \qquad R^2 = .95$$

Krishnamurty

$$I_p^{me} = -2.15 + \underset{(*)}{.10}U_{-1} + \underset{(*)}{.026}\Pi_{-1} - 1.2R_{-1} \qquad \bar{R}^2 = 0.73$$

$$I_p^{c} = -5.63 + \underset{(*)}{.04}Y^d + \underset{(**)}{10.97}U_{-1} \qquad \bar{R}^2 = .90$$

Krishnamurty-Choudhry

$$I_p^{me} = -2.35 + .017Y^d + 9.93U_{-1} - 1.23R_1 \qquad \bar{R}^2 = .43$$

Marwah I & A-1

$$I = \underset{(**)}{14.69} + \underset{(*)}{0.057}\frac{(Y + Y_{-1})}{2} + 4.78R_1 \qquad \bar{R}^2 = .80$$

(for 1947-60 sub-sample)

$$I = \underset{(*)}{6.27} + \underset{(*)}{.089}Y + \underset{(**)}{.74}I_{-1} \qquad R^2 = .90$$

II

$$I_g = -11.35 + .125Y + .3I_{-1} + 1.20Z_1 \qquad \bar{R}^2 = .94,\ d = 1.21$$

Mammen

$$I'_p/P_i = 1.3312 + \underset{(**)}{1.0084}\Pi'/P_i + \underset{(**)}{0.64}\ 1FK'/P_m \qquad \bar{R}^2 = .75,\ d = 1.25$$

$$I'_g/P_i = 0.3991 + \underset{(**)}{.1002}(Y^* - Y) + \underset{(*)}{0.4580}(I'_g/P_i)_{-1} \qquad \bar{R}^2 = .91,\ d = 1.31$$

Agarwala

$$I_a = \underset{(**)}{159.71} + \underset{(**)}{.055}\frac{P_aY_a - T_a}{Pinv} - \underset{(**)}{300.1357}\frac{P_a}{Pinv} \qquad \bar{R}^2 = .76,\ VN = 1.36$$

$$I_i = GC\left[\underset{(**)}{-1219.723} + \underset{(**)}{.3633}\frac{P_iY_i - T_i}{Pinv} + \underset{(**)}{20.9242}\frac{\dot{P}}{P}.100\right]$$

$$\bar{R}^2 = .78,\ VN = 2.85$$

Table 6.6 (continued)

Selected Investment Functions

Bhattacharya	$I'_p = .655 + \underset{(**)}{.089Y'} - \underset{(**)}{.175R_1} + .189I'_{-1}$	$\bar{R}^2 = .74,\ d = 2.18$
Pandit	$I_p = -3.15 + \underset{(**)}{0.07Y} + \underset{(**)}{0.52I_{p-1}}$	$\bar{R}^2 = .88,\ d = 1.88$
Gupta	$I'_p = \underset{(**)}{20666.5} + \underset{(*)}{.1212Y'_{-1}} - \underset{(**)}{5460.72i_g} + \underset{(*)}{.8640(LI-LI_{-1})'}$	
	$+ \underset{(**)}{.5025BPD'} + \underset{(**)}{746.9t} - \underset{(*)}{.058K'_{p-1}}$	$R^2 = .93,\ d = 2.60$
UNCTAD-LINK	$ICF = \underset{(**)}{-218.55} + \underset{(**)}{7.1421Y_{na}} + \underset{(**)}{0.2957}\ \frac{EF'}{P}$	$\bar{R}^2 = .91,\ d = 2.49$
	$H = -93.8 + \underset{(*)}{2.0095Y'}$	$\bar{R}^2 = .34,\ d = 1.48$

Notes: (i) Z_1 = dummy variable, 1 for post-1950 years, zero elsewhere;

(ii) in Bhattacharya's model monetized levels of relevant variables are used;

(iii) GC = percentage of capital issues sanctioned to total amount applied for;

(iv) Y* = target income.

In some cases, either one-year or half-year lags have been utilised for some variables, e.g. we have $\frac{Y + Y_{-1}}{2}$ in Marwah and $\frac{\Pi + \Pi_{-1}}{2}$ in Choudhry to incorporate half year lags. Although lag structures have not been fully explored, one encounters quite a variety of formulations and choice of variables for the investment functions.

Demand for money functions are surveyed in Table 6.7. Not all models have a money-demand function. A decomposition of this demand has been attempted only in a few models, e.g. Mammen, Gupta and UNCTAD-LINK. Income and short-term rate of interest appear in almost all functions. The specifications closely follow the Keynesian hypothesis. For time-deposits a longer-term rate of interest has been used. The influence of price-expectations has been incorporated in Choudhry and Krishnamurty-Choudhry. Only in Bhattacharya's model a wealth effect is introduced, with the help of a net worth (NW) variable which is defined as the sum of real monetized capital stock, money-supply, time-deposits and government securities held by public.

Table 6.7

Demand for Money: Selected Functions

Choudhry I	$M' = -0.028 + \underset{(**)}{0.16Y'} - \underset{(*)}{0.07}\frac{\Delta P}{P_{-1}} - 0.07R_{-1} + 0.048P^{s}$	$R^2 = .92$
III	$M' = \underset{(**)}{-3.91} + \underset{(**)}{0.17Y'} - \underset{(*)}{0.058}\frac{\Delta P}{P_{-1}} - 0.052P^{s}$	$R^2 = .92$
Krishnamurty-Choudhry	$M' = -2.35 + \underset{(**)}{.22Y} - 0.11R + 2.15\frac{P}{P_{-1}}$	$\bar{R}^2 = .76$
Mammen -M	$C' = 1.64 + \underset{(**)}{.1065Y'} = \underset{(**)}{3.0\ 96}\log \dot{R} + \underset{(**)}{.5818t}$	$\bar{R}^2 = .99$, $d = 1.45$
	$D' = .4007 + \underset{(**)}{.0551Y'} - \underset{(*)}{0.4073}\log R$	$\bar{R}^2 = .93$, $d = 1.15$
	$T' = -4.5393 + \underset{(**)}{.1199Y'} - \underset{(**)}{1.9075}R_1 + \underset{(**)}{1.3283}R_{12}$	$\bar{R}^2 = .99$
Bhattacharya	$M' = .472 + \underset{(**)}{.104Y^{d'}} - \underset{(**)}{.082R} + \underset{(**)}{.264NW}$	$\bar{R}^2 = .995$, $d = 1.262$
Gupta	$C'_p = \underset{(**)}{4081.07} + \underset{(**)}{.0924Y'} - \underset{(**)}{1336.24}i_T + 644.60t$	$R^2 = .98$, $d = 1.86$
	$D'_p = \underset{(**)}{-2005.80} + \underset{(**)}{.2397}Y'_{na} - \underset{(**)}{1395.89}i_1$	$R^2 = .98$, $d = .77$
	$T'_p = -287.37 + \underset{(**)}{.2599}Y'_{na} + \underset{(*)}{863.74}i_T - \underset{(**)}{2135.56}i_g$	
	$-\underset{(**)}{1457.54}i_1 + \underset{(**)}{300.62t}$	$R^2 = .99$, $d = 1.92$
UNCTAD-LINK	$C' - C'_{-1} = 0.4189\,GD' + \underset{(**)}{58.3337}$	$R^2 = .394$, $d = 1.50$
	$(M'/_p) = \underset{(**)}{.7605}\ln Y - \underset{(*)}{0.2749}\ln R_{g_1} + \underset{(**)}{.5563}\ln\left(\frac{M'}{P}\right)_{-1}$	
	$\underset{(**)}{-1.8225}$	$\bar{R}^2 = .92$, $d = 1.44$
	$T' = \underset{(**)}{6.6866}Y'_{na} + \underset{(**)}{238.9561}R_{12} - \underset{(**)}{361.6561}R_{g1}$	
	$+ \underset{(**)}{.6936}T'_{-1} + \underset{(**)}{551.4968}$	$\bar{R}^2 = .99$, $d = 1.70$

Notes: (i) C_p, D_p, T_p refer respectively to private non-bank demand for currency, demand deposits and time-deposits;

(ii) in Bhattacharya's equation, Y^d and NW are at their monetized levels.

Selected import functions are tabulated in Table 6:8. Whereas Narasimham, Mammen, Pandit and Gupta have an aggregate import function, other models have tried some sort of decomposition. In Bhattacharya's model imports are treated as part of autonomous expenditure. The decomposition of imports has been in terms of (i) food and non-food imports; (ii) capital and consumer goods imports; (iii) food, machinery and 'other' imports, and in Marwah II, (iv) imports have been divided into seven categories relating to S.I.T.C. Demand for import has been specified primarily as a function of an income or investment variable. A constraint is sometimes recognised in terms of the availability of foreign exchange reserves or current inflow of foreign exchange in the form of capital or aid. A terms of trade variable has also sometimes been incorporated. A price variable is there in Mammen; a relative price variable in Pandit and UNCTAD-LINK. In Choudhry's model, prices appear in a deviation form. In UNCTAD-I, the equation for non-food imports is interesting. The operative equation in any one year is one of two formulations: in one case a foreign exchange constraint is operative, and in the other case, it is not.

Another set of functions surveyed here are production functions. Although only a few models contain production functions, the importance of introducing supply side influences has generally been recognised, and an attempt to introduce these via production functions has consistently been made in at least the more recent of the models surveyed here. Basically, either an aggregate production function or a disaggregation between agricultural and non-agricultural output has been attempted. Capital stock, sometimes distinguished between agricultural and non-agricultural sectors, has been taken into account. Capital has been considered to be the primary constraint on production. Only Agarwala and Gupta try log-linear fit thus using Cobb-Douglas type of formulations. Others have employed a straightforward linear fit.

Table 6.8

Selected Import Functions

Choudhry I	$IM' = -0.20 + \underset{(**)}{.057Y'} - .012(P-P_f) + .08\,FE'$	$R^2 = .82$
II & III	$IM'_f = 16.99 - \underset{(**)}{1.16Y'_a} + .004A' - .0015W$	
II	$IM'_m = .0074 + \underset{(**)}{.053I'_p} - .0001A' - .0005W$	$R^2 = .27$ VN = 2.08
	$(IM-IM_f-IM_m)' = .56 + \underset{(**)}{.097\,\alpha Y'} - \underset{(**)}{.026(P-P_f)} + \underset{(**)}{.024W}$	$R^2 = .60$ VN = 1.22
Krishnamurty	$IM_p = -21.03 + \underset{(**)}{.18\,Y^d} + \underset{(**)}{.96\,FE}$	$R^2 = .70$
Krishnamurty-Choudhry	$(IM_k+IM_R) = -.26 + \underset{(**)}{.73}(I_p^{me} + I_p^c + I_g^{me} + I_g^c) + .11\,\frac{(FE+A)'}{P_m}$	$R^2 = .72$
Mammen	$IM'/P_m = 6.34 + \underset{(**)}{.0259Y} + 1.0316\,\frac{IFK'}{P_m} - \underset{(**)}{.232}\,P_m$	$\bar{R}^2 = .95$ d = 1.57
Agarwala	$IM_k = \underset{(*)}{59.96} + \underset{(**)}{.2778}\,I_i$	$\bar{R}^2 = .68$ VN = 1.75
	$\frac{IM_c}{Y_{na}} = .055 + \underset{(**)}{.00227}\,t$	$\bar{R}^2 = .68$ VN = 2.50
UNCTAD I	$IM_{of} = \text{Minimum}\,(IM^1_{of}, IM^2_{of})$ where	
	$IM^1_{of} = \underset{(*)}{-468.17} + \underset{(**)}{.097Y}$	$\bar{R}^2 = .613$ d = 2.138
	$IM^2_{of} = \underset{(**)}{-1023.4} + \underset{(**)}{.664Y} + \underset{(**)}{1.567}\,X.R.\frac{P_x}{P_m}$	$\bar{R}^2 = .71$ d = 2.49
Gupta	$IM' = 159.90 + \underset{(**)}{.0808Y^{d'}} + .074\,IM'_{-1}$	$\bar{R}^2 = .91$ d = 1.74
UNCTAD-LINK	$IM_f = \underset{(**)}{9.557} - \underset{(**)}{.078}\,QAF + \underset{(**)}{.1476}Y_{na} - .0639\,\frac{P_{mf}}{P_w}$	$\bar{R}^2 = .76$ d = 1.06
	$IM_{of} = 30.9 + \underset{(**)}{2.0025}Y_{na} - \underset{(**)}{.0056}\,P_{mnf} + 0.1964\left(\frac{FE'}{P_m}\right)_{-1}$	$\bar{R}^2 = .38$ d = 1.92

Notes: α = coefficient of liberality defined to vary between 0.25 and 1; W = % decline in shipping during War, 1940-42 average over 1937-39 average; R = official exchange rate; QAF = index of food production.

Table 6.9

Selected Production Functions

Choudhry I	$Y = \underset{(**)}{21.32} + \underset{(*)}{.087K} + .031N$	$R^2 = .67$
II & III	$Y_a = \underset{(**)}{18.96} - \underset{(**)}{.016N_a} - \underset{(**)}{.29}\,R_1$	$R^2 = .75$, VN = 1.46
	$Y_{na} = \underset{(*)}{7.76} + .08K + .06\,N_i + .11\,t$	$R^2 = .98$, VN = .46
Krishnamurty	$Y_a = -63.71 + \underset{(**)}{.16L} + \underset{(*)}{3.30}\,L/L_i(\%)$	$R^2 = .83$
	$Y_{na} = 26.77 + \underset{(**)}{.17}\left[\frac{K + K_{-1}}{2}\right]U + \underset{(*)}{.06N_i}$	$R^2 = .99$
Krishnamurty-Choudhry	$Y_a = -56.72 + \underset{(**)}{.50\,A} + \underset{(*)}{203.90}\,A_i/A$	$\overline{R}^2 = .90$
	$Y_{mm} = \underset{(*)}{.17N_i} + .18\left[95.8 + \left(\frac{CI + CI_{-1}}{2}\right)\right].U$	$\overline{R}^2 = .96$
Agarwala	$\ln Y_a = \underset{(**)}{2.914} + \underset{(**)}{.6543}\ln K_a - \underset{(*)}{.034}\ln R_1$	$\overline{R}^2 = .72$, VN = 1.86
	$\ln Y_{na} = \underset{(**)}{-.4407} + \underset{(**)}{.5599}\ln (K_{na})_{-\frac{1}{2}} + .4401\ln N_{na}$	$\overline{R}^2 = .87$, VN = .71
UNCTAD I	$Y = \underset{(**)}{10097.99} + \underset{(**)}{.3175}\,CI_{-1}$	$\overline{R}^2 = .97$, d = 1.64
	$Y_a = \underset{(**)}{2051.02} + .3274Y$	$\overline{R}^2 = .93$, d = 1.39
Gupta	$\ln Y_{na} = 1.43 + .9777\ln\left(\frac{K + K_{-1}}{2}\right) + .0078\,t$	$R^2 = .997$, d = .753
Marwah II	$Y = 76.7 + .27\,(U)\,K_{-1}$	$\overline{R}^2 = .91$, d = 1.23
UNCTAD-LINK	$Y_a = \underset{(**)}{14.36} + \underset{(**)}{.2579}\,QAF + \underset{(**)}{.1230}\,QANF$	$\overline{R}^2 = .997$, d = 1.21
	$Y_s = \underset{(**)}{12.09} + \underset{(**)}{.0044}\,KGD_{-1} + \underset{(**)}{.1388}\,Y$	$\overline{R}^2 = .996$, d = 1.15

Notes: CI = cumulative (net) investment, Y_s = output of services; Y_{mm} = output in mining, manufacturing and trade; QAF = index of food production; QANF = index of non-food production; KGD = cumulative government investment in departmental undertakings.

In the agricultural output functions, Agarwala and Choudhry introduce the influence of rainfall, whereas Krishnamurty and Krishnamurty-Choudhry make a distinction between acreage which is irrigated and that which is not. Labour or employment has also sometimes been taken as a constraint especially in the non-agricultural production functions. There are no output considerations in Bhattacharya, Pandit, Marwah I and Mammen models. In Narasimham's model, supply considerations are reflected in the price-equations.

A relatively more disaggregated treatment to output has been provided in Marwah II and UNCTAD-LINK. In Marwah II, apart from the aggregate output, there are also sectoral output variables for foodgrains, agricultural output, industrial output, manufactured goods, raw materials and semi manufactured goods. In UNCTAD-LINK aggregate output is divided between agricultural output, manufacturing output in corporate sector, manufacturing output in non-corporate sectors and the output of services. Within the agricultural output a distinction is made between foodgrains and non-foodgrains production. Selected production functions are tabulated in Table 6.9.

6.9 Summary

In this Chapter, the main trends in the macroeconometric model building activity for the Indian economy are explored. A summary of the main features of models built by twelve authors/institutions is provided in terms of their sample sizes, estimation techniques, sectoral emphasis etc.. A number of mainstream functions like those relating to consumption, investment, production etc. are brought together to provide an overview of the variety of specifications used and explored.

In the next three chapters we propose to consider available models individually in a somewhat greater detail followed by a critical review in Chapter 10.

Chapter 7

Models of the Indian Economy: I

In this chapter the main features of models by Narasimham, Choudhry, Krishnamurty, Krishnamurty-Choudhry and Marwah are considered. Except for Marwah II, these models were built in the late fifties and early sixties. They represent the first wave of macroeconometric models of the Indian economy.

7.1 Narasimham Model

Narasimham's (1956) model was intended primarily for policy analysis. The model contains 18 equations out of which 11 are stochastic. These were estimated with data for the period 1919-52. Variables were measured as deviations from 9-year moving averages. Relevant income-expenditure variables are all at current prices. The structure of the model is defined in the following equations. In this and subsequent models, variable names for endogenous variables are defined, in general, while specifying the equations relating to them.

7.1.1 Specification of the Model

(a) Stochastic Equations

1.	Private consumption	$U = f\left[(L^W + L^S), (F - T^f), (Z^{NC} - T^{NC})\right]$
2.	Private Investment	$V = f\left[(Z^{NC} - T^{NC})_{-\frac{1}{2}}, (Z^C - T^C)_{-\frac{1}{2}}\right]$
3.	Imports	$M = f\,(Y - T^f - T^{NC} - T^C)$
4.	Consumer's prices	$p^u = f(1, p^f, U')$
5.	Investment goods' prices	$p^v = f(1_{-\frac{1}{2}}, p^{mv}_{-1}, V'_{-1})$
6.	Farm prices	$p^f = f(f'_{-\frac{1}{2}}, p^W)$
7.	Profits of non-corporate enterprises	$Z^{NC} = f(U + U^g)$
8.	Profits of corporate enterprises	$Z^C = f(V + V^g)$

9. Demand for labour $a = f(U', V'_{-\frac{1}{2}})$

10. Wage rate $l = f(p^{v}, a/N, t)$

11. Direct taxes on non-corporate incomes $T^{NC} = f(Z^{NC}, Z^{NC}_{-1})$

(b) Definitions and Identities

12. Real income* $y = U' + V'$

13. Money income $Y = U + U^{g} + V + V^{g} - M + E - T$

14. Profits of non-corporate enterprises $Z^{NC} = Y - L^{s} - L^{w} + F - Z^{C}$

15. Prices of all output $p = 0.94\ p^{u} + 0.06\ p^{v}$

16. Money income $Y = 1.65\ y + 0.29\ p$

17. Farm income $F = 1.93\ f + 0.08\ p^{f}$

18. Non-farm wage-bill $L^{w} = 1.53\ a + 0.01\ l$

The endogenous variables of the system are, U, V, M, a, p^{u}, p^{v}, l, p^{f}, Z^{NC}, Z^{C}, T^{NC}, y, Y, p, U', V', L^{w} and F. In subsequent discussion, only 16 variables are taken as endogenous. Variables p^{f} and Z^{NC} listed above are taken as exogenous, and the equations relating to these are ultimately dropped.

The exogenous variables which are listed below are divided into two categories: instruments and data. The instruments are:
p^{f} = farm prices, U^{g} = govt. consumption of goods and services, V^{g} = govt.investment expenditure, T^{NC} = direct taxes on non-corporate profits, T^{f} = direct taxes on farm incomes, T = indirect taxes.

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* U' and V' respectively refer to volume of consumer and investment goods.

Other exogenous variables are:

p^w = price level in countries importing from India, p^m = price of imported materials for investment goods, E = exports, f = farm production, f' = available farm production (farm production minus exports), N = population, L^s = govt. salary bill, t = time trend.

The lagged endogenous variables are: V'_{-1}, Z^{NC}_{-1}, Z^C_{-1}, l_{-1}, T^{nc}_{-1}. There are some lagged exogenous variables as well. These are p^{mu}_{-1}, T^c_{-1} and f'_{-1}.

Half year lags are introduced by taking the average of current and last year's values.

The definitional equations are derived in the following manner. The general price level is taken as a weighted average of price of consumer and investment goods, the former having a weight of 94% and the latter a weight of 6%. This is on the basis that during 1919-52 it was found that on the average outputs of consumer and investment goods accounted respectively for 94 and 6 per cent of the total output. Money income Y, is related to its volume y and price-level p in another definitional equation. This is derived as follows. Since Y, y, p are all defined as deviations from their mean levels, $\overline{Y}$, $\overline{y}$, $\overline{p}$, one can relate their absolute values $\overline{\overline{Y}}$, $\overline{\overline{y}}$, $\overline{\overline{p}}$ as follows:

$$\overline{\overline{Y}} = 0.01\ \overline{\overline{y}} \times \overline{\overline{p}}$$

i.e. value is volume times price divided by 100, price being measured as a percentage of its base level. This is rewritten as

$$(\overline{Y} + Y) = 0.01\ (\overline{y} + y)\ (\overline{p} + p) \qquad \text{from which}$$

$$Y = 0.01\ (\overline{p}\ .\ y + \overline{y}\ .\ p)$$

ignoring the term p, y, being the product of deviations. Substituting mean values $\bar{p}$ and $\bar{y}$ for the period 1919-52, we get

$$Y = 1.65\ y + 0.29\ p$$

Similarly, the value of farm production is written as a function of its volume and price

$$F = 0.01\ (\bar{p}\ .\ f + \bar{f}\ .\ p^t)$$

Substituting mean values, we get

$$F = 1.93\ f + 0.08\ p^f$$

The non-farm labour income is determined by the wage rate l and volume of non-farm employment N. Thus,

$L^W = 0.01\ (\bar{l}\ .\ a + \bar{a}\ .\ l)$, and substituting mean values,

$$L^W = 1.53\ a + 0.01\ l.$$

7.1.2 Considerations in Specification

Disposable incomes are considered the primary determinants of consumption expenditure. Consumer prices were not considered important. Disposable incomes were divided into four categories: (i) non-farm labour incomes ($L^W + L^S$), (ii) disposable farm incomes ($F - T^f$), disposable non-corporate incomes ($Z^{NC} - T^{NG}$) and disposable corporate incomes ($Z^C - T^C$). The first category was directly taken as a measure of disposable income as taxes paid out of these were considered negligible. The last category of income was finally dropped as its coefficient consistently came out to be negative. It was assumed that the marginal propensity to consume out of corporate incomes was zero. For the first category, i.e. non-farm labour incomes, the m.p.c. came out to be greater than one. Hence, in the final estimate, it was given *a priori* a value of unity.

For the determination of private investment expenditure, the profit theory was chosen. Profits were defined as balance of national income after labour and farm incomes are paid out. Taxes were also taken out as disposable profits were considered more relevant. These were sub-divided into disposable corporate and non-corporate profits so that a distinction could be made between the marginal propensities to invest from the two categories of profits. After testing, a lag of six months was introduced in both the explanatory variables.

Disposable national income was used as the explanatory variable in the import function. A terms-of-trade variable was not used on the ground that imports were predominantly of consumer durables and investment goods which could not be produced at home.

There are four price equations in the model, viz., p^u, p^v, p^f and p. Consumer prices (p^u) are explained in terms of the wage-rate (l), which is a direct cost element, the farm prices (p^f), which represents the cost of raw materials, and the volume of production of consumer goods and services (U'). Thus, all these variables represent supply side considerations.

Investment prices (p^V) are made a function of the wage rate ($l_{-\frac{1}{2}}$), import price of investment goods (p^{mv}_{-1}) and the output of investment goods (V'_{-1}) with lags as indicated. A half-year lag in the wage rate is incorporated because employment in this sector is generally on a contractual and salary basis. A one year lag in import prices is used because of the long duration of the production process. A lag, greater than the lag for wage rate is chosen on the ground that materials are bought before labour is employed. A lag of one year in the output of investment goods is used on the assumption that there is a lag in producers' reactions to changing trade.

The equation for farm prices is made to depend on lagged farm output ($f'_{-\frac{1}{2}}$) and foreign prices (p^W). p^W refers to the price-level in countries which import farm products and the hypothesis is that any increase in this external price leads to an increase in domestic farm prices. Both f' and p^W are exogenous variables, and thus, farm prices are really determined outside the main model. Later, it was treated as exogenous in the model.

In order to explain the profit variables in the model, both corporate (Z^C) and non-corporate (Z^{NC}) demand factors were found to be more important than the cost factors. Thus the total demand for investment goods ($U + U^g$), and consumer goods ($V + V^g$) were used for both the profit variables. The former was found significant for Z^C and the latter for Z^{NC}.

Labour demand is estimated from the employment output relationship. Output of consumer goods and investment goods were used as separate explanatory variables, the latter with an *a priori* lag of half a year. It is supposed that variation in output is in advance of variation in employment in this industry. In determining the wage rate, the cost of living index (p^u), the proportion of population employed ($^a/_N$) and a time-trend (t) were used. It was assumed that the higher the prices of consumer goods, the higher will be the pressure on increasing the wage rate. Similarly, the higher the proportion of population employed, the greater is the power of labour to bargain for higher wages. A trend was used to represent long-run influences such as changes in population, habits and productivity.

7.1.3 Estimates of Stochastic Equations

Equations were estimated by OLS. These are given below. Figures in brackets refer to standard errors for the coefficients.

1. $U = (L^W + L^S) + \underset{(0.08)}{0.96} (F - T^f) + \underset{(0.05)}{0.85} (Z^{NC} - T^{NC}) - 0.12 \qquad R = .99$

2. $V = \underset{(0.0075)}{0.0228} (Z^{NC} - T^{NC})_{-\frac{1}{2}} + \underset{(0.103)}{1.230} (Z^C - T^C)_{-\frac{1}{2}} - 3.32 \qquad R = .93$

3. $M = \underset{(0.02)}{0.11} (Y - T^f - T^{NC} - T^C) + 4.32 \qquad R = .76$

4. $p^u = 0.20\ l + \underset{(0.08)}{0.62}\ p^f + \underset{(2.28)}{6.42}\ U' + 1.62 \qquad R = .88$

5. $p^v = 0.16\ l_{-\frac{1}{2}} + \underset{(0.10)}{0.35}\ p^{mv}_{-1} + \underset{(16.40)}{46.40}\ V'_{-1} - 0.46 \qquad R = .71$

6. $p^f = \underset{(10.20)}{-17.90}\ f'_{-\frac{1}{2}} + \underset{(0.38)}{2.72}\ p^W + 4.18 \qquad R = .91$

7. $Z^{NC} = \underset{(0.06)}{0.58} (U + U^g) + 0.28 \qquad R = .95$

8. $Z^C = \underset{(0.05)}{0.52} (V + V^g) + 0.04 \qquad R = .92$

9. $a = \underset{(0.01)}{0.03}\ U' + \underset{(0.06)}{0.20}\ V'_{-\frac{1}{2}} + 0.03 \qquad R = .81$

10. $l = \underset{(0.07)}{0.23}\ p^u + \underset{(0.25)}{0.84} (a/N) - \underset{(0.16)}{0.70} t - 2.32 \qquad R = .87$

11. $T = \underset{(0.006)}{0.022} Z^{NC} + \underset{(0.006)}{0.028}\ Z^{NC}_{-1} + 0.064 \qquad R = .85$

7.1.4 Framework of Policy Analysis

In order to attain short-term targets, Narasimham considers not only the adequate values of instruments but also their efficiency. This latter consideration is important when instruments exceed the number of targets. The policy problem is set up in the following manner:

(i) target variables are chosen;

(ii) desirable levels of these targets are assigned for the succeeding year;

(iii) forecasts for these target variables are generated from the model assuming an unchanged economic policy by holding all instruments at their old levels;

(iv) the deviations between these forecasts and most desirable levels of target variables are taken to represent the given changes that have to be brought about by the use of policy instruments.

Once these deviations are estimated, the model is solved to yield adequate values of policy instruments given the most desired target levels. For this purpose, however, the general model is reduced to a decision model so that relationships only between targets and instruments are shown after eliminating all other variables.

Thus, national product (y), price-level (p) and deficit in balance of payments (D = M - E) were considered target variables. The instrument variables were p^f, U^g, V^g, T, T^f, T^{NC} and T^C out of which first p^f, T and T^{NC} were chosen. The general system is solved and rewritten in the following form:

$$p^f = -1.66y + 1.66p$$

$$T = -0.86y - 0.92p + 6.55\ D$$

$$T = y + p - 9.10\ D$$

These estimates were obtained by putting variations in the date variables equal to zero and under other specified assumptions. This model is solved for p^f, T and T^{NC} for alternative values of y, p and D. Values of $\frac{\partial p^f}{\partial y}$, $\frac{\partial T}{\partial y}$ etc. are taken to indicate relative efficiency of instruments. Other alternative set-ups were also tried in a similar manner.

7.2 Models by Choudhry

Choudhry (1963) constructs four alternative models within the same analytical framework. These models may be denoted as Choudhry I, II, III and Revised-III (R-III). The reduced form of the last model is used for policy analysis.

Equations are estimated by LIML and OLS for the sample period 1930-55. OLS estimates are used for comparisons. Except for output variables, all components of income and expenditure are at current prices unless otherwise indicated.

7.2.1 Choudhry I

This is a small model of 10 equations out of which 7 are stochastic. The equations are specified as below.

(a) Stochastic Equations

1.	Price level	$P = f(Y, O)$
2.	Real Income	$O = f(K, N)$
3.	Private consumption expenditure	$C = f(Y^d, L, NP, C_{-1})$
4.	Investment expenditure	$I = f(Y^d, Y_{-1}, iL_{-1}, L, K_{-1})$
5.	Imports	$M = f(Y, (P-P_f), E)$
6.	Demand for Liquidity	$L = f(Y, \frac{\Delta P}{P_{-1}}, iL_{-1}, P^s)$
7.	Exports	$X = f(Y^W, (P - P_f))$

(b) Definitions and Identities

8.	National Income	Y	$= C + I + G + X - M - T^{ind}$
9.	Disposable income	Y^d	$= Y + T^r - T_y - S^b$
10.	Real capital stock	K	$= K_{-1} + I/P$

The exogenous variables (11) are the following:

NP = population, iL_{-1} = lagged long-term rate of interest, E_f = foreign exchange reserves, P_f = foreign prices, P^s = prices of industrial securities, Y^w = world income, $G - T^{ind}$ = government expenditure net of indirect taxes, T_y = direct taxes, T^r = transfer payments, S^b = business savings, N = total employment.

The lagged endogenous variables are C_{-1}, Y_{-1}, K_{-1}, and P_{-1}.

7.2.2 Choudhry II

In model II, certain disaggregations are incorporated. Aggregate output is divided between agricultural and non-agricultural outputs; imports are divided into three categories: (i) food, (ii) machinery, and (iii) others; and exports are divided into four categories. Other main changes relate to the specification of consumption and investment functions. The equations, now 20 in number, are specified as below:

(a) Stochastic Equations

1.	General price level	P	$= f(O,Y)$
2.	Agriculture Output	O_1	$= f(N_1,R_1)$
3.	Non-agricultural output	O_2	$= f(K,N_2,t)$
4.	Private consumption expenditure	C	$= f(L,Y^d,C_{-1},NP)$
	(alternate equation)	C	$= f(L,Y^d,C_0)$

5./

5. Private Investment expenditure $I_p = f(L, Y^d, Y_{-1}, iL_{-1}, K_{-1})$

6. Food Imports $M_1 = f(O_1, A, W)$

7. Imports of machinery $M_2 = f(I_p, A, W)$

8. Other imports $M_3 = f\left[(P-P_f), \alpha Y, W\right]$

9. Demand for liquidity $L = f(Y, \frac{P-P_{-1}}{P_{-1}}, iL_{-1}, P^s)$

10. Tea exports $X_1 = f(t, P_{X1}, Y^w)$

11. Jute exports $X_2 = f(t, Y^w)$

12. Cotton exports $X_3 = f(t, Y^w)$

13. Other exports $X_4 = f\left[(P-P_f), t, Y^w\right]$

(b) Definitions and Identities

14. Total real output $O = O_1 + O_2$

15. National Income $Y = C + I + G + X - M - T^{ind}$

16. Real capital stock $K = K_{-1} + I/P$

17. Disposable income $Y^d = Y + T^r - T_y - S^b$

18. Total investment $I = I_p + I_g$

19. Total imports $M = M_1 + M_2 + M_3$

20. Total exports $X = X_1 + X_2 + X_3 + X_4$

The exogenous variables which are common to Model I are iL_{-1}, P_f, P^s, Y^w, T^{ind}, T_y, T_r, S^b and NP (in one formulation of consumption function). New exogenous variables are given below:

N_1 = current agricultural employment, R_1 = % deviation from 'ideal' rainfall, N_2 = current employment in non-agricultural section, t = time-trend, c_0 = previous highest consumption (in the alternative formulation of the consumption function), A = foreign aid, W = incidence of War on shipping measured by % decline in ships docking at Indian ports over 1937-39 (non-zero values for this variable are used only for the period 1940-46), α = coefficient of import liberalization assumed to vary between 0.25 and 1.00, P_{X1} = index of tea prices and I_g = government investment.

The lagged endogenous variables are C_{-1}, Y_{-1}, K_{-1} and P_{-1}.

7.2.3 Considerations in Specification

In specifying agricultural output in model II capital stock is not used as in the general production function in model I and non-agricultural production function in model II. This is because of non-availability of adequate data for capital stock in Indian agriculture. Also, it was assumed that capital has not changed in this sector significantly. However, the percentage deviation from an 'ideal' rainfall is taken as an explanatory variable. Thus, too much rain has been considered as bad as too little.

Among alternative formulations of private consumption expenditure, in one case a lagged endogenous variable C_{-1} is tried, and in another, the highest previous consumption (C_0). The former was used in view of the permanent income hypothesis and the latter for the relative income hypotheses. In model I, investment referred to total investment, whereas in model II, it refers to private investment expenditure. Disposable income, long-rate of interest, liquidity, lagged income and lagged capital stock enter as explanatory variable in the investment equations.

Imports and exports are disaggregated in model II. Food imports are made to depend on agricultural output, foreign aid and the incidence of war on shipping. It is expected that the higher the domestic output, the lower the level of foreign aid and the lower the shipping, the lower will be the level of imports. Imports for machinery depend on a similar set of factors. However, rather than agricultural output, total investment demand is used here. For explaining 'other' imports, the deviation between domestic and foreign prices $(P - P_f)$ is used to incorporate a competitiveness factor. The higher is this deviation, the greater is likely to be the demand for imports. Another important variable is αY. 'α' is an index of import liberalization. It is itself estimated as a function of the deviation actual and desired foreign exchange reserves $(E_f - E^*_f)$. The latter are seen to have two components: (i) a trend value of foreign exchange (E^*_{f1}) and (ii) the expected adverse balance in two subsequent years (E^*_{f2}). E^*_{f1} is calculated simply by regressing E_f on time. Thus, its estimate is given by,

$$\hat{E}_{f1} = -2.781 + .6077\ t \qquad (R = .72)$$

E^*_{f2} was defined as $\frac{M-X}{Y} \cdot (Y^* + Y^*_{-1})$. $\left(\frac{M-Y}{Y}\right)$ was calculated for each quinquennium. Thus,

$\frac{M-X}{Y}$ (1930-35) = -.023, (36-40) = -.02, (41-45) = -.003, (46-50) = .011, and (51-55) = +.07. Y^* was estimated by regressing current national income on past 5 national incomes. The estimated equation was

$$Y^* = 2.94 + 1.31Y_{-1} - .36Y_{-2} - .19Y_{-3} + .61Y_{-4} - .38Y_{-5}$$

From these $E_f - E_f^*$ was calculated. It had a maximum range of variation between -8.82 to 10.37. It was assumed that when $E_f - E_f^* = 10.37$, $\alpha = 1$ and $E_f - E_f^* = -8.82$, $\alpha = .25$. The equation,

$\alpha = -.25 + .0308\ (E_f - E_f^*)$ was used for generating the values of α over time.

In the export equations, world income and a time-trend are common explanatory variables to all export categories. A price variable is used in two equations: P_{X1} as an index of tea prices in the equation for tea exports, and $(P - P_f)$ the deviation between domestic and foreign prices in the equation for other exports. For jute and cotton prices, the price effects were not considered important.

7.2.4 Estimates of Stochastic Equations in Choudhry I and II†

Choudhry I

1. $$P = \underset{}{343.05} + \underset{(**)}{4.39}\ Y - \underset{(**)}{13.03}\ O \qquad R^2 = .965,\ \overline{S} = 25.3$$

2. $$O = \underset{(**)}{21.32} + \underset{(*)}{.087}\ K + .031\ N \qquad R^2 = .67,\ \overline{S} = 1.51$$

3. $$C = 13.41 + \underset{(**)}{.92}\ Y^d + \underset{(*)}{.074}\ L - .14\ NP + .024\ C_{-1} \qquad R^2 = .99,\ \overline{S} = 8.30$$

4. $$I = \underset{(*)}{-3.05} + \underset{(*)}{.028}\ Y^d + \underset{(*)}{.02}\ Y_{-1} + .14\ iL_{-1} + \underset{(**)}{.13}L + \underset{(**)}{.098}\ K_{-1}$$
$$R^2 = .94,\ \overline{S} = .58$$

5. $$M = -.20 + \underset{(**)}{.057}\ Y - \underset{(*)}{.012}\ (P-P_f) + .08\ E \qquad R^2 = .82\quad \overline{S} = 1.05$$

6. $$L = -.028 + \underset{(**)}{.16}\ Y - \underset{(*)}{.07}\ \frac{P-P_{-1}}{P_{-1}} - .078\ iL_{-1} + \underset{(**)}{.048}\ P^S \qquad R^2 = .92,\ \overline{S} = 2.28$$

7. $$X = \underset{(**)}{-1.27} + \underset{(**)}{.07}\ Y^W + \underset{(**)}{.0016}\ (P-P_f) \qquad R^2 = .87,\ \overline{S} = .61$$

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† The expressions in paranthesis relate to t-values. ** indicates a t-value greater than 2. * indicates a t-value between 1 and 2. In the absence of a *, a k-value lower than 1 is indicated. $\overline{S}$ refers to the standard error of estimate.

Choudhry II

1. $P = 343.05 + 4.39\,Y - 13.03\,O$ $\qquad R^2 = .965$

2. $O_1 = \underset{(**)}{18.96} - \underset{(**)}{.016}\,N_1 - \underset{(**)}{.29}\,R_1$ $\qquad R^2 = .75,\ VN = 1.46$

3. $O_2 = \underset{(*)}{7.76} + .08\,K + .06\,N_2 + .11\,t$ $\qquad R^2 = .89,\ VN = .46$

4. $C = \underset{(*)}{-.94} + \underset{(**)}{.88}\,Y^d + \underset{(*)}{.17}\,L + \underset{(*)}{.05}\,C_0$ $\qquad R^2 = .99,\ VN = 1.02$

5. $I_P = .27 + \underset{(**)}{.024}\,Y^d - \underset{(**)}{.15}\,L + \underset{(*)}{.016}Y_{-1} - .16iL_{-1} + \underset{(**)}{.05}\,K_{-1}$
$R^2 = .94,\ VN = 1.13$

6. $M_1 = 16.99 + .004\,A - \underset{(**)}{1.16}\,O_1 - .0015\,W$ $\qquad$ (R^2 etc. not given)

7. $M_2 = .0074 + \underset{(**)}{.053}\,I_P - .0001\,A - .0005\,W$ $\qquad R^2 = .27,\ VN = 2.08$

8. $M_3 = .56 + \underset{(**)}{.097}\alpha Y - \underset{(**)}{.026}(P-P_f) - \underset{(**)}{.024}\,W$ $\qquad R^2 = .60,\ VN = 1.22$

9. $L = .70 + \underset{(**)}{.16}\,Y - \underset{(*)}{.073}\,\dfrac{P-P_{-1}}{P_{-1}} - \underset{(*)}{.91}\,iL_{-1} + \underset{(**)}{.047}\,P^s$ $\qquad R^2 = .92,\ VN = .60$

10. $X_1 = \underset{(**)}{-.42} + \underset{(**)}{.011}\,Y^W + \underset{(**)}{.0011}P_{X1} - .0045\,t$ $\qquad R^2 = .92,\ VN = 1.79$

11. $X_2 = \underset{(*)}{-.56} + \underset{(*)}{.017}\,Y^W + .014\,t$ $\qquad R^2 = .65,\ VN = 1.89$

12. $X_3 = \underset{(**)}{-0.33} + \underset{(*)}{.0065}\,Y^W + \underset{(*)}{.014}\,t$ $\qquad R^2 = .71,\ VN = 1.56$

13. $X_4 = \underset{(*)}{-.46} + \underset{(**)}{.055}\,Y^W + \underset{(*)}{.0026}(P-P_f) - .097\,t$ $\qquad R^2 = .64,\ VN = 1.23$

It will be observed that a relatively lower R^2 is obtained for equations for real output in model I, and for equations 7, 8, 11 and 13 relating respectively to two of the import and two of the export components in model II.

Except for X_4 ('other' exports), all other export equations belong to the recursive part of the model being explained by a time-trend and world-income, and in case of tea exports, also tea-prices. All these explanatory variables are exogenous to the model. There is some simultaneity in the equation for X_4 because of the variable P but its coefficient is not very significant, the estimated t-value being less than 2.

7.2.5 Choudhry III

In essence, Choudhry's model III is the same as his model II. The models differ in the following respects.

(i) Profits (Π) and non-profit income (non-Π) are added as additional endogenous variables.

(ii) Non-profit income is explained as a function of rate of change in prices ($\Delta P/P_{-1}$), agricultural employment (N_1) and non-agricultural employment (N_2). Thus,

$$\text{non-}\Pi = f\left(\frac{\Delta P}{P_{-1}}, N_1, N_2\right)$$

(iii) Profit income is determined by the identity

$$\Pi + \text{non } \Pi = Y$$

(iv) The equation for investment is written in a different form. Now $\frac{\Delta P}{P_{-1}}$ is introduced as an explanatory variable. Also, the average of current and lagged profits $\frac{(\Pi+\Pi_{-1})}{2}$ is used. The terms Y_{-1} and K_{-1}, which were used on the basis of acceleration principle are used differently. Now the capital-output ratio is used which is the inverse of capacity utilisation.

(v) The definition of M_2 - imports of machinery - is changed to include industrial raw materials which are taken away from the 'other' imports category.

(vi) In the equation of demand for liquidity, the interest-rate variable is dropped.

Now, the system has the following 22 endogenous variables: P, O_1, O_2, C, I_P, M_1, M_2, M_3, L, X_1, X_2, X_3, X_4, O, Y, K, Y^d, I, M, X, Π and non-Π.

The exogenous variables are P_f, α, N, R, N_2, t, C_0, A, W, P^s, Y^W, I_g, G, T^{ind}, T^r, T_x, S^b. Variables iL_{-1} and NP which had appeared earlier as exogenous variables are dropped. The lagged endogenous variables are L_{-1}, Y_{-1}, I_{-1}, P_{-1} and Π_{-1}.

7.2.6 Reduced-form Analysis of Choudhry III

Since there are a number of non-linearities involved in the structural equations, a reduced-form was obtained for a linearized version of the model. An analysis of the reduced-form showed a number of anomalies. It was found, for example, that as government expenditures increase, non-profit incomes fall. A positive change in foreign aid showed a negative effect on national income. An increase in agricultural employment produced generally bad results for the economy. An increase in non-agricultural employment also did not provide satisfactory results.

7.2.7 Revised-III Model

In view of these observations, it was proposed to revise model III. Now agricultural output (O_1) and the quantity of liquidity were assumed to be exogenous. The non-agricultural production function was re-estimated using OLS. The explanatory variables were capital stock (K) and an estimated employment index (N). The non-profit income was re-estimated with the new employment index

$$\text{non-}\Pi = f(N, P, P_{-1})$$

Thus, the equation was made linear by not using $(P-P_{-1})/P_{-1}$ as an explanatory variable.

The new employment index was constructed on the supposition that the available employment index is compiled largely from data on industrial employment and that agricultural and non-agricultural labour force ratio had remained roughly at 72:28. With this information it was possible to generate a distribution of employment index among the two sectors.

These changes helped remove some of the problems faced in model III. The revised model III has 20 equations in as many endogenous variables. The list of exogenous variable has two additions (L, O_1) while N_1 is dropped. In place of N_2 a new index N is used. P^S is also dropped as L is itself being treated as exogenous. In effect, the number of exogenous variables/is increased by one.

7.2.8 Estimates of Stochastic Equations in Choudhry III and R-III

Choudhry III

The model adopts most of the equation of Choudhry II. The following equations are substituted:

5a $$I_p = \underset{(**)}{-3.46} + 1.8\,\frac{\Pi+\Pi_{-1}}{2} - .0001\,\frac{(P-P_{-1})}{P_{-1}} + \underset{(**)}{1.39}\,\frac{K_{-1}}{Y_{-1}} - \underset{(*)}{.039}\,L_{-1} \qquad R^2 = .95$$

7a $$M_2 = \underset{(*)}{.12} + \underset{(**)}{.14}\,I + \underset{(**)}{.014}\,A - \underset{(**)}{.0078}\,W \qquad R^2 = .93,\ VN = 1.94$$

8a $$M_3 = \underset{(*)}{.61} + \underset{(**)}{.067}\,\alpha Y - \underset{(*)}{.0093}\,(P-P_f) - \underset{(**)}{.017}\,W \qquad R^2 = .73,\ VN = 1.53$$

9a $$L = \underset{(**)}{-3.91} + \underset{(**)}{.17}\,Y - \underset{(*)}{.058}\,\frac{(P-P_{-1})}{P_{-1}} - \underset{(**)}{1.052}\,P^s \qquad R^2 = .92$$

The following stochastic equation is added.

21 $$\text{Non-}\Pi = -50.88 + \underset{(*)}{.042}\,N_2 + \underset{(*)}{0.41}\,\frac{P-P_{-1}}{P_{-1}} \qquad R^2 = .89$$

This along with the identity

22 $$\Pi = Y - \text{Non-}\Pi$$

makes the system complete in 22 endogenous variables.

Choudhry R-III

The model adopts equations of Choudhry III except the following two equations.

$$3b \quad O_2 = 6.325 + \underset{(**)}{.1344}\,K + \underset{(**)}{.153}\,N \qquad R^2 = .89$$

$$21b \quad \text{Non-}\Pi = -37.4 + \underset{(**)}{.4788}\,N + \underset{(*)}{.0873}\,P + \underset{(*)}{.0964}P_{-1} \qquad R^2 = .96$$

Equations for agricultural output (eqn.2 in model II) and demand for liquidity (eqn.9a in model III) are dropped. These variables are now considered exogenous making the system complete in 20 endogenous variables.

7.2.9 Forecasting within the Sample-Period

Four variables, viz. P, O_2, I_P and Y were chosen for estimation within the sample period 1930-1955. Predictions, 26 in number, were generated with model III and revised model III, for changes in these varaibles. The performance as to correct prediction of the direction of change is summarised in the following table. The predicted magnitudes of changes were wide off the mark.

Table 7.1

Number of Direction of Changes Correctly Predicted: Choudhry-III & R-III

Variables	Model III	Revised Model III
	No. of Correctly Predicted Changes in Direction*	
P	16	18
O_2	19	19
I_P	14	13
Y	11	15

* Note: out of 26 predictions

The author himself acknowledges that the model is "somewhat disappointing as an effective tool for predicting changes in the important endogenous variables of the system."

Beyond-sample predictions were also generated for the period 1956-1959 for changes in four variables taken above. Although about 75% of the direction of changes were predicted accurately, the magnitudes of changes were again widely divergent from the realized changes.

7.3 Models by Krishnamurty

Krishnamurty's (1964) analysis also results in four models. The larger model with which he starts contains 21 endogenous variables. Later a revised version of this model in 19 endogenous variables is presented. Subsequently, he chooses two sub-models from this framework containing respectively 9 and 12 endogenous variables.

Krishnamurty's analysis is intended to be growth-oriented rather than for short-term policy analysis. The special features of his model are a disaggregated treatment to capital formation and an endogenous determination of population.

Models are estimated by 2SLS and OLS. In general, equations are estimated over the period 1948-1961. The demographic equations are estimated over a longer period. Since the sample period was limited, 2SLS estimates were obtained by using a subset of 7 predetermined varaibles in the first stage. The variables were chosen in terms of their importance on economic grounds and together they were able to explain about 95% of the variation in the endogenous variables. All relevant income and expenditure variables are measured at constant prices.

7.3.1 Specification of Krishnamurty I and Revised I

(a) Stochastic Equations

1.	Per capita consumption† (to determine total consumption)	$C/N = f(Y_p^d/N,\ Y_a/Y_i,\ t)$
2.	Private investment in machinery and equipment	$I_p^{me} = f(U_{-1},\ \Pi_{-1},\ iL_{-1})$
3.	Private investment in construction	$I_p^C = f(Y_p^d,\ U_{-1})$
4.	Inventory stock	$\sum_{i=1}^{t} (I_p^s)_i = f\left[(Y - I_p^s - I_g^s)\right]$

† Refers to the private sector.

5.	Agricultural Output	$Y_a = f(L, L_i/L)$
6.	Non-agricultural output (to determine non-agricultural employment, N_i)	$Y_i = f\left[\frac{(K+K_{-1})}{2} U, N_i\right]$
7.	Capacity output in non-agricultural sector	Y_i^C is obtained from equation 6, by setting U=1, and raising N_i by I.
8.	Depreciation in non-agricultural sector	$D_i = f\left[\frac{(K+K_{-1})}{2} . U\right]$
9.	Private imports	$M_p = f(Y^d, F)$
10.	Long-term rate of interest	$iL = f(i_s, iL_{-1})$
11.	Short-term rate of interest	$i_s = f(ER/RR, BR)$
12.	Index of industrial profits	$\Pi = f(W, U)$
13.	Index of wage earnings (money-wage rate)	$W = f(t)$
14.	Birth rate	$\log b = f\left[\log \frac{1}{4} \sum_{i=4}^{1} (Y/N)_{-i}, \log t_1, D_1\right]$
15.	Death rate	$\log d = f\left[\log \frac{1}{4} \sum_{i=4}^{1} (Y/N)_{-i}, GW, t_1, D_1\right]$

(b) Definitions and Identities

16.	Investment	$K-K_{-1} = I_p^{me} + I_g^{me} + I_p^C + I_g^C + I_g^{rp} - D_i = I$
17.	Non-agricultural income	$Y_i = Y - Y_a$
18.	Utilisation of capacity in non-agricultural sector	$U = 100(Y_i/Y_i^C)$

19. Personal disposable income $Y_p^d = Y + T - T_d$

20. Population $N = N_{-1}\,(1 + b - d)$

21. National income
$$Y = C + I_P^{me} + I_g^{me} + I_P^C + I_g^C + I_P^s + I_g^s + I_g^{rp} - D_i + G' + E - M_P - M_g - T_i + S.$$

The exogenous variables of the system are the following: t = time-trend, I_g^s = government investment in inventories, $(I_g^{me} + I_g^C + G')$ = sum of government investments in machinery and equipment and construction and expenditure other than investment, L = gross acreage sown, L_i = gross acreage irrigated, F = foreign exchange reserves at the beginning of period, ER/RR = ratio of excess reserves to required reserves, BR = bank-rate, t_1 = trend variable starting 1922; D_1 = dummy variable taking a value 0 for 1922-1947 and 1 for 1948-1961; GW = government welfare expenditure; I_g^{rp} = government expenditure on repair and maintenance, E = exports; T = national debt interest payments plus other small transfer items; T_d = direct taxes, M_g = government imports, T_i = indirect taxes, S = subsidies. There are six lagged endogenous variables.

When this system was estimated in the form specified above a number of unreasonable and unrealistic results were obtained. In particular, very high impact multipliers for certain exogenous variables were obtained. Hence a number of revisions were made. The consumption function, the function for private investment in machinery and equipment, the equations for inventory stocks and the non-agricultural production function which was used to determine non-agricultural employment were respecified. The new equations are given below.

Private consumption	$C = f(Y_p^d)$
Private investment in machinery and equipment	$I_p^{me} = f(U_{-1}, iL_{-1})$
Inventory-stock of the private sector	$\sum_{i=1}^{t} I_{p_i}^{s} = f\left[(Y-I_p^{s}-I_g^{s}), \sum_{i=1}^{t-1}(I_p^{s})_i\right]$
Non-agricultural employment	$N_i = f(Y_i)$

Equations for the industrial profits and wage rate were dropped to produce a system of 19 equations in the revised form to determine the following endogenous variables:

C, I_p^{me}, I_p^{C}, I_p^{s}, D, Y_i, Y_a, Y, N_i, iL, i_s, M_p, b, d, N, Y_i^C, U, Y_p^d and K.

The one-year lagged endogenous variables are N_{-1}, K_{-1}, iL_{-1}, U_{-1} and Y_{-1} and $\sum_{i=1}^{t-1}(I_p^{s})_i$ apart from the ones in the demographic equations.

Lagged profits are now exogenous.

7.3.2 Considerations in Specification

As in most other models, a single consumption function was envisaged because of lack of disaggregated data over groups of individuals or commodities. However, unlike many other models, the consumption variable is defined in per capita terms. Per capita real disposable income is taken to be its primary determinant. In model I, an attempt was made to distinguish between agricultural and non-agricultural incomes by introducing a ratio (Y_a/Y_i) variable. There is also a time trend in this model. Various forms and lag-structures were tried. Ultimately, however, in revised model I, the simplest of all consumption functions $\left[C = a + b.Y^d_p\right]$ was found most appropriate.

The major feature of Krishnamurty's study is a disaggregated treatment to private gross investment which is divided into investment in (i) machinery and other equipment, (ii) construction, and (iii) change in stocks. In explaining investment, the profit principle and the capacity version of the acceleration principle were followed either jointly or independently.

In the absence of any data as to the non-wage component of national income, the available index of industrial profits was used to capture the profit variable in the investment equation (I_p^{me}) in model I. No hypothesis regarding profits expectations is formed except that last year's profits are the best guide to current profits. Industrial utilization of capacity is captured by an index (U) which was computed on the basis of utilization rates of about 64 industries. A lag of one year is introduced. In addition, an interest rate (iL) is also used to incorporate long-term costs of borrowing funds.

For investment in 'construction', personal disposable income (Y^d_p) was used to explain residential constructions, whereas the capacity utilization variable was used to explain non-residential construction. The former variable alone explained about 86% of the variations in I_P^C.

The third component of investment is inventories. Here, rather than explaining 'change in stocks', it was chosen to explain 'level of stocks' because preliminary studies showed a very poor explanation for the former dependent variable. The initial level of stocks at the end of 1947-48 was assumed to be zero and the inventory investments in subsequent periods were cumulatively added to obtain the current level of stocks. Except for constant terms, this procedure does not affect the parameters of the linear functions. Levels of stocks were explained using the acceleration principle.

An agricultural production function is estimated. In view of the problems of lack of data on capital stock and employment in this sector as also the problem of disguised employment, the land variable was used for this function. The two explanatory variable are gross area sown and the proportion of gross area irrigated to gross area sown. Irrigated areas give some account of investment in agriculture. A time trend was also added. The non-agricultural output is determined as a residual $(Y-Y_a)$. There is a production function for this sector, but it is used to determine non-agricultural employment. Production in this sector is taken to be a function of capital stock utilized and employment in non-agricultural activities. An employment index is constructed to cover the latter variable on the basis of a few related employment series in this sector.

Two other functions of interest are imports and long-term rate of interest. Government imports are assumed exogenous, and private imports are explained by private disposable income and foreign exchange reserves. Long-term rate of interest is a distributed lag function of short-term interest rate. A Koyck type transformation is used resulting in a lagged dependent variable (iL_{-1}) on the right-hand side apart from i_s. After experimenting with a number of alternative short-term rates, the average of treasury bill rate and the 3-month deposit rate of scheduled banks was taken to be the best indicator of the short-term rate.

The unique feature of Krishnamurty's model is an endogenous demographic sector. This contains equations for birth rate (b_t) and death rate (d_t) and an identity to provide the current level of population

$$N_t = N_{t-1}(1 + b_t - d_t)$$

Birth rate is related to (i) the average of past four years' per capita incomes, and (ii) a time-trend. A fall in the birth rate is expected with a rise in either of the two variables. Death rate is a function of (i) the average per capita income variable, (ii) government expenditure on education and public health, and (iii) a time-trend. A negative sign was expected for the coefficients of all these variables. These equations were estimated over a longer period (1922-1961) than the rest of the model. Therefore, a dummy variable was introduced in both equations to take account of the partition in 1947.

7.3.3 Estimates of Stochastic Equations in Krishnamurty I and R-I

1. $$C/N = -17.54 + \underset{(**)}{.81}\, Y_p^d/N + \underset{(*)}{.54}\, Y_a/Y_i - \underset{(*)}{1.70}\, t \qquad R^2 = .85,\ \bar{S} = 3.65$$

(Revised version) $$C = 17.15 + \underset{(**)}{.75}\, Y_p^d \qquad R^2 = .98$$

2. $$I_p^{me} = -2.15 + \underset{(*)}{.10}\, U + .026\, \Pi_{-1} - \underset{(*)}{1.2}\, iL_{-1} \qquad R^2 = .73 \quad \bar{S} = .47$$

3. $$I_p^C = \underset{(*)}{-6.15} + .04\, Y_p^d + \underset{(**)}{.11}\, U_{-1} \qquad R^2 = .92 \quad \bar{S} = 1.36$$

4. $$\sum_{i=1}^{t} (I_p^s)_i = -30.18 + \underset{(*)}{.38}\, (Y - I_p^s - I_g^s) \qquad R^2 = .93 \quad \bar{S} = 1.36$$

5. $Y_a = -63.71 + \underset{(**)}{.16}\, L + \underset{(*)}{3.30}\, L_i/L$ $\qquad R^2 = .83 \quad \overline{S} = 1.94$

6. $Y_i = 26.77 + \underset{(**)}{.17}\, \frac{K+K_{-1}}{2}\, U + \underset{(*)}{.06}\, N_i$ $\qquad R^2 = .99 \quad \overline{S} = .57$

7. Y_i^c = obtained from equation 6, with $U=1$ and $N_i = (N_i)^I$

8. $D = 5.60 + \underset{(**)}{.007} \frac{(K+K_{-1})U}{2}$ $\qquad R^2 = .76 \quad \overline{S} = .18$

9. $M_p = -21.03 + \underset{(**)}{.18}\, Y_p^d + \underset{(**)}{.96}\, F$ $\qquad R^2 = .70 \quad \overline{S} = .76$

10. $iL = 1.45 + \underset{(**)}{.23}\, i_s + \underset{(**)}{.48}\, iL_{-1}$ $\qquad R^2 = .96 \quad \overline{S} = .08$

11. $i_s = -1.34 + \underset{(**)}{.04}\, ER/RR + \underset{(**)}{1.39}\, BR$ $\qquad R^2 = .95 \quad \overline{S} = .63$

12. $\Pi = -47.89 - \underset{(**)}{0.79}\, W + \underset{(**)}{2.46}\, U$ $\qquad R^2 = .74 \quad \overline{S} = 8.4$

13. $W = 100.90 + \underset{(**)}{2.93}\, t$ $\qquad R^2 = .56 \quad \overline{S} = .74$

14. $\log b = 4.62 - \underset{(**)}{1.21} \log \frac{1}{4} \sum_{i=4}^{1} (Y/N)_{-i} - \underset{(*)}{0.022} \log t_1 - \underset{(**)}{0.07}\, D_1$

$R^2 = .71 \quad \overline{S} = .02$

15. $\log d = 6.05 - \underset{(**)}{1.79} \log \frac{1}{4} \sum_{i=4}^{1} (Y/N)_{-i} - \underset{(**)}{0.10}\, GW - \underset{(**)}{0.04} \log t_1 - \underset{(**)}{.18}\, D_1$

$R^2 = .93 \quad \overline{S} = .03$

In the revised model, the following equations were substituted.

1a $C = 17.15 + \underset{(**)}{.75}\, Y_p^d$ $\qquad R^2 = .98$

2a $I_p^{me} = -3.64 + \underset{(**)}{.16}\, U_{-1} - \underset{(**)}{1.75}\, iL_{-1}$ $\qquad R^2 = .68$

4a $\sum_{i=1}^{t} (I_p^s)_i = -24.77 + .32\, (Y - I_p^s - I_g^s) + .17 \sum_{i=1}^{t-1} (I_p^s)_i$ $\qquad R^2 = .93$

6a $N_i = 12.90 + \underset{(**)}{2.15}\, Y_i$ $\qquad R^2 = .95$

Equations 14-15 were estimated over the sample period 1922-1961. Equations 5 and 9 were estimated respectively over the sample periods 1948-59 and 1949-6

7.3.4 Krishnamurty-II and -III

Simulations of revised model-I were not satisfactory inasmuch as they produced a declining trend in national income. The downswing was traced back to the dependence of private investment equations on the industrial utilization of capacity which declined sharply from 1961-62. This variable was therefore dropped in the equations of private investment in 'machinery and equipment' and in 'construction'. A sub-model containing 9 equations was created. Its specification is given below.

1. Private consumption $$C = f(Y_p^d)$$

2. Private investment in machinery and equipment $$\sum_{i=1}^{t} (I_p^{me})_i = f\left[Y_p^d, \sum_{i=1}^{t-1} (I_p^{me})_i\right]$$

3. Private investment in construction $$I_p^C = f(Y_p^d)$$

4. Private inventory-stock $$\sum_{i=1}^{t} (I_p^s)_i = f\left[(Y-I_p^s-I_g^s), \sum_{i=1}^{t-1} (I_p^s)_i\right]$$

5. Private imports $$M_p = f(Y_p^d, F)$$

6. Depreciation $$D = f\left[\left(\sum_{i=1}^{t} (I)_i + \sum_{i=1}^{t-1} (I)_i\right)/2\right]$$

7. Increment in capital stock (net investment) $$K_t - K_{t-1} = I_p^{me} + I_p^C + Z - D = I$$

8. National income $$Y = C + I_p^{me} + I_p^C + I_g^s + Z + G + E - M_p - M_g - T_i + S - D$$

9. Personal disposable income $$Y_p^d = Y + T - T_d$$

The exogenous variables of the system are:

I_g^s, F, T_i, T_d, G, E, M_g, T, Z (= government investment other than in stocks) and S (= subsidies). Other variables have the same meaning as in model-I.

The lagged endogenous variables are:

$$\sum_{i=1}^{t-1} (I_p^{me})_i \ , \quad \sum_{i=1}^{t-1} (I_p^{S})_i \ , \quad \sum_{i=1}^{t-1} (I)_i$$

Although this model is complete in itself, it did not contain any production function and was based purely on the expenditure approach. In specifying Krishnamurty-III, an agricultural production function from model-I was reintroduced. In addition, the private investment equation for 'machinery and equipment' is further revised. Now a stock-adjustment form is introduced. The gross capital-stock in this sector is treated as a function of non-agricultural output with a lag of one year along with a lagged dependent variable. The equation for private imports is also revised. They are now made a function of current non-agricultural output and the level of foreign exchange reserves at the beginning of the period. Income originating in the non-agricultural sector was considered more relevant than total income because the latter has a rural component which has little to do with imports. Further, an equation for demand for labour in the non-agricultural sector is also brought in. This is made a function of non-agricultural output and capital-stock so that features of labour-capital substitution may be incorporated in the analysis.

In model-III, there are 12 endogenous variables. These are: C, I_P^{me}, I_P^{C}, I_P^{s}, M_P, Y_P^d, D, K, Y, Y_i, Y_a, and N_i. The revised and additional equations compared to model-II are specified as below.

(i) <u>Revised equations</u>

2a Investment in machinery and equipment $\sum_{i=1}^{t} (I_P^{me})_i = f\left[(Y_i)_{-1}, \sum_{i=1}^{t-1} (I_P^{me})_i\right]$

5a Private imports $M_P = f(Y_i, F)$

(ii) <u>New equations</u>

10. Agricultural income $Y_a = f(L, L_i/L)$

11. Employment in non-agricultural sector $N_i = f\left[Y_i, \left(\sum_{i=1}^{t} I_i + \sum_{i=1}^{t-1} I_i\right)/2\right]$

12. Non-agricultural income $Y_i = Y - Y_a$

The exogenous variables in Kirhsnamurty-III are:

L, E, G, I_g^{s}, Z, F, L_i, T, T_d, T_i, S and M_g.

The lagged endogenous variables are:

$\sum_{i=1}^{t-1} (I_P^{me})_i$, $\sum_{i=1}^{t-1} (I)_i$ and $\sum_{i=1}^{t-1} (I_P^{s})_i$.

Simulations with OLS estimates of model-II and -III were performed under specified assumption about the exogenous variable. Model II was used to generate national income (Y) projections for the five years from 1961-62 to 1965-66. Model III was used to generate Y, Y_i and Y_a. The results are summarised in Table 7.2.

Table 7.2

National Income Projections with Krishnamurty-II and -III

Year	Model II	Model III		
	Y	Y	Y_i	Y_a
1961-62	127.51	132.26	71.39	60.87
1962-63	131.77	134.74	72.05	62.69
1963-64	136.29	135.23	71.32	64.57
1964-65	142.21	138.23	71.72	66.51
1965-66	148.18	144.17	75.67	68.50

Note: variables are at 1948-49 prices Rs.abja.

7.3.5 Estimates of Stochastic Equations in Krishnamurty-II and -III

OLS estimates for model-II are given below. The sample/period is 1948-61.

1. $$C = \underset{}{17.1469} + \underset{(.0316)}{.7509}\, Y^d_p \qquad R^2 = .98$$

2. $$\sum_{i=1}^{t} (I_P^{me})_i = -3.7178 + .0718\, Y^d_p + .9607 \sum_{i=1}^{t-1} (I_P^{me})_i \qquad R^2 = .99$$

3. $$I_P^C = -3.5127 + \underset{(.0127)}{.1052}\, Y^d_p \qquad R^2 \doteq .91$$

4. $$\sum_{i=1}^{t} (I_P^s)_i = -24.7683 + \underset{(.1493)}{.3166}\, (Y - I_P^s - I_g^s) + .1722 \sum_{i=1}^{t-1} (I_P^s)_i \qquad R^2 = .93$$

5. $$M_P = -22.6777 + \underset{(.0401)}{.1966}\, Y^d_p + \underset{(.2336)}{1.0233}\, F \qquad R^2 = .73$$

6. $$D = 6.0156 + .0078 \left[\left(\sum_{i=1}^{t} (I)_i + \sum_{i=1}^{t-1} (I)_i\right) \Big/ 2\right] \qquad R^2 = .80$$

Figures in brackets are standard errors of estimated coefficients.

For model-III, the revised equations were:

2a $$\sum_{i=1}^{t} (I_P^{me}) = -20.1432 + \underset{(.1832)}{.5437}\, (Y_i)_{-1} + \underset{(.1215)}{.6926} \sum_{i=1}^{t-1} (I_P^{me})_i \qquad R^2 = .99$$

5a $$M_P = -18.0892 + \underset{(.0678)}{.2988}\, Y_i + \underset{(.2419)}{.9373}\, F \qquad R^2 = .68$$

And, the new equations were:

10 $$Y_a = -63.7114 + \underset{(.0268)}{.1610}\, L + \underset{(2.0270)}{3.2995}\, (L_{i/L})\% \qquad R^2 = .83$$

11 $$N_i = 4.5361 + \underset{(1.4210)}{2.3434}\, Y_i - \underset{(.3236)}{.0439} \left[\left(\sum_{i=1}^{t} (I)_i + \sum_{i=1}^{t-1} (I_i)\right) \Big/ 2\right] \qquad R^2 = .94$$

These OLS estimates were first adjusted for the constant term on the basis of the residuals of the respective equations for the last three years of the sample period. The adjusted models were then simulated. Model-III was also estimated by 2SLS using a subset of seven predetermined variables in the first stage.

7.4 Krishnamurty-Choudhry Model

The efforts of Krishnamurty (1964) and Choudhry (1963) have been put together to produce a more comprehensive model in order to overcome some of their individual shortcomings.

It was recognised by the authors that their models had different sets of limitations. For example, in Choudhry's model, although aggregate demand and supply factors were considered, the former were all measured in nominal terms. This tended to exaggerate the multicollinearity among variables in the inflationary sample period. Further, his investment equation was not very satisfactory and his money-demand equation was almost outside the rest of this system. In Krishnamurty's model, on the other hand, there was no demand for money equation, nor an explanation for the price level. Since aggregate demand and supply were both measured in real terms, the determination of price-level fell outside the system. Although monetary policy was introduced in the system, it had very weak feedbacks. As such these models were complementary in nature. The combined model contains a theory of price-level, a fuller treatment of investment and money-demand, and an endogenous demographic sector. In the form in which it is finally presented, it contains 20 stochastic equations and 10 identities. There are 26 predetermined variables. The sample period was 1948-61.

Since the number of predetermined variables exceeded the number of observations, 2SLS estimation procedures were modified by using a set of principal components of the predetermined variables in the first stage.

7.4.1 Specification of the Model

(a) Stochastic Equations

1. Agricultural output $$O_{ag} = f(A_i/A,\ A)$$

2. Irrigated acreage $$A_i = f(CI_{-1})$$

3. Output in mining, manufacturing and trade (to determine employment (n) in this sector) $$O_{mm} = f\left(n,\ \frac{CI + CI_{-1}}{2},\ U\right)$$

4. Capacity utilization $$U = f\left[\frac{y + y_{-1}}{2},\ (M_1 + M_2)\right]$$

5. Residual output $$O_r = f(N)$$

6. Per capita private consumption $$\frac{C}{N} = f\left(\frac{y^d}{N},\ \frac{y - O_{ag}}{y},\ t\right)$$

7. Private investment in construction $$I_P^{C} = f(y^d,\ U_{-1})$$

8. Private investment in machinery & equipment $$I_P^{me} = f(i_L,\ y^d,\ U_{-1})$$

9. Level of private stocks $$CI_P^{s} = f\left[(y - I_P^{s} - I_g^{s}),\ (CI_P^{s})_{-1}\right]$$

10. Imports of capital goods and raw materials $$M_1 + M_2 = f\left[(I_P^{me} + I_g^{me} + I_P^{s} + I_g^{s}),\ (E_{XA} - FA)^{def}\right]$$

11. Imports of food and 'other' items $$M_3 = f(PL\ 480)$$

12. Depreciation $$D = f(CI_{-1},\ U_{-1})$$

13.	Direct taxes	$T^d = f(y)$
14.	Indirect taxes less subsidies	$T^{i-s} = f(y)$
15.	Birth-rate	$\ln b = f(\ln A(y/N)_{-1}, \ln t)$
16.	Death-rate	$\ln d = f(\ln A(y/N)_{-1}, \ln Gw, \ln t)$
17.	Price-level	$P_y = f(P_{y_{-1}}, \frac{O_{ag}}{y_{-1}})$
18.	Long-term rate of interest	$i_L = f(i_s, i_{L_{-1}})$
19.	(Supply fn. for) demand deposits	$\frac{DD+DD_{-1}}{2} = f(i_s/i_B, RES)$
20.	Demand for money (to determine i_s)	$L = f(y, i_s, P_y/P_{y_{-1}})$

(b) <u>Identities and Definitions</u>

21.	Net investment	$I\text{-}D = I_p^C + I_g^C + I_p^{me} + I_g^{me} + G'\text{-}D$
22.	Net national product at constant prices (to determine O_{mm})	$y = O_{ag} + O_{mm} + O_r$
23.	Total private consumption	$C = (\frac{C}{N}).N$
24.	Private investment in inventories	$I_p^s = CI_p^s - (CI_p^s)_{-1}$
25.	Personal disposable income	$y^d = y + T - T^d$

26. National income $y = C + (I-D) + I_g^s + I_P^s + X-M+G-T^{i-s}$

27. Total imports $M = M_1 + M_2 + M_3$

28. Population $N = N_{-1}\ (1 + b + d).\ 10^6$

29. Money income $Y = P_y .\ y$

30. Supply of money $L = DD + PC - L_{-1}$

The definitions for cumulated and average variables are:

$$CI_P^s = \sum_{i=1}^{t}(I_P^s)_i \ ; \ CI = \sum_{i=1}^{t}(I-D)_i \ ; \ A(y/N)_{-1} = \frac{1}{4}\sum_{i=t-4}^{t-1}(y/N)_i$$

The endogenous variables in the system are:

O_{ag}, A_i, n, U, O_r, C/N, I_P^C, I_P^{me}, CI_P^s, (M_1+M_2), M_3, D, T^d, T^{i-s}, b, d, P_y, i_L, DD, i_s, I, O_{mm}, C, I_P^s, y^d, M, N, y, L and Y.

There are 16 exogenous variables. These are defined below.

A = total cropped area; t = time-trend; ln t = natural logarithm of the time-trend; I_g^{me} = government investment in machinery and equipment; I_g^s = government investment in inventories; RES = reserves of scheduled banks with RBI (average of weeks); T = interest on national debt + transfer payments + net private donations from abroad + residuals; X = exports; PC = currency with public at the end of the period; $(E_{XA} + FA)^{def}$ = the sum of foreign aid (U.S. only) and foreign exchange reserves deflated by the import price index; PL 480 = rupee loans to India by U.S. under Public Law 480; ln GW, where GW = government welfare expenditure deflated by 1952-53 wholesale price index; I_g^C = government gross investment in construction; G' = government expenditure on maintenance; G = government current expenditure; and, i_B = bank-rate.

I_g^s, I_g^{me}, I_g^C, G', G are all measured at 1948-49 prices.

There are 10 lagged endogenous variables:

CI_{-1}, y_{-1}, U_{-1}, $CI_{P_{-1}}^{s}$, $A(y/N)_{-1}$, $P_{y_{-1}}$, $i_{L_{-1}}$, DD_{-1}, N_{-1} and L_{-1}.

7.4.2 Considerations in Specification

On the supply side, there is a three-fold division of output between agriculture, 'mining, manufacturing and trade', and a residual. Agricultural output is seen to depend on cultivated acreage and irrigation, the latter influence being incorporated in a ratio form as in Krishnamurty-I. Irrigation itself is endogenously explained as a function of capital. Output in mining, manufacturing is dependent on employment in this sector and capital-stock, the latter variable weighted by an index of capacity utilization. The residual output is dependent of labour (population) alone. Thus, land, labour and capital have all been brought in but they are seen as constraints in different sectors. There is an endogenous explanation for capacity utilization in terms of real output (average of current and last year's levels) and imports of producer goods. Together equations 1-5 constitute the production sector. Equation 3, however, is used to determine employment in the mining etc. sector, output of which is later determined in an identity.

On the demand side, aggregate consumption expenditure is determined via a per capita consumption function which has per capita disposable income, the ratio of non-agricultural income to total income and a time trend as explanatory variables. Private investment expenditure is disaggregated into construction, machinery and equipment and change in level of stocks, as in Krishnamurty models, and follow a similar pattern of explanation.

Imports are divided into two categories, viz., producer goods (M_1+ M_2) and food and other items (M_3). The former are dependent on investment demand both from the private and the government sector, and foreign exchange reserves and foreign aid. The latter variable was not found to be statistically significant in the sample period. Imports of food etc. were explained by PL 480 funds. This explanation was very partial and $\bar{R}^2$ in the estimated equation was very low.

The demographic sector equations are inherited from Krishnamurty's model and use a similar set of explanatory variables. In the statistical fit expected signs are obtained. In particular, it is observed that a rise in per capita income has a negative impact on both birth and death-rates, however, the decline in the latter is much faster than the former.

There is an endogenous explanation for the long-term rate of interest, supply of demand deposits, and the demand for money. Long-term rate of interest is explained by a distributed lag of short-term rates. The demand deposit equation is built on the hypothesis that the banks increase the supply of demand deposits as commercial bank reserves rise or as the ratio of short-term interest rate to the bank-rate rises.

The demand for liquidity is a function of income and rate of interest. In addition, there is a price variable $P_y/P_{y_{-1}}$ on the hypothesis that as prices rise people shift to goods and physical assets.

Another important equation is the price equation used to determine the implicit price deflator. This is made a function of (i) the agricultural output, and (ii) last year's prices. In the estimated equation $\bar{R}^2$ is very low. This equation was not integrated with the monetary sector of the model.

7.4.3 Estimates of Stochastic Equations

1. $$O_{ag} = -56.72 + \underset{(**)}{.50}\,A + \underset{(*)}{203.96}\,A_i/A \qquad \bar{R}^2 = .90$$

2. $$A_i = 22.44 + \underset{(**)}{.0612}\,CI_{-1} \qquad \bar{R}^2 = .92$$

3. $$O_{mm} = \underset{(*)}{.17}\,n + .18\left[95.8 + \frac{CI+CI_{-1}}{2}\right]U \qquad \bar{R}^2 = .96$$

4. $$U = .26 + \underset{(**)}{.0045}\,\frac{y+y_{-1}}{2} + \underset{(**)}{.030}\,(M_1+M_2) \qquad \bar{R}^2 = .92$$

5. $$O_r = -26.18 + \underset{(**)}{111.49}\,N \qquad \bar{R}^2 = .99$$

6. $$\frac{C}{N} = -17.54 + \underset{(**)}{.81}\,\frac{y^d}{N} + \underset{(*)}{.54}\,\frac{y-O_{ag}}{y} - \underset{(*)}{1.70}\,t \qquad \bar{R}^2 = .85$$

7. $$I_p^C = -5.63 + \underset{(*)}{.04}\,y^d + \underset{(**)}{10.97}\,U_{-1} \qquad \bar{R}^2 = .898$$

8. $I_P^{me} = -2.35 + .017\ y^d + 9.93\ U_{-1} - 1.23\ i_L$ $\quad \bar{R}^2 = .43$

9. $CI_P^s = -25.78 + \underset{(*)}{.33}\ (y - I_P^s - I_g^s) + .1069\ CI_{P_{-1}}^s$ $\quad \bar{R}^2 = .90$

10. $M_1 + M_2 = -.26 + \underset{(**)}{.73}\ (I_P^{me} + I_g^{me} + I_P^s + I_g^s) + .11\ (E_{XA} - FA)^{def}$ $\quad \bar{R}^2 = .72$

11. $M_3 = 2.70 + \underset{(**)}{.0040}\ PL\ 480$ $\quad \bar{R}^2 = .53$

12. $D = 5.53 + \underset{(**)}{.0074}\ \left[95.8 + CI_{-1}\right] U_{-1}$ $\quad \bar{R}^2 = .74$

13. $T^d = -.46 + \underset{(**)}{.030}\ y$ $\quad \bar{R}^2 = .68$

14. $T^{i-s} = -10.28 + \underset{(**)}{.16}\ y$ $\quad \bar{R}^2 = .94$

15. $\ln b = 3.88 - \underset{(**)}{.93}\ \ln A(y/N) - \underset{(*)}{.043}\ \ln t$ $\quad \bar{R}^2 = .50$

16. $\ln d = 5.81 - \underset{(**)}{1.78}\ \ln A(y/N) - \underset{(*)}{.040}\ \ln GW - .049\ \ln t$ $\quad \bar{R}^2 = .88$

17. $P_y = 153.91 - \underset{(*)}{2.11}\ O_{ag}/y_{-1} + \underset{(**)}{.50}\ P_{y_{-1}}$ $\quad \bar{R}^2 = .41$

18. $i_L = 1.44 + \underset{(**)}{.21}\ i_s + \underset{(**)}{.50}\ i_{L_{-1}}$ $\quad \bar{R}^2 = .94$

19. $\dfrac{DD + DD_{-1}}{2} = 3.05 + \underset{(*)}{.91}\ i_s/i_B + \underset{(**)}{4.99}\ (ER + RR)$ $\quad \bar{R}^2 = .74$

20. $L = -2.35 + \underset{(**)}{.22}\ y - .11\ i_s + 2.15\ P_y/P_{y_{-1}}$ $\quad \bar{R}^2 = .76$

No simulations were performed with model. The authors confess that much more experimentation was needed before a final shape could be given to the model. The relatively weak equations in the system in statistical terms (low $\bar{R}^2$ etc.) are those relating to I_P^{me} (eq.8), M_3 (eq.11), birth-rate (eq.15) and the price level (eq.17).

7.5 Models by Marwah

Marwah's attempts (1963, 1972) towards building a macroeconometric model of India has resulted in three models, all aimed primarily towards a study of price behaviour in India. The former two models (1963) have an aggregate price variable being explained in one case by excess demand, and in the other, by incorporating elements of the quantity theory. These aggregate models are respectively of 7 and 9 equations. A sectoral model explaining prices of manufactured goods, semi-manufactures, industrial raw materials and food grains is appended to the aggregate models. The aggregate models together with the sectoral model, we shall call Marwah-I and alternate -I(A-I).

The latter attempt (1972) results in a much more disaggregated model with a number of sectoral prices. Further, this model, to be called Marwah-III, gives primary importance to aggregate supply considerations. The income-expenditure identity is used merely to close the system.

7.5.1 Marwah-I and A-I

The models are estimated by 2SLS and OLS. The sample period is 1939-60. Some equations are respecified and re-estimated for a sub-sample period of 1947-60. The models are specified as follows.

(a) Model-I: Aggregate part

Stochastic Equations

1.	Private consumption expenditure	$\frac{C}{P} = f\left(X, \frac{L_{-2}}{P}, N\right)$
2.	Private investment expenditure	$I/P = f\left(\frac{X+X_{-1}}{2}, R_l\right)$
3.	Long-term rate of interest	$R_l = f(R_s, R_{l_{-1}})$
4.	Money (demand)	$M = f(Y, R_l, P_s)$
5.	Price-level	$P = f\left[(Y-Y_{-1}), P_{-1}\right]$

Identities

6.	Real income	$X = Y/P$
7.	Money income	$Y = C + I + H$

(b) Model A-I: Aggregate part

Stochastic Equations

Equation 1-3, same as in Model-I.

4.	Price-level	$P = f\left[(\Delta M_A - \Delta M_s), P_{-1}\right]$
5.	Inverse of income-velocity of money	$\theta = f\left(\frac{1}{R_l}, P_s\right)$

Identities

Identities 6-7, same as in Model-I.

8.	Money supply	$M = \theta . P . X$

9. Excess of actual money-supply over 'normal' or 'safe' money-supply

$$\Delta M_A - \Delta M_s = (M - M_{1953}) - \left[\frac{X-X_{1953}}{X_{1953}} + \frac{\theta-\theta_{1953}}{\theta_{1953}}\right] . M_{1953}$$

(c) Model-I and A-I: Sectoral Part

Stochastic Equations

1. Price of manufactured goods $P_m = f\left[\frac{W.N_L}{X_m}, P_{im}\right]$

or, for 1947-60 sub-sample $P_m = f\left[\left(\frac{S_m}{X_m}\right)_{-1}, P_{m_{-1}}\right]$

2. Output of manufactured goods $X_m = f(N_L)$

3. Wage-rate $W = f(\Delta P_C, \frac{X_m.P_m}{N_L}, W_{-1})$

4. Cost of living index $P_c = f(P)$

5. Price of semi-manufactures $P_{sm} = f\left[\left(\frac{S_{sm}}{X_{sm}}\right)_{-1}, P_{sm_{-1}}\right]$

6. Price of industrial raw materials $P_r = f(X_i, \frac{Fe}{X_r}, P)$

7. Demand for foodgrains $D_f = f(Y_{-1}, N)$

8. Domestic output of food-grains $X_f = f(X_g, P_{f_{-1}}, Z_1)$

9. Price of foodgrains $P_f = f(F_{if}, P_{f_{-1}}, Z_2)$

Identities

10. Imports of foodgrains $F_{if} = D_f - X_f$

Thus, together with the sectoral model, Model-I is a system of 17 equations, and Model A-I, a system of 19 equations.

The endogenous variables in Model-I are the following:

C, I, M (or R_s), R_1, P, X, Y, P_m, X_m, W, P_c, P_{sm}, P_r, D_f, X_f, P_f and F_{if}.

The additional endogenous variables in Model A-I are θ and $(\Delta M_A - \Delta M_s)$.

The exogenous variables in Model-I are the following: N = population, L_{-2} = currency + demand deposits + saving deposits (lagged 2 years); P_s = price of industrial securities; R_s = short-term rate of interest (or M); N_L = index of factory employment; P_{im} = unit value of manufactured imports; S_{sm}/X_{sm} = stock of semi-manufactured goods divided by its output; F_e = exports; X_r = output of raw materials; X_i = industrial output; X_g = agricultural output; Z_1 = dummy for market shifts in food supply; Z_2 = dummy having value 1 for world war and other years of rationing and controls and zero everywhere else; H = balance of income after private consumption and investment expenditure.

In another version, using alternative forms for equations for private investment and price of manufactured goods, S_m = stocks of manufactured goods enters as an exogenous variable in the place of P_{im}.

In Model A-I, the additional exogenous variables are Y_{1953}, θ_{1953} and M_{1953}.

7.5.2 Considerations in Specification

The aggregate part of Model-I is couched in the income-expenditure approach in a straightforward manner. Price-level is determined by a lagged structure of increases in aggregate nominal demand. Demand for consumption expenditure is determined by real income, real balances and population. Demand for investment goods is determined by real income with half a year's lag, and long-term rate of interest. The long-term rate, in turn, is determined by a lag-structure of short-term rates. Demand for money is a function of income, cost of holding money as entered in long-term interest rate and the price of a substitute asset, viz., industrial securities.

In the quantity-theoretic version of the determination of price-level, the explanatory variable is the excess of actual money-supply or a normal or safe level. This 'safe' level of money-supply is determined by the rate of change in real output and income-velocity measured relative to 1953 levels. There is a lagged price variable on the right-hand side.

Whereas, the general price-level affects sectoral prices, via the equations for the cost of living index and the price of industrial raw materials, the aggregative model is not affected by sectoral prices or outputs. The aggregate models can thus be treated as independent of the sectoral models.

7.5.3 Estimates of Stochastic Equations in Marwah-I and A-I

The two versions of the aggregate part of the model are estimated as below:

(a) Marwah-I

1. $$C/P = \underset{(**)}{17.21} + \underset{(**)}{0.87}\, X + .23\, \frac{L_{-2}}{P} + .046\, N \qquad \bar{R}^2 = .98$$

2. $$I/P = \underset{(**)}{14.69} + \underset{(*)}{.057}\, \frac{X+X_{-1}}{2} + 4.78\, R_1 \qquad \bar{R}^2 = .80$$

For 1947-60 sub-sample,

$$I/P = \underset{(*)}{6.27} + \underset{(*)}{.089}\, X + .74\, (I/P)_{-1} \qquad R^2 = .90$$

3. $$R_1 = .89 + \underset{(**)}{.10}\, R_s + \underset{(**)}{.65}\, R_{1_{-1}} \qquad R^2 = .87$$

4. $$M = 3.13 + \underset{(**)}{.17}\, Y - .83\, R_1 + \underset{(**)}{.073}\, P_s \qquad R^2 = .95$$

5. $$P = 5.03 + \underset{(**)}{0.68}\, (Y - Y_{-1}) + \underset{(**)}{.95}\, P_{-1} \qquad R^2 = .98$$

(b) Marwah A-I

Equations 1-3 are as above.

4. $$P = 20.57 + 0.53\, (\Delta M_A - \Delta M_s) + \underset{(*)}{.83}\, P_{-1} \qquad R^2 = .95$$

5. $$\theta = .053 + .45\, \frac{i}{R_1} + .0012\, P_s \qquad R^2 = .80$$

(c) Sectoral Part

The stochastic equations of the sectoral part common to Model-I and A-I are given below:

1. $$P_m = \underset{(**)}{19.31} + .33\, \frac{W.N_L}{X_m} + \underset{(**)}{.48}\, P_{im} \qquad R^2 = .96$$

or, for 1947-60 sub-sample:

$$P_m = \underset{(**)}{75.86} + .15\, \left(\frac{S_m}{X_m}\right)_{-1} + \underset{(**)}{.56}\, P_{m_{-1}} \qquad R^2 = .84$$

2. $X_m = \underset{(**)}{45.84} + \underset{(**)}{.51} N_L$ $\quad R^2 = .70$

3. $W = \underset{(**)}{4.44} + \underset{(*)}{.16} \Delta P_c + .17 \frac{X_m . P_m}{N_L} + \underset{(**)}{.99} W_{-1}$ $\quad R^2 = .99$

4. $P_c = \underset{(*)}{4.65} + \underset{(**)}{.93} P$ $\quad R^2 = .97$

5. $P_{sm} = 48.56 + \underset{(**)}{11.25} \left(\frac{S_{sm}}{X_{sm}}\right)_{-1} + \underset{(**)}{.65} P_{sm_{-1}}$ $\quad R^2 = .79$
(for 1947-60)

6. $P_r = 33.68 + \underset{(**)}{.32} X_i + 47.16 \frac{F_e}{X_r} + \underset{(**)}{.51} P$ $\quad R^2 = .96$

7. $D_f = 548.16 + \underset{(**)}{3.59} Y_{-1} + \underset{(**)}{2.16} N$ $\quad R^2 = .96$
(for 1944-60)

8. $X_f = 21.79 + \underset{(**)}{6.07} X_g + .40 P_{f_{-1}} - \underset{(**)}{45.97} Z_1$ $\quad R^2 = .98$

9. $P_f = 32.87 + \underset{(*)}{.41} F_{if} + \underset{(**)}{.60} P_{f_{-1}} - \underset{(*)}{7.93} Z_2$ $\quad R^2 = .80$

7.5.4 Marwah -II

Although Marwah's later model (1972) has evolved from the earlier models, it is much more disaggregated and it gives greater emphasis to supply-considerations, making aggregate demand play a passive role. Its primary concern continues to be the determination of aggregate and sectoral prices. Estimation is done, in general, with reference to annual data for the period 1939-1965. For certain sectors, however, the sample is smaller. One unique feature in the estimation of some equations is a pooling of cross-section data for some Asian countries with the time-series data for India.

The method of estimation is a 'modified' 2SLS procedure. In the first stage, solutions for only that part of the model are obtained for which the full sample is used. These solutions are utilized in the second stage irrespective of the sample size wherever necessary. For the recursive part of the model OLS is used.

The model has 48 endogenous variables out of which 26 are national income components, 19 are price variables, 11 are production and stock variables, 10 are financial variables and the remaining 16 are other 'miscellaneous' variables. There are 39 stochastic equations and 9 identities.

An interesting feature of the model is that there are some variables which are relevant for but do not strictly pertain to the Indian economy for which an endogenous explanation is offered. These are imports of LDCs from LDCs and imports of DCs from LDCs.

7.5.5 Specification of Marwah-II

(a) Stochastic Equations

1. Real output (income) $\quad X = f\left(\frac{X}{XC} \cdot K_{-1}\right)$

2. Private consumption $\quad C/P = f(Y/P, \frac{L_{-2}}{P}, N)$

3. Total investment expenditure $\quad I/P = f\left[(X, (I/P)_{-1}, Z)\right] S_1 + f\left[Y/P, F_{i(7)}/P\right] S_2$

4. Government investment $\quad I_g/P = f\left[X, (I/P)_{-1}, Z\right]$

5. Total (merchandise) imports $\quad F_i/P_i = f\left[Y/P, GFR_{-1}, P/P_i\right]$

6. Imports of Foods and Beverages
$$\frac{F_{i(0+1)}}{P_{i(0+1)}} = f\left[Y/P, \frac{Y}{PN}, P_f, \frac{NDPNA}{NDPA} \right]$$

7. Imports of raw materials
$$\frac{F_{i(2+4)}}{P_{i(2+4)}} = f\left[\frac{Y}{P}, GFR_{-1} \right]$$

8. Imports of fuels
$$F_{i(3)}/P_{i(3)} = f\left[\frac{Y}{PN}, N, \frac{P_3}{P}, Z_w \right]$$

9. Imports of chemicals
$$F_{i(5)}/P_{i(5)} = f\left[Y/P, \frac{P_5}{P_{i(5)}(1 + D_i)} \right]$$

10. Imports of manufactured goods
$$\frac{F_{i(6)}}{P_{i(6)}} = f\left[\frac{Y}{P}, \frac{Y}{PN}, \frac{P_m}{P_{i(6+8)(1+D_i)}}, \frac{NDPNA}{NDPA} \right]$$

11. Imports of machinery and transport equipment
$$\frac{F_{i(7)}}{P_{i(7)}} = f\left[\frac{Y}{P}, GFR_{-1}, \frac{P_7}{P} \right]$$

12. Imports of LDCs coming from LDCs
$$\frac{F_{iLDLD}}{P_{LDe}} = f\,(GDP^*_{LD})$$

13. Imports of DCs coming from LDCs
$$\frac{F_{iDLD}}{P_{LDe}} = f\,(GDP^*_{D})$$

14. Exports
$$F_e/P_e = f\left[\frac{F_{iLDLD} + F_{iDLD}}{P_{LDe}}, \frac{P_{LDe}}{P_e} \right]$$

15. Unit value index of exports
$$P_e = f\left[P, P_{e_{-1}} \right]$$

16. Indirect taxes
$$T = f(Y)$$

17. (Demand for) Money and Saving deposits $\frac{M + \Delta SD}{P} = f\left[\frac{1}{i_r}, i_{sh}, \left(\frac{1}{v}\right)_{-1}\right]$

18. Government bond yield (% p.a.) $i_r = f(i_s, i_{r_{-1}})$

19. Price-level $P = f\left[(\Delta M_A - \Delta M_s), P_i, P_f\right]$

20. Saving deposit to money-ratio $\frac{SD}{M} = f\left[i_r, \left(\frac{SD}{M}\right)_{-1}\right]$

21. Capacity utilisation $\frac{X}{XC} = f\left[(L/Y)_{-1}, \left(\frac{S_m}{X_m}\right)_{-1}, \left(\frac{X}{XC}\right)_{-1}\right]$

22. Agricultural output $X_g = f(X, X_{g_{-1}})$

23. Demand for foodgrains $D_f = f\left[\left(\frac{Y}{P}\right)_{-1}, N, P_f\right]$

24. Domestic output of foodgrains $S_{df} = f\left[X_g, (F_{if})_{-1}\right]$

or $S_{df} = f\left[X_g, (P_f)_{-1}\right]$

25. Price of foodgrains $P_f = f\left[F_{if}, (P_f)_{-1}, Z_2\right]$

26. Output of manufactured goods $X_m = f(X, X_{m_{-1}})$

27. Employment in factories $N_L = f(N_{L_{-1}}, X_m)$

28. Price of manufactured goods $P_m = f\left[\frac{WN_L}{X_m}, \frac{X_m}{XC_m}, P_f, P_i, Z\right]$

29. Average wage earnings $W = f(\Delta P_c, W_{-1})$

30. Cost of living index = $P_c = f(P_f, P)$

31. Index of capacity utilisation in the manufactured sector

$$\frac{X_m}{XC_m} = f\left[\frac{L}{Y}, \left(\frac{S_m}{X_m}\right)_{-1}, \frac{X}{XC}\right]$$

32. Stocks of manufactured goods as a ratio of output

$$\frac{S_m}{X_m} = f\left[\frac{H}{P}, \left(\frac{S_m}{X_m}\right)_{-1}\right]$$

33. Output of semi-manufactures $X_{sm} = f(X)$

or $X_{sm} = f(X, X_{sm_{-1}})$

34. Price of semi-manufactures $P_{sm} = f\left[\left(\frac{S_{sm}}{X_{sm}}\right)_{-1}, P, (P_{sm})_{-1}\right]$

35. Output of raw materials $X_r = f\left[X, (X_r)_{-1}\right]$

36. Price of raw materials $P_r = f\left[X_{ind}, \frac{F_e}{X_r}, P\right]$

37. Price of fuels $P_3 = f(P)$

38. Price of chemicals $P_5 = f(P)$

39. Price of machinery and transport equipment $P_7 = f(P)$

(b) <u>Definitions and Identities</u>

40. Private investment $I_P = I - I_g$

41. 'Other' imports $F_{i(OTH)} = F_i - \left[F_{i(0+1)} + F_{i(2+4)} + F_{i(3)} + F_{i(5)} + F_{i(6+8)} + F_{i(7)}\right]$

42. Liquidity $L = M + SD$

43. Imports of foodgrains $F_{if} = D_f - S_{df}$

44. General price level to determine P_{misc}

$$P = W_1P_f + W_2P_m + W_3P_{sm} + W_4P_r + W_5P_{misc}.$$

45. Money income $Y = PX$

46. Inverse of income-velocity of money $\frac{1}{v} = \frac{M}{PX}$

47. Real capital stock $K = K_{-1} + I/P$

48. Money income $Y = C + I + F_e - F_i - T + H$

where $H = G + F_{es} - F_{is} + NFI + I_{inv}$ is defined as the sum of government current expenditure plus exports of services minus imports of services plus net foreign income plus investment in inventories.

The endogenous variables of the system are:

X, C, I, I_g, F_i, $F_{i(0+1)}$, $F_{i(2+4)}$, $F_{i(3)}$, $F_{i(5)}$, $F_{i(6)}$, $F_{i(7)}$, $F_{i(LDLD)}$, $F_{i(DLD)}$, F_e, P_e, T, M, i_r, P, SD, $\frac{X}{XC}$, X_g, D_f, S_{df}, P_f, X_m, N_L, P_m, W, P_c, X_m/XC_m, S_m/X_m, X_{sm}, P_{sm}, X_r, P_r, P_3, P_5, P_7, I_P, $F_{i(OTH)}$ L, F_{if}, P_{misc}, Y, $\frac{1}{v}$, K, H.

The exogenous variables (26) are defined as follows:

N = population; D_i = rate of import duties; GDP^*_D = gross domestic product of developed countries in 1958 constant prices; GDP^*_{LD} = gross domestic product of less developed countries in 1958 constant prices; i_s = short term interest rate (% p.a.) computed from Bazar bill and call money rates;

i_{sh} = index of industrial share prices;GFR = gold and foreign exchange reserves at the end of the period; X_{ind} = index of industrial production; P_i = unit value index of total imports; P_{LDe} = price index of exports of less developed regions of the world; NDPA = % of net domestic product originating in agriculture; NDPNA = % of net domestic product originating in non-agricultural sectors; M_{1953}, X_{1953}, and $(\frac{1}{v})_{1953}$ from which to generate M_A and M_s; W_1, W_2, W_3, W_4, W_5 = sectoral price-weights; Z = 0 for 1939-60, 1 for 1961-65; Z_1 = 0 for 1939-43, 1 for 1944-48 and 53-65, 2 for 1949-52, Z_2 = 1 for World War and other years of rationing and control, zero everywhere else, Z_W = 1 for border War with China, zero everywhere else, S_1 = 1 for 1939-60, 0 for 1961-65, S_2 = 0 for 1939-60, 1 for 1961-65.

The lagged endogenous (19) variables are:

K_{-1}, L_{-2}, I_{-1}, P_{-1}, $P_{e_{-1}}$, $(\frac{1}{v})_{-1}$, $i_{r_{-1}}$, $S_{P_{-1}}$, M_{-1}, $(S_m)_{-1}$, $(X_m)_{-1}$,

$(\frac{X}{XC})_{-1}$, $(X_g)_{-1}$, $(F_{if})_{-1}$, $(P_f)_{-1}$, $(N_L)_{-1}$, $(S_{sm})_{-1}$, $(P_{sm})_{-1}$, and $(X_{sm})_{-1}$.

It seems that some of the variable definitions could have been better organised. Thus, imports of food and beverages are determined in a stochastic equation, and imports foodgrains in an identity. It is the difference between these two categories of imports which should be determined in the identity. Similarly, industrial production (X_{ind}) is exogenous whereas total output (X) and agricultural output (X_a) are endogenous. Clearly, industrial output could be determined in the system by defining an appropriate identity. In the investment function, two dummies are used with the same meaning. Other variables listed like t(= time trend) and T_w (= index of world trade) have not been explicitly used in the model anywhere. It is also not explained why the liquidity variable in the consumption function is given a two year lag.

7.5.6 Causal Structure of the Model

The model starts by postulating that the existing capital-stock is not fully utilised. A capacity utilisation rate is determined with the help of a given inventory stock and credit availability. Given capital stock and the capacity utilization rate, output is determined through a production function. From output, the general price-level, financial variables and the food-market variables are determined. Having obtained the price-level and the output, the value of output is determined from which various expenditure flows are generated. The difference between the sum of these expenditures flows (aggregate demand) and value of output (aggregate supply) is taken to indicate supply shortage or a dis-equilibrium gap. The real value of this gap is used in determining the inventory stock which again determines the capacity utilization rate with the help of credit conditions and the system restarts. The national income identity is used here merely to close the system.†

In the overall or global model, various sectoral models are embedded. Sectoral prices are affected by sectoral and global forces. But global prices are not affected by sectoral forces.

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† As an alternative, the income expenditure approach was followed by suppressing the production function but the results were not encouraging and this alternative was dropped.

7.5.7 Considerations in Specification

(a) Production, Consumption and Investment Functions

Capital-stock is the only binding constraint on the expansion of output. Capital-stock itself is adjusted by the capacity utilization ratio. This gives the production function. Private consumption is a function of total income, real money balances and population. Aggregate investment expenditure and government investment expenditure are explained and private investment is determined as a residual. In the equation for aggregate investment, two sub-relations are obtained, one, for the pre-1960 years based time-series data, and second, for 1960 and later years based on TS-CS data. The latter sub-relation gives the distribution of total investment expenditure between (i) imported machinery plus transportation equipment; and (ii) home-produced capital goods. It is shown that total investment is severely limited by the capacity to import from abroad.

(b) Foreign Trade Sector and Taxation

Total import demand and its breakdown into a number of S.I.T.C. commodity classes such as food and beverages, raw materials, fuels, chemicals, manufactured goods, and machinery and transportation equipment are estimated with reference to explanatory variables like real income, relative prices, import duties, ratio of national income originating in agricultural and non-agricultural sectors, population, and gold and foreign exchange reserves. Import demand equations are estimated by a pooling of CS - TS data.

Exports are explained by world demand for goods produced in less developed countries and Indian export prices in relation to prices of these countries. In the study of the foreign trade, the role of relative prices is predominant.

(c) Money Market and General Price Level

Equations 17 through 20 specify the money market conditions and the general price-level. The demand for money is formulated in the general Keynesian framework. The reciprocal of income-velocity of money is made the dependent variable and is explained by a distributed lag pattern in the rate of interest and share-prices. A relationship between the short- and long-term rates of interest is also estimated through a Koyck transformation of the lag-structure. The general price-level is a function of excess money-supply (excess over a safe level), imports and food prices. The impact of the food-sector on the general price-level is seen to be the largest.

(d) Sectoral Outputs and Prices

Apart from the identities, equations 23 through 25 refer to the foodgrains sector, 27 through 32 refer to the manufactures, 32 and 33 refer to semi-manufactures, 35 and 36 refer to raw materials and 37 through 39 refer to other prices. Thus, there is considerable dis-aggregation of output and prices in the model.

For the foodgrains sector a cobweb type of model is used. Imports of foodgrains are utilised as a measure of excess demand in this sector. No relative price variable is used in estimating total food demand as substitution effects are not considered relevant in this sector. The price elasticity of food demand is found to be low. Foodgrains production has been made a simple linear function of total agricultural production in the country on the assumption that the same conditions characterise both.

In the framework of a cobweb model, foodgrains prices are also introduced with a one-period lag. However, its coefficient turns out to be negative which is not expected theoretically. It is also significant statistically. This might be because of the subsistence nature of Indian farming, where farmers are forced to increase production in the wake of a fall in prices so that they may maintain at least their subsistence income.

Total demand in the manufactured goods sector is generated by global forces outside the sector. Level of employment in this sector is then obtained from the production function, using the total demand already estimated. Prices in this sector are explained by interactions of global forces and demand and cost factors in the manufacturing sector. The equation uses a unit cost mark-up. Adjustments in the mark-up factors are made by the demand conditions reflected through a capacity operating rate. Mark-ups are used on unit wage cost, food prices and import prices.

In the sector of semi-manufactures and raw materials, production level is closely related to the aggregate production in the economy. The price-equation for semi-manufactures is based on market-clearing behaviour. As price moves up, inventories are depleted. In addition, the general price level is also introduced as an explanatory variable to introduce the effect of general economic conditions in the analysis. For determining the price-level of industrial raw materials, both demand and supply factors are explicitly introduced via the index of industrial production and general price level for reflecting direct and global demand factors respectively, and the value of exports relative to total production of raw materials for reflecting supply conditions.

The model explicitly determines the general price-level and four sectoral prices. The remaining price, viz., the price for miscellaneous items is then obtained as a residual, by using appropriate weights.

7.5.8 Estimates of Stochastic Equations

Except where otherwise specified, equations are estimated with 2SLS and for the full sample period (1939-65). Figures in brackets are sampling errors of individual coefficients.

1. $$X = \underset{}{76.6981} + \underset{(.017)}{.2685}\,(X/XC)\,K_{-1} \qquad \bar{R}^2 = .91, d=1.23$$

2. $$C/P = 10.6394 + \underset{(.143)}{.8422}\,Y/P + \underset{(.163)}{.2655}\,L_{-2}/P + 31.8\,N \qquad \bar{R}^2 = .96, d=1.42$$

3. $$I/P = \left[-5.7799 + \underset{(.023)}{.0824}\,X + \underset{(.1005)}{.7594}\,(I/P)_{-1} - 2.0531\,Z\right] S_1 + \left[-.135 + \underset{(.047)}{.063}\,Y/P + F_{i(7)}/P\right] S_2$$

S_1 = 1 (for 39-60), 0 (for 60-65); S_2 = 1 (for 60-65), 0 (for 39-60)

CS-TS data for S_2; this part estimated with OLS.

$\bar{R}^2$ = .89, d = 1.46

4. $$I_g/P = -11.3479 + \underset{(.0177)}{.1252}\,X + \underset{(.0767)}{.3024}\,(I/P)_{-1} + \underset{(.575)}{1.1949}\,Z \qquad \bar{R}^2 = .94, d=1.21$$

Equations 5-11 use CS-TS data and are estimated with OLS.

5. $$F_i/P_i = -3.6146 + \underset{(.004)}{.0434}\,Y/P + \underset{(.638)}{2.3231}\,GFR_{-1} + \underset{(.046)}{6.668}\,P/P_i \qquad \bar{R}^2 = .94(CS), .97(TS)$$

6. $$F_{i(0+1)}/P_{i(0+1)} = -4.5171 + \underset{(.001)}{.011}\,Y/P + \underset{(.0005)}{.00076}\,Y/PN + \underset{(.589)}{3.2042}\,P_f + \underset{(.627)}{1.2867}\,NDPNA/NDPA \qquad \bar{R}^2 = .83(CS), .95(TS)$$

7. $$F_{i(2+4)}/P_{i(2+4)} = 3.8227 + \underset{(.0013)}{.0057}\,Y/P + \underset{(.205)}{.337}\,GFR_{-1} - \underset{(.009)}{.1367}\,t \qquad \bar{R}^2 = .72(CS), .98(TS)$$

8. $F_{i(3)}/P_{i(3)} = -.9761 + \underset{(.00005)}{.00015}\ Y/PN + \underset{(.1319)}{1.4251}\ N$

$+ \underset{(.898)}{1.1699}\ P_{(3)}/P + \underset{(.073)}{.1949}\ Z_W$ $\qquad \bar{R}^2 = .86(CS), .76(TS)$

9. $F_{i(5)}/P_{i(5)} = -.6225 + \underset{(.0005)}{.0036}\ Y/P + \underset{(.0916)}{1.1425}\ P_5\ /P_{i_{(5)}} \cdot (1+D_i)$

$\bar{R}^2 = .76(CS), .97(TS)$

10. $\dfrac{F_{i(6+8)}}{P_{i(6+8)}} = .5174 + \underset{(.0015)}{.0163}\ Y/P + \underset{(.00003)}{.00023}\ Y/PN + \underset{(1.67)}{1.0763}\ P_M/P_{i(6+8)} \cdot (1+D_i)$

$-\underset{(.409)}{1.1125}\ NDPNA/NDPA$ $\qquad \bar{R}^2 = .90(CS), .86(TS)$

11. $\dfrac{F_{i(7)}}{P_{i(7)}} = 2.7813 + \underset{(.0015)}{.01579}\ Y/P + \underset{(.23)}{.2603}\ GFR_{-1} - \underset{(1.05)}{1.9596}\ P_7/P$

$\bar{R}^2 = .91(CS), .36(TS)$

Equations 12-14 are estimated with OLS. Sample: 1952-64 for equations 12 and 13 and 1951-65 for equation 14.

12. $\dfrac{F_{iLDLD}}{P_{LDe}} = 1.384 + \underset{(.0013)}{.0282}\ GDP^*_{LD}$ $\qquad \bar{R}^2 = .98,\ d=1.26$

13. $\dfrac{F_{iDLD}}{P_{LDe}} = -5.4381 + \underset{(.0084)}{.0271}\ GDP^*_{D}$ $\qquad \bar{R}^2 = .99,\ d=1.58$

14. $$F_e/P_e = -.354 + \underset{(.0189)}{.1455}\frac{(F_{iLDLD} + F_{iDLD})}{P_{LDe}} + \underset{(1.402)}{2.99}\ P_{LDe}/P_e \qquad \bar{R}^2 = .82, d=1.26$$

15. $$P_e = .0621 + \underset{(.117)}{.2974}\ P + \underset{(.121)}{.6228}\ (P_e)_{-1} \qquad \bar{R}^2 = .89,\ d=1.83$$

16. $$T = -5.8493 + \underset{(.0058)}{.12008}\ Y \qquad \bar{R}^2 = .97,\ d=.95$$

(Sample 1951-65)

17. $$\frac{M + \Delta SD}{PX} = -0.0189 + \underset{(.198)}{.2423}\ 1/i_r + \underset{(.0002)}{.001}\ i_{sh} + .1967\ (\frac{1}{v})_{-1}$$

$$\bar{R}^2 = .75,\ d=1.76$$

18. $$i_r = 1.5954 + \underset{(.059)}{.266}\ i_s + \underset{(.186)}{.2577}\ i_{r-1} \qquad \bar{R}^2 = .94,\ d=1.38$$

Sample 1951-65, OLS

$$\bar{R}^2 = .75,\ d=1.76$$

19. $$P = .1315 + \underset{(.002)}{.0058}\ (\Delta M_A - \Delta M_S) + \underset{(.046)}{.1864}\ P_i + \underset{(.055)}{.7161}\ P_f$$

$$\bar{R}^2 = .99,\ d=1.57$$

20. $$SD/M = -.0488 + \underset{(.019)}{.0223}\ i_r + \underset{(.109)}{.8766}\ (S/M)_{-1} \qquad \bar{R}^2 = .91,\ d=.90$$

21. $$X/XC = .7974 - \underset{(.223)}{.3367}\ (L/Y)_{-1} - \underset{(.048)}{.0502}\ (S_m/X_m)_{-1} + \underset{(.264)}{.2709}\ (X/XC)_{-1}$$

$$\bar{R}^2 = .41,\ d=1.93$$

Sample 1947-65

22. $$X_g = -16.2048 + \underset{(.224)}{.625}\ X + \underset{(.171)}{.583}\ X_{g-1} \qquad \bar{R}^2 = .93,\ d=2.55$$

Equations 23 through 25 are estimated with OLS.

23. $$D_f = \underset{}{-56.882} + \underset{(.134)}{.2266}\,(Y/P)_{-1} + \underset{(54.0)}{279.8}\,N$$ $\bar{R}^2 = .95$, $d=1.80$ Sample 1944-65

24. $$S_{df} = 1.8345 + \underset{(.2270)}{.6021}\,X - \underset{(.322)}{.4887}\,(F_{if})_{-1}$$ $\bar{R}^2 = .98$, $d=1.44$ Sample 1944-65

OR

$$S_{df} = -5.2733 + \underset{(.3580)}{.7010}\,X_g - \underset{(.337)}{.9547}\,P_{f_{-1}}$$ $\bar{R}^2 = .98$, $d=1.47$ Sample 1944-65

25. $$P_f = .2847 + \underset{(.0191)}{.0556}\,F_{if} + \underset{(.166)}{.6202}\,P_{f_{-1}}$$ $\bar{R}^2 = .90$, $d=1.96$ Sample 1944-65

26. $$X_m = -21.2881 + \underset{(.1399)}{.5021}\,X + \underset{(.0997)}{.7523}\,X_{m_{-1}}$$ $\bar{R}^2 = .97$, $d=1.94$

27. $$N_L = -1.786 + \underset{(.067)}{.8244}\,N_{L_{-1}} + \underset{(.063)}{.2206}\,X_m$$ $\bar{R}^2 = .98$, $d=2.72$ Sample 1940-65

28. $$P_m = -0.1971 + \underset{(.097)}{.270}\,\frac{WN_L}{X} + \underset{(.1127)}{.3395}\,\frac{X_m}{XC} + \underset{(.0718)}{.3305}\,P_f + \underset{(.094)}{.3165}\,P_i$$
$$+ \underset{(.032)}{.0764}\,Z$$ $\bar{R}^2 = .99$, $d=1.23$

29. $$W = .0562 + \underset{(.097)}{.182}\,\Delta P_e + \underset{(.018)}{.9746}\,W_{-1}$$ $\bar{R}^2 = .99$, $d=1.83$, OLS

30. $$P_c = .0298 + \underset{(.127)}{.553}\,P_f + \underset{(.129)}{.3964}\,P$$ $\bar{R}^2 = .99$, $d=.96$

31. $$X_m/XC_m = \underset{}{-0.0475} + \underset{(2.77)}{.4875}\ L/Y - \underset{(.049)}{.127}\ (S_m/X_m)_{-1}$$ $\bar{R}^2 = .69, d=1.66$

Sample 1947-65, OLS

32. $$S_m/X_m = .522 - \underset{(.016)}{.025}\ H/P + \underset{(.238)}{.4806}\ (S_m/X_m)_{-1}$$ $\bar{R}^2 = .45, d=1.61$

Sample 1951-65, OLS

33. $$X_{sm} = -120.223 + \underset{(.165)}{2.2408}\ X$$ $\bar{R}^2 = .93, d=1.06$

Sample 1951-65

OR

$$X_{sm} = 29.0616 - \underset{(.276)}{.4356}\ X + \underset{(.126)}{1.2082}\ X_{sm_{-1}}$$ $\bar{R}^2 = .99, d=2.1$

Sample 1951-65

34. $$P_{sm} = -0.0036 - \underset{(.0298)}{.0641}\ (S_{sm}/X_{sm})_{-1} + \underset{(.1106)}{.7698}\ P + \underset{(.099)}{2967}\ P_{sm_{-1}}$$ $\bar{R}^2 = .97, d=1.55$

Sample 1947-65

35. $$X_r = -38.899 + \underset{(.314)}{.849}\ X + \underset{(.174)}{.5961}\ X_{r_{-1}}$$ $\bar{R}^2 = .90, d=2.55$

36. $$P_r = -.3376 + \underset{(.0007)}{.00296}\ X_{ind} + \underset{(1.80)}{7.50}\ F_e/X_r + \underset{(.131)}{.631}\ P$$ $\bar{R}^2 = .98, d=1.48$

Equations 37-39 are estimated with OLS.

37. $$P_3 = .254 + \underset{(1.120)}{.784}\ P$$ $\bar{R}^2 = .89$

38. $$P_5 = .543 + \underset{(.105)}{.447}\ P$$ $\bar{R}^2 = .77$

39. $$P_7 = .497 + \underset{(.071)}{.522}\ P$$ $\bar{R}^2 = .91$

7.5.9 Applications of the Model

The estimated† model is used for both forecasting and policy analysis. Forecasts are generated for three years: 1965, 66 and 67. The first of these is within the sample, and the last two are beyond-sample. Beyond sample forecasts are generated by using actual values of exogenous and predetermined variables wherever available. Model estimates of predetermined variables are used where actual observations are not available.

Forecasts for 1965 and 1966 closely follow the realizations except for agricultural output and raw materials output. The deviations between predictions and realizations are relatively larger for the 1967 forecasts.

Conclusions regarding policy are obtained from magnitudes of short-term and, in a few cases, long-term multipliers. Non-linearities are evaluated around alternative fixed points. Impact multipliers show food prices and import prices to have the largest effect on other prices. The influence of money market on the prices is also substantial. Interest rate changes do not much affect the real sector of the economy except through their effect on prices. Capital accumulation has the largest impact on the output of semi-manufactures followed by raw materials, manufactures and agricultural commodities.

7.6 Summary

In this Chapter we have summarised the models and, where applicable, their alternate versions by Narasimham, Choudhry, Krishnamurty, Krishnamurty-Choudhry and Marwah. The main features of these models, their uses, the considerations in the specifications and the considerations in the provision of alternate versions have been noted. The specification of stochastic equations and definitions and identities is provided along with the estimates of stochastic equations. For easy comparability with the original sources, the symbolic representation of variables closely follows the original

† Where alternate versions of equations are provided, it is not clear which equations were used in the solutions.

Chapter 8

Models of the Indian Economy: II

In this chapter the main features of models by Mammen, Pandit, Bhattacharya and Gupta are considered. These models were built in the sixties and early seventies with sample periods beginning in the post-independence period. In three of these models, viz. Mammen, Bhattacharya and Gupta, the monetary sector of the economy plays a predominant role.

8.1 Models by Mammen

Mammen's study (1967) is primarily that of the monetary sector of the economy. A real sector is grafted on the monetary sector in order to account for some of the interactions. The sectors are first estimated independently. Later a simultaneous estimation of both sectors is also provided. The sample period relates to 16 annual observations from 1948-49 to 1963-64. Both OLS and 2SLS estimates are provided for the sectoral models. The joint model is estimated by using principal components of the predetermined variables in the first stage.

8.1.1 Monetary Sector

The framework of the monetary model is provided by the following equations.

(a) Stochastic Equations

1.	Demand for currency (to determine 3-month deposit interest-rate, R_3)	$C = f(Y, R_3, t, Z')$
2.	Demand for demand deposits	$D = f(Y_{na}, R_3)$; later $D = f(Y, R_3)$
3.	Demand for time deposits	$T = f(Y, R_i, R_{12})$
4.	Yield on variable dividend industrial securities	$R_i = f(R_g, PR)$; later dropped.
5.	Rate of interest offered by commercial banks on 12 mths' deposits	$R_{12} = f(R_g, R_{12-1})$
6.	Long-term rate of interest (yield on central govt. conversion loan 1986 or later)	$R_g = f(R_3, G_s)$
7.	Ratio of commercial banks holding of central and State govt.securities to their total credit	$BHS/BCR = f\left[R_3/R_g, (BHS + BCR)\right]$
8.	Ratio of excess reserves to total deposits	$ER/(D+T) = f(R_b)$
9.	Call money rate	$R_b = f(R_3, BR)$
10.	Required reserves	$RR = f\left[(D + T)\right]$

(b) Identities

11. Commercial banks' balance-sheet $BHS + BCR \triangleq (D + T)$

12. Public holding of Central and State government securities $PHS = GS - BHS$

The 12 endogenous variables are R_3, D, R_i, R_{12}, T, R_g, BHS, PHS, BCR, ER, R_b and RR.

The exogenous variables are C = currency, Y = income (net national income at factor cost at current prices), Y_{na} = non-agricultural income at current prices, PR = profit at current prices, t = time trend, BR = rate of borrowing by scheduled banks from RBI, GS = government interest bearing securities outstanding, Z' = dummy, taking a value 1 from 1958-59 and zero for earlier years.

There is only one lagged endogenous variable, R_{12-1}.

8.1.2 Considerations in Specification of the Monetary Sector

Demands for currency, demand-deposits and time-deposits are specified separately in equations 1 through 3. In each case an income variable enters. For currency and demand-deposits the short-term rate of interest is considered relevant. For time deposits, a longer-term rate of interest and yield on a substitute, viz., industrial securities are brought into analysis. Since currency (C) is taken as exogenous, equation 1 is used to determine R_3. Other rates of interest are R_g, R_{12}

and R_b. R_b, the inter-bank borrowing rate, is another short-term rate and is seen to depend on the 3-months deposit rate and the bank-rate. R_g, which is the only long-term rate is a function short-term rate (R_3) and the exogenous variable, G . The short-rate is included because the investors have a choice of investing for short periods, and it also reflects the influence of expectations. In explaining R_{12}, a competitive long-term rate R_g and a lag to represent expectations are introduced.

In explaining asset-holdings of commercial banks, the ratio BHS/BCR is taken to be the dependent variable since commercial banks' holding of central and State governments' securities (BHS) and commercial banks' credit (BCR) are substitutes to a certain extent. The ratio BHS/BCR is affected by the relative yield of these assets (R_3/R_g) and total amount of assets (BHS + BCR). R_3 is taken to be a proxy variable for the yield on BCR, and thus R_3/R_g is taken to indicate the relative yield on the two types of assets. Excess reserves (ER) and required reserves (RR) are explained following Brunner's (1961) analysis. Excess reserves are a function of rate of interest (R_b) as it represents the opportunity cost of holding reserves.

8.1.3 Real Sector

The structure of the model is defined by the following equations.

(a) Stochastic Equations

1. Real public consumption $\quad C_p/P_i = f(X)$

2. Real public investment $\quad I_p/P_i = f(PR/P_i, IFK/P_m)$

3. Real government investment $\quad I_g/P_i = f\left[(X^*-X), (I_g/P_i)_{-1}\right]$

4.	Real imports	$M/P_m = f\left[X, IFK/P_m, P_m\right]$
5.	Indirect taxes	$T_i = f(Y)$
6.	Real Profits	$PR/P_i = f\left[X_{na}, (PR/P_i)_{-1}\right]$
7.	Implicit price deflator	$P_i = f(P_w)$
8.	Wholesale price index	$P_w = f(P_a)$

(b) <u>Definitions and Identities</u>

9.	Real non-agricultural income	$X_{na} = X - X_a$
10.	Real national income	$X = Y/P_i$
11.	National Income at current prices	$Y = C_p + I_p + C_g + I_g + E - M - T_i$
12.	Inflow of capital at current prices	$IFK = M - E + Z$

The exogenous variables are X^* = target real income; X_a = real agricultural income; C_g/P_i = real government consumption expenditure; E = exports at current price; Z = balance of payments (=IFK+E-M); P_a = index of agricultural prices; P_m = index of import prices.

The lagged endogenous variables are $(I_g/P_i)_{-1}$ and $(PR/P_i)_{-1}$.

The system is estimated with OLS and 2SLS.

8.1.4 Considerations in Specification of the Real Sector

Private or non-governmental consumption in real terms is simply a function of real income. Although the role of liquid assets (currency, demand and time-deposits) was explored, it was not found statistically significant. Real private investment is related positively with profits (PR) and inflow of capital (IFK). Government investment is affected by long-term objectives in planning. Therefore, it is hypothesized that government investment decisions are affected by the difference between realized real income (X) and a target real income (X*). A lag-structure is also introduced through $(I_g/P_i)_{-1}$ to take account of past performance. The demand for imports is studied as subject to the constraints of real income, price of imports and the inflow of foreign capital. There are two price indices, viz., the implicit price deflator (P_i) and the wholesale price index (P_w) which are explained within the model. P_w enters as the explanatory variable for P_i, and in turn, is itself explained by the agricultural price index which is taken as exogenous.

As such, the model is one of aggregate demand and ignores the supply side. It has a weak fiscal sector.

Both the monetary and real sector models contain a number of ratio variables and thus provide a non-linear system.

The primary purpose of building the real sector model was to generate values for PR, Y, and Y_{na} which could be fed back to the monetary sector model.

8.1.5 The Joint Model

The real and monetary sectors of 12 equations each are joined together to produce a 26-equation complete model. The two additional endogenous variables are (PR/P_i) = real profits and Y_{na} = non-agricultural income. These are generated by adding two identities in the system.

The complete model is constructed in order to incorporate feedbacks from the monetary sector to the real sector. However, only one such feedback is explicitly introduced. This concerns the wholesale prices which are affected by the financial assets (C + D + T). Two alternatives estimates are made: one takes (C + D + T) as a proportion of money income, and the other takes it as a proportion of real income. No other equations of the segmented models are changed in the complete model.

The total list of 26 endogenous variables in the system is given below.

C/P_i, I_p/P_i, I_g/P_i, M/P_m, T_i, PR/P_i, PR, Y, Y_{na}, X, X_{na}, P_i, P_w, D, T, BHS, BCR, PHS, ER, RR, R_3, R_{12}, R_g, R_b, R_i and IFK.

There are now 15 predetermined variables compared to 9 for the monetary model and 9 for the real sector model. These are,

X^*, X_a, C_g/P_i, E, P_a, P_m, Z, C, G_s, BR, Z', t and $(I_g/P_i)_{-1}$ and R_{12-1}.

Y_{na}, Y and PR, which were exogenous to monetary sector have now become endogenous in the complete system. The system now contains 7 identities which are listed below.

1. $Y_{na} = (P_i . X_{na})/100$

2. $X_{na} = X - X_a$

3. $X = (Y/P_i).100$

4. $Y = C_p + I_p + I_g + C_g + E - M - T_i$

5. $IFK = M - E + Z$

6. $PR = (PR/P_i).p_i/100$

7. $PHS = G_s - BHS$

There are 19 stochastic equations. These are estimated by 2SLS-PC where the first 8 principal components of the predetermined variables are used in the first stage. In general, there are no appreciable differences in the magnitude of coefficients in the complete model as compared to the segmented models except in the equations concerning time-deposits (T). However, the t-values in the complete model are relatively lower.

The estimated equations for the joint model are given below.

8.1.6 Estimates of Stochastic Equations in the Joint Model

Figures in brackets are t-ratios

	Equation	$\bar{R}^2$	d
1.	$C_p/P_i = 11.4753 + \underset{(45.586)}{.8148}\ X$	.993	1.313
2.	$I_p/P_i = 1.2195 + \underset{(3.13)}{1.1834}\ PR/P_i + \underset{(1.51)}{.5377}\ IFK/P_m$	.734	1.303
3.	$I_g/P_i = .4255 + \underset{(2.35)}{.1075}(X^*-X) + \underset{(1.54)}{.4118}(I_g/P_i)_{-1}$	.91	1.28
4.	$M/P_m = 6.2181 + \underset{(2.21)}{.026}\ X + \underset{(7.3)}{1.0315}\ IFK/P_m \underset{(-3.75)}{-.0223}\ P_m$	.95	1.62
5.	$T_i = -8.858 + \underset{(25.83)}{.1484}\ Y$	.98	2.15
6.	$PR/P_i = -1.8248 + \underset{(2.26)}{.0502}\ X_{na} + \underset{(4.0)}{.7226}\ (PR/P_i)_{-1}$	.96	1.54
7.	$P_i = 42.6210 + \underset{(12.451)}{.5740}\ P_w$	.91	1.78
8a.	$P_w = -17.9551 + \underset{(18.66)}{.9242}\ P_a + \underset{(3.39)}{1.357}\frac{(C+D+T)\%}{Y}$	.98	.99
8b.	$P_w = -0.8633 + \underset{(12.56)}{.8077}\ P_a + \underset{(3.96)}{.8984}\ \frac{(C+D+T)\%}{X}$	.98	.74
9a.	$D = .9941 + \underset{(11.47)}{.0965}\ Y_{na} - \underset{(-1.83)}{.4100}\ \log R_3$	.93	1.03
9b.	$D = .6424 + \underset{(10.029)}{.0522}\ Y - \underset{(-1.207)}{.2995}\ \log R_3$	.91	1.26
10a.	$T = -3.6451 + \underset{(6.39)}{.0903}\ Y - \underset{(-3.112)}{.6880}\ R_i + \underset{(2.317)}{.9623}\ R_{12}$	.98	2.6
10b.	$T = -5.0250 + \underset{(6.878)}{.1126}\ Y - \underset{(-1.303)}{1.4805}\ R_g + \underset{(1.458)}{1.2163}\ R_{12}$	.97	2.45

11.	$R_i = .6717 + \underset{(4.173)}{1.7054}\ R_g - \underset{(-4.294)}{0.5515}\ PR$	.56	1.43
12a.	$R_{12} = -1.6972 + \underset{(2.399)}{.7888}\ R_g + \underset{(2.878)}{.5545}\ R_{12-1}$	.93	1.77
12b.	$R_{12} = .1740 + \underset{(1.537)}{.1580}\ R_3 + \underset{(6.534)}{.8272}\ R_{12-1}$	.93	1.975
13	$R_g = 2.0232 + \underset{(13.023)}{.3824}\ R_3 + \underset{(5.365)}{.0632}\ G_s$	.95	.98
14.	$BHS/BCR = 1.2837 - \underset{(8.044)}{.8169}\ R_3/R_g - \underset{(2.798)}{.0128}(BHS+BCR)$	.91	1.785
15.	$(ER/D+T)\% = 8.8847 - \underset{(-10.708)}{1.2763}\ R_b$	.89	1.919
16.	$R_b = -1.6275 + \underset{(4.92)}{.6975}\ R_3 + \underset{(3.48)}{.6330}\ BR$	.91	1.26
17a.	$RR = .0984 + \underset{(8.56)}{.0333}\ (D+T)$	.83	1.51
17b.	$RR = \underset{(29.223)}{.0404}\ (D+T)$	.80	1.24
18.	$C = 3.424 + \underset{(4.46)}{.0842}\ Y - \underset{(6.85)}{3.5981}\ \log R_3 + \underset{(5.41)}{.7386}\ t - \underset{(1.52)}{.8093}\ Z'$	.986	1.78
19.	$(BHS + CR) \hat{=} (D + T)$		
	$(BHS + CR) = \underset{(118.5\ )}{.9956}\ (D + T)$	.99	.85

The remaining seven equations in the system are identities.

In the separate demand functions for currency, demand- and time deposits, interest elasticities are found to be low and differences in income-elasticities between these categories are observed. Time deposits are highly income-elastic whereas income-elasticity for currency and demand-deposits is near unity.

Variations in supply of currency have substantial effects on all types of interest-rates. Further, a fall in short-term rate is indicated to be steeper than a fall in long-term rate following an increase in currency.

The real sector of the model has been envisaged to be entirely independent of the monetary sector variables except the influence of financial assets on prices (equation 8). This is the main limitation of the model.

Although some simulations were performed, the main purpose of the exercise was the comparision of individual elasticity and propensity coefficients and testing of specification.

8.2 Bhattacharya's Model

Bhattacharya's (1975) model of short-term income determination is a system of 9 equations with 7 exogenous and 2 lagged endogenous variables. The sample consists of 19 years from 1949-50 to 1967-68. The system contains 6 stochastic equations.

This model has two special features: (i) money-supply is endogenous to the system, and (ii) monetized income and expenditure levels are used as compared to total levels. But the link is established only via a relationship between short- and long-term rates of interest. A direct relationship via consumption and investment functions has not been established.

8.2.1 The Structure of the Model

The model consists of the following functional relationships and identities.

Monetized consumption	$C = f_1(Y^d)$
Monetized Investment	$I = f_2(Y, r^l, I_{-1})$
Demand for money (to determine short-term interest-rate, r)	$M = f_3(Y^d, r, NW)$
Supply of money	$M = f_4(U, q, l)$
Tax-revenue	$T_t = f_5(Y, t)$
Long-term rate of interest	$r^l = f_6(r, r^l_{-1})$
Monetized income	$Y = C + I + E_n$
Monetized personal disposable income	$Y^d = Y - T_t + T_r$
Excess of discount rate over short term interest rate	$q = r^d - r$

The exogenous variables are defined to be the following: (i) NW = net worth of the private sector defined as the sum of real monetized capital stock, money supply inclusive of time deposits and government securities held by public; (ii) t = time trend; (iii) T_r = transfers; (iv) U = unborrowed monetary base; (v) r^d = the discount-rate; (vi) l = liquidity ratio and (vii) E_n = autonomous expenditure plus exports minus imports. The lagged endogenous variables are (i) r^l_{-1} = lagged long-term rate of interest and I_{-1} = lagged investment. The absence of any production function, import-demand function, prices or wages functions constitutes the major limitation of the model.

8.2.2 Considerations in Specification of the Main Stochastic Equations

The simplest of all consumption functions, viz., consumption as a function of income is found most appropriate. For investment, rather than using the basic Keynesian approach where investment is largely dependent upon the short-term rate of interest, a synthetic approach is adopted where income, following the permanent income hypothesis, and capital-stock, following the acceleration principle are used as explanatory variables in addition to the cost of borrowed capital, viz., the rate of interest.

It is argued that the long-term rate of interest is more relevant for the investment function than the short-term rate for two reasons: (i) since the propensity to save is low, borrowing is an important determinant, and (ii) since inflation has been the rule of the day investors take into account expected rate of change in prices which is reflected more appropriately in the long-term rate rather than the short-term rate. Statistical tests with capital stock as an explanatory variable did not produce plausible results which is contrary to the findings of Krishnamurty (1964) and Krishnamurty-Choudhry (1968). The capital-stock variable was finally dropped and an alternative approach was followed. Current investment was taken as a function of expected income and expected long-term rate of interest. A Koyck transformation was used to generate a lagged dependent variable on the right-hand side. This equation was retained although the coefficient of this lagged term was not found to be statistically significant.

Formulation of an endogenous money supply function is a major feature of the model. In this context, the theories developed by Brunner (1961) and Crouch (1968), where money-supply is a function of the monetary base, the currency-deposit ratio and the reserve ratio, the approach followed by Polak and White (1955) and Teigen (1964), where interest-rate is explicitly introduced in the money supply function, and the approach of Meigs (1962) where interest rate and discount rate both enter the function were considered and found to be inappropriate in the Indian conditions.

In India, commercial banks borrow heavily from the Reserve Bank of India to meet the increasing demand for credit especially in busy seasons. For the banking system as a whole, the two sources of change in their portfolios are the public and the RBI. Inter-bank loans get cancelled out for the system as a whole. Faced with uncertainties in the credit market, the commercial banks try to adjust their excess or free reserves, i.e. reserves held in excess of required reserves. The free reserves are shown to be a function of the difference between the discount rate (r^d) and the short-term interest rate (r). In addition, free reserves are also affected by the liquidity ratio (l), i.e. the ratio of liquid assets to deposits. The banks tend to hold more excess reserves if the cost of borrowing reserves, r^d, goes up, or the yield goes down. Thus, in addition to the unborrowed monetary base, ($r^d - r$) and l also enter the money-supply function.

For the demand for money function, apart from income and short-term rate of interest, the net worth of the private sector is also used as an explanatory variable. Net worth is a proxy for wealth because no reliable time series of wealth is available in India. Another explanatory variable, p^e, the expected rate of change in prices was also tried in this function in the first instance but was dropped because of its multi-collinearity with r.

A link between the real and the monetary sectors is established with a term-structure of interest rate function. This postulates the long-term interest rate as a distributed lag function of short-term rate. A Koyck transformation is used resulting in the lagged long-term rate on the right-hand side apart from the short-term rate. Since the short-term rate enters the money-demand function and the long-term rate the investment function, a link between the real and monetary sectors is established via this interest rate function.

From the structural equations, a matrix of derived reduced-form coefficients is obtained for purposes of forecasting. Impact multipliers, elasticities corresponding to impact multipliers, "delay" and total multipliers have also been calculated.

Based on the magnitudes of impact and delay multipliers and corresponding elasticities calculated at mean levels, it is contended that fiscal policy variables (E_n and T_r) have larger effects on income in the short run while the monetary policy variable (U, r^d and l) are more effective in the long run.

Sample period forecasts are found to be satisfactory, while the margin of error is larger for beyond sample forecasts.

8.2.3 Estimates of Stochastic Equations

2SLS estimates with the sample period 1949-50 to 1967-68 for the stochastic equations of the system are given below.

1. $$C = .428 + \underset{(14.023)}{.848}\, Y^d \qquad \bar{R}^2 = .998 \quad d=2.243$$

2. $$I = .655 + \underset{(3.838)}{.089}\, Y - \underset{(-2.070)}{.175}\, r^1 + \underset{(.654)}{.189}\, I_{-1} \qquad \bar{R}^2 = .744 \quad d=2.177$$

3. $$T = .401 + \underset{(4.414)}{.130}\, Y + \underset{(1.933)}{.049}\, t \qquad \bar{R}^2 = .940 \quad d=1.176$$

4. $M = .472 + .104\, Y^d - .082\, r + .264\, NW$ $\qquad \bar{R}^2 = .995 \quad d=1.262$
$\qquad\qquad (4.421) \quad (-2.160) \quad (6.816)$

5. $M = -1.268 + 1.832\, U - .138\, q + .088\, l$ $\qquad \bar{R}^2 = .997 \quad d=1.859$
$\qquad\qquad (35.606) \quad (-4.245) \quad (6.439)$

6. $r^1 = 1.572 + .162\, r + .689\, r^1_{-1}$ $\qquad \bar{R}^2 = .408 \quad d=1.501$
$\qquad\qquad (.468) \quad (2.661)$

There are two equations with lagged dependent variables on the right hand side (eqns. 2 and 6). In these, simple 2SLS estimates are not appropriate. Further, the interest rate equation is the only link between the real and monetary sectors, but its $\bar{R}^2$ is very low (= .41)

The procedure of taking monetized levels and income and expenditure components is a unique feature of the model: but it necessitates some arbitrary estimation of monetization in the economy and imposes the assumption that there is no interaction between the monetized and non-monetized parts of the economy.

8.3 Pandit Model

Pandit (1973) has suggested a model of 16 equations out of which 11 are stochastic. The model is estimated with 2SLS over the sample period 1950-51 to 1965-66. It is built within the familiar income-expenditure approach with a view to predict variations in the GNP components, prices, interest-rate and wages.

8.3.1 Specification of the Model

(a) Stochastic Equations

1.	Private real consumption	C^p	$= f(Y^d, \theta)$
2.	Private real fixed investment (gross)	I^p	$= f(X, I^p_{-1})$
3.	Real imports	I_m	$= f(I, P_i/P_f)$
4.	Real exports	E	$= f(P_e/Q, I_w, Z)$
5.	Unit value index of exports	P_e	$= f(P_m, P_{e_{-1}})$
6.	Money supply	M	$= f(D_{-1}, r)$
7.	Short term interest rate	r	$= f(V_n, s)$
8.	Index of wage rate prevailing in factories and plantations	W	$= f(X_n, P_m, L)$
9.	Index of the price of manufactures	P_m	$= f(W, P_r, \pi)$
10.	Index of food prices	P_f	$= f(Y, F_{-1}, P_r)$
11.	Change in implicit GNP deflator	ΔP	$= f(\Delta P_f, M/X)$

(b) Identities and Definitions

12.	Real Disposable income	Y^d	$= Y - T^d/P$
13.	Real income (real net national product	Y	$= X - Dep$
14.	Real gross national product (where $H = C^g + I^g + I_i + Dep + S_d$)	X	$= C^p + I^p + E - I_m + H$

15. Real gross non-agricultural product $X_n = X - X_a$

16. Money velocity with respect to non-agricultural income $V_n = P.X_n/M$

The model has 15 (13 current and 2 lagged) exogenous variables. These are:

I = Real fixed investment (gross)

P_i = Unit value index of imports

Q = Unit value index of Asian exports

I_w = World imports excluding those of Oceania, South Africa and Latin America

Z = Dummy variable for the Korean War

S = Total security holdings by the public

L = Employment in factories and plantations

P_r = Index of the prices of raw material

Π = Index of industrial profits

T^d = Direct taxes

Dep = Depreciation

H = Sum of real government consumption and investment (gross) expenditure, total investment in inventories, depreciation and a statistical discrepancy term (S_d)

X_a = real gross agricultural output

D_{-1} = deficit expenditure defined as total expenditure minus total revenue by the central government in the previous year

F_{-1} = total domestic output of foodgrains in the previous year

The lagged endogenous variables of the system are: I^{p}_{-1} and $P_{e_{-1}}$

The ratio of agricultural to non-agricultural output (Θ) is taken as exogenous. It should have defined in an identity ($\Theta = X_a/X_n$) with the system as it uses one endogenous and one exogenous variable.†

Important features to note are that the entire government expenditure (consumption and investment) is taken to be exogenous. There is no determination of taxes within the system, i.e. tax-income relationships have not been incorporated. In effect, the government sector and its interrelations with the private sector have not been explored at all.

There are four price variables endogenous to the model, viz., P_e, P_m, P_f, ΔP. All expenditure and output variables are considered in real terms.

† The system contains 13 exogenous, 2 lagged exogenous and 2 lagged endogenous variables, i.e. 17 predetermined variables apart from the constant term. It is not clear how the system was estimated by 2SLS with only 15 observations (1950-51 to 1965-66) especially when it is made clear (Pandit, 1973, p.454) that no principal component method or other modifications were used for the first stage. Presumably in the first stage, endogenous variables were regressed only on the 13 current exogenous variables.

8.3.2 Considerations in Specification

(a) Consumption and investment functions

In the equation for real private consumption apart from real private disposable income, the ratio of agricultural income to non-agricultural income (Θ) is introduced on the ground that the marginal propensity to consume is higher in the former sector and hence any shift of the distribution of income in favour of the agricultural sector should lead to a rise in consumption.

Population, lagged consumption and liquid assets were also tried in the consumption function for preliminary investigation with OLS but were not found to be statistically significant.

The estimate of m.p.c. (0.73) is somewhat lower than earlier econometric studies. The author claims that partly the reason is the particular sample period chosen in this study compared to other studies.

The demand for gross private fixed investment is a function of GNP and its past values introduced via the lagged investment term. Other familiar explanatory variables for this function like the rate of interest, rate of profit and liquidity were tried for preliminary investigation but were not found to be statistically significant in OLS estimates. This may be due to the particular time series chosen to reflect some of these v ariables.

(b) Imports, Exports and Export-Prices

Although a less developed economy does not have a control over import prices in terms of international currency, it tries to manipulate this price by changes in exchange rate and by import taxes. India's policy has been one of considerable import control in most of the years in the sample. Whereas consumption goods except food imports have been restricted, the remaining imports are largely determined by investment requirements. The explanatory variables for import-demand are therefore chosen to be real investment, unit price of imports and domestic price level for foodgrains. In a year of food shortage, high food imports are thus explained by high domestic food prices.

In estimating export demand, an index of growth in world imports is introduced. A dummy variable is introduced to cover the extraordinary exports in years affected by the Korean War. In addition, a relative price variable, viz., unit price of Indian exports relative to the unit price of exports from other Asian countries is introduced in view of the consideration that these are the countries offering strong competition to Indian exports.

The export-price variable is also endogenously determined. It is said that the dominant part of India's exports consists of manufactured and semi-manufactured goods. The unit price of exports which is desired is thus a function of the price of domestic manufactured and semi-manufactured goods (P_m). However, since observed export price (P_e) is not necessarily equal to desired price (P_e^*) an adjustment mechanism is needed.

The adjustment which is implied in the model is as follows:

$$\Delta P_e = \lambda(P_e^* - P_{e_{-1}})$$

$$P_e^* = \alpha + \beta\, P_m$$

Hence,

$$P_e = \lambda\alpha + \lambda\beta\, P_m + (1-\lambda)\, P_{e_{-1}}$$

From the estimated coefficients, $\lambda = 0.66$, $\alpha = 40.30$, $\beta = 0.51$. The value of λ implies that two-thirds of the adjustment takes place in one year.

(c) Money Supply and Short-Term Rate of Interest

The RBI definition of money supply, viz., that it is currency with the public plus demand deposits plus that part of time-deposits which can be withdrawn without notice, has been adopted. The bazar bill rate which is the discount-rate of 3 months trade bills has been adopted as the short-term rate of interest.

Money supply is affected, apart from this rate of interest, by the deficit expenditure by the central government in the previous year. The deficit is defined simply as expenditure minus revenue for the central government. The author admits that the lag should be less than a year. The estimated coefficient is less than unity for this deficit variable but this must be due to the particular definition adopted for deficit expenditure. Part of this deficit is met by borrowing and not by increasing the supply of high-powered money.

The equation determining the rate of interest is really a demand for money equation. The transactions demand for money is reflected by the velocity variable defined as one relating the real stock of money to real non-agricultural income. The only other variable in this demand for money function is stock of securities held by public (S) and this represents an alternative monetary asset.

(d) Wages and Prices

The wage rate in the organised sector of the economy is determined by the price of food items, real output in the non-agricultural sector and the level of employment. It is contended that it is only in this sector that a market for labour formally exists. Wage rates in other sectors like agricultural and informal organised sectors are not considered primarily because of lack of data.

In addition to the export prices, there are three other prices which find an endogenous determination in the model. These are, P_m, P_f and ΔP respectively, the prices of manufactures, food articles and change in the implicit deflator of GNP.

The price of manufactures is determined by a mark-up equation the cost of production as indicated by raw material prices and wages along with the margin of profits enter as explanatory variables. Demand considerations are not present on the assumption that demand exists for any level of prices and if there were to be a situation of deficient demand, producers would curtail production rather than/reduce prices given the monopolistic structure of the market.

The price of food is determined by both supply and demand considerations. It is thus looked upon as the reduced form of the following 3 equations:

Food demand	F^d	$= \phi(Y, P_f)$
Food supply	F^s	$= \psi(F_{-1}, P_r, P_f)$
Market clearing Identity	F^d	$= F^s$

Thus,

$$P_f = f(Y, P_r, F_{-1})$$

The demand function assumes that food demand is price elastic. The supply function assumes that there is/some lag in market arrivals and a portion of last year's output is sold this year.

The changes in general price level in this model are related to (i) variations in money supply per unit of real output and to (ii) changes in the level of food prices. Thus,

$$\Delta P = \alpha \left[\frac{M}{X} - \left(\frac{M}{X}\right)_0 \right] + \beta \Delta P_f$$

$$= \gamma + \frac{\alpha M}{X} + \beta \Delta P_f$$

It turns out that food prices are a major determinant of the general price level as the second variable explains the major portion of variations in the general price level. The first term is a quantity theory term and it plays only a marginal role in the determination of P.

$\left(M/X\right)_0$ is the money supply-real GNP ratio which is consistent with the stationary price level ($\Delta P = 0$) provided P_f is constant ($\Delta P_f = 0$). The estimated coefficients imply this ratio to be equal to about 18.5 per cent.

8.3.3 Choice of the Model

The author finds that a 'modified' Keynesian framework would be suitable for LDCs. He finds the effective demand is a crucial determinant of nominal as also real income, and although some effect on the price level of changes in effective demand is also there. For explaining inflation he gives prominence to the 'structural' hypothesis in view of the consideration that structural factors like availability of foodgrains are crucial to the course of inflation.

In order further to test the applicability of Keynesian and quantity theory frameworks, the author conducts experiments similar to those of Friedman and Meiselman (1963). He fits a set of 3 alternative models for the autonomous expenditure hypothesis and the quantity of money hypothesis. The root mean squared errors in each case are given in Table 8.1.

Table 8.1.

Pandit's Model: Preliminary Tests of Hypotheses

(Y = GNP at current prices)	Root Mean Squared Errors		
Models	$Y= a+bX$	$Y= a+bX+CY_{-1}$	$Y= a+b\Delta X$
1. X = autonomous expenditure at current prices	6.47	5.29	6.54
2. X = nominal money supply	7.36	6.63	8.60

From this it is concluded that the superiority of either specification is not clearly indicated.

8.3.4 Estimates of Stochastic Equations

The 2SLS estimates of the 11 stochastic equations are given below. Figures in brackets are t-values.

	Equation	d	$\bar{R}^2$
1.	$C_p = \underset{(-0.47)}{-9.85} + \underset{(9.02)}{0.73}\, Y^d + \underset{(1.47)}{0.18}\theta$	1.16	0.96
2.	$I_p = \underset{(-0.84)}{-3.15} + \underset{(2.88)}{0.07}\, X + \underset{(2.42)}{0.52}\, I_{p-1}$	1.88	0.88
3.	$I_m = \underset{(3.05)}{15.93} + \underset{(1.11)}{0.12}\, I - \underset{(-2.79)}{0.10}\, P_i/P_f$	2.44	0.89
4.	$E = \underset{(8.57)}{9.39} - \underset{(-3.69)}{0.05}\, P_e/Q + \underset{(5.56)}{0.02}\, I_w + \underset{(2.13)}{0.63}\, Z$	1.97	0.70
5.	$P_e = \underset{(1.74)}{26.40} + \underset{(3.38)}{0.34}\, P_m + \underset{(1.71)}{0.35}\, P_{e-1}$	0.92	0.71
6.	$M = \underset{(-4.31)}{-10.73} + \underset{(2.40)}{0.52}\, D_{-1} + \underset{(4.84)}{1.05}\, r$	1.50	0.64
7.	$r = \underset{(1.82)}{4.26} + \underset{(1.65)}{0.02}\, V_n + \underset{(6.57)}{0.06}\, S$	1.06	0.83
8.	$W = \underset{(4.25)}{30.49} + \underset{(2.81)}{1.27}\, X_n + \underset{(1.79)}{0.16}\, P_m - \underset{(-1.10)}{0.64}\, L$	1.27	0.96
9.	$P_m = \underset{(5.08)}{38.32} + \underset{(2.77)}{0.39}\, W + \underset{(3.72)}{0.25}\, P_r + \underset{(1.74)}{0.06}\, \Pi$	1.56	0.97
10.	$P_f = \underset{(-0.58)}{-5.94} + \underset{(3.62)}{0.83}\, Y - \underset{(-2.62)}{0.84}\, F_{-1} + \underset{(5.92)}{0.62}\, P_r$	1.84	0.94
11.	$\Delta P = \underset{(-2.18)}{-10.47} + \underset{(5.24)}{0.41}\, \Delta P_f + \underset{(2.29)}{0.56}\, M/X$	2.50	0.85

8.3.5 Sample-period Simulations

Static and dynamic simulations were performed with the model over the sample period. Estimated values for two variables, viz., price level and real output have been reported. Static simulations are made by feeding in actual values of lagged endogenous variables and dynamic simulations, by feeding values of these variables as estimated from the model. Dynamic predictions were made with 1953-54 as the base. The largest margin of error in these cases is about 10 per cent of the true value. Mostly the error is around 5 per cent for both variables. For static estimates the worst predictions err by about 6 per cent, and in general, the error is about 3 per cent for both variables.

8.3.6 Policy Simulations

In order to compare the effectiveness of alternative policy measures 1964-65 was adopted as the base year. Alternative assumptions were made. Estimated values for real GNP and the price-level under the assumptions listed below are given in Table 8.2.

(i) Direct taxes are raised to an amount equal to 10 per cent of actual money GNP and resources thus raised are used to augment government investment expenditure.

(ii) Government investment is restricted to Rs 12.0 billion in 1965-66 as well as in 1966-67.

(iii) Foodgrains output increases in 1964-65 and 1965-66 by 5 per cent over the previous year.

(iv) All exogenous variables are taken at their actual levels.

Table 8.2

Pandit's Model: Policy Simulations

	Real GNP		Price Level	
	1965-66	1966-67	1965-66	1966-67
(A)	167.20	157.47	131.86	151.61
(B)	150.72	135.10	128.04	147.85
(C)	165.00	162.46	132.93	147.31
(D)	165.22	161.56	140.35	162.30
Actual	160.23	163.07	135.40	154.33

Direct taxes seem to have little effect. Public investment has a substantial effect. Foodgrains output has a substantial effect on inflation. Public expenditure affects both prices and income.

8.4 Gupta Model

Gupta's (1973) model is predominantly that of the monetary sector of India with a view to analysing the role of monetary policy. Some real sector equations are introduced only because this sector contains most of the target variables. The monetary sector is considerably more disaggregated compared to the two other models emphasizing the monetary sector, viz., Mammen and Bhattacharya models.

The model is divided into four sectors, viz., (i) government including the Reserve Bank of India, (ii) commercial banking, (iii) State co-operative banking, and the private non-banking sector. The subscripts g, cb, sb and p are used respectively to distinguish between variables relating to these sectors. Budget constraints are recognised for each sector. It is one of the few models in the Indian context which makes the government budget constraint an integral part of the study.

The model contains 6 kinds of financial assets, viz., currency, bank, reserves, government bonds, demand deposits, time deposits and private non-banking liabilities (bank loans). In each case a demand, supply and a market-clearing condition is considered. Apart from these, components of national income and other balance sheet identities have been included.

In all there are 42 equations out of which 17 are stochastic equations. 27 endogenous variables relate to the monetary sector and 15 to the real sector. Out of the 27 equations relating to financial endogenous variables 13 are stochastic, and out of the 15 equations for the real sector variables, only 4 are stochastic.

There are 26 current exogenous and 4 lagged exogenous variables. These contain 9 monetary policy variables and 5 fiscal policy variables. There are 14 lagged endogenous variables.

The model is estimated from a sample period 1948-49 to 1967-68 which provides 20 annual observations. Thus, the sample relates to the pre-bank-nationalization period. The model is estimated by a modified 2SLS procedure which uses a subset of predetermined variables in the first stage. For different type of equations, different subsets were used each containing 13 predetermined variables.

The causal structure of the model is simultaneous and all stochastic equations are overidentified by the order condition.

8.4.1 Specification of the Model

(a) Stochastic Equations of the Financial Sector

Demand for excess reserves of -

1. Commercial banks $ER^{cb} = f(DD^{cb}, DT^{cb}, i_g, i_1, i_{B1}, ER^{cb}_{-1})$

2. State co-operative banks $ER^{sb} = f\left[(DD^{sb} + DT^{sb}), (i_g - i_B), ER^{sb}_{-1}\right]$

Demand for government bonds by -

3. Commercial banks $GS^{sb} = f\left[DD^{cb}, DT^{cb}, i_g, i_1, GS^{cb}_{-1}\right]$

4. State co-operative banks $GS^{sb} = f\left[(DD^{sb} + DT^{sb}), i_g, GS^{sb}_{-1}\right]$

Demand for private non-bank liabilities by -

5. Commercial banks $LI^{cb} = f\left[DD^{cb}, DT^{cb}, i_g, i_1, LI^{cb}_{-1}\right]$

6. State co-operative banks $LI^{sb} = f\left[(DD^{sb} + DT^{sb}), i_g, LI^{sb}_{-1}\right]$

Borrowing from RBI (supply) of -

7. Commercial banks $B^{cb} = f\left[(DD^{cb} + DT^{cb}),\ (i_1 - i_{B1}),\ d_B,\ B^{cb}_{-1}\right]$

8. State co-operative banks $B^{sb} = f\left[(DD^{sb} + DT^{sb}),\ (i_g - i_B),\ B^{sb}_{-1}\right]$

9. Time deposit rate† $i_T = f\left[DD^{cb},\ DT^{cb},\ i_g,\ i_1,\ i_{T_{-1}}\right]$

10. Private non-bank currency holdings (demand) $CU^p = f\left[(Y,\ i_T,\ T)\right]$

11. Private non-bank demand deposits demand $D^b = f\left[Y_{na},\ i_1\right]$

12. Private non-bank time deposits' demand ≠ $T^b = f\left[Y_{na},\ i_T,\ i_g,\ i_1, T\right]$

13. Private non-bank borrower's own liabilities (supply= total bank credit) $LI = f\left[Y_{na},\ i_1,\ T\right]$

This equation is used to determine i_1, LI being determined in equation 22.

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† Defined as a simple average of the average rate on 3-months time deposits with the State Bank of India and that with the other major scheduled commercial banks in Bombay, Calcutta and Madras.

≠ Net time deposits held by the private non-bank sector less P.L. Funds' counterpart deposits.

(b) Stochastic Equations of the Real Sector*

14. Real non-agricultural income (X_n) $\ln X_n = f\left[\ln \frac{k+k_{-1}}{2}, T\right]$

15. Private consumption expenditure $CO^p = f\,(Y_d)$

16. Private (net) investment demand $I^p = f\left[Y_{-1}, i, (LI-LI_{-1}), BPD\right]$

17. Imports** $IM = f\,(Y_d, IM_{-1})$

(c) Financial Sector Identities

18. All banks' required reserves $RR^b = r_d^{cb}(\alpha_1 D^{cb}) + r_d^{sb}(\alpha_2 D^{sb}) + r_t^{cb}(\beta_1 T^{cb}) + r_t^{sb}(\beta_2 T^{sb})$

19. Unborrowed reserve money $URM = CU^p + RR^b + ER^{cb} + ER^{sb} - B^{cb} - B^{sb}$

20. Government securities (to determine GS^p - govt. securities held by the private sector) $GS = FS^{cb} + GS^{sb} + GS^p$

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* X_n is measured at 1948-49 prices; CO^p, I^p, and IM are measured at current prices.

** Includes earned income paid abroad.

21. All (net) time deposits $T^b = \hat{T}^b + PLF$

22. Private non-bank liabilities $LI = LI^{cb} + LI^{sb} + LI^g$

23. Commercial bank demand deposits $D^{cb} = D^b - D^{sb}$

24. State co-operative bank demand deposits $D^{sb} = \alpha D^{cb}$

25. Commercial bank time deposits $T^{cb} = T^b - T^{sb}$

26. State co-operative bank time deposits $T^{sb} = \beta T^{cb}$

Disposable demand deposits -

27. of commercial banks $DD^{cb} = D^{cb} - r_d^{cb}\ (\alpha_1 D^{cb})$

28. of State co-operative banks $DD^{sb} = D^{sb} - r_d^{sb}\ (\alpha_2 D^{sb})$

Disposable time deposits -

29. of commercial banks $DT^{cb} = T^{cb} - r_t^{cb}\ (\beta_1 T^{cb})$

30. of State co-operative banks $DT^{sb} = T^{sb} - r_t^{bs}\ (\beta_2 T^{sb})$

31. Government budget constraint:

$$CO^g + I^g + GS.\frac{ig}{100} = T_d + (T_i - S) + \Delta(URM) + \Delta GS - \Delta(LI^g) + \Delta(OS^g)$$

(d) Real Sector Identities

32. Direct taxes $T_d = t_d.Y$

33. Indirect taxes less of subsidies $(T_i - S) = t_i.CO^p$

34. Aggregate demand (income) at current prices

$$Y = CO^p + I^p - (T_i - S) - IM + EX + CO^g + I^g$$

35. Aggreage supply (output) at constant price

$$X = X_{na} + X_a$$

36. Private nominal disposable income $Y_d = Y + U - T_d$

37. Nominal non-agricultural output $Y_{na} = Y - P_a X_a$

38. NNI price deflator $P_y = Y/X$

39. Non-agricultural income price deflator $P_{na} = Y_{na}/X_{na}$

40. Agricultural income price deflator $P_a = \gamma P_y$

41. Aggregate real capital stock $k = k_{-1} + I/P_y + I^g/P_y$

42. Balance of payments $BPD = IM - EX$

The exogenous variables of the system belong to three categories: (i) monetary policy, (ii) fiscal policy, and (iii) others. These are listed below.

Monetary policy variables:

1. i_{B1} = weighted average (effective) bank rate on borrowings by the scheduled commercial banks from RBI

2. i_B = bank-rate

4. d_B = Bill Market Scheme dummy variable†

5. GS = Total stock of government securities held outside the government sector.

6. r_d^{cb} = Reserve requirements ratio against demand liabilities of commercial banks

7. r_d^{sb} = Reserve requirements ratio against demand liabilities of State co-operative banks

8. r_t^{cb} = Reserve requirements ratio against time liabilities of commercial banks

9. r_t^{sb} = Reserve requirements ratio against time liabilities of State co-operative banks

Fiscal Policy Variables

10. CO^g = Government current (consumption expenditure) on commodities and services at current price

11. I^g = Government investment expenditure at current prices

12. t_d = Direct tax rate (= T_d/Y)

13. t_i = Indirect tax (net of subsidies)

14. OS^g = Net "other" (miscellaneous) sources of government finance (equation 31)

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† Constructed on author's subjective evaluation of the qualitative changes in the Bill Market Scheme. d_B = 0 for 48-49, 49-50, 50-51

d_B = 1 for 1951-52. For remaining years 1952-53 through 1967-68, the assigned values are 1, 1.1, 1.42, 1.52, 1.52, 1.92, 2.02, 2.12, 2.12, 2.42, 2.42, 2.42, 2.42 and 2.42 respectively.

Other Exogenous Variables

15. α = ratio of D^{sb} to D^{eb}

16. β = ratio of T^{sb} to T^{cb}

17. α_1 = ratio of demand liabilities to net demand deposits of commercial banks

18. α_2 = ratio of demand liabilities to net demand deposits of State co-operative banks

19. β_1 = ratio of time liabilities to net time deposits: commercial banks

20. β_2 = ratio of time liabilities to net time deposits for State co-operative banks

21. PLF = time deposits of banks held by the U.S. Government in counterpart to PL Funds receipts.

22. X_a = Agricultural income at 1948-49 prices

23. EX = Exports of commodities and services (including earned income received from abroad) at current prices

24. U = National debt interest + transfer payments + net private donations from abroad - income from domestic product accruing to government, at current prices.

25. γ = ratio of P_a to P_y.

26. K^P_{-1} = Private capital stock at the end of the previous period at current prices†

27. T = Time-trend

Three other lagged exogenous variables are LI^g_{-1}, S^g_{-1} and GS_{-1}

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† Defined as a multiple of NNI price deflator, and aggregate real net capital stock less cumulated government real net investment beginning with the year 1919 i.e. $k^g_{1918} = 0$, where k^g is the government real capital stock.

Lagged Endogenous Variables

The lagged endogenous variables in the model are given below.

ER^{cb}_{-1}, ER^{sb}_{-1}, GS^{cb}_{-1}, GS^{sb}_{-1}, LI^{cb}_{-1}, LI^{sb}_{-1}, B^{cb}_{-1}, B^{sb}_{-1}, $(i_T)_{-1}$, k_{-1},

Y_{-1}, LI_{-1}, IM_{-1},

8.4.2 Considerations in Specification and Interpretation of Equations

There are six markets in the monetary sector. These relate to bank reserves, currency, government bonds, demand deposits and time-deposits.

For bank reserves, components of demand are specified in equations 12 and 18, and those of supply, in equations 7 and 8. The supply of unborrowed reserves is taken as exogenous. There is no demand function for RBI's (lender's) demand for bank borrowings which is assumed to be unlimited.

The market for currency is represented by equation 10 alone. As this asset has zero yield, both demand and supply are denoted by the same notation.

The market clearing conditions for bank reserves and currency are represented together in equation 19. The government sector is assumed to supply a given amount of unborrowed reserve money (unborrowed bank reserves plus currency), the distribution of which into its components is determined by its holders.

For government bonds, the demand is given by equations 3 and 4 and the market clearing condition in equation 20. Supply is assumed exogenous.

For demand deposits, supply is assumed unlimited and demand alone is identified in equation 11. No market clearing is required. For time deposits, demand is represented in equation 12, supply in equation 9 and equilibrium condition is given in equation 21. Demand for time deposits comes from the private non-bank sector and U.S. Government (P.L. funds counterpart deposits). The latter variable is assumed exogenous.

The market for private non-bank liabilities is defined by equations 5 and 6 (demand), 13 (supply) and 22 (market clearing condition). Government sector's demand for private non-bank liabilities is assumed exogenous.

Equations 23-30 are definitional equations of the money sector. The government budget constraint is given by equation 31.

The policy-variables in this budget constraint are CO^g, I^g, GS, t_d, t_i, URM, LI^g and OS^g. The constraint limits one degree of freedom by implying the endogenous determination of one of the variables it contains. URM is chosen as endogenous with a view to analyse the effects of government tax rates, government expenditures, open market operations etc. on the target variables by leaving them exogenous. The change in the stock of government securities held outside the government sector (ΔGS) is taken to measure open market operations.

In the real sector of the model there is a supply equation for non-agricultural output and demand equations for private consumption, private investment and imports. Agricultural output is exogenous. Tax equations are introduced such that income is taken as the base of direct taxes and private consumption, the base of indirect taxes. Tax revenues and bases are related in identities by rates t_d and t_i which are ex-post average rates rather than 'true' policy variables.

The monetary sector affects the real sector through the equations for private investment where the bond yield and availability of bank loans (i.e. change in private non-bank liabilities) appear.

The real sector affects the monetary sector via

(i) demand for currency (eqn. (1) where NNI enters

(ii) private non-bank sector's demand for demand deposits (eqn.11) where non-agricultural income enters

(iii) private non-bank sector's demand for time deposits (eqn.12) where non-agricultural income enters, and

(iv) the supply of private non-bank liabilities where also non-agricultural income enters.

Further, the government budget restraint has two endogenous monetary sector variables (i_g and URM) and two endogenous real sector variables $\left[T_d \text{ and } T_i - S\right]$.

8.4.4 Impact Multipliers

Since the model contains some non-linear relations, its reduced form was obtained under the following consecutive steps:

(a) differentiation of the estimated model, (b) linearization of its otherwise non-linear coefficients at the point of last sample period (i.e. 1967-68 values), and (c) solution of the resultant model for the endogenous variables.

The estimates of impact multipliers indicate that policy multipliers are high with respect to money magnitudes and prices, and rather low with respect to real magnitudes. Omission of the government budget constraint from the policy model approximately doubles the impact multipliers of monetary policy variables and halves those of fiscal policy variables.

In general, private non-bank liabilities with the government sector (i.e. RBI's loans to private non-bank through the State co-operative banks) is the most efficient monetary policy variable, and open market operations are more powerful than government investment expenditure with respect to financial variables, while quite the reverse holds true with respect to non-financial variables.

The study suggests that the RBI should go ahead in advancing loans to the agriculturists through the State co-operative banks, and though the government should continue its efforts to increase its investment expenditure, it should try to finance the same through increased indirect tax rate.

8.5 Summary

In this Chapter, three models of the Indian economy emphasizing the monetary sector of the economy have been brought together. These have been constructed respectively by Mammen, Bhattacharya and Gupta. The special feature of Mammen's work is the construction of a real sector model and a monetary sector model separately before joining them together. Bhattacharya's model uses 'monetized' income-expenditure levels. Gupta's model is highly disaggregated on the monetary side and highly aggregated on the side of the real sector.

Chapter 9

Models of the Indian Economy: III

In this chapter, three models are considered. Two of these have been built by the UNCTAD secretariat (1968, 1973) and one is built by Agarwala. In all these models supply side considerations have been incorporated.

9.1 Agarwala Model

Agarwala's (1970) effort is distinguished from other econometric models of India inasmuch as it is based on the aggregate supply approach and it incorporates the dualistic nature of the economy *vis-a-vis* industry and agriculture. His model is a system of 24 equations out of which 17 are stochastic. There are 13 exogenous variables and 7 lagged endogenous variables. Among the exogenous variables, 5 are under the control of the domestic government, 6 are under the control of the rest of the world, and 2 are non-economic.

The sample period for most of the estimates is of 13 years from 1948-49 to 1960-61. The whole model is estimated by OLS although for certain sub-systems 2SLS and LIML estimates are also provided.

The following functional relations and identities are postulated in the model. It is the only model which does not contain a consumption function.

9.1.1 Specification of the Model

(a) Stochastic Equations

1.	Agricultural Output	$Y_a = f(K_a, R_2)$
2.	Output of Foodgrains	$F_a = f(Y_a)$
3.	Non-agricultural output	$Y_i = f(K_i, L_i)$
4.	Employment in non-agricultural sector	$L_i = f(F_a, (M-X)_a)$
5.	Money supply	$M = f(C, t)$
6.	Money income	$pY = f(M, pY_{-1})$
7.	Price-Level of consumption goods	$p_c = f(p_a, p_{c_{-1}})$
8.	Price-Level of industrial goods	$p_i = f(p_{im})$
9.	Price-Level of manufactured goods	$p_{im} = f\left[w_i / \frac{\delta Y_i}{\delta L_1}, (p_{im})_{-1} \right]$
10.	Investment cost index	$p_{inv} = f(p_{im}, w_i)$
11.	Industrial wages	$w_i = f(p_c, (w_i)_{-1})$
12.	Investment in the agricultural sector	$I_a = f\left[\frac{p_a Y_a - T_a}{p_{inv}}, \frac{p_a}{p_{inv}} \right]$
13.	Investment in the industrial sector	$I_i = GC.\ f\left[\frac{p_i Y_i - T_i}{p_{inv}}, \frac{\dot{p}}{p} \right]$

14. Export price $p_X = f\left[p, p_{XU}\right]$

15. Exports $X = f\left[Y_w, p_X/p_{XU}\right]$

16. Imports of capital goods $M_K = f\,(I_i + G_i)$

17. Import of consumer goods as a proportion of non-agricultural output $\frac{M_c}{Y_i} = f\,(t)$

(b) Identities

18. Output $Y = Y_a + Y_i$

19. Price Level $p = p_a \cdot \frac{Y_a}{Y} + p_i \frac{Y_i}{Y}$

20. Value of Imports $p_m.MT = p_X.X + FC$

21. Imports $MT = M_K + M_c + M_o$

22. Government investment in non-agricultural sector $G_i = (I_i + G_i) - I_i$

23. Capital in agricultural sector $K_a = \dfrac{K_a^o + K_a^o + G_a + I_a}{2}$

24. Capital in industrial $K_i = \dfrac{K_i^o + K_i^o + G_i + I_i}{2}$

Income, output and other aggregates like Y_a, K_a, Y_i, I_a, I_i, MT, M_c, X, G_i, M_k, M_c, K_a^o, K_i^o are measured at 1948-49 prices. All price indices have 1948-49 as the base year except p_{XU}, the export price index of under-developed countries for which the base year is 1958. Y_w, the index of world GNP at constant factor cost has also 1958 as the base year.

The exogenous variables classified as under the control of domestic government are:

T_a = tax in agriculture; T_i = tax in the non-agricultural sector; G_a = government investment in agriculture; GC = government control on private investment indicated by capital issues sanctioned as a percentage of applications; and C = notes and currency.

Those under the control of the rest of the world are:

p_{XU} = export price index of underdeveloped countries; FC = net inflow of foreign capital at current prices; p_m = price index of imports of India; M_o = import of foodgrains and miscellaneous goods, $(M-X)_a$ = net imports of foodgrains.

The two non-economic exogenous variables are:

t = time-trend (1948-49 = 0), and R_2 = % deviation of actual from 'normal' rainfall.

The lagged endogenous variables are pY_{-1}, $p_{c_{-1}}$, $(p_{im})_{-1}$, $(w_i)_{-1}$ (p) and K_a^o and K_i^o . The last two refer respectively to capital in agriculture and non-agriculture at the beginning of the period.

9.1.2 Considerations in Specification

(a) Production Functions and Employment

Cobb-Douglas type production functions are used both for the agricultural and the industrial sectors. This is in preference to the fixed coefficients type or variable-elasticity of substitution type production functions. The former type do not allow for substitution of factors and the latter type are difficult to estimate. Cobb-Douglas production functions permit substitution of factors and diminishing returns to factors.

Data used for production is that of value-added. For labour, total number of persons in employment rather than number of days is used. A distinction between skills is not drawn. For capital, a variation of the perpetual inventory method is used. In this method an inventory is maintained at the base year prices by adding gross investment and by deducting capital consumption annually. For capital consumption depreciation figures are used.

In the agricultural sector, production is constrained by only two factors, viz., capital (K_a) and climate. This latter factor is taken into account by an index of variation in rainfall (R_2) which is the absolute value of percentage deviation of actual annual rainfall from an estimated normal annual rainfall.

Foodgrains production is calculated simply as a function of the value-added in agriculture.

Employment in the non-agricultural is seen to depend on the availability of foodgrains which is the sum of domestic foodgrains production (F_a) and net imports of foodgrains $(M-X)_a$.

In estimating the non-agricultural production, an additional restriction is made, viz., that there are constant returns to scale. Otherwise, the sum of the elasticities of output with respect to labour and capital came out to be unrealistically low.

(b) Money-income, Money-supply, Wages and Prices

It is contended that the quantity theory is applicable in the Indian context and that the income-velocity of money may be stable although one has to allow for changing degrees of monetization. Various forms were tried to test the stable velocity hypothesis and ultimately the Friedman-Meiselman form with a distributed lag mechanism using a Koyck transformation was considered most appropriate.

An attempt is made to determine money-supply endogenously with the help of notes and currency (C) and a time-trend (t) as the explanatory variables. Two other variables, viz., the rate of change in prices $\dot{p}/p$ and the rate of interest i were also tried but were dropped on statistical grounds.

Money wages are worked upon with the hypothesis that real wages are constant. This hypothesis is substantiated with data up to 1930. After 1930 there is an upward trend in real wages but this is explained as primarily due to a fall in prices during the Great Depression. On the whole, money-wages seemed to move only with prices. Various forms are tried for estimating the money-wage price relationship and the one which is ultimately chosen incorporates a distributed lag hypothesis with a Koyck transformation.

Various internal prices and world-demand forces were tried for explaining the export prices. In general, internal prices offer a poor explanation of changes in export prices. The equation finally selected contains the general price-level (p) and price of exports of under-developed countries.

(c) Investment and Foreign Trade

Demand is not considered to be a limiting factor for investment in the agricultural sector, hence the acceleration hypothesis is not considered valid here. A lag structure in investment was not favoured on statistical grounds. Government expenditure in agriculture was also not found to be statistically significant. The yield on 3 per cent bonds was used as a proxy for rate of interest in the rural sector for want of any other reliable series. But this was also not found to be statistically significant. The equation finally chosen contains disposable agricultural income and a relative price variable (p_a/p_{inv}).

For the investment function in the non-agricultural sector three hypotheses were considered: the interest-rate hypothesis, the profit hypothesis, and the acceleration hypothesis. The first approach did not prove promising. Individually, the latter two approaches did well. But when they are combined the results were not satisfactory. Ultimately, the profit index as an explanatory variable was chosen in preference to rate of change of income on grounds that it was slightly superior statistically and was considered more plausible theoretically in a less developed country where supply constraints are more relevant. As an indicator of supply of investible funds, disposable income of the non-agricultural sector deflated by the investment price index $(p_iY_i - T_i)/p_{inv}$ was considered relevant. In addition, price expectations were introduced by $\dot{p}/p$ and the influence of government control on private investment was brought about the variable, GC, which has been defined earlier.

In estimating an export function, an index of world income (Y_W) is used to indicate the level of economic activity in the rest of the world. For incorporating relative prices, the ratio of the price index of Indian exports and the export price index of underdeveloped countries is used (p_X/p_{XU}). The underdeveloped countries considered were India's competitors in the world market.

Imports are considered in three parts: imports of capital goods (M_k); import of consumer goods (M_c), and imports of foodgrains and miscellaneous goods (M_o). While M_o is taken to be exogenous to the system, M_k and M_c are endogenously determined. Foreign exchange position is tried as an explanatory variable but the expected signs are not obtained. Imports seem to be increasing while foreign exchange reserves are falling. Introduction of a lag did not help either. So the hypothesis that imports are dependent upon the capacity to import is dropped. Rather, investment in the non-agricultural sector ($I_i + G_i$) is taken to be a determining factor for import of capital goods, and income and a time trend for import of consumer goods.

9.1.3 Estimates of Stochastic Equations

Unless otherwise mentioned, the sample period is 1948-49 to 1960-61. OLS estimates of the stochastic equations are given below. Figures in brackets are standard errors.

		$\bar{R}^2$	δ^2/s^2
1.	$\log Y_a = \underset{(.575)}{2.9142} + \underset{(.0785)}{.6543} \log K_a - \underset{(0.0785)}{.0347} \log R_2$	.72	1.86
2.	$F_a = \underset{(780.48)}{-38.23.301} + \underset{(.1554)}{2.0675} Y_a$	.94	.977
3.	$\log \frac{Y_i}{L_i} = - \underset{(.0734)}{.4407} + \underset{(.0628)}{.5599} \log \frac{K_i}{L_i}$	.87	.71

from which

$$\log Y_i = -.4407 + .5599 \log K_i + .4401 \log L_i$$

4.	$L_i = \underset{(279.1535)}{1803.044} + \underset{(.0393)}{.3347}\, F_a + \underset{(.3044)}{1.1844}\, (M-X)_a$	.87	2.37
5.	$M = \underset{(14.260)}{-59.545} + \underset{(.028)}{1.466}C + \underset{(.85)}{1.405}\, t$	.995	.213
	(1901/2 - 1960/61)		
6.	$pY = \underset{(60.35)}{115.3565} + \underset{(.213)}{1.14997}\, M + \underset{(.0478)}{.8005}\, (pY)_{-1}$	.992	2.147
	(1901/2 - 1960/61)		
7.	$p_c = \underset{(16.196)}{-20.7097} + \underset{(.1212)}{.37629}\, p_a + \underset{(.1495)}{.84368}\, (p_c)_{-1}$	.85	1.71
8.	$p_i = \underset{(8.004)}{73.1858} + \underset{(.0726)}{.27133}\, p_{im}$	.55	.82
9.	$p_{im} = \underset{(19.498)}{3.105} + \underset{(23.23)}{25.26}\, w_i/(\delta Y_i/\delta L_i) + .7068(p_{im})_{-1}$	.74	1.96
10.	$p_{inv} = \underset{(5.21)}{-3.0835} + \underset{(.0785)}{.7858}\, p_{im} + \underset{(.0314)}{.2354}\, w_i$	.99	2.14
11.	$w_i = \underset{(9.57)}{-2.5414} + \underset{(.0673)}{.2683}\, p + \underset{(.0562)}{.7702}(w_i)_{-1}$	.99	1.81
	(1939/40 - 1961/2)		
12.	$I_a = \underset{(62.95)}{159.71} + \underset{(.012)}{.055}\, Y_{da} - \underset{(57.61)}{300.357}\, p_a/p_{inv}$	.76	1.36
13.	$I_i = \frac{GC}{100}\left[\underset{(525.77)}{-1219.723} + \underset{(.1105)}{.3633}\, (p_iY_i - T_i)/p_{inv} + \underset{(9.143)}{20.9242}\, \frac{\dot{p}}{p}\, 100\right]$	.75	2.85
14.	$p_X = \underset{(54.65)}{-219.7205} + \underset{(.412)}{1.2599}\, p + \underset{(.3123)}{1.9791}\, p_{XU}$	.79	2.085
15.	$X = \underset{(146.79)}{575.3957} + \underset{(.8764)}{2.0848}\, Y_w - \underset{(115.47)}{208.5028}\, p_X/p_{XU}$	.35	2.775
16.	$M_k = \underset{(41.771)}{59.9567} + \underset{(.0535)}{.2778}\, (I_i + G_i)$	.68	1.75
17.	$\frac{M_c}{Y_i} = \underset{(.00315)}{.05523} - \underset{(.000445)}{.00227}\, t$	.67	2.497

The model is estimated with OLS. Simultaneous equation methods of estimation were not applied because of the non-uniformity of the sample and because of having 20 predetermined variables with 12 common observations. As an exploratory exercise two smaller sub-systems were estimated with simultaneous equation methods.

9.1.4 Applications of the Model

The entire system is not solved for the reduced-form in order to obtain impact multipliers etc. This is because of a number of non-linearities in the model. Linearization of the non-linear equations was also not favoured. Instead a simulation approach is followed whereby feeding in specified values of exogenous variables, year-by-year values of endogenous variables are obtained for the sample period. No beyond sample forecasting has been done. Predictions are generated within the sample period using the simultaneous equation system and feeding back predicted values of the lagged endogenous variables rather than their actual values. The random term in each independent equation is ignored, although it is recognised that this may create problems for a non-linear system because although disturbances may be individually zero on average their covariation need not be zero on the average. In addition, the error terms may be serially correlated.

Since the results of simulation will depend to a certain extent on the year with which simulation begins, it was chosen by turn/ to use all the years in the sample beginning from 1949-50 as the initial years.

9.2 UNCTAD I Model

The UNCTAD (1968) model was constructed primarily for projecting internal and external resource requirements of the Indian economy. It approaches the determination of expenditure from the savings side rather than the consumption side which has been the usual practice. It is contended that data on savings is more reliable. It also permits a breakdown between household, corporate and government savings.

The model contains 32 equations in 31 endogenous variables. Its overdeterminancy is however, intended. It gives rise to a domestic resources gap and a trade gap by suppressing alternative equations. In all there are 17 stochastic equations.

The model is estimated with OLS. The number of observations vary but the sample starts at 1950-51 and extends in some cases to 1964-65. In other cases it stops earlier.

9.2.1 Specification of the Model

Variables at current prices are marked with an asterisk. Other income-expenditure variables are at 1960/1 prices.

(a) Stochastic Equations

1.	National income (to determine investment requirements)	$Y = f(CI_{-1})$
2.	Agricultural income	$Y_a = f(Y)$
3.	Government savings	$S_g = f(T_X, S_{g_{-1}})$
4.	Corporate savings	$S_c = f(Y_m)$

5. Household savings $S_h = f(Y, P_{nya}/P_{ya}, S_{h_{-1}})$

6. Jute exports $X_j = f(X_w, P_{xj})$

7. Other exports $X_o = f(X_w, P_{xo}/P_{zow})$

8. Unit value index of other exports $P_{xo} = f\left[P_{wnf}, \frac{(X_o)_{-1}}{Y_{anfm}}\right]$

9. Non-food imports $M_{of} = \text{Minimum } (M^1_{of}, M^2_{of})$ where

$$M^1_{of} = f(Y, X.\frac{P_x}{P_m}.R) \text{ and } M^2_{of} = f(Y)$$

10. Agricultural income deflator $P_{ya} = f(\frac{Y_{a_{-1}}}{Y_{na}}, P_{ya_{-1}})$

11. National income deflator $P_y = f(P_{ya}, L/Y, P_m)$

12. Index of wholesale prices of non-food articles $P_{wnf} = f(P_y)$

13. Non-agricultural income deflator $P_{yna} = f(P_y)$

14. Income in mining and manufacturing sectors $Y_m = f(Y)$

15. Non-food agricultural output plus mining and manufacturing output $Y_{anfm} = f(Y)$

16. Government tax revenues $T_x = f(Y)$

17. Net (money) payments of factor income $E^*_2 = f(CTG^*_{-1})$

(b) Identities and Definitions

18.	Tea exports	$X = X_{T,0}\,(1.012)^{t}$
19.	Unit value index of exports	$P_x = .628\,P_{xo} + .175\,P_{xT} + .197\,P_{xj}$
20.	Real income	$Y = Y_a + Y_{na}$
21.	Total savings	$S = S_g + S_c + S_h$
22.	Cumulative investment	$CI = I + CI_{-1}$
23.	Real resources gap	$RG = I - S$
24.	Nominal resources gap	$RG^* = P_y\,(I - S)$
25.	Total exports	$X = X_T + X_j + X_o$
26.	Total imports	$M = M_f + M_{nf}$
27.	Nominal trade gap (inclusive of factor incomes and other invisibles)	$TG^* = P_m.M + E_2^* - P_x.X - E_1^*$
28.	Real trade gap	$TG = M + \frac{E_2^*}{P_m} - \frac{X.P_x}{P_m} - \frac{E_1^*}{P_m}$
29.	Cumulative nominal trade gap	$CTG^* = CTG^*_{-1} + TG^*$
30.	Cumulative nominal resources gap	$CRG^* = CRG^*_{-1} + RG^*$
31.	Income identity	$TG^* = RG^*$
32.	Foreign capital inflow	$F^* = TG^*$

There are 11 exogenous variables in the system:

X_w = volume index of total world exports; P_m = unit value index of Indian imports; F = foreign capital inflow (in real terms; L = currency and deposits with the public including time-deposits; E_1^* = net receipt of invisibles excluding factor incomes; M_f = value of food imports; P_{xj} = unit value index of Indian jute exports; P_{xT} = unit value index of Indian tea exports; P_{xo} = world price index of 'other' exports; t = time-trend; R = index of official exchange rate (U.S. cents per Indian rupee) with the rate for 1960-61 = 1.0; this rate does not vary in the sample period).

There are 8 lagged endogenous variables:

CI_{-1}, $S_{g_{-1}}$, $S_{h_{-1}}$, $(X_o)_{-1}$, $(Y_a)_{-1}$, $(P_{ya})_{-1}$, CTG^*_{-1} and CRG^*_{-1}.

The list of the 31 endogenous variables is given below:

Y, Y_{na}, S_g, S_c, S_h, S, CI, I, RG, RG*, X, P , M_{of}, M, P_{ya}, P_y, P_{wnf}, P_{yna}, Y_m, T_x, E_2^*, TG*, TG, CTG*, CRG*, P_{xo}, X_o, Y_a, Y_{anfm}, X_T, and X_j.

9.2.2 Considerations in Specification

In estimating a production function for aggregate output, it is recognised that the relevant constraint is capital and that a distinction between imported capital and domestic capital should be made. Employment or labour supply is not recognised as a constraint. Following this hypothesis, the production function was estimated with lagged cumulative investment (indicative of capital stock at the beginning of period), and imports of capital goods and raw materials as explanatory variables. The latter variable was however dropped as its coefficient tended to obtain a negative sign with a low t-value.

Production function in the agricultural sector is estimated simply as a linear function of total output. This simplistic method was resorted to for want of adequate data on agricultural inputs.

Savings are divided into household, corporate, and government savings. In the equation relating to household and government savings a lagged dependent variable enters on the right-hand side. In addition, in government saving function, tax-revenues, and in household saving function, national income and a relative price variable are used. The price ratio between agricultural and non-agricultural outputs is used to incorporate to some extent a distinction between rural and urban saving propensities. The inclusion of lagged terms on the right-hand side is for the purpose of identifying long run propensities to save within the framework of a Nerlove type rigidity model where either household or the government is not able to save in a current year all/it wants to save because of institutional and other rigidities.

It was observed over the sample period that the share of corporate savings in total savings was small and has remained more or less steady. Yet it was accorded a separate treatment because this sector is growing in importance and it has a very high marginal propensity to save out of net profits. Corporate savings are related to income originating in the manufacturing sector. No data on an appropriate profit series was available.

Exports are divided into three categories: tea, jute and 'others'. The two traditional exports have been explained separately; tea, in terms of a simple time-trend and jute, in terms of (i) world economic activity as indicated by total world exports and (ii) international prices of Indian jute exports. For tea a growth rate of 1.2% per annum was used. In the residual category of exports, eleven items have been cited as important. These are: cotton textiles, iron ore, manganese, oil cakes, vegetables oils, hides and skins, leather, pepper, tobacco, wool and sugar. A world price index was especially constructed with reference to these items. First, international price indices for each of these items were constructed taking quotations from representative world markets. These indices were then weighted proportional to the respective shares of exports of each commodity in total exports in this group. This provides a world price index of 'other' exports (P_{xow}). The ratio (P_{xo}/P_{xow}), the numerator relating to the unit value index of 'other' exports, is used as an explanatory variable along with world exports in the 'other' exports demand function. The supply of this group is taken to be predetermined. Together with demand for this category, this helps develop a price-formation equation for P_{xo}.

P_{xo} is explained in terms of wholesale price of non-food items (P_{wnf}) which reflects domestic costs and lagged exports $(X_o)_{-1}$ as a ratio of income generated in the agricultural sector excluding food and in the mining and manufacturing sector (Y_{anfm}). $(X_o)_{-1}$ reflects demand considerations and Y_{anfm} reflects supply side influences. Y_{anfm} is wider in definition than 'other' exports, but has been used for statistical convenience.

The prices of jute and tea exports are taken as exogenous and subsequently exports prices of all categories are determined as a weighted average of prices of the three separate categories.

Imports are divided into food- and non-food imports. Food imports are taken to be exogenous as there is a considerable degree of uncertainty about these and past experience is not a very reliable guide. Non-food imports, consisting mainly of raw materials and capital goods required in production, are estimated in two alternative ways. In one case, the purchasing power of exports - given by the value of exports relative to price of imports adjusted by a coefficient indicating the official exchange rate $(X.\frac{P}{\overline{P_m}}.R)$ is taken to be a constraint. This variable is taken to reflect to some extent the import restrictions which distort the income-import relationship. In both alternatives income appears as the primary determinant. Since, in the years when purchasing power of exports is sufficiently high, it would not operate as a constraint, imports are derived from the following:

$$M_{of} = \text{Minimum } (M^1_{of}, M^2_{of}) \quad \text{where}$$

$$M^1_{of} = f(Y), \quad M^2_{of} = f(Y, X.\frac{P_x}{P_m}.R)$$

Apart from export prices, there are four other endogenous price variables, viz., agricultural income deflator (P_{ya}), national income deflator (P_y), index of wholesale prices of non-food articles (P_{wnf}) and non-agricultural income deflator (P_{yna}).

Agricultural prices are explained in terms of lagged agricultural output relative to non-agricultural output and previous year's agricultural prices. Thus, the supply of agricultural output reflected in $(Y_a)_{-1}$ and the demand for agricultural output reflected by the non-agricultural income, influence agricultural prices. The introduction of a lag implies that not only the current levels but also the imbalances between the rates of growth in agricultural and non-agricultural outputs go to determine the agricultural prices.

For the general price level (P_y) agricultural prices are taken as a pace-setter. In addition, the ratio of money supply to real national income and the import prices are also entered as explanatory variables. The remaining two price variables, P_{wnf} and P_{yna} are taken simply to be linear functions of P_y itself.

9.2.3 Estimation of Resources and Trade Gaps

The resources gap is estimated on the assumption that foreign exchange is not a constraint, i.e. trade gap is zero; the trade gap is estimated on the assumption that saving is not a bottleneck. In each case, the relevant identity is suppressed. Thus, the model of 32 equations becomes determinate in 31 endogenous variables in each case. In effect, there are two models different in specification as far as the estimation of gap is concerned.

If gaps were not to be calculated the model works like this: given the path of exogenous variables (one of which is, say, foreign capital inflow) what is the model consistent path for GDP and other endogenous variables. When gaps are to be calculated the question asked is the reverse. Given the path of GDP growth, and the time path of other exogenous variables, what is required flow of foreign capital (or domestic savings) consistent with the model.

9.2.4 Estimates of Stochastic Equations†

The sample begins in 1950-51. The number of observations listed under "No." indicate how far it extends in each case. Figures in brackets are standard errors.

	Equation	No.	$\bar{R}^2$	dw
1.	$Y = \underset{(139.68)}{10097.99} + \underset{(.0162)}{.3175}\, CI_{-1}$	12	.97	1.64
2.	$Y_a = \underset{(332.0437)}{2051.016} + \underset{(.0260)}{.3274}\, Y$	13	.93	1.39
3.	$S_g = \underset{(60.3518)}{-8.6489} + \underset{(.0748)}{.1506}\, T_x + \underset{(.3064)}{.6018}\,(S_g)_{-1}$	12	.72	1.734
4.	$S_c = \underset{(58.337)}{-116.59} + \underset{(.0252)}{.0762}\, Y_m$	12	.425	1.63
5.	$S_h = \underset{(708.11)}{-2071.875} + \underset{(.0379)}{.1066}\, Y_t + \underset{(435.65)}{1129.958}\, \frac{P_{nya}}{P_{ya}} + \underset{(.2169)}{.3909}\,(S_h)_{-1}$	12	.806	2.176
6.	$X_T = X_T\,(1.012)^t$			
7.	$X_j = \underset{(9.8329)}{110.9642} + \underset{(.0875)}{.5068}\, X_w - \underset{(.1079)}{.4060}\, P_{xj}$	15	.712	1.584
8.	$X_o = \underset{(82.0397)}{419.3627} + \underset{(.372)}{2.8128}\, X_w - \underset{(90.2395)}{274.5188}\, P_{xo}/P_{zow}$	15	.80	2.046
9.	$P_{xo} = \underset{(20.83)}{16.029} + \underset{(.1461)}{.2443}\, P_{wnf} + \underset{(133.42)}{659.0244}\,(X_o)_{-1}/Y_{anfm}$	13	.65	2.11
10.	$P_x = .628 + .175\, P_{xT} + .197\, P_{xj}$			
11.	$M_{of} = \text{Minimum}\,(M^1_{of}, M^2_{of})$ where			
	$M^1_{of} = \underset{(365.8)}{-1023.3849} + \underset{(.0247)}{.0656}\, Y_t + \underset{(.7628)}{1.5672}\, \frac{X.P_x.R}{P_m}$	12	.71	2.49
	$M^2_{of} = \underset{(281.05)}{-464.1648} + \underset{(.0225)}{.0965}\, Y$	12	.61	2.14

† Equations 6 and 10 are definitions.

			No.	$\bar{R}^2$	dw
12.	P_{ya} =	$\underset{(37.237)}{132.9351} - \underset{(26.41)}{83.8572}(Y_a)_{-1}/Y_{na} + \underset{(.2026)}{.4378}(P_{ya})_{-1}$	12	.67	2.012
13.	P_y =	$\underset{(5.0581)}{29.0923} + \underset{(.0406)}{.5894}\, P_{ya} + \underset{(14.0165)}{29.7182}\, L + \underset{(.0424)}{.0497}\, P_m$	12	.99	.95
14.	P_{wnf} =	$\underset{(18.17)}{-28.1870} + \underset{(.1876)}{1.223}\, P_y$	12	.78	1.68
15.	P_{yna} =	$\underset{(6.5765)}{48.78} + \underset{(.0688)}{.5207}\, P_y$	12	.83	1.714
16.	Y_m =	$\underset{(105.43)}{-185.8474} + \underset{(.0083)}{.2008}\, Y$	12	.98	1.431
17.	Y_{anfm} =	$\underset{(170.039)}{-192.7146} + \underset{(.0133)}{.3568}\, Y$	13	.98	1.66
18.	T_x =	$\underset{(195.9621)}{-1551.1849} + \underset{(.0157)}{.2295}\, Y$	12	.95	2.08
19.	E_2^* =	$\underset{(4.213)}{1.2091} + \underset{(.0031)}{.0334}\, CTG^*_{-1}$	11	.922	1.134

It would be observed that in general the sample period is relatively small with a minimum of 11 and a maximum of 15 observations. A high $\bar{R}^2$ is generally obtained. A relatively low $\bar{R}^2$ is for equation 4 relating to corporate savings. The equations for government and private savings as also that for agricultural prices contain lagged dependent variables. In these cases, tests other than the d-w statistics should have been adopted for testing serial correlation in errors.

9.2.5 Projections with the Model

Projections are made under alternative assumptions of a low growth and a high growth of GDP. In the low growth variant, it is assumed that income will grow at 4% p.a. and food imports will be 9 million tons. In the high growth variant, income grows at 5% p.a. and food imports are 5 million tons. In addition, the following assumptions, common to the two variants are made.

(i) All exogenous variables are constant at their base year level in 1962-63.

(ii) World exports grow at 6% p.a.

(iii) The exchange-rate index (variable R) is changed from its pre-1966 level of 1.0 to .65 for post-1966 years to account for the devaluation in this year.

The projections for 1975-76 for selected variables along with corresponding 1962-63 values under the two variants are given below.

Table 9:1

UNCTAD I : Projections for 1975-76 — Million $, 1960 prices

	1962-63	1975-76	
		Low growth	High growth
GDP	31300	52117	59020
Gross domestic savings	3240	7640	9650
Gross domestic investment	3940	6310	8850
Savings gap	700	-1330	-800
Imports of goods	2380	4970	5370
Exports of goods*	1640	2840	2870
Import surplus	740	2130	2500
Factor income payments	210	-	-
Trade gap**	950	-	-

* Excluding factor income payments

** Including factor income payments

9.3 UNCTAD (LINK) Model

A model for the Indian economy† was prepared by the UNCTAD secretariat as indicative of the types of models for developing countries to be ultimately used in the LINK project. The model is presented in Ball (ed.) (1973). Basic data and simulation results are not given and the model is considered still to be in an experimental stage.

The system is defined by 71 (+ 3) equations with 51 stochastic equations. There are 18 (+ 2) exogenous and 23 lagged endogenous variables. The sample period is 1950-51 to 1968-69.

9.3.1 Specification of the Model

Variables marked with an asterisk (*) are in current prices.

Production

1.	Area under food crops	$AF = f(PF/PNF, AF_{-1})$
2.	Total area under crops	$A = f(PAW/P, A_{-1})$
3.	Average yield per acre for food crops	$AYF = f(R, t)$
4.	Average yield per acre (non-food crops)	$AYNF = f(R, t)$
5.	Capacity output in manufacturing (corporate sector)	$YMCP = f(KCG_{-1})$
6.	Capacity output in manufacturing (non-corporate sector)	$YMNCP = f(KNC_{-1})$
7.	Capacity utilisation	$CU = f(YA_{-1}, MOF, KCG_{-1}, CU_{-1})$
8.	Net national product in agricultural sector	$YA = f(QAF, QANF)$
9.	Net output of services	$YS = f(KGD_{-1}, Y)$

† The model draws partly on the work by Pani (1971)

Savings and Investment

10. Savings of household sector $SH = f(YD, \frac{PNA}{PA}, SH_{-1})$

11. Savings of government sector $SG = f(T, SG_{-1})$

12. Savings of corporate sector $SC^* = f(CPR^*, SC^*_{-1})$

13. Investment in fixed assets by corporate sector $ICF = f(YNA, EF^*/P)$

14. Investment in inventories $H^* = f(Y^*)$

15. Depreciation $DEP^* = f(KGP_{-1})$

Prices and Wages

16. Wholesale prices of food articles $PF = f(SF_{-1}, SF_{-2}, YNA, \frac{L^*}{Y})$

17. Wholesale prices of agricultural raw materials $PNF = f(\frac{QANF_{-1}}{YMC + YMNC}, \frac{L^*}{Y})$

18. Wholesale prices of manufactured articles $PFG = f(PNF, CU, PFG_{-1})$

19. Average annual earnings per worker in factories $W^* = f(PCL, YMC)$

20. Wholesale price index $PW = f(PF, PNF, PFG)$

21. Implicit GNP deflator $P = f(PW)$

22. Consumer price index $PCL = f(PF, PFG)$

23. Wholesale prices of agricultural commodities $PAW = f(PF, PNF)$

24. Implicit deflator of income in agricultural sector $PA = f(PAW)$

25. Implicit deflator of income in non-agricultural sector $PNA = f(P)$

26. Unit value index of other exports $PXO = f(PW, PXO_{-1})$

27. Profits after tax in the corporate sector $CPR^* = f(YNA^*)$

Government and Monetary Sectors

28.	Direct taxes of central and State governments	$TD^* = f(RDT, YNA^*)$
29.	Indirect taxes of central and State governments	$TI^* = f(RIT1, RIT2, Y^*, MG^*, XG^*)$
30.	'Other' current receipts of central & State governments	$TO^* = f(Y^*)$
31.	Total holdings of government securities excluding RBI holdings	$GSBP^* = f(GD^*, GSBP^*_{-1})$
32.	Currency held by public (C^*)	$C^* - C^*_{-1} = f(GD^*)$
33.	Money supply (currency + demand deposits) with the public (L^*)	

a. $\ln (L^*/P) = f(\ln Y, \ln RG1, \ln \left(\frac{L^*}{P}\right)_{-1})$

b. $\ln (L^*/P) = f(\ln Y, \ln RSH, \ln (L^*/P)_{-1})$

34.	Time deposits	$T^* = f(YNA^*, R_{12}, RG1, T^*_{-1})$
35.	Borrowings of commercial banks from the Reserve Bank	$BR^* = f(RG, RB, BL^*)$
36.	Commercial banks loan to private sector	$BL^* = f(YNA^*, ICF^*, RA)$
37.	Total reserves of commercial banks with the Reserve Bank	$TR^* = f(D^*, T^*, RS1)$
38.	Commercial banks advance rate	$RA = f(RB, BL^*)$
39.	Call money rate	$RS1 = f(TR/DT, RB, L^*)$
40.	Yield on long-term government bonds	$RG1 = f\left(\frac{GSBP^*}{YNA^*}, R12\right)$

41.	External finance of the corporate sector	EF*	=	$f\left[\Delta(L^* + T^*)\right]$
42.	12-month time deposit rate	R12	=	f(RS1, R12)
43.	Average rate of return on government securities held by commercial banks	RG	=	f(RG1)

<u>External Trade</u>

44.	Imports of foodgrains	MF	=	$f(QAF, YNA, \frac{PMF}{PW})$
45.	Imports other than food	MOF	=	$f\left[YNA, PMOF, (\frac{FR}{PM})_{-1}\right]$
46.	Exports of tea	XT	=	$XT_{65-69}\,(1.01)^t$
47.	Exports of jute	XJ	=	$XJ_{65-69}\,(1.01)^t$
48.	Exports of engineering goods	XEG	=	f [PXEG.ER]
49.	Other exports	XO	=	f [EW, PXO.ER]
50.	Exports of services (XS*)	ln XS*	=	f (t)
51.	Factor income paid abroad	YF*	=	$f\,(CDF_{-1})$

<u>Definitions and Identities</u>

52.	Area under non-food crops	ANF	=	A-AF
53.	Agricultural output (food crops)	QAF	=	AYF.AF
54.	Agricultural output (non-food crops)	QANF	=	AYNF.ANF
55.	Net output in manufacturing (corporate sector)	YMC	=	CU.YMCF
56.	Net output in manufacturing (non-corporate sector)	YMNC	=	CU.YMNCP

57.	Net output in non-agricultural sector	$YNA = YMC + YMNC + YS$
58.	Net output in non-a agricultural sector at current prices	$YNA^* = PNA.YNA$
59.	Net output in agriculture at current prices	$YA^* = PA.YA$
60.	Net national product at current prices	$Y^* = YA^* + YNA^*$
61.	Net national product at constant prices	$Y = YA + YNA$
62.	NNP at current prices	$Y^* = PY$
63.	Total imports at current prices	$MG^* = PMF.MF + PMOF.MOF$
64.	Total value of exports	$XG^* = PXT.XT + PXJ.XJ + PXEG.XEG + PXO.XO$
65.	Total supply of foodgrains	$SF = 0.88\ QAF + MF$
66.	Money supply with the public	$L^* = C^* + D^*$
67.	Sum of demand and time deposits	$D^* + T^* = EL^* + GSB^*$
68.	Public (other than RB) holdings of Govt.securities	$GSP^* = GSBP^* - GSB^*$
69.	Total current receipts of the Govt.sector at constant prices	$T = \frac{TD^* + TI^* + TO^*}{P}$
70.	Investment-saving identity to determine investment in fixed assets by the non-corporate sector (INCF)	$P(SH+SG) + SC^* + DEP^* + YF^* = PI(ICF + INCF + IG) + H^* + XG^* - MG^* + XS^*$
71.	Foreign capital inflow	$F^* = MG^* - XG^* - XS^* + FER^* - FER^*_{-1}$

The following three identities have been defined outside the model. However, since endogenous variables are used in these, they are appropriately defined inside the model.

72. Capital stock in govt.sector other than in departmental undertakings and in fexed assets in corporate sector

$$KCG = KCG_{-1} + ICF + IGO$$

73. Capital stock in fixed assets in the non-corporate sector

$$KNC = KNC_{-1} + INCF$$

74. Capital stock in govt.sector in departmental undertakings

$$KGD = KGD_{-1} + IGD$$

The exogenous variables of the system are the following. C^* = currency with public, F = Foreign capital inflow, GD^* = government deficit on current account, IG^* = total investment in the government sector, IGD^* = government sector investment in departmental undertakings, IGO^* = Government sector investment other than in departmental undertakings, PM = unit value index of total imports, PMF = unit value index of food imports, PMOF = unit value index of other imports, PXEG = unit value index of exports of jute manufactures, XT = unit value index of exports of tea, R = rainfall, expressed as percentage of normal, RB = Reserve Bank discount rate, RDT = average rate of direct taxes, expressed as a percentage of national income, RIT1 = average rate of other indirect taxes, expressed as percentage of national income, RIT2 = average rate of customs duties, expressed as a percentage of the value of imports + exports, t = time trend; XT_{65-69} = average tea exports during 65-69; XJ = average jute exports during 65-69. The lagged endogenous variables are: AF_{-1}, A_{-1}, KCG_{-1}, KNC_{-1}, YA_{-1}, CU_{-1}, KGD_{-1}, SH_{-1}, SG_{-1}, SC^*_{-1}, KGP_{-1}, SF_{-1}, SF_{-2}, PFG_{-1}, PXO_{-1}, $GSBP^*_{-1}$, T^*_{-1}, P^*_{-1}, $R12_{-1}$, $\left(\frac{FR}{PM}\right)_{-1}$, CDF_{-1} FER^*_{-1}.

9.3.2 Considerations in Specification

There is a relatively greater disaggregation of output in this model than in others where supply enters at all. Output is disaggregated into (i) agriculture, (ii) manufacturing and (iii) residual (mainly services) sectors. Supply of agricultural output is considered by dividing it into the food and non-food production. In both cases output is determined by area under crops and yield per acre, thus decomposing a relatively stagnant factor (area) from a changing factor (yield) in agricultural production. Manufacturing production is divided between corporate and non-corporate sector. In both cases, production is related to capital stock (cumulative investment) in each sector and capacity utilisation.

Consumer expenditure is again explained, as in UNCTAD-I model, from the savings side. This is because data on savings is considered more reliable than on consumption as far as the Indian economy is considered, and it also permits a sectoral breakdown between household, corporate and government sectors. Household savings are a function of disposable income, terms of trade between agricultur and non-agricultural sectors as represented by the ratio of national income deflators of the two sectors and lagged savings. The terms of trade variable reflects distribution of income between rural and urban sectors. If income changes in favour of the former sector, it is expected that savings will go down as in rural areas the propensity to consume is expected to be high. Also, to the extent a change in the terms of trade reflects a scarcity of the supply of food relative to other consumption goods, savings will be reduced as food is the most important item of consumption. Government and corporate savings are related respectively to tax revenues and corporate savings.

Investment expenditure is also divided between these three sectors. Government investment is taken to be exogenous. Corporate investment is related to the level of non-agricultural income and by the availability of real external finance. For the non-corporate sector, there is a greater dependence on internal finance. Non-corporate investment is determined as a residue implying that saving and investment decisions are one and the same for this sector.

There are 9 domestic price variables and 7 external trade variables in the model. For the agricultural sector there are the wholesale prices of (i) food articles (PF); (ii) agricultural raw materials (PNF); (iii) agricultural commodities (PAW) and (iv) the implicit deflator of agricultural income (PA). For the non-agricultural sector there are (i) wholesale prices of manufactured articles (PFG) and (ii) implicit deflator of non-agricultural income (PNA). For the economy as a whole there is a (i) wholesale price index (PW); (ii) a consumer price index (PCL) and (iii) an implicit GNP deflator (P). In addition there are unit value indices for (i) total imports (PM), food imports (PMF); (ii) 'other' imports (PMOF), and for exports of engineering goods (PXEG), jute manufactures (PXJ), tea (PXT) and 'other' exports (PXO). Except PMOF, other import and export price variables are taken as exogenous. Prices of non-food articles in the agricultural sector are determined by excess demand and ratio of cash balances with the public to total real output. The latter variable reflects the pressure on prices from the monetary side. Prices of manufactured goods are determined by raw material costs and a mark-up which varies directly with capacity utilization. Food prices are determined by lagged supply variables, non-agricultural income and ratio of cash-balances to total income. The influence of lagged supply variables is explained as being partly due to the annual nature of crops and the difference between crop year (July - June) and the financial year (April - March) and partly due to rigidities in adjustment. All other prices are weighted functions of food and non-food prices in the agricultural sector and prices of manufactured goods in the non-agricultural sector.

Government revenues are divided into direct taxes, indirect taxes and other miscellaneous receipts. Direct taxes are a function of an 'average' tax rate and income in the non-agricultural sector and indirect taxes are a function of 'average' rates of excise and customs duties, total income at current prices and current values of imports and exports. It is not clear how the 'average' tax-rates are calculated. If they are obtained by just dividing the tax-revenues by income, imports and exports etc., the tax revenues are more appropriately determined in an identity. Miscellaneous receipts are just a function of income.

In the monetary sector, there is a demand equation for real cash balances in two alternative forms. In both versions real income and interest rates appear. In addition in one case, yield on long term government securities, and in the other case, yield on corporate equities are used. There is also an equation for demand for time deposits and other equations explain the behaviour of commercial banks.

Imports are divided between food and non-food imports. The former is dependent on domestic food output, real income of the non-agricultural sector and real prices of food imports. Non-agricultural income and import prices and foreign exchange reserves explain non-food imports. Exports are divided into tea, jute, engineering goods and others. For the former two, simple time-trends are used. Exports of engineering goods are explained by the price of engineering goods in the world market and remaining exports are explained by world income and unit value of exports of these other goods.

9.3.3 Estimated Stochastic Equations

Figures in brackets are t-ratios.

		$\bar{R}^2$	dw
I.	**Production**		
1.	$AF = \underset{(1.13)}{10.7215} + \underset{(3.0)}{.2094}\ PF/PNF + \underset{(9.43)}{.7452}\ AF_{-1}$	.876	2.19
2.	$A = \underset{(1.48)}{30.3224} + \underset{(4.59)}{.1353}\ PAW/P + \underset{(3.2)}{.6237}\ A_{-1}$	.87	2.43
3.	$AYF = \underset{(5.1)}{63.67} + \underset{(2.6)}{.3057}\ R + \underset{(5.6)}{1.5589}\ t$	.66	1.91
4.	$AYNF = \underset{(9.6)}{79.5221} + \underset{(1.12)}{.0863}\ R + \underset{(5.98)}{1.1014}\ t$	.67	2.43
5.	$YMCP = \underset{(29.4)}{7.5884} + \underset{(26.75)}{.0008}\ KCG_{-1}$	.98	.47
6.	$YMNCP = \underset{(12.96)}{12.6484} + \underset{(1.88)}{.0009}\ KNC_{-1}$	.78	.40
7.	$CU = \underset{(1.47)}{8.37} + \underset{(3.18)}{.3116}\ YA_{-1} + \underset{(3.07)}{.0547}\ MOF - \underset{(4.32)}{.0005}\ KCH_{-1} + \underset{(8.49)}{.627}\ CU_{-1}$	.92	1.74
8.	$YA = \underset{(24.16)}{14.357} + \underset{(28.82)}{.2579}\ QAF + \underset{(16.20)}{.1230}\ QANF$	.997	1.21
9.	$YS = \underset{(4.22)}{12.0855} + \underset{(10.74)}{.0044}\ KGD_{-1} + \underset{(4.9)}{.1388}Y$	.996	1.15
II.	**Savings and Investment**		
10.	$SH = \underset{(2.06)}{-1485.72} + \underset{(2.955)}{14.394}\ YD + \underset{(1.47)}{4.0726}\ PNA/PA + \underset{(1.57)}{.3414}\ SH_{-1}$	.81	2.09
11.	$SG = \underset{(.1645)}{7.8322} + \underset{(1.2355)}{.0588}\ T + \underset{(2.36)}{.5666}\ SG_{-1}$	.70	1.21
12.	$SC^* = \underset{(.1638)}{2.6423} + \underset{(2.07)}{.2775}\ CPR^* + \underset{(1.26)}{.3060}\ SC^*_{-1}$	.55	1.61

		Equation	$\bar{R}^2$	dw
13.	ICF	$= \underset{(4.96)}{-218.55} + \underset{(8.66)}{7.1421\ YNA} + \underset{(2.49)}{.2957\ EF^*/P}$	.91	2.49
14.	H*	$= \underset{(.877)}{-93.816} + \underset{(8.12)}{2.0095\ Y^*}$	.34	1.48
15.	DEP*	$= \underset{(16.85)}{367.14} + \underset{(25.53)}{2.7972\ KGP_{-1}}$	.97	.87

III. Prices and Wages

		Equation	$\bar{R}^2$	dw
16.	PF	$= \underset{(3.17)}{92.359} - \underset{(3.30)}{1.66\ SF_{-1}} - \underset{(4.67)}{2.397\ SF_{-2}} + \underset{(4.17)}{2.583\ YNA} + \underset{(4.82)}{6.249\ L^*/Y}$	.97	1.42
17.	PNF	$= \underset{(.709)}{33.3296} - \underset{(1.708)}{10.515\ \frac{QANF_{-1}}{(YMC + YMNC)}} + \underset{(9.38)}{6.849\ \frac{L^*}{Y}}$	.96	1.37
18.	PFG	$= \underset{(.743)}{16.0645} + \underset{(6.43)}{.31\ PNF} + \underset{(2.39)}{.4623\ CU} + \underset{(3.87)}{.4538\ PFG_{-1}}$	.98	
19.	W*	$= \underset{(7.585)}{339.6} + \underset{(8.853)}{6.5895\ PCL} + \underset{(2.3)}{17.8942\ YMC}$	.97	1.97
20.	PW	$= \underset{(1.67)}{-1.8836} + \underset{(82.4)}{.5317\ PF} + \underset{(14.6)}{.1550\ PNF} + \underset{(16.6)}{.3298\ PFG}$	.999	1.8
21.	P	$= \underset{(6.41)}{28.6471} + \underset{(20.35)}{.6544\ PW}$	.96	.43
22.	PCL	$= \underset{(.1459)}{.8756} + \underset{(14.66)}{.5931\ PF} + \underset{(4.99)}{.4473\ PFG}$	.995	1.52
23.	PAW	$= \underset{(3.97)}{9.75} + \underset{(11.27)}{.6138\ PF} + \underset{(4.87)}{.2870\ PNF}$	.995	1.52
24.	PA	$= \underset{(3.655)}{-12.84} + \underset{(38.9)}{.9695\ PAW}$	.989	1.50

	Equation	$\bar{R}^2$	dw
25.	$PNA = \underset{(9.19)}{58.6261} + \underset{(9.31)}{.4993}\ P$	.84	.40
26.	$PXO = \underset{(.983)}{14.777} + \underset{(3.99)}{.5627}\ PW + \underset{(1.053)}{.2084}\ PXO_{-1}$	.74	1.81
27.	$CPR^* = \underset{(.336)}{9.9064} + \underset{(5.77)}{2.0561}\ YNA^*$	.655	.49

IV. <u>Government and Monetary Sectors</u>

	Equation	$\bar{R}^2$	dw
28.	$TD^* = \underset{(16.995)}{-443.592} + \underset{(13.89)}{139.3652}\ RDT + \underset{(33.4)}{6.5171}\ YNA^*$	.99	1.44
29.	$TI^* = \underset{(30.9)}{-1285.959} + \underset{(14.1)}{277.223}\ RIT1 + \underset{(13.2)}{101.0271}\ RIT2 + \underset{(14.2)}{6.9731}\ Y^* + \underset{(2.57)}{.0018}\ MG^* + \underset{(3.44)}{.0042}\ XG^*$	.999	2.11
30.	$TO^* = \underset{(1.31)}{-49.3945} + \underset{(9.4)}{2.1327}\ Y^*$	.84	1.24
31.	$GSBP^* = \underset{(1.25)}{149.0526} + \underset{(1.8)}{.4592}\ GD^* + \underset{(11.29)}{.9228}\ GSBP^*_{-1}$	.96	2.12
32.	$C^*-C^*_{-1} = \underset{(2.12)}{58.3337} + \underset{(3.47)}{.4189}\ GD^*$	.39	1.50
33.a	$\ln\left(\frac{L^*}{P}\right) = \underset{(2.17)}{-1.8225} + \underset{(2.45)}{.7605}\ \ln Y - \underset{(1.06)}{.2749}\ \ln RG1 + \underset{(2.46)}{.5563}\ \ln\left(\frac{L^*}{P}\right)_{-1}$	.93	1.44
33.b	$\ln\left(\frac{L^*}{P}\right) = \underset{(2.57)}{1.524} + \underset{(2.32)}{.5901}\ \ln Y - \underset{(2.15)}{.2738}\ \ln RSH + \underset{(2.22)}{.5143}\ \ln\left(\frac{L^*}{P}\right)_{-1}$	.94	
34.	$T^* = \underset{(2.24)}{551.497} + \underset{(3.36)}{6.687}\ YNA^* + \underset{(3.77)}{238.96}\ R12 - \underset{(3.35)}{361.66}\ RGi + \underset{(7.26)}{.6936}\ T^*_{-1}$	.99	1.7

No.	Equation	$\bar{R}^2$	dw
35.	$BR^* = -158.1568 + 111.4452\ RG - 34.4719\ RB + .0133\ BL^*$	.597	1.93
	(1.92) (2.66) (1.76) (.91)		
36.	$BL^* = -275.797 + 27.493\ YNA^* + 1.012\ ICF^* - 107.02\ RA$	.995	1.93
	(2.04) (15.64) (5.3) (3.5)		
37.	$TR^* = 55.2665 + .0458\ D^* + .0641\ T^* - 10.2119\ RS1$	.98	2.51
	(8.22) (3.757) (6.27) (4.04)		
38.	$RA = 2.1967 + .9464\ RB + .0004\ BL^*$	.88	1.67
	(2.78) (3.66) (1.67)		
39.	$RS1 = 3.92 - .5545\ TR/DT + 1.17\ RB - .0004\ L^*$	.87	1.60
	(2.34) (3.72) (4.14) (2.08)		
40.	$RG1 = 3.2042 - .0287 \dfrac{GSBP^*}{YNA^*} + .4892\ R12$	.96	2.22
	(8.54) (2.3) (16.6)		
41.	$EF^* = 113.91 + .4131\ \Delta(L^* + T^*)$	.42	1.42
	(2.44) (3.7)		
42.	$R12 = .1457 + .2565\ RS1 + .7614\ R12_{-1}$	.93	2.54
	(.567) (2.17) (7.26)		
43.	$RG = .7950 + .5636\ RG1$	.87	2.36
	(3.57) (11.1)		

V. External Trade

No.	Equation	$\bar{R}^2$	dw
44.	$MF = 9.557 - .078\ QAF + .1476\ YNA - .0639\ PMF/PW$	.76	1.06
	(2.49) (2.82) (4.11) (1.84)		
45.	$MOF = 30.91 + 2.0025\ YNA - .0056\ PMOF + .1964 \left(\dfrac{FR}{PM}\right)_{-1}$	.38	1.92
	(.71) (2.92) (2.63) (3.21)		
46.	$XT = XT_{65-69}\ (1.01)^t$		
47.	$XJ = XJ_{65-69}\ (1.01)^t$		

			$\bar{R}^2$	dw
48.	XEG	$= \underset{(6.29)}{747.57} - \underset{(5.03)}{5.8362}\ (PXEG.ER)$	.68	1.44
49.	XO	$= \underset{(3.83)}{49.685} + \underset{(6.99)}{.5651}\ EW - \underset{(.87)}{.1565}\ (PXO.ER)$	.84	1.72
50.	ln XS*	$= \underset{(2.75)}{.2612} + \underset{(7.32)}{.0625}\ t$	.76	1.18
51.	YF*	$= \underset{(1.17)}{-.059} + \underset{(21.4)}{.0536}\ CDF_{-1}$	.96	1.02

Equations 46 and 47 are basically definitions. A low value of $\bar{R}^2$ is observed for equations 12 (corporate savings), 14 (inventory investment), 32 (currency held by public), 35 (commercial banks' borrowings from RBI), 41 (external finance of corporate sector) and 45 (non-food imports). Among these, the $\bar{R}^2$ in equations 14, 32, 41 and 45 is near or below .40. This indicates that a substantial amount of variation still needs to be explained. A low value of d-w is observed in equations 5, 6, 15, 21, 25 and 27 indicating possible positive autocorrelation. These equations need further testing. There are many equations containing lagged dependent variables. Thus the two equations relating to area under crops (equations 1 and 2), the capacity utilisation equation (equation 7), the three savings' equations (equations 10, 11 and 12) the price equations (18 and 26), four equations in the government and monetary sector (31, 33, 34 and 42), and the non-food import equation (equation 45) contain lagged dependent variables. In some of these cases d-w statistic is reported and in some cases it is not. Since this statistic cannot adequately guide as to the presence of auto-correlation in these cases, further exploration is needed.

No simulation and prediction results have been reported in the paper by th UNCTAD Staff in Ball (ed., 1973).

9.4 Summary

In this Chapter, Agarwala model and the two UNCTAD models have been surveyed. Compared to other models of the Indian economy, these models place a relatively greater emphasis on supply side considerations. All these models attempt to distinguish between the agricultural and non-agricultural sectors. The UNCTAD (LINK) model is a highly disaggregated but it is still in an experimental stage.

Chapter 10

Models of the Indian Economy: A Review

In Chapter 6, the main characteristics and distinguishing features of twelve models of the Indian economy and, in some cases, their alternate versions were surveyed. In Chapters 7, 8 and 9, these models were considered individually. In this chapter a review of these models is undertaken with a view to derive some guidelines for future model-building activity for the Indian economy. This review is concerned mainly with the following issues:

(i) whether mainstream behavioural relations show structural stability in post 1950 data;

(ii) should expenditure functions be estimated with variables measured at constant or at current prices?;

(iii) could some lessons be derived as to the appropriate choice of exogenous variables in a model of income determination; and

(iv) what has been the forecasting performance of some of these models?

10.1 Structural Stability of Parameters

It has been argued in Chapter 6 that except for functions analysing long-run relations, it is best not to extend one's sample back into the pre-planning and pre-independence period because apart from any structural breaks there are difficulties arising out of the partition, and from changes in the quality and coverage of standard income, expenditure and price series. Although model-builders in the later fifties and early sixties were forced to use pre-1950 data in order to get an acceptable sample size, such a situation would not prevail today.

The question remains as to whether the post-1950 period represents a homogenous sample. Two considerations are relevant here. First, the major revision of the series of national income and its components by the C.S.O. already referred to in Chapter 6, provides data beginning from 1960/61 except for some variables where it goes further back. This implies that certain arbitrary procedures have to be resorted to if one wants to take all the relevant series back to 1950/51. Secondly, even on the supposition that consistent data beginning from 1950-51 are available, one has to consider whether or not there have been shifts in the parameters of structural relations in the later years. Such an exercise is undertaken here with respect to four mainstream demand functions which occur in most of the models surveyed here. These functions relate respectively to the private demand for consumption and investment and the demand for money and imports.

Ideally, shifts in parameters should be tested for a model as a whole between a sample period and a forecast period. To some extent this aspect has been covered in this chapter in the section on forecast evaluation for selected models. The tests presented here relate to individual functions and can at best be taken as preliminary evidence or direct tests on these mainstream functions which tend to be a common denominator for all the models in question.

The test procedure is based on the familiar Chow test. Notes on data are given in Appendix 2. The sample available to us relates to the period 1950-51 to 1974-75. This was divided into two sub-samples of 12 and 13 observations respectively. The tests have been carried out for alternative specifications and forms of the functions, consistent with those in the models we have reviewed.

Private demand for consumption goods is estimated with respect to the following alternative specifications:*

(i) $C_p = f(Y^d)$

(ii) $C_p/P = f(Y^d/P)$

(iii) $\ln C_p/P = f(\ln Y^d/P, \ \ln (NW/P)_{-1}, \ \ln P)$

In many models, disposable income has been favoured as the only explanatory variable as it alone explains more than 90 per cent of variations in private consumption irrespective of whether variables are measured in real or nominal terms. In other models, wealth effects or real balance effects have also been used. We have captured this by using a 'net worth of the private sector' (NW) variable. Incorporation of this or such other variables usually throws up wrong signs and insignificant coefficients. The third specification listed above also tests for a non-homogenous impact of prices. This is taken up in greater detail in a subsequent section. The question of the specification of an aggregate private consumption expenditure function is further examined in Chapter 11.

Estimates of parameters and related Chow-statistics along with the relevant critical F-value for a 95 per cent confidence level are given in Table 10:1. We look both at the Chow-statistic and at the magnitudes of coefficients in the first and second parts of the sample to see if they are widely divergent and individually significant.

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* Symbols used in this Chapter are defined in the Notes appended to this Chapter.

Table 10.1

Consumption Function Parameters: Stability Tests

Sample	Dependent Variable	Independent Variables: Const.				Chow Stat.	Critical F-value*
		Const.	Y^d				
Pooled	C_p	919.73 (9.74)	.7994 (222.5)			.5539	F(2,21) = 3.467
First		1163.24 (4.068)	.7798 (29.91)				
Second		758.11 (3.15)	.8035 (116.4)				
	C_p/P		Y^d/P				
Pooled		21.89 (13.36)	.7106 (68.13)			.8796	F(2,21) = 3.467
First		18.75 (4.24)	.7373 (20.87)				
Second		18.06 (4.16)	.7301 (30.65)				
	$\ln C_p/P$		$\ln Y^d/P$	$\ln(NW/P)_{-1}$	$\ln P$		
Pooled		1.039 (5.56)	.8703 (25.98)	-.0751 (-1.93)	-.0174 (-1.05)	.8682	F(4,17) = 2.965
First		.985 (3.2)	.8894 (15.76)	-.0883 (-1.45)	-.0072 (-.12)		
Second		.644 (1.83)	.943 (16.7)	-.0664 (-1.11)	-.0248 (-1.24)		

* Tabulated F-values are at 95% confidence level.

There is no evidence in these functions for private consumption to indicate statistically significant parameter shifts. The magnitudes of coefficients also do not indicate and substantiate shifts. Thus, it could be hypothesised that the marginal propensity to consume or the marginal savings-ratio has not shown any systematic tendency to change as far as the private sector is concerned.

A similar study for the private investment function has been done with respect to following specifications:

(i) $I_p = f(Y^d, R_1)$

(ii) $I_p = f(Y^d, R_1, I_{p_{-1}})$

(iii) $(I_p/P) = f(Y^d/P, R_1)$

(iv) $(I_p/P) = f(Y^d/P, R_1, (I_p/P)_{-1})$

These variations account for specifications defined in nominal and in real terms. The long-term rate of interest has commonly been used in investment functions. The lagged term takes account of some distributed lag specifications of the investment function. The estimated coefficients along with their t-values and the related Chow-statistics for these alternative specifications are given in table 10:2.

Again it will be observed that the estimated Chow-statistics are below the critical F-values. Hence, in these cases also the null hypothesis that parameters have not shifted cannot be rejected. In some cases, however, the magnitude of coefficients indicate a relatively wider shift. Thus with respect to the fourth function, the marginal investment-income ratio shifts from .09 to .28 between the earlier and the latter samples and in the second function, it shifts from .10 to .15. The coefficients of the long-term rate of interest also show considerable variation between the truncated samples in these fits. However, in some cases the coefficients are not statistically significant judged from the t-values. The reason for this kind of volatility in the magnitudes may be multi-collinearity among the regressors. The equations containing lagged investment variables may also suffer from serial correlation. Thus, although jointly considered the paramters do not appear to have significantly shifted statistically, where individual coefficients are important, additional care needs to be undertaken in the estimation procedures.

Table 10:2

Private Investment Functions: Stability Tests

Sample	Dependent Variable	Independent Variables				Chow Stat.	Critical F-value
	I_p	Const.	Y^d	R_1	$I_{p\,-1}$		
Pooled		-471.2 (-.1124)	.1328 (29.75)	-7.70 (-.122)		.2571	F(3,19) = 3.127
First		47.45 (.078)	.141 (4.98)	-102.83 (-1.23)			
Second		-316.73 (.387)	.1293 (13.73)	-9.631 (-.09)			
Pooled		-539.78 (-1.296)	.1420 (6.56)	1.269 (.0188)	-.0878 (-.433)	.5092	F(4,17) = 2.965
First		180.26 (.3034)	.10196 (3.04)	-119.74 (-1.48)	.3211 (1.315)		
Second		-370.06 (-.443)	.1539 (4.72)	5.347 (.048)	-.2548 (-.791)		
	I_p/P		Y^d/P	R_1	$(I_p/P)_{-1}$		
Pooled		-11.906 (-4.23)	.1805 (13.67)	-.0061 (-.014)		1.2795	F(3,19) = 3.127
First		1.818 (.207)	.1232 (2.92)	-1.256 (-1.19)			
Second		-8.175 (-1.27)	.1610 (5.53)	.0074 (-.015)			
Pooled		-7.846 (-2.18)	.1298 (4.03)	-.1855 (-.436)	.3137 (1.71)	2.4378	F(4,17) = 2.965
First		2.4138 (.349)	.0905 (2.53)	-1.547 (-1.84)	.5085 (2.54)		
Second		-17.24 (-2.03)	.2799 (3.37)	.195 (.417)	-.676 (-1.52)		

The demand for money is another mainstream function which occurs in one form or another in the simultaneous block of most of the Indian models. Again, variables like money and income have been measured in nominal and real terms. Apart from income, short-term rate of interest is generally included. Sometimes a wealth effect or real-balance effect has also been included. In order to capture these alternative specifications, the following equations have been estimated here:

(i) $M = f(Y^d, R)$

(ii) $M = f(Y^d, R, NW_{-1})$

(iii) $M/P = f(Y^d/P, R)$

(iv) $M/P = f(Y^d/P, R, (\frac{NW}{P})_{-1})$

The net worth variable is a proxy for wealth of the private sector. Its lagged value indicates wealth at the beginning of the period. The relevant estimates and test-statistics are tabulated in table 10:3. Again, the null hypothesis of no-shift of parameters cannot be rejected for these demand functions. The marginal money-income relationships in alternative formulations do not indicate wide changes in the magnitudes of coefficients between the earlier and later samples. Correct signs for the rate of interest variable is obtained in all but two cases. There is some indication of wider differences in the magnitudes of coefficients, but since this variable is measured in percentages, its smaller units compared to units of other variables imply that the differences in magnitudes need not be over-emphasized.

Table 10.3

Demand for Money Functions: Stability Tests

Sample	Dependent Variable	Independent Variables				Chow Stat.	Critical F-value
	M	Const.	Y^d	R	NW_{-1}		
Pooled		258.79 (.702)	.1775 (22.25)	-43.15 (-.236)		.7609	F(3,19)= 3.127
First		-20.77 (-.082)	.1867 (8.43)	32.796 (.314)			
Second		598.63 (.624)	.1830 (10.87)	-209.99 (-.489)			
Pooled		-1167.8 (-2.22)	.0831 (2.86)	-53.86 (-.356)	.0678 (3.33)	.7738	F(4,17)= 2.965
		-1154.4 (-4.2)	.0232 (.639)	-87.68 (-1.414)	.0899 (4.759)		
		-2612.4 (-1.74)	.04398 (.765)	206.02 (.534)	.095 (2.49)		
	M/P		Y^d/P	R	$(NW/P)_{-1}$		
Pooled		3.146 (1.7)	.1772 (9.73)	-.6299 (-.626)		.7291	F(3,19)= 3.127
First		-2.178 (-.547)	.2134 (5.11)	-.3174 (-.204)			
Second		.4728 (.075)	.1964 (5.17)	-1.078 (-.767)			
Pooled		-8.79 (-2.23)	.1602 (10.09)	-.4879 (-.583)	.0321 (3.28)	1.172	F(4,17)= 2.965
First		-6.06 (-1.38)	.1767 (3.95)	-.6875 (-.474)	.0213 (1.61)		
Second		-29.48 (-2.72)	.1626 (5.40)	.4814 (.416)	.0689 (3.06)		

For import functions, the following specifications were used:

(i) $IM = f(Y, F_{-1})$

(ii) $IM = f(Y, P_m/P, F_{-1})$

(iii) $IM/P_m = f(Y/P, (F/P_m)_{-1})$

These are consistent with the commonly used explanatory variables for estimating aggregate import-demand in the models considered. Apart from the income-variable, a relative price variable is used to capture substitution effects, and availability of foreign exchange at the beginning of the period reflects whether a foreign exchange constraint was operative and also how vigorously import controls were resorted to.

The results for this function are given in table 10:4. This is the only case among the four aggregate functions considered here, where there is ground for not accepting the null hypothesis of no-change in the estimated parameters, especially when variables are measured at current prices. In the first two formulations the critical F-value at 95 per cent confidence level is greater than the estimated Chow-statistic. In terms of magnitudes of coefficients also, substantial changes in some cases may be observed. When variables are measured in constant prices, however, the null hypothesis of no-change may be accepted.

A number of suggestions can be made regarding the import function. First, a non-systematic part of imports is food-imports, which may first be set aside and this equation should be estimated only for non-food imports. In some models, like the UNCTAD models, this has been done. Second, exchange-rate variations need to be incorporated. In the earlier part of the sample India has had a fixed exchange rate. There was a devaluation in 1966. Now a system of 'controlled float' is operative and the Rupee has been linked to a 'basket' of external currencies. This variable could not have been separately included in the above specifications because for the first sub-sample a fixed-exchange rate was perfectly collinear with the constant term.

Table 10.4

Aggregate Import Functions: Stability Tests

Sample	Dependent Variable	Independent Variables				Chow Stat.	Critical F-value
	IM	Const.	Y	P_m/P	F_{-1}		
Pooled		422.6 (2.86)	.0479 (13.33)		-.0899 (-.457)	4.2686	F(3,19) = 3.127
First		-786.2 (-.6795)	.1246 (1.767)		-.4695 (.8557)		
Second		-133.4 (-.531)	.0827 (6.93)		-2.16 (-2.88)		
Pooled		-648.9 (-1.423)	.054 (13.3)	1071.2 (2.46)	-.481 (-2.01)	5.3773	F(4,17) = 2.965
First		-176.98 (-.1252)	.1083 (1.45)	-358.33 (-.786)	.478 (.853)		
Second		-2039.8 (-3.39)	.0617 (6.24)	2513.2 (3.32)	-1.0856 (-1.75)		
	IM/P_m		Y/P		$(F/P_m)_{-1}$		
Pooled		8.4 (2.93)	.0285 (2.1)		-.3945 (-2.24)	.7224	F(3,19) = 3.127
First		-6.7 (.43)	.1150 (1.24)		.1538 (.275)		
Second		11.2 (2.25)	.0154 (.532)		-.4474 (-.89)		

The upshot of the considerations in this section is that the period beginning from 1950-51 offers a reasonably homogenous sample period, as far as most of the commonly used aggregate stochastic equations regarding consumption, investment, money-demand are concerned. In the case of import-demand one observes some unsystematic elements but there are *a priori* grounds to believe that these could be modelled by modifying the dependent variable and by using other explanatory variables without being forced to truncate the sample.

10.2 Non-homogenous Impact of Price-level Variations

A second question regarding which some guidelines are needed is concerned with the question of using variables, both dependent and independent, measured at current, and at constant prices. Some models like Narasimham, Choudhry and Bhattacharya have used nominal variables in their expenditure functions while most others have used constant-price or deflated series.

The choice is dictated first by the objectives and basic theoretical underpinnings of the model. If a model of income-determination is built in the modern monetarist tradition, it is not envisaged as to how movements in the nominal variables would be decomposed into their real and price counterparts. In such cases, working with the nominal variables may be a prerequisite.

In the more mainstream model building tradition in the IS-LM framework or other approaches of a hybrid variety, one relevant question is whether price movements have a non-homogenous impact. The primary source for such an impact could be money illusion or misperception of the price-level and its changes on the part of consumers and other behavioural units of the economy. Most Indian models assume the absence of money illusion, i.e. they assume that an equi-proportionate change in prices, money income and money wealth would not alter their real demands. Another source for a non-homogenous impact may be the formation of price expectations based on observed price movements in the current and past years.

If a non-homogenous impact of price-movements can be established for relevant demand functions, it would indicate that in models using only deflated variables, the impact of price-level changes has been under-emphasized. In these cases, either the use of nominal variables may be justified or prices may need to enter as separate explanatory variables. To some extent this choice would further depend on statistical considerations of multicollinearity among regressors in an inflationary sample period and whether separate information regarding the impact of prices is wanted or not.

Some preliminary tests as to the non-homogenous impact of prices have been performed here with respect to three mainstream demand functions, viz., private consumption and investment functions, and the demand for money function. For the sake of convenience and easier interpretation, the log-linear form has been adopted for these demand functions in the tradition of these type of tests (e.g. Branson and Klevorick, 1969). The interpretation of the tests below is cast in terms of the consumption function. A similar interpretation can be extended to other functions.

A direct test, without the introduction of any lags, which would be in line with the kind of functions used in the models surveyed here, is to conduct the regression

$$\ln c_t = \beta_o + \beta_1 \ln y_t + \beta_2 \ln nw_t + \beta_3 \ln P_t + u_t \tag{10.1}$$

and see if β_3 is significantly different from zero, where c, y and nw refer respectively to the deflated values of private consumption, private disposable income and net worth of the private sector at the beginning of the period.

Moregenerally, one can use some distributed lag mechanisms. For example, one can test for $\sum_{k=0}^{K} \eta_k > 0$, in the regression

$$\ln c_t = \beta_0 + \sum_{i=0}^{I} \gamma_i \ln y_{t-i} + \sum_{j=0}^{J} \delta_j \ln nw_{t-j} + \sum_{k=0}^{K} \eta_k \ln P_{t-k} + \varepsilon_t \qquad \ldots\ldots \; (10.2)$$

It can be shown that $\Sigma\, \eta_k$ would be positive because of money-illusion. It would be zero if in the formation of expectations, the adjustment mechanism is either level-based or change-based (see e.g. Branson and Klevorick, 1969) but it may be different from zero if expectations are formed in other ways.

Since there is no *a priori* knowledge of either the nature or the length of distributed lag mechanisms, if any, the best one can do is test with alternative lengths of lags and inclusion and exclusion of various variables. In doing so, no smoothing restrictions on the coefficients of the lagged variables were imposed.

First, however, we did the test for $\beta_3=0$ in equation 10.1 for the three functions involved. As indicated earlier, the specification in this equation is more in line with the functions surveyed in the models we are concerned with here.

Variables were measured in one set of equations as natural logs, and in another, as first differences of the logs. This kind of filter was used in the hope that if some multicollinearity and serial correlation problems exist in the first set, they would be eliminated in the second set.

Table 10.5

Impact of Price-Level Variations: Some Specifications

Dependent Variable	Independent Variables					$\bar{R}^2$	dw
I. Variables measured as logs.	II. Variables measured as first difference of logs						
C_p/P	Const.	Y^d/P	NW_{-1}/P		P		
I	1.044 (5.3)	.8652 (21.5)	-.0073 (-1.91)		-.0159 (-.852)	.994	1.55
II	-.0114 (-2.52)	1.044 (15.99)	-.038 (-1.06)		.0857 (2.154)	.94	2.30
I_p/P				R_1	P		
I	-12.53 (-5.43)	1.362 (2.96)	1.165 (2.63)	.058 (.30)	.239 (1.14)	.885	1.33
II	.0433 (.85)	-.191 (-.26)	.913 (2.24)	-.219 (.95)	.142 (.312)	.112	1.92
M/P				R	P		
I	-4.0 (-4.9)	1.012 (5.88)	.452 (2.86)	.0945 (.61)	-.104 (-.907)	.93	2.1
II	.072 (5.03)	.128 (.65)	.793 (.72)	-.088 (-1.096)	-.817 (-6.83)	.77	1.28

It is difficult to say what should be the *a priori* expectation about the sign of P. In the standard static theory of consumer's behaviour, at higher price-levels, money incomes are perceived to be higher and money-illusion should have a positive effect. On the other hand, if a non-homogenous impact of prices is due to formations of expectations regarding future price movements, a higher price-level, when it is associated with a fall in prices in future, may lead to a negative impact on current demands.

In the consumption function the coefficient of the price-level turns out to be significantly positive (at 5 per cent level of significance), when variables are measured as first differences in logs. For the investment function, the equation estimated with dirst differences gives a very poor fit with $\bar{R}^2$ as low as .112. When variables are measured as logs the fits are much better. In this case although the sign for the coefficient of the P variable is positive, it is significant only at the 15 per cent significance level. In both the demand for money functions there is a negative sign for the price variable. It is significant at 1 per cent level of significance when variables are measured as first differences in logs. These results are supplemented with F-tests on the price-variables given in Table 10:6.

Working with the first differences in logs, more tests have been performed with alternative lag profiles. Since, *a priori*, it is not known what should be the distributed lag pattern in the income, wealth and other variables, alternative specifications have been adopted for the lags. The length of lag was not increased beyond two years to conserve degrees of freedom. Results for the investment functions consistently give very poor fit for all such estimations. The results of F-tests on the price terms with various specifications are reported in Table 10:6. Since no derivations for the structural equations used here is provided, they should be interpreted only as auxiliary equations.

Table 10.6

F-Tests on Price Terms

Dependent Variable	Independent Variables	F*	Critical Value of F at 95 % confidence leve
Variables Measured as Logs			
C_p	Y^d, NW_{-1}, P	1973.3	4.381
I_p	Y^d, NW_{-1}, R_1, P	1.294	4.414
M	Y^d, NW_{-1}, R, P	.822	4.414
Variables Measured as First Differences of Logs			
C_p	Y^d, NW_{-1}, P	241.04	4.381
C_p	$Y^d, Y^d_{-1}, Y^d_{-2}, NW, NW_{-1}, NW_{-2}, P, P_{-1}, P_{-2}$	.823	3.411
C_p	$Y^d, Y^d_{-1}, NW, P, P_{-1}, P_{-2}$	1.364	3.239
C_p	$Y^d, Y^d_{-1}, Y^d_{-2}, NW, P, P_{-1}$	1.288	3.634
I_p	Y^d, NW_{-1}, R_1, P	.098	4.414
I_p	$Y^d, Y^d_{-1}, NW, NW_{-1}, R_1, P, P_{-1}, P_{-2}$	.1515	3.344
M	Y^d, NW_{-1}, R, P	46.61	4.414
M	$Y^d, Y^d_{-1}, Y^d_{-2}, NW, P, P_{-1}, P_{-2}$	.8775	3.287

Notes: (1) F* is defined as $\dfrac{(\Sigma\,\hat{\hat{y}}^2 - \Sigma\,\hat{y}^2)/(K-M)}{\Sigma\,e^2\,/(N-K)}$ where $\Sigma\,\hat{\hat{y}}$ and $\Sigma\,e^2$ respectively measure the explained and unexplained variations when all regressors are used, and $\Sigma\,\hat{y}^2$ measures the explained variation when regressors excluding the price terms are used. K is the number of parameters in the first case, M is the number of parameters in the second case and N (=23) is the number of observations.

(2) The critical value of F refers to (K-M, N-K) degrees of freedom.

(3) Various other lags were used in all the three types of functions but the additional explanatory power of the price terms was not found to be significant.

In conclusion, it seems that when only current terms are used, as is the case in most of the models reviewed, there is evidence that the price variable has an independent and statistically significant impact on consumption and money demand function. When income, wealth and other variables are lagged, price terms, either current and lagged taken together do not seem to have a significant effect in any of the functions.

In conducting the regressions reported above the simultaneity problem has not been taken account of. As such these tests cannot be taken as more than a preliminary evidence. They do seem to point out that at least in the mainstream consumption functions, the role of the price-variable needs to be explored further.

10.3 Tests of Exogeneity for Selected Variables

An appropriate choice of the exogenous variables is an important aspect of the specification of a model. Such a choice would affect both the forecasting performance of the model as also the analysis of policy options. Forecasting performance depends to a large extent on how well the exogenous variables have been predicted outside the model. If a model takes as endogenous some variables whose interdependence on other endogenous variables is insignificant or weak, the forecaster loses the option of using either informal forecasts for these, or formal forecasts but derived from other specialised sources, in having attempted a formal explanation for these variables in his own model.

On the other hand, model misspecification may also occur when variables which are truly endogenous are mistakenly considered as exogenous. This may especially subvert the policy analysis. Many times policy variables are jointly dependent on other endogenous variables of the model and in such cases they should not be taken as completely exogenous.

From this point of view we have examined six major variables common to most models of income determination in the Indian context. These are: external aid, exports, money-supply, unborrowed money reserves, government consumption expenditure and government investment expenditure. What we want to examine is whether or not these variables are jointly dependent with income, i.e. whether changes in these are affected by changes in income.

The test procedures adopted for this purpose are the now familiar Granger-Sims (Granger, 1969; Sims, 1972) type of tests. A review of these tests was undertaken in Chapter 3. Most people working with these tests acknowledge that there are a number of situations when the tests would be fooled. The results reported here also cannot be taken as definitive statements but merely as qualitative guides to formulation of models in the Indian context. Another point to note is that whereas in most empirical work in this context quarterly data has been used, we are constrained to use annual data. However, annual intervals are not inappropriate for variables like aid, government consumption and investment expenditures etc. where decisions are taken with a longer time perspective.

The operational part of the test boils down to conducting regressions, with various lags and leads, between the two variables in question; in our case, between income and one of the six variables listed earlier. Thus, if the two variables are X and Y, one has to regress Y on past, present and future values of X and vice-versa. Apart from looking into the significance of the past coefficients, the tests are based on the significance or otherwise of future coefficients as a group for 'proper causality', i.e. where the cause shows its effect after a period; and, on the present and future coefficients where contemporaneous causation is included. Before the regressions are carried, however, one generally needs to filter the data in order to approximate the requisite conditions of working with covariance-stationary series and serial-correlation-independent residuals. We have used the same filter as the one proposed in Sims (1972) in view of evidence that this filter has been found to reduce the data to stationarity for most economic time-series. Thus, we have first taken the logarithms of all relevant series and then transformed these by the filter $y_t = Y_t - 1.5Y_{t-1} + .5625Y_{t-2}$. It must be acknowledged that the results tend to be sensitive to the type of filter used and further experimentation is possible. Furthermore, we have limited the lags and leads to 3 years in both directions in order to preserve degrees of freedom. A constant term is included in all regressions.

10.3.1 Exogeneity of Exports

If exports are dependent primarily on developments in the rest of the world and exogenous domestic policy measures, they are best treated as exogenous to the model of income determination. On the other hand, if they are affected in a major way by other variables in the model, their endogeneity may not be ignored.

First of all let us recapitulate as to how exports have been treated in the models we have reviewed. This variable enters as exogenous in models by Narasimham, Krishnamurty, Krishnamurty-Choudhry, Marwah -I, Mammen, Bhattacharya and Gupta.

In Choudhry's model exports are divided into 4 categories referring respectively to tea, jute, cotton and 'other' exports. However, these are explained mostly by world income and a time trend which are themselves exogenous to the system. The only part of exports bearing some relation to the rest of the system is 'other' exports where a domestic price variable enters but its coefficient is not necessarily significant ($t < 2$).

In Marwah II, exports are endogenous but again they depend on variables which are either exogenous like export prices of less developed regions or which are entirely dependent on exogenous variables like imports of LDCs and DCs coming from LDCs. The only variable which makes exports jointly dependent with the rest of the system is unit value of exports, the coefficient of which is not necessarily significant.

In Pandit's model, exports are explained by world imports and a dummy which are exogenous. There is also a relative price variable containing unit value of Indian exports which relates this equation to the rest of the model. The coefficient in this case has a t-value of -3.69 which indicates statistical significance.

In UNCTAD I model, exports are divided between tea, jute and 'other' exports: tea exports are a predefined function of time, jute exports are a function of world exports and jute prices which are both exogenous, and 'other' exports are a function of world exports, unit value of other exports and world price index of 'other' exports. The only variable which relates it to the rest of the system is unit value of 'other' exports which depends on non-agricultural output. In UNCTAD-LINK model, exports are divided into tea, jute, engineering, and 'other' categories. Tea and jute exports are predefined functions of time. Engineering goods' exports are explained by the price of engineering goods in the world market, and the remaining residual category is explained by world income and the unit value of exports of these other goods. This last variable links the export sector to other internal forces. However, its coefficient obtains a low t-value indicating that it may not be significantly different from zero.

This evidence indicates that the joint dependence of exports and its components with other domestic endogenous variables is very weak and that this variable may be treated as exogenous at least to the simultaneous block of the model. It may be explained in the recursive block or a separate block for it may be constructed outside a model if a formal explanation is offered at all. The evidence indicates that this variable is affected basically by changes in the international economy and government export policy and rather than tying oneself to a formal explanation within the model, it is best to have it outside so that a degree of flexibility is at hand for introducing autonomous changes affecting this variable whenever they may occur.

More direct evidence is provided here to substantiate that exports are exogenous to income in terms of the Granger-Sims tests, the results of which are reported in Table 10:7.

To recapitulate, a unidirectional causality from exports to income would require that in the regression of income on exports, the future coefficients be insignificant and the past and current coefficients be significant. And in the regression of exports on income, the future coefficients as a group should be significant whereas the past coefficients may or may not be significant. The evidence presented in Table 10:7 clearly substantiates the exogeneity of exports to income.

Table 10.7

Income and Exports: Tests of Exogeneity

Regressions:	Income on Exports		Exports on Income	
Lags	With current exports	Without current exports	With current income	Without current income
	I	II	I	II
-3	.471 (2.25)	.345 (1.92)	.155 (.34)	.29 (.79)
-2	.187 (1.10)	.119 (.734)	-.78 (1.49)	-.68 (1.44)
-1	.155 (.99)	.142 (.90)	-.15 (.28)	-.26 (.54)
0	.227 (1.12)		.344 (.54)	
1	.048 (.278)	-.025 (.155)	1.61 (2.498)	1.43 (2.68)
2	.035 (.219)	.038 (.236)	-.302 (.59)	-.28 (.57)
3	.110 (.707)	.159 (1.05)	.291 (.56)	-.46 (1.14)

F-tests on Future Coefficients	F*	F*	Critical Value of F at 95% confidence level
Current and Future Coefficients in I	.204	3.439	3.48
Future coefficients in I	.611	2.624	3.71
Future coefficients in II	.384	3.634	3.36

Note : Figures in brackets are t-ratios.

10.3.2 Exogeneity of International Aid

A second variable under the control of the rest of the world regarding which a similar idea needs to be formed is 'aid'. Decisions regarding international aid are taken outside the economy but they may or may not be independent of developments in the domestic economy. One may expect aid to be negatively linked to the (per capita) income levels and positively linked to the rate of growth of income, the latter relation indicating favourable aid for viable customers. Although expressly these economic considerations may arise, they may still be overridden by political and other non-economic considerations. In such a case again one may find a unidirectional causality from aid to income and not vice versa. The models we have reviewed invariably take aid as exogenous. The evidence in terms of the Granger-Sims tests indicates that such a treatment of the aid variable is not unjustified. The relevant results are given in Table 10:8.

There is some evidence of causality from aid to income in terms of the magnitudes of future coefficients in the regression of aid on income. But the relevant t-values and F-value for the future coefficients as a whole are not significant. There is no evidence of bidirectional causality. On the whole, this evidence suggests lack of causality either way. This indicates that aid is not affected by domestic income, hence it is justifiably treated as exogenous. Furthermore, the impact of aid and on income also seems very weak. This may be due to its relatively small magnitude compared to income.

Table 10.8

Income and Aid: Tests of Exogeneity

Regressions	Income on Aid		Aid on Income	
	With current aid	Without current aid	With current income	Without current income
Lags	I	II	I	II
-3	.0098 (.187)	.0129 (.266)	-.584 (.113)	-2.04 (.49)
-2	-.0249 (.385)	-.016 (.306)	.790 (.133)	-.313 (.058)
-1	.0049 (.0675)	.021 (.537)	-.72 (.119)	.454 (.084)
0	-.018 (.258)		-3.7 (.51)	
1	-.025 (.453)	-.014 (.433)	1.27 (.174)	3.21 (.533)
2	.0196 (.584)	.024 (.887)	-2.34 (.401)	-2.58 (.459)
3	.0189 (.796)	.021 (.977)	3.62 (.614)	1.81 (.398)

F-tests on Future Coefficients	F*	F*	Critical value of F for 95% confidence level.
Current & Future terms in I	.557	.321	3.48
Future terms in I	.459	.291	3.71
Future terms in II	.644	.323	3.36

10.3.3 Money Supply and Unborrowed Money Reserves: Tests of Exogeneity

The question as to whether money-supply should be taken as exogenous has been extensively explored for some of the developed countries like U.S.A., Canada and U.K. and conclusions are not uniform between countries. The results seem sensitive to the empirical context. In the Indian context, most of the models use money supply to be exogenous. There are some exceptions like Mammen, Bhattacharya and Gupta who treat it as endogenous. Their approaches differ in terms of degree of disaggregation but a bidirectional causality arises in all cases because of interest-rate variables which affect money supply and are affected by income. In Bhattacharya's model, which endogenously determines aggregate money supply, the explanatory variables are unborrowed money reserves (UMR), net liquidity ratio, and the excess of Reserve Bank's discount rate over short-term interest rate. In other models further disaggregation of the money supply variables has been offered. We propose to test here whether money-supply could itself be treated as exogenous and whether unborrowed money reserves should be taken as exogenous. The relevant results are respectively given in Tables 10.9 and 10.10.

Table 10.9

Income and Money-Supply: Tests of Exogeneity

Regressions	Income on Money-Supply		Money-Supply on Income	
	With current money-supply	Without current money-supply	With current income	Without current income
Lags	I	II	I	II
-3	.737 (2.21)	.737 (2.32)	.065 (.674)	.116 (1.44)
-2	-.947 (-1.92)	-.935 (2.01)	-.082 (.74)	-.044 (.434)
-1	1.409 (3.10)	1.388 (3.32)	-.120 (1.07)	-.161 (1.55)
0	-.074 (.178)		.128 (.94)	
1	-.607 (1.48)	-.635 (1.76)	.550 (4.03)	.483 (4.16)
2	.489 (.82)	.474 (.843)	.255 (2.34)	.264 (2.44)
3	-.056 (.116)	-.057 (1.24)	.161 (1.46)	.223 (2.55)

F-tests on Future Coefficients	F*	F*	Critical values of F
Current & Future Coefficients in I	.787	13.79	3.48
Future Coefficients in I	.737	11.01	3.71
Future Coefficients in II	1.067	14.53	3.36

Table 10.10

Income and Unborrowed Money Reserves: Tests of Exogeneity

Regressions	Income on UMR		UMR on Income	
	With current term	Without current term	With current term	Without current term
Lags	I	II	I	II
-3	.642 (2.38)	.580 (1.95)	-.131 (.89)	-.139 (1.19)
-2	.885 (1.77)	.838 (1.51)	.017 (.103)	.011 (.073)
-1	1.071 (2.25)	1.36 (2.72)	.035 (.202)	.041 (.274)
0	.576 (1.90)		.021 (.102)	
1	-.067 (.214)	-.104 (.301)	.568 (2.74)	.579 (3.43)
2	.128 (.338)	-.180 (.473)	.279 (1.69)	.278 (1.77)
3	-.397 (1.49)	-.407 (1.37)	.189 (1.13)	.178 (1.40)

F-tests on Future Coefficients	F*	F*	Critical value of F at 95% confidence level
Current & Future Coefficients	1.22	3.697	3.48
Future Coefficients in I	1.58	.068	3.71
Future Coefficients in II	.73	.096	3.36

In the regression of income on money-supply the past coefficients are observed to be significant in terms of the t-values. The future coefficients are not significant in terms of individual t-values, and the F-statistic for the future coefficients as a group. On the other hand, in the regression of money supply on income, the future coefficients seem definitely significant both in terms of individual values and the F-statistic for the group as a whole. The F-value is higher when the current term is also included along with future terms. This evidence supports unidirectional causality from money-supply to income. The effects of changes in money-supply appear to have effects in future periods as also in the current period.

Thus, most of the models are justified in treating money-supply as exogenous. This is not to say that an expanded endogenous treatment of money-supply would not be justified if such is the purpose of the model. However, in all likelihood such an expanded treatment is likely to be very weakly linked to the simultaneous block of the model and would belong to a recursive block.

A similar set of tests are conducted with reference to unborrowed money reserves in order to justify whether its treatment as exogenous is useful. In terms of the t-values of individual coefficients, in the regression of income on UMR, past coefficients are significant but not the future coefficients. While in the reverse regression, the future coefficients are significant in terms of t-values. The F-statistic is significant for the future coefficients when taken together with the current term indicating possible contemporaneous but still unidirectional causality.

The conclusion is that it is not altogether unjustified to treat money-supply as exogenous. However, it should be recalled that the Granger-Sims tests cannot be used as definitive statements on the question of exogeneity. Further discussion on the question of exogeneity of money-supply in the Indian context is provided in Chapter 11.

10.3.4 Government Consumption and Investment Expenditures: Tests of Exogeneity

The treatment of government revenue and expenditure components in the models is both interesting and important in view of the increasing importance of public sector activities.

Many of models reviewed here take both government revenues and expenditures to be exogenous to the domestic income levels. Thus, they are not able to distinguish between automatic and discretionary components of tax-revenues. Furthermore, expenditures of all kinds are taken to be entirely independent of government's automatic revenues which depend on domestic income levels. Also, the government budget constraint has generally been ignored implying that the government can freely choose values for all its instruments.

We propose to test here whether government consumption and investment expenditures show bidirectional causality. Domestic incomes are affected by government expenditure via the multiplier process. But on the reverse, government's income and budget-levels depend to a considerable extent on domestic income levels, which in turn may affect decisions regarding government expenditure levels. The relevant results are reported in Tables 10:11 and 10:12.

Table 10.11

Income and Government Consumption: Tests of Exogeneity

Regressions	Income on Govt. consumption		Govt. consumption on Income	
	With current term	Without current term	With current term	Without current term
Lags	I	II	I	II
-3	-.119 (.572)	-.093 (.436)	-.023 (.065)	.424 (1.214)
-2	.233 (1.16)	.255 (1.23)	-.429 (1.05)	-.091 (.204)
-1	.114 (.612)	.058 (.309)	.715 (1.73)	.355 (.791)
0	.239 (1.32)		1.14 (2.27)	
1	.392 (1.999)	.318 (1.64)	.849 (1.69)	.254 (.506)
2	-.085 (.37)	-.101 (.428)	.337 (.842)	.411 (.875)
3	.043 (.142)	.133 (.437)	-.743 (1.834)	-.189 (.499)

F-tests on Future Terms	F*	F*	Critical values of F at 95% confidence level
Current & Future Coefficients	2.052	1.994	3.48
Future Coefficients in I	1.688	1.758	3.71
Future Coefficients in II	1.563	.269	3.36

In the regression of income on government consumption the future coefficients are not significant. On the other hand, in the regression of government consumption on income the future coefficients are also not significant on the strength of the F-tests and in terms of t-values when current terms are not used. When current terms are used the t-values for the current term and two of the future coefficients indicate significance. The magnitudes of the coefficient are also meaningfully different from zero. This evidence does not categorically establish absence of bidirectional causality.

Table 10.12

Income and Government Investment: Tests of Exogeneity

Regression	Income on Govt.Investment		Govt.Investment on Income	
	With current term	Without current term	With current term	Without current term
Lags	I	II	I	II
-3	-.031 (.135)	-.057 (.249)	-.194 (.414)	-.045 (.119)
-2	.367 (1.57)	.294 (1.28)	-.491 (.917)	-.378 (.784)
-1	.315 (1.32)	.195 (.889)	-.520 (.956)	-.640 (1.313)
0	.290 (1.17)		.378 (.575)	
1	.012 (.053)	-.068 (.317)	.507 (.770)	.309 (.568)
2	-.055 (.262)	-.003 (.015)	.773 (1.469)	.798 (1.568)
3	.075 (.366)	.162 (.839)	.049 (.092)	.233 (.567)

F-tests on Future Terms	F*	F*	Critical values of F fo a 95% confidence level
Current & Future Coefficients in I	.097	.788	3.48
Future Coefficients in I	.548	.670	3.71
Future Coefficients in II	.269	.834	3.36

In a similar regression between income and government investment expenditures, although the future coefficients are not significant in the regression of income on government investment, they are also not significant in the reverse regression. Thus no clear cut evidence for assuming unidirectional causality from government investment to income is provided.

10.4 Forecasting Performance: Selected Models

The tests which we have hitherto considered touch upon certain aspects of macro modelling of the Indian economy. Their individual limitations warrant that only notional guidance can be taken from them. Furthermore, the information provided by them needs to be complemented by consideration of models as a whole. The usefulness and validity of a macroeconometric model as a whole for the purpose of prediction and future policy making lie in its forecasting performance.

In general, the models reviewed here have been abandoned by their respective builders after their first efforts except a few like the UNCTAD models which may be in current use. It is very difficult therefore to compare 'own' forecasts against realizations. Furthermore, there are difficulties of varying sample-periods and estimation techniques which imply the use of different information sets in different models and thereby make comparisons between their forecasting performance difficult to interpret. An additional difficulty relates to the revision of the series of national income and its components. Whereas the models are estimated with reference to the conventional series, realizations for later years would only be available in revised terms thus invalidating comparisons between predictions and realizations. In some cases, where beyond-sample values of exogenous variables are available only in revised terms, it would not be possible to make any predictions in the first place. For all these reasons, the old estimates of the models cannot be used for forecasting.

One way out of this situation is to re-estimate some of the models with a uniform sample period, uniform estimation techniques and data series which would relate to the revised C.S.O. estimates for national income and its components. This is the alternative we have followed here. However, for this purpose we have chosen a limited number of models. It is difficult to include all the models in the analysis. Apart from the almost prohibitive resource requirements for this purpose, in many models a number of series were model builders' own creation corresponding to which any established series are not available. Also, for some of the bigger models, the number of predetermined variables exceed the number of observations we can set aside for estimation. This calls for modifications in estimation techniques and thus a deviation from the objective of following a common estimation technique as far as possible.

The models we have re-estimated here are Choudhry-I, Marwah -I (aggregate part) and the Bhattacharya model. These models belong to the mainstream model building tradition for the Indian economy. These are relatively smaller in size and produce forecasts for a nearly similar set of variables. They do differ, however, in the specification of individual equations although their basic theoretical foundation is common in terms of the IS-LM framework. In two of the models (Choudhry -I and Marwah -I), production functions are utilized and the supply constraint plays a part while in Bhattacharya's model aggregate demand is the active force but its special feature is an endogenous money supply function.

All these models are re-estimated by 3SLS for a common sample period from 1950/51 to 1969/70. The forecasts are generated for the five years from 1970/1 to 1974/5. Two sets of forecasts are obtained. In both cases, actual values of the exogenous variables have been used. However, in one case, actual values of lagged endogenous variables, and in the other case, their model-generated values are used. This latter variety thus provides dynamic forecasts from the models.

In constructing the models, similar specifications as in the original versions are used. However, in some instances minor changes are involved. These changes are explicitly mentioned later when the re-estimated models are presented. The re-estimated models need only be interpreted as adapted versions rather than strict replicas of the original models.

The forecasts are compared against realizations as also against forecasts from some autoregressive models which use only past history of a series and can serve as 'bench mark' predictions.

In presenting the adapted models, we have used, as far as possible, the same variable names as in the original models.

10.5 The Re-estimated Models

2SLS estimates of the models for the period 1950-51 to 1969-70 are given below. Figures in brackets are t-ratios.

10.5.1 Adapted Bhattacharya Model

1. $$C_p = \underset{(9.81)}{925.395} + \underset{(151.05)}{.80027}\, Y^d \qquad \bar{R}^2 = .999,\quad d = 1.391$$

2. $$I_p = \underset{(-1.27)}{-361.226} + \underset{(4.89)}{.1184}\, Y - \underset{(-.687)}{34.978}\, R_1 + \underset{(.291)}{.0647}\, I_{p_{-1}} \qquad \bar{R}^2 = .974,\quad d = 1.91$$

3. $$M = \underset{(-1.849)}{-600.7867} + \underset{(2.829)}{.092693}\, Y^d - \underset{(-.166)}{14.3195}\, R + \underset{(2.816)}{.04953}\, NW \qquad \bar{R}^2 = .991,\quad d = 1.29$$

4. $$M = \underset{(-.938)}{-337.6033} + \underset{(33.28)}{1.76824}\, U - \underset{(-2.33)}{99.1919}\, q - \underset{(-.739)}{4.67}\, l \qquad \bar{R}^2 = .996,\quad d = 2.04$$

5. $$R_1 = \underset{(1.385)}{1.4067} + \underset{(1.307)}{.53844}\, R + \underset{(2.579)}{.534036}\, R_{1_{-1}} \qquad \bar{R}^2 = .507,\quad d = 1.208$$

6. $T = \underset{(-7.33)}{-450.239} + \underset{(12.03)}{.1031\ Y} + \underset{(2.75)}{35.3235\ t}$ $\quad \bar{R}^2 = .989$, $d = 1.255$

7. $Y = C_p + I_p + E_n$

8. $Y^d = Y - T + TR$

9. $q = R_d - R$

In the original model, 'monetized' levels of income and expenditure components were used where as these estimates refer to the total levels rather than the monetized levels. Apart from the fact that certain arbitrary procedures have to be adopted for deriving the monetized counterparts from the official data series, which we have abstracted from, such a change was also necessary from the point of view of comparability with the other models for the present exercise.

The nine endogenous variables for this system are C_p, I_p, R, M, R_1, T, Y, Y^d and q. The exogenous variables are R_d, 1, NW, t, U, E_n and TR which together with the two lagged endogenous variables $I_{p_{-1}}$ and $R_{1_{-1}}$ provide 9 predetermined variables.

10.5.2 Adapted Choudhry I Model

The re-estimated model is given below.

1. $P = \underset{(11.22)}{78.5803} + \underset{(18.80)}{.004741\ Y} - \underset{(-4.94)}{.349399\ O}$ $\quad \bar{R}^2 = .99$, $d = 1.019$

2. $O = \underset{(-8.05)}{-102.732} + \underset{(1.67)}{.060626\ K/P} + \underset{(19.58)}{.524013\ N}$ $\quad \bar{R}^2 = .975$, $d = 2.022$

3. $C_p = \underset{(.359)}{329.017} + \underset{(17.38)}{.98895\ Y^d} - \underset{(-3.516)}{1.1622\ M} + \underset{(.908)}{2.683\ N} + \underset{(.305)}{.0124\ CP_{-1}}$ $\quad \bar{R}^2 = .999$, $d = 1.69$

4. $I_p = \underset{(.0578)}{-39.867} + \underset{(.817)}{.0602787\ Y^d} - \underset{(-1.16)}{.066714\ Y_{-1}} - \underset{(-.397)}{21.773\ R_{1_{-1}}} + \underset{(1.95)}{.820757\ M} - \underset{(-1.017)}{1.6262\ (K/P)_{-1}}$ $\quad \bar{R}^2 = .975$, $d = 1.81$

5. $IM = \underset{(2.487)}{595.896} + \underset{(1.85)}{.032622}\, Y + \underset{(.466)}{4.3491}\,(P-P_W) - \underset{(-.814)}{.18769}\, F_{-1}$ $\quad \bar{R}^2 = .80$, $d = 1.29$

6. $M = \underset{(-.392)}{-174.698} + \underset{(22.125)}{.15795}\, Y + \underset{(1.27)}{684.159}\, \frac{(P-P_{-1})}{P_{-1}}$

$- \underset{(-.2899)}{13.5314}\, R_{1_{-1}} + \underset{(1.88)}{8.293}\, P^s$ $\quad \bar{R}^2 = .908$, $d = 2.108$

7. $EX = \underset{(4.58)}{293.592} + \underset{(3.92)}{.47306}\, Y_W - \underset{(-.1667)}{.6807}\,(P-P_W)$ $\quad \bar{R}^2 = .865$, $d = 1.122$

8. $Y = C_p + I_p + G + EX - IM - T^{ind}$

9. $Y^d = Y - TRR$

10. $K = K_{-1} + I_p + I_g$

In the original model world income and prices (Y_W and P_W) were used in the export function. We have used world imports and unit value of world imports for these variables which seem to be more directly related the variable being explained and data for which is more reliable. Total capital stock is used in the output equation and government investment is introduced separately in the capital-stock identity. Other features of the original model are maintained. The model contains non-linearities and appropriate procedures are used for its solution in the forecast period.

The endogenous variables of the system are:

P, O, C_p, I_p, IM, M, X, Y, Y^d and K.

The predetermined variables are N, $R_{1_{-1}}$, F_{-1}, P^s, Y_w, P_w, $(G-T^{ind})$, TRR, I_g, Y_{-1}, $C_{p_{-1}}$, P_{-1} and K_{-1}.

10.5.3 Adapted Marwah I Model: Aggregate Part

The estimated equations along with the identities for the first version (non-quantity theory version) of the model are given below.

1. $C_p/P = \underset{(3.92)}{42.3907} + \underset{(5.45)}{.76241}\ X - \underset{(-4.00)}{.10789}\ \frac{L_{-1}}{P} - .079109\ N$ $\qquad \bar{R}^2 = .987$, $d = .963$

2. $I_p/P = \underset{(-3.6)}{-8.25159} + \underset{(4.24)}{.107172}\ X + \underset{(2.31)}{.39715}\ (I_p/P)_{-1}$ $\qquad \bar{R}^2 = .899$, $d = 1.95$

3. $M = \underset{(-1.70)}{-1342.2} + \underset{(15.77)}{.14628}\ Y + \underset{(1.52)}{105.665}\ R_1 + \underset{(2.44)}{16.773}\ P^s$ $\qquad \bar{R}^2 = .990$, $d = 1.60$

4. $R_1 = \underset{(1.360)}{1.3745} + \underset{(1.51)}{.58548}\ R + \underset{(2.57)}{.519097}\ R_{1_{-1}}$ $\qquad \bar{R}^2 = .51$, $d = 1.192$

5. $P = \underset{(1.68)}{8.6612} + \underset{(4.89)}{.00614}\ (Y - Y_{-1}) + \underset{(14.92)}{.8768}\ P_{-1}$ $\qquad \bar{R}^2 = .973$, $d = 2.45$

6. $Y = C_p + I_p + H$

7. $X = Y/P$

8. $L = M + TD$

9. $L = L_{-1} + DL$

Compared to the original model the only difference is the use of a one-year lagged liquidity variable rather than a two-year lag. Total liquidity is defined as money supply plus time deposits and appropriate identities are added.

The endogenous variables of the system are: C_p, I_p, M, R_1, P, Y, X, L and DL.

The exogenous variables are: H, TD, PS, N and R, and the lagged endogenous variables are: $I_{p_{-1}}$, P_{-1}, L_{-1}, $R_{1_{-1}}$ and Y_{-1}. A multiplication by P in appropriate equations reduces the system to linearity among endogenous variables for purposes of solution.

10.5.4 Autoregressive Benchmark Models

There are four variables (C_p, I_p, M and Y) common to all the three models and three variables (R_1, Y^d and P) common at least to two models. We concentrate on these seven variables for purposes of a comparative evaluation of the forecasting performance of these models. The output variable (O = Y/P) is common to two models, but since forecasts for Y and P are separately being evaluated, we have not included variable O into this analysis.

For each of the seven variables mentioned above, autoregressive benchmark models have been constructed. The models are estimated in first differences, with a maximum lag-length of five periods. The information used refers to the period 1950-51 to 1969-70. A maximum of five lagged terms are used in order to conserve degrees of freedom. The selection of lagged terms included in any equation was based on the significance of the individual coefficient and its contribution towards explaining the variation in the dependent variable. All these equations should be interpreted only as auxiliary equations as no economic meaning is attached in these choices. It will be noticed that the highest R^2 would be obtained when all the five lagged terms are used. In a step-wise regression procedure we have selected those combinations of lagged terms which have a R^2 near to this highest value but the highest $\bar{R}^2$.
A constant term is used in all the models. Predicted levels of variables are then obtained by integrating predicted changes with previous years' levels. This is done dynamically in one case. In the static case actual levels of previous years' values are used.

The estimated autoregressive models are given below. Since one observation is lost in first differencing and five in allowing for lags, the effective sample in each case consists of fourteen observations from 1956/7 to 1969/70.

1. $C_p = \underset{(2.126)}{769.736} - \underset{(-.434)}{.0916}\, C_{p_{-2}} + \underset{(2.976)}{.8113}\, C_{p_{-3}}$ $\qquad R^2 = .4611$

2. $I_p = \underset{(.909)}{99.9679} + \underset{(1.664)}{.5066}\, I_{p_{-3}} + \underset{(.995)}{.4277}\, I_{p_{-5}}$ $\qquad R^2 = .2564$

3. $M = \underset{(.4581)}{25.3567} + \underset{(2.261)}{.53312}\, M_{-1} + \underset{(.688)}{.19306}\, M_{-3} + \underset{(1.614)}{.4290}\, M_{-4}$ $\qquad R^2 = .783$

4. $R_1 = \underset{(.889)}{.2401} - \underset{(-.909)}{.2962}\, R_{1_{-2}} - \underset{(-.557)}{.2006}\, R_{1_{-3}} - \underset{(-1.058)}{.3928}\, R_{1_{-4}}$ $\qquad R^2 = .3472$

$\qquad - \underset{(-1.078)}{.3892}\, R_{1_{-5}}$

5. $P = \underset{(2.698)}{7.2771} - \underset{(-.584)}{.1624}\, P_{-1} - \underset{(.889)}{.25185}\, P_{-4}$ $\qquad R^2 = .078$

6. $Y = \underset{(2.188)}{771.842} + \underset{(4.449)}{.91337}\, Y_{-3}$ $\qquad R^2 = .6226$

7. $Y^d = \underset{(2.011)}{682.001} + \underset{(4.264)}{.91684}\, Y^d_{-3}$ $\qquad R^2 = .5692$

Forecasts generated from the selected adapted models and the extrapolative models for the period 1970/1 to 1974/5 are given in Tables 10.13 and 10.14.

Table 10.13

Actuals and Static Predictions from Adapted Models and Extrapolation

Variable	Years	Actuals	Static Predictions: Marwah I	Choudhry I	Bhattacharya	Auto-regressive model
C_p	70/71	29725	30166.3	28120.4	30886.9	31407
	71/2	31415	33706.0	35553.8	36038.5	30822
	72/3	33556	36298.2	46046.5	41579.9	33590
	73/4	42904	40652.3	68892.3	48147.3	37441
	74/5	49798	41210.7	79906.2	52654.7	44364
I_p	70/71	4486	4687.0	4063.4	4573.9	4454
	71/2	4546	5277.1	5797.7	5470.1	4999
	72/3	5210	5666.9	8380.8	6344.5	5157
	73/4	5248	6503.8	14088.1	7540.2	5429
	74/5	8105	6195.3	15470.4	8056.4	5969
M	70/71	7140	6955.0	6679.0	7020.2	6992
	71/2	8139	7686.9	8387.5	7983.9	7743
	72/3	9412	8483.1	10836.0	9038.7	8788
	73/4	10836	9637.6	15962.1	10311.7	10161
	74/5	11530	9444.1	16521.6	11252.8	11637
R_1	70/71	5.53	6.168		8.148	6.01
	71/2	6.49	6.294		6.767	6.32
	72/3	6.99	6.793		6.866	7.21
	73/4	5.59	7.298		4.487	7.91
	74/5	5.10	6.905		10.284	5.76
P	70/71	181.50	183.718	185.201		173.6
	71/2	188.80	203.033	231.853		184.1
	72/3	207.50	216.998	296.411		196.1
	73/4	254.70	249.911	436.870		212.0
	74/5	313.60	233.713	504.107		260.8
Y	70/71	40535	41177.3	38472.0	41784.9	42355
	71/2	43251	46273.2	48486.9	48798.6	42357
	72/3	47022	50221.1	62284.3	56180.4	47368
	73/4	57678	56682.2	91973.5	65213.5	52489
	74/5	68457	57959.9	105575.9	71265.0	60114
Y^d	70/71	36175		34112.0	37439.3	38167
	71/2	38340		43575.9	43876.6	37519
	72/3	41748		57010.3	50801.0	41983
	73/4	51781		86076.5	59007.5	46892
	74/5	61087		98205.9	64639.8	53695

Table 10.14

Dynamic Predictions from Adapted Models and Extrapolation

Variable	Years	Dynamic Predictions			
		Marwah I	Choudhry I	Bhattacharya	Autoregressive Model
C_p	70/71	30166.3	28120.4	30886.9	31407
	71/2	34209.3	37211.2	35863.9	32503
	72/3	37690.5	44593.6	41684.4	34678
	73/4	42649.4	64920.1	48396.0	38563
	74/5	45844.8	55734.7	53321.1	40023
I_p	70/71	4687.0	4063.4	4573.9	4454
	71/2	5413.1	6374.5	5401.4	4968
	72/3	6065.8	7422.3	6385.6	5579
	73/4	7008.8	11967.3	7638.0	5797
	74/5	7649.1	7814.9	8318.5	6518
M	70/71	6954.9	6679.0	7020.2	6992
	71/2	7815.4	8742.2	7966.2	7596
	72/3	8752.5	10207.2	9049.3	8244
	73/4	9996.6	14592.2	10336.9	8993
	74/5	10424.8	12361.1	11320.2	9794
R_1	70/71	6.168		8.148	6.01
	71/2	6.626		8.069	6.81
	72/3	6.863		7.767	7.54
	73/4	7.232		5.038	8.45
	74/5	7.758		10.356	8.63
P	70/71	183.718	185.201		173.6
	71/2	204.959	242.265		176.2
	72/3	219.682	285.244		183.5
	73/4	245.313	409.032		188.0
	74/5	253.616	354.168		194.2
Y	70/71	41177.3	38472.0	41784.9	42335
	71/2	46912.4	50599.9	48555.3	44178
	72/3	52012.3	60003.5	56325.9	48294
	73/4	59184.1	86208.7	65560.0	53762
	74/5	64047.9	75484.7	72193.6	56198
Y^d	70/71		34112.0	37439.3	38167
	71/2		45688.9	43658.3	39510
	72/3		54729.5	50931.5	43154
	73/4		80311.7	59318.2	48298
	74/5		68114.7	65472.6	50212

10.6 Evaluation of Forecasting Performance

Overall variable-wise comparisons across the models are made by using the inequality coefficient defined as $\left[\Sigma(P_t - A_t)^2/\Sigma A^2_t\right]^{1/2}$. These are given in Table 10.15 both for static and dynamic predictions. It would be observed that in general, the autoregressive benchmark models outperform the adapted macroeconometric models. A few exceptions to this feature of the results may, however, be noted.

In predicting M, the Bhattacharya model outperforms the extrapolative model both for static and dynamic predictions.

Among the three models considered here, the best prediction performance is shown by the Marwah model which has the lowest inequality coefficients for most of the variables in question. Bhattacharya model performs nearly as well as the Marwah model in terms of the magnitude of the inequality coefficients and the differences are too small to attach any significance. Although the Choudhry-I model does consistently worse than these two, it is a model which incorporates greater structural detail and is the larger of three systems considered here and is also non-linear in endogenous variables. Some of the errors may have multiplied for these reasons.

Comparing static to dynamic predictions for individual models, it would appear that the performance of the Marwah model is improved for C_p, I_p, M and P in dynamic predictions. For the Bhattacharya model, the performance is marginally deteriorated for all the variables and in the case of Choudhry-I model there is an improvement for C_p, I_p, Y, Y^d and P.

Table 10.15

Inequality Coefficients

Static Predictions

Models	C_p	I_p	R_1	M	Y	Y	P
Marwah I	.1119	.1931	.1930	.1229	.0976	-	.1562
Choudhry I	.490	.9459	-	.3424	.4530	.5077	.5370
Bhattacharya	.1295	.2144	.4426	.0341	.1147	.1280	-
Extrapolative	.0925	.1727	.1851	.0476	.0856	.0873	.1325

Dynamic Predictions

Models	C_p	I_p	R_1	M	Y	Y	P
Marwah I	.0747	.1732	.2389	.0740	.0663	-	.1221
Choudhry I	.3044	.5770	-	.1873	.2817	.3158	.3537
Bhattacharya	.1332	.2211	.4603	.0321	.1183	.1319	-
Extrapolative	.1279	.1395	.3449	.1332	.1117	.1122	.2664

Since the extrapolative models were built in a rigorous way in the sense that they produce near optimal results for the sample period when a maximum of five lags are used, it is not surprising that they perform well *vis a vis* forecasts of individual variables. The fact that Marwah I and Bhattacharya models do nearly as well as the extrapolative models should, therefore, be considered as a very satisfactory feature of these models. Furthermore, the fact that the dynamic predictions do not deteriorate fast and in some cases even improve should also be considered as a desirable property of these models.

In order to further extend this analysis in terms finding the relative importance of different sources of errors for the predictions generated from the three adapted models, we have decomposed the mean square errors *vis a vis* the two alternative schemes of decomposition in terms of inequality proportions as suggested by Theil (1961, 1966). The results are reported in Tables 10.16 and 10.17.

Table 10.16

Decomposition of Mean Square Error: Static Predictions

Vari-able	Model	Fraction of Error Due to: Bias	Diff Variation	Diff Covariation	Slope Error	Disturbance Error
C_p	B	.063	.660	.277	.450	.487
	C	.576	.412	.012	.420	.005
	D	.782	.0019	.216	.0125	.205
I_p	B	.0180	.3906	.5914	.0154	.9666
	C	.5663	.3478	.0859	.4109	.0228
	D	.5205	.0015	.4779	.0351	.4444
R_1	B	.4241	.0586	.5173	.2300	.3459
	C	-	-	-	-	-
	D	.268	.2071	.5248	.6743	.0577
M	B	.6830	.2694	.0476	.2418	.0752
	C	.4799	.5064	.0138	.5145	.0056
	D	.7940	.0995	.1066	.0925	.1137
Y	B	.0327	.6200	.3473	.4179	.5494
	C	.5728	.4104	.0167	.4205	.0067
	D	.7648	.0032	.2320	.0149	.2203
Y^d	B	-	-	-	-	-
	C	.5728	.4122	.0150	.4215	.0056
	D	.7923	.0078	.1999	.0222	.1855
P	B	.1029	.5127	.3844	.1691	.7280
	C	.6521	.3262	.0216	.3392	.0087
	D	-	-	-	-	-

Note: B, C and D refer respectively to Marwah I, Choudhry I and Bhattacharya models.

Table 10.17

Decomposition of Mean Square Error: Dynamic Predictions

Vari-able	Model	Fraction of Error Due to Bias	Dift Variation	Dift. Covariation	Slope Error	Disturbance Error
C_p	B	.0490	.5020	.4490	.3537	.5973
	C	.5501	.2142	.2357	.3226	.1273
	D	.7979	.0084	.1937	.0232	.1789
I_p	B	.4314	.0736	.4950	.0018	.5667
	C	.3763	.1438	.4798	.4686	.1551
	D	.5661	.0015	.4324	.0553	.3786
R_1	B	.4797	.0116	.5087	.3205	.1998
	C	-	-	-	-	-
	D	.4945	.1319	.3735	.4448	.0607
M	B	.7751	.2190	.0058	.2175	.0073
	C	.3813	.3957	.2230	.4920	.1267
	D	.7936	.0638	.1425	.0569	.1494
Y	B	.1352	.3711	.4937	.2547	.6101
	C	.5315	.2075	.2610	.3232	.1453
	D	.7850	.0123	.2027	.0282	.1869
Y^d	B	-	-	-	-	-
	C	.5315	.2214	.2470	.3395	.1289
	D	.8059	.0185	.1756	.0358	.1583
P	B	.0735	.6789	.2476	.4707	.4559
	C	.6328	.1308	.2364	.2355	.1317
	D	-	-	-	-	-

Note: B, C and D refer respectively to Marwah I, Choudhry I and Bhattacharya models.

For Marwah model, the bias error contributes little as far as the variable C_p, I_p, Y and P are concerned in the case of static predictions. But for R_1 and M respectively 42 and 68 per cent of the error is due to a misprediction of the mean. Errors due to different variation are strongest for C_p, Y and P. Error due to different covariation is strongest for I_p and R_1 accounting for more than 50 per cent to the total error, and for Y and P, where more than 30 per cent is accounted for due to this source of error.

For Bhattacharya model, the bias error is important in the case of C_p, I_p, M, D and Y^d. This is seen to be the most important source of error accounting for more than 70 per cent of the error for four out of six variables.

For Choudhry model, both the bias as well as error due to different variation appear to be generally important while error due to different covariation in one decomposition, and the disturbance error in the other, are very small in all cases.

In the case of dynamic predictions the bias error is important for I_p, R_1 and M for Marwah -I model, for C_p, Y, Y^d and P for Choudhry -I model and for C_p, I_p, R_1, M, Y and Y^d for the Bhattacharya model.

In view of overwhelming importance of bias as a source of error, we have reported the mean errors in Table 10.18 from which the direction and magnitude as to how the prediction mean differs from the actual mean can be gauged.

Table 10.18

Mean Errors $(\overline{A} - \overline{P})$

Models	Variables						
	C_p	I_p	R_1	M	Y	Y^d	P
Marwah -I	1073	-147	-.7516	970.1	925.9		11.75
Choudhry-I	-14220	-4041	-	-2266	-17970	-17970	-101.7
Bhattacharya	-4382	-878	-1.37	289.9	-5360	-5327	-

Dynamic Predictions

Marwah -I	-632.5	-645.8	-.9894	622.6	-1278	-	7.762
Choudhry-I	-8636	-2009	-	-1105	-10770	-10770	-65.96
Bhattacharya	-4551	-944.5	-1.936	272.8	-5495	-5538	-

For Marwah -I model, the predicted mean exceeds the actual mean for I_p and R_1 in the case of static predictions, and for C_p, I_p, R_1 and Y in the case of dynamic predictions. Similarly, the mean is over-predicted in Choudhry -I model, for all variables both for static and dynamic predictions. For Bhattacharya model except for M, the mean is overpredicted in both the static and dynamic cases for all the variables. In contrast, for the extrapolative models, as one would expect, the means are underpredicted as can be observed directly from the predictions given in Tables 10.13 and 10.14.

10.7 Summary

In this Chapter, a review of macroeconometric model building activity for the Indian economy has been undertaken with a view to derive some guidelines for future models as also to evaluate the forecasting performance of some of the existing models. For the latter purpose, adapted versions of three aggregative models, viz., Choudhry I, Bhattacharya, and Marwah I models were re-estimated with common data, sample period and estimation technique. These were then used to generate forecasts for a common forecast period with actual values of exogenous variables. Subsequently, the forecasting performance of these models is evaluated by comparing the forecasts against realizations as also against the forecasting performance of extrapolative models.

Notes

Variable names for symbols used in this Chapter are defined below:

C_p = private consumption expenditure

Y^d = disposable income

I_p = private investment expenditure

Y = GDP at current market prices

R_l = long-term rate of interest

M = Money-supply

U = unborrowed money reserves

NW = Net worth of the private sector

R = short-term rate of interest

R_d = discount-rate

I_g = government investment expenditure

q = excess of discount-rate over short-term interest-rate

l = net liquidity ratio of commercial banks

TD = time-deposits

P = wholesale price index

N = population

IM = imports

P_W = unit value index of world imports

F = foreign exchange reserves at the end of the period

P^s = index of prices of industrial securities

EX = exports

Y_W = world imports

T^{ind} = indirect taxes less subsidies

L = money-supply plus time deposits

T = total tax revenues

P_m = unit value index of imports

K = capital-stock

O or X = output (= Y/P)

Variables G, TR, TRR, H and DL are remainder terms in definitions and are interpreted accordingly.

Units and data used are explained in Appendices 1 and 2.

PART III

A MODEL OF THE INDIAN ECONOMY WITH EMPHASIS ON FISCAL POLICY

Chapter 11

A Macroeconometric Model of the Indian Economy: Specification and Estimation

One purpose of the review of available models of the Indian economy undertaken in Part II of this work was to enable one to use the information and experience provided by them for future model-building exercises which can be viewed as constituting a continually evolving process. As a part of this exercise, one macroeconometric framework for the purposes of forecasting and policy simulation is presented here.

11.1 Introductory Features of the Model

Although this model draws upon the earlier works, it does differ from them in certain respects, and its distinguishing features which also provide a justification for the undertaking to construct it, may be noted at the outset.

First, most available models of the Indian economy have become dated inasmuch as their samples do not extend beyond 1967-68. As such their estimated parameters refer to the 'conventional' C.S.O. estimates of national income and its components and forecasts from these cannot be generated with reference to the 'revised' series in terms of which post 1967-68 data are available. Hence, at the very least, these models need to be re-estimated.

Secondly, although various aspects of the economy have been considered in some detail in one model or another, a general weakness of all these models is a somewhat under-explored fiscal sector.

This is because, first, on the expenditure side, generally just one aggregate variable is used and the entire government expenditure is considered exogenous without allowing for any simultaneity with income and other endogenous variables of the model. Secondly, on the revenue side, tax revenues are taken either entirely as discretionary or entirely as automatic . Both these extreme options seem inappropriate. A more useful option is to introduce tax-rates and tax-bases explicitly in the model. As explained later, there are various difficulties in doing this but we are able to achieve this objective to a certain extent.

The model proposed here gives due attention to both aggregate supply and demand aspects. On the supply side it distinguishes between agricultural and non-agricultural production while on the demand side, it provides for a distinction between government and private expenditures. Thus, conceptually its framework analyses both types of dualities which have been considered important for a developing economy.

The model provides for an extended treatment of the fiscal sector. It contains a government budget constraint which is missing in most of the earlier models. It is used to determine government borrowing requirements from the private sector. Furthermore, government expenditure is divided between consumption and investment expenditures. While investment expenditure is taken as exogenous, consumption expenditure is endogenously explained. Tax revenues are divided into four categories: taxes on non-corporate incomes, direct taxes other than tax on non-corporate incomes, customs duties, and indirect taxes other than customs duties. For the first category, tax-rates have been explicitly introduced, as explained in a subsequent section. For other categories, the more general way of regressing revenues on bases has been followed so that the estimated coefficient is interpreted as an average effective rate. However, rather than using just an aggregate tax-base viz., income, as in other models, we have distinguished between tax-bases, so that different average effective rates are estimated for each category. Subsequently policy simulations may be performed by changing these average rates.

Production functions have been estimated for the agricultural and non-agricultural sectors separately. In both cases, capital is considered to be the primary constraint while labour is not considered a constraint at all in view of the excess availability of skilled and unskilled labour alike.

The model is non-linear in endogenous variables. The non-linearity arises, one, from the functional forms of the production functions which are Cobb-Douglas type and estimated as log linear functions, and two, from considering some variables at both constant and current prices which leads to multiplicative or ratio terms.

There are, in all, six price variables. The price deflators refer to investment goods, agricultural goods, non-agricultural goods, imports, and a general income deflator which refers to GDP at factor cost and which is used for deflating all other categories. A general price index is introduced to capture overall price variations and is defined as a function of the income deflator.

The model is specified as a system of 34 equations, 18 of which are stochastic. The specification is given below.

11.2 Specification of Structural Equations

Structural equations of the model, both stochastic and definitional, are given below. Variables marked with an asterisk are measured at current prices. Other income-expenditure variables are at constant 1960/1 prices. Endogenous variables are defined while specifying the equations.

11.2.1 Stochastic Equations

M.1 Private consumption expenditure

$$CP = f(YD, YA/YNA)$$

M.2 Private investment expenditure

$$IP = f(YD, RL, KP_{-1})$$

M.3 Government consumption expenditure

$$CG^* = f(TVR^*, CG^*_{-1})$$

M.4 Demand for money (to determine short term interest rate, R)

$$M^* = PY.\ f(YD, R, FA_{-1})$$

M.5 Supply of money

$$M^* = f(U^*, RD-R, NLR)$$

M.6 Demand for time-deposits

$$TD = f(YD, R, RL, FA_{-1})$$

M.7 Imports

$$IMP^*/PM = f(Y^*/PY, FS^*/PM, PM/P)$$

M.8 Long term rate of interest

$$RL = f(R, RL_{-1})$$

M.9 Tax on non-corporate incomes (personal income taxes)

$$TDY^* = f(Y^*, BTR, ITR)$$

M.10 Direct taxes other than tax on non-corporate incomes

$$NTDY^* = f(Y^*, t)$$

M.11 Customs duties

$$CD^* = f(EXP^* + IMP^*)$$

M.12 Indirect taxes other than customs duties

$$NCD^* = f(Y^*)$$

M.13 Agricultural output

$$\ln YA = f(\ln KA)$$

M.14 Non-agricultural output

$$\ln YNA = f(\ln KNA)$$

M.15 Investment in agriculture

$$IA = f(YA, PA/PNA, IA_{-1})$$

M.16 General price index

$$P = f(PY)$$

M.17 Price deflator for investment goods

$$PI = f(PY)$$

M.18 Income deflator

$$PY = f(PA, PNA)$$

11.2.2 Definitions and Identities

M.19 Income (GDP at market prices)

$$Y^* = CP.PY + CG^* + IP.PI + IG^* + EXP^* - IMP^*$$

M.20 GDP at market prices (to determine non-agricultural price deflator, PNA)

$$Y^* = YA.PA + YNA.PNA + (CD^* + NCD^*)$$

M.21 Disposable income

$$YD^* = Y^* - TVR^* + TR^*$$

M.22 Unborrowed money reserves

$$U^* = U^*_{-1} + D^* + (F^* - F^*_{-1}) + RES^*$$

M.23 Total government current borrowing (government budget constraint)

$$ZBP^* = CG^* + IG^* + TR^* - TVR^* - A^* - D^*$$

M.24 Real disposable income

$$YD = YD^*/PY$$

M.25 Foreign exchange reserves

$$F^* = FS^* - IMP^*$$

M.26 Potential foreign exchange availability (in the current year)

$$FS^* = F^*_{-1} + EXP^* + A^*$$

M.27 Financial assets with the private sector

$$FA = M + TD + BP^*/PY$$

M.28 Total government borrowing from the private sector

$$BP^* = (BP^*_{-1} + ZBP^*)$$

M.29 Private capital stock†

$$KP = IP + .98198\ KP_{-1}$$

M.30 Government capital stock

$$KG = IG^*/PI + .98818\ KG_{-1}$$

M.31 Total capital stock

$$K = KP + KG$$

M.32 Capital stock in agriculture

$$KA = IA + .95633\ KA_{-1}$$

M.33 Capital stock in non-agriculture

$$KNA = K - KA$$

M.34 Total tax revenues

$$TVR^* = TDY^* + NTDY^* + CD^* + NCD^*$$

11.2.3 Listing of Variables

The thirty four endogenous variables, already defined, may be listed as follows:

CP, CG*, IP, M*, TD, ZBP, IMP*, RL, TDY*, NTDY*, CD*, NCD*, YA, YNA, IA, P, PI, PY, Y*, PNA, YD*, YD, U*, ZB*, F*, FS*, FA, BP, KP, KG, K, KA, KNA and TVR*.

† The terms for replacement capital in three equations, M.29, M.30 and M.32 are actually stochastic. These terms were estimated as functions of previous year's capital stock. Thus, if DKP, DKG and DKA refer respectively to replacement capital in private, government and agricultural sectors, all at constant 60-61 prices, we have the following estimates:

$$DKP = \underset{(27.24)}{.01802}\ KP_{-1} \qquad R^2 = .9699$$

$$DKG = \underset{(50.11)}{.01182}\ KG_{-1} \qquad R^2 = .9909$$

and $$DKA = .04037\ KA_{-1} \qquad R^2 = .9798$$

The exogenous variables of the model are defined below.

RD	=	discount rate (average of effective rates)
NLR	=	net liquidity ratio
BTR	=	basic tax-rate of personal income taxes
ITR	=	incremental tax-rate of personal income taxes
t	=	time trend, beginning 1950-51 = 1.
EXP*	=	exports
PM	=	unit value of imports
IG*	=	government investment expenditure
PA	=	agricultural price deflator
TR*	=	net transfers to the private sector
RES*	=	residual in the unborrowed reserves definition
A*	=	net aid
D*	=	government borrowing from the Reserve Bank of India

The lagged endogenous variables of the system are listed below.

CG^*_{-1}, KP_{-1}, FA_{-1}, RL_{-1}, U^*_{-1}, F^*_{-1}, BP^*_{-1}, KG_{-1}, KA_{-1}.

11.3 Sample Period and Estimation Technique

The sample period is not uniform. For most of the estimates the sample extends from 1950-51 to 1974-75 thus providing 25 observations. For some functions especially, the production functions, lack of consistent data for some of the earlier years force us to use only 15 observations from 1960-61 to 1974-75. For one period lagged variables, in the first case, data for 1949-50 were used. In the second case, however, one observation is lost in equations using lagged variables.

Mixed estimation procedures have been followed. In general two-stage least squares with some modifications has been followed. In the first stage, rather than using all predetermined variables for generating values of the endogenous variables, subsets of the predetermined variables have been used. This procedure was resorted to because of non-uniformity of the sample and because of our intention later to save some observations in order to obtain a beyond-sample forecast period.

For equations for which the full sample was used, the following predetermined variables were used in the first stage.

RD, NLR, BTR, ITR, T, EXP*, IG*, TR*, A*, PM

CG^*_{-1}, KP_{-1}, FA_{-1}, U^*_{-1}, F^*_{-1}, KG_{-1}, RL_{-1}

For equations for which the smaller sample was used, the following predetermined variables were used in the first stage.

PA, T, EXP*, IG*, A*, TR*, PM

KP_{-1}, KG_{-1}, RL_{-1}

In addition a constant term was used in both subsets.

The method of using a subset of predetermined variables rather than a set of their principal components was resorted to in view of the greater flexibility it provides during construction and revisions of the model building exercise. The choice of variables in both cases is chiefly arbitrary but care is taken to include those variable in each of the two cases which are more directly relevant for the equations in question. All the important exogenous variables are used in any case, and their number is found to be large enough to explain 95 to 100 per cent of the variation in endogenous variables occurring on the right-hand side of the equations.

The estimation procedure is further modified for equations containing lagged dependent variables. Here, the lagged term is generated first by regressing the endogenous variables on the subset of predetermined variables as listed above.† The estimated values of the endogenous variable are lagged one period and used in the second stage of estimation. Furthermore, in order to correct for any serial correlation introduced because of the use of the lagged dependent terms or otherwise, Cochrane-Orcutt procedures are used to transform data before estimation is done. First order transformation, e.g. transformation of Y_t into $Y_t - \rho Y_{t-1}$ is resorted to. Tests for second order serial correlation are also subsequently performed. Final value of ρ is determined by iteration until it changes by less than .005. The constant terms of the equations are readjusted to refer to the original equation, before the estimates are presented.

Because of various non-linearities and a non-uniform and limited size of the sample period, estimation procedures like 3SLS or FIML which can take into account contemporaneous correlation between the error terms have not been resorted to.

† excluding own lagged term, where necessary.

Where, on the right-hand side of an equation, composite variables like (RD - R) or (EXP* + IMP*) etc. are used, or where transformation of endogenous variables like ln KNA or ln KA is involved, first an estimate of the endogenous variable is obtained by regression on the set of predetermined variables as specified in this section, and then transformation etc. is done.

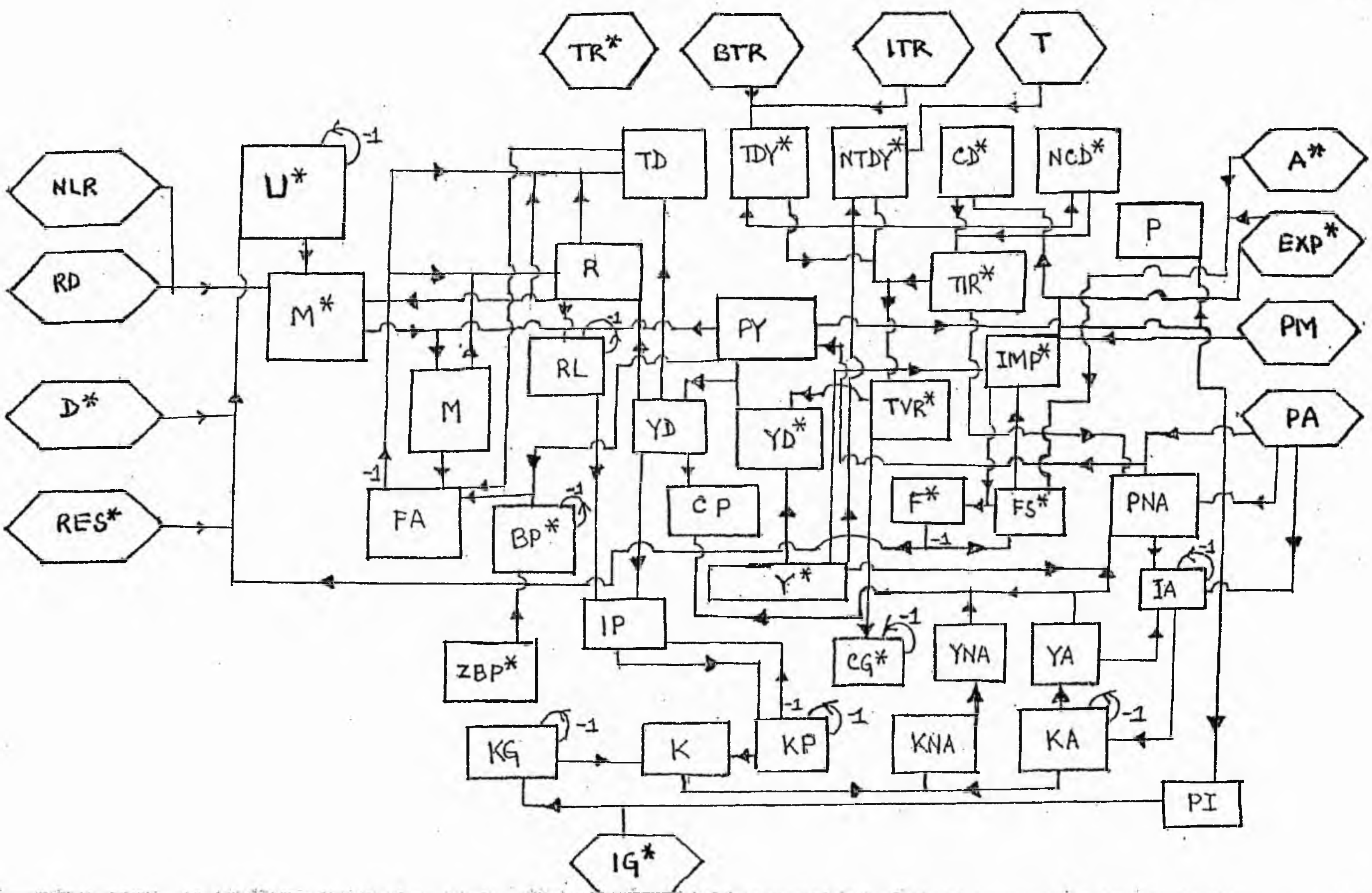
11.4 The Causal Structure of the Model

The causal structure is simultaneous. There are feedbacks from the real sector to the monetary sector and vice versa. The related mechanism can be studied through the flow-chart which is given in Fig.11.1.

The interdependence of the monetary and real sectors can be traced in the following manner. In the monetary sector, in equations like those for demand for money and time deposits, the feedback arises from both income and wealth effects. For the money supply equation the feedback arises from the definition of unborrowed money reserves. For the real sector equations, feedback from the monetary sector arises from the long-term rate of interest for the investment function. The long term rate is related to the short-term rate via the term structure.

The government budget identity is used to determine government private borrowing requirements. In the model, the government is free to choose the levels of its investment expenditure and its borrowing from the Central bank. Its consumption expenditure is determined within the model given the level of tax-revenues.

Fig.11.1 Flow Chart: Structural Relations
Hexagonal blocks contain exogenous variables



The crowding-out phenomenon takes place via the private investment function. Given the previous years' total borrowing from the private sector, the financial assets of the private sector are changed which affect the short-term interest rate through the money demand equation and the long-term rate through the term structure equation. These, in turn, affect the current investment decisions. On the other hand, government's transfer payments which include interest payments alter the level of disposable income which affects both the consumption and investment functions of the private sector.

We move from the expenditure streams to the output side in the following manner. Investment decisions by the private and government sectors provide us with the total capital stock. Investment decisions for the agricultural sectors are generated by using a stochastic equation which incorporates effects of agricultural income, relative prices and institutional factors. Capital stock in the non-agricultural sector is then generated as a residual. Given the capital-stocks in the two sectors, we have an output stream, which together with the expenditure stream and exogenous agricultural prices, is used to determine the non-agricultural prices.

The main dynamic links in the model can be traced as below. On the output side, previous years' capital stock along with current investment decisions produce current capital-stock. In the monetary sector, previous years' level of unborrowed money reserves and current inflow of foreign exchange along with government borrowing from the Reserve Bank is used to generate current unborrowed money reserves. Financial assets of the private sector at the beginning of the year are used in the money-demand and time-deposits equation and the current levels of the financial assets are determined through the budgetrestraint and current decisions on holding money and time-deposits.

11.5 Comments on the Choice of Exogenous Variables

Of the 13 variables listed as exogenous, four, viz., BTR, ITR, IG* and TR* are fiscal policy instruments, and three, viz., RD, NLR and D* are monetary policy instruments. Agricultural prices (PA) are also considered exogenous because of the considerable control exerted by the government over these through the agricultural prices commission. The net aid term A* is under the control of the rest of the world. Exports (EXP*) have also been considered exogenous. Some justification for this treatment for both these variables has been provided in Chapter 10 by the use of Granger-Sims tests and by considerations arising from a review of the treatment of these variables in other preceding models which may provide grounds for an *a priori* justification.

Other exogenous variables are a time-trend (T), a statistical discrepancy term (RES*), and unit value of imports (PM). From a policy point of view these are irrelevant variables.

For forecasting we will need to independently predict values for A*, EXP* and PM*. T takes obvious values, and RES* can be set at a reasonable level. For the remaining eight variables under the control of the domestic government various combinations can be chosen for generating simulations and conditional forecasts.

It is possible to visualise some of the policy variables as interdependent in the sense that decisions regarding them are taken jointly. This implies a comprehensive or a 'synoptic' approach of the policymakers rather than taking policy as a series of ad hoc decisions. For our purposes, however, as long as the feedback from endogenous variables on the policy instruments can justifiably be treated as weak, it is not inappropriate to take them as exogenous. At the state of policy simulations, however, we may introduce joint changes in some of the policy instruments.

11.6 Comments on the Specification and Estimation of Individual Functions

11.6.1 Private Consumption Expenditure

One of the simplest explanations for the private demand for consumption expenditure derives from the absolute income hypothesis where personal disposable income is used as the single explanatory variable. There are, however, many alternatives to this approach using concepts and hypotheses like habit-persistence, permanent income, relative income, wealth, net worth, liquid assets etc. Various versions of these hypotheses can be invoked depending on different adjustment mechanisms and parameter restrictions. It can be shown, however, that in most cases the estimable form of the function can be reduced to a form like $C = F(YD, C_{-1}) + U$ (e.g. see Surrey, 1974). In the place of C_{-1}, sometimes a direct estimate of wealth, or equivalently, a rate of interest variable, is used in order to avoid problems of estimation arising out of the use of a lagged dependent variable.

The moderation of the consumption-income relationship by one or more of these devices has two important implications. First, by introducing wealth effects, the interdependence between the monetary and real sectors as contained in the rest of the system, is reinforced. Thus, when wealth in private portfolios increases as a result of lending to the government, there are positive effects on consumption which affects bond-financed government expenditure multipliers. Secondly, the introduction of a modifying influence via a term like the lagged consumption variable, acts as a link between short- and long-term m.p.cs. The higher is the coefficient of the lagged consumption term, the greater would be the difference between the short- and long-run relations, and the longer it will take for the full impact of a fiscal policy measure on aggregate demand to be felt via the consumption function.

It can be said that from the viewpoint of estimation, since different hypotheses may be reduced to a similar estimable form, the choice between one or the other is difficult on purely statistical grounds. Furthermore, from the viewpoint of policy analysis, as long as a similar impact is created by the influence moderating the short-term consumption-income relationship, such a choice is also not too critical. This does not imply, of course, that one theory can therefore be taken as the equivalent of another. In particular, their differences would still arise from the way the error term is entered. In some cases there would be serial correlation in error by specification as in most adaptive expectations mechanisms, and in some cases, this would not be so as, for example, in some versions of the habit persistence hypothesis.

Our 2SLS estimates, using first, only disposable incomes as the explanatory variable, provide the following consumption function:

$$CP = \underset{(9.39)}{1801.09} + \underset{(58.79)}{.7307}\,YD \qquad (11.1)$$

$$\bar{R}^2 = .9931, \quad F(1,23) = 3455.87, \quad d\text{-}w = 2.61$$

When an indicator of wealth or lagged consumption is used, one expects a positive sign for their coefficients and the coefficient of YD is expected to be reduced. As an indicator of wealth, we have used, alternatively, liquid financial assets of the private sector (FA = M + TD + BP) or net worth of the private sector (NW = FA + KP) at the beginning of the period. As an alternative to both of these, lagged consumption (CP_{-1}) was used where this term was derived by first regressing CP on the set of predetermined variables specified in section 11.3 and then lagging the estimates by one year.

Our results, however, consistently indicate a negative and mostly insignificant coefficients for all of these influences. Their inclusion leads to an increase in the short-run m.p.c. and the purpose of their inclusion is not only defeated but reversed. There is no reason to expect consumption expenditure to be inferior with respect to wealth if it is normal with respect to income. Some of these results are given below. Unless otherwise indicated, the sample is from 1950/1 to 1974/5.

$$\underset{(6.27)}{CP = 1618.93} + \underset{(11.82)}{.8001\ YD} - \underset{(1.05)}{.106\ FA_{-1}} \tag{11.2}$$

$\bar{R}^2 = .9931$, $F(2,22) = 1735.67$, d-w = 2.66

$$\underset{(7.79)}{CP = 1771.55} + \underset{(7.87)}{.7547\ YD} - \underset{(-.253)}{.0066\ NW_{-1}} \tag{11.3}$$

$\bar{R}^2 = .9928$, $F(2,22) = 1657.02$, d-w = 2.59

$$\underset{(7.34)}{CP = 2364.95} + \underset{(8.43)}{.950\ YD} - \underset{(-1.98)}{.3106\ CP_{-1}} \tag{11.4}$$

$\bar{R}^2 = .9934$, $F(2,21) = 1726.22$, d-w = 2.435 (SMPL 1951/2-74/5)

One reason for getting the wrong sign could be multicollinearity among the regressors because of the presence of a strong trend in all of these. In order to explore whether or not multicollinearity is harmful in these cases, we have used Farrar and Glauber's (1967) suggestion which involves a comparison between the overall coefficient of multiple determination ($R^2_{Y.X_1\ X_2 \ldots X_K}$) or its square root, with similar quantities obtained by regressing one explanatory variable on all the others ($R^2_{X_i}$, X_1, ... X_K) for each of these. When the former quantity is smaller than the latter, multicollinearity is said to be harmful. In itself, this test is more in the nature of a rule of thumb, and has been supplemented by other tests suggested by Farrar and Glauber where necessary.

For the function $CP = f(YD, CP_{-1})$, we find that

$R^2_{CP.YD,CP_{-1}} = .9934$ and $R^2_{YD.CP_{-1}} = .9985$ (SMPL, 1951/2-74/5)

Thus, multicollinearity is indicated to be harmful. For the function, $CP = f(YD, FA_{-1})$, we have,

$$R^2_{CP.YD.FA_{-1}} = .9931 \quad \text{and} \quad R^2_{YD.FA_{-1}} = .9945$$

and again the former quantity is smaller than the latter. A similar result is obtained for the third case.

In order to reduce the correlation between the regressors, we have then used first differences. Now the estimated functions are:

$$\underset{}{\Delta CP} = \underset{(7.031)}{1.012\ \Delta YD} - \underset{(-2.536)}{.4287\ \Delta CP_{-1}} \tag{11.5}$$

$\bar{R}^2 = .704$, $F(1,21) = 53.313$, d-w = 3.04, SMPL (52/3-74/5)

In this case $R^2_{\Delta YD.\Delta CP_{-1}} = .4190$. Although multicollinearity does not look harmful we are still getting the wrong sign for CP_{-1}. Similarly, for the alternative using financial assets, we have

$$\Delta CP = \underset{(5.29)}{.8867\ \Delta YD} - \underset{(-.757)}{.1645\ \Delta FA_{-1}} \tag{11.6}$$

$\bar{R}^2 = .6297$, $F(1,22) = 40.106$, d-w = 2.95 SMPL (51/2-74/5)

In this case $R^2_{\Delta YD.\Delta FA_{-1}} = .3332$

Again, whereas multicollinearity does not look to be harmful, a wrong sign is obtained for the wealth variable. Similar results are obtained when net worth is used as the explanatory variable.

Results do not improve when all variables are measured as deviations from a time-trend. For the case with CP_{-1}, for example, we have

$$\underset{}{CP} = \underset{(-.04)}{-1.885} + \underset{(6.57)}{.858}\ YD - \underset{(-1.76)}{.289}\ CP_{-1} \tag{11.7}$$

$\bar{R}^2 = .6541$, $F(2,22) = 2367$, d-w = 2.38

As a second approach to the problem, we have worked from the side of the rate of interest. The expected sign for the coefficient of real rate of interest is negative since it represents the opportunity cost of consumption. Since income and wealth are related via the rate of interest, its inclusion is equivalent to that of wealth. In this case, we obtain the following estimates:

$$CP = \underset{(7.304)}{1817.58} + \underset{(29.315)}{.7221}\ YD + \underset{(.271)}{42.819}\ RR \tag{11.8}$$

$\bar{R}^2 = .9992$, $F(2,21) = 1457.60$, d-w = 2.65, SMPL (1951/2-74/5)

where $RR = R-(P-P_{-1})/P_{-1}$.

Again the wrong sign for RR is obtained, although $R^2_{YD.RR} = .9826$ which is smaller than the overall R^2 and multicollinearity may not be harmful. It should be remarked that if real rate of interest is constructed from expected rate of change in prices,

$$RR' = R-(P_{-1} - P_{-2})/P_{-2}$$

then also similar results are obtained.

For want of any results indicating otherwise, we are tempted to conclude that due to the subsistence nature of a large part of consumption expenditure in the Indian economy, the difference between short- and long-run m.p.cs is very small and that the interdependence between the real and monetary sectors via the consumption function is weak. This conclusion may need to be modified if there are some significant price effects. As a next step we have therefore attempted to include these in the analysis.

Two influences seem to be relevant here. First, the expected rate of inflation may influence current consumption, and second, the price-level itself may incorporate a money-illusion effect. Both these are expected to have a positive influence. We have proxied the expected rate of inflation by the observed rate of price change in the previous year, viz.,

$$(P_{-1} - P_{-2})/P_{-2}$$

The following results are obtained:

$$\underset{(8.57)}{CP = 1866.06} + \underset{(49.32)}{7263\ YD} + \underset{(.234)}{160.99}\ (P_{-1} - P_{-2})/P_{-2} \qquad (11.9)$$

$\bar{R}^2 = .9922,\quad F(2,21) = 1456.3,\quad d\text{-}w = 2.671$

$$\underset{(6.947)}{CP = 1929.17} + \underset{(21.781)}{.715\ YD} + \underset{(.0815)}{61.156}\ (P_{-1} - P_{-2})/P_{-2} + \underset{(.379)}{.8199\ P} \qquad (11.10)$$

$\bar{R}^2 = .9918,\quad F(3,20) = 931.3,\quad d\text{-}w = 2.68$

Although expected signs are obtained, the coefficients of terms indicating the price effects are not indicated to be significantly different from zero.

A different approach to the estimation of a consumption function, relevant especially for a developing economy, is to distinguish between m.p.c.'s in the agricultural and non-agricultural sectors. Since subsistence consumption is mainly in the agricultural sector, one would expect that over a long period, as the proportion of non-agricultural income in the total income increases, the overall m.p.c. would decline. We have included the ratio of agricultural to non-agricultural income (YA/YNA) in the function to introduce this influence. The equation was estimated for the smaller sample period (1960/1-74/5) for which data for YA and YNA was used in the rest of the model. Estimates of YD, as derived for the full sample, in the first stage of estimation, were used for the relevant period in this equation. The following results are obtained:

$$CP = \underset{(-.909)}{-1250.64} + \underset{(20.515)}{.8068}\, YD + \underset{(2.245)}{2404.53}\, (YA/YNA) \qquad (11.11)$$

$\bar{R}^2 = .9858 \quad F(2,12) = 486.945, \quad d\text{-}w = 2.19$

This result seems to be the best we have obtained. In the framework of this hypothesis, the long-period m.p.c. is expected to be smaller than the short-period m.p.c. This may also explain why for a subsistence economy, the usual relation, where the opposite of this holds, may not be obtained. It may be remarked that the function we have used is similar to that of Pandit (1973) but whereas in his model the ratio YA/YNA was taken as exogenous, in our model, it is endogenous. Comparable formulations, though not entirely similar, are present in the specification of the consumption/saving functions in the two UNCTAD models.

Mention may be made of the possibility, although we have not pursued it here, that it may be desirable because of the formidable collinearity problems among other things, that the entire problem be cast in terms of consumption (saving) income ratios or rates of growth of consumption (saving). With these as the dependent variables, rates of growth of income, prices and wealth etc. may be used as the explanatory variables. This would be in line with the recent suggestions made in Deaton (1977) and Surrey (1974). This alternative however, is not very attractive for simultaneous equation models if the rest of the system is cast in terms of the levels of variables. The use of ratios and rates would considerably increase the non-linearities of the system and would raise problems at the later stages of solution and interpretation.

11.6.2 Government Consumption Expenditure

In contrast to the other models of the Indian economy, we have an endogenous explanation of the government consumption expenditure. In terms of the Granger-Sims tests carried out in Chapter 10, we have argued that the assumption of exogeneity of government consumption expenditure is questionable and its joint dependence with the rest of the system can not be ruled out. The estimable equation for this function which we have adopted is:

$$CG^* = a + b\ TVR^* + c\ CG^*_{-1} + u$$

We have used variables defined in nominal terms because a constant price tax-revenue series is not available. The Indian C.S.O. provides a constant price series for almost all income expenditure components but this. The main difficulty arises because of a mixed system of specific and *ad valorem* indirect taxes many of which have also kept on changing from specific to *ad valorem* within our sample period.

The tax-revenue term, which is endogenous to the system is responsible for the joint determinancy of government consumption. The lagged term can enter in one of many ways. In particular it can enter, if a first-order expectation hypothesis via TVR* is used or if a habit persistence hypothesis is used.

On *a priori* grounds we favour the habit persistence hypothesis. Aggregate government consumption expenditure is determined by claims from individual departments which are processed and voted through the budget. Each department makes sure that it puts in at least as many claims as the ones approved in the previous year for these will have an easy passage. Habit persistence would seem strong in this case.

This hypothesis is further substantiated on statistical grounds. If the term CG^*_{-1} is incorporated in other ways, in most cases, serial correlation in the error term would enter by specification. However, we do not find a significant serial correlation in our estimates. The estimated function is reported below,

$$CG^* = \underset{(.9066)}{36.1169} + \underset{(3.45)}{.26902}\,TVR^* + \underset{(6.111)}{.72035}\,CG^*_{-1} \qquad (11.12)$$

$$\bar{R}^2 = .9955, \quad F(2,22) = 2632.56, \quad d\text{-}w = 2.02$$

The high magnitude of the coefficient of CG^*_{-1} indicates that the difference between short- and long-run m.p.c.'s for the government is considerable. The long-run m.p.c. is estimated to be .9620.

The implications of using an endogenous government consumption function of this type are the following:

(i) Since a part of government expenditure is no more exogenous, total autonomous expenditures and thereby the scope of fiscal policy is reduced;

(ii) since the difference in short- and long term m.p.c.'s is substantial for government expenditures, it is likely to produce small multiplier effects in the short run but bigger multipliers in the long run compared to a situation where the difference between these short- and long-term m.p.c.'s may be small;

(iii) since government's m.p.c. in the long run is much greater than the private sector's long run m.p.c., increased government consumption expenditure by additional taxation would imply a lower investment for the economy as a whole in the long run. This is in sharp contrast with the usual argument provided for increasing levels of taxation and government expenditure activities whereby the government is supposed to increase investment. For a capital-scarce developing economy, this has important implications.

11.6.3 Private Investment Expenditure

There are various alternative approaches to the specification of a demand function for investment goods for the private sector such as the acceleration principle, the permanent income approach, the neo-classical approach, and the simple Keynesian approach where investment is just a function of the rate of interest. We have used a mixed approach. The estimable form of this relation contains, in our model, private disposable income, long-term rate of interest and lagged private capital-stock as explanatory variables. Earlier models of the Indian economy have also used a function of this type or have considered it useful. Income is used to reflect the level of aggregate demand and as an index of profits. Long-term rate of interest represents the cost of borrowing. Preference has been shown in favour of the long-term rather than the short-term rate in specifications used in models like those of Bhattacharya, Choudhry, Krishnamurty and Marwah.

It reflects the costs of long-term borrowing. Furthermore, since the long-term rate is expected to be sensitive to the expected rate of change in prices, in an inflationary climate, it is more relevant for investment decisions.

Our 2SLS estimates, as given below, however throw up the wrong sign for the interest rate variable. A positive sign for this does not agree with any *a priori* hypothesis, nor does it agree with any of the earlier estimates of the investment functions.

$$\underset{}{IP} = \underset{(-2.79)}{-1587.39} + \underset{(2.767)}{.1525\ YD} + \underset{(1.080)}{49.334\ RL} + \underset{(.534)}{.01705\ KP_{-1}} \qquad (11.13)$$

$$\bar{R}^2 = .9213, \quad F(3,21) = 94.7141, \quad d\text{-}w = 1.598$$

The wrong sign for the coefficient of RL may be due to multicollinearity.

We have

$$R^2_{IP.YD,\ RL,\ KP_{-1}} = .9213, \qquad R^2_{RL.YD,\ KP_{-1}} = .9682$$

$$R^2_{YD.RL,KP_{-1}} = .9923 \quad \text{and} \quad R^2_{KP_{-1}.YD,RL} = .9954$$

Variables YD and RL are as estimated in the first stage by using the set of predetermined variables. All the coefficients above are adjusted for degrees of freedom. Multicollinearity among the regressors is indicated to be harmful. Furthermore, in terms of the F-test suggested by Farrar and Glauber, RL is at least one variable where it is significantly located.

We have, for RL[†],

$$F^*_{(1,23)} = 731.10$$

which is significant at the 1% level.

When the equation is estimated in terms of first differences, in order to reduce the correlations between the regressors, the following result is obtained.

$$\underset{}{\Delta IP} = \underset{(.286)}{.03978}\,\Delta YD - \underset{(-.357)}{22.507}\,\Delta RL + \underset{(.526)}{.0427}\,\Delta KP_{-1} \qquad (11.14)$$

$$\bar{R}^2 = -.0085 \quad F(2,21) = .9032, \quad d\text{-}w = 2.406$$

Although a negative sign is obtained for RL, the overall equation gives a poor fit. Similar conclusions are derived when a constant term (indicating a time trend) is also included in the above equation. Since we are interested in explaining the level of variables, we have re-estimated the investment function with reference to levels but with the coefficient of RL constrained to take the value as obtained in the above equation.

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†
$$F^* = \frac{\left(R^2_{RL.YD,KP_{-1}}\right)\Big/(k-1)}{\left(1 - R^2_{RL.YD,KP_{-1}}\right)\Big/(n-k)}$$

The investment function now is:

$$IP = \underset{(-2.256)}{-1166.85} + \underset{(2.912)}{.1643}\ YD - 22.5069\ RL + \underset{(.392)}{.01288}\ KP_{-1} \qquad (11.15)$$

$\bar{R}^2 = .9177 \quad F(2,22) = 134.727, \quad d\text{-}w = 1.386$

The magnitudes and signs of the coefficients in this estimate are more acceptable.*

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* The positive sign for KP_{-1} causes some concern. In the flexible accelerator models, if one hypothesises that capital stock in existence depends on the past values of output,

$$K_t = v(1-\mu)y_t + v(1-\mu)\ Y_{t-1} + v(1-\mu)\mu^2 y_{t-2} + \ldots \quad (0<\mu<1)$$

one gets, using the Koyck transformation,

$$K_t - \mu K_{t-1} = v(1-\mu)\ y_t$$

or

$$K_t - K_{t-1} = v(1-\mu)\ y_t - (1-\mu)\ K_{t-1}$$

The expression on the left hand side is net investment. If replacement investment is assumed proportional to the existing capital stock, we have

$$I_t^G = I_t^n + \lambda K_{t-1} = v(1-\mu)\ y_t - (1-\mu-\lambda)\ K_{t-1}$$

If $\mu + \lambda > 1$, one could get a positive sign for K_{t-1}, in the equation explaining gross investment. The use of previous years' capital stock on the right hand side, however, lends itself also to a more general formulation of the investment function. Thus, if it represents a supply side influence, a positive sign would imply that the higher is the economy's capacity to produce investment goods, the higher would be its current demand.

11.6.4 Tax Equations

Taxes are divided into four categories. Direct taxes are decomposed into personal income taxes and a residual category. Similarly, indirect taxes are divided between customs duties and a residual category. In the first case, the distinction is made because for personal income taxes we have been able to explicitly incorporate tax-rate changes into the equation. In the second case, the distinction is made in order to distinguish between tax-bases. For customs duties, it is possible to clearly specify the tax bases as imports and exports.

For the personal income tax or, more precisely, tax on non-corporate incomes, the procedure for incorporating tax-rates can be outlined as follows. Since, in any given year, the number of tax-rates is large, varying from one income-bracket to another, all these tax-rates cannot be introduced separately in the revenue function because of the problems of loss of degrees of freedom, possible violation of the order conditions in a macroeconomic model of a small size and multicollinearity among the tax-rates. We have, therefore, attempted first to reduce the rate-structure of any given year to two parameters which may be called a 'basic'tax rate (α) and an 'incremental' tax-rate (β). This is done by regressing the tax-rates r_j, $j=1,2,\dots J$, on the mid-points of income brackets or their ranks in ascending order. For example, α and β could be extracted from

$$r_j = \alpha + \beta j + u \qquad (j=1,2,\dots J) \qquad (11.16a)$$

or

$$r_j = \alpha + \beta m_j + u \qquad (11.16b)$$

where r_j refers to the tax-rate for the jth income-slab. J is the total number of income-slabs in the rate schedule and m_j are the mid-points for each income slab. u is the error term assumed normal, $N(0,\sigma^2)$. Estimates of α and β are interpreted as a basic tax-rate and an incremental rate by which the basic rate increases as one moves from a lower to a higher income bracket. If the revenue generating function for the tax is written as

$$R = \Sigma r_j Y_j$$

where Y_j is the aggregate of income taxed at the rate r_j, we can write using, for example, equation (11.16a), the total revenue as

$$R = \Sigma(\alpha + \beta j + u) Y_j$$

or

$$R = \alpha\Sigma Y_j + \beta\Sigma jY_j + u\Sigma Y_j$$

Taking expectations,

$$R = \alpha\Sigma Y_j + \beta\Sigma jY_j$$

or

$$R = \alpha Y + \beta\Sigma jY_j \qquad (11.17)$$

Y is the total income subject to tax and jY_j is a weighted distribution of this income. In the first term,revenue effects of changes in the distribution between the aggregate taxed and untaxed income would be reflected, while in the second term, effects of changes in the distribution of income among the tax-brackets are reflected. For estimation purposes, therefore one needs, apart from α and β, variables which can reflect changes in the total income taxed and its distribution among the tax-brackets. One such scheme is to use per capita income and population as separate regressors . In such a case,

$$R = f(\alpha,\beta, Y/N, N)$$

It is argued that if per capita income is held constant and total income increases as population increases, total taxed income would increase, while, if per capita income increases with population held constant, people would be thrown in higher income-brackets and the distribution of income between tax-brackets would be affected.

The estimates of α and β which we shall refer to as BTR and ITR are given in Appendix ~~2~~ 3.

The estimated equation for this function using per capita income and population as separate regressors is,

$$\underset{(.769)}{TDY^* = 90.915} + \underset{(19.06)}{10.743\, Y^*/N} - \underset{(-1.11)}{7.037\, BTR} + \underset{(4.11)}{73.71\, ITR} - .792\, N \quad (11.18)$$

$$\bar{R}^2 = .9906, \quad F(4,20) = 632.951, \quad d\text{-}w = 2.33$$

The signs for BTR and N are negative which is not expected. A negative sign for BTR may be due to tax-evasion and avoidance which may increase if the basic tax rate goes up. But the negative sign for N cannot be explained. It seems primarily due to multicollinearity as the correlation between per capita income and population is very high. This means that it is difficult to separate out effects of changes in these two variables. Hence we have lumped them together and used aggregate income in their place. The estimated equation is,

$$\underset{(.391)}{TDY^* = -50.291} + \underset{(24.99)}{.0128\, Y^*} - \underset{(-.8379)}{5.2712\, BTR} + \underset{(2.507)}{53.246\, ITR} \quad (11.19)$$

$$\bar{R}^2 = .9868, \quad F(3,21) = 599.96, \quad d\text{-}w = .959$$

For the three other categories of tax-revenues, respective tax-bases are used as explanatory variables. For domestic indirect taxes, i.e. indirect taxes other than customs duties, and also direct taxes other than tax on non-corporate incomes, the income level, Y*, is used as a proxy for the tax-base. For customs duties (IMP* + EXP*) are used as the tax-base. In all these cases, the variety of rate-categories, the mixture of specific and *ad valorem* taxes, the changeover from specific to *ad valorem* taxes within the sample period, and the increasing coverage of commodities brought under indirect taxation, make it difficult to introduce tax-rates separately in the equations. The estimated equations are given in Section 11.7.

11.6.5 Money-Supply and Demand

Whether or not money-supply should be treated as endogenous to the model of income determination in India is debatable. Bhattacharya (1975) has argued strongly in favour of its endogenous determination within the model. The relationship which he suggests is,

$$M^* = f(U^*, RD-R, NLR)$$

In his model, since U*, the unborrowed money reserves, and NLR and RD, respectively, the net liquidity ratio and the discount rate are all exogenous to the model, the joint determinary of money-supply with the rest of the system arises from R, the short-term rate of interest. In our model, however, since U* is also endogenous, the simultaneity of M* arises both from this term and the short-term rate of interest. The estimated function is the following:

$$M^* = \underset{(-2.945)}{-897.215} + \underset{(53.69)}{1.8608}\, U^* - \underset{(-.480)}{34.6391}(RD-R) + \underset{(.44)}{2.5629}\, NLR \qquad (11.20)$$

$$\bar{R}^2 = .9964, \quad F(3,21) = 2229.24, \quad d\text{-}w = 1.946$$

In order to test how strong the simultaneity is, we have first dropped R and re-estimated the function in order to do an F-test on R to see how significant its contribution is. In this case, the estimates are

$$M^* = \underset{(-2.42)}{-929.38} + \underset{(37.98)}{1.8547}\, U^* - \underset{(-.1175)}{7.3797}\, RD + \underset{(.458)}{2.9887}\, NLR \qquad (11.21)$$

$\bar{R}^2 = .9961$, $F(3,21) = 2068.73$, $d\text{-}w = 1.9075$

In this equation, when R is added and constrained to take of a value equal to the magnitude for the coefficient of RD but opposite in sign, as in equation (11.20), the improvement in the fit can be tested by the F-ratio,

$$F^* = \frac{(\Sigma\hat{\hat{y}}^2 - \hat{y}^2)/K-M}{\Sigma e^2/\ N-K}$$

which comes out to be .3624 which is not significant for the appropriate degrees of freedom.

In our model, U* is defined as

$$U^* = U^*_{-1} + D^* + F^* - F^*_{-1} + RES^*$$

and, further $F^* - F^*_{-1} = EXP^* + A^* - IMP^*$

so that, $U^* = U^*_{-1} + D^* + EXP^* - IMP^* + RES^*$

Since, U^*_{-1} is predetermined, and D* and RES* have been taken as exogenous, the terms which cause simultaneity in M* via the U* can be taken as (EXP* - IMP*). Hence we have further tested whether the inclusion of these terms cause improvement in the overall fit of the money-supply function by defining a new term,

$$UZ^* = U^* - (EXP^* - IMP^*)$$

and running the regression with OLS, since all terms are now exogenous. We have,

$$M^* = \underset{(-.173)}{-135.86} + \underset{(18.35)}{2.063}\ UZ^* - \underset{(-2.998)}{445.127}\ RD + \underset{(.541)}{7.147}\ NLR \qquad (11.22)$$

$$\bar{R}^2 = .9842, \quad F(3,21) = 500.535 \quad d\text{-}w = 1.2948$$

When this is compared to equation (11.20), the significance of the additional terms in equation (11.20) can be judged by forming the F* ratio as before which comes out to be 15.9147 which is significant for the appropriate degrees of freedom at 1% level of confidence.

On the question of simultaneity of money-supply with the rest of the system we now have three types of evidence. First, in terms of the Granger-Sims tests carried out in Chapter 10, the endogeniety of money-supply is not clearly established. Second, in terms of the specification of the Bhattacharya model, if only R is responsible for the endogeneity then again it is not clearly established. In addition to R, if some terms in the definition of unborrowed money reserves are also responsible for it, then the evidence is more favourable for the simultaneous determination of money supply within the system. We have chosen equation (11.20) to be included in the model noting that even if simultaneity is weak this equation can still be used to relate money-supply to the directly controllable monetary policy instruments like the discount-rate and the net liquidity ratio.

For demand for money, the usual formulation incorporating income, short-term interest rate and wealth proxied by financial assets of the private sector, provides acceptable results in terms of signs and significance of individual coefficients and a satisfactory overall fit. The estimated function is:

$$M^*/PY = \underset{(2.875)}{468.879} + \underset{(1.611)}{.07155}\ YD - \underset{(-1.403)}{114.986}\ R + \underset{(3.611)}{.2262}\ FA_{-1} \qquad (11.23)$$

$$\bar{R}^2 = .9673, \quad F(3,21) = 237.317 \quad d\text{-}w = 1.224$$

11.6.6 Imports

The usual explanation for imports is offered in terms of domestic income (expected positive effect), foreign exchange reserves at the beginning of the period (expected positive effect) and import prices relative to domestic prices with an expected negative effect.

An important consideration arising in the Indian context is that imports have generally been financed from current exports earnings and foreign aid rather than by depleting foreign exchange reserves.† For this purpose we have used a variable defined as potential foreign exchange availability

$$FS^* = F^*_{-1} + EXP^* + A^*$$

This variable, when used in conjunction with income and relative prices provides acceptable results. The 2SLS estimates for the import function with variables appropriately deflated are given below:

$$\frac{IMP^*}{PM} = \underset{(3.106)}{17.752} + \underset{(.714)}{.0000774}\, Y^*/PY + \underset{(1.112)}{.2069}\, \frac{FS^*}{PM} - \underset{(-3.872)}{10.2604}\, \frac{PM}{P}$$

$$\bar{R}^2 = .6930, \quad F(3,21) = 19.1055, \quad d\text{-}w = 1.12 \qquad (11.24)$$

Among various other formulations, this equation gives all the expected signs.

Another useful consideration for the import equation is whether or not exchange-rate variations have had an independent impact. Until 1969-70, a fixed exchange-rate system has been followed in India, with one devaluation in 1966. In more recent years a system of quasi-flexible exchange-rate has been followed where the Rupee is linked to a basket of currencies and changes in its value are periodically announced. The use of an exchange-rate variable, in addition to those in equation 1, and in place of the relative price variable, did not provide useful results in terms of increase in $\bar{R}^2$ or significance for its own coefficient. Some

† Except in recent years.

of these results are reported below:

$$\frac{IMP^*}{PM} = \underset{(2.79)}{16.915} + \underset{(.789)}{.0001719}\, Y + \underset{(1.185)}{.2325}\, \frac{FS^*}{PM} - \underset{(-3.413)}{9.7776}\, \frac{PM}{P} - \underset{(.503)}{.2666}\, ER$$

$$\bar{R}^2 = .6822, \quad F(4,20) = 13.88, \quad d\text{-}w = 1.02 \tag{11.25}$$

$$\frac{IMP^*}{PM} = \underset{(-.351)}{-1.246} + \underset{(3.249)}{.000659}\, Y + \underset{(2.115)}{.4747}\, \frac{FS^*}{PM} - \underset{(-1.426)}{.8768}\, ER$$

$$\bar{R}^2 = .521, \quad F(3,21) = 9.70, \quad d\text{-}w = .61 \tag{11.26}$$

Equation (11.24) was finally accepted for inclusion in the model.

11.6.7 Term-Structure of Interest Rates

The long-term interest rate is visualised as a distributed lag function of the short-term rate. A geometrically declining set of weights for the lag function provides useful results. The function was hypothesized as

$$RL = a + bR + \theta b\, R_{-1} + \theta^2 b\, R_{-2} + \ldots\ldots + u$$

From which we have

$$\theta RL_{-1} = \theta a + \theta b\, R_{-1} + \theta^2 bR_{-2} + \ldots\ldots + \theta u_{-1}$$

The usual Koyck transformation provides,

$$RL = a(1 - \theta) + bR + \theta RL_{-1} + (u - \theta U_{-1}) \tag{11.27}$$

Thus, RL is estimated as a function of current short-term interest rate and lagged long-term rate. However, since there is the possibility of serial correlation in the error term by specification, we have used the Cochrane-Orcutt iterative procedures to correct for first order autocorrelation. Furthermore, for RL_{-1}, the lagged dependent variable, we have used an estimated series which is derived by regressing RL on the subset of predetermined variables as used in other equations and then lagging it one-period.

The results for 2SLS and 2SLS corrected for first order auto-correlation (2SLS - AR1) are given below. In the latter case $\bar{R}^2$ is improved considerably.

$$\text{2SLS} \quad RL = \underset{(1.698)}{1.960} + \underset{(.377)}{.11978}\, R + \underset{(2.939)}{.6141}\, RL_{-1} \tag{11.28}$$

$$\bar{R}^2 = .3215 \qquad F(2,21) + 6.45, \qquad d\text{-}w = 1.23$$

$$\text{2SLS- AR1} \quad RL = \underset{(1.90)}{3.3024} + \underset{(.545)}{.2691}\, R + \underset{(1.359)}{.3097}\, RL_{-1} \tag{11.29}$$

$$\bar{R}^2 = .4383 \qquad F(2,20) = 9.583, \qquad d\text{-}w = 1.51$$

As an alternative to the geometric lag-scheme, polynomial distributed lags with varying degrees for the polynomial and number of lagged terms were used to see if the results can be improved. It can be observed from Table 11:1 that the use of the Almon scheme for estimation of the terms structure of interest rate does not improve results in terms of R^2. Estimates of R and RL, as derived from a regression or the subset of predetermined variables indicated earlier, were used as instruments for short-term rate and lagged long-term rates.

Table 11:1

Term-Structure of Interest Rates: Polynomial Distributed Lags

Sample	Dependent Variable: RL		
	Degree of Polynomial	Length of Lags	R^2
53/4 - 74/5	2	5	.1346
	3	5	.1669
	4	5	.2257
54/5 - 74/5	2	6	.0997
	3	6	.1869
	4	6	.2106
55/6 - 74/5	2	7	.0732
	3	7	.1790
	4	7	.1823

Equation (11.29) was therefore accepted for inclusion in the model.

11.6.8 Production Functions

Output is divided between agricultural and non-agricultural outputs. In both cases, labour was not considered a constraint. The primary constraint in both cases is capital. For easier interpretation, the log-linear forms were used for both the production functions.

For the non-agricultural sector, using capital alone as an explanatory variable, we get the following estimates:

$$\ln YNA = \underset{(3.239)}{-1.3355} + \underset{(25.691)}{.9726} \ln KNA \tag{11.30}$$

$\bar{R}^2 = .9792$ $F(1,13) = 660.094$, d-w = .7341 SMPL (60/1-74/5)

The use of a time-trend to capture changes in technology etc. did not improve $\bar{R}^2$ and threw up a negative sign for KNA. Hence it was not included. In the above estimate, however, there are two problems: one, of autocorrelation in the error term, and second, a negative value for the constant. Corrections for autocorrelation using first and second-order adjustments yielded the following results:

$$\ln YNA = \underset{(-.0404)}{-.0284} + \underset{(13.33)}{.85389} \ln KNA \tag{11.31}$$

$\bar{R}^2 = .9887$ $F(1,12) = 1140.47$, d-w = 1.87 2SLS-AR1

$$\ln YNA = \underset{(-.988)}{-.4582} + \underset{(21.057)}{.8928} \ln KNA \tag{11.32}$$

$\bar{R}^2 = .9878$ $F(1,11) = 970.082$, d-w = 2.36, 2SLS-AR2

Although the Durbin-Watson statistic looks much better, we are not able to get rid of the negative sign for the constant. Since this is taken as a parameter for efficiency, it is difficult to interpret a negative value for it. Finally we have dropped the constant, the estimated equations now are,

$$\ln YNA = \underset{(978.79)}{.84999} \ln KNA \tag{11.33}$$

$\bar{R}^2 = 1.00, \quad$ d-w = .4365

and

$$\ln YNA = \underset{(833.05)}{.85133} \ln KNA \tag{11.34}$$

$\bar{R}^2 = 1.00, \quad$ d-w = 1.879, (2SLS-AR1)

For the agricultural output, apart from capital-stock in this sector, acreage and fertilizer inputs were used as additional variables but their inclusion did not satisfactorily improve the results. Hence, the equation containing only capital-stock was accepted. Some of the other results are also reported here for purposes of comparison.

$$\ln YA = \underset{(5.556)}{4.728} + \underset{(4.903)}{.45499} \ln KA \tag{11.35}$$

$\bar{R}^2 = .6252 \quad F(1,13) = 24.3511, \quad$ d-w = 1.583 SMPL (60/1-74/5)

$$\ln YA = \underset{(-.881)}{-6.238} + \underset{(.1032)}{.0296} \ln KA + \underset{(1.558)}{3.20} \ln AC \tag{11.36}$$

$\bar{R}^2 = .6623 \quad F(2,12) = 14.726, \quad$ d-w = 2.189 SMPL (60/1-74/5)

In this equation, the inclusion of acreage, renders the constant negative and the coefficient of capital goes down both in terms of magnitude and significance. The inclusion of fertilizers also has problems.

$$\ln YA = \underset{(-1.087)}{-7.746} + \underset{(.839)}{.3214} \ln KA + \underset{(1.499)}{3.052} \ln AC - .667 \ln FRT \qquad (11.37)$$

$\bar{R}^2 = .070 \quad F(3,11) = 10.474, \quad d\text{-}w = 2.35$

The signs for the constant term and for FRT are not right. Inclusion of a time-trend also did not prove useful. Hence equation (11.35) was accepted for inclusion in the model.

11.6.9 Investment in Agriculture

Investment in agriculture could be explained by agricultural income to capture within section investment, and a relative price-variable (PA/PNA) to capture attraction of investment from the non-agricultural sector. A lagged investment term (IA) is also used. This may accommodate either a first order partial adjustment hypothesis on expectations regarding income and/or the relative prices. It may also reflect the influence of institutional factors. In our view the latter influence is considerable in Indian agriculture.

2SLS estimates are given below:

$$IA = \underset{(-1.367)}{-474.161} + \underset{(1.617)}{.06859} YA + \underset{(1.154)}{215.50} PA/PNA + \underset{(2.124)}{.5373} IA_{-1} \qquad (11.38)$$

$\bar{R}^2 = .8982 \quad F(3,10), \quad d\text{-}w = 1.49$

The usual caution about the lagged dependent variable was followed and first an estimate of IA was generated using the specified set of predetermined variables and this estimate, lagged one year, was used in this equation.

A significant serial correlation was not observed by direct tests on the error term. Hence this equation was accepted for the model.

11.6.10 Other Equations

Demand for time-deposits was explained using both short and long-term rates apart from disposable incomes and financial assets of the private sector. Expected signs are obtained.

$$\underset{}{TD} = \underset{(-2.731)}{-720.507} + \underset{(1.870)}{.0986\ YD} + \underset{(1.037)}{105.712\ R} - \underset{(-2.431)}{96.226\ RL} + \underset{(1.224)}{.0911\ FA_{-1}} \tag{11.39}$$

$$\bar{R}^2 = .935, \quad F(4,30) = 87.33, \quad d\text{-}w = .83$$

Whereas other features of this equation are satisfactory, significant positive serial correlation was observed from the d-w statistic and from direct tests on the error term. Hence the equation was re-estimated with appropriate correction for first order autocorrelation before inclusion in the model.

Other stochastic equations relate to prices. Non-agricultural prices (PNA) are generated from an identity, and these, together with the exogenous agricultural prices (PA) are used to determine the income-deflator which in turn is related to the deflator for investment goods and the general price-level.

11.7 Estimates of Stochastic Equations

In view of the considerations given above, the following estimates, subject to further revisions in view of serial correlation in some cases, as indicated in the next section, were chosen to be included in the model.

E.1 Private consumption expenditure:

$$CP = \underset{(-.909)}{-1250.64} + \underset{(20.515)}{.8068}\ YD + \underset{(2.245)}{2404.536}\ YA/YNA$$

$\bar{R}^2 = .9859 \quad F(2,12) = 486.945 \quad d\text{-}w = 2.195 \quad SMPL\ (60/2 - 74/5)$

or

E.1a $CP = \underset{(9.39)}{1801.09} + \underset{(58.79)}{.7307}\ YD$

$\bar{R}^2 = .9931 \quad F(1.23) = 3455.87 \quad d\text{-}w = 2.606$

E.2 Private investment expenditure:

$$IP = \underset{(-2.26)}{-1166.85} + \underset{(2.91)}{.1643}\ YD - \underset{(\text{constrained})}{22.5069}\ RL + \underset{(.392)}{.01288}\ KP_{-1}$$

$\bar{R}^2 = .9177 \quad F(2,22) = 134.727 \quad d\text{-}w = 1.387$

E.3 Government consumption expenditure:

$$CG^* = \underset{(.9066)}{36.1169} + \underset{(3.45)}{.26902}\ TVR^* + \underset{(6.111)}{.72035}\ CG^*_{-1}$$

$\bar{R}^2 = .9955 \quad F(2,22) = 2632.56 \quad d\text{-}w = 2.02$

E.4 Demand for money:

$$M = \underset{(2.875)}{468.879} + \underset{(1.611)}{.07155}\ YD - \underset{(-1.403)}{114.986}\ R + \underset{(3.611)}{.2262}\ FA_{-1}$$

$\bar{R}^2 = .9673 \quad F(3,21) = 237.317 \quad d\text{-}w = 1.224$

E.5 Supply of money:

$$M^* = -897.215 + 1.8608\ U^* - 34,6391\ (RD-R) + 2.5629\ NLR$$
$$(-2.945) \quad (53.69) \quad (-.480) \quad (.414)$$

$\bar{R}^2$ = .9964 F(3,21) = 2229.24 d-w = 1.946

E.6 Demand for time-deposits:

$$TD = 720.507 + .0986\ YD + 105.712\ R - 96.226\ RL + .0911\ FA_{-1}$$
$$(-2.731) \quad (1.870) \quad (1.037) \quad (-2.431) \quad (1.224)$$

$\bar{R}^2$ = .935 F(4,20) = 87.33 d-w = .83

E.7 Imports:

$$IMP = 17.752 + .000077\ Y^*/PY + .2069\ FS^*/PM - 10.26\ PM/P$$
$$(3.106) \quad (.714) \quad (1.112) \quad (-3.872)$$

$\bar{R}^2$ = .6936 F(3,21) = 19.1055 d-w = 1.121

E.8 Long-term rate of interest:

$$RL = 3.3024 + .2691\ R + .3097\ RL_{-1}$$
$$(1.90) \quad (.545) \quad (1.359)$$

$\bar{R}^2$ = .4383 F(2,20) = 9.583 d-w = 1.51 2SLS - AR1

Final value of ρ = .5392652; t(ρ) = 3.071

E.9 Tax on non-corporate incomes:

$$TDY^* = -50.291 + .0128\ Y^* - 5.2712\ BTR + 53.246\ ITR$$
$$(.391) \quad (24.99) \quad (-.8379 \quad (2.507)$$

$\bar{R}^2$ = .9868 F(3,21) = 599.96 d-w = .959

E.10 Direct tax other than tax on non-corporate incomes:

$$NTDY^* = -54.983 + .01086\ Y^* + 14.905\ T$$
$$(-2.939) \quad (7.783) \quad (4.779)$$

$\bar{R}^2$ = .9755 F(2,22) = 478.343 d-w = .5507

E.11 Customs duties:

$$CD^* = \underset{(-4.05)}{-138.619} + \underset{(17.93)}{.1944}\,(EXP^* + IMP^*)$$

$\bar{R}^2$ = .9304 F(1,23) = 321.683 d-w = .9375

E.12 Indirect taxes other than customs duties:

$$NCD^* = \underset{(-11.0)}{-527.848} + \underset{(51.86)}{.084834}\,Y$$

$\bar{R}^2$ = .9911 F(1,23) = 2687.13 d-w = 1.16

E.13 Agricultural output:

$$\ln YA = \underset{(5.556)}{4.728} + \underset{(4.903)}{.45499}\,\ln KA$$

$\bar{R}^2$ = .625 F(1,13) = 24.35 d-w = 1.583

Sample: 1960-61 to 74-75

E.14 Non-agricultural output:

$$\ln YNA = \underset{(978.79)}{.8499}\,\ln KNA$$

$\bar{R}^2$ = 1.0 d-w = .4365

Sample: 1960-61 to 74-75

E.15 Investment in agriculture:

$$IA = \underset{(-1.367)}{-474.161} + \underset{(1.607)}{.0686}\,YA + \underset{(1.154)}{215.50}\,PA/PNA + \underset{(2.124)}{.5373}\,IA_{-1}$$

$\bar{R}^2$ = .898 F(3,10) = 39.2165 d-w = 1.492

Sample: 1961-62 to 74-75

E.16 General Price index:

$$P = \underset{(-9.629)}{-22.319} + \underset{(7.160)}{114.36}\ PY$$

$\bar{R}^2$ = .9953   F(1,23) = 5127.1   d-w = 1.44

E.17 Price deflator for investment goods:

$$PK = \underset{(.534)}{.0183} + \underset{(38.57)}{.91161}\ PY$$

$\bar{R}^2$ = .9841   F(1,23) = 1486.68   d-w = .811

E.18 Implicit income deflator:

$$PY = .0161 + .3881\ PA + .6018\ PNA$$

$\bar{R}^2$ = .9998   F(2,12) = 46382.6   d-w = 1.49

Sample: 1960-61/1974-75

11.8 Tests for Serial Correlation and Some Revised Estimates

In view of various limitations of the d-w statistic, we have conducted some direct tests on the error-terms of the equation 1 through 18 of the previous section. First and second order serial correlation are tested by fitting the equations,

$$e_t = \rho\ e_{t-1}\ ; \quad e_t = \rho\ e_{t-1} \text{ and } e_t = \rho_1 e_{t-1} + \rho_2 e_{t-2}$$

where e's refer to the estimated errors. Standard t-tests on the significance of ρ_1, ρ_2 etc. are performed. The results are given in Table 11:2.

Table 11:2

Tests for Serial Correlation

Variable	Sample Size	$e_t = \rho e_{t-1}$	$e_t = \rho e_{t-2}$	$e_t = \rho_1 e_{t-1} + \rho_2 e_{t-2}$	
CP	14/13	-.207 (-.762)	-.192 (-.623)	-.253 (-.872)	-.245 (-.770)
IP	24/23	.2535 (1.333)	-.0171 (-.0797)	.2413 (1.105)	-. 936 (-.417)
CG*	24/23	-.0124 (-.059)	.274 (-1.123)	-.011 (-.050)	-.273 (-1.10)
M*/PY	24/23	.376 (1.857)	.0504 (.233)	.452 (2.05)	-.142 (-.637)
M*	24/23	-.117 (-.494)	(.025) (.093)	-.1225 (-.4894)	.0446 (.1599)
TD	24/23	.057 (3.388)	.1575 (.755)	.712 (3.370)	-.250 (-1.189)
IMP*/PM	24/23	.328 (1.843)	.192 (.458)	.571 (2.777)	-1.13 (-.621)
RL	22/21	.225 (1.032)	-.293 (-1.246)	.2695 (1.22)	-.3371 (-1.432)
TDY*	24/23	.353 (1.78)	.322 (1.606)	.243 (1.120)	.236 (1.106)
NTDY*	24/23	.714 (4.939)	.401 (2.063)	.936 (4.322)	-.278 (-1.302)

Table 11:2 (continued)

CD*	24/23	.531 (2.951)	-.0337 (-.156)	.784 (3.893)	-.464 (-2.297)
NCD*	24/24	.4157 (2.1897)	-.0137 (-.0612)	.4995 (2.350)	-.2299 (-1.027)
ln (YA)	14/13	.214 (.794)	-.192 (-.684)	.255 (.869)	-.251 (-.861)
ln (YNA)	14/13	+.4811 (2.175)	-.0975 (-.3786)	.6798 (2.39)	-.402 (-1.59)
IA	13/12	.058 (.160)	-.502 (-1.43)	.107 (.282)	-.518 (-1.397)
P	24/23	.215 (.936)	-.0022 (-.0092)	.236 (.956)	-.055 (-.223)
PY	14/13	.1614 (.557)	.139 (.472)	.0642 (.1961)	.1249 (.3945)

In order to test whether individual coefficients are significantly different from zero, we have used a two-tailed test. The needed critical values of t for a 95% confidence level are given below:

degrees of freedom	23	22	21	20	19	13	12	11	10
Critical value of t-	2.069	2.074	2.080	2.086	2.093	2.160	2.179	2.201	2.22

Significant positive first order autocorrection in the error is observed in the equations for variables TD, NTDY*, CD*, NCD* and ln YNA. In these cases we have re-estimated the equations using Cochrane-Orcutt procedures. The 2SLS - AR1 estimates for these equations are given below:

(ER.6) $$\underset{(-2.261)}{TD = -1281.89} + \underset{(2.051)}{.1084\ YD} + \underset{(.433)}{39.563\ R} - \underset{(-.237)}{7.519\ RL} + \underset{(1.396)}{.0984\ FA_{-1}}$$

$\bar{R}^2 = .9699$ $F(4,19) = 186.04$ d-w = 1.612

Final value of ρ = .802318; $t(\rho) = 6.585$

(ER.10) $$NTDY^* = \underset{(-1.526)}{-102.969} + \underset{(4.6298)}{.0103\ Y^*} + \underset{(2.578)}{18.9137\ T}$$

$\bar{R}^2 = .9881$ $F(2,21) = 957.347$ d-w = 1.54

Final value of ρ = .7219059; $t(\rho) = 5.11$

(ER.11) $$CD^* = \underset{(-.592)}{-45.645} + \underset{(9.487)}{.1652}\ (EXP^* + IMP^*)$$

$\bar{R}^2 = .9521$ $F(1,22) = 458.003$ d-w = 1.499

Final value of ρ = .7126234; $t(\rho) = 4.976$

(ER.12) $$NCD^* = \underset{(-5.84)}{-491.61} + \underset{(33.61)}{.08351\ Y^*}$$

$\bar{R}^2 = .994$ $F(1,22) = 3824.82$ d-w = 1.81

Final value of ρ = .535877; $t(\rho) = 3.109$

(ER.14) $$\ln YNA = \underset{(833.05)}{.85133}\ \ln KNA$$

$\bar{R}^2 = 1.00$ d-w = 1.879

Final value of ρ = .7528283; $t(\rho) = 2.614$

The model as given in section (11.7)and as revised by these estimates is finally used for analysis and forecasting. Alternative simulations without the autoregressive error corrections have also been performed.

11.9 Summary

In this Chapter, a macroeconometric model of the Indian economy consisting of eighteen stochastic equations has been suggested. In the specification of individual equations, preliminary ideas were derived from earlier studies as also from direct tests. Following the custom in presenting such exercises, many of the negative results regarding individual equations have not been reported.

The proposed model is intended for forecasting and policy simulations. It was solved by using Powell's (1968) subroutine in all its applications.

Chapter 12

Analysis of Forecasting Performance

In this Chapter the forecasting performance of the model is evaluated and the forecast errors are analysed. This is done in two stages. First, the model-predicted values of endogenous variables within the sample period are compared with corresponding realizations both for static and dynamic predictions. Secondly, the model is re-estimated for a smaller sample period, saving some observations to provide a forecast period. Predictions for the remaining period, which is now outside the sample used for estimation, are then compared against realizations. Either of these precedures cannot be taken as an adequate guide to the 'true' forecasting performance of the model which is appropriately judged only with reference to a 'genuine' forecast period. Such an exercise, however, has to await some passage of time. Meanwhile, we propose to show that the model at least performs satisfactorily within the sample period and in a 'pseudo' forecast period.

The analysis is done both with respect to predicted levels of variables and predicted changes in the levels of variables. Static predictions are generated by using sample values of both the exogenous and lagged endogenous variables. In the dynamic predictions, model-generated values of the lagged endogenous variables are used.

For the within sample-period analysis model's prediction performance is compared against 'naive' extrapolative models. For levels of variables, these benchmark models are taken as

$$P_t = A_{t-1} \tag{12.1}$$

and for changes in levels, the benchmark models are taken as

$$\Delta P_t = \Delta A_{t-1} \tag{12.2}$$

Comparisons of predictions with realizations are done variable by variable. However, rather than using all the 34 endogenous variables, we have presented results for 24 of these. This choice is made in terms of importance and interest in the variables. Also, when various variables are linked through identities like

$$Y^* = Y.\ PY\ ,$$

evaluation of forecasting performance with reference to any two would suffice for the third. Inclusion of all variables can only overstate the forecasting performance. The variables chosen for the present analysis are:

CP, IP, R, TD, IMP*, M*, CG*, RL, TDY*, NTDY*,
CD*, NCD*, YA, YNA, IA, P, PNA, Y*, U*, ZBP*,
F*, KP, KNA and K.

The model is taken as defined by equations E.1a-E.5, ER.6, E.7-E.9, ER.10, 11, 12, E.13, ER.14, E.15-18 plus the identities M.19 to M.34. This we shall call model A. Subsequently we also present results for the model defined by equations E.1a to E.18 and identities M.19 to M.34. This, we shall call, model B. The difference between these two models is that in the first case some corrections for serial correlation have been incorporated. Later, in Appendix ~~3~~ 4, we also present results when the alternative consumption function 1b. is used.

12.1 Sample Period Predictions: Overall Performance

The overall performance for each variable is indicated by Theil's inequality coefficient (U) and the root-mean-square error (RMS), defined respectively as

$$U = \left[\Sigma(P_t - A_t)^2 / \Sigma A_t^2 \right]^{1/2}$$

and

$$RMS = \left[\frac{1}{n}\Sigma(P_t - A_t)^2 \right]^{1/2}$$

In general, the rankings obtained by the two indices for any variable for different models or forecasts would be the same. Either of these could be used for comparisons across the models variable-by-variable. The inequality coefficient has the added advantage that it can be used for comparisons across variables for a given model, as it is independent of the unit of measurement because of the division by ΣA^2. The RMS measure, on the other hand, is useful in the sense that it offers some indication of the average size of error.

Rather than using the whole sample period we have used the period from 1961-62 to 1974-75. Values of lagged endogenous variables are derived from the observations for 1960-61. We are constrained to use data from 1960-61 to 1974-75 only as this period provides a sample common to all variables.

A summary of the prediction performance of Model A is provided in Table 12.1. In predicting the levels of variables, the model does better than the extrapolative benchmark for both the static and dynamic predictions for the following variables:

CP, IP, IMP*, CG*, TDY*, CD*, NCD*, YA, P, Y*, ZBP*, KP, K.

For TD, M* and YNA the static predictions are better than the benchmark but the dynamic predictions are slightly worse off. For R, RL, NTDY*, IA, PNA and F*, the model does not do better than the benchmark predictions as far as the levels of variables are concerned. Out of these, for RL and NTDY*, the model does better than the extrapolative benchmark when predicted changes in levels are considered. The relevant statistics for prediction performance *vis a vis* changes in levels indicates that except for R, TD, M*, F* and PNA, the model does better than the extrapolative benchmark for all other variables.

Table 12.1

Forecasting Performance: Sample Period Predictions

Variable	Forecast	Levels of Variables		Changes in Levels	
		RMS	U	RMS	U
CP	Static	493.4	.0330	661.8	.8761
	Dynamic	449.2	.0300	545.5	.8654
	Benchmark	598.5	.0491	1043.0	1.3807
IP	Static	182.0	.0799	244.8	.9080
	Dynamic	175.2	.0769	240.4	.8918
	Benchmark	186.4	.1141	409.5	1.5189
R	Static	3.016	.9222	2.015	5.566
	Dynamic	6.819	2.085	2.247	6.208
	Benchmark	.3493	.1068	.3231	1.0738
TD	Static	127.9	.0827	104.7	.6745
	Dynamic	163.8	.0938	118.3	.7620
	Benchmark	151.6	.0869	95.25	.6135
IMP*	Static	282.7	.1293	309.7	.5982
	Dynamic	293.8	.1344	326.0	.6298
	Benchmark	500.1	.2287	423.7	.8185
M*	Static	440.7	.0644	403.5	.5436
	Dynamic	1059.0	.1548	403.5	.5436
	Benchmark	716.8	.1048	250.2	.5819
CG*	Static	158.1	.0455	213.0	.5092
	Dynamic	254.3	.0732	174.5	.4170
	Benchmark	404.5	.1164	243.5	.5819
RL	Static	1.417	.2222	.8931	.9089
	Dynamic	3.268	.5127	1.197	1.2178
	Benchmark	.9486	.1488	1.203	1.2246
TDY*	Static	30.53	.0656	29.24	.4348
	Dynamic	40.33	.0866	26.60	.3956
	Benchmark	64.80	.1391	34.71	.5161

Table 12.1 (continued)

Vari-able	Forecast	Levels of Variables		Changes in Levels	
		RMS	U	RMS	U
NTDY*	Static	199.0	.3091	36.41	.4593
	Dynamic	184.4	.2864	36.23	.4572
	Benchmark	77.49	.1204	51.36	.6481
CD*	Static	105.1	.1608	88.18	.6205
	Dynamic	109.8	.1680	90.95	.6399
	Benchmark	137.4	.2102	97.43	.6856
NCD*	Static	256.9	.0939	200.9	.4874
	Dynamic	332.0	.0121	197.8	.4799
	Benchmark	460.1	.1683	278.4	.6755
YA	Static	432.1	.0581	563.7	.9916
	Dynamic	415.4	.0559	548.8	.9653
	Benchmark	548.0	.0737	878.3	1.545
YNA	Static	285.1	.0259	252.5	.5044
	Dynamic	533.4	.0485	242.3	.4840
	Benchmark	499.3	.0454	333.2	.6654
IA	Static	71.51	.1112	74.20	1.2523
	Dynamic	133.9	.2083	73.71	1.244
	Benchmark	57.77	.0898	63.54	1.072
P	Static	13.63	.0762	9.935	.4233
	Dynamic	18.64	.1042	7.164	.3052
	Benchmark	22.62	.1264	11.35	.4834
Y*	Static	2558.0	.0676	1815.0	.3585
	Dynamic	3374.0	.0892	1636.0	.3231
	Benchmark	4886.0	.1292	2515.0	.4967
U*	Static	282.8	.0684	309.8	.7130
	Dynamic	694.2	.1679	302.9	.6972
	Benchmark	420.0	.1016	342.7	.7887
ZBP*	Static	118.5	.1555	130.5	.4043
	Dynamic	130.2	.1710	130.7	.4048
	Benchmark	314.3	.4127	471.2	1.4598
F*	Static	282.5	.7311	309.8	2.5102
	Dynamic	695.0	1.796	303.0	2.4548
	Benchmark	120.2	.3108	159.2	1.289
PNA	Static	.1974	.1263	.1228	.8150
	Dynamic	.2817	.1802	.9466	.6284
	Benchmark	.1452	.0929	.0711	,4722

Table 12.1 (continued)

Vari-able	Forecast	Levels of Variables		Changes in Levels	
		RMS	U	RMS	U
KP	Static	168.5	.0036	237.8	.1660
	Dynamic	936.9	.0203	201.8	.1409
	Benchmark	1398.0	.0302	258.7	.1806
KNA	Static	313.1	.0055	212.3	.0851
	Dynamic	2804.0	.0497	362.7	.1454
	Benchmark	2443.0	.0433	207.1	.0829
K	Static	205.3	.0029	217.5	.0751
	Dynamic	2441.0	.0367	303.4	.1048
	Benchmark	2832.0	.0426	229.6	.0793

The values of inequality coefficients can be used to make comparison regarding prediction performance across variables. The inequality coefficient, as far as the predicted levels in the static case are concerned, ranges from the lowest value of .0036 for KP to the highest of .9222 for R. We have ranked variables by different ranges of values for the inequality coefficient for static predictions of the levels in order to get an idea as to what variables the model predicts relatively better compared to others. This information is provided in Table 12.2.

Table 12.2

Static Predictions (Levels): Ranking by Inequality Coefficients

Inequality coefficient	⩽.05	.05 to .10	.10 to .30	.30 to .50	.50 to .75	.75 to 1.0	⩾1.0
Variables	CP,CG* YNA, K, KNA, K.	IP,TD, M*, TDY*, NCD*, YA,P, Y*,U*	IMP*, RL, CD*, IA, PNA, ZBP*	NTDY*	F*	R	—

It is observed that the model does very well in predicting private and government consumption expenditures and the non-agricultural output as also the capital stock variables. The performance regarding private investment expenditure, time-deposits, money-supply, taxes on non-corporate incomes, agricultural output, GDP and unborrowed reserves are also very satisfactory. Even for the few cases where the extrapolative benchmarks outperform the model, the magnitude of the inequality coefficient is relatively low. The one variable where the model performs worst is the short-term rate of interest. Even in this case, the inequality coefficient is less than one, remembering that the definition of the coefficient which we have used permits a range from zero to infinity. On observation we find that the short-term interest-rate is forecasted relatively better during the later years in the sample and hence for beyond-sample forecasting it should not cause too many problems. Further analysis where the mean square error is decomposed indicates that there is scope for subjective corrections in the equation for the short-term interest-rate should this be desired for a forecast period.

The prediction performance of Model B, which has a similar specification but which does not contain corrections for first-order serial correlation in some of the equations is summarised in Table 12.3.

Table 12.3

Prediction Performance: Static Predictions (Model B)

Variable	Levels		Changes in Levels	
	RMS	U	RSM	
CP	498.8	.0334	665.1	.8803
IP	182.8	.0802	200.4	.9013
R	2.133	.6523	1.958	5.408
TD	164.9	.0945	165.1	1.063
IMP*	265.5	.1214	305.2	.5894
M*	422.3	.0617	394.0	.5308
CG*	166.8	.0480	213.8	.5200
RL	1.234	.1936	.8971	.9129
TDY*	30.25	.0650	29.70	.4416
NTDY*	41.33	.0642	36.20	.4568
CD*	92.49	.1415	87.23	.6138
NCD*	240.2	.0878	201.0	.4877
YA	432.5	.0582	563.5	.9912
YNA	283.7	.0258	253.6	.5066
IA	68.26	.1062	77.74	1.312
P	11.97	.0669	10.41	.4435
PNA	.1737	.1111	.1299	.8623
Y*	2309.0	.0610	1834.0	.3623
U*	265.5	.0642	305.2	.7023
ZBP*	111.8	.1468	132.6	.4107
F*	265.5	.6863	305.3	2.4737
KP	162.3	.0035	237.6	.1659
KNA	265.0	.0046	205.3	.0827
K	160.5	.0024	212.4	.0733

A comparison of results for Model A and B indicates that while the latter performs relatively better for most variables in predicting levels, the former is somewhat better in predicting changes in levels. It is difficult to see which is the stricter criterion but in general, in the literature on forecasting, appropriate prediction of changes has been considered the more desirable property. Either way the differences in the prediction performance of the two versions of the model are marginal. We have carried further analysis in terms of Model A.

12.2 Prediction of Direction of Changes

Whereas the overall performance regarding predictive accuracy may be judged by statistics like the root-mean-square error and the inequality coefficient, this information needs to be supplemented by some considerations regarding the ability of the model to correctly predict direction of changes. This could be observed directly from the time-series plots of predicted and realized changes included in this Chapter. In Table 12.4, we have summarised this information under three headings: number of positive changes correctly predicted out of the total number of positive changes; number of negative changes correctly predicted out of the total number of negative changes; and number of negative changes predicted when they do not occur.

Except for a few variables, the direction of change has in general been in the upwards direction. This property of the realizations is very closely reflected by the model both for static and dynamic predictions. There are some variables where some downward changes have occurred and are also important like ZBP*, P, F* etc. In these cases also the model performs very satisfactorily.

Table 12.4

Prediction of Direction of Changes

Variable	Static Predictions			Dynamic Predictions		
	Positive changes correctly predicted	Negative changes correctly predicted	Negative changes incorrectly predicted	Positive changes correctly predicted	Negative changes correctly predicted	Negative changes incorrectly predicted
CP	11/11	0/3	-	11/11	0/3	-
IP	9/9	0/5	-	9/9	0/5	-
R	2/6	3/4	-	4/6	4/4	-
TD	12/13	0/1	-	12/13	0/1	-
IMP*	10/10	1/3	-	10/10	0/3	-
M*	13/13	0/0	-	13/13	0/0	-
CG*	13/13	0/0	-	13/13	0/0	-
RL	5/7	3/6	-	3/7	2/6	-
TDY*	12/13	0/0	1	12/13	0/0	1
NTDY*	12/12	0/1	-	12/12	0/1	-
CD*	9/10	0/3	1	10/10	0/3	-
NCD*	13/13	0/0	-	13/13	0/0	-
YA	8/8	0/6	-	8/8	0/6	-
YNA	13/13	0/0	-	13/13	0/0	-
IA	6/9	0/4	3	7/9	1/4	2
P	12/12	1/1	-	12/12	1/1	-
PNA	11/13	0/0	2	12/13	0/0	1
Y*	13/13	0/0	-	13/13	0/0	-
U*	11/12	0/1	1	11/12	0/1	1
ZBP*	9/9	3/4	-	9/9	3/4	-
F*	5/7	5/7	-	5/7	3/7	-
KP, KNA, and K	13/13	0/0	-	13/13	0/0	-

For short-term interest rate the model predicts 3 out of 4 downward changes correctly and for the long-term rate of interest, 3 out of 6 negative changes. For the price-level, the model perfectly follows the movement of direction of changes. For ZBP* and F*, variables which are determined in identities, again the direction of changes are very satisfactorily predicted. In other variables, positive changes are predominant and adequately captured in both static and dynamic predictions. Incorrectly predicted downturns are few and far between.

In Figures 12.1 to 12.6 predicted and realized changes denoted respectively by symbols P and A have been plotted over time. The units are decided in each case by the minimum and maximum values to be plotted. The column indicated by 'o' represents a value of 0.0. Thus, if both P and A are on the same side of this line, it implies that the direction of change has been correctly predicted. In drawing these diagrams, a few of the extreme values had to be ignored in some cases. If for any year, only the symbol A is plotted, it implies that the predicted change coincides with the realized change. The years should be read as 62-63 for 63, 63-64 for 64 etc.

12.3 Diagnostic Checks

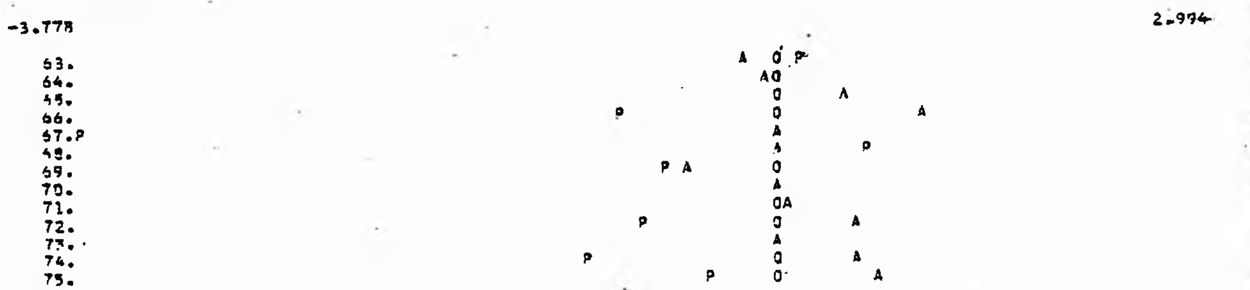
The mean square errors for the predicted levels for static predictions of model A have been decomposed in order to gain an idea as to the relative importance of different sources of error. Sources of error are identified as due to bias, different variation and different covariation in one decomposition, and as due to slope and disturbance errors in addition to that of bias in the other decomposition. Inequality proportions attached to these sources, as suggested in Theil (1961, 1966) have been calculated for the 24 variables included for forecasting analysis in the earlier sections. The interpretation of the inequality proportions has been discussed in Chapter 4. It will be recalled that systematic errors like those of bias, slope and different variation are of the type the forecaster can do something about. On the other hand, errors due to different covariation of the predicted and realized series or the disturbance error are of the type that a forecaster cannot do much about. Table 12.5 summarises information about the inequality proportions.

Time-Series Plots: Predicted Changes (P) and Realised Changes (A)

Variable CP

Variable IP

Variable R



Variable TD

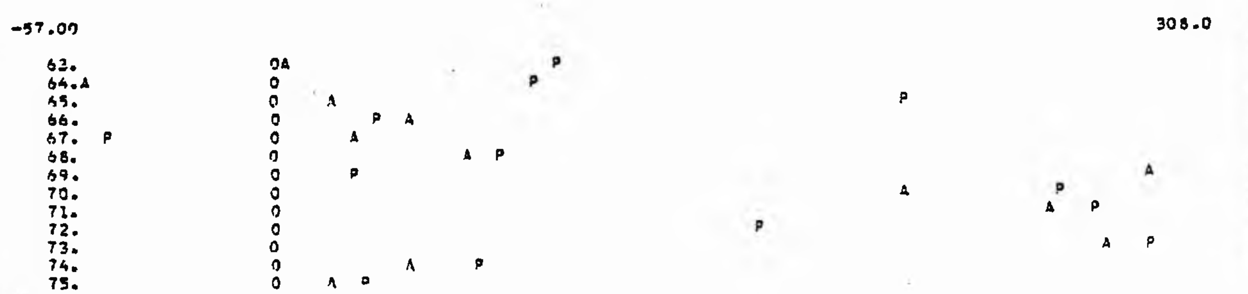
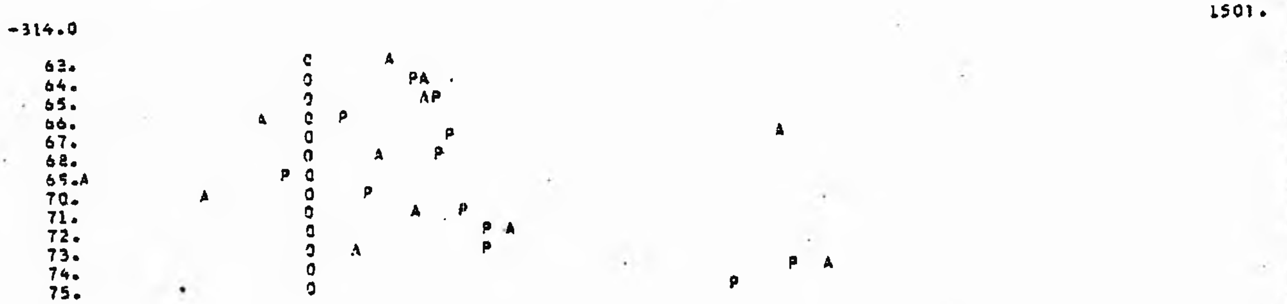


Figure 12.1

Variable IMP*



Variable M*

Variable CG*

Variable RL

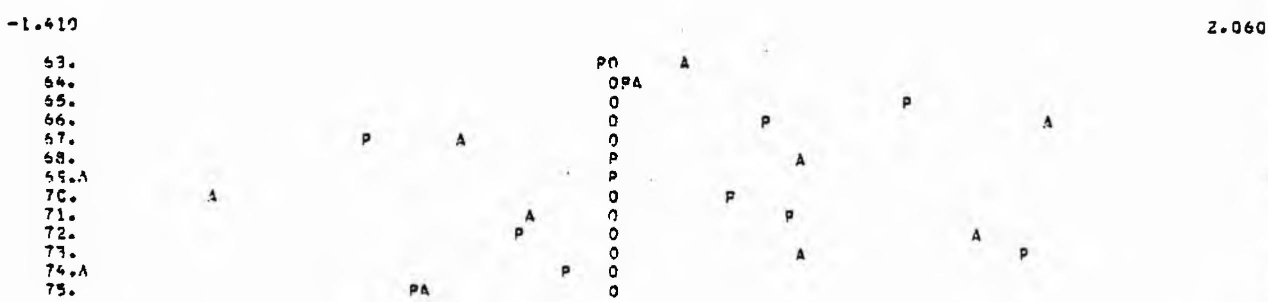
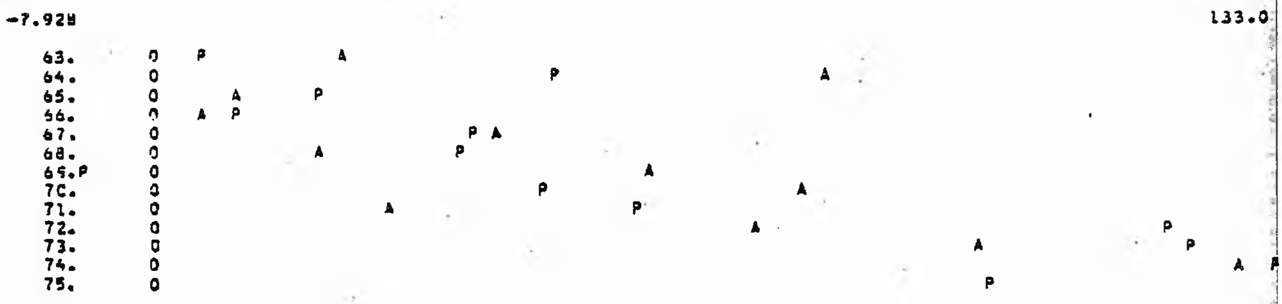


Figure 12.2

Time-Series Plots: Predicted Changes (P) and Realised Changes (A)

Variable TDY*



Variable NTDY*

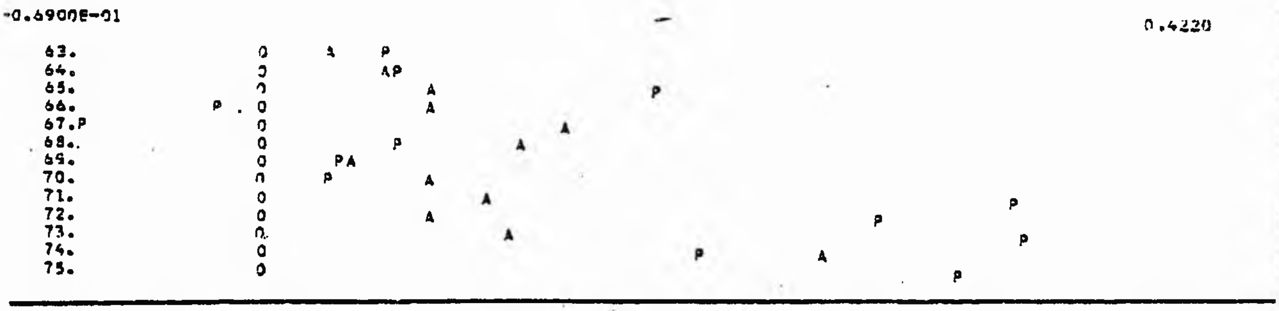
Variable CD*

Variable NCD*

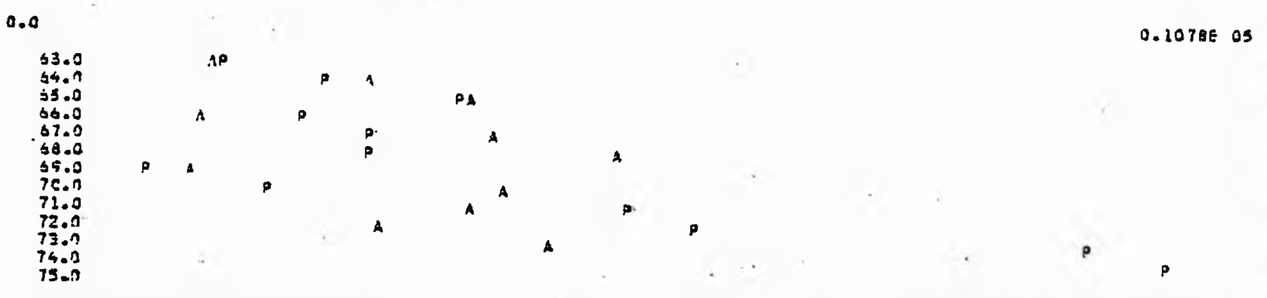
Figure 12.3

Time-Series Plots: Predicted Changes (P) and Realised Changes (A)

Variable PNA



Variable Y*



Variable U*

Variable ZBP*

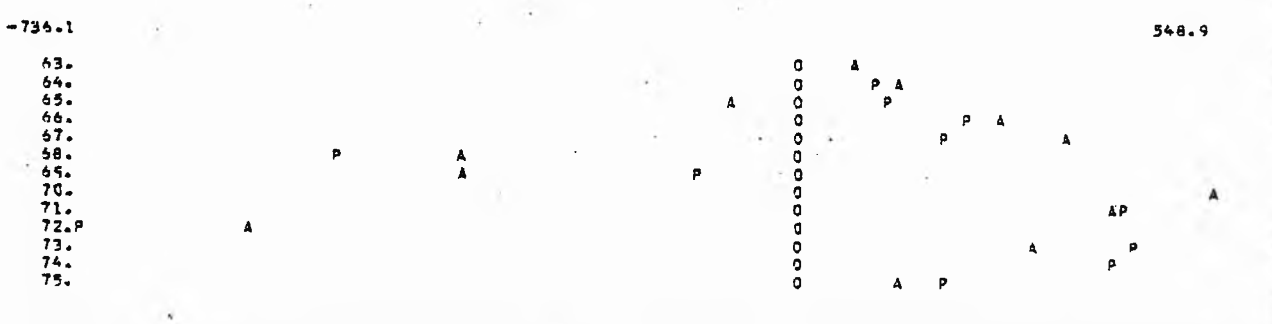


Figure 12.4

Time-Series Plots: Predicted Changes (P) and Realised Changes (A)

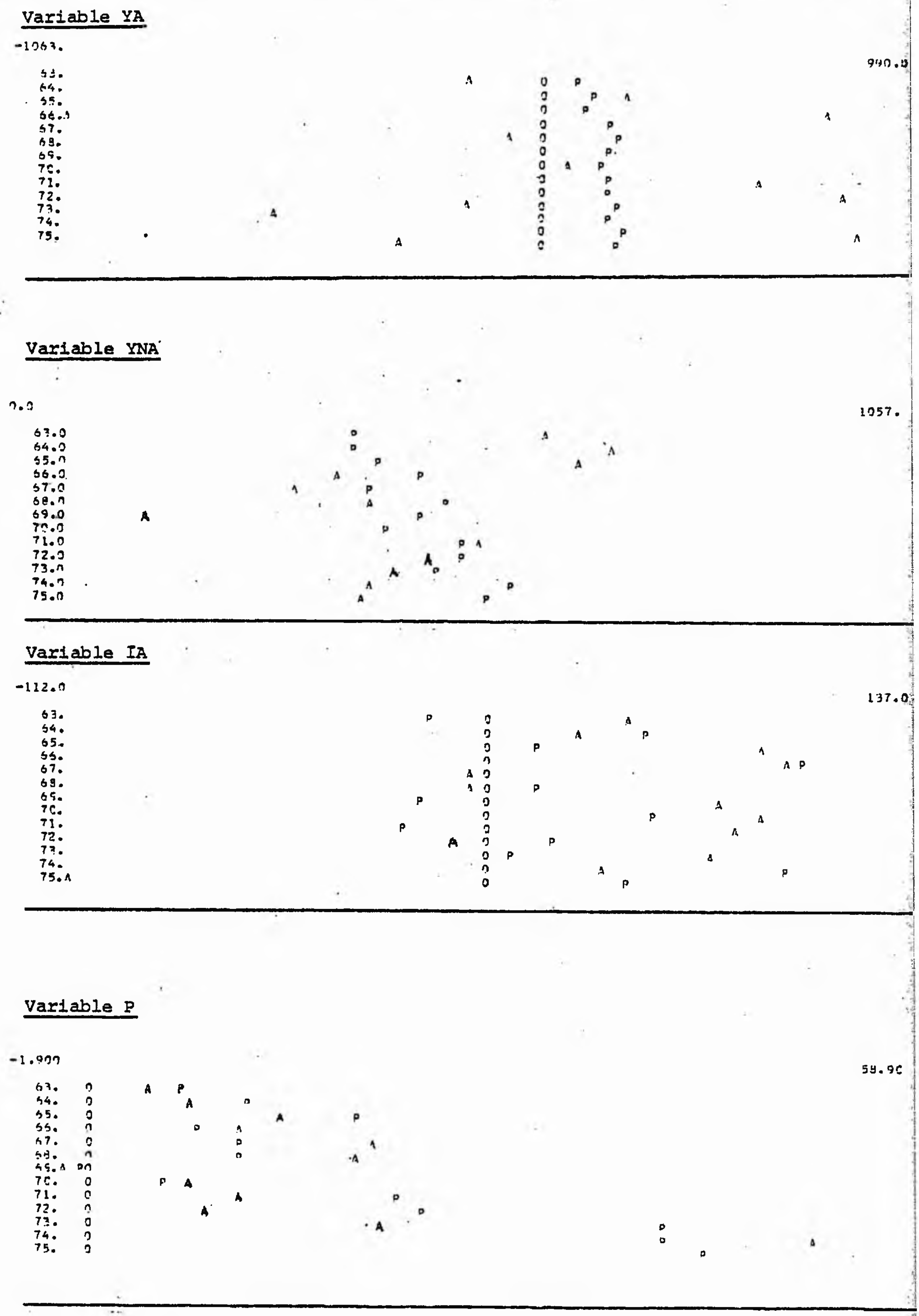
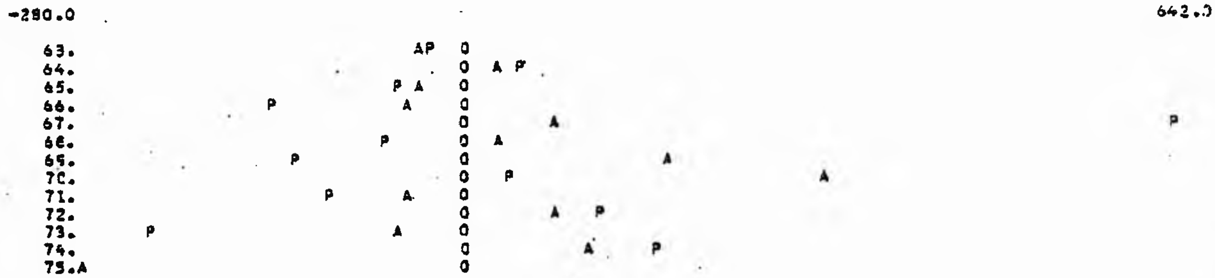


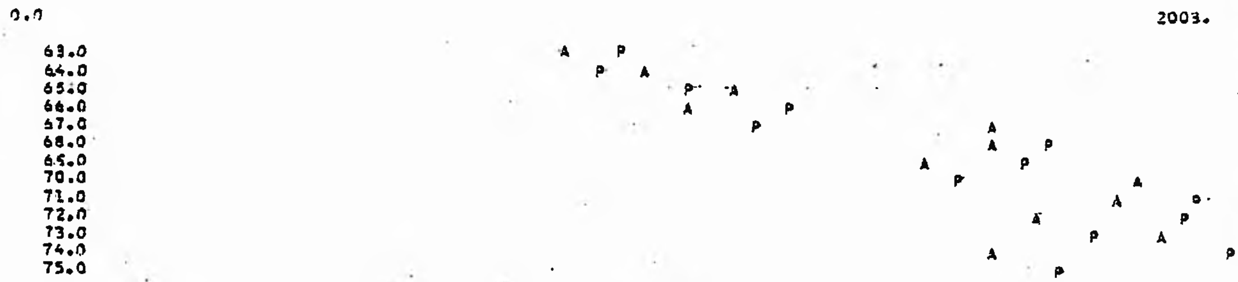
Figure 12.5

Time-Series Plots: Predicted Changes (P) and Realised Changes (A)

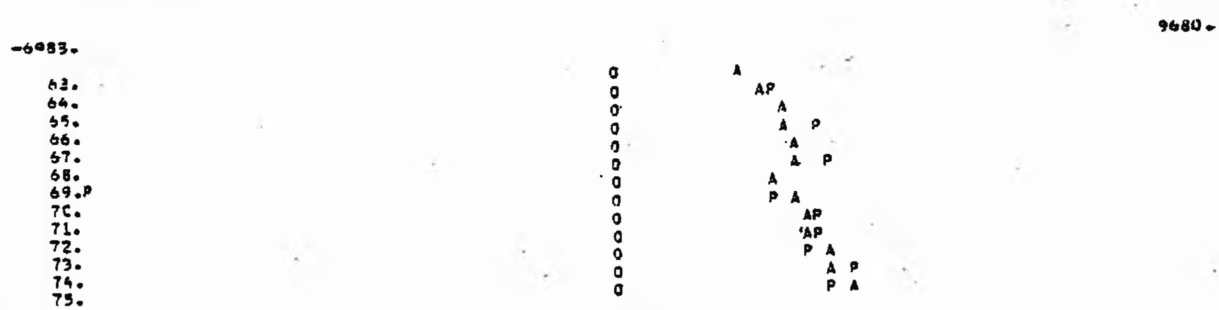
Variable F*



Variable KP



Variable KNA



Variable K

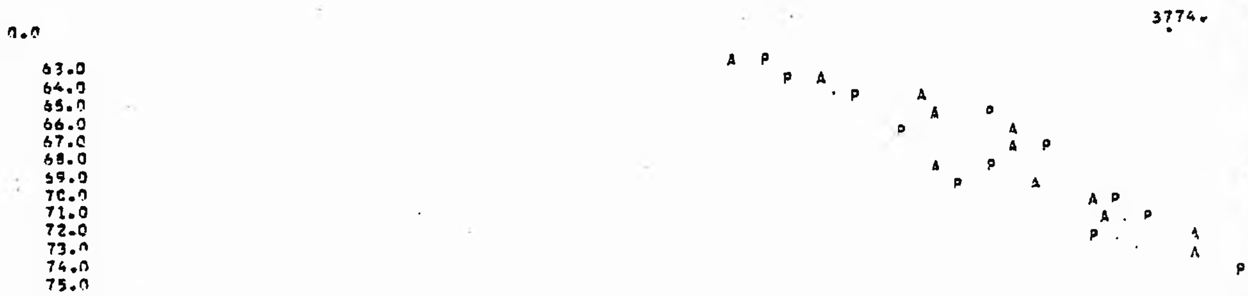


Figure 12.6

The bias error contributes less than ten per cent of the total mean square error in the case of CP, IP, TD, IMP*, M*, CG*, TDY*, YA, U*, F*, and KP. Its contribution is between ten and twenty per cent for variables CD*, NCD*, YNA, Y*, and between twenty and twenty-five per cent for RL, P, PNA and ZBP*. For R, about forty per cent of the error is due to bias and for NTDY* nearly all the error is due to bias. This suggests that at least for these two variables, for a true forecast period, it may be desirable to subjectively change the constant term in the equation governing these two variables by the amount of the mean error $(\overline{P} - \overline{A})$. The sign and magnitudes of the mean error are given in Table 12.6.

Errors due to different variation of the predicted and realized series around their means are observed to be relatively low for all cases except R and IMP*. The same is true for the slope error in the alternative decomposition. This is a satisfactory situation since the variation in the predicted series reflected by (σ_P) closely follow variation in the actual series (σ_A).

The main source of error is observed to be different covariation in one decomposition and the disturbance or the residual error in the other decomposition. These do not offer much scope for correction. On the whole, this analysis reveals that the model is robust enough to leave little room for systematic errors.

The nature and magnitude of the systematic errors can further be quantitied by calculating mean error $(\overline{A} - \overline{P})$ and the coefficient (β) in the regression of actual on the predicted series. It will be recalled that it is desirable for such a regression line to be as close to the line of perfect forecast as possible. This would be achieved when $(\overline{A} - \overline{P})$ is zero and when $\beta = 1$. This informatio for the predicted levels in the static case of model A are given in Table 12.6.

Table 12.5

Diagnostic Checks: Levels of Variables

Variable	Inequality Proportions due to				
	Bias	Diff variation	Diff covariation	Slope	Residual
CP	.0465	.0808	.8722	.0249	.9280
IP	.0213	.0785	.9001	.0079	.9708
R	.3969	.4370	.1660	.5753	.0278
TD	.0031	.3936	.6032	.2826	.7142
IMP*	.0881	.0248	.8870	.0000	.9118
M*	.0583	.0144	.9274	.0017	.9401
CG*	.0032	.0681	.9286	.0974	.8993
RL	.2240	.1273	.6487	.1012	.6747
TDY*	.0349	.0034	.9618	.0002	.9649
NTDY*	.9457	.0201	.0342	.0244	.0299
CD*	.1801	.2061	.6138	.1050	.7149
NCD*	.1397	.0387	.8215	.0766	.7837
YA	.0120	.1551	.8329	.0036	.9843
YNA	.1587	.2603	.5811	.2117	.6296
IA	.1015	.0354	.8630	.0008	.8977
P	.2248	.1554	.6198	.2126	.5625
PNA	.2216	.2219	.5565	.3396	.4388
Y*	.1608	.1223	.7169	.1662	.6730
U*	.0872	.0099	.9027	.0002	.9126
ZBP*	.2217	.0037	.7746	.0401	.7382
F*	.0876	.0004	.9120	.3597	.5527
KP	.0909	.0009	.8987	.0018	.8978
KNA	.7628	.0159	.2055	.0151	.2063
K	.3871	.1386	.2791	.1367	.2809

Table 12.6

Mean and Slope Errors

Variable	CP	IP	R	TD	IMP*	M*
$(\bar{A} - \bar{P})$	106.4	26.57	1.90	8.072	83.93	-106.4
β	1.046	1.038	.1103	1.169	.9987	1.007
	CG*	RL	TDY*	NTDY*	CD*	NCD*
$(\bar{A} - \bar{P})$	8.928	.6705	5.703	-193.5	44.61	96.00
β	.9656	.3379	.9978	.8689	1.137	.9462
	YA	YNA	IA	P	PNA	Y*
$(\bar{A} - \bar{P})$	47.36	-113.6	-22.79	6.462	.0929	1026.0
β	1.048	1.079	.9861	.9003	.7660	.9341
	U*	ZBP*	F*	KP	KNA	K
$(\bar{A} - \bar{P})$	-83.50	55.77	-83.71	-50.79	-273.4	-127.7
β	1.001	.9279	.2385	.9987	1.004	1.007

Note: $(\bar{A} - \bar{P})$ = mean error; β = regression coefficient of actual on predicted levels.

The predicted mean exceeds the realized mean for M*, YNA, IA, U*, KNA, K F*, KP, NTDY*. The predicted mean falls short of the realized mean for CP, IP, R, TD, IMP*, CG*, RL, TDY*, CD*, NCD*, YA, P, PNA, ZBP* and Y*. Except for a few variables, the fitted line in the regression of the actual on predicted levels is sloped very closely to the line of perfect forecasts.

12.4 Evaluation of Pseudo-Forecasts

The analysis of the prediction performance of a macroeconometric model within the sample period is useful inasmuch as it sheds light on the peculiarities and properties of the model. By itself, however, such an analysis can serve only as a rough guide to the predictive prowess of a model. Forecasting performance is best judged when the model is used to predict outside the sample. This requires a passage of time and such an exercise cannot be properly done at the stage of model construction. What can, however, be done at this stage is to save some observations in the sample and re-estimate the model with a truncated sample period. Predictions generated from this re-estimated model can then be compared with available realizations for the remaining period. We have pursued this exercise here although we realize that such forecasts cannot be taken as 'true' forecasts. As Christ (1975) has argued, the information of the whole sample qualitatively goes in the specification of the model even when estimated with the truncated sample period. In other words, the specification of the model would not have been the same if information about only the truncated sample were available. For this reason forecasts generated from the truncated sample estimates have sometimes been termed as 'pseudo-forecasts'. To the extent that the 'pseudo-forecasts' use more information qualitatively than what is contained in the truncated sample, forecasting performance of the model would be overstated. On the other hand, if the artificial forecast period contains short-term peculiarities for extraneous reasons, the forecasting performance of the model would be understated.

In re-estimating the model we have used the same sets of predetermined variables for each equation as in the earlier case. The model is estimated for the period 1950/1 to 1969/70 for most equations and for the period 1960/1 to 1969/70 for equations for which the sample began in 1960/1 in the original estimation. In this latter case, since the number of predetermined variables used in the first stage of estimation exceeds the number of available observations, we have used a set of principal components of these predetermined variables in the first stage of estimation. The first seven principal components were used and together they explain about 98 per cent of the variation in the subset of predetermined variables.

The model is re-estimated with the original specification. In some cases first-order correction for serial correlation in the error is obtained by using the Cochrane-Orcutt iterative procedures. The model is presented below. No comments on individual coefficients are made to conserve space. In general, the same signs for the coefficients are obtained as in the full sample estimation. The overall quality of individual equations in terms of $\bar{R}^2$ etc. also remains satisfactory. For the consumption function, the specification given by equation E.1(b) of Chapter 11 was used. The term YA/YNA is dropped as its sign was not acceptable and the coefficient was not significantly different from zero. Without this term it was possible to estimate this equation for the sample beginning 1950/51.

12.4.1 Estimates of Equations for the Truncated Sample

The estimated equations are given below.

(T.1a) $$CP = \underset{(6.003)}{1702.5305} + \underset{(29.939)}{.74065}\ YD - \underset{(-.4943)}{93.8092}\ YA/YNA$$

$$\bar{R}^2 = .992 \qquad F(2,7) = 1219.6, \quad d\text{-}w = 2.4216$$

(T.1b) $$CP = \underset{(9.004)}{1799.828} + \underset{(50.46)}{.73086}\ YD$$

$$\bar{R}^2 = .993 \quad F(1,18) = 2545.9, \quad d\text{-}w = 2.279$$

(T.2) $$IP = \underset{(-3.07)}{-3425.73} + \underset{(.305)}{.02649}\ YD - \underset{(-.1101)}{5.704}\ RL + \underset{(1.899)}{.1214}\ KP_{-1}$$

$$\bar{R}^2 = .897 \quad F(3,16) = 56.05, \quad d\text{-}w = 1.480$$

(T.3) $$CG^* = \underset{(-.032)}{-.788} + \underset{(5.806)}{.60385}\ TVR^* + \underset{(2.124)}{.3014}\ CG^*_{-1}$$

$$\bar{R}^2 = .997 \quad F(2,16) = 2604.3, \quad d\text{-}w = 2.213$$

(T.4) $$M^*/PY = \underset{(3.789)}{617.9802} + \underset{(1.120)}{.0451196}\ YD - \underset{(-.4083)}{29.4766}\ R + \underset{(4.034)}{.2215}\ FA_{-1}$$

$$\bar{R}^2 = .960 \quad F(3,16) = 151.01, \quad d\text{-}w = 1.698$$

(T.5) $$M^* = \underset{(-1.004)}{-354.10} + \underset{(3.355)}{1.769}\ U^* - \underset{(-2.387)}{95.982}\ (RD-R) - \underset{(-.711)}{4.419}\ NLR$$

$$\bar{R}^2 = .997 \quad F(3,16) = 1797.6, \quad d\text{-}w = 2.028$$

(T.6) $$TD = \underset{(-2.718)}{-573.26} + \underset{(2.304)}{.10776}\ YD + \underset{(.316)}{30.204}\ R - \underset{(-1.688)}{61.511}\ RL + \underset{(.7416)}{.0475}\ FA_{-1}$$

$$\bar{R}^2 = .897 \quad F(4,15) = 42.17, \quad d\text{-}w = .953$$

(T.7) $$IMP^*/PM = \underset{(1.514)}{12.5698} + \underset{(1.0204)}{.000206}\ Y^*/PY + \underset{(1.795)}{.3802}\ FS^*/PM - \underset{(-2.633)}{9.5137}\ PM/P$$

$$\bar{R}^2 = .724 \quad F(3,16) = 17.607, \quad d\text{-}w = 1.089$$

(T.8) $$RL = \underset{(-.1324)}{-.2223} + \underset{(2.659)}{2.0257}\ R + \underset{(.14428)}{.1062}\ RL_{-1}$$

$$\bar{R}^2 = .6574 \quad F(2,15) = 17.313, \quad d\text{-}w = 1.586 \qquad (2SLS\text{-}AR1)$$

(T.9) $$TDY^* = \underset{(-.715)}{-77.779} + \underset{(13.18)}{.01046}\ Y^* + \underset{(.449)}{2.6166}\ BTR + \underset{(.780)}{21.493}\ ITR$$

$$\bar{R}^2 = .9585 \quad F(3,16) = 147.105, \quad d\text{-}w = 1.360$$

(T.10) $$NTDY^* = \underset{(-1.9386)}{-45.142} + \underset{(2.4794)}{.007943}\ Y^* + \underset{(3.9937)}{19.005}\ T$$

$$\bar{R}^2 = .9410 \quad F(2,17) = 152.438, \quad d\text{-}w = .3275$$

$$(T.11)\quad CD^* = \underset{(-2.108)}{-116.807} + \underset{(7.583)}{.1812}\,(EXP^+ + IMP^*)$$

$$\bar{R}^2 = .7484 \quad F(1,18) = 57.5071, \quad d\text{-}w = 0.7203$$

$$(T.12)\quad NCD^* = \underset{(-9.151)}{-472.256} + \underset{(3.0872)}{.0807}\,Y^*$$

$$\bar{R}^2 = .9804 \quad F(1,18) = 952.585, \quad d\text{-}w = 1.1145$$

$$(T.13)\quad \ln YA = \underset{(2.334)}{5.21767} + \underset{(1.555)}{.39972}\,\ln KA$$

$$\bar{R}^2 = .1870, \quad d\text{-}w = 1.8758, \quad \text{2SLS-AR1.}$$

$$(T.14)\quad \ln YNA = \underset{(589.36)}{.851127}\,\ln KNA$$

$$\bar{R}^2 = 1.00, \quad d\text{-}w = 1.78, \quad \text{2SLS-AR1}$$

$$(T.15)\quad IA = \underset{(-.1904)}{-97.9055} + \underset{(.2715)}{.01448}\,YA + \underset{(.03186)}{13.354}\,PA/PNA + \underset{(1.6325)}{1.032}\,IA_{-1}$$

$$\bar{R}^2 = .7813 \quad F(3,5) = 10.525, \quad d\text{-}w = 1.8766$$

$$(T.16)\quad P = \underset{(-5.022)}{-16.7799} + \underset{(38.553)}{109.51796}\,PY$$

$$\bar{R}^2 = .9874 \quad F(1,18) = 1485.28 \quad d\text{-}w = 1.8156$$

$$(T.17)\quad PI = \underset{(1.499)}{.0806} + \underset{(18.671)}{.85362}\,PY$$

$$\bar{R}^2 = .9481 \quad F(1,18) = 348.49 \quad d\text{-}w = .3565$$

$$(T.18)\quad PY = \underset{(-.632)}{-.01425} + \underset{(17.207)}{.37994}\,PA + \underset{(14.620)}{.63599}\,PNA$$

$$\bar{R}^2 = .9997 \quad F(2,7) = 14488.2 \quad d\text{-}w = 2.4774$$

Replacement Terms

$$DKA = \underset{(10.3180)}{.0371158}\,KA_{-1} \qquad R^2 = .9221$$

$$DKP = \underset{(19.998)}{.0168899}\,KP_{-1} \qquad R^2 = .9546$$

$$DKG = \underset{(47.744)}{.0125501}\,KG_{-1} \qquad R^2 = .9917$$

12.4.2 Autoregressive Benchmark Models

We have used predictions generated from auxiliary autoregressive models to serve as benchmarks. In the forecast evaluation exercise presented in Section 12.1 the benchmark models used were 'naive' no-change or constant-change models. For the exercise with reference to the pseudo-forecast period, we have constructed more powerful extrapolative models. These are estimated for the 24 variables for which the forecast evaluation exercise has been done in Section 12.1. In each case, the extrapolative model is estimated by regressing first differences of variables on their own lagged terms. A maximum of five lagged terms are allowed for variables where the sample extends back to 1950-51 and a maximum of three lagged terms are allowed for variables where the sample extends back to 1960-61. In all cases the last observation used is for 1969/70, leaving five observations to provide a forecast period as before. In a step-wise regression procedure we choose only those lagged terms which contribute significantly towards explaining variation in the dependent variable. The estimated models are given below. Variables refer to first differences.

AR1

$$CP = \underset{(3.135)}{2919.405} - \underset{(-3.407)}{1.2683}\, CP_{-1} - \underset{(-3.331)}{1.8441}\, CP_{-2} - \underset{(2.515)}{1.8685}\, CP_{-3} - \underset{(-2.180)}{1.6139}\, CP_{-4} - \underset{(-.988)}{.6256}\, CP_{-5}$$

$$R^2 = .6554$$

AR2

$$IP = \underset{(2.42)}{288.1599} - \underset{(-1.28)}{.4543}\, IP_{-1} - \underset{(-1.86)}{.6067}\, IP_{-2} - \underset{(-.738)}{.2539}\, IP_{-3} - \underset{(-.975)}{.33795}\, IP_{-4}$$

$$R^2 = .3072$$

AR3

$$CG^* = \underset{(2.074)}{115.55} + \underset{(.763)}{.2055}\, CG^*_{-1} + \underset{(.834)}{.2281}\, CG^*_{-4} + \underset{(.731)}{.2166}\, CG^*_{-5}$$

$$F^2 = .2736$$

AR4 $$M^* = \underset{(.4581)}{25.3567} + \underset{(2.261)}{.533119}\, M^*_{-1} + \underset{(.6862)}{.19306}\, M^*_{-3} + \underset{(1.614)}{.4290}\, M^*_{-4}$$

$$R^2 = .783$$

AR5 $$TD = \underset{(2.342)}{96.9527} + \underset{(.5972)}{.16688}\, TD_{-1} - \underset{(-1.448)}{.45299}\, TD_{-5} \qquad R^2 = .195$$

AR6 $$R = \underset{(1.822)}{.1097} - \underset{(-3.381)}{.8252}\, R_{-3} \qquad R^2 = .4879$$

AR7 $$IMP^* = \underset{(2.057)}{184.3869} - \underset{(-1.5646)}{.5065}\, IMP^*_{-2} - \underset{(-.7860)}{.2525}\, IMP^*_{-3} - \underset{(-.8103)}{.3080}\, IMP^*_{-4}$$

$$- \underset{(-1.356)}{.48112}\, IMP^*_{-5}$$

$$R^2 = .3375$$

AR8 $$RL = \underset{(.889)}{.240096} - \underset{(-.909)}{.2962}\, RL_{-2} - \underset{(-.557)}{.2006}\, RL_{-3} - \underset{(-1.058)}{.3928}\, RL_{-4} - \underset{(-1.078)}{.3892}\, RL_{-5}$$

$$R^2 = .3472$$

AR9 $$TDY^* = \underset{(.968)}{11.132} + \underset{(1.278)}{.4123}\, TDY^*_{-1} - \underset{(-.613)}{.2114}\, TDY^*_{-2} + \underset{(1.386)}{.4364}\, TDY^*_{-3}$$

$$- \underset{(-1.369)}{.4584}\, TDY^*_{-4} + \underset{(1.824)}{.6262}\, TDY^*_{-5}$$

$$R^2 = .426$$

AR10 $$NTDY^* = \underset{(2.408)}{31.219} + \underset{(1.117)}{.32798}\, NTDY^*_{-1} + \underset{(.688)}{.2350}\, NTDY^*_{-3} - \underset{(-1.545)}{.4941}\, NTDY^*_{-4}$$

$$R^2 = .276$$

AR11 $$CD^* = \underset{(.977)}{16.7595} + \underset{(2.493)}{.6187}\, CD^*_{-1} - \underset{(-.511)}{.1842}\, CD^*_{-3} - \underset{(-.964)}{.3168}\, CD^*_{-4}$$

$$R^2 = .4899$$

AR12 $$NCD^* = \underset{(1.298)}{48.391} + \underset{(.819)}{.2384}\, NCD^*_{-1} + \underset{(.244)}{.0766}\, NCD^*_{-2} - \underset{(-2.655)}{1.0057}\, NCD^*_{-3}$$

$$+ \underset{(1.288)}{.7787}\, NCD^*_{-4} + \underset{(1.845)}{1.18499}\, NCD^*_{-5}$$

$$R^2 = .863$$

AR13 YA $= \underset{(.735)}{244.336} - \underset{(-.6825)}{.3534}\ YA_{-1} - \underset{(-.944)}{.4822}\ YA_{-2}$ $R^2 = .2714$

AR14 YNA $= \underset{(1.386)}{830.6697} - \underset{(-.906)}{.9878}\ YNA_{-1} + \underset{(.944)}{1.5367}\ YNA_{-2} - \underset{(1.055)}{1.4994}\ YNA_{-3}$

$R^2 = .4938$

AR15 IA $= \underset{(5.528)}{85.751} - \underset{(-2.967)}{.87686}\ IA_{-2} - \underset{(-1.157)}{.2850}\ IA_{-3}$ $R^2 = .7857$

AR16 P $= \underset{(2.698)}{7.2771} - \underset{(-.584)}{.1624}\ P_{-1} - \underset{(.889)}{.25185}\ P_{-4}$ $R^2 = .078$

AR17 Y* $= \underset{(1.977)}{696.914} + \underset{(4.546)}{.93896}\ Y^*_{-3}$ $R^2 = .6326$

AR18 ZBP* $= \underset{(4.719)}{246.217} - \underset{(-2.574)}{.5262}\ ZBP^*_{-1} - \underset{(-6.156)}{1.4274}\ ZBP^*_{-2} - \underset{(-3.106)}{1.145}\ ZBP^*_{-3}$

$- \underset{(3.162)}{1.1114}\ ZBP^*_{-4} - \underset{(-3.648)}{1.2249}\ ZBP^*_{-5}$ $R^2 = .8625$

AR19 K $= \underset{(1.624)}{356.8206} + \underset{(2.593)}{.5998}\ K_{-1} + \underset{(.843)}{.2342}\ K_{-4} + \underset{(.455)}{.1392}\ K_{-5}$ $R^2 = .856$

AR20 KP $= \underset{(-.2119)}{-35.167} + \underset{(2.493)}{.5543}\ KP_{-1} + \underset{(.969)}{.2951}\ KP_{-4} + \underset{(1.535)}{.5290}\ KP_{-5}$ $R^2 = .801$

AR21 KNA $= \underset{(2.595)}{2509.596} - \underset{(-.267)}{.1451}\ KNA_{-1} - \underset{(-.526)}{.4991}\ KNA_{-2} + \underset{(+.833)}{.6137}\ KNA_{-3}$ $R^2 = .786$

AR22 U* $= \underset{(.656)}{30.1194} - \underset{(-1.897)}{.4829}\ U^*_{-1} + \underset{(1.834)}{.4736}\ U^*_{-2} + \underset{(3.497)}{.8376}\ U^*_{-3}$

$+ \underset{(1.001)}{.2668}\ U^*_{-4}$ $R^2 = .7057$

AR23 PNA $= \underset{(1.675)}{.0867} + \underset{(.646)}{.3742}\ PNA_{-1} - \underset{(-1.144)}{1.0206}\ PNA_{-2}$

$+ \underset{(.884)}{.6994}\ PNA_{-3}$ $R^2 = .4024$

AR24 F* $= \underset{(.059)}{2.3278} + \underset{(2.624)}{.8871}\ F^*_{-1} - \underset{(-1.160)}{.4789}\ F^*_{-2} + \underset{(.7397)}{.2714}\ F^*_{-3}$ $R^2 = .4091$

It would be observed that the fit obtained for individual equations in terms of R^2 shows substantial variation between equations, ranging from R^2s as high as .863 to as low as .078. Since these models directly predict changes in levels, levels themselves can be predicted by adding to previous years levels. We have used two options here. Static predictions for levels are generated by adding the predicted change to actual previous year's level of the dependent variable and dynamic predictions are obtained by successive integration with only 1969/70 actual levels of the dependent variables being used as starting points.

Although the prediction performance of these models would differ from equation to equation, in general they are expected to do better than the naive no-change or constant-change extrapolative models. As such they provide relatively better benchmark models than the ones used in Section 12.1. In view of the very limited size of the sample for some of the equations estimated with the truncated sample, and in view of the fact that we are constrained not to deviate from the specification of the original model whereas slight application of what Howrey et al (1974) have called 'tender loving care (TLC)' in terms of such deviations could improve the results, the forecasting results in this section should be liberally interpreted. The general experience regarding comparisons between macroeconometric model forecasts and extrapolative model forecasts, where the latter are interpreted as 'sparring partners', has been described by Granger and Newbold (1975) as one where the sparring partner normally outperforms the champion. As such if the forecasting performance of our model is nearly as well as the extrapolative benchmark we have used, it should be interpreted as a highly satisfactory situation.

In Table 12.7 we have reported inequality coefficients for static and dynamic forecasts of levels of variables for the main model as also those for extrapolative forecasts and a set of no-change benchmark forecasts. Compared to this last category both model-forecasts and autoregressive model forecasts nearly always do better. For the static predictions, model forecasts do better than the extrapolative forecasts for variables CP, IMP*, M*, TDY*, NTDY*, CD*, YA, YNA, IA, ZBP*, KP, KNA and K. For the dynamic predictions, this holds for variables CP, IMP*, TDY*, NTDY*, YNA, ZBP*, KP, KNA and K. Thus, as far as the prediction of levels of variables is concerned, the model performs in the pseudo-forecast period nearly as well as the autoregressive models if not better. In Table 12.8, we have compared predicted changes in levels of variable for model forecasts and extrapolative forecasts. For the static case, the model forecasts do better for variables CP, IP, IMP*, M*, NTDY*, CD*, NCD*, YA, YNA, IA, Y*, ZBP*, KP and KNA. This is so for nearly all these variables in the dynamic case also. Thus, on the whole, it can be said that the model is able to predict more closely the realizations for nearly fifty per cent of the variables under consideration. In view of the various difficulties associated with the estimation within a truncated sample period, we have listed earlier, we contend that the prediction performance of the model as revealed by Table 12.7 and 12.8 is highly satisfactory.

Table 12.7

Prediction Performance in Pseudo-Forecast Period:
Inequality Coefficients for Predicted Levels of Variables

Variable	Model Predictions		Autoregressive Benchmark model		'Naive' No Change Extrapolation
	Static	Dynamic	Static	Dynamic	
CP	.0366	.0364	.0737	.0678	.0450
IP	.1652	.2167	.0912	.0835	.1118
R	4.160	3.863	.1185	.1315	.0987
TD	.8689	.9723	.0539	.1778	.0867
IMP*	.1970	.1650	.2273	.3083	.2613
M*	.0398	.1621	.0476	.1332	.1117
CG*	.1641	.3518	.0648	.1182	.1169
RL	5.163	5.170	.1851	.3449	.1383
TDY*	.0303	.1863	.0802	.1872	.1402
NTDY*	.0425	.1161	.0960	.1827	.1317
CD*	.0786	.7843	.4762	.0711	.1745
NCD*	.1622	.3735	.0699	.1291	.1469
YA	.0659	.0679	.0672	.0486	.0650
YNA	.0125	.0153	.8480	.8607	.0307
IA	.1032	.1287	.1024	.0875	.0831
P	.1964	.3899	.1325	.2664	.1501
PNA	.3618	.7022	.0692	.0949	.1139
Y*	.1824	.3806	.0871	.1189	.1389
U*	.1020	.2533	.0765	.1558	.1142
ZBP*	.1628	.1687	.4896	.5578	.3908
F*	1.0215	2.539	.3265	.6358	.2488
KP	.0108	.0365	.0136	.0382	.0321
KNA	.0052	.0138	.0077	.0167	.0426
K	.0037	.0091	.0018	.0019	.0425

Table 12.8

Prediction Performance in Pseudo-Forecast Period: Inequality Coefficients for Predicted Changes in Levels

Variable	Model Predictions		Autoregressive Benchmark Model	
	Static	Dynamic	Static	Dynamic
CP	.9076	.9346	2.452	1.638
IP	1.087	1.118	1.298	.8142
R	11.593	11.577	.8665	1.201
TD	2.452	2.890	.4251	.6215
IMP*	.3728	.3653	.7391	.8696
M*	.2307	.4461	.3627	.4264
CG*	.7293	1.415	.6091	.5555
RL	10.086	11.665	1.527	1.335
TDY*	.2899	.6091	.2633	.5716
NTDY*	.4334	.4758	.6304	.7284
CD*	.4127	.4551	.6140	.7843
NCD*	.6158	1.125	.6493	.4759
YA	.9935	.9957	1.700	1.055
YNA	.2565	.3319	4.084	3.673
IA	1.241	1.219	1.576	1.240
P	.5499	.8581	.4401	.8818
PNA	1.144	2.053	.3649	.6060
Y*	.5244	1.025	.4344	.6271
U*	.4415	.5940	.8046	.6696
ZBP*	.4232	.5625	1.015	1.252
F*	2.030	2.732	1.616	1.312
KP	.1860	.4004	.1869	.4232
KNA	.0708	.0858	.1296	.1793
K	.0720	.0658	.0513	.0428

12.5 Summary

In this Chapter the forecasting performance of the model proposed in Chapter 11 has been analysed. This analysis has been done both with respect to the predictions within the sample period, and predictions in a 'pseudo' forecast period. In both cases static and dynamic predictions were used. It is seen that the model performs satisfactorily compared to autoregressive benchmark models.

Chapter 13

Policy Simulations and Expenditure Multipliers

Apart from forecasting, the model proposed in Chapter 11 can also be used for policy analysis. We have analysed the effects of policy changes on the endogenous variables by dynamic simulations of the model. Since the model is non-linear, the impact multiplier matrix cannot be directly derived by the inversion of the matrix of structural coefficients attached to the endogenous variables as would have been the case in a linear model.

The procedure we have adopted is to first hold all exogenous variables at their sample levels, and find out dynamic solutions to the model. These solutions which we may call control solutions, are then compared with similar solutions when one by one a step change in one or a combination of the exogenous policy variables or a step change in the autonomous component of an equation is introduced for all years in the simulation period. The difference between the simulated solution and the control solution is interpreted as being due to the policy change introduced.

Since dynamic solutions are sensitive to the choice of the initial year, we have first solved the system successively with initial years changing from 1961/2, 62/3 .... to 1973/74. These solutions use the first reading of the lagged endogenous variables from the actual levels of their respective previous years. On observations we find that 1964/65 provides a good starting point. Dynamic solutions closely emulate the realizations and a sample of 11 observations is provided for studying the time-profile of effects of policy changes. It is also a 'normal' year on *a priori* grounds. Years just before and after this are affected by the Indo-Pakistan Wars and may not be desirable as starting points.

In all the simulations we have solved for the deterministic part of the model keeping the stochastic terms at zero. For all these simulations, we have reported results as deviations from the control solutions for the following nine variables:

CP, IP, CG*, IMP*, YA, YNA, P, Y* and M*

These variables are of immediate interest to the policy maker. Results for all the 34 endogenous variables of the model are not reported in order to conserve space.

13.1 Fiscal Policy Changes

On the expenditure side we have introduced an increase of Rs 100.0 cr. in government investment expenditure (IG*) on its own in one case, and accompanied by a similar increase in government deficit financing in another. In another option we have introduced a step increase of Rs 100.0 cr. in government transfer payments.

On the revenue side, in one option, the basic and incremental tax rates for personal income taxes are increased one-by-one by prespecified amounts. In another option, the constant term in the equation for indirect taxes other than customs duties is reduced by Rs 100.0 These are some useful fiscal policy changes for consideration although the existence of many other interesting possibilities is readily admitted.

First, we consider increases in government investment expenditure. One variant here is where IG* is increased by Rs 100.0 cr. in all years from 1964/65 to 74/75 holding other exogenous variables at their sample levels. In terms of the model, an increase in IG* on its own implies that the entire increase is financed by a rise in government borrowing from the private sector initially. Later as tax-bases go up, part of the increase would be financed by rising tax-revenues. The impact of such an increase is summarised in Table 13.1.

As government borrowing increases,

private investment expenditure goes down as a consequence. As IG* goes up, capital stock increases but nominal income directly increase faster than output leading to a rise in the price level. Real disposable incomes go down because of this and because of a rise in tax-revenues which are related to nominal tax-bases. As a consequence real private consumption expenditure goes down. As long as agricultural prices are not allowed to deviate from their sample levels, investment in agriculture and agricultural output also show a movement in the negative direction. The expansionary effects of an increase in IG* on nominal income would be more if TR* which includes government interest payments to the private sector is allowed to increase. Also, since no wealth effect is directly incorporated in the private consumption function, a positive effect via this channel is excluded by the model.

A rise in government investment expenditure of Rs 100.0 cr. accompanied by a rise in government borrowing from the central bank (D*) of a similar amount implies that government borrowing from the private sector (ZBP*) does not increase initially and may go down subsequently if there is an expansionary effect on nominal tax-bases. The resultant impact of this deficit-financed increase in government investment expenditure is summarised in Table 13.2.

Consequent upon an increase in nominal government investment expenditure financed by borrowing from the Central Bank, nominal income increases on the one hand via the usual multiplier effects. Since a rise in D* leads to a rise in the unborrowed money reserves, money supply increases and short-term interest rate falls. This has a positive effect on real private investment, and the effect increases in magnitude

Table 13.1

Effects of a Sustained Increase in IG* of Rs 100.00 crores

Years beginning 1964/5	CP	IP	CG*	IMP*	YA	YNA	P	Y*	M*
0	-4.5078	-5.0691	15.380	23.6572	-3.7227	9.0273	3.244	499.277	-17.2773
1	-4.9376	-6.3782	27.996	19.8518	-10.1563	17.5273	3.479	558.719	-52.6289
2	-12.3594	-9.1802	35.355	29.8987	-18.7773	27.7383	2.931	482.895	-100.555
3	-19.4414	-12.5811	42.225	17.3789	-29.0703	36.8594	3.335	556.953	-121.969
4	-22.7344	-15.1023	47.006	16.1362	-39.4766	46.3047	3.233	553.074	-143.230
5	-29.3086	-17.6372	50.577	11.0669	-49.8633	55.1211	3.262	565.309	-159.043
6	-24.5938	-16.5710	53.234	4.0378	-58.3359	62.0547	3.171	579.125	-169.188
7	-22.3281	-15.8960	55.194	-2.5042	-65.0430	67.5234	3.095	590.855	-165.828
8	-27.5117	-16.0610	55.891	-7.7791	-70.4570	71.9805	2.978	574.031	-156.566
9	-34.3047	-16.3318	53.242	-3.2249	-74.5508	75.2969	2.451	457.359	-156.219
10	-35.8906	-15.8040	47.891	1.1321	-77.2852	78.0898	1.829	326.938	-162.293
Average	-21.29	-13.33	43.99	9.968	-45.16	49.77	3.00	522.23	-127.71

Table 13.2

Effects of a Sustained Increase in IG* and D* of Rs 100.00 cr. each

Year starting 1964/5	CP	IP	CG*	IMP*	YA	YNA	P	Y*	M*
0	-4.824	.7688	16.517	25.360	-3.984	9.672	3.484	536.254	132.094
1	-5.461	6.655	31.624	23.459	-11.262	19.602	4.053	651.012	246.035
2	-13.895	10.893	42.046	38.739	-21.664	32.625	3.699	611.270	340.727
3	-22.254	13.809	53.445	26.060	-34.945	45.379	4.558	766.535	466.637
4	-25.677	18.682	63.320	27.337	-49.246	59.695	4.750	822.523	582.461
5	-33.258	23.137	72.910	22.979	-64.555	74.332	5.178	915.555	705.727
6	-25.707	29.952	82.991	15.356	-77.930	87.102	5.553	1037.890	846.965
7	-20.695	34.765	93.878	65.369	-89.418	98.320	6.003	1177.830	1011.52
8	-26.449	36.250	105.121	-1.712	-99.770	108.688	6.500	1309.200	1195.18
9	-35.172	35.443	112.184	11.847	-108.766	117.750	6.146	1252.160	1361.67
10	-37.731	34.628	114.457	29.743	-115.859	126.480	5.394	1126.060	1496.65
Average	-22.83	22.27	71.68	20.52	-60.67	70.88	5.029	927.85	762.24

Table 13.3

Effects of a Sustained Increase in TR* by Rs 100.0 cr.

Year starting 1964/5	CP	IP	CG*	IMP*	YA	YNA	P	Y*	M*
0	51.328	5.602	14.997	23.718	-3.746	-3.711	3.267	485.82	-16.08
1	40.941	1.721	27.551	20.778	-10.398	-7.516	3.645	551.41	-51.77
2	26.777	-2.873	34.951	32.372	-19.551	-8.961	3.168	476.17	-103.54
3	10.797	-8.499	42.150	19.917	-30.785	-10.969	3.725	560.56	-128.53
4	.8125	-12.796	47.239	19.341	-42.433	-12.816	3.713	558.07	-154.69
5	-13.438	-17.225	51.207	14.342	-54.375	-15.098	3.855	576.36	-175.89
6	-25.336	-19.735	54.331	7.566	-64.465	-18.751	3.976	596.07	-192.40
7	-37.383	-22.143	56.935	.5278	-72.824	-23.297	4.134	619.28	-194.43
8	-54.481	-24.720	58.598	-5.594	-80.035	-27.746	4.294	621.36	-189.95
9	-71.406	-26.991	56.422	2.395	-86.016	-32.109	3.921	491.49	-200.67
10	-84.328	-28.856	50.339	12.671	-90.543	-36.008	3.375	318.06	-227.95
Average	-14.16	-14.23	44.97	13.46	-50.47	-17.91	3.73	532.24	-148.72

over time. The increase in both private and government investment together increases capital-stock and real output, agricultural and non-agricultural taken together, increases. But the increase in output is not sufficient to match the increase in nominal demand, and the main result of this deficit-financed increase is on the price-level which moves up. As long as agricultural prices are held at their sample levels, ratio of agricultural to non-agricultural income goes down, real private consumption expenditure shows a decline. On the other hand, government consumption expenditure, which is linked to the nominal tax-bases shows an increase.

An increase in transfer payments of Rs 100.0 cr. is introduced as a third option. The results are summarised in Table 13.3. If transfer payments to the private sector are increased financed by borrowing from the private sector, the net effects indicate that money incomes increase but the real income is not able to increase. The reason is an upward movement in prices which implies a lower government investment in real terms if IG* is held at its sample level in nominal terms. The initial impact then is on the prices. Private consumption and investment expenditures initially go up because of the direct increase in disposable incomes. But as the rate of interest rises because of the build up of financial assets due to an increase in ZBP*, private investment also falls and real output falls further and prices increase more. Ultimately, real disposable incomes begin to fall and private consumption expenditure also falls. The net effect on money supply is negative despite the rise in short term interest rate. Increase in nominal incomes has a positive effect on imports. With exports held at sample levels, foreign exchange reserves are depleted and unborrowed money reserves go down leading to a fall in the money-supply. Since tax-bases in nominal terms, like Y* and IMP* show an increase, total tax-revenues and consequently government consumption expenditure also shows an increase. The magnitude of this rise increases over time until the last two years where the positive effects on Y^* etc. become of a smaller magnitude.

On the revenue side, effects of changes in what we have called the incremental and the basic tax-rates for the tax on non-corporate incomes have been separately examined. First, a step increase in ITR of 0.25 is introduced. The results are given in Table 13.4.

An increase in the incremental tax-rate implies that the tax structure becomes more progressive and incomes at higher levels are taxed at relatively higher rates. The impact of such an increase raises tax-revenues from non-corporate incomes and leads to a fall in personal disposable incomes. The initial impact is a fall in private consumption and investment expenditures. Money incomes also fall. Over time the magnitude of the impact on consumption and investment from the private sector goes down and ultimately they start to increase. Under the assumption that agricultural prices are not allowed to deviate from their sample levels, agricultural output increases, and non-agricultural output increases first and declines subsequently. Government consumption expenditure shows a rise initially. The magnitude of this rise increases over time.

Because of the rise in tax-revenues, government borrowing from the private sector declines. Financial assets are reduced, demand for money goes down lowering the rate of interest and the money supply is increased. The net impact is a lower short-term rate of interest and private investment begins to increase when the long-run rate responds. Most of the increase in investment goes towards the agricultural sector as with unresponsive prices relative to decline in the non-agricultural prices, it becomes more attractive. The magnitude of the impact on real agricultural output increases, and non-agricultural output actually falls in the later stages.

Table 13.4

Effects of a Sustained Increase in ITR of .25

Years starting 1964/5	CP	IP	CG*	IMP*	YA	YNA	P	Y*	M*
0	-7.5078	-1.052	2.1580	-2.3086	.3711	.2695	-.3080	-46.0117	1.5117
1	-6.6914	-.8372	3.9519	-1.3999	.9102	.3984	-.2580	-39.0313	4.4961
2	-5.6133	-.5962	5.6309	-1.5603	1.5195	.3047	-.1740	-25.3867	7.1992
3	-4.6055	-.3420	6.9678	-.4761	2.1055	.2148	-.1600	-22.5703	7.7266
4	-4.3086	-.2612	8.1172	-.1519	2.6094	.9766	-.1250	-16.5859	7.8789
5	-3.7305	-.1599	9.0920	.2099	3.0078	0.0	-.1010	-12.0586	7.5664
6	-3.0781	-.0952	9.9300	.5139	3.2852	-.0703	-.0780	-7.7734	6.9414
7	-2.4180	-.0339	10.6611	.6921	3.4570	-.1133	-.0570	-3.6289	5.9531
8	-1.4102	.0881	11.3320	.7561	3.5664	-.1406	-.0390	1.3281	4.8906
9	-.3945	.2173	11.9844	.6850	3.6133	-.1484	-.0200	7.3867	3.9492
10	.2500	.2798	12.6055	.6868	3.6133	-.1406	-.0049	1.2500	2.9570
Average	-3.59	-.254	8.403	.214	2.619	.061	-.120	-13.80	5.55

Table 13.5

Effects of a Sustained Increase in BTR of 1.00

Years starting 1964/5	CP	IP	CG*	IMP*	YA	YNA	P	Y*	M*
0	2.9688	.4160	-.8540	.9111	-.1445	-.1094	.1230	18.238	-.6016
1	2.6484	.3301	-1.565	.5527	-.3594	-.1602	.1030	15.473	1.773
2	2.2227	.2358	-2.229	.6177	-.5938	-.1172	.0690	10.058	2.8477
3	1.8203	.1359	-2.758	.1880	-.8359	-.0898	.0630	8.949	3.0547
4	1.7070	.1038	-3.214	.0601	-1.0273	-.0429	.0490	6.574	3.1172
5	1.4805	.0649	-3.599	-.0830	-1.1914	0.0	.0399	4.781	2.9883
6	1.2227	.0388	-3.931	-.2029	-1.3008	.0234	.0299	3.090	2.7539
7	.9609	.0149	-4.220	-.2739	-1.3711	.0430	.0299	1.441	2.3594
8	.5703	-.0337	-4.484	-.2981	-1.4102	.0508	.0150	-.5117	1.9414
9	.1602	-.0859	-4.746	-.2720	-1.4258	.0625	.0081	-2.906	1.5625
10	-.0898	-.1108	-4.988	-.2719	-1.4336	.0586	.0020	-4.875	1.1679
Average	1.425	.1008	-3.326	-.084	-1.009	-.026	.048	5.483	2.088

Table 13.6

Effects of a Sustained Reduction in Indirect Taxes (NCD*) of Rs 100.00 cr.

Years starting 1964/5	CP	IP	CG*	IMP*	YA	YNA	P	Y*	M*
0	-9.773	-6.033	-17.121	19.957	-3.270	-5.102	2.836	309.910	-14.137
1	-14.754	-7.573	-31.323	12.646	-8.184	-8.938	2.462	256.086	-41.551
2	-21.981	-9.435	-44.860	15.117	-13.875	-10.609	1.729	137.020	-67.750
3	-29.277	-11.635	-55.551	5.401	-19.738	-12.234	1.667	119.379	-74.094
4	-34.695	-13.237	-64.835	2.818	-24.910	-13.645	1.384	68.203	-77.758
5	-40.555	-14.541	-72.727	-0.472	-29.375	-15.133	1.209	31.344	-77.367
6	-43.239	-14.683	-79.543	-3.339	-32.648	-16.887	1.053	-3.625	-74.254
7	-45.930	-14.800	-85.492	-5.320	-34.988	-18.742	.928	-36.809	-66.773
8	-49.711	-15.000	-90.984	-6.359	-36.666	-20.504	.826	-77.371	-58.191
9	-53.102	-15.125	-96.700	-4.863	-37.773	-22.207	.667	-140.660	-52.133
10	-55.063	-15.093	-102.566	-3.439	-38.434	-23.781	.518	-204.063	-48.031
Average	-36.19	-12.47	-67.428	2.93	-25.44	-15.25	1.389	41.765	-59.276

In the second option here, the basic tax-rate (BTR) of the tax on non-corporate incomes is increased by 1.00 over and above its sample levels. This implies that marginal tax-rates for all brackets of income go up by this amount. The resultant impact is summarised in Table 13.5. There is a positive impact on nominal incomes initially but this declines in magnitude over time and in the end becomes negative. Private consumption and investment expenditure shows a positive effect in real terms despite a net rise in the price level. Real output, however, declines in the net although the non-agricultural output shows an increase in the later years. It should be recalled that in the relevant tax equation we have permitted a negative sign attached to the term BTR on the ground that as the basic tax rate goes up, tax-revenue may actually decline because of a rise in tax evasion and avoidance. The above results, therefore, can only be relied upon to the extent that this hypothesis is correct.

A third version of tax changes is introduced by reducing domestic indirect taxes (NCD*) by a constant amount equal to Rs 100.0 cr. This is done by reducing the constant term of the relevant equation. Since there is no change in the effective tax-rate, this reduction may be interpreted as an increase in autonomous exemptions etc. The results are summarised in Table 13.6.

The immediate impact of such a reduction is on government tax-revenues which fall. Since government consumption responds to this but only slowly, initially the government has to compensate for loss of revenues by borrowing from the public. As the private sector's financial assets increase, the rates of interest go up and private investment falls. Real output falls as a consequence. Although initially there is a beneficial effect on nominal incomes, real output does not increase and prices increase as a result. Real disposable incomes show a decline leading to a fall in private expenditures. Over time government consumption expenditure falls in increasing magnitudes, and government borrowing requirements are reduced. The rate at which real output was falling is halted, and price increases become smaller and smaller in magnitude which is also the case for nominal income.

13.2 Monetary Policy Changes

The three monetary policy options considered here relate to (i) a sustained unit increase in the discount rate, (ii) a sustained increase of 5.00 in the net liquidity ratio, and (iii) a sustained rise of Rs 100.0 cr. in government borrowing from the Central Bank. The resulting changes in nine endogenous variables being considered here over and above their control solutions have been respectively summarised in Tables 13.7, 13.8 and 13.9.

The effects of an increase in the discount-rate (RD) work through the money supply function. An increase in RD leads to a fall in the money-supply which, other things remaining the same, leads to a rise in the short and long-term rates of interest. Private investment and non-agricultural output decrease. Non-agricultural prices go up, and as long as agricultural prices can be held at their sample levels, investment in agriculture, and agricultural output increases. The net effect on money income of a rise in the discount rate is shown to be negative. Real private consumption, however, increases, as long as the fall in money income is smaller than the fall in the price-level, and because of the positive effects on the ratio of real agricultural to non-agricultural income.

Table 13.7

Effects of a Sustained Increase in RD by 1.00

Years starting 1964/5	CP	IP	CG*	IMP*	YA	YNA	P	Y*	M*
0	.0703	-1.117	-.2099	-.3318	.0547	-.1289	-.0440	-6.8086	-27.945
1	.0898	-1.271	-.4289	-.3472	.0160	-.2734	-.0560	-9.1094	-28.176
2	.2148	-1.234	-.5710	-.5454	.3047	-.4570	-.0510	-8.3242	-27.629
3	.3438	-1.155	-.7148	-.3472	.4922	-.6289	-.0599	-10.0195	-27.355
4	.4102	-1.144	-.8210	-.3308	.6875	-.8047	-.0599	-10.1445	-26.738
5	.5469	-1.114	-.9121	-.2529	.8828	-.9766	-.0620	-10.7656	-26.379
6	.4570	-1.081	-1.002	-.1450	1.0546	-1.1211	-.0650	-11.8047	-26.477
7	.4063	-1.025	-1.0932	-.0281	1.1914	-1.2383	-.0670	-12.9023	-26.769
8	.5000	-.925	-1.1797	.0720	1.3164	-1.3398	-.0710	-13.8359	-27.309
9	.6133	-.806	-1.2227	-.0710	1.4179	-1.4258	-.0649	-12.6602	-27.613
10	.6484	-.713	-1.1953	-.2441	1.4883	-1.5000	-.0551	-10.8125	-27.527
Average	.391	-1.053	-.850	-.234	.823	-.8995	-.0596	-10.65	-27.27

Table 13.8

Effects of a Sustained Rise in NLR of 5.00

Years starting 1964/5	CP	IP	CG*	IMP*	YA	YNA	P	Y*	M*
0	-.0234	.4099	.0771	.1211	-.0195	.0469	.0170	2.496	10.234
1	-.0313	.4690	.1570	.1279	-.0547	.1016	.0210	3.363	10.426
2	-.0781	.4581	.2111	.2019	-.1094	.1716	.0190	3.074	10.223
3	-.1328	.4280	.2639	.1281	-.1797	.2305	.0220	3.700	10.129
4	-.1563	.4226	.3032	.1223	-.2500	.3008	.0220	3.750	9.891
5	-.1992	.4131	.3372	.0940	-.3281	.3594	.0229	3.984	9.770
6	-.1680	.4001	.3699	.0530	-.3906	.4102	.0230	4.371	9.797
7	-.1484	.3801	.4050	.0100	-.4414	.4570	.0250	4.770	9.990
8	-.1826	.3420	.4375	.0261	-.4844	.4961	.0260	5.117	10.098
9	-.2266	.2981	.4492	.0261	-.5195	.5273	.0242	4.691	10.021
10	-.2344	.2629	.4492	.0909	-.5547	.5625	.0208	4.000	10.018
Average	-.144	.389	.315	.086	-.303	.333	.022	3.94	10.08

Table 13.9

Effects of a Sustained Increase in D* by Rs 100.00 cr.

Years starting 1964/5	CP	IP	CG*	IMP*	YA	YNA	P	Y*	M*
0	-.3438	5.9348	1.119	1.761	-.2813	.6797	.2360	36.258	148.68
1	-.6875	13.2319	3.570	3.741	-1.1797	2.1875	.5650	91.035	297.21
2	-1.9765	20.2861	6.579	9.052	-3.0664	5.1289	.7549	125.598	439.32
3	-3.7695	26.5842	11.029	8.962	-6.2500	8.9297	1.2050	205.264	585.82
4	-4.4375	33.9517	16.022	11.553	-10.3789	14.0078	1.4960	263.587	722.03
5	-6.2031	40.854	21.927	12.310	-15.6016	20.0703	1.8940	342.814	860.22
6	-3.5508	46.561	29.254	11.645	-20.7578	26.1133	2.3630	450.911	1011.20
7	-1.0078	50.619	38.104	9.262	-25.7500	32.0117	2.8960	579.040	1172.04
8	-1.9336	52.128	48.597	6.147	-30.8555	38.0313	3.5190	727.379	1346.81
9	-3.9531	51.473	58.225	15.117	-35.8750	43.8555	3.6909	785.603	1512.34
10	-4.6484	50.083	65.841	28.523	-40.2852	49.8359	3.5569	788.541	1654.94
Average	-2.956	35.61	27.298	10.734	-17.298	21.896	2.016	399.59	886.42

The increase in the net liquidity ratio (NLR) causes the money supply to rise. This is because banks are better able to face uncertainty and to substitute interest-earning liquid assets for excess reserves when their liquid assets relative to the deposits rise. Also, with a higher net liquidity ratio, banks are left with a greater surplus of liquid assets over the limit prescribed by the RBI which may then be converted into interest earning assets. For these reasons a positive impact on money supply of an increase in the net liquidity ratio has been postulated. This option, therefore, permits an expansionary monetary policy.

As money supply increases, the short-term, and often a lag, the long-term interest rates go down, private investment is positively affected, non-agricultural output goes up and non-agricultural prices go down. To the extent agricultural prices are not allowed to respond, investment in the agricultural sector goes down, and consequently also its output. The net impact on money income is positive. Money imports also show a positive response. Prices also rise.

Over time, the impact on money-income rises in magnitude at first, then the strength of the impact declines, and then rises again.

An increase in the government borrowing from the central bank, without a corresponding increase in any of the other exogenous variables implies, in terms of the model we have got, that the government borrowing from the private sector goes down by a similar amount initially.

The first impacts of these changes are reflected on the money-supply which goes up via an increase in the unborrowed money reserves. Demand for money goes down because of a negative impact on the financial assets of the private sector. The net effect is a reduction in the short and often a lag, in the long-term interest rates. This increases private investment expenditure. There is a positive impact on the non-agricultural output and in the absence of a response in agricultural prices, a negative impact on agricultural output as investment is directed towards the non-agricultural sector.

The net impact on money income is positive. There is also a positive impact on the price-level. Money incomes increase faster than real income and the price-level goes up because of an increase in the non-agricultural prices.

13.3 Other Exogenous Changes

One interesting possibility is to let A* increase by Rs 100.0 in all years. This variable is partially under the control of the rest of the world. The option of net aid receipts (A*) going up by 100.00 with government investment expenditure levels held at the sample values means that the government reduces its liabilities to the private sector by the amount by which aid increases. The net effect is shown to be beneficial for real output although money incomes show a decline. As aid increases, imports increase via the FS* variable and there is negative impact on money income. Since for any given real output prices would then decline, real disposable incomes increase leading actually to a rise in private investment and real output. Real consumption expenditure also increases. Since government consumption expenditure is associated with nominal tax-revenues and nominal incomes, it shows a decline. As the financial assets of the private sector decline, the short-term interest rate and subsequently the long-term interest rate fall, leading to a rising magnitude of private investment over time. The money-supply is also shown to be positive as unborrowed reserves directly increase because of the rise in aid levels. Table 13.10 summarises these results.

Table 13.10

Effects of a Sustained Rise in A* by Rs 100.00 cr

Years starting 1964/5	CP	IP	CG*	IMP*	YA	YNA	P	Y*	M*
0	2.992	6.581	-1.761	16.104	.738	2.027	-.616	-86.262	122.164
1	6.867	13.663	-4.336	30.211	2.605	5.668	-1.080	-153.633	227.203
2	12.222	20.226	-6.141	36.828	5.801	9.512	-1.165	-162.148	325.191
3	19.402	26.504	-8.534	50.009	10.406	14.238	-1.571	-220.621	401.176
4	26.684	32.782	-10.573	56.922	15.746	19.535	-1.737	-242.281	463.508
5	35.418	38.345	-12.433	64.916	21.746	25.449	-1.937	-268.305	516.984
6	43.570	41.438	-13.961	73.188	27.082	32.254	-2.118	-287.621	567.250
7	51.510	43.514	-15.312	81.024	31.715	39.555	-2.312	-308.539	604.508
8	60.088	44.185	-16.160	88.038	35.898	46.523	-2.484	-315.105	635.820
9	67.914	43.709	-14.707	86.231	39.496	52.969	-2.324	-240.152	673.320
10	75.334	43.285	-10.801	82.060	42.316	58.781	-2.068	-134.250	716.098
Average	36.55	32.20	-10.429	60.50	21.23	27.86	-1.765	-219.9	477.6

13.4 Long-Term Elasticities of Selected Policy Changes

Although in the preceding sections we have considered the time profile of the magnitude of effects of specified policy changes, it is necessary to consider elasticities for making comparisons *vis a vis* the efficacy of different policy changes in affecting certain objective variables. For this purpose we have considered elasticities calculated with the help of averages over the eleven year period from 1964/5 to 1974/5 over which the policy simulations have been done and interpreted these as long-term elasticities. These are calculated with reference to two target variables, viz., income Y* and prices, P. The policy changes relate to government investment expenditure (IG*), government transfer payments (TR*), incremental and basic tax-rates (BTR and ITR). These are all fiscal policy instruments. We also consider the discount-rate (RD), the net liquidity ratio (NLR) and deficit-financing (D*) which are interpreted as monetary policy instruments. Net aid levels (A*) are also considered as a policy instrument partially under the control of the rest of the world.

Elasticities are calculated by dividing the average impact of change in income (or the price-level) as a proportion of average level of income (or prices) in the control solution by the average change in the policy instrument as a proportion of its average level in the simulation period. The relevent results are given in Table 13.11.

Table 13.11

Effects on Income (Y*) and Prices (P): Long-Term Impact and Elasticities for Selected Policy Changes

Policy Variable	Average level of policy variable	Average increment in policy variable	Impact on Prices	Price elasticities *	Impact on Income	Income elasticities
IG*	3002.36	100.0	3.00	0.5135	522.23	0.4224
TR*	404.0	100.0	3.73	.0859	532.24	.0579
ITR	2.51	0.25	-.120	(-).00687	-13.80	(-).00373
BTR	20.17	1.00	.048	.00552	5.483	.00298
RD	5.974	1.00	-.6558	(-).0223	-117.187	(-).01886
NLR	26.876	5.00	.022	.00067	3.94	.00057
D*	1192.55	100.0	2.016	.1371	399.59	.01272
A*	475.55	100.0	-1.765	(-).0479	-219.9	(-).0282

* with premultiplied signs

In affecting the price-level, the more effective instruments seem to be government investment expenditure and deficit financing. These variables are relatively more effective in affecting nominal incomes as well. Tax-rate variations have relatively smaller impact on the system. On the whole expenditure instruments of fiscal policy have higher elasticities than those of monetary policy instruments.

13.5 Expenditure Multipliers

The policy analysis of the previous sections can be supplemented by an analysis of expenditure multipliers. These have been calculated with reference to unit increases in government investment expenditure (IG*), exports (EXP*) and in the autonomous components of CP, IP, and CG*. Similarly, multipliers for a unit reduction in the autonomous component of imports (IMP*) have also been calculated. The procedure we have adopted is to introduce a step increase of 100.00 in nominal terms in the two exogenous variables IG* and IMP*. For the endogenous variables, CP, IP, CG* and IMP*, the constant terms were interpreted as auxiliary variables and to make the multipliers comparable, again step changes of 100.00 in nominal terms were introduced. The resulting changes in nominal income Y* which is GDP at market prices, and in real income (YA + YNA) which is GDP at factor cost at constant 1960/1 prices were calculated by taking the deviations in each case from the control solutions described in the previous section. Effects relative to unit changes are then obtained by dividing these deviations by 100.00.

The time profile of the multipliers beginning with year 1964/65 from which the control solution starts is presented in Table 13.12. This Table relates to the impact on nominal income of unit increases in current Rs in IG*, EXP*, CP, IP, CG* and a unit decline in current Rs in IMP*.

Table 13.12

Expenditure Multipliers for Nominal Income

Year beginning 1964/5	Effects on Y* of sustained unit changes in					
	IG*	EXP*	CP*	IP*	CG*	IMP*
0	4.993	3.707	5.100	4.993	5.100	5.990
1	5.587	3.819	5.856	5.704	9.549	5.906
2	4.829	3.124	5.120	5.056	10.637	4.533
3	5.570	3.464	6.130	5.999	14.644	4.889
4	5.531	3.289	6.180	6.109	16.314	4.457
5	5.653	3.278	6.492	6.443	18.334	4.290
6	5.791	3.351	6.883	6.911	20.290	4.235
7	5.909	3.449	7.346	7.432	22.307	4.220
8	5.740	3.483	7.649	7.793	23.743	4.096
9	4.574	2.898	6.389	6.964	20.679	3.099
10	3.269	2.112	4.510	5.816	15.722	1.848
Average	5.222	3.270	6.150	6.293	16.120	4.324

The average impact over a period of eleven years, which can be interpreted as long-term multipliers, indicates that a high multiplier is obtained if there is a step increase in private investment expenditure. Ranked according to magnitude the multipliers for CP*, IG*, IMP*, and EXP* follow suit. The highest multipliers are obtained for increases in government consumption expenditures. Also, import multipliers are higher than export multipliers indicating that a reduction in imports is better than an increase in exports of a similar magnitude as far as nominal income is concerned.

The time profiles of the impact multipliers are also of interest. The multipliers for government consumption expenditure are initially high but undergo a dramatic decline after two years and later rise and settle down at a relatively low level. All multipliers show a rise in the beginning and a decline near the end of the eleven year period. The time profiles of these multipliers have been plotted in Fig. 13.1.

Table 13.13

Expenditure Multipliers for Real Income

Year beginning 1964/5	Effects on (YA + YNA) of sustained unit changes in					
	IG*	EXP*	CP*	IP*	CG*	IMP*
0	.0530	-.0168	-.1022	.0530	-.1022	-.1116
1	.0737	-.1275	-.2348	.0738	-.3054	-.2298
2	.0896	-.1811	-.3673	.0902	-.5552	-.3274
3	.0779	-.2407	-.5275	.0778	-.8988	-.4325
4	.0683	-.2923	-.6906	.0673	-1.281	-.5243
5	.0526	-.3407	-.8614	.0500	-1.709	-.6093
6	.0372	-.3820	-1.026	.0327	-2.143	-.6817
7	.0248	-.4163	-1.180	.0180	-2.569	-.7421
8	.0152	-.4443	-1.319	.0059	-2.965	-.7914
9	.0075	-.4672	-1.443	-.0050	-3.326	-.8319
10	.0080	-.4822	-1.543	-.0053	-3.628	-.8601
Average	.0462	-.3124	-.8449	.0417	-1.7711	-.5584

For purposes of comparison we have also calculated the impact on real output given by the sum of agricultural and non-agricultural outputs of autonomous expenditure changes similar to those in Table 13.12. These results are given in Table 13.13. We observe that most of the positive effects on nominal income as given in Table 13.12 must have been due to price rises. In terms of the impact on real output, only government investment and autonomous changes in private investment seem to have a positive effect with the former multiplier slightly bigger than the latter in the long run. For all other expenditure categories the multipliers are negative. The largest such multipliers relate to government consumption expenditure. It can be seen that an autonomous increase in government consumption expenditure has a large impact on nominal incomes via an increase in prices and has a large detrimental effect on growth of real output. The time profiles of these impact multipliers can be observed from Fig.13.2.

Autonomous Expenditure Multipliers of Real Income (YA + YNA)

Effects on Real Income of sustained unit increments in IG*(1), CP*(2), IP*(3), CG*(5), EXP*(6) and decrease in IMP*(4).

Years should be read as 64-65 for 65, 65-66 for 66 etc.

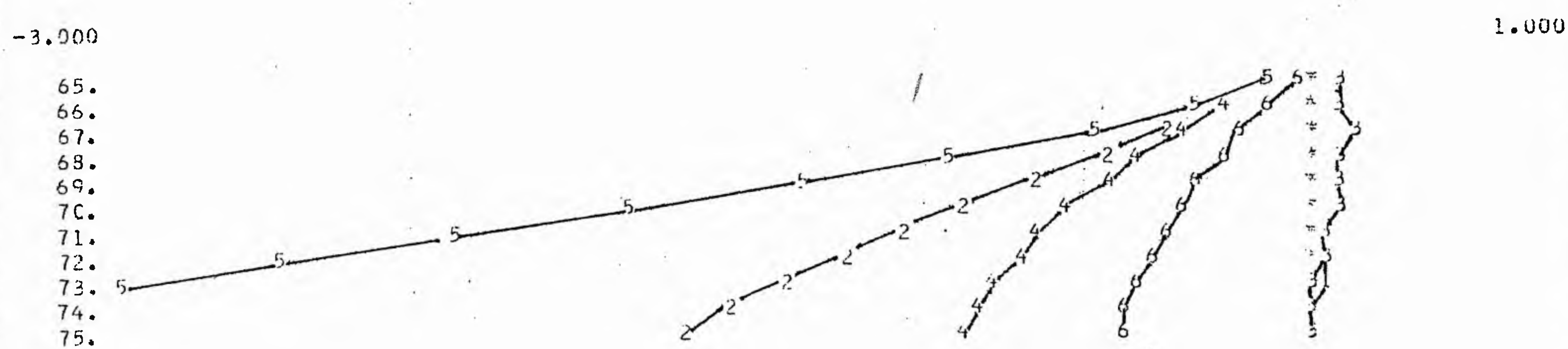


Fig. 13.2

Autonomous Expenditure Multipliers of Nominal Income (Y*)

Effects on Nominal Income of sustained unit increments in the autonomous components of IG*(1), CP*(2), IP*(3), CG*(5), EXP*(6) and decrease in IMP*(4).

Years should be read as 64-65 for 65, 65-66 for 66 etc.

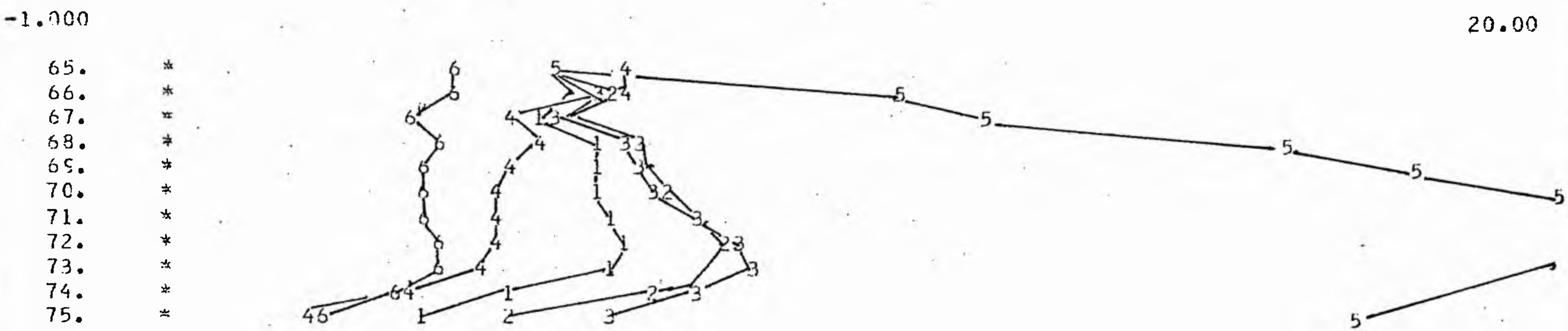


Fig. 13.1

For both government and private investment multipliers there is a rise initially and subsequently the magnitude of the multiplier falls. In the latter case it actually becomes negative in the end.

13.6 Summary

The discussion contained in the preceding pages can be summarised from the view point of (i) direction and magnitude of effects of alternative policy profiles, (ii) the efficacy of different instruments in affecting prices and nominal incomes from the viewpoint of elasticities, and (iii) the multiplier effects of increases in autonomous expenditures of different types on nominal and real incomes. We take up these aspects of policy analysis in turn.

A positive impact on prices occurs following increases in government investment expenditure, whether financed by increases in deficit or by borrowing from the private sector. Prices also increase if government transfer payments increase, if there is a rise in the net liquidity ratio indicating an expansionary monetary policy, if indirect taxes are reduced encouraging private spending, and if government deficit financing increases on its own. Prices fall if the discount rate increases indicating a contractionary monetary policy or if there is an increase in the progressivity of income tax indicated by a rise in variable ITR. Real output in the aggregate is positively affected by a rise in IG*, financed in either of the two ways considered. It is also positively affected by a rise in ITR although the magnitude is very small. Increases in NLR, D*, and A*, also have beneficial effects on real output as a whole, although its distribution between agricultural and non-agricultural sectors is affected differently in all the above cases. Real output falls in the aggregate if transfer payments increase, if the basic tax rate increases, and if there is a reduction in the autonomous component of indirect taxes. Changes in tax-rates and the discount rate have very small effects on output in terms of magnitude.

In comparing the efficacy of different instruments within a long-term perspective from the viewpoint of elasticities, the ranking according to the magnitudes of elasticities in affecting the price-level would be in the following order:

IG*, D*, TR*, A*, RD, ITR, NLR, BTR

A similar ranking in affecting nominal incomes would be in the following order:

IG*, TR*, A*, RD, D*, ITR, BTR, NLR.

Thus expenditure instruments seem to be most effective and tax-rate and discount-rate type of instrumentsdo not seem to be very effective.

From the viewpoint of multipliers for a similar increase in nominal terms in the autonomous part of different type of expenditures provides a ranking in decreasing magnitude of long-term multipliers in the following order:

CG*, IP*, CP*, IG*, IMP*, EXP*.

Although increases in government consumption expenditure have/the highest multiplier effects for nominal income, most of their impact comes from rising prices. Its impact is negative when real income effects are considered. Real income multipliers reported below are negative for the last four variables. The ranking is as follows:

IG*, IP*, EXP*, IMP*, CP*, CG*.

In very broad terms, the conclusion seems to be that rather than playing around with tax-rates and discount-rates, the government should primarily look into the expenditure variables under its control and the level of deficit financing it permits. Furthermore, rather than increasing all types of expenditures, it must look into the proportion in which its expenditure is divided between consumption and investment activities, increases in the former being definitely detrimental to the interests of the economy.

Chapter 14

Conditional Forecasts and Conclusions

The model proposed in Chapter 11 has been used for generating conditional predictions for the five year period from 1975/6 to 1979/80. The version of the model used for this purpose is the one labelled model A in Chapter 12.

Even though we have used a prediction period of five years, since the model is primarily for short-to-medium term forecasting, a greater degree of reliance may only be attached to predictions for the first two-to-three years. Alternative forecasts have been generated under alternative assumptions about the future path of the exogenous variables. Although we have used a number of profiles of the future path of both policy and non-policy exogenous variables, we have not exhausted by any means all the interesting options.

The exercise presented here should be interpreted as highly tentative. It does go on to show, however, that the model predicts reasonably acceptable results in terms of direction and magnitude of changes.

14.1 Exogenous Variables in the Forecast Period

In order to be able to forecast, one must independently obtain or predict using either formal or informal models, values of the exogenous variables for the prediction period. We have divided the exogenous variables into two categories. For one category which consists primarily of fiscal and monetary policy variables, we use various combinations and alternative profiles. For most of the remaining exogenous variables we have adopted simple autoregressive models to predict their values. The autoregressive models are estimated over the sample period 1949-50 to 1974-75 in terms of first differences for variables EXP*, PM, A*, IG* and TR*, and over the sample period 1960-61 to 1974-75 for PA for which information for this smaller sample was used in the rest of the model. A maximum of five lags is permitted in the first case and a maximum of three lags in the second case. Thus we lose six observations, one for differencing and five for lags in the first case and four observations in the second case.

Changes in levels are dynamically predicted for 1975/6 to 1979/80 and levels are then predicted by successive integration with actual 1974-75 levels used for initiating the process.

In choosing the autoregressive models those lagged terms are dropped which do not contribute significantly towards explaining the variation in the dependent variable. The estimated models in terms of changes are given below.

$$EXP^* = \underset{(.6739)}{32.1886} + \underset{(3.194)}{.8874}\, EXP^*_{-1} + \underset{(.6884)}{.2369}\, EXP^*_{-2}$$

$$R^2 = .4706, \quad \bar{R}^2 = .4083$$

$$PM = \underset{(.884)}{6.1504} + \underset{(2.368)}{.7819}\, PM_{-1} + \underset{(.9996)}{.4827}\, PM_{-2}$$

$$R^2 = .2561, \quad \bar{R}^2 = .1686$$

$$A^* = \underset{(1.2157)}{7.85999} - \underset{(-1.237)}{.2958}\, A^*_{-1} + \underset{(.881)}{.1849}\, A^*_{-2}$$

$$R^2 = .1268, \quad \bar{R}^2 = .0241$$

$$IG^* = \underset{(1.172)}{133.8042} + \underset{(1.548)}{.40122}\, IG^*_{-1} + \underset{(.745)}{.2726}\, IG^*_{-2} + \underset{(1.745)}{.6492}\, IG^*_{-3}$$

$$- \underset{(-1.024)}{.4211}\, IG^*_{-4} - \underset{(-.990)}{.5031}\, IG^*_{-5}$$

$$R^2 = .5039, \quad \bar{R}^2 = .3268$$

$$TR^* = \underset{(.762)}{20.9671} + \underset{(1.7438)}{.4518}\ TR^*_{-1} + \underset{(.9964)}{.26704}\ TR^*_{-2} + \underset{(.3419)}{.1231}\ TR^*_{-3}$$

$$- \underset{(-.6008)}{.25105}\ TR^*_{-4} + \underset{(1.3515)}{.5961}\ TR^*_{-5}$$

$$R^2 = .3858, \quad \bar{R}^2 = .1665$$

$$PA = \underset{(3.041)}{.3086} - \underset{(-.7937)}{.2368}\ PA_{-2} - \underset{(-.8473)}{.2562}\ PA_{-3}$$

$$R^2 = .0892, \quad \bar{R}^2 = -0.0625$$

Table 14.1

Projected Values of Some Exogenous Variables

Variable	1975/6	1976/7	1977/8	1978/9	1979/80
EXP*	4352.30	5357.18	6489.16	7763.90	9195.43
PM 1	476.34	639.92	836.89	1076.02	1364.23
PM 2	377.0	501.0	678.0	929.0	1288.0
IG*	6086.83	7029.72	7535.82	7563.89	7835.45
A*	581.68	668.48	574.66	672.22	708.17
TR*	1357.15	1574.14	1943.59	2237.49	2608.91
PA	3.592	3.639	3.804	4.081	4.339

Projected levels of variables from these models are given in Table 14.1. For PM, the unit value index of imports, an alternative set of projections are derived from the following model:

$$PM = \underset{(-2.121)}{-44.0705} + \underset{(.248)}{.26627}\ T + \underset{(6.102)}{1.4252}\ PM_{-1} \qquad R^2 = .7982, \quad \bar{R}^2 = .7799$$

where T is a time trend. The initial year was taken to 1973-74 in this case. This provides somewhat lower levels of import prices, which for PM 1 have become very high in the later years. It does seem that consequent upon the oil crisis, world prices are going to be very high and thus the values of either PM 1 or PM 2 do not seem to be unrealistic.

The remaining exogenous variables are RES*, T, BTR, ITR, RD, NLR, D* and TR*. The time-trend takes obvious values and RES*, which is a statistical discrepancy term is set at its 1974-75 level which is the last year of our sample. For the remaining variables alternative values are used for simulations.

IG*, TR*, BTR and ITR are fiscal policy variables and RD, NLR and D* are taken as monetary policy variables. EXP* is subject to international conditions and to some extent on domestic commercial policy. A* is subject to international aid policy. We have introduced policy changes by varying BTR, ITR, RD, NLR and D* from their 1974/75 levels and by varying other variables like IG*, TR*, A* by varying them from their normal growth path defined in Table 14.1.

For the lagged endogenous variables the initial reading comes from the 1974-75 observations and subsequent values are generated by the model.

The projections are presented here for only a few key variables. The space requirements for presenting results for all the 34 variables under varying conditions are considerable. We have chosen to present results for eight variables only, viz., CP, IP, IMP*, RL, P, Y*, YA, YNA.

14.2.1 Conditional Forecast 1: Control Solution

In this version, exogenous variables take values as in Table 14.1 with PM 2. The policy variables BTR, ITR, NLR, D* and RD are set at their 1974-75 levels. This option therefore forecasts values of endogenous variables if the policies of 1974-75 were continued. The values can therefore be taken as points of comparison.

Table 14.2

Forecast 1

Variable	1975/6	1976/7	1977/8	1978/9	1979/80
CP	18275	18857	19470	20052	20710
IP	3094	3244	3418	3612	3841
IMP*	4347	4984	5286	4934	3941
RL	6.655	6.883	6.396	4.794	2.355
P	348.9	404.5	469.7	550.5	671.7
Y*	81704	97270	115928	139178	174213
YA	8403	8448	8476	8489	8487
YNA	14357	14952	15547	16126	16699

14.2.2 Conditional Forecast 2

In this version, all exogenous variables take values as in conditional forecast 1 except government investment expenditure (IG*) which is given a step increase of Rs 100.0 cr over its normal growth path given in Table 14.1 in each year of the prediction period. The results are reported in Table 14.3.

Table 14.3

Forecast 2

Variable	1975/6	1976/7	1977/8	1978/9	1979/80
CP	18281	18864	19479	20063	20722
IP	3093	3243	3417	3610	3840
IMP*	4372	5011	5316	4966	3973
RL	6.735	6.994	6.536	4.964	2.542
P	351.0	406.6	471.7	552.2	673.1
Y*	82198	97790	116447	139677	174698
YA	8403	8446	8473	8486	8483
YNA	14360	14959	15557	16138	16713

14.2.3 Conditional Forecast 3

In this version the discount rate is increased by 1.00 from its 1974-75 level and the net liquidity ratio is lowered by 1.00 for all the years of the prediction period. This indicates a contractionary monetary policy. PM is set at PM 2. The results are given in Table 14.4.

Table 14.4

Forecast 3

Variable	1975/6	1976/7	1977/8	1978/9	1979/80
CP	18275	18857	19470	20052	20709
IP	3093	3244	3417	3611	3841
IMP*	4346	4984	5285	4933	3940
RL	6.679	6.907	6.417	4.811	2.369
P	348.9	404.4	469.7	550.5	671.6
Y*	81695	97260	115917	139167	174201
YA	8403	8448	8476	8489	8487
YNA	14356	14951	15547	16125	16698

14.2.4 Conditional Forecast 4

In this version the tax-rate for personal income taxes are lowered by introducing a reduction in BTR from 21.74 to 20.74 and in ITR from 2.31 to 2.29. This version therefore introduces an expansionary fiscal policy compared to the 1974-75 levels. PM is set at PM 2. The results are summarised in Table 14.5

Table 14.5

Forecast 4

Variable	1975/6	1976/7	1977/8	1978/9	1979/80
CP	18274	18856	19469	20052	20709
IP	3094	3244	3418	3612	3841
IMP*	4346	4984	5285	4933	3941
RL	6.653	6.880	6.392	4.790	2.351
P	348.8	404.4	469.7	550.5	671.6
Y*	81889	97259	115919	139171	174208
YA	8403	8448	8476	8489	8487
YNA	14357	14952	15547	16126	16699

14.2.5 Conditional Forecast 5

In this version the changes in monetary and fiscal policies introduced in Forecast 3 and 4 are combined. This provides a combination of expansionary fiscal policy with contractionary monetary policy. Thus RD, NLR, BTR and ITR are set respectively at 9.00, 30.94, 20.74 and 2.29 whereas in their 1974/75 levels are 8.00, 31.94, 21.74 and 2.31. PM is set at PM 2. IG*, TR*, EXP*, A* etc. follow their normal growth path given in Table 14.1. The results are summarised in Table 14.6.

Table 14.6

Forecast 5

Variable	1975/6	1976/7	1977/8	1978/9	1979/80
CP	18274	18856	19469	20051	20709
IP	3093	3243	3417	3611	3841
IMP*	4345	4983	5285	4932	3940
RL	6.676	6.904	6.413	4.808	2.365
P	348.788	404.389	469.648	550.461	671.599
Y*	81681	97249	115908	139161	174197
YA	8403	8448	8476	8489	8487
YNA	14356	14951	15547	16125	16698

14.2.6 Conditional Forecast 6

In this version A* is stepped up by Rs 100.0 cr over its normal growth. PM is set at PM 2. The results are given in Table 14.7.

Table 14.7

Forecast 6

Variable	1975/6	1976/7	1977/8	1978/9	1979/80
CP	18275	18858	19472	20055	20714
IP	3097	3249	3425	3620	3850
IMP*	4363	5013	5324	4978	3991
RL	6.544	6.673	6.116	4.472	2.020
P	348.5	403.9	469.0	549.8	670.9
Y*	81623	97142	115777	139027	174076
YA	8404	8448	8476	8490	8488
YNA	14357	14953	15549	16130	16705

14.3 A Comparison of Selected Policy Changes

The effects of policy changes compared to the control solutions are compared with reference to three key variables, viz., price-level (P), real output (YA + YNA) which gives GDP at factor cost at 1960-61 prices, and nominal income (Y*) which gives GDP at market prices. The results for the six policy changes introduced in Sections 14.2.1 through 14.2.6 are compared in Table 14.8.

Table 14.8

Predicted Effects of Policy Changes on Key Variables

Policy	Variable	Effects on Selected Variables Measured as Deviations from Control Solution				
		1975/6	1976/7	1977/8	1978/9	1979/80
A	P	2.107	2.073	1.913	1.674	1.426
	(YA+YNA)	3.066	5.535	7.714	9.738	11.473
	Y*	494.1	520.0	518.8	499.6	485.4
B	P	4.199	4.205	3.960	3.553	3.141
	(YA+YNA)	1.477	2.273	2.973	3.758	4.430
	Y*	969.1	1017.3	1007.5	954.9	901.8
C	P	-.035	-.041	-.040	-.037	-.034
	(YA+YNA)	-.055	-.102	-.156	-.195	-.238
	Y*	-8.06	-10.25	-10.94	-11.0	-11.18
D	P	-1.063	-.048	-.035	-.025	-.018
	(YA+YNA)	.035	.066	.074	.086	.086
	Y*	-14.19	-11.19	-8.50	-6.13	-4.25
E	P	-.098	-.089	-.076	-.062	-.052
	(YA+YNA)	-.012	-.039	-.078	-.109	-.152
	Y*	-22.31	-21.44	-19.44	-17.13	-15.44
F	P	-.379	-.600	-.711	-.744	-.745
	(YA+YNA)	.789	2.109	3.656	5.285	6.863
	Y*	-80.44	-128.19	-150.31	-150.88	-136.88

Notes: The control solution is obtained by keeping BTR, ITR, RD, NLR, D* and RES* at their 1974/75 levels and permitting normal growth in EXP*, IG*, PM, PA, A*, TR* and T. The values taken by these are given in Table 14.1 with PM set at PM 2.

Policy A: IG* is stepped up by Rs 100.0 cr compared to above.

Policy B: IG* and TR* are stepped up by Rs 100.0 cr each.

Policy C: RD is stepped up by 1.00 and NLR is stepped down by 1.00 compared to their 1974-75 levels.

Policy D: BTR is reduced by 1.00 and ITR by .02 compared to their 1974-75 levels.

Policy E: C & D above are combined.

Policy F: A* is stepped up by Rs 100.0 over its normal growth.

It can be observed from Table 14.8 that increases in government investment expenditures with and without increases in transfer payments lead to rises in the price-level, real output and nominal incomes. The highest increases in real output are obtained when government investment increases on its own. Since this increase is financed by borrowing from the private sector, part of transfer payments are now assigned to meet increased interest payments and keeping TR* at its normal growth means most of this growth goes to meet these payments.

Contractionary monetary policy (Policy C) leads to a fall in the price-level but also in real output. Expansionary fiscal policy introduced by lowering of tax-rates of personal income tax has a beneficial effect on real output and price level also declines. The magnitude of the effects are however small, and when combined with a contractionary monetary policy the net effects are negative for all the three variables, only for one of which, viz., the price-level, it is beneficial.

Stepping up of net aid receipts over its normal growth has positive effects on real output and lowers the price-level compared to the control solution and thus has beneficial effects.

Some other conditional forecasts of interest using PM 1 from Table 14.1 as a possible growth path for the unit value of imports and keeping RD, NLR, BTR, ITR, D* and RES* at their 1974-75 levels are given below.

14.4.1 Conditional Forecast 7

In this version both government investment expenditure and government transfer payments go up by Rs 100.00 over and above their normal growth path. Both the increases are financed by government borrowing from the private sector and a substantial part of the rise in transfer payments can be seen as meeting the additional interest payments on the increased governmental debt. The results are given in Table 14.9.

Table 14.9

Forecast 7

Variable	1975/6	1976/7	1977/8	1978/9	1979/80
CP	18308	18887	19498	20078	20733
IP	3097	3245	3417	3609	3838
IMP*	4397	5040	5349	5003	4012
RL	6.816	7.109	6.685	5.145	2.745
P	353.085	408.683	473.684	554.076	674.792
Y*	82672	98288	116935	140133	175114
YA	8401	8445	8471	8483	8480
YNA	14360	14957	15554	16135	16710

14.4.2 Conditional Forecast 8

In this version the domestic policy variables RD, NLR, BTR, ITR, D*, IG*, TR* are kept at their 1974-75 levels. A*, PM and PA are allowed to follow their growth path as defined in Table 14.1 with PM 1 for the unit value index of imports. This solution can also be interpreted as the results when no domestic policy changes are introduced in the prediction period. It can also serve as a point of comparison. The results are summarised in Table 14.10.

Table 14.10

Forecast 8

Variable	1975/6	1976/7	1977/8	1978/9	1979/80
CP	18190	18657	19146	19665	20263
IP	3090	3242	3421	3629	3872
IMP*	4154	4401	4318	3767	2484
RL	6.035	5.066	3.183	0.334	-3.293
P	333.623	368.819	442.840	503.505	622.130
Y*	78157	88526	103751	126129	159406
YA	8409	8466	8510	8537	8547
YNA	14337	14879	15412	15936	16458

14.4.3 Conditional Forecast 9

In this version IG* is stepped up by Rs 100.00 cr over its normal growth path. PM is set at PM 1 in Table 14.1. The results are given in Table 14.11.

Table 14.11

Forecast 9

Variable	1975/6	1976/7	1977/8	1978/9	1979/80
CP	18292	18884	19503	20081	20706
IP	3098	3260	3448	3653	3882
IMP*	4070	4422	4514	4206	3841
RL	6.634	6.444	5.376	3.278	0.607
P	362.8	431.2	507.8	591.8	696.1
Y*	84797	103406	124965	149313	180346
YA	8399	8435	8453	8457	8448
YNA	14353	14940	15528	16105	16683

14.4.4 Conditional Forecast 10

In this version, A* is stepped by Rs 100.00 cr over its normal growth path. PM is set at PM 1 in Table 14.1 The results are reported in Table 14.12

Table 14.12

Forecast 10

Variable	1975/6	1976/7	1977/8	1978/9	1979/80
CP	18287	18878	19495	20072	20698
IP	3101	3266	3456	3662	3892
IMP*	4052	4415	4516	4218	3861
RL	6.445	6.123	4.959	2.792	0.083
P	360.7	428.9	505.5	589.6	694.0
Y*	84296	102846	124374	148711	179734
YA	8399	8436	8455	8460	8451
YNA	14350	14935	15521	16097	16675

14.4.5 Conditional Forecast 11

In this version government investment expenditure increases by 100.0 cr over its normal growth and this increase is entirely financed by government borrowing from the Central bank. Thus IG* increase by 100.00 in all years in the prediction period over its normal growth path given in Table 14.1, accompanied by a step increase in D*. PM is set at PM 1. The results are summarised in Table 14.13.

Table 14.13

Forecast 11

Variable	1975/6	1976/7	1977/8	1978/9	1979/80
CP	18292	18886	19505	20084	20711
IP	3101	3265	3456	3662	3892
IMP*	4073	4429	4526	4224	3864
RL	6.582	6.222	5.072	2.919	0.220
P	363.0	431.6	508.3	592.5	696.8
Y*	84833	103489	125101	149502	180593
YA	8399	8435	8453	8456	8447
YNA	14353	14941	15530	16108	16688

14.6 Overall Summary and Conclusions

This work was undertaken both with a view to construct a macroeconometric model of the Indian economy for purposes of forecasting and policy analysis and to provide relevant background information for the model building activity in India in particular and in developing countries in general.

In order to meet the latter objective, we have first surveyed the techniques of forecasting and policy analysis currently in use with the main emphasis on the construction and uses of macroeconometric models. We deal with the nature and uses of macroeconometric models, problems of specification, identification, estimation and the preparation of forecasts. We also consider available techniques for the evaluation of forecasts and forecasting models, and the techniques by which such models can be used as a guide in policy decisions. Specific issues that arise in the context of developing countries are examined both in the light of current experience in this field and on *a priori* grounds. The choices that arise and the considerations that are relevant in making appropriate decisions regarding these in the context of developing countries have been discussed.

In Part II of this work, we devote our attention to a survey and appraisal of macroeconometric models of the Indian economy. We have covered models built by twelve authors/institutions including alternate versions of models in many cases. This list does not purport to be an exhaustive list of all macroeconometric works for the Indian economy, but it does cover most of the ground. In the presentation of these models through Chapter 7 to 9 we have followed a uniform scheme of going through the main features of the model, the specification, relevant considerations in the specification, the estimated equations and the main conclusions derived from the model.

In the presentation of the models, we have used as far as possible the same scheme of variable names as in the original models. This tends to be cumbersome at stages. However, since in a short space, we have intended to cover relatively lengthy works, we feel that those interested in greater details of one or the other model may need to look into the original works. At that stage comparability in variable names would have an advantage.

The models which we have covered were constructed by Narasimham, Choudhry, Krishnamurty, Krishnamurty-Choudhry, Marwah, Mammen, Pandit, Gupta, Bhattacharya and UNCTAD. All these models have some special features which may be summarised here. Narasimham's model was one of the first attempts to build a macro model for policy analysis. It uses a sample beginning in the pre-1950 years and the deviations from a 9-yearly moving average were used for purposes of estimation. Choudhry's model focusses on short-term forecasting while Krishnamurty's model has a longer-term focus with an endogenous demographic sector. Models by Mammen, Bhattacharya and Gupta have a special emphasis on the monetary sector of the economy. Bhattacharya's model deals with 'monetized' income-expenditure levels. Most of these models underemphasise or ignore supply side considerations. In Marwah's models as also in those of Agarwala, Pandit and UNCTAD models, the supply side plays a more active role. Marwah's models focus primarily on the determination of sectoral prices while the UNCTAD models expand on the foreign trade side. Agarwala's model underplays the demand aspects and incorporates the dualistic nature of the economy into analysis. On the whole, although models belong generally to the IS-LM framework, they offer a considerable variety in terms of different sectoral emphasis and degree of disaggregation.

A review of some aspects of macro-modelling of the Indian economy is undertaken in Chapter 10. We observe that, with reference to some mainstream functions, there is some preliminary evidence of structural stability in the post 1950 years and it seems a good starting point for a sample for estimation purposes. We also observe that it may be more useful to specify demand relations in terms of appropriately deflated variables where possible rather than in nominal terms if the approach followed is within the IS-LM framework. We have also considered the question of the choice of exogenous variables. Based on *a priori* considerations, some Granger-Sims tests, and the treatment in earlier models, we identify certain variables which could be treated in recursive blocks or as exogenous. Finally, we have evaluated the forecasting performance of adapted versions of three models, viz., Choudhry-I, Bhattacharya and Marwah-I under conditions of a common sample period and estimation technique and for a common forecast period. For nearly all variables considered for this purpose, the forecasts by autoregressive models outperform the macroeconometric models. But in two cases, viz., Marwah-I and Bhattacharya, the performance is nearly as good as the benchmark models. The performance of the adapted Choudhry-I model is slightly worse for a few variables but it contains a somewhat greater disaggregation.

Whereas all these models have many interesting features, their continued use for forecasting and simulation is not envisaged because of some common shortcomings and also because the estimates of their stochastic equations have become dated in view of major data revisions.

Some of their shortcomings may be noted. Except a few, these models do not specify a government budget restraint leaving all government income-expenditure variables to be determined freely and under-emphasizing the interdependence of monetary and fiscal sectors. In general, the fiscal sector has been under-explored. Aggregate tax categories are used and treated either entirely as discretionary or entirely as automatic. Similarly, highly aggregated government expenditure categories are used and treated entirely exogenous. We have argued at various places that these procedures are not entirely satisfactory.

As a sequel to this, and also as a part of continuing work in the field of macroeconometric forecasting, we specify and estimate a fresh model with eighteen stochastic equations. Its special features are an endogenous treatment of government consumption expenditure, an explicit treatment of tax-rates in the personal income tax function, and a disaggregation of tax-revenues permitting a distinction between different tax-bases. Furthermore, a budget restraint is used, and through this as also through other equations, the interdependence of the monetary and real sector is emphasized. There is an endogenous treatment of the money supply which takes account of its interdependence with the external sector. Also, a distinction between agricultural and non-agricultural sectors is made in terms of prices, outputs and investments. Different deflators are used for different income-expenditure categories. The model is non-linear in specification and is estimated by mixed estimation procedures. In particular, two-stage least squares, with subsets of predetermined variables in the first stage, are used with first-order autoregressive correction in certain cases.

The model is used for forecasting and policy simulation. Analysis of forecasting performance is done in two stages. First, static and dynamic model estimates are generated within the sample period and compared against realizations and against 'naive' benchmark models. Secondly, some of the observations are saved from the available sample to provide a 'pseudo' forecast period and the model is re-estimated with the remaining observations. Forecasts from this are generated statically and dynamically for the 'pseudo' forecasts period and compared against realizations and against relatively powerful autoregressive models. In both these exercises, the model is observed to perform highly satisfactorily. Diagnostic checks on the forecast errors have also been performed and the incidence of misprediction of direction of changes has been noted.

The model is then used for policy simulation. Control solutions are dynamically generated with an appropriate initial year and with sample levels of exogenous variables. Subsequently, in the simulations, policy variables are changed in a prespecified manner, and the deviations of simulated solutions from the control solutions are interpreted as due to the policy changes introduced. Expenditure multipliers have also been calculated. From the policy analysis, the following main conclusions emerge.

Prices increase following increases in government investment expenditure or transfer payments, a reduction in indirect taxes encouraging private spending and the pursuit of an expansionary monetary policy. Real output is positively affected by a rise in government investment expenditure, expansionary monetary policy, increases in aid, and deficit financing. Real output falls if the basic tax rate for personal income tax increases. Changes in tax rates and discount rate have a very small impact in terms of magnitudes whereas policy changes affecting expenditure variables and government deficit financing have a larger impact judged from elasticities. Effects on nominal incomes are largest for autonomous increases in government consumption expenditure. But the impact is primarily on prices and real output actually goes down in this case.

Although we find that the model performs satisfactorily *vis a vis* forecasting, and provides reasonable results *vis a vis* policy simulation, we offer it only as one of many possible macroeconomic profiles, and we also acknowledge the use of relevant information provided by many of its now dated predecessors. The exercise of macroeconometric model building has an evolutionary nature and the experience of developed countries in this context indicates that there is room for many alternative models inasmuch as they use different information sets or exploit the same information in different ways. Furthermore, as data becomes more streamlined and there is sharing of data and other relevant information among different model builders, the costs of constructing and updating models is also likely to be substantially reduced. In the end, large models will have to be constructed and maintained by institutions rather than individual authors.

It is customary to suggest some directions of future research in an exercise of the present nature. We feel that the model we have presented could be extended in one or more of the following directions: integration with an input-output block, integration with a flow of funds analysis, further disaggregation of some of the sectors and randomization of the estimated coefficients. The properties of the model as it stands, could also be further analysed by stochastic simulations. In the end, we may conclude by saying that while more high-powered vehicles would occupy the scene when more streamlined data and longer sample periods are available, we offer this work in the hope that it will provide a useful stepping stone in a long climb.

APPENDICES

AND

BIBLIOGRAPHY

Appendix 1 : Data

Data used in estimation and testing of the model described in Part III are given in Table A.1. These are compiled from the following official and non-official sources.

Sources

A. C.S.O. - Estimates of National Income, 1948/9 - 1962/3;
Estimates of National Product, 1969, 1970, 1971;
National Accounts Statistics, 1976.

B. R.B.I. - Reports on Currency and Finance (Annual)

C. I.M.F. - International Financial Statistics, 1951 - 1975;
1953 - 1977.

D. Government of India - Budget Documents.

E. Government of India - Economic Survey, 1976.

F. C.S.O. - Basic Statistics relating to the Indian Economy, 1950/1 - 1967/68; 1950/1 - 1972/3; 1950/1 - 1974/5.

G. Bhattacharya, B.B. (1975), Short-Term Income Determination, McMillan, New Delhi; A Model of the Indian Economy (forthcoming).

H. Lal, R.N. (1977), Capital Formation and its Financing in India, Allied Publishers, New Delhi.

Units

Units for data given in Table A.1 are as follows.

(i) Crores of Rs. : CP, IP, CG*, IMP*, Y*, IG*, A*, EXP*, RES*, TR*, TDY*, NTDY*, CD*, NCD*, TVR*, M*, U*, BP*, ZBP*, F*, D*, FS*, DKP, DKG, Y*(a), CP*(a), IP*, YD*(a), TD, YD*, FA, KP, KG, NW*(a), YA, YNA, IA, KA, KNA, DKA.

Variables marked by * are at current prices; other variables in the list above are at constant 1960/1 prices.

(ii) Billions of Rs. : A*(a)

(iii) % per annum : RD, R, RL.

(iv) Units : BTR, ITR.

(v) % ratio : NLR.

(vi) Millions : N

(vii) '000 tons : FRT

(viii) Billion US dollars: YW

(ix) Indices : 1960/1 = 100.0 : PM, P, AC

1960/1 = 1.0 : PI, PY, PNA, PA

1970 = 100.0 : PW, PS

Notes on Series in Table 1

1. CP, Private consumption expenditure at constant 1960/1 prices,

$$= (Y^* - CG^* - IP^* - IG^* - EXP^* + IMP^*)/PY$$

2. IP, Private investment expenditure at constant 1960/1 prices

$$= IP^*/PI$$

3. CG*, Government consumption expenditure at current prices, Source A

Table A.1

Data Used for Estimation and Testing

Year	CP (1)	IP (2)	CG* (3)	IMP* (4)	Y* (5)
1950/51	8278	939	558	710	9503
51/2	8755	893	575	1040	10004
52/3	8946	585	599	700	9741
53/4	9560	670	630	650	10498
54/5	9664	735	674	750	9811
55/6	9788	1050	605	840	10367
56/7	10380	1267	789	1170	11773
57/8	10482	1040	919	1300	12005
58/9	11284	915	982	1100	13356
59/60	11296	936	1036	1010	13845
60/61	11803	1442	1086	1236	15018
61/2	12151	1447	1206	1100	15977
62/3	12278	1505	1458	1209	17099
63/4	12778	1650	1876	1365	19656
64/5	13985	1820	2005	1527	23044
65/6	13236	1781	2296	1465	24112
66/7	13330	2292	2501	2119	27662
67/8	14596	2304	2785	2212	32294
68/9	14964	2252	3050	1898	33279
69/70	15657	2671	3417	1748	36951
70/71	16517	2654	3773	1899	40375
71/2	16665	2558	4435	2179	42983
72/3	16297	2779	4748	2241	47037
73/4	17692	2451	5057	2949	57678
74/5	17436	2987	5979	4450	68457

Table A.1 contd

Year	IG* (6)	A* (7)	EXP* (8)	PM (9)	RES* (10)
1950/51	233	-68	740	94	0.0
51/2	289	137	850	130	-54
52/3	317	-9	710	113	-225
53/4	334	56	640	104	-160
54/5	406	44	700	101	19
55/6	548	103	760	99	-33
56/7	659	123	770	103	7
57/8	722	205	800	80	133
58/9	862	324	720	102	-77
59/60	995	236	780	95	-114
60/61	1140	400	784	100	168
61/2	1146	232	802	102	-189
62/3	1444	347	835	98	-174
63/4	1681	411	985	101	-132
64/5	1949	483	1015	103	-398
65/6	2216	498	932	108	-221
66/7	2126	859	1325	156	123
67/8	2331	730	1508	142	-544
68/9	2167	452	1597	147	-851
69/70	2260	388	1625	146	-795
70/71	2811	220	1639	153	-687
71/2	3249	464	1785	155	-1208
72/3	3606	53	2143	160	-1567
73/4	4760	388	2658	231	-1740
74/5	5551	696	3474	346	-1895

Table A.1 contd

Year	RD (11)	NLR (12)	BTR (13)	ITR (14)	TR* (15)
1950/51	3.00	43.93	17.43	2.37	-21
51/2	3.25	44.84	18.30	2.49	-26
52/3	3.50	44.22	18.30	2.49	-3
53/4	3.50	43.94	18.30	2.49	26
54/5	3.50	43.87	18.30	2.49	24
55/6	3.50	42.78	19.41	2.68	56
56/7	3.50	34.59	18.59	2.79	51
57/8	4.00	33.33	17.55	2.45	72
58/9	4.00	40.90	17.55	2.45	92
59/60	4.00	43.29	17.55	2.45	129
60/61	5.12	35.66	17.55	2.45	44
61/2	4.81	32.49	16.33	2.59	156
62/3	5.19	29.16	18.70	2.62	56
63/4	5.15	28.58	18.90	2.94	-22
64/5	5.18	26.22	19.59	2.54	53
65/6	5.87	28.65	18.47	2.12	100
66/7	6.27	28.17	20.31	2.33	180
67/8	6.00	27.83	20.31	2.33	229
68/9	4.85	25.38	20.31	2.33	159
69/70	4.54	23.29	19.90	2.38	188
70/71	5.50	20.38	21.06	2.33	254
71/2	6.00	23.50	20.07	2.98	436
72/3	7.00	28.17	20.07	2.98	775
73/4	6.50	32.11	20.07	2.98	915
74/5	8.00	31.94	21.74	2.31	1155

Table A.1 contd

Year	R (16)	RL (17)	P (18)	PI (19)	PY (20)
1950/51	1.12	5.48	86.3	.766	.962
51/2	1.52	5.54	91.6	.821	.982
52/3	2.22	5.72	80.1	.846	.930
53/4	2.45	6.07	83.8	.818	.941
54/5	2.53	5.09	78.1	.811	.847
55/6	2.52	5.01	74.1	.816	.862
56/7	2.53	5.68	84.4	.862	.928
57/8	2.48	6.88	86.9	.902	.947
58/9	2.52	6.27	90.4	.934	.978
59/60	2.59	5.27	83.8	.956	.987
60/61	2.65	4.88	100.0	1.0	1.0
61/2	2.58	4.67	100.2	1.038	1.0197
62/3	2.42	4.85	104.0	1.067	1.056
63/4	2.35	4.90	110.4	1.120	1.145
64/5	2.72	6.96	122.5	1.165	1.250
65/6	3.50	8.11	131.9	1.241	1.354
66/7	3.50	7.71	150.2	1.388	1.549
67/8	3.50	8.22	167.6	1.466	1.679
68/9	3.00	6.81	165.7	1.498	1.670
69/70	3.00	5.75	171.9	1.579	1.736
70/71	3.09	5.53	181.5	1.691	1.790
71/2	3.50	6.49	188.8	1.777	1.869
72/3	3.50	6.99	207.5	1.874	2.060
73/4	3.92	5.59	254.7	2.141	2.425
74/5	4.49	5.10	313.6	2.714	2.856

Table A.1 contd

Year	TDY* (21)	NTDY* (22)	CD* (23)	NCD* (24)	TVR* (25)
1950/1	133.89	97.11	157.16	229.8	618
51/2	146.19	96.81	232.33	253.7	729
52/3	141.73	106.6	174.04	251.0	673
53/4	122.84	117.2	158.13	299.9	698
54/5	122.26	114.7	182.43	327.6	747
55/6	131.36	123.6	165.74	375.3	796
56/7	151.74	141.3	169.19	447.8	910
57/8	163.70	159.3	177.77	561.2	1062
58/9	172.01	164.0	137.37	630.6	1104
59/60	148.85	222.2	150.71	708.3	1230
60/61	167.38	252.6	167.73	779.3	1367
61/2	165.39	301.6	210.91	869.1	1547
62/3	185.96	388.0	248.02	1016	1838
63/4	258.60	448.4	337.62	1228	2273
64/5	266.55	496.5	403.21	1381	2547
65/6	271.80	503.2	549.83	1532	2857
66/7	308.69	523.3	602.12	1580	3014
67/8	325.62	533.4	538.50	1886	3283
68/9	378.47	531.5	475.90	2255	3641
69/70	448.45	594.6	402.54	2656	4102
70/71	473.00	618.0	490.00	3033	4614
71/2	537.00	738.0	716.00	3356	5347
72/3	625.47	844.5	856.64	3722	6049
73/4	741.37	911.6	996.43	4163	6812
74/5	874.41	1093.0	1332.90	5225	8525

Table A.1 contd

Year	M* (26)	U* (27)	BP* (28)	ZBP* (29)	F* (30)
1950/51	1948	1515	2074	86	860
51/2	1767	1329	2125	51	807
52/3	1725	1310	2169	44	808
53/4	1750	1343	2258	89	854
54/5	1882	1424	2500	242	848
55/6	2146	1596	2628	128	871
56/7	2276	1605	2815	187	594
57/8	2353	1728	2976	161	299
58/9	2463	1808	3271	295	243
59/60	2669	1954	3711	440	249
60/61	2868	2049	4235	524	197
61/2	3044	2169	4589	354	131
62/3	3308	2328	5002	413	104
63/4	3750	2558	5523	521	135
64/5	4078	2654	5977	454	106
65/6	4529	2990	6642	665	71
66/7	4949	3170	7584	942	136
67/8	5349	3389	8179	595	162
68/9	5779	3720	8431	252	313
69/70	6386	3879	9117	686	578
70/71	7140	4141	10132	1015	538
71/2	8139	4854	10590	458	608
72/3	9412	5562	11297	707	563
73/4	10836	6540	12544	1247	660
74/5	11530	6474	13899	1355	380

Table A.1 contd

Year	D* (31)	FS* (32)	DKP (33)	DKG (34)
1950/51	134	1570	232	45
51/2	-79	1847	269	46
52/3	205	1508	290	46
53/4	147	1504	321	48
54/5	71	1598	285	51
55/6	182	1711	376	68
56/7	279	1764	403	70
57/8	285	1599	482	73
58/9	213	1343	560	73
59/60	254	1259	591	77
60/61	-21	1433	620	116
61/2	375	1231	681	131
62/3	360	1313	774	158
63/4	330	1500	831	170
64/5	523	1633	906	206
65/6	592	1536	1012	217
66/7	-8	2255	1141	261
67/8	737	2374	1246	312
68/9	1031	2211	1350	336
69/70	689	2326	1527	388
70/71	989	2437	1685	421
71/2	1851	2787	1828	460
72/3	2320	2804	1955	538
73/4	2285	3609	2134	665
74/5	2109	4830	2668	746

Table A.1 contd

Year	Y*(a) (alternate) (35)	CP*(a) (alternate) (36)	IP* (37)	A*(a) (38)	TD (39)	FA (40)
1950/51	9559	8019	719	.2	312.9	4494
51/2	10104	8697	733	.4	331.0	4294
52/3	9694	8273	495	.12	365.6	4552
53/4	10349	8847	548	.17	371.9	4631
54/5	9457	7831	596	.15	484.1	5658
55/6	10037	8107	857	.38	547.6	6086
56/7	11517	9377	1092	.27	575.4	6061
57/8	11794	9715	938	.15	819.4	6447
58/9	13210	10890	856	.18	989.9	6852
59/60	13773	11077	895	.38	1198	7662
60/61	15018	11803	1441	.30	1095	8198
61/2	15977	12390	1533	.34	1186	8671
62/3	17099	12965	1606	.66	1188	9057
63/4	19656	14631	1848	.71	1131	9230
64/5	23044	17481	2121	1.31	1147	9191
65/6	24112	17922	2211	.68	1185	9436
66/7	27618	20604	3181	.84	1208	9299
67/8	32402	24614	3376	.18	1263	9320
68/9	33552	25263	3373	.90	1517	10026
69/70	37214	27443	4217	.35	1699	10630
70/71	40535	29725	4486	.38	1926	11575
71/2	43251	31415	4546	.22	2234	12255
72/3	47022	33556	5210	.25	2477	12530
73/4	57678	42904	5248	16.74	2517	12158
74/5	68457	49798	8105	1.01	2531	11434

Table A.1 contd

Year	PW (41)	YW (42)	PS (43)	N (44)	NW*(a) (alternate) (45)
1950/51	82	770	76.7	360.18	32970
51/2	67	804	65.1	366.30	31505
52/3	58	767	62.9	372.75	32044
53/4	59	797	73.2	379.57	34781
54/5	58	892	80.0	386.62	35811
55/6	58	982	82.9	394.22	37122
56/7	60	1083	69.6	402.23	38593
57/8	58	1010	69.9	410.69	40011
58/9	59	1072	78.9	419.61	41464
59/60	63	1202	88.9	429.02	43219
60/61	64	1255	94.1	439.00	45296
61/2	61	1328	97.3	451.01	47473
62/3	61	1446	87.4	462.03	49801
63/4	61	1621	87.9	472.13	52653
64/5	65	1761	79.7	482.71	55695
65/6	85	1939	78.6	493.39	59193
66/7	98	2031	79.2	504.34	64002
67/8	94	2264	79.3	515.60	68622
68/9	101	2582	92.4	527.18	78869
69/70	100	2971	100.0	539.07	79017
70/71	102	3322	96.7	551.02	85770
71/2	112	3884	95.1	562.67	92501
72/3	136	5348	105.0	574.43	100617
73/4	170	7854	114.3	586.27	109538
74/5	192	8163	94.1	598.10	121512

Figures for PW, PS relate to calendar years, e.g. 1950/51 = 1951

Population (N) figures are mid year estimates

Table A.1 contd

Year	YA (46)	YNA (47)	IA (48)	PNA (49)	PA (50)	AC (51)	FRT (52)	DKA (53)
1960/1	6151	7320	395.00	1.00	1.00	100.0	292	225.18
61/2	6808	7801	362.39	1.017	1.023	101.0	385	201.47
62/3	6639	8351	401.06	1.045	1.071	101.9	560	207.74
63/4	6814	8983	425.45	1.097	1.209	102.6	631	235.73
64/5	7440	9573	500.35	1.167	1.356	102.1	673	286.86
65/6	6377	9888	580.94	1.237	1.536	101.6	769	315.02
66/7	6293	10155	575.58	1.356	1.861	102.2	1351	369.10
67/8	7283	10507	571.78	1.459	1.997	103.0	2047	360.81
68/9	7335	10598	633.64	1.497	1.929	104.8	1946	380.08
69/70	7805	11655	707.88	1.566	1.991	105.6	1819	451.49
70/1	8454	12131	774.83	1.656	1.983	106.7	1688	483.85
71/2	8276	12546	765.97	1.726	2.085	105.8	2227	457.99
72/3	7675	12927	827.40	1.823	2.457	106.5	2580	516.84
73/4	8373	13280	858.58	2.047	3.026	106.1	2627	509.50
74/5	8044	13626	746.92	2.469	3.513	108.7	3114	501.06

4. IMP*, Imports at current prices, Source A

5. Y*, GDP at current market prices, consistent with revised series of C.S.O. based on C.S.O. estimates, taken from Source H.

6. IG*, Government investment expenditure = gross domestic capital formation in the public sector, Sources A & H

7. A*, Net Aid, calculated as $(F^* - F^*_{-1} - EXP^* + IMP^*)$

8. EXP*, Exports of goods and services valued at current domestic prices, Source A

9. PM, unit value index of imports, Source F

10. RES*, residual, a discrepancy term, calculated as $(U^* - U^*_{-1} - D^* - F^* + F^*_{-1})$

11. RD, discount-rate, since 1960-61 figures are weighted averages of different discount rates, Source G

12. NLR, Net liquidity ratio of commercial banks, defined as 100 (Bank reserves + Bank cash)/Bank deposits, Source G

13. & 14. BTR and ITR, basic and incremental tax-rates of tax on non-corporate incomes, computed as explained in Appendix 2.

15. TR*, net transfer payments to the private sector, Source A

16. R: short-term rate of interest defined as yield on 3-months' treasury bills, figures for 1950-1, 51/2 and 57/8 - years in which treasury bills were not floated were estimated on the basis of a regression line between yield on three months treasury bills and the inter-bank call money-rate, Source G.

17. RL: Long-term rate of interest, defined as yield on variable dividend industrial securities. Source G

18. Wholesale Price index, Source F

19. PI: Implicit price deflator for investment goods computed as gross domestic capital formation at current prices by the same at 1960/61 prices. Source H

20. PY: Implicit income deflator; for 1950/51 to 59/60 obtained by dividing GDP at market prices minus indirect taxes less subsidies by GDP at factor cost at 1960/61 prices both series consistent with revised income series; Source H for above; for 1960/61 and later years, directly from Source H.

21. TDY*: Tax revenue from taxes on non-corporate incomes Sources C & A

22. NTDY*: Direct taxes other than TDY*, Source C

23. CD*: Custom duties (taxes on imports and exports) Source C & F

24. NCD*: Indirect taxes less subsidies other than CD*, Source C & A

25. TVR*: Total tax revenues less subsidies = TDY* + NTDY* + CD* + NCD*

26. M*: Money-supply with the public = currency plus demand deposits, Source C & G

27. U*: Unborrowed money reserves defined as currency in circulation plus bank reserves minus borrowing of commercial banks for R.B.I. Source C & G

28. BP*: Government outstanding debt to private sector, Source G

29. ZBP^*: Government current borrowing from the private sector

$= BP^* - BP^*_{-1}$

30. F^*: Foreign exchange reserves at the end of the year,

Source G

31. D^*: Government deficit financing $= CG^* + IG^* - TR^* - TVR^* - A^*$

32. FS^*: Potential foreign exchange availability defined as

$= F^*_{-1} + EXP^* + A^*$

33. DKP: replacement capital in the private sector at constant 1960-61 prices, computed as replacement capital in the private sector at current prices (Source H) divided by PI.

34. DKG: replacement capital in the public sector at constant 1960/61 prices, computed as replacement capital in the public sector at current prices (Source H) divided by PI.

35. $Y^*(a)$: GDP at current market prices, alternate series, Source G

36. $CP^*(a)$: private consumption expenditure, alternate series related to $Y^*(a)$, Source G

37. IP^*: private investment expenditure at current prices, Source A

38. Foreign Aid, alternate series, Source D.

39. TD: time-deposits with the commercial banks, Source G

40. FA: financial assets of the private sector $= TD + (M^* + BP^*)/PY$

41. PW: unit value index of world imports, Source D

42. YW: world imports, Source D

43. PS: index of prices of industrial securities, Source D

44. N: mid-year population estimates, Source D

45. NW*: Net worth of the private sector, calculated as

$$KP^*_{-1} + IP^* + M^* + TD.\ PY + BP^*$$

46. YA: Gross domestic product at factor cost in agriculture at constant 1960/1 prices, Source A

47. YNA: Gross domestic product at factor cost in non-agriculture at constant 1960/1 prices, Source A

48. IA: Gross investment in agricultural sector at constant 1960/1 prices, compiled from Sources A and H using PI as the deflator.

49. PNA: implicit price deflator for non-agricultural output computed from relevant series from Source A

50. PA: implicit price deflator for agricultural output, computed from Source A

51. AC: Index of area under cultivation, Source E

52. FRT: Fertiliser inputs, imports plus domestic production, Source E

53. DKA: depreciation in agriculture, estimated from IA, using the ratio DKP*/IP* assuming that it bears the same ratio for agriculture as the one for the private sector. Gross investment in agriculture at current prices is obtained from Sources A & H.

Other series used in Chapter 11 are generated from the data given in Table A.1. Thus, disposable income with the private sector, YD is generated from $(Y^* - TVR^* + TR^*)/PY$; KP, capital stock in the private sector is computed as $KP_{-1} + IP - DKP$ with the initiating value taken as 37098 (Source G) for the year 1960/1; KG, capital stock in the public sector is computed as $KG_{-1} + IG^*/PI - DKG$ with the initiating value taken as 9524 (Source G) for the year 1960/1. Both KP and KG are at constant 60/1 prices.

KA, capital stock in agriculture at constant 60/1 prices is computed as KA + IA - DKA. The initiating value for 1960/1 is obtained by dividing total capital stock for this year by using the ratio .16058 derived from

$$\sum_{t=60-61}^{74-75} IA_t \Bigg/ \sum_{t=60-61}^{74-75} INA_t$$

on the assumption that capital accumulation in the agricultural and non-agricultural sectors bears the same ratio in pre-1960/1 years as in post-1960/1 years. KNA is obtained as a residual = KP + KG - KA. Data for INA, gross investment in the non-agricultural sector is obtained from Sources A & H.

Appendix 2

Notes on Data Used in Chapter 10

In Chapter 10 we have conducted certain tests and re-estimated some existing macroeconometric models of the Indian economy under common sample conditions. In this chapter, the symbolic representation of variables conforms in general to the original models. As such often the variable names differ from those given in Table A.1. In the following table we explain what series were used for the variables occurring in Chapter 10.

Table A.2.

Variable Name	Series No.	Variable Name	Series No.
C_p	36	1	12
Y^d	38	T	25
I_p	37	P	18
Y	35	N	48
R_1	17	IM	4
M	26	P_W	45
U	27	F	30
NW	49	P^s	47
R	16	EX	8
R_d	11	Y_W	46
I_g	6	T^{ind}	(23 + 24)
TR	15	L	(26 + 40)

The following variables were calculated from identities:

$Y^d = Y + TR - T$; $EN = (Y - C_p - I_p)$; $G = (Y - C_p - I_p - EX + IM + T^{ind})$; $TRR = Y^d - Y$; $DL = L - L_{-1}$; $H = Y - C_p - I_p$; O or $X = Y/P$; and $q = R_d - R$.

Capital stock K was obtained by accumulating investment at current price (series 6 + 37) with the initiating value for the year 1960/1. The time-trend t is initiated in 1950-51 with a value 1. For foreign aid, series (38), for government consumption and investment expenditures, respectively series (3) and (6), for exports, series (8) and for unit value index of imports (P_m), series 9 were used.

Appendix 3

Table A.3

Estimates of the Basic Tax Rate (BTR=γ) and the Incremental Tax Rate (ITR=β)

Year	γ	β	R^2	t_γ	t_β
1949/50	18.39	2.42	.94	8.66	20.85
1950/51	17.43	2.37	.89	6.37	15.88
1951/52*	18.30	2.49	.89	6.37	15.88
1955/56	19.41	2.68	.91	6.70	16.94
1956/57	18.59	2.79	.92	6.73	18.54
1957/58**	17.55	2.45	.81	4.46	11.43
1961/62	16.33	2.59	.85	4.42	12.87
1962/63	18.70	2.62	.83	4.80	12.35
1963/64	18.90	2.94	.90	5.78	16.46
1964/65	19.59	2.54	.84	5.23	12.40
1965/66	18.47	2.12	.83	5.76	12.12
1966/67§	20.31	2.33	.83	5.76	12.12
1969/70	19.90	2.38	.84	5.75	12.63
1970/71	21.06	2.33	.85	6.38	12.95
1971/72§§	20.07	2.98	.88	5.41	14.73
1945/75	21.74	2.31	.79	5.51	10.72

* and 52/3, 53/4, 54/5

** and 58/9, 59/60, 60/61

§ and 67/8, 68/9

§§ and 72/3, 73/4

Estimates are derived from $r_j = \gamma + \beta j + u$; $j = 1,2,\ldots,J$, where j's are the ranks of income-brackets, r_j's, the % tax-rates applicable to 'individuals' in the personal income tax schedules for non-corporate incomes. The income-brackets are kept uniform over time although they do not always represent an equal income-range. In all 31 income-brackets (J = 31) are used for each of the years.

Appendix 4

Sample-Period Estimates of Endogenous Variables with Model A with Consumption Function E.1(a).

Table A.4

	61-62	62-63	63-64	64-65	65-66	66-67	67-68
CP	12366.150	12574.289	12847.587	13248.658	13706.314	14173.070	14572.700
IP	1575.754	1631.889	1701.654	1786.151	1893.939	2027.508	2136.424
R	0.438	0.885	0.965	3.777	2.821	-1.172	-0.733
TD	1072.140	1166.831	1247.923	1429.869	1453.248	1394.005	1457.225
IMP*	1005.439	1119.731	1267.010	1444.863	1486.441	1687.408	1855.419
M*	3246.219	3526.042	3971.063	4212.368	4594.088	5618.646	5910.277
CG*	1297.239	1434.354	1702.944	2110.942	2256.864	2563.926	2808.741
TDY*	204.168	212.011	256.775	276.161	283.887	314.641	348.777
NTDY*	493.845	527.917	570.112	624.877	663.356	705.948	752.475
CD*	252.792	277.112	326.198	360.518	353.678	451.752	509.710
NCD*	829.105	951.238	1138.822	1427.699	1585.339	1776.129	1998.616
RLZ	4.932	4.987	5.064	5.836	6.217	5.499	5.493
P	89.917	96.855	107.860	126.195	132.682	140.359	150.657
PI	0.913	0.968	1.056	1.202	1.254	1.315	1.397
YA	6583.355	6651.967	6744.283	6834.679	6968.866	7129.120	7258.828
YNA	8468.429	8799.171	9127.253	9489.719	9900.644	10255.226	10688.932
IA	423.151	404.422	444.069	455.795	540.761	679.765	692.006
PY	0.981	1.042	1.138	1.299	1.355	1.423	1.513
Y*	15814.877	17277.359	19523.592	22982.752	24870.420	27155.039	29819.212
PNA	0.944	1.014	1.085	1.257	1.235	1.137	1.199
YD*	14190.967	15365.082	17209.685	20346.497	22084.160	24086.568	26438.635
U*	2263.561	2417.269	2654.990	2736.137	2968.559	3601.592	3745.581
ZBP*	212.329	259.077	329.037	417.688	596.605	770.455	292.164
F*	225.561	193.269	232.990	188.137	49.559	567.592	518.581
FS*	1231.000	1313.000	1500.000	1633.000	1536.000	2255.000	2374.000
FA	8911.304	9202.685	9419.686	9243.009	9692.793	10554.701	10571.915
BP*	4447.329	4848.077	5331.037	5940.688	6573.605	7412.455	7876.164
TVR*	1779.910	1968.277	2291.907	2689.254	2886.260	3248.471	3609.577
KP	33005.112	38867.449	39703.156	40678.306	41811.277	42893.435	44444.874
KG	10666.649	11869.151	13160.457	14522.906	16147.302	17588.588	18967.453
K	48671.761	50736.600	52863.613	55201.212	57958.579	60482.023	63412.327
KA	7583.224	7756.719	7995.262	8232.704	8592.137	9032.387	9397.515
KNA	41088.537	42979.882	44868.331	46968.508	49366.442	51449.637	54014.812
M	3307.660	3383.610	3488.522	3243.642	3389.527	3949.836	3907.487
YD	14459.559	14744.423	15118.465	15667.379	16293.735	16932.548	17479.490

Table A.4 contd

	68-69	69-70	70-71	71-72	72-73	73-74	74-75
CP	14868.725	15216.537	15659.965	16157.601	16682.984	17176.341	17669.932
IP	2221.435	2311.054	2423.182	2563.670	2678.652	2816.505	2960.012
R	-1.273	1.340	4.132	3.190	5.861	4.534	3.871
TD	1481.736	1704.168	1936.661	2068.581	2311.459	2360.716	2375.728
IMP*	1817.624	1887.951	2094.188	2326.319	2561.985	3209.278	3760.150
M*	6027.085	6008.862	6449.627	7823.506	8887.673	10801.981	12371.851
CG*	3034.691	3286.036	3699.159	4136.564	4885.989	5428.576	5995.098
TDY*	341.116	382.041	434.087	540.148	647.779	764.029	849.056
NTDY*	777.984	813.309	881.423	953.914	1059.893	1172.843	1296.516
CD*	518.165	534.400	570.765	633.200	731.224	923.138	1148.832
NCD*	2051.757	2183.989	2580.425	3012.116	3713.636	4471.331	5315.426
RLZ	5.506	5.772	6.195	5.873	6.890	6.687	6.075
P	149.852	154.031	174.075	194.498	230.224	265.784	304.577
PI	1.391	1.424	1.584	1.747	2.031	2.315	2.624
YA	7373.642	7513.913	7651.971	7810.422	7952.989	8132.587	8292.399
YNA	11097.719	11471.756	11924.373	12379.712	12811.268	13325.686	13809.607
IA	676.562	724.499	707.132	730.495	740.202	823.612	864.035
PY	1.506	1.542	1.717	1.896	2.208	2.519	2.858
Y*	30455.555	32038.974	36786.101	41955.393	50355.759	59428.796	69536.428
PNA	1.231	1.252	1.548	1.779	2.058	2.208	2.458
YD*	26925.533	28313.235	32573.400	37252.015	44978.227	53012.456	62081.598
U*	3800.376	3739.049	3945.812	4706.681	5241.015	6279.722	7163.850
ZBP*	188.670	743.297	1088.459	367.186	741.457	1099.236	1286.268
F*	393.376	438.049	342.812	460.681	242.015	399.722	1069.850
FS*	2211.000	2326.000	2437.000	2787.000	2804.000	3609.000	4830.000
FA	11043.104	11550.192	11634.944	11732.906	11467.368	11566.291	11542.193
BP*	8367.670	9174.297	10205.459	10499.186	11331.457	12389.236	13830.268
TVR*	3689.022	3913.739	4466.701	5139.378	6152.532	7331.340	8609.830
KP	45957.679	47373.947	49158.382	50926.005	52543.410	54385.974	55958.257
KG	20218.940	21455.347	22814.138	24296.764	25763.166	27660.979	29610.485
K	66176.619	68829.294	71972.520	75222.769	78306.576	82046.952	85568.742
KA	9727.308	10138.652	10552.591	11038.820	11486.528	12064.363	12591.559
KNA	56447.311	58690.643	61419.929	64183.949	66820.048	69982.590	72977.182
M	4003.346	3896.646	3755.628	4126.517	4024.640	4237.763	4323.128
YD	17884.635	18360.658	18967.543	19648.618	20367.669	21042.887	21716.426

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Abbreviations, where used, are either self-explanatory or pertain to those used in the Journal of Economic Literature.

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