

Entrepreneurship, Agency Frictions and Redistributive Capital Taxation

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Abstract

Motivated by the observation that among OECD countries redistribution is negatively correlated with entrepreneurial activity, we examine the implications of entrepreneurial financial frictions for optimal linear capital taxation, in a setting where the government is concerned with redistribution. By including financial frictions, we emphasize the effect of a new channel affecting the equity-efficiency trade-off of redistribution: taxes affect the allocative efficiency of capital and, ultimately, total factor productivity. We find that high tax rates are optimal, provided that they are applied to wealth, rather than risky capital. Under plausible parameter values, we find that the optimal tax on risky capital is lower than that on wealth, and roughly in line with current U.S. levels. This suggests welfare gains from taxing wealth at a higher rate than risky capital.

1 Introduction

[Guvenen et al. \(2017\)](#) – I just put a random citation here so that this would compile

2 Model

In this section we describe the economic environment in detail and present the model assumptions.

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2.1 Environment

There is a continuum of two types of households: entrepreneurs and workers. In addition, there is a continuum of competitive final goods producers, and a continuum of competitive financial intermediaries. Entrepreneurs own capital, while workers live hand to mouth and supply inelastic labor. Entrepreneurs invest some of their capital in risky projects, which yield a stochastic amount of an intermediate good, and lease the rest of their capital directly to final goods producers. Entrepreneurs may convert the intermediate goods they produce directly into units of final output, or may sell them to the final goods producers. Final goods producers produce output using labor, capital and intermediate goods. Financial intermediaries allocate finance between entrepreneurs. **The government levies (possibly negative) taxes on the agents and funds the fixed (exogenously given) level of government spending \bar{G} . – must change this sentence to account for gov't being able to borrow.**

Timing Time is infinite. Workers and final goods producers are infinitely lived. Entrepreneurs die at the end of each period with probability $\gamma \in (0, 1)$ and their capital is equally distributed among a γ measure of newly born entrepreneurs. Financial intermediaries live for one period and in each period new intermediaries are born. Every period is divided into three sub-periods: morning, afternoon and evening. In the morning, entrepreneurs buy and sell capital amongst themselves through financial intermediaries and each entrepreneur devotes some of her own capital to her own risky project. We will refer to this as risky investment. She leases the rest of her capital to the competitive final goods firms. We will refer to this as risk-free capital investment. In the afternoon, each entrepreneur draws an idiosyncratic shock which will affect the intermediate goods produced by her risky project. In the evening, entrepreneurs either convert the intermediate goods they produce into units of final output or sell them to the final goods firms, and workers are employed by these firms.¹ Output of the final good is produced. Financial intermediaries die, and then entrepreneurs divide their remaining resources between consumption and investment.

Technology An entrepreneur i starts the period with k_{it} units of capital. We assume that there exists some $\underline{k} > 0$ such that all entrepreneurs have an initial capital at least equal to \underline{k} . **Do we still need this? Does this mean that we must restrict k' to be greater than \underline{k} ?** Entrepreneurs vary in their entrepreneurial ability. At the start of each period, before capital is traded, entrepreneur i draws publicly observable ability $\theta_{it} \in [\underline{\theta}, \bar{\theta}]$, which affects the productivity of her risky project. The draw of θ is correlated over time and all entrepreneurs draw from an identical distribution, with cdf $G(\theta) = A_0 - \frac{A_1}{\theta}$

¹As we discuss below, the quantity of intermediate output that entrepreneurs convert directly into final output is unobserved by others. This technology, which allows entrepreneurs to secretly divert resources for personal use, represents the ability of firm managers to divert firms' funds for personal use in reality. This creates a financial market friction.

and associated pdf $g(\theta)$.² The constants A_0 and A_1 are chosen so that $G(\underline{\theta}) = 0$ and $G(\bar{\theta}) = 1$.³ We refer to θ_{it} as the entrepreneur's 'type'. If an entrepreneur with type θ_{it} invests k_{it}^E in her private project in the morning, then in the afternoon the project yields a number of units of intermediate goods equal to $y_{it}^E = \epsilon_{it}\theta_{it}k_{it}^E$, where ϵ_{it} is an idiosyncratic productivity shock distributed according to the cumulative distribution function $H(\epsilon)$, with probability distribution function $h(\epsilon)$. We assume that $h(0) > 0$, $h(\epsilon) = 0$ for all $\epsilon < 0$, $h(\cdot)$ is continuously differentiable for $\epsilon > 0$, $\mathbb{E}(\epsilon) = 1$ and, for all $v \geq 1$ and $x \geq 0$, $\frac{\partial^2}{\partial x^2} \left(\frac{\int_{\epsilon} (1+x\epsilon)^{-v} h(\epsilon) d\epsilon}{\int_{\epsilon} (1+x\epsilon)^{-v} \epsilon h(\epsilon) d\epsilon} \right) \geq 0$.⁴

Any capital of the entrepreneur's which she does not invest in her risky project, she leases to the final goods producer directly. Let k_{it}^F be the amount leased to the final goods producer. Note that, for entrepreneur i , $k_{it}^E + k_{it}^F$ is not equal to k_{it} , because entrepreneurs sell capital among themselves in the morning of each period. In addition to selling intermediate goods to the final goods producer, the entrepreneur can convert them directly into units of final output. If she does this, she can convert one unit of the intermediate good into $\rho \in (0, 1)$ units of the final good.

We assume that the final goods producer buys intermediate goods from the entrepreneur at price r_t^E per unit, and rents capital from the entrepreneur directly at rental rate r_t^F per unit. The final goods producer also hires workers at wage rate w_t . The representative final goods producer produces final output according to the production function:

$$Y_t = F\left(Y_t^E, K_t^F, N\right),$$

where Y_t is the aggregate final output, N is aggregate labor, Y_t^E is aggregate input of the intermediate good and K_t^F is aggregate capital leased directly to the final goods producer. As is standard, we assume that F is concave and increasing in all arguments and that for each factor $i \in \{Y_t^E, K_t^F, N\}$: $\lim_{i \rightarrow 0} F_i = \infty$ and $F = 0$ at $i = 0$. Furthermore, we assume that $\lim_{N \rightarrow \infty} F_N = 0$, and $\lim_{K_t^F \rightarrow \infty} F_{K_t^F} = 0$. However, in contrast to the standard Inada conditions, we assume that $\lim_{Y_t^E \rightarrow \infty} F_{Y_t^E} > \rho > 0$.⁵

The device of having a final goods producer that uses some intermediate goods and some capital goods directly is a simple way to allow entrepreneurs to choose between

²The assumption that an entrepreneur's type is independent of her initial capital appears quite restrictive. Its practical applicability may be greater, however, if we interpret k_{it} as the number of efficiency units of capital that the entrepreneur entered the period with, and imagine that this is commonly observed. θ_{it} then refers to a shock that the entrepreneur receives, which does not depend on the initial quantity of efficiency units of capital she possesses.

³The distributional assumptions on G imply that the distribution of entrepreneurial ability resembles a Pareto distribution, except for the upper bound $\bar{\theta}$. That ability has an upper bound is essential in our setting, because it can readily be shown that the financial friction will cease to be relevant if there are types with sufficiently high θ . Pareto distributions with a parameter close to 2 are common in the literature.

⁴Numerically, we find that the latter assumption holds for many distributions with non-negative support: exponential, lognormal, Pareto, generalized Pareto, gamma and chi-square.

⁵This assumption is to guarantee that entrepreneurs will all use their private projects to produce intermediate goods in any constrained efficient allocation.

investing capital in a risky way (the private project) or risk-free way (leasing it directly to the final goods producer). This is designed to capture the idea that some investment projects are more risky than others and that capital owners must take into account the risks associated with different projects when making investment decisions.

Preferences Workers have a constant labor endowment $N = 1$ which they supply inelastically. Entrepreneurs do not work. Entrepreneurial consumption is denoted C_{it} and workers' consumption is denoted c_t . The period utility of entrepreneur i is $u(C_{it}) = \frac{C_{it}^{1-\sigma}}{1-\sigma}$ or $u(C_{it}) = \log(C_{it})$ in the event $\sigma = 1$. We assume $\sigma \geq 1$.⁶ The period utility of workers is $u(c_t)$, i.e. the same utility function as entrepreneurs.

Government **Must change the government tab to account for gov't borrowing** The government taxes the undepreciated part of capital at the end of the period at tax rate τ_W , where capital depreciates at rate δ . The government taxes intermediate goods that are sold to the final goods firm at tax rate τ_E and taxes capital leased directly to the final goods firm at tax rate τ_F . It is assumed that the government is unable to tax intermediate goods that are converted directly into consumption by the entrepreneurs.⁷ It taxes labor at rate τ_N . Any of these tax rates can be negative.

The government has to finance exogenous expenditure \bar{G} , and must balance its budget. Taxes are paid in the evening and government spending also takes place in the evening. The government is not allowed to trade in financial assets at any time. This implies the following government budget constraint in the evening:

$$\bar{G} = w\tau_N + \tau_E r_E Y_E + \tau_F r_F K_F + \tau_W(1 - \delta)K \quad (1)$$

Financial Markets and Budget Constraints Workers live hand to mouth. Thus, aggregate worker consumption and labor supply satisfy:

$$c_t = w_t(1 - \tau_N) \quad (2)$$

$$N = 1. \quad (3)$$

Entrepreneurs may fund capital purchases in the morning by selling financial claims on the return of that capital. These claims are bought by risk neutral perfectly competitive financial intermediaries. Financial intermediaries make zero profits in equilibrium.⁸

⁶Aspects of the model solution jump discontinuously when $\sigma < 1$, because if entrepreneurs are sufficiently close to risk neutral they will not seek to avoid zero consumption. $\sigma \geq 1$ represents the typical case assumed in calibrated macroeconomic models.

⁷As discussed below, the conversion of intermediate goods directly into consumption is privately observed by the entrepreneur, and so the government cannot tax this. This may also be viewed as representing the possibility of tax evasion on the part of entrepreneurs.

⁸Since financial intermediaries make zero profits, it makes no difference to the equilibrium behavior of the economy who owns the banks. We may assume that they are owned either by workers or by entrepreneurs.

Financial intermediaries raise the funds to buy these financial claims by issuing bonds to other entrepreneurs.⁹ We refer to an entrepreneur as a borrower if she sells financial claims to the financial intermediary in the morning in order to invest, and a saver if she instead buys bonds.

Let $b_t(k, \theta)$ denote the amount this entrepreneur obtains in the morning by selling financial claims. This is positive if the entrepreneur is a borrower in the morning and negative if the entrepreneur is a saver. Let $k_t^E(k, \theta)$ denote the level of capital the entrepreneur born with endowment k and type θ invests in her own business, and let $k_t^F(k, \theta)$ denote the level of capital she invests in the risk-free sector. None of these objects depend on ϵ because ϵ is not revealed until the afternoon. Therefore, in the morning the budget constraint of an entrepreneur is:

$$k_t^E(k, \theta) + k_t^F(k, \theta) = k_t + b_t(k, \theta). \quad (4)$$

Let $C_t(k, \theta, \epsilon)$ and $I_t(k, \theta, \epsilon)$ be the consumption and investment of an entrepreneur, which depend not only on initial capital k and entrepreneurial type θ , but also on the productivity realization ϵ . Let $\hat{b}_t(k, \theta, \epsilon)$ denote the amount the entrepreneur pays in the evening to the financial intermediaries that bought financial claims in the morning. This will be negative if the entrepreneur was a saver in the morning. Let $y_t^E(k, \theta, \epsilon)$ denote the total amount of intermediate goods sold to the final goods producer by the entrepreneur. Given the assumptions about financial markets, the budget constraint of an entrepreneur in the evening is:

$$C_t(k, \theta, \epsilon) + (1 + \tau_I) I_t(k, \theta, \epsilon) + \hat{b}_t(k, \theta, \epsilon) = (1 - \tau_E) r_t^E y_t^E(k, \theta, \epsilon) + \rho \left(\theta \epsilon k_t^E(k, \theta) - y_t^E(k, \theta, \epsilon) \right) + r_t^F (1 - \tau_F) k_t^F(k, \theta) - \tau_W (1 - \delta) \left(k_t^E(k, \theta) + k_t^F(k, \theta) \right), \quad (5)$$

where

$$I_t(k, \theta, \epsilon) = k_{t+1}(k, \theta, \epsilon) - (1 - \delta) \left(k_t^E(k, \theta) + k_t^F(k, \theta) \right) \quad (6)$$

and $y_t^E(k, \theta, \epsilon)$ satisfies:

$$y_t^E(k, \theta, \epsilon) \leq \theta \epsilon k_t^E(k, \theta), \quad (7)$$

Note that an entrepreneur who sells all her units of intermediate goods to the final goods producer receives after-tax income $(1 - \tau_E) \theta \epsilon r_t^E$ from doing so, and an entrepreneur who converts them directly into units of final output secures after-tax income $\theta \epsilon \rho$ from this.

Recall that the entrepreneur decides how to divide resources between consumption

⁹The role of intermediaries is for mathematical convenience. Simply allowing the entrepreneurs to trade financial claims among themselves would suffice. In that sense the intermediaries are no more essential to the model than are the competitive final goods producers.

and investment at the end of the period, after the financial intermediary is dead. This means the latter has no influence on this decision. Let $z_t(k, \theta, \epsilon)$ denote the “cash on hand” that the entrepreneur has at the end of the period to spend on consumption and investment. Combining (5) and (6), the budget constraint of the entrepreneur in the evening can be rewritten as:

$$\begin{aligned} z_t(k, \theta, \epsilon) &= C_t(k, \theta, \epsilon) + (1 + \tau_I) k_{t+1}(k, \theta, \epsilon) & (8) \\ z_t(k, \theta, \epsilon) + \hat{b}_t(k, \theta, \epsilon) &= (1 - \tau_E) r_t^E y_t^E(k, \theta, \epsilon) + \rho \left(\theta \epsilon k_t^E(k, \theta) - y_t^E(k, \theta, \epsilon) \right) \\ &+ r_t^F (1 - \tau_F) k_t^F(k, \theta) + (1 - \tau_W) (1 - \delta) \left(k_t^E(k, \theta) + k_t^F(k, \theta) \right) & (9) \end{aligned}$$

The value of $z_t(k, \theta, \epsilon)$ is specified by the entrepreneur’s contract with the financial intermediary, which we describe shortly.

Financial contract An entrepreneur’s realization of ϵ , her output of intermediate goods, the quantity of intermediate goods she converts directly into consumption and her consumption are all private information. In particular, after observing the shock ϵ , an entrepreneur can choose to honestly report her output of intermediate goods, but she can also lie by under-reporting the quantity of intermediate goods she produces and converting more intermediate goods directly into final output than she admits to. The quantity of intermediate goods the entrepreneur sells to the final goods producer is public information however.¹⁰

When selling financial claims in the morning, the market will expect the entrepreneur to repay $\hat{b}_t(k, \theta, \epsilon)$ in the evening, given ϵ . In equilibrium, the market must be correct in expecting this, and so the entrepreneur must have an incentive to repay this amount, rather than lying about ϵ and repaying too little. Therefore, it is without loss of generality to restrict attention to contracts where the entrepreneur honestly reports her ϵ , and pays the promised amount $\hat{b}_t(k, \theta, \epsilon)$. Such a contract is only incentive compatible if it is optimal for the entrepreneur to report ϵ honestly, rather than lying by reporting some $\hat{\epsilon} \neq \epsilon$ and converting more intermediate goods directly into final output. This gives rise to the following incentive compatibility constraint:

$$\frac{\partial z_t(k, \theta, \epsilon)}{\partial \epsilon} = \rho \theta k_t^E(k, \theta) \quad (10)$$

for any (k, θ, ϵ) . I’m not sure how to interpret this given that up to hear we haven’t set up the value functions in the infinite horizon notes. Also, in the slides you haven’t set the problem in terms of z so this constraint looks different there. I like the z notation because it makes it clear that consumption-investment happens after the bank is dead.

The additional constraint that the entrepreneur faces in selling financial claims to the

¹⁰In the extreme case $\rho = 0$ there would be no informational friction, since the entrepreneur has no incentive to convert intermediate goods directly into consumption.

financial intermediaries is the participation constraint of the intermediaries. Specifically, financial intermediaries will only buy financial claims sold by the entrepreneur if these claims receive at least $(1 - \tau_F) r_t^F + (1 - \delta)(1 - \tau_W)$ return in the evening in expectation, where $(1 - \tau_F) r_t^F + (1 - \delta)(1 - \tau_W)$ is the market risk-free rate, which is equal to the post-tax return on capital that is leased directly to the final goods firms. This implies the following constraint:

$$\int_{\epsilon} \hat{b}_t(k, \theta, \epsilon) h(\epsilon) d\epsilon \geq \left[(1 - \tau_F) r_t^F + (1 - \delta)(1 - \tau_W) \right] b_t(k, \theta). \quad (11)$$

Since there is perfect competition between the (risk-neutral) financial intermediaries, the entrepreneur will be able to sell any financial claims she issues, provided the incentive compatibility constraint and equation (11) are satisfied. Therefore, the entrepreneur can choose $b_t(\cdot)$ and $\hat{b}_t(\cdot)$ to maximize her expected utility, subject to these constraints. This implies that the bank participation constraint will bind with equality.

Entrepreneur's Optimization Problem Let $V_t(k, \theta)$ denote the value function in the morning for an entrepreneur with capital k and ability θ , and let $\tilde{V}_t(z, \theta)$ denote the value function in the evening of an entrepreneur with cash on hand z and ability θ who is yet to choose between consumption and investment. Then $V_t(k, \theta)$ and $\tilde{V}_t(z, \theta)$ evolve according to the following Bellman equations:

$$\tilde{V}_t(z, \theta) = \max_C u(C) + \beta(1 - \gamma) \mathbb{E}_{\theta'} \left(V_{t+1} \left(\frac{z - C}{1 + \tau_I}, \theta' \right) \mid \theta \right) \quad (12)$$

$$V_t(k, \theta) = \mathbb{E}_{\epsilon} (\tilde{V}_t(z, \theta)) \quad (13)$$

where the equation (12) uses the budget constraint (8) and the $1 - \gamma$ reflects the probability that the entrepreneur may die. Substituting (13) into (12) we obtain:

$$\tilde{V}_t(z, \theta) = \max_C u(C) + \beta(1 - \gamma) \mathbb{E}_{\theta', \epsilon} (\tilde{V}_{t+1}(z', \theta') \mid \theta) \quad (14)$$

where $z' = z \left(\theta', \frac{z - C}{1 + \tau_I}, \epsilon' \right)$. Then, the entrepreneur writes a contract with the financial intermediary aiming to maximize her expected end of period value. That is, the optimal financial contract specifies $\left\{ z_t(k, \theta, \epsilon), k_t^E(k, \theta), k_t^F(k, \theta), b_t(k, \theta), \hat{b}_t(k, \theta, \epsilon), y_t^E(k, \theta, \epsilon) \right\}$ which solve

$$\sup_{\epsilon} \int_{\epsilon} \tilde{V}(z(k, \theta, \epsilon), \theta, X) h(\epsilon) d\epsilon \quad (15)$$

subject to

$$k_t^E(k, \theta) + k_t^F(k, \theta) = k_t + b_t(k, \theta) \quad (16)$$

$$z_t(k, \theta, \epsilon) + \hat{b}_t(k, \theta, \epsilon) = (1 - \tau_E)r_t^E y_t^E(k, \theta, \epsilon) + \rho \left(\theta \epsilon k_t^E(k, \theta) - y_t^E(k, \theta, \epsilon) \right) \\ + r_t^F (1 - \tau_F) k_t^F(k, \theta) + (1 - \tau_W) (1 - \delta) \left(k_t^E(k, \theta) + k_t^F(k, \theta) \right) \quad (17)$$

$$\int_{\epsilon} \hat{b}_t(k, \theta, \epsilon) h(\epsilon) d\epsilon \geq \left[(1 - \tau_F)r_t^F + (1 - \delta)(1 - \tau_W) \right] b_t(k, \theta) \quad (18)$$

$$k_t^E(k, \theta) \geq 0 \quad (19)$$

$$y_t^E(k, \theta, \epsilon) \in [0, \theta \epsilon k_t^E(k, \theta, \epsilon)] \quad (20)$$

$$z_t(k, \theta, \epsilon) \geq 0 \quad (21)$$

$$\frac{\partial z_t(k, \theta, \epsilon)}{\partial \epsilon} = \rho k_t^E(k, \theta) \theta. \quad (22)$$

After financial intermediaries die, the entrepreneur chooses $\{C_t(k, \theta, \epsilon), k_{t+1}(k, \theta, \epsilon)\}$ to maximize (14) subject to (8).

Market clearing The labor and asset market clearing conditions are:

$$N = 1 \quad (23)$$

$$\int_i b_t(k_{it}, \theta_{it}) di = 0 \quad (24)$$

$$\int_i \hat{b}_t(k_{it}, \theta_{it}, \epsilon_{it}) di = 0. \quad (25)$$

where k_{it} denotes the initial capital of entrepreneur i , and θ_{it} and ϵ_{it} denote the values of θ and ϵ drawn by entrepreneur i in period t . In fact, since ϵ_{it} is an i.i.d. draw for each entrepreneur i , equation (25) can be shown to follow from (24) and (11), so it is redundant.

The capital market must clear in the morning of each period:

$$K_t = K_t^E + K_t^F, \quad (26)$$

where K_t^E denotes total capital invested in entrepreneurs' private projects, and K_t^F is total capital in the risk-free sector. These are given, respectively, by:

$$K_t^E = \int_i k_t^E(k_{it}, \theta_{it}) di \quad (27)$$

$$K_t^F = \int_i (k_{it} - b_t(k_{it}, \theta_{it}) - k_E(k_{it}, \theta_{it})) di. \quad (28)$$

and the goods market clearing condition then follows by Walras' law:¹¹

$$\int_i C(k_{it}, \theta_{it}, \epsilon_i) di + c_t + \bar{G} = F(Y_t^E, K_t^F, 1) + (1 - \delta)(K_t^E + K_t^F),$$

Check for changes due to gov't borrowing where the total amount of intermediate goods sold to the final goods producer, Y_t^E , is given by:

$$Y_t^E = \int_i y_t^E(k_{it}, \theta_{it}, \epsilon_{it}) di. \quad (29)$$

The first order conditions of the representative final goods producer imply that the (before tax) returns on capital and wage rate are given by:

$$r_t^E = F_1(Y_t^E, K_t^F, N) \quad (30)$$

$$r_t^F = F_2(Y_t^E, K_t^F, N) \quad (31)$$

$$w_t = F_3(Y_t^E, K_t^F, N). \quad (32)$$

Equilibrium Definition For given tax rates $\{\tau_W, \tau_E, \tau_F, \tau_N\}$, an equilibrium \mathcal{E} of this economy is a set of (non-negative) prices $\{r_t^E, r_t^F, w_t\}$ and decision rules for the entrepreneurs $\{C_t(k, \theta, \epsilon), k_{t+1}(k, \theta, \epsilon), z_t(k, \theta, \epsilon), k_t^E(k, \theta), k_t^F(k, \theta), b_t(k, \theta), \hat{b}_t(k, \theta, \epsilon), y_t^E(k, \theta, \epsilon)\}$ and for the workers c , and (non-negative) aggregate quantities K_t^F, Y_t^E, N , such that:

1. The Government's budget constraint (1) is balanced, **must be changed to account for gov't borrowing**
2. Worker consumption and labor supply satisfy (2) and (3),
3. Entrepreneurs' decision rules are given by the solution to the entrepreneur's problem in (14) and (15),
4. K_t^F and Y_t^E are the aggregate of entrepreneurs' decisions, given by (28) and (29),
5. Returns of capital r_t^E and r_t^F , and wages w_t are determined by the first order conditions of the final goods producers (30)-(32).

¹¹In particular, the goods market clearing condition can be obtained by summing the budget constraints of workers, entrepreneurs and government, substituting the other market clearing conditions and using that F displays constant returns to scale.

References

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