# Monetary policy and financial stability in the long run: A simple game-theoretic approach☆

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**Abstract**

Many theoretical central bank models use short horizons and focus on a single tradeoff. However, in reality, central banks play complex, long-horizon games and face more than one tradeoff. We account for these strategic interactions in a simple infinite-horizon game with a novel tradeoff: tighter monetary policy deters financial imbalances, but looser monetary policy reduces the likelihood of insolvency. We term these factors discipline and stability effects, respectively. The central bank's welfare decreases with dependence between real and financial shocks, so it may reduce costs with correlation-indexed securities. An independent central bank cannot in general attain both low inflation and financial stability.

*JEL Codes:* E50, G21, G28

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## 1. Introduction

The aftermath of the 2008 crisis featured a large effort by central banks and monetary authorities to address the question of financial fragility. In particular, central banks attempted to utilize novel methods to shore up the financial system and stave off potential incipient crises. A natural challenge concerns the extent to which central banks can actually achieve the goal of monitoring financial stability, while conducting more traditional roles of managing price and output stability. Our paper attempts to address this issue.

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Practically, a number of central banks have grappled with this challenge. For example, after the crisis, the Norwegian Central Bank explicitly included an additional "financial stability" term in the loss function of their analytical framework, using the deviation of the monetary policy rate from the long term equilibrium rate as a proxy for financial fragility. This consideration is exactly captured in our model, indicating that the central bank should internalize the social cost of financial instability in their decision making. Worldwide, regulators have been raising required capital ratios for banks, and imposing capital surcharges on systemically important financial institutions. These approaches can be seen in our model as reducing the number of undercapitalized banks, thereby ameliorating the social cost from bank failures.

What does a central bank do if it finds itself in a situation that is ex ante suboptimal for the banking system? There is a growing literature on optimal central bank policy towards banking intermediaries. Much of the game theoretic literature uses three or four period examples of the tension between central bank and intermediary incentives, which often allows for attractive optimality results.[1] The relevant challenge which we tackle is that many game-theoretic models of monetary policy and stability consider short horizons, and only analyze a single tradeoff. This setting is problematic because real world central banks face multiple tradeoffs with long horizons. Such multiple tradeoffs are particularly onerous because many central banks have one main policy tool during normal times, the interbank interest rate. The question we address is "What are the limits of monetary policy for a one-tool central bank which faces multiple policy tradeoffs that manifest over long horizons?"

While our paper adds to existing central bank literature that is based on games, it is complementary to another strand of research, namely dynamic stochastic general equilibrium (DSGE) models. This latter, non-game based literature, has recently started to tackle the issue of multiple tradeoffs between price and financial stability.[2] Another related literature uses dynamic general equilibrium models to address the issue of credit market imperfections and systemic externalities that derive from bank-level financial fragility.[3] This research has recently begun to uncover important limitations and extensions of previous work on optimal monetary policy.

An important strength of the DSGE approach is that it permits formalization of infinite horizon problems in a straightforward manner. Until recently, the DSGE approach was not utilized in studying strategic interaction and sustainable cooperation between intermediaries and central banks. While DSGE is an excellent tool for analysing dynamic equilibrium, for questions of purely strategic issues, a game theory approach has been shown to be complementary in the literature on central bank policy, as shown in the line of literature beginning with Barro and Gordon (1983). Because of the above reasons, we pursue this complementary approach, and highlight the importance of strategic interaction with a game-based analysis. Our game-theoretic framework may provide an alternative, complementary perspective to the abovementioned, ongoing DSGE research.

We complement the existing literature in several dimensions. First, as mentioned above, recently there have been papers analyzing the strategic interaction between the banking sector and the central bank in a DSGE framework, such as Cuciniello and Signoretti (2015), who analyzes

[1] See Freixas et al. (2000); Chapter 7 of Allen and Gale (2007); and Allen et al. (2009).

[2] See Faia and Monacelli (2007); Carlstrom et al. (2010); Otrok et al. (2012); Angelini et al. (2012); Angeloni and Faia (2013); De Fiore and Tristani (2013); and Unsal (2013).

[3] See Jeanne and Korinek (2010); Gilchrist and Zakrajsek (2011); Korinek (2011); Agénor et al. (2012); and Jeanne and Korinek (2013). For an overlapping generations approach, see Galí (2014). After the working paper version of our paper was distributed, we have become of other recent research in this area, including Collard et al. (2012); Kannan et al. (2012); Ueda (2013); Rabanal and Quint (2014); and Ueda and Valencia (2014).

the interaction of intermediaries and central banks, using a DSGE setup.[4] The latter authors derive an amplification mechanism that is positively correlated with the level of the central bank's inflation targeting. In their paper, the large banks' pricing on loans affects macroeconomic outcomes, which generates a feedback on monetary policy. Therefore this line of research mainly deals with the banks' role in amplifying real volatilities, instead of the fragility of the economy. Our paper is complementary to Cuciniello and Signoretti (2015), since we focus on the fragility issue rather than inflation alone. Specifically, in our paper we address the impact of banks' strategic behavior on financial stability, which allows us to analyze the central bank's tradeoff between price stability and financial fragility. In Gertler and Kiyotaki (2015), financial fragility arises from the fact that depositors can run on the banks when a sufficiently large negative shock on fundamentals is anticipated. However, the modeling framework is complex enough that it abstracts from business cycles. Thus, although it is a proper framework for designing policy for banking crises, it tends to say less about the central bank's tradeoff between price stability and financial fragility in normal times.

Banks have the potential to affect financial stability through their risk-taking behavior, while banks' incentive in risk-taking is partially driven by monetary policy. Consequently, such risk-taking is a crucial link for the feedback between monetary policy and financial stability. Angeloni et al. (2015) is one of the seminal papers on this linkage. In this approach, banks' risk-taking behavior is completely undesirable socially, which is not always the case in reality. Indeed, banking is inherently about risk-taking: through maturity transformation, banks take liquidity risk and improve risk sharing in the economy. The truly undesired result is not banks' risk-taking per se, but rather, excessive risk-taking. One contribution in this paper is to explain how banks' excess risk-taking endogenously arise from fire sale externalities, i.e., the individual bank does not internalize the impact of its failure on social fire sale cost. Banks' risk-taking in equilibrium enters the central bank's decision problem through the classic Barro and Gordon (1983) model, while the dynamic game theoretic approach makes it tractable to analyze the central bank's tradeoffs in an analytical solution.

Our paper obviously builds on the important work of Kashyap and Stein (2012). We add value to the Kashyap-Stein framework in three dimensions. First, we deepen their insight by formalizing the tradeoff between fragility and inflation, allowing for potential correlation of real and financial shocks. This setup delivers a feedback effect to central banks. Second, we endogenize the aforementioned feedback effect, based on a strategic model of interaction between the real economy and the banking sector. Third, we develop a simple long run framework and characterize the conditions under which the central bank can support a cooperative solution.

More broadly, our paper's main contribution to the game-theoretic banking literature is that we characterize the tradeoffs between inflation, investment and fragility that plague monetary policy, in a simple game-theoretic framework which accounts for long horizons. Moreover, we extend research on central bank independence, by demonstrating the limits on a central bank that attempts to achieve both low inflation and financial stability.

We proceed by modelling excessive systemic risk-taking arising from a fire-sale externality, and then summarizing the central bank's tradeoffs. We then model an economy with production and banking sectors that experience exogenous, correlated shocks, which feed back to the central bank. Subsequently, we endogenize feedback between the real economy and banking sector in a static and dynamic game, and demonstrate how a central bank can credibly support a cooperative

[4] We are grateful to an anonymous referee for providing this reference.

equilibrium with minimal fragility. A more detailed breakdown of our approach is provided in the next subsection.

### *1.1. Overview of the paper*

Since this paper covers a lot of ground, a brief overview to guide the reader is presented here. To establish a theoretical framework, in Section 2 we present a general form of the fragility-investment tradeoff, in which banks' investment decisions affect the likelihood of systemic events. This negative externality is not internalized by individual banks, and the resulting excessive aggregate investment in the banking sector increases systemic risk.

We further develop this idea in Section 3 by modeling excessive systemic risk-taking arising from a fire-sale externality, building on Kashyap and Stein (2012). We proceed in two steps. First, we describe the basic setup, where banks are financing long term projects with a mixture of short and long term debt. While short term debt is cheaper, banks are exposed to liquidity risks when they need to roll over the debt. If creditors refuse to roll over the debts in the intermediate term, there will be bank failures and banks will have to engage in fire sales, thereby liquidating premature projects. This fire-sale cost increases with the total assets on sale in the market, but individual banks do not internalize such costs when they make their initial investment decisions, which leads to excessive investment. The market equilibrium thus features excessive financial fragility and inefficiently high social welfare costs from fire sales. Second, we motivate the central bank's tradeoff between financial fragility, output, and price stability. To start with, we formalize this scenario with a production sector and banking sector that experience exogenous output shocks and liquidity shocks, respectively, where these two shocks are potentially correlated. A social welfare maximizing central bank therefore faces dual tradeoffs between financial fragility and inflation, when setting short term rates to stabilize output. The covariance of real and financial shocks reflects a feedback effect that directly affects the central bank's expected loss. In Section 3, such feedback effect is taken as exogenous.

In Section 4 we present an extended model, where we explicitly model the feedback between the real economy and banking sector, accounting for banks' strategic response to monetary policy. We proceed in three steps. First, we present the basic framework, with banks that invest in firms' capital via short- and long-term debt as before. In this framework, an intermediate output shock to real economy affects bank returns, making debt refinancing feasible. When the central bank sets short term interest rates to cushion an output shock, it affects both output and financial fragility. Second, the changing output level implies a change in banks' profit level, affecting their ability to refinance. Moreover, the short-term interest rate set by the central bank changes banks' refunding costs, which in turn changes the probability of bank failure. When there are more bank failures, more firms need to be liquidated, thus generating a loss in aggregate output. The key insight from Section 4 is that this feedback effect presents the central bank with a huge dilemma: when the policy rate needs to be raised to cool down an overheated economy, the central bank cannot raise the rate too high because that would increase the number of bank failures. Banks understand the central bank's dilemma, and therefore from the start of the game engage in excessive investment, which increases equilibrium financial fragility. Third, at the end of Section 4, we analyze the central bank's problem in an infinitely repeated game. We show in the longer run how the central bank can restrict banks' aggregate investment, lower financial fragility and restore the first best solution by adhering to the socially optimal interest rate. The deviating banking sector will be punished by the central bank's best response, and we characterize the range of central bank discount rates that ensures this cooperative solution.

*1.2. A dual policy tradeoff*

Modern central banks have tended to focus on one policy tool during normal times, the interbank interest rate r. However, central banks face a variety of policy objectives. For the purposes of this paper, we consider three policy objectives–inflation, employment or investment, and financial stability. Since these objectives often conflict, the central bank faces a dual policy tradeoff.

The first tradeoff, between inflation and unemployment, is well documented and understood. This basic tradeoff is known as the Phillips curve, first documented by Phillips (1958), then placed in a micro-founded setting by Lucas (1972) and Woodford (2003). The Phillips curve's ramifications for central bank policy are examined by Barro and Gordon (1983). The second tradeoff, between investment and financial stability, has only been recently analyzed, see Cao and Illing (2015); Chollete and Jaffee (2012); and Kashyap and Stein (2012). The crux of this tradeoff is that an interest rate policy which encourages investment has an externality effect of excess credit supply, which in turn increases financial instability. In this paper we analyze both tradeoffs from the perspective of the central bank.

*1.3. Contributions*

Our paper contributes to the literature on game theoretic analysis of optimal monetary policy by formalizing the investment fragility tradeoff faced by a central bank. We then characterize the dual tradeoff from inflation-investment-fragility that plagues monetary policy, in a simple static game. Finally, we extend our results to a dynamic game setting. Our main contribution to the game-theoretic banking literature is that we characterize the long horizon tradeoffs between inflation, investment and fragility that plague monetary policy. In a related sense, we extend research on central bank independence (e.g. Rogoff (1985)), by demonstrating that when a central bank attempts to achieve financial stability, it is restricted in how to address inflation, and will face nontrivial effects on financial risk.

The remainder of the paper is organized in the following manner. Section 2 motivates the relationship between financial fragility and investment. Section 3 motivates the central bank's tradeoff between fragility and investment, where the feedback between the real and financial factors comes from the correlation of shocks. Section 4 studies an extended model that accounts for a feedback mechanism between the real and financial sectors, and Section 5 is the conclusion.

## 2. Motivation for fragility-investment relation

In order to motivate the dual tradeoff approach to macroprudential policy, we need a functional relation for the second tradeoff[5], between fragility $f$ and aggregate investment liabilities $I$. That is, we require a simple micro-foundation for the function $f = f(I)$. A natural way to do this is to consider the well-documented *fire-sale externality* phenomena of banks: during normal times, banks overinvest. Since every bank does this, during extreme events the whole system is adversely affected.

[5]For the first tradeoff, the Phillips curve, see Phillips (1958); Lucas (1972); and Woodford (2003).

*2.1. Leverage-based fragility measure*

This approach models an externality from excessive investment, as in Fisher (1933); Keynes (1936); Allen and Gale (2007); and Kashyap and Stein (2012). We summarize the approach of Kashyap and Stein (2012). Consider a large number of banks that play an infinitely repeated game, in periods denoted by $t$, for $t = 0, 1, ...\infty$. In order to model revelation of information about systemic risk fragility, each period $t$ is further subdivided into three dates $t_0$, $t_1$, and $t_2$. Every bank $i$ holds exogenously determined equity $k^i$, whose value is uniformly distributed over the interval $[0, K]$.

Banks are in the business of investing, which can funded in two ways – short term or long term debt. Specifically, each period the representative bank has an investment opportunity, which, during *normal times*, transforms investment $I^i$ made at initial date $t_0$ into $\theta I^i$ two dates later at $t_2$, for $\theta > 1$. During extreme periods, investment yields no profit, and $\theta = 0$. In order to fund $I^i$, bank $i$ may issue an amount $mI^i$ of short-term debt and $(1 - m)I^i$ of long-term debt. The gross interest rates on these two types of debts are $r_1$ and $r_1 + \delta$, respectively. Therefore the bank's costs in normal times are $mI^i r_1 + (1 - m)I^i(r_1 + \delta) = I^i(r_1 + \delta - m\delta)$, implying net profits of $I^i(\theta - r_1 - \delta + m\delta)$.

At date $t_1$ there is a public signal of the $t_2$ return from investment. With probability $1 - p$ it will be a normal state in $t_2$, with investment return of $\theta I^i$, while with probability $p$ (for systemic) it will be in the crisis state, with investment return of 0. In a normal state, debtors will roll over the banks' debt, while in a crisis state, debtors refuse to roll over debt. A bank goes bankrupt if its equity value $k^i$ is less than the value of short term debt $mI^i$ at date $t_1$. Since $k^i$ is uniformly distributed over $[0, K]$ the probability that bank $i$ will fail is simply $\frac{mI^i}{K}$. For simplicity, the likelihood of systemic risk is assumed to be linear in aggregate investment $I \equiv \sum_i I^i$. Similarly, the cost $C^i$ that each bankrupt firm imposes on society is also linear, $C^i = \gamma \cdot \sum_i I^i$, for $\gamma > 0$. Under the insolvency of bank $i$, its entire assets will be sold at a depressed price $p$, which applies to all other banks' assets and implies a cost for bank $i$ of $C^i = \gamma \sum_i I^i$. Thus bank $i$'s costs during extreme periods are given by $\frac{pmI^i\gamma \sum I^i}{K}$.

Such a fire-sale externality implies that each bank does not internalize the cost it imposes on the other banks when it fails. The problem for bank $i$ at the beginning of period $t$, date $t_0$ is to maximize expected profit $\Pi^i$:

$$\max_{I^i} \Pi^i = I^i(\theta - r_1 - \delta + m\delta) - \frac{pmI^i\gamma \sum_i I^i}{K}. \tag{1}$$

The solution to (1) is given by

$$I^c = (\theta - r_1 - \delta + m\delta)\frac{K}{pm\gamma}, \tag{2}$$

where the $c$ denotes the competitive outcome, and we remove superscript $i$ for simplicity. The socially optimal value maximizes joint profits $\sum_i \Pi^i = \sum_i I^i(\theta - r_1 - \delta + m\delta) - \frac{pmI^i\gamma \sum_i I^i}{K}$. The corresponding optimal investment[6] is given by

$$I^p = (\theta - r_1 - \delta + m\delta)\frac{K}{2pm\gamma}, \tag{3}$$

[6]For further details, see p. 272 of Kashyap and Stein (2012).

where $p$ denotes a Pareto optimum. Similar to the results in the above subsection, the competitive level of investment $I^c$ exceeds the optimum $I^p$.

**Fragility Measure** $f$. In this framework, financial fragility $f$ is represented by the probability of bankruptcy due to excess leverage. With a continuum of competitive banks, this probability is the mass of banks with capital less than $mI^c$, that is, $\frac{mI^c}{K}$. We therefore define our main fragility measure $f$ as below.

**Definition 1** The l**everage-based fragility** measure is $f(I) = \frac{mI}{K}$.

We summarize the relevance of the fragility measures in Proposition 1, and the inefficiencies that motivate central bank intervention in Proposition 2, below.

**Proposition 1** *In a competitive banking system, financial fragility $f$ can be represented as a function of equilibrium aggregate bank investment $I = \sum_i I^i$.*

**Proposition 2** *In a competitive banking system with fire-sale externalities, the leverage-based fragility $f$ is inefficiently high.*

Now that we have established the concept of financial fragility, we utilize it in the remainder of the paper in a game theoretic setup.

## 3. Modelling tradeoffs between fragility, investment and output

In this section we develop a simple model to establish a theoretical framework, and throughout we focus on describing the second, financial fragility tradeoff. As a first step, the linkage between financial sector and real economy is established by the correlation between financial and technological shocks. The explicit macro-finance feedback mechanism will be further developed in the succeeding section. Since the first tradeoff of unemployment and inflation is well understood, we utilize existing results from that literature directly. The setting is standard, where the central bank knows the payoff functions of banks.[7] The central bank interacts with banks and entrepreneurs, and plays the game repeatedly. Such repetitions are cumbersome to represent, hence we simplify compute by focusing on a stage game, which is a one-shot version of the repeated game. If the full game is played $T$ times, then the payoff for each player is simply the discounted sum of the payoffs in each stage game.

**Notation**. This paper uses several types of notation in the next section. In order to ease the reader's understanding of the paper, we present below the most frequently used parameters and symbols.

- $a$: Cost to central bank of missing inflation or interest target
- $b$: Cost to central bank of missing investment target
- $\beta$: Phillips curve-based cost to central bank of missing interest target
- $\pi^*$: Target inflation rate
- $\phi$: The inverse of the rate of risk aversion
- $I^*$: Socially optimal level of investment
- $K$: Total equity in the banking system

[7]For earlier work, see Kydland and Prescott (1977), and Barro and Gordon (1983). For details on repeated games, see Fudenberg and Tirole (1991); Gibbons (1992); and Mas-Colell et al. (1995).

- $\gamma$: Multiplier for systemic costs relative to $I$, which measures fire-sale costs
- $m$: Fraction of short-term debt in banking system, which measures system illiquidity
- $\varepsilon_y$: Output shock
- $G\left(E\left[\varepsilon_y^{i*}\right], \hat{r}\right)$: The number of banks that are expected to fail

*3.1. A base model without feedback effects*

First, banks and companies form expectations of financial fragility, $\hat{f}$, and inflation, $\hat{\pi}$, respectively. Second, the policymaker assesses expectations and chooses the actual fragility $f$ and inflation $\pi$. Banks and companies receive payoffs of $-(f-\hat{f})^2$ and $-(\pi-\hat{\pi})^2$, respectively. That is, banks (companies) desire to anticipate fragility (inflation) as accurately as possible, achieving their maximal payoff when $f = \hat{f}$ $(\pi = \hat{\pi})$. It is the policymaker's desire for fragility, inflation, and output to close to target levels, $f^*$, $\pi^*$ and $y^*$.

The policymaker's loss function is defined as

$$\min_{\pi} L = (y-y^*)^2 + a\,(\pi-\pi^*)^2 + b\,[f(I)-f^*]^2\,, \tag{4}$$

where $a > 0$ and $b > 0$ reflect costs to the central bank of missing its targets. All terms in (4) depend on the inflation rate $\pi$, in a manner that we now make explicit via the inflation-output and fragility-investment tradeoffs, below. The inflation-output relationship is defined by the Phillips curve,

$$y = \overline{y} + \beta\left(\pi - \hat{\pi} + \varepsilon_y\right), \tag{5}$$

where $\beta > 0$, and $\varepsilon_y$ is a zero-mean shock to output, $\varepsilon_y \sim [0, \sigma_y^2]$, and the relationship between real interest rate and output is charaterized by the standard $IS$ curve[8]

$$y = -\phi\,(r_1 - \pi) + \overline{y}. \tag{6}$$

We model the relationship between financial investment and the short-term interest rate by building on the Kashyap and Stein (2012) specification, from equation (2) above:

$$I = \left(\theta - r_1 - \delta + m\delta + \varepsilon_f\right)\frac{K}{pm\gamma}.$$

Note that, unlike Kashyap and Stein (2012), we account for random financial shocks via the term $\varepsilon_f$, $E[\varepsilon_f] = 0$. The covariance matrix $V$ for $\varepsilon_y$ and $\varepsilon_f$ is of the form $V = \begin{bmatrix} \sigma_y^2 & \rho_{y,f} \\ \rho_{y,f} & \sigma_f^2 \end{bmatrix}$. Financial fragility is defined as the cost of a fire sale, namely $f(I) = \left(\theta - r_1 - \delta + m\delta + \varepsilon_f\right)\frac{K}{pm}$. And the socially optimal financial fragility $f^*$ is $f^* = \gamma I^* = \gamma\left(\theta - r_1 - \delta + m\delta + \varepsilon_f\right)\frac{K}{2pm\gamma}$, or

$$f^* = \left(\theta - r - \delta + m\delta + \varepsilon_f\right)\frac{K}{2pm}. \tag{7}$$

[8] Motivated by Clarida et al. (1999).

Therefore, the central bank's decision problem in (4) can be rewritten in terms of $\pi$, using (5), (6), and (7), as

$$\min_{\pi} L = \left[\overline{y} + \beta\left(\pi - \hat{\pi} + \varepsilon_y\right) - y^*\right]^2 + a\left(\pi - \pi^*\right)^2 + b\left[\left(\theta - r_1 - \delta + m\delta + \varepsilon_f\right)\frac{K}{2pm}\right]^2. \quad (8)$$

This objective is solved in Appendix A to yield expressions for optimal inflation rate, denoted by $\pi^c$, and output $y$:

$$\pi^c = \frac{1}{B}\left[-\beta\left(\overline{y} - y^*\right) + a\pi^* + \left(\theta + \frac{y - \overline{y}}{\phi} - \delta + m\delta\right)C\right] - \frac{1}{A}\left[\beta^2\varepsilon_y - C\varepsilon_f\right] \quad (9)$$

and

$$y = \overline{y} + \frac{\beta}{A}\left(B\varepsilon_y + C\varepsilon_f\right), \quad (10)$$

where the coefficients $A$, $B$ and $C$ capture the relative weights attached to various forms of over- or undershooting: $A = a + \beta^2 + b\left(\frac{K}{2pm}\right)^2$, $B = a + b\left(\frac{K}{2pm}\right)^2$, and $C = b\left(\frac{K}{2pm}\right)^2$.

The expected welfare loss $E[L^n]$ is also computed in Appendix B, to be

$$\begin{aligned} E[L^n] &= (\overline{y} - y^*)^2 + a\left[\frac{-\beta\left(\overline{y} - y^*\right) + (\theta + \frac{y-\overline{y}}{\phi} - \delta + m\delta - \pi^*)C}{B}\right]^2 \\ &\quad + C\left[\frac{-\beta\left(\overline{y} - y^*\right) + a\pi^* + a(\theta + \frac{y-\overline{y}}{\phi} - \delta + m\delta)}{B}\right]^2 + \frac{B^2 + \beta^2(a + C)}{A^2}\beta^2\sigma_y^2 \\ &\quad + \frac{(a + \beta^2)^2(C^2 + C)}{A^2}\sigma_f^2 + 2C\beta^2\frac{B + \beta^2}{A^2}\rho_{y,f}. \end{aligned}$$

where $\sigma_y^2$, $\sigma_f^2$ and $\rho_{y,f}$ are the real shock variance, financial shock variance, and correlation of the two shocks, respectively. In contrast, the cooperative, expected welfare loss $E[L^c]$ under perfect anchoring is computed in the Appendix to be

$$E[L^c]. = E\left[\left(\overline{y} - y^* + \beta\varepsilon_y\right)^2 + \left(\frac{K}{2pm}\varepsilon_f\right)^2\right] = (\overline{y} - y^*)^2 + \beta^2\sigma_y^2 + \frac{C}{b}\sigma_f^2.$$

**Comments on the solution**. Two aspects of the above solutions appear cogent. First, the volatility of shocks to both the financial and real sectors matter for optimal loss of the central bank. Second, and perhaps more novel, the covariance of real and financial shocks, $\rho_{y,f}$, directly affects the central bank's expected loss. In particular, it is not enough for the central bank to focus on real or financial shocks: it must *also* identify the dependence between real and financial shocks. We summarize this latter observation in a proposition, below.

**Proposition 3**: *In a dual tradeoff model, the central bank's expected loss increases monotonically with the dependence between financial and real shocks.*

*3.2. Explanatory power and novel implications*

By establishing a macro-finance linkage through correlation between financial and real shocks, our theoretical model provides a simple method to incorporate financial and real considerations, simultaneously, in a tractable form for central bank policy. It also offers a potential explanation of several interesting phenomena. In particular, a large, positive real economy shock $\varepsilon_y$ shifts the production function up and raises marginal productivity. Since marginal productivity determines marginal return on capital, banks increase their supply of credit. Consequently real shocks have financial effects, and our model explains the empirically observed procyclicality of credit.

Similarly, a large negative financial shock $\varepsilon_f$ reduces the amount of credit $I$ that banks supply. This, in turn, reduces firms' investment and therefore results in output contraction. Hence, our model explains the empirically observed occasional spillovers from financial crises to the real economy. These spillovers will be exacerbated, the larger the correlation between real and financial shocks.

**Implications**. Our model also has some novel implications. In particular, it suggests a dual tradeoff between enhancing economic output and maintaining financial fragility. The reason is that the central bank's policy instrument $r_1$, the short rate, relates positively to the inflation rate as in the $IS$ curve but negatively in the financial credit supply, equation (2). Central banks will experience the bite of this tradeoff, the larger the correlation between real and financial shocks.

**Potential policy tools**. In light of the above analysis, effective policy instruments in the face of this dual tradeoff must reduce real-financial correlations $\rho_{y,f}$. One possible approach is for central banks to market *correlation-indexed bonds*, i.e. securities that are indexed to the level of dependence between the real and financial sector. While these securities are appealing from a theoretical perspective, the central bank would first require accurate measures of dependence. One possibility is to utilize robust multivariate dependence measures (Joe (1997), Nelsen (1998)) such as those explored by Patton (2006) and Chollete et al. (2009).

## 4. A model with feedback between banks and firms

Thus far, we have modelled a simple link between banks and firms, via the correlation of shocks, $\rho_{f,y}$. More realistically, banks create credit $I$ that firms use to produce real output $y$. Hence the central bank's choice of interest rate $r$ affects bank credit, which affects real output. In turn, the real output is observed, which affects the central bank's choice of interest rate. In this section, we attempt to build a mechanism that captures such dual direction feedback.

*Feedback from the real economy to the financial sector*, or, *business driven credit cycles*. Our framework allows us to account for feedback from the real economy, because the banking sector decide the amount of credit $I$ available depending on its expected marginal return. This marginal return, $\theta$ from the Kashyap and Stein (2012) specification, is equal in competitive markets to the marginal productivity of capital in the real economy.[9]

*Feedback from financial sector to the real economy*, or, *credit driven business cycles*. This framework also allows us to capture the other direction of the feedback, which is less well understood in existing literature. It endogenously accounts for the real effect of financial instability, since it reduces $I$ via fire sales. In turn, the reduced $I$ decreases real output.

[9]This approach is similar to the credit transmission channel of Bernanke and Gertler (1989), known as financial accelerators, on which a large share of the current literature on macro-finance linkages is based.

| $t_0$ | $t_1$ | $t_2$ |
|---|---|---|
| Bank $i$ provides credit $I^i$ to firms, based on its expectation of central bank's choice on inflation rate $\hat{\pi}$;<br><br>The credit $I^i$ is financed by both short term and long term debt contract, with interest rates $r_1$ and $r_1 + \delta$, respectively;<br><br>The firms use the credit to buy capital input, and start production. | The shock to the production sector $\varepsilon_y$ gets revealed;<br><br>The central bank decides the new inflation rate $\pi^E$, based on the shock, inflation, and cost of bankruptcies in the financial sector;<br><br>The bank gets bankrupt if its equity value becomes negative, and its assets are sold at depressed price;<br><br>If the bank survives, it repays the existing short-term debt contracts, and rolls over part of the debts by issuing new short-term debts with interest rate $r^E$ under $\pi^E$. | The surviving banks get return from its investment on the firms, and repay both the short and long term debt contracts;<br><br>The failed banks get dissolved, and debts cleared. |

Figure 1: Timeline of events in the model

We therefore revisit the Phillips curve and financial investment equations from the previous section. In light of the above observations, we propose to account for a macro-finance linkage via the standard Phillips curve, that the aggregate output $y$ is given by

$$y = \overline{y} + \beta\left(\pi - \hat{\pi} + \varepsilon_y\right), \tag{11}$$

where $\overline{y}$ denotes the natural rate of output, and firms' output is produced from the capital input which comes from banks' credit supply, $I$, as well as the standard $IS$ curve

$$y = -\phi\left(r_1 - \pi\right) + \overline{y}.$$

We retain our previous specification of the relationship between financial investment and the short-term interest rate from Section 3 above:

$$I = \left(\theta(y) - r_1 - \delta + m\delta + \varepsilon_f\right)\frac{K}{pm\gamma}. \tag{12}$$

Hence the financial sector and real sector are linked in three ways. First, as discussed above, the two equations (11) and (12) are linked via the function $\theta(y)$, which represents the marginal return on bank capital in (12), and is determined by the marginal product of $y$ from equation (11). Second, they are linked by the available credit for investment $I$, which is a component of real output. Finally, they are linked via the short rate $r_1$, which is set by the central bank in response to output shock $\epsilon_y$.

### *4.1. Formalization of the model*

Consider a large number of banks and firms that play an infinitely repeated game with a central bank, in periods denoted $t$, for $t = 0, 1, \ldots, +\infty$. Each period $t$ is further subdivided into three dates $t_0$, $t_1$, and $t_2$. The timing of the model is summarized in Figure 1.

**Stage** $t_0$: Each bank $i$ decides its loan or credit supply $I^i$ to firms, given its expectation on the future inflation rate $\hat{\pi}$ (hence the central bank's short-term (one-period) policy rate $\hat{r}$) and the firms' expected output. To finance $I^i$, bank $i$ may issue an amount $mI^i$ of short-term debt and $(1-m)I^i$ of long-term debt, the later of which lives from $t_0$ to $t_2$. The short-term debt is rolled over at $t_1$ with expected rate $\hat{r}$. With similar notation to the previous section, the gross interest rates on these two types of debt are $r_1$ and $r_1+\delta$, respectively, where compounding yields $r_1 = \hat{r}^2$.

The firms use the loan $I^i$ as an input. The return from the bank's investment in firms, $\theta$, is determined by the expected aggregate output $E[y]$, and $\theta(E[y])$ is an increasing function of $E[y]$. The bank's expected costs are therefore $C(m, r_1) = mI^i r_1 + (1-m)I^i(r_1+\delta)$, implying expected net profits of $\theta(E[y])I^i - C(m, r_1)$, if it survives till $t_2$. Without much loss of generality, assume that $\theta(E[y]) = \omega E[y]$.

**Stage** $t_1$: At $t_1$ there is a publicly observed signal of the aggregate shock on date $t_2$'s return from firms. The shock is denoted by $\varepsilon_y \sim N\left(0, \sigma^2_{\varepsilon_y}\right)$. After the shock, the central bank sets its inflation rate $\pi^E$, based on its loss function:

$$\min_{\pi^E} \qquad L = (y-y^*)^2 + a\left(\pi^E - \pi^*\right)^2 - \gamma \sum_i I^i G\left(E\left[\varepsilon^{i*}_y\right], \hat{r}\right),$$

where the superscript $E$ denotes equilibrium. The term $G\left(E\left[\varepsilon^{i*}_y\right], \hat{r}\right)$ denotes the number of banks expected to fail, which depends on both real and monetary shocks, as explained later in this section. This policy rate will affect aggregate output at $t_2$. Actual output is determined by the input $I$, subject to price stickiness. We define this inflation-output relationship by the Phillips curve

$$y\left(\pi^E, \varepsilon_y\right) = \bar{y} + \beta\left(\pi^E - \hat{\pi} + \varepsilon_y\right)$$

and the goods market is cleared by the $IS$ curve

$$y\left(\pi^E, \varepsilon_y\right) = -\phi\left(r^E - \pi^E\right) + \bar{y}$$

where $r^E$ is the monetary policy rate through which the central bank implements $\pi^E$.

The investment under natural output $\bar{y}$ is denoted by $\bar{I}$. In a state when the bank's return is higher than $r^E$, debtors will roll over banks' debt, while in other states, debtors refuse to roll over debt. The bank fails if its equity value $k^i$ (uniformly distributed on $[0, K]$) is less than the value of short term debt $mI^i$ net the expected value of its assets at date $t_2$. If the bank fails, all of its assets will liquidated in a fire sale, subject to the fire sale cost.

**Stage** $t_2$: If the bank survives, the firms will repay the loans, and its debtors withdraw.

The equilibrium of the model is featured by the following set of functions:

(1) Given $\pi^E$, the firms' aggregate output at $t_2$ is $y\left(\pi^E, \varepsilon_y\right)$, implying that the gross return to the bank's investment is $\theta = \omega y\left(\pi^E, \varepsilon_y\right)$.

At $t_1$, the bank will fail if $mI^i\hat{r}r^E + (1-m)I^i(r_1+\delta) - \omega y I^i > k^i$. Define $y^{i*}$ and $\varepsilon^{i*}_y$ such that $mI^i\hat{r}r^E + (1-m)I^i(r_1+\delta) - \omega y^{i*}I^i = k^i$, and $y^{i*} = \bar{y} + \beta\left(\pi - \hat{\pi} + \varepsilon^{i*}_y\right)$. Then the ex post probability that the bank will fail is $G\left(\varepsilon^{i*}_y\right)$.

(2) If the bank fails at $t_1$, its assets will be sold at the depressed price. The fire sale cost is $\gamma \sum_i I^i$.

At $t_0$ the bank's optimal decision on investment is determined by maximizing its profit

$$\max_{I^i} L_B^i = \omega(E[y])I^i - C(m, r_1) - \gamma \sum_i I^i G\left(E\left[\varepsilon_y^{i*}\right], \hat{r}\right),$$

where the $B$ superscript denotes the bank, $G\left(E\left[\varepsilon_y^{i*}\right], \hat{r}\right)$ denotes the ex ante probability of insolvency, and rational expectations imply that $\hat{\pi} = E\left[\pi^E\right]$.

At stage $t_0$ the following relation holds: $\omega(E[y]) = \omega E\left[\bar{y} + \beta\left(\pi - \hat{\pi} + \varepsilon_y\right)\right] = \omega I$, where $I$ is the aggregate credit supply, taken as given for each individual bank.

Given the bank's expectations for central bank policy, $\hat{\pi}$, the bank's expected probability of becoming insolvent[10] at $t_1$ is $G\left(E\left[\varepsilon_y^{i*}\right], \hat{r}\right) = \frac{mI^i\hat{r}^2 + (1-m)I^i(r_1+\delta) - \omega II^i}{K}$. Therefore, the bank's optimal decision problem at $t_0$ is

$$\max_{I^i} L_B^i = \omega II^i - mI^i r_1 - (1-m)I^i(r_1+\delta) - \gamma \sum_i I^i \frac{mI^i\hat{r}^2 + (1-m)I^i(r_1+\delta) - \omega II^i}{K}.$$

Note that since we are in a two period model, the short rate $\hat{r}$ and long rate $r_1$ are related by $r_1 = \hat{r}^2$.

**Bank's competitive equilibrium outcome** $I^E$. Under the above assumptions, the bank's first order condition yields

$$\frac{\partial L_B^i}{\partial I^i} = \omega I - mr_1 - (1-m)(r_1+\delta) - \gamma I \frac{m\hat{r}^2 + (1-m)(r_1+\delta) - \omega I}{K} = 0.$$

This condition is satisfied by the equilibrium investment $I^E$ such that

$$\omega I^E K - r_1 K - K\delta + Km\delta - \gamma I^E r_1 - \gamma I^E \delta + \gamma I^E m\delta + \gamma(I^E)^2\omega = 0, \tag{13}$$

where the superscript $E$ denotes equilibrium. However, the bank does not take into account the fire-sale externality it imposes to the entire financial system, which leads to excessive investment.

**Bank's social optimum** $I^p$. We now account for fire-sale externalities. If a planner forces the bank to consider fire-sale costs, the bank will solve the following decision problem:

$$\max_{I^p} L_B^p = \omega(E[y(I^p)])I^p - C(m, r_1) - \gamma \sum_i I^p G\left(\varepsilon_y^{i*}, \hat{r}\right)(i),$$

where the superscript $p$ denotes planner. This maximization problem can be rewritten

$$\max_{I^p} L_B^p = \omega(I^p)^2 - mI^p r_1 - (1-m)I^p(r_1+\delta) - \gamma I^p \frac{mI^p\hat{r}^2 + (1-m)I^p(r_1+\delta) - \omega(I^p)^2}{K}.$$

The first order condition yields

$$\frac{\partial L_B^p}{\partial I^p} = 2\omega I^p - mr_1 - (1-m)(r_1+\delta) - \frac{2\gamma I^p m\hat{r}^2 + 2\gamma I^p(1-m)(r_1+\delta) - 3\gamma\omega(I^p)^2}{K} = 0,$$

[10] Our formulation of $G(\cdot)$ therefore generalizes the Kashyap and Stein (2012) framework, since the short rate enters $G(\cdot)$. This formalizes the notion that the central bank can affect the likelihood of default.

or the $I^p$ such that

$$2I^p\omega K - r_1 K - K\delta + Km\delta - 2I^p\gamma r_1 - 2I^p\gamma\delta + 2I^p\gamma m\delta + 3(I^p)^2\gamma\omega = 0. \tag{14}$$

As summarized in Proposition 4 below, the bank's equilibrium investment is larger than the social optimum, the latter of which accounts for excess fragility.

**Proposition 4: Inefficiency of bank's competitive equilibrium investment**

*Part A: In market equilibrium, the bank's investment is larger than the planner's solution, i.e.,* $I^E > I^p$*. Moreover,* $\sqrt{3}I^p < I^E < 2I^p$*.*

*Part B: In market equilibrium, liquidity risk provides some market discipline, but investment is still excessive.*

Part A is straightforward, and provides an upper-bound on the magnitude of over-investment. Part B of the proposition says that a higher share of short-term debt (higher $m$) reduces banks' investment. Equivalently, liquidity risk restricts banks' risk-taking in the market equilibrium, thereby providing some market discipline. Nevertheless, due to the fire-sale externality, market discipline is not sufficient. Compared with the planner's solution, the competitive equilibrium still features excessive investment.

**Central bank's decision.** We now derive the central bank's optimal strategy for short rates. To establish the theoretical framework, we focus on the case where banks choose the fragility-based excessive credit supply $I^E$ solved above[11]. The central bank's optimal decision at $t_1$, after observing $\varepsilon_y$, is to solve the following program:

$$\min_{\pi^E} L_C = (y - y^*)^2 + a\left(\pi^E - \pi^*\right)^2 + b\gamma \sum_i I^i G\left(\varepsilon_y^{i*}\right)(i),$$

where the subscript $C$ denotes the central bank, subject to the Phillips curve and the $IS$ curve.

From the Appendix C, the central bank's optimal inflation rate $\pi^E$ is proved to satisfy

$$\begin{aligned}\pi^E \quad = \quad & \frac{2aK}{2aK + b\gamma(I^E)^2 m}\pi^* + \frac{2\beta\phi K}{2aK + b\gamma(I^E)^2 m}(y^* - \overline{y}) + \frac{mb\gamma\left(I^E\right)^2}{2a\phi K + b\phi\gamma(I^E)^2 m}\left(I^E - \overline{I}\right) \\ & + \frac{b\gamma\left(I^E\right)^2\omega\beta}{2aK + b\gamma(I^E)^2 m} - \frac{\beta^2}{a + \beta^2}\varepsilon_y.\end{aligned} \tag{15}$$

The solution for $\pi^E$ in (15) can be related to extant models. For example, as in Barro and Gordon (1983), $\pi^E$ should partially respond to demand shocks, which is intuitive. There are, however, some important differences between $\pi^E$ and the short rate obtained by a traditional central bank concerned with inflation targeting. We discuss these differences below.

*4.2. Implications for macroprudential policy*

In conventional inflation-targeting frameworks, the central bank's decision problem does not account for fire sales. Specifically, the central bank's problem is

[11] Given the symmetric structure of the problem, the central bank will solve the same program when banks supply the socially optimal credit $I^p$, except that everywhere in the program it would replace $I^E$ with $I^p$.

$$\min_{\pi^0} L = (y - y^*)^2 + a\left(\pi^0 - \pi^*\right)^2,$$

subject to the Phillips curve

$$y\left(\pi^0, \varepsilon_y\right) = \overline{y} + \beta\left(\pi^0 - \hat{\pi} + \varepsilon_y\right)$$

in which $\pi^0$ is the inflation set by a pure inflation targeting central bank. The first order condition yields

$$\frac{\partial L}{\partial \pi^0} = 2\beta\left[\overline{y} - y^* + \beta\left(\pi^0 - \hat{\pi} + \varepsilon_y\right)\right] + 2a\left(\pi^0 - \pi^*\right) = 0,$$

or

$$\pi^0 = \frac{a\pi^* + \beta\left(y^* - \overline{y}\right) + \beta^2\hat{\pi} - \beta^2\varepsilon_y}{a + \beta^2}.$$

The consistency condition $E\left[\pi^0\right] = \hat{\pi}$ implies that $\hat{\pi} = E\left[\pi^0\right] = E\left[\frac{a\pi^*+\beta(y^*-\overline{y})+\beta^2\hat{\pi}-\beta^2\varepsilon_y}{a+\beta^2}\right] = \frac{a\pi^*+\beta(y^*-\overline{y})+\beta^2\hat{\pi}}{a+\beta^2}$, or $\hat{\pi} = \pi^* + \frac{\beta}{a}\left(y^* - \overline{y}\right)$. By substituting this condition into the expression for $\pi^0$, we obtain the optimal inflation rate $\pi^0$ for an inflation-targeting central bank:

$$\pi^0 = \pi^* + \frac{\beta}{a}\left(y^* - \overline{y}\right) - \frac{\beta^2}{a + \beta^2}\varepsilon_y. \tag{16}$$

Recall that for a central bank which takes financial stability into account, the optimal short rate $r^E$ from (15) satisfies

$$\begin{aligned}\pi^E \quad = \quad & \frac{2aK}{2aK + b\gamma(I^E)^2 m}\pi^* + \frac{2\beta K}{2aK + b\gamma(I^E)^2 m}\left(y^* - \overline{y}\right) + \frac{mb\gamma\left(I^E\right)^2}{2a\phi K + b\phi\gamma(I^E)^2 m}\left(I^E - \overline{I}\right)\\ & + \frac{b\gamma\left(I^E\right)^2\omega\beta}{2aK + b\gamma(I^E)^2 m} - \frac{\beta^2}{a + \beta^2}\varepsilon_y.\end{aligned}$$

**Observations on the solution.** By comparing the solution for $\pi^0$ in (16) to the one for $\pi^E$ above, we can see the role of financial stability in the central bank's decision making. In particular, we discern two effects.

The first is a *discipline effect*: in the first two terms of $\pi^E$, $\frac{2aK}{2aK+b\gamma(I^E)^2 m} < 1$ and $\frac{2\beta K}{2aK+b\gamma(I^E)^2 m} < \frac{\beta}{a}$, which indicates that the central bank is cautious in stimulating the economy, in order to restrict excessive investment. This effect is in line with suggestions by John Taylor and others, who argued after the 2008 global financial crisis that a central bank which is concerned about financial stability should be more aggressive (relative to what the Taylor rule suggests) in containing inflation in normal times, in order to deter build-up of financial imbalances. Here we do see such an incentive.

The second is the *stability effect*: in the following terms of $\pi^E$, $\frac{mb\gamma\left(I^E\right)^2}{2a\phi K+b\phi\gamma(I^E)^2 m}\left(I^E - \overline{I}\right) +$

$\frac{b\gamma(I^E)^2\omega\beta}{2aK+b\gamma(I^E)^2 m} > 0$, which indicates that the central bank's "stability target" inflation rate should be higher, once it cares about financial fragility. Thus, the central bank has the incentive to overheat the economy in order to increase the return and reduce the insolvency risk in the banking sector.

More broadly, these findings extend the literature on central bank independence (e.g. Rogoff (1985)). This literature argues that in order to achieve price stability, monetary policy should be delegated to a "conservative" central bank that only cares about inflation or, in the context of our model, $a \to +\infty$. In such a case, our model implies the following condition: $\lim_{a\to+\infty} \pi^0 = \pi^E = \pi^*$.

What does our solution imply about dynamics? There is obviously the potential for moral hazard. If banks have a systematical expectation on loose monetary policy and higher $I^i$ at $t_0$, the central bank will be, ex post, forced to follow a loose monetary policy if it has a low discount factor. We discuss this issue further in the repeated game formulation below.

### *4.3. An impossibility result*

Our framework may be valuable to clarify the limitations on central banking imposed by attempts to control both inflation and fragility. Intuitively, the only time a central bank's dual targets of inflation control and stability are compatible is when the inflation rates $\pi^E$ and $\pi^0$ are equal. The central bank can use the same rate to control both inflation and targeting only if $\pi^E - \pi^0$ is identically zero. We solve this condition in the Appendix, which yields the following proposition:

**Proposition 5: General impossibility of both fragility control and inflation commitment.** *Even if a central bank is independent, it is generally unable to control simultaneously fragility and inflation. Such control is only possible in the knife-edge case of* $\pi^* = \frac{\beta}{a}(y^* - \overline{y}) + \frac{1}{\phi}\left(I^E - \overline{I}\right) + \frac{\omega\beta}{m}$, *i.e., where the target inflation rate is a specific linear function of the output gap and the investment gap.*

### *4.4. Infinitely repeated game*

Now consider the following infinitely repeated version of the game above. One solution above sets the vector $[I^E, \pi^E]$ such that, given banks' expectations, the marginal cost and benefits to the central bank from surprise investment offset each other. This outcome is subgame perfect, since the central bank is expected to allow a positive amount of fragility and indeed does so. However, the central bank would be better off if it could commit to a socially optimal scenario of $\pi^p$ (where $\pi^p \neq \pi^E$) that results in the optimal credit supply $I^p$. This is an inherent inconsistency problem, since the policymaker and banks have an incentive to deviate from the socially optimal investment and short rate $[I^p, \pi^p]$. A simple way to assess the implications of such deviations is to examine an infinite horizon game.

**Infinite game formulation.** Let policymakers, firms and banks share a common discount factor $d$. To solve this game, we shall clarify the player payoffs and focus on pure strategies, then derive conditions under which $\pi = \hat{\pi} = \pi^p$ and $I = I^p$ every period, in a subgame perfect Nash equilibrium.

**Payoff functions for central bank and banks.** Denote the banks' choice of credit supply in equilibrium and under the social (planner) optimum as $I^E$ and $I^p$, respectively. Then the central bank's payoff as a function of short rates and the banks' credit supply $I^i \in \{I^E, I^p\}$, from the previous section, is

$$L_C(\pi^c,\hat{\pi},I^i) = \left[(\bar{y}-y^*)+\beta\left(\pi^c-\hat{\pi}+\varepsilon_y\right)\right]^2 + a\,(\pi^c-\pi^*)^2 \quad (17)$$
$$+b\gamma I^i \frac{mI^i\hat{r}r^c + (1-m)\,I^i\,(r_1+\delta) - \omega\left[\bar{y}+\beta\left(\pi^c-\hat{\pi}+\varepsilon_y\right)\right]I^i}{K},$$

where $\pi^c \in \{\pi^E, \pi^p\}$. The bank's payoff function is

$$L_B(\hat{r}, I^i) = \omega II^i - mI^i r_1 - (1-m)I^i\,(r_1+\delta) - \gamma\sum_i I^i \frac{mI^i r_1 + (1-m)\,I^i\,(r_1+\delta) - \omega II^i}{K}. \quad (18)$$

We shall use the payoffs in (17) and (18) to compute optimal strategies and credible punishments for deviations.

**Strategies for the central bank and bank.** Given a bank's credit supply $I^i \in \{I^E, I^p\}$, the central bank's optimal monetary policy, from the previous section, is given by

$$\pi^i = \frac{2aK}{2aK+b\gamma(I^i)^2m}\pi^* + \frac{2\beta K}{2aK+b\gamma(I^i)^2m}\,(y^*-\bar{y}) + \frac{mb\gamma\left(I^i\right)^2}{2a\phi K+b\phi\gamma(I^i)^2m}\left(I^i-\bar{I}\right) \quad (19)$$
$$+\frac{b\gamma\left(I^i\right)^2\omega\beta}{2aK+b\gamma(I^i)^2m} - \frac{\beta^2}{a+\beta^2}\varepsilon_y$$

where $\pi^i \in \{\pi^E, \pi^p\}$.

Also from the section 4.1 equation (13), the bank's equilibrium strategy for investment is $I^E$ such that the following relation holds:

$$\omega I^E K - Kr_1 - K\delta + Km\delta - \gamma I^E r_1 - \gamma I^E\delta + \gamma I^E m\delta + \gamma(I^E)^2\omega = 0. \quad (20)$$

By contrast, from the section 4.1 equation (14), the socially optimal investment $I^p$ allows for fire-sale externalities, and $I^p$ is such that the following relation holds:

$$2\omega I^p K - Kr_1 - K\delta + Km\delta - 2\gamma I^p r_1 - 2\gamma I^p\delta + 2\gamma I^p m\delta + 3\gamma(I^p)^2\omega = 0. \quad (21)$$

The central bank wishes to encourage socially optimal investment $I^p$ defined in (21), and stop intermediary banks from doing $I^E$ in (20). To accomplish this objective, the central bank can choose a trigger strategy, which sets $\pi^c = \pi^p$ computed from (19), as long as the intermediaries behave and choose credit supply equal to $I^p$. Otherwise, the central bank assumes banks are going to opt for their most profitable deviation of $I^E$ and sets $\pi$ equal to the best response $\pi^E$, computed from (19). In our model, the banks' expectations are correct on average, so $I^E$ and $I^p$ are in turn best responses to $\pi^E$ and $\pi^p$, respectively. Thus the central bank's strategy is supported in a subgame perfect Nash equilibrium.

The repeated game involves evaluating stage game payoffs to various strategies from the central bank and bank. These payoffs may be represented in the following matrix, Table 1. In Table 1, $L_C^p(\cdot)$ denotes the central bank's payoff in the socially optimal setting (where financial

Table 1: Stage game payoffs to central bank and intermediary bank

| | | Bank | |
|---|---|---|---|
| | | *Cooperate*: $\pi = \pi^p$ | *Deviate*: $\pi = \pi^E$ |
| **Central bank** | *Cooperate*: $I = I^p$ | $(L_C^p, L_B^p)$ | $(\tilde{L}_C^E, \tilde{L}_B^E)$ |
| | *Deviate*: $I = I^E$ | $(\tilde{L}_C^p, \tilde{L}_B^E)$ | $(L_C^E, L_B^E)$ |

fragility is minimized), and $L_C^E(\cdot)$ denotes the central bank's payoff in the inefficient equilibrium, where financial fragility is a problem. The tilde denotes payoffs from a deviation. Thus $\tilde{L}_B^E$ is what the bank obtains by deviating to the equilibrium investment $I^E$.

More specifically, we consider two types of subgames that the central bank faces. The first type is one in which banks have chosen the socially optimal credit supply $I^p$ and the central bank sets the inflation rate to $\pi^c$ in (19), for the present and for all previous periods. The second type is any one in which intermediary banks have deviated by choosing the individually rational but fire-sale provoking credit level $I^E$. The central bank and intermediary bank share a common discount factor $d$.

**Banks:** In the first period, banks hold the expectation $\hat{\pi} = \pi^p$, and supply the socially optimal credit $I^p$. In subsequent periods they expect $\hat{\pi} = \pi^p$, if and only if all prior credit supply was $I^p$ and inflation rates were actually $\pi^p$. Otherwise, bank expectations $\hat{\pi}$ are set equal to $\pi^E$, which we defined as optimal for the central bank if it wants to punish banks for excess credit supply in the stage game.

**Central Bank:** The central bank chooses $\pi = \pi^p$ if and only if current bank and firm expectations satisfy $\hat{\pi} = \pi^p$, all previous expectations have been $\hat{\pi} = \pi^p$, and all previous actual investment and short rates have been $I^p$ and $r^p$. Otherwise, the central bank assumes banks are building up excess credit $I^E$ and chooses $\pi^c = \pi^E$ from its best response (19).

**Infinite game equilibrium.** Let banks' first-period expectations be $\hat{\pi} = \pi^p$. Further, as in Table 1, let the central bank's loss function be $L_C^p(\pi^p, I^p)$. Now, given the bank's strategy, the central bank can focus on two possible paths of financial fragility. The first path involves $\hat{\pi} = \pi^p$, which results the next period in $\hat{\pi} = \pi^p$ and $I^p$. Consequently the central bank makes the same decision in the next period. The second alternative involves expectations $\hat{\pi} = \pi^E$ and credit supply $I = I^E$. This results in $(I^E, \pi^E)$ forever.

The payoffs from these two strategies are as follows: The first strategy ($\pi = \pi^p$ in this period), yields the central bank a payoff of $L_C^p(\pi^p, I^p)$ forever, for a total of $\frac{1}{1-d}L_C^p(\pi^p, I^p)$. The second strategy ($\pi = \pi^E$ this period) yields the central bank $\tilde{L}_C^p(\pi^p, I^p)$ this period, then $L_C^E(\pi^E, I^E)$ forever, for a total of $\tilde{L}_C^p(\pi^p, I^p) + \frac{d}{1-d}L_C^E(\pi^E, I^E)$. Therefore the central bank's strategy is a best response to the banks' strategy if the following condition holds: $\frac{1}{1-d}L_C^p(\pi^p, I^p) \geq \tilde{L}_C^p(\pi^p, I^p) + \frac{d}{1-d}L_C^E(\pi^E, I^E)$. This condition can be rewritten as

$$d \geq \frac{\tilde{L}_C^p(\pi^p, I^p) - L_C^p(\pi^p, I^p)}{\tilde{L}_C^p(\pi^p, I^p) - L_C^E(\pi^E, I^E)} \equiv \overline{d}, \tag{22}$$

which has a ready economic implication. Economically speaking, the central bank finds it optimal not to deviate from its commitment to minimize future financial fragility, if and only if its discount rate (i.e., its willingness to sacrifice present pain for future gain from enforcing low fragility) is large enough, larger than the threshold $\overline{d}$.

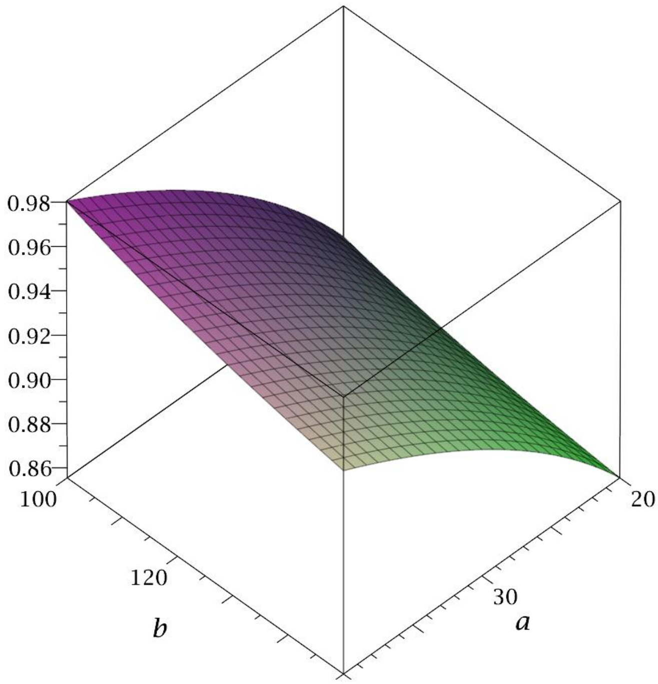


Figure 2: Threshold discount rate for infinite-horizon game. The figure shows the threshold discount rate $d$ from equation (22). The calibration is as in Appendix D. The parameters $a$ and $b$ denote the central bank's weight on inflation and fragility, respectively.

In order to illustrate our results visually, we depict the threshold discount rate $\overline{d}$, aggregate investment $I$, and the optimal inflation rate in traditional and fragility-based settings, i.e., equations (20) and (21), respectively. This communicates the difference between our approach and previous research in an intuitive manner.

Details of our calibration are in the Appendix D. The surface for the threshold discount rate is displayed in Figure 2. Intuitively, $d$ is more likely to be below 1 when the weight of financial stability gets higher in the central bank's loss function. We show results of our investment calibration in Figure 3. Evidently investment is too low and unresponsive when the central bank only targets inflation. Figure 4 displays the optimal inflation rate for a central bank that cares about fragility or only inflation. The upper flat surface shows that the optimal inflation rate is unresponsive to fragility concerns, which is straightforward. The lower curved surface corresponds to a relatively large $m$ (illiquidity). It is lower than the case (upper curved surface) of small $m$, indicating attenuation of optimal inflation rates in the case of a highly illiquid financial sector. In sum, the calibration results are quite reasonable. Nevertheless, we do not place much emphasis on this aspect of the paper, since our focus is to illustrate, in a tractable theoretical framework, the forces at work when a central bank faces the dual tradeoffs of inflation and fragility.

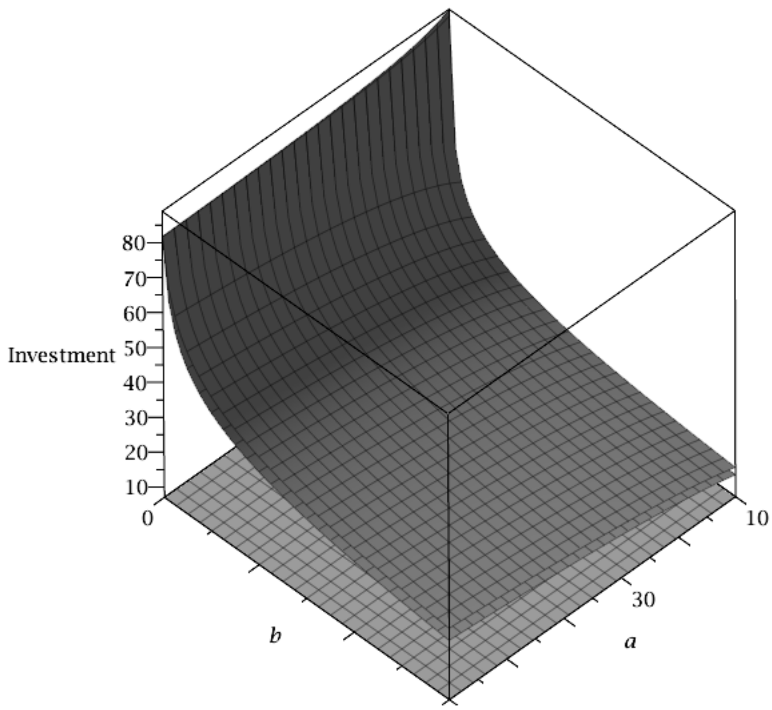


Figure 3: Aggregate investment with fragility-conscious central bank. The figure shows the average investment surface from equations (13) and (14). The calibration is as in Appendix D. The parameters $a$ and $b$ denote the central bank's weight on inflation and fragility, respectively. The curved surfaces correspond to equation (14), and show the optimal investment when the central bank targets both inflation and fragility – the upper curved surface corresponds to $m = 0.4$ while the lower curved surface corresponds to $m = 0.6$. The bottom surface corresponds to equation (13), and shows the optimal investment when the central bank only targets inflation.

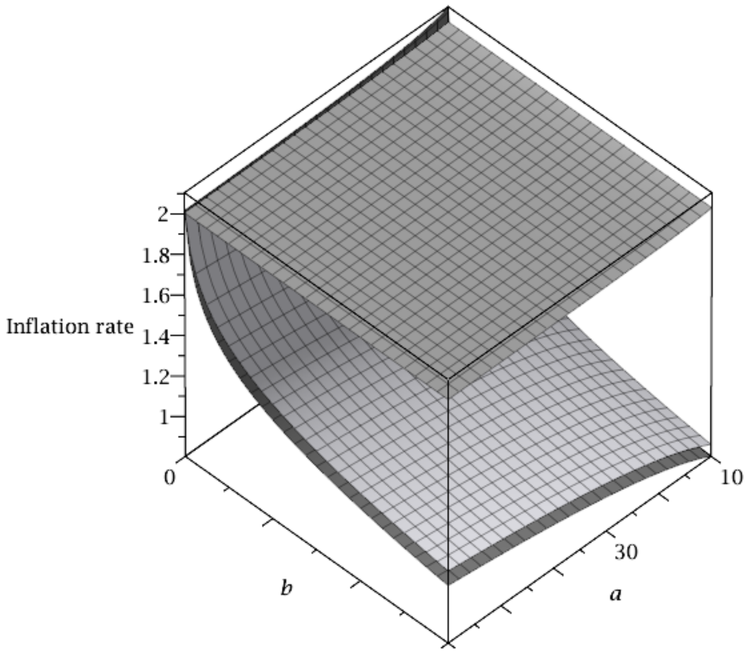


Figure 4: Optimal inflation rates under traditional targeting and fragility concerns. The figure shows the optimal inflation rates $\pi^0$ and $\pi^E$ from equations (16) and (15). The calibration is as in Appendix D. The parameters $a$ and $b$ denote the central bank's weight on inflation and fragility, respectively. The curved surfaces correspond to equation (15), and depict the optimal inflation rate when the central bank targets both inflation and fragility – the light corresponds to $m = 0.4$, while the dark corresponds to $m = 0.6$. The top surface corresponds to equation (16), and shows the optimal inflation when the central bank only targets inflation.

## 5. Conclusions

We develop a framework that characterizes central banks' dual tradeoff between financial stability and inflation control. Since our approach focuses on strategic interaction, it is complementary to recent research on financial fragility that is based on dynamic general equilibrium and related frameworks. We analyze two models: a base model where there is no feedback, and an extended model with feedback effects between the real and financial sectors. The base model offers a potential explanation of several interesting phenomena. In particular, a large, positive real economy shock shifts the production function upward and raises marginal productivity. Since marginal productivity determines the marginal return on capital, banks increase their supply of credit. Consequently real shocks have financial effects, and our model explains the empirically observed procyclicality of credit. Similarly, a large negative financial shock reduces the credit banks supply. This, in turn, reduces firms' investment and therefore results in output contraction. Hence, our model explains the empirically observed occasional spillovers from financial crises to the real economy. These spillovers will be more prominent, the larger the correlation between real and financial shocks.

In the extended model, we account for both intermediaries' and central banks' incentives to deviate from appropriate bailout, liquidity, and interest rate targets. Our model introduces a novel tradeoff between enhancing economic output and maintaining financial stability. Central banks will experience the bite of this tradeoff, the larger the correlation between real and financial shocks. Specifically, tight monetary policy deters buildup of financial imbalances, but loose monetary policy reduces the likelihood of insolvency. We term these factors the discipline effect and stability effect, respectively. We show that the central bank's welfare loss increases with dependence between the real and financial shocks. Thus, a central bank may be able to reduce tradeoff costs by marketing correlation-indexed securities. Evidently, in order to market such securities, the central bank would need to estimate robust measures of dependence, perhaps related to those explored by Patton (2006) and Chollete et al. (2009).

Our findings extend the literature on central bank independence (e.g. Rogoff (1985)). This literature demonstrates that in order to achieve price stability, monetary policy should be delegated to a conservative central bank whose sole concern is inflation. Our framework suggest a further impossibility result: in our model, even with an independent central bank, the attainment of both low inflation and financial stability is generally not possible. An additional financial stability arm, such as financial supervision authority, is a necessary complementary institution to central bank to reduce financial instability.

## Appendix

*A. Derivation of optimal policy response $r^c$ of Section 3.1*

Recall from equation (8) that the the central bank's decision problem is

$$\min_{\pi} L = \left[\bar{y} + \beta\left(\pi - \hat{\pi} + \varepsilon_y\right) - y^*\right]^2 + a\left(\pi - \pi^*\right)^2 + b\left[\left(\theta - \left(\pi - \frac{y-\bar{y}}{\phi}\right) - \delta + m\delta + \varepsilon_f\right)\frac{K}{2pm}\right]^2.$$

The first-order condition is

$$\frac{\partial L}{\partial \pi} = 0 = 2\beta\left[\bar{y} + \beta\left(\pi - \hat{\pi} + \varepsilon_y\right) - y^*\right] + 2a\left(\pi - \pi^*\right)$$

$$+2b\left[\left(\theta-\left(\pi-\frac{y-\overline{y}}{\phi}\right)-\delta+m\delta+\varepsilon_f\right)\frac{K}{2pm}\right]\left(-\frac{K}{2pm}\right),$$

which can be solved to obtain the central bank's best response, denoted $\pi^c$:

$$\pi^c=-\frac{1}{a+\beta^2+b\left(\frac{K}{2pm}\right)^2}[\beta(\overline{y}-y^*)-\beta^2\hat{\pi}+\beta^2\varepsilon_y-a\pi^* \tag{23}$$

$$-b\left(\frac{K}{2pm}\right)^2\left(\theta+\frac{y-\overline{y}}{\phi}+\varepsilon_f\right)+(1-m)\delta b\left(\frac{K}{2pm}\right)^2].$$

To compute the above expression, we require the inflation rate expectations $\hat{\pi}$, to which we now turn. If expectations are on average correct in an equilibrium, then $\hat{\pi}=E[\pi^c]$, whence we can derive the following relation:

$$\hat{\pi}=-\frac{1}{a+\beta^2+b\left(\frac{K}{2pm}\right)^2}\left[\beta(\overline{y}-y^*)-\beta^2\hat{\pi}-a\pi^*-b\left(\frac{K}{2pm}\right)^2\left(\theta+\frac{y-\overline{y}}{\phi}\right)+(1-m)\delta b\left(\frac{K}{2pm}\right)^2\right].$$

This expression can be simplified to

$$\hat{\pi}=-\frac{1}{a+b\left(\frac{K}{2pm}\right)^2}\left[\beta(\overline{y}-y^*)-a\pi^*-b\left(\frac{K}{2pm}\right)^2\left(\theta+\frac{y-\overline{y}}{\phi}\right)+(1-m)\delta b\left(\frac{K}{2pm}\right)^2\right],$$

or, equivalently,

$$\hat{\pi}=\frac{1}{a+b\left(\frac{K}{2pm}\right)^2}\left[\beta(y^*-\overline{y})+a\pi^*+\left(\theta+\frac{y-\overline{y}}{\phi}-\delta+m\delta\right)\left(\frac{K}{2pm}\right)^2b\right]. \tag{24}$$

By substituting the $\hat{\pi}$ term from (24) into (23) we obtain the equilibrium policy rate, namely,

$$\pi^c=-\left\{\frac{1}{a+b\left(\frac{K}{2pm}\right)^2}\left[\beta(\overline{y}-y^*)-a\pi^*-b\left(\frac{K}{2pm}\right)^2\left(\theta+\frac{y-\overline{y}}{\phi}\right)+(1-m)\delta b\left(\frac{K}{2pm}\right)^2\right]\right\}$$

$$-\frac{1}{a+\beta^2+b\left(\frac{K}{2pm}\right)^2}\left\{\beta^2\varepsilon_y-b\left(\frac{K}{2pm}\right)^2\varepsilon_f\right\}.$$

Now define $A=a+\beta^2+b\left(\frac{K}{2pm}\right)^2$, $B=a+b\left(\frac{K}{2pm}\right)^2$, and $C=b\left(\frac{K}{2pm}\right)^2$. Then the above expression implies

$$\pi^c=\frac{1}{B}\left[-\beta(\overline{y}-y^*)+a\pi^*+b\left(\frac{K}{2pm}\right)^2\left(\theta+\frac{y-\overline{y}}{\phi}\right)-(1-m)\delta b\left(\frac{K}{2pm}\right)^2\right]-\frac{1}{A}\left[\beta^2\varepsilon_y-b\left(\frac{K}{2pm}\right)^2\varepsilon_f\right]$$

or

$$\pi^c = \frac{1}{B}\left[-\beta(\overline{y}-y^*) + a\pi^* + \left(\theta + \frac{y-\overline{y}}{\phi} - \delta + m\delta\right)C\right] - \frac{1}{A}\left[\beta^2\varepsilon_y - C\varepsilon_f\right]. \tag{25}$$

Thus $\pi^c$ and $\hat{\pi}$ have the following relation, $\pi^c = \hat{\pi} - \frac{1}{A}(\beta^2\varepsilon_y - C\varepsilon_f)$.

We also need to obtain the term for output $y$. From the Phillips curve relation (5), actual output is

$$y = \overline{y} + \beta\left(\pi - \hat{\pi} + \varepsilon_y\right),$$

which upon substitution yields

$$y = \overline{y} + \frac{a + b\left(\frac{K}{2pm}\right)^2}{a + \beta^2 + b\left(\frac{K}{2pm}\right)^2}\beta\varepsilon_y + \frac{b\left(\frac{K}{2pm}\right)^2}{a + \beta^2 + b\left(\frac{K}{2pm}\right)^2}\beta\varepsilon_f.$$

And finally,

$$y = \overline{y} + \frac{B}{A}\beta\varepsilon_y + \frac{C}{A}\beta\varepsilon_f = \overline{y} + \frac{\beta}{A}\left(B\varepsilon_y + C\varepsilon_f\right). \tag{26}$$

*B. Derivation of expected welfare loss $E[L^n]$ in Section 3.1*

To compute this moment, we first calculate the welfare loss expression $L^n$, by substituting (25) and (26) into the objective (8):

$$\begin{aligned}
L &= \left[\overline{y} + \frac{B}{A}\beta\varepsilon_y + \frac{C}{A}\beta\varepsilon_f - y^*\right]^2 \\
&\quad + a\left[\frac{1}{B}\left[-\beta(\overline{y}-y^*) + a\pi^* + \left(\theta + \frac{y-\overline{y}}{\phi} - \delta + m\delta\right)C\right] - \frac{1}{A}\left[\beta^2\varepsilon_y - C\varepsilon_f\right] - \pi^*\right]^2 \\
&\quad + b\left(\frac{K}{2pm}\right)^2\left[\left(\frac{\beta(\overline{y}-y^*) - a\pi^* + a(\theta + \frac{y-\overline{y}}{\phi} - \delta + m\delta)}{B} + \frac{\beta^2\varepsilon_y + (a+\beta^2)\varepsilon_f}{A}\right)\right]^2 \\
&= \left[\overline{y} + \frac{B}{A}\beta\varepsilon_y + \frac{C}{A}\beta\varepsilon_f - y^*\right]^2 + a\left[\frac{-\beta(\overline{y}-y^*) + (a-B)\pi^* + (\theta + \frac{y-\overline{y}}{\phi} - \delta + m\delta)C}{B} - \frac{\beta^2\varepsilon_y - C\varepsilon_f}{A}\right]^2 \\
&\quad + b\left[\left(\frac{B(\theta + \frac{y-\overline{y}}{\phi} - \delta + m\delta) - \left[-\beta(\overline{y}-y^*) + a\pi^* + \left(\theta + \frac{y-\overline{y}}{\phi} - \delta + m\delta\right)C\right]}{B} + \frac{\beta^2\varepsilon_y + (A-C)\varepsilon_f}{A}\right)\frac{K}{2pm}\right]^2.
\end{aligned}$$

To simplify, note that $(a-B)\pi^* = -C\pi^*$ in the fourth line; and $(B-C)(\theta + \frac{y-\overline{y}}{\phi} - \delta + m\delta) = a(\theta + \frac{y-\overline{y}}{\phi} - \delta + m\delta)$, and $(A-C)\varepsilon_f = (a+\beta^2)\varepsilon_f$, for the fifth line. Thus we can rewrite the above expression as

$$L = \left[\overline{y} + \frac{B}{A}\beta\varepsilon_y + \frac{C}{A}\beta\varepsilon_f - y^*\right]^2 + a\left[\frac{-\beta(\overline{y}-y^*) + (\theta + \frac{y-\overline{y}}{\phi} - \delta + m\delta - \pi^*)C}{B} - \frac{\beta^2\varepsilon_y - C\varepsilon_f}{A}\right]^2$$

$$
\begin{aligned}
&+b\left(\frac{K}{2pm}\right)^2\left[\left(\frac{\beta\left(\bar{y}-y^*\right)-a\pi^*+a(\theta+\frac{y-\bar{y}}{\phi}-\delta+m\delta)}{B}+\frac{\beta^2\varepsilon_y+(a+\beta^2)\varepsilon_f}{A}\right)\right]^2 \\
= &\left[\bar{y}+\frac{B}{A}\beta\varepsilon_y+\frac{C}{A}\beta\varepsilon_f-y^*\right]^2+a\left[\frac{(1-\alpha)\beta y^*+(\theta-\delta+m\delta-r^*)C}{B}-\frac{\beta^2\varepsilon_y-C\varepsilon_f}{A}\right]^2 \\
&+C\left[\left(-\frac{-\beta\left(\bar{y}-y^*\right)+a\pi^*+a(\theta+\frac{y-\bar{y}}{\phi}-\delta+m\delta)}{B}+\frac{\beta^2\varepsilon_y+(a+\beta^2)\varepsilon_f}{A}\right)\right]^2. \qquad (27)
\end{aligned}
$$

Now that we have the expression for welfare loss $L$, we can compute its moments, which rely on joint stochastic properties of the real and financial shocks $\varepsilon_y$ and $\varepsilon_f$. Recall that the distribution of the shocks is the following, $\varepsilon \equiv [\varepsilon_y, \varepsilon_f]'$, then $\varepsilon \sim [0, V]$ where the covariance matrix $V$ is of the form $V = \begin{bmatrix} \sigma_y^2 & \rho_{y,f} \\ \rho_{y,f} & \sigma_f^2 \end{bmatrix}$.

Then we can take expectations of (27). In the following, we first express the constant terms, then the stochastic ones, i.e., those that depend on the moments of $\varepsilon_y$ and $\varepsilon_f$. Specifically,

$$
\begin{aligned}
E[L] = &\left(\bar{y}-y^*\right)^2+a\left[\frac{-\beta\left(\bar{y}-y^*\right)+(\theta+\frac{y-\bar{y}}{\phi}-\delta+m\delta-\pi^*)C}{B}\right]^2 \\
&+C\left[\frac{-\beta\left(\bar{y}-y^*\right)+a\pi^*+a(\theta+\frac{y-\bar{y}}{\phi}-\delta+m\delta)}{B}\right]^2 \\
&+\left(\left[\frac{B}{A}\right]^2\beta^2+a\left[\frac{\beta^2}{A}\right]^2+C\left[\frac{\beta^2}{A}\right]^2\right)\sigma_y^2+\left(\left[\frac{C}{A}\right]^2\beta^2+a\left[\frac{C}{A}\right]^2+C\left[\frac{(a+\beta^2)}{A}\right]^2\right)\sigma_f^2 \\
&+2\left(\frac{B}{A}\frac{C}{A}\beta^2-a\frac{\beta^2}{A}\frac{C}{A}+C\frac{\beta^2}{A}\frac{(a+\beta^2)}{A}\right)\rho_{y,f} \\
= &\left(\bar{y}-y^*\right)^2+a\left[\frac{-\beta\left(\bar{y}-y^*\right)+(\theta+\frac{y-\bar{y}}{\phi}-\delta+m\delta-\pi^*)C}{B}\right]^2 \\
&+C\left[\frac{-\beta\left(\bar{y}-y^*\right)+a\pi^*+a(\theta+\frac{y-\bar{y}}{\phi}-\delta+m\delta)}{B}\right]^2 \\
&+\frac{B^2+a\beta^2+C\beta^2}{A^2}\beta^2\sigma_y^2+\frac{C^2(\beta^2+a)+C(a+\beta^2)^2}{A^2}\sigma_f^2+2C\beta^2\frac{B+\beta^2}{A^2}\rho_{y,f},
\end{aligned}
$$

or

$$
E[L^n] = \left(\bar{y}-y^*\right)^2+a\left[\frac{-\beta\left(\bar{y}-y^*\right)+(\theta+\frac{y-\bar{y}}{\phi}-\delta+m\delta-\pi^*)C}{B}\right]^2
$$

$$+C\left[\frac{-\beta\left(\bar{y}-y^{*}\right)+a\pi^{*}+a(\theta+\frac{y-\bar{y}}{\phi}-\delta+m\delta)}{B}\right]^{2}$$

$$+\frac{B^{2}+\beta^{2}(a+C)}{A^{2}}\beta^{2}\sigma_{y}^{2}+\frac{(a+\beta^{2})^{2}(C^{2}+C)}{A^{2}}\sigma_{f}^{2}+2C\beta^{2}\frac{B+\beta^{2}}{A^{2}}\rho_{y,f}. \quad (28)$$

In contrast, the cooperative solution features two conditions $\hat{\pi}=\pi^{c}=\pi^{*}$ and $\pi^{*}=\theta-(1-m)\delta$. Substituting these conditions into the objective function (8) and taking expectations yields the expected loss under perfect anchoring, we can derive $E[L^{c}]$

$$\begin{aligned} E[L^{c}] &= E\left[\left(\bar{y}-y^{*}+\beta\varepsilon_{y}\right)^{2}+\left(\frac{K}{2pm}\varepsilon_{f}\right)^{2}\right] \\ &= (\bar{y}-y^{*})^{2}+\beta^{2}\sigma_{y}^{2}+\frac{C}{b}\sigma_{f}^{2}, \end{aligned} \quad (29)$$

where the second line uses the fact that $C=b\left(\frac{K}{2pm}\right)^{2}$. The difference in expected loss $\Delta=E[L]-E[L^{c}]$ is calculated from (28) and (29) to be

$$\begin{aligned} \Delta &= a\left[\frac{-\beta\left(\bar{y}-y^{*}\right)+(\theta+\frac{y-\bar{y}}{\phi}-\delta+m\delta-\pi^{*})C}{B}\right]^{2}+C\left[\frac{-\beta\left(\bar{y}-y^{*}\right)+a\pi^{*}+a(\theta+\frac{y-\bar{y}}{\phi}-\delta+m\delta)}{B}\right]^{2} \\ &\quad +\frac{B^{2}+\beta^{2}(a+C)-A^{2}}{A^{2}}\beta^{2}\sigma_{y}^{2}+\frac{b(a+\beta^{2})^{2}(C^{2}+C)-A^{2}C}{bA^{2}}\sigma_{f}^{2}+2C\beta^{2}\frac{B+\beta^{2}}{A^{2}}\rho_{y,f}. \end{aligned} \quad (30)$$

*C. Proofs*

*Proposition 1*

*In a competitive banking system, financial fragility $f$ can be represented as a function of equilibrium aggregate bank investment $I=\sum_{i}I^{i}$.*

**Proof of Proposition 1:** By Definition 1 in Section 2.

*Proposition 2*

*In a competitive banking system with fire-sale externalities, the leverage-based fragility $f$ is inefficiently high.*

**Proof of Proposition 2:** In a competitive banking system with fire-sale externalities, the total investment is higher than the socially optimal level, $I^{c}>I^{p}$. By Definition 1 in Section 2, $f(I^{c})>f(I^{p})$.

*Proposition 4: Inefficiency of bank equilibrium investment*

*Part A: In market equilibrium, the bank's investment is larger than the planner's solution, i.e., $I^{E}>I^{p}$. Moreover, $\sqrt{3}I^{p}<I^{E}<2I^{p}$. Part B: In market equilibrium, liquidity risk provides some market discipline, but investment is still excessive.*

**Proof of Part A:** Subtracting (14) from (13), one can get

$$\gamma\omega\left(3(I^{p})^{2}-(I^{E})^{2}\right)+\left(2I^{p}-I^{E}\right)(\omega\hat{r}K-\gamma r_{1}-\gamma\delta+\gamma m\delta)\equiv0.$$

The banks borrow short-term debt only if long-term debt is too costly, or $\omega < \omega I^p \leq r_1 + \delta$. Therefore

$$\omega \hat{r} K - \gamma (r_1 + \delta) + \gamma m \delta \geq \omega (\hat{r} K - \gamma) + \gamma m \delta > 0$$

as long as $\hat{r} K - \gamma > 0$. This latter condition is fairly weak: since $\hat{r} > 1 > \gamma$, the inequality strictly holds if $I^p > K > 1$.

Suppose that $I^E < I^E$, then $\gamma\omega\left(3(I^p)^2 - (I^E)^2\right) + \left(2I^p - I^E\right)(\omega\hat{r}K - \gamma r_1 - \gamma\delta + \gamma m\delta) > 0$. A contradiction.

Suppose that $I^p \leq I^E \leq \sqrt{3}I^p$, then $\gamma\omega\left(3(I^p)^2 - (I^E)^2\right) + \left(2I^p - I^i\right)(\omega\hat{r}K - \gamma r_1 - \gamma\delta + \gamma m\delta) > 0$. A contradiction.

Suppose that $I^E \geq 2I^p$, then $\gamma\omega\left(3(I^p)^2 - (I^E)^2\right) + \left(2I^p - I^i\right)(\omega\hat{r}K - \gamma r_1 - \gamma\delta + \gamma m\delta) < 0$. A contradiction.

Therefore, $\sqrt{3}I^p < I^E < 2I^p$.

**Proof of Part B:** This part of the proof involves evaluating comparative statics of the liquidity effect. Using the implicit function theorem, we can take the derivative of (17) with respect to $m$ to get

$$\frac{\partial I^E}{\partial m} = -\frac{K\delta + \gamma I\delta}{\omega K - \gamma r_1 - \gamma\delta + \gamma m\delta + 2I^E\gamma\omega}.$$

The denominator can be rewritten as

$$\omega K + \gamma(\omega I^E - r_1) + \gamma(\omega I^E - \delta) + \gamma m\delta.$$

Since $\omega I^E$ is the return on investment, $\omega I^E > r_1 > \delta$ by assumption. Therefore the denominator is positive, and $\frac{\partial I^E}{\partial m} < 0$. ■

*Derivation of optimal inflation rate $\pi^E$ in model of section 4.2*

Recall that the central bank's program is

$$\min_{\pi^E} L_C = (y - y^*)^2 + a\left(\pi^E - \pi^*\right)^2 + b\gamma \sum_i I^i G\left(\varepsilon_y^{i*}\right)(i),$$

subject to the Phillips curve

$$y = \overline{y} + \beta\left(\pi^E - \hat{\pi} + \varepsilon_y\right)$$

and the $IS$ curve

$$y = -\phi\left(r^E - \pi^E\right) + \overline{y}.$$

For simplicity, we have assumed that bankruptcy formally occurs in the end of the period, $t_2$. Thus, the share of bankrupted banks can be written as

$$\frac{mI^E\hat{r}r^E + (1-m)I^E(r_1 + \delta) - \omega\left[\overline{y} + \beta\left(\pi^E - \hat{\pi} + \varepsilon_y\right)\right]I^E}{K}.$$

The first order condition is linear in $\pi^E$ so that we can solve $\hat{\pi}$ by taking expectations. The first order condition yields

$$\frac{\partial L_C}{\partial \pi^E} = 2\beta\left[\overline{y} - y^* + \beta\left(\pi^E - \hat{\pi} + \varepsilon_y\right)\right] + 2a\left(\pi^E - \pi^*\right) + \frac{b\gamma I^E}{K}\left(mI^E\hat{r} - \omega I^E\beta\right) = 0.$$

Since in a rational expectations equilibrium $E\left[\pi^E\right] = \hat{\pi}$, we take expectations on the first order condition, in order to obtain

$$2\beta\left(\overline{y} - y^*\right) + 2a\left(\hat{\pi} - \pi^*\right) + \frac{b\gamma I^E}{K}\left[mI^E\left(\hat{\pi} - \frac{I^E - \overline{I}}{\phi}\right) - \omega I^E\beta\right] = 0,$$

which implies that interest rate expectations satisfy

$$\hat{\pi} = \frac{2a\phi K\pi^* + 2\beta\phi K\left(y^* - \overline{y}\right) + mb\gamma(I^E)^2\left(I^E - \overline{I}\right) + b\phi\gamma(I^E)^2\omega\beta}{2a\phi K + b\phi\gamma(I^E)^2 m}.$$

We now insert $\hat{\pi}$ into the first order condition, and solve for the central bank's optimal short rate to obtain

$$\pi^E = \frac{2aK}{2aK + b\gamma(I^E)^2 m}\pi^* + \frac{2\beta K}{2aK + b\gamma(I^E)^2 m}\left(y^* - \overline{y}\right) + \frac{mb\gamma\left(I^E\right)^2}{2a\phi K + b\phi\gamma(I^E)^2 m}\left(I^E - \overline{I}\right) + \frac{b\gamma\left(I^E\right)^2\omega\beta}{2aK + b\gamma(I^E)^2 m} - \frac{\beta^2}{a + \beta^2}\varepsilon_y,$$

which is presented as equation (15) of the text.

*Proof of the impossibility result, Proposition 5*

We have to show that when the central bank's rates are equal, $\pi^E = \pi^0$, then the target inflation rate is a linear function of the investment gap: $\pi^* = \frac{\beta}{a}(I^E - I^*) + \frac{\omega\beta}{m}$.

The central bank can use the same rate to control both inflation and targeting only if $\pi^E - \pi^0$ is identically zero. We solve this condition by taking the difference $\pi^E - \pi^0$:

$$\begin{aligned}
\pi^E - \pi^0 &= \tfrac{2aK}{2aK+b\gamma(I^E)^2m}\pi^* + \tfrac{2\beta K}{2aK+b\gamma(I^E)^2m}\left(y^* - \overline{y}\right) + \tfrac{mb\gamma\left(I^E\right)^2}{2a\phi K+b\phi\gamma(I^E)^2m}\left(I^E - \overline{I}\right) \\
&\quad + \tfrac{b\gamma\left(I^E\right)^2\omega\beta}{2aK+b\gamma(I^E)^2m} - \tfrac{\beta^2}{a+\beta^2}\varepsilon_y - \left[\pi^* + \tfrac{\beta}{a}\left(y^* - \overline{y}\right) - \tfrac{\beta^2}{a+\beta^2}\varepsilon_y\right] \\
&= \left[\tfrac{2aK}{2aK+b\gamma(I^E)^2m} - 1\right]\pi^* + \beta\left(y^* - \overline{y}\right)\left[\tfrac{2K}{2aK+b\gamma(I^E)^2m} - \tfrac{1}{a}\right] \\
&\quad + \tfrac{mb\gamma\left(I^E\right)^2}{2a\phi K+b\phi\gamma(I^E)^2m}\left(I^E - \overline{I}\right) + \tfrac{b\gamma\left(I^E\right)^2\omega\beta}{2aK+b\gamma(I^E)^2m} \\
&= \left[\tfrac{2aK-2aK-b\gamma(I^E)^2m}{2aK+b\gamma(I^E)^2m}\right]\pi^* + \beta\left(y^* - \overline{y}\right)\left[\tfrac{2aK-2aK-b\gamma(I^E)^2m}{a[2aK+b\gamma(I^E)^2m]}\right] \\
&\quad + \tfrac{mb\gamma\left(I^E\right)^2}{2a\phi K+b\phi\gamma(I^E)^2m}\left(I^E - \overline{I}\right) + \tfrac{b\gamma\left(I^E\right)^2\omega\beta}{2aK+b\gamma(I^E)^2m} \\
&= \left[\tfrac{-b\gamma(I^E)^2m}{2aK+b\gamma(I^E)^2m}\right]\pi^* - \beta\left(y^* - \overline{y}\right)\left[\tfrac{b\gamma(I^E)^2m}{a[2aK+b\gamma(I^E)^2m]}\right] \\
&\quad + \tfrac{mb\gamma\left(I^E\right)^2}{2a\phi K+b\phi\gamma(I^E)^2m}\left(I^E - \overline{I}\right) + \tfrac{b\gamma\left(I^E\right)^2\omega\beta}{2aK+b\gamma(I^E)^2m} \\
&= \left[\tfrac{-b\gamma(I^E)^2m}{2aK+b\gamma(I^E)^2m}\right]\pi^* - \left[\tfrac{b\beta\gamma(I^E)^2m}{a[2aK+b\gamma(I^E)^2m]}\right]\left(y^* - \overline{y}\right) \\
&\quad + \tfrac{mb\gamma\left(I^E\right)^2}{2a\phi K+b\phi\gamma(I^E)^2m}\left(I^E - \overline{I}\right) + \tfrac{b\gamma\left(I^E\right)^2\omega\beta}{2aK+b\gamma(I^E)^2m},
\end{aligned}$$

Table 2: Description of variables in investment and monetary policy

| Parameter | Interpretation |
|---|---|
| 1. $a$ | Cost to central bank of missing inflation or interest target |
| 2. $b$ | Cost to central bank of missing investment target |
| 3. $\beta$ | Phillips curve-based cost to central bank of missing interest target |
| 4. $\pi^*$ | Target interest rate |
| 5. $I^E - \overline{I}$ | Gap between optimal and the investment level under natural rate of output |
| 6. $\overline{I}$ | Investment level under natural output |
| 7. $\phi$ | Inverse of the rate of risk aversion |
| 8. $K$ | Total equity in banking system |
| 9. $\gamma$ | Multiplier for systemic costs relative to $I$, which measures fire-sale costs |
| 10. $m$ | Fraction of short-term debt in banking system, which measures system illiquidity |
| 11. $\varepsilon_y$ | Output shock |

$$\pi^E - \pi^0 = \left[\frac{b\gamma(I^E)^2}{2aK + b\gamma(I^E)^2 m}\right]\left[m\left(\frac{\beta}{a}(y^* - \overline{y}) + \frac{1}{\phi}\left(I^E - \overline{I}\right) - \pi^*\right) + \omega\beta\right]. \tag{31}$$

The central bank can use the same rate to control both inflation and targeting only if the expression in (31) is identically zero, i.e., at least one of the terms on the RHS is zero. Let us examine the first term. This term is non-zero because the central bank cares about fragility ($b > 0$), fire sales occur ($\gamma > 0$), investment is positive ($I^E > 0$), and there is always some short term debt in the economy ($m > 0$). Therefore it suffices to examine only the second term. This second term is zero if the target inflation rate is a specific linear function the investment gap: $\pi^* = \frac{\beta}{a}(y^* - \overline{y}) + \frac{1}{\phi}\left(I^E - \overline{I}\right) + \frac{\omega\beta}{m}$ as was to be shown. ■

*D. Calibrations for optimal investment, short-rates, and threshold discount rate*

We calibrate the key variables from Section 4, in order to illustrate them graphically.

*Calibration of investment and short-rates*

In this part we calibrate variables from Section 4.2 and 4.3. Aggregate investment is represented in equations (13}) and (14), while short rates under fragility and traditional central banking are in equations (16) and (15). The main parameters are described below.

We approach determination of the parameter values through a combination of natural constraints, theoretical considerations, and banking practice. Notice that:

- Typical inflation targeting central banks put much higher weight on inflation loss than output loss, i.e., $a >> 1$;
- The ratio $\frac{I^E}{k}$ reflects the bank's leverage ratio. The typical leverage in the banking industry is about 12.5, rarely below 5; therefore, $\frac{I^E}{K} = 5$ (the lowest banking leverage) may be a good guess;
- The value $\gamma\Sigma_i I^i$ is the fire sale loss of one unit liquidated asset, therefore $\gamma I^E < 1$ — even in the worse case when all the bank assets are liquidated, the banks can still recover some value ($1 - \gamma I^E$ per unit);

- The reliance on short-term debt varies quite a lot across banks, and depends mainly on how much investment banking business one bank gets involved. However, post-crisis banks tend to focus more on the mainstream banking, and $m$ is mostly between 20% and 40% in Europe and US.

*Calibrating the threshold discount rate*

We now calibrate the threshold discount rate $d$ for the infinitely repeated game from Section 4.4. The discount rate is given by expression (22), reproduced here:

$$d \geq \frac{\tilde{L}_C^p(\pi^p, I^p) - L_C^p(\pi^p, I^p)}{\tilde{L}_C^p(\pi^p, I^p) - L_C^E(\pi^E, I^E)} \equiv \bar{d}.$$

The expressions for $\tilde{L}_C^p$, $L_C^p$ and $L_C^E$ depend on similar parameters to those for the optimal inflation rates. In particular, equation (22) gives the central bank's loss function as

$$L_C(\pi^c, \hat{\pi}, I^i) = \left[(\bar{y} - y^*) + \beta\left(\pi^c - \hat{\pi} + \varepsilon_y\right)\right]^2 + a\left(\pi^c - \pi^*\right)^2 + b\gamma I^i \frac{mI^i \hat{r} r^c + (1-m)\, I^i\,(r_1+\delta) - \omega\left[\bar{y} + \beta\left(\pi^c - \hat{\pi} + \varepsilon_y\right)\right] I^i}{K},$$

where $\pi^c \in \{\pi^E, \pi^p\}$ and $I^i \in \{I^E, I^p\}$.

Preliminary results for computing threshold

Before we compute the threshold $\bar{d}$, recall that $\pi^E$ and $\pi^p$ are calculated from equation (19) in the text, as

$$\begin{aligned}\pi^E = {} & \frac{2aK}{2aK + b\gamma(I^E)^2 m}\pi^* + \frac{2\beta K}{2aK + b\gamma(I^E)^2 m}(y^* - \bar{y}) + \frac{mb\gamma\left(I^E\right)^2}{2a\phi K + b\phi\gamma(I^E)^2 m}\left(I^E - \bar{I}\right) \\ & + \frac{b\gamma\left(I^E\right)^2 \omega\beta}{2aK + b\gamma(I^E)^2 m} - \frac{\beta^2}{a+\beta^2}\varepsilon_y\end{aligned}$$

and

$$\begin{aligned}\pi^P = {} & \frac{2aK}{2aK + b\gamma(I^P)^2 m}\pi^* + \frac{2\beta K}{2aK + b\gamma(I^P)^2 m}(y^* - \bar{y}) + \frac{mb\gamma\left(I^P\right)^2}{2a\phi K + b\phi\gamma(I^P)^2 m}\left(I^P - \bar{I}\right) \\ & + \frac{b\gamma\left(I^P\right)^2 \omega\beta}{2aK + b\gamma(I^P)^2 m} - \frac{\beta^2}{a+\beta^2}\varepsilon_y.\end{aligned}$$

Further, $I^E$ and $I^p$ satisfy the following conditions from equations (20) and (21) in the text:

$$\omega I^E K - r_1 K - K\delta + Km\delta - \gamma I^E r_1 - \gamma I^E \delta + \gamma I^E m\delta + \gamma (I^E)^2 \omega = 0,$$

and

$$2\omega I^p K - r_1 K - K\delta + Km\delta - 2\gamma I^p r_1 - 2\gamma I^p \delta + 2\gamma I^p m\delta - 3\gamma (I^p)^2 \omega = 0,$$

respectively.

Computing the threshold $\bar{d}$

Now we can compute the three different elements of the threshold discount rate in (22) – namely $L_C^p$, $L_C^E$, and $\tilde{L}_C^p$. The first two are straightforward since they involve the bank and central bank both playing cooperate or deviate:

$$\begin{aligned} L_C^p(\pi^p, \hat{\pi}, I^p) &= \left[(\bar{y} - y^*) + \beta\left(\pi^P - \hat{\pi} + \varepsilon_y\right)\right]^2 + a\left(\pi^P - \pi^*\right)^2 \\ &\quad + b\gamma I^P \frac{mI^P \hat{r} r^P + (1-m) I^P (r_1 + \delta) - \omega\left[\bar{y} + \beta\left(\pi^P - \hat{\pi} + \varepsilon_y\right)\right] I^P}{K}; \end{aligned} \tag{32}$$

and

$$\begin{aligned} L_C^E(\pi^E, \hat{\pi}, I^E) &= \left[(\bar{y} - y^*) + \beta\left(\pi^E - \hat{\pi} + \varepsilon_y\right)\right]^2 + a\left(\pi^E - \pi^*\right)^2 \\ &\quad + b\gamma I^E \frac{mI^E \hat{r} r^E + (1-m) I^E (r_1 + \delta) - \omega\left[\bar{y} + \beta\left(\pi^E - \hat{\pi} + \varepsilon_y\right)\right] I^E}{K}. \end{aligned} \tag{33}$$

The third payoff, $\tilde{L}_C^p$, is different because it reflects the central bank's payoff (by choosing $\pi = \pi^p$) if the bank deviates ($I = I^E$) for one period:

$$\begin{aligned} \tilde{L}_C^p(\pi^p, \hat{\pi}, I^E) &= \left[(\bar{y} - y^*) + \beta\left(\pi^P - \hat{\pi} + \varepsilon_y\right)\right]^2 + a\left(\pi^P - \pi^*\right)^2 \\ &\quad + b\gamma I^E \frac{mI^E \hat{r} r^P + (1-m) I^E (r_1 + \delta) - \omega\left[\bar{y} + \beta\left(\pi^P - \hat{\pi} + \varepsilon_y\right)\right] I^E}{K}. \end{aligned} \tag{34}$$

Thus the threshold discount factor $\bar{d}$ in (22) can be computed from equations (32) to (34), and the short rate and investment expressions from Section 2.1.

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