FREGE'S CASE FOR THE LOGICALITY OF HIS BASIC LAWS

Alexander R. Yates

A Thesis Submitted for the Degree of PhD
at the
University of St Andrews

2017

Full metadata for this item is available in
St Andrews Research Repository
at:
http://research-repository.st-andrews.ac.uk/

Please use this identifier to cite or link to this item:
http://hdl.handle.net/10023/12252

This item is protected by original copyright

Frege's Case for the Logicality of his Basic Laws

Alexander R. Yates

University of
St Andrews

This thesis is submitted in partial fulfilment for the degree of
PhD
at the
University of St Andrews

23/09/2016

1. Candidate's declarations:

I, **Alex Yates**, hereby certify that this thesis, which is approximately **67,000**. words in length, has been written by me, and that it is the record of work carried out by me, or principally by myself in collaboration with others as acknowledged, and that it has not been submitted in any previous application for a higher degree.

I was admitted as a research student in **September, 2012** and as a candidate for the degree of **PhD** in [September, 2012]; the higher study for which this is a record was carried out in the University of St Andrews between **2012** and **2016**.

(*If you received assistance in writing from anyone other than your supervisor/s*):
I, **Alex Yates**, received assistance in the writing of this thesis in respect of **typographical error identification**, which was provided by **Sebastian Becker, Femke van Coile, Ravi Thakral, Noah Friedman-Biglin, Caroline Touborg, and Matthew McKeever**

Date **19/09/2016** signature of candidate ………

2. Supervisor's declaration:

I hereby certify that the candidate has fulfilled the conditions of the Resolution and Regulations appropriate for the degree of **PhD** in the University of St Andrews and that the candidate is qualified to submit this thesis in application for that degree.

Date **19/09/2016** signature of supervisor ………

3. Permission for publication: (*to be signed by both candidate and supervisor*)

In submitting this thesis to the University of St Andrews I understand that I am giving permission for it to be made available for use in accordance with the regulations of the University Library for the time being in force, subject to any copyright vested in the work not being affected thereby. I also understand that the title and the abstract will be published, and that a copy of the work may be made and supplied to any bona fide library or research worker, that my thesis will be electronically accessible for personal or research use unless exempt by award of an embargo as requested below, and that the library has the right to migrate my thesis into new electronic forms as required to ensure continued access to the thesis. I have obtained any third-party copyright permissions that may be required in order to allow such access and migration, or have requested the appropriate embargo below.

The following is an agreed request by candidate and supervisor regarding the publication of this thesis:

PRINTED COPY
a) Embargo on all or part of print copy for a period of **2** years on the following ground(s):
- Publication would be commercially damaging to the researcher, or to the supervisor, or the University
- Publication would preclude future publication

Supporting statement for printed embargo request if greater than 2 years:

N/A

ELECTRONIC COPY
a) Embargo on all or part of electronic copy for a period of **2** years on the following ground(s):
- Publication would be commercially damaging to the researcher, or to the supervisor, or the University
- Publication would preclude future publication

Supporting statement for electronic embargo request if greater than 2 years:

N/A

ABSTRACT AND TITLE EMBARGOES

If you have selected an embargo option indicate below if you wish to allow the thesis abstract and/or title to be published. If you do not complete the section below the title and abstract will remain embargoed along with the text of the thesis.

a) I agree to the title and abstract being published YES
b) I require an embargo on abstract NO
c) I require an embargo on title NO

Date **19/09/2016** signature of candidate …… signature of supervisor ………

.

Abstract

Frege wanted to show that arithmetical truths are logical by proving them from purely logical basic laws. But how do we know that these basic laws are logical? Frege uses generality and undeniability to make a prima facie case for logicality—if a truth is general and undeniable, then it's likely logical. I argue that Frege could, did, and had to make a deeper case for why we're right in recognizing his basic laws as logical. Implicit in his work is a view of logical laws as epistemically analytic—his arguments for his basic laws serve to elicit a reflective awareness of the fact that understanding them is sufficient for recognizing them to be true. This view both fits with Frege's comments concerning the connection between logic, truth, and normativity, and serves to explain why and in what sense he took logic to be general and conceptually undeniable.

In my view, semantics must play a distinctive role in any rational reconstruction of Frege's case for logicality—the aforementioned "reflective awareness" must be an explicit appreciation of how the truth of formulas expressing Frege's laws follows quickly from his stipulations governing terms which figure in those formulas. Opposing this view is the elucidatory interpretation of Thomas Ricketts, Warren Goldfarb, and Joan Weiner, which holds that Frege's arguments for his basic laws can't be taken at face value, and must serve the merely elucidatory purpose of easing us into the language. Another reading is the correctness interpretation of Richard Heck and Jason Stanley, which holds that Frege's primary purpose in his arguments is justifying the claim that Frege's axioms, qua formulas, are true. I argue against both of these interpretations, and in doing so clarify the role and limits of semantics in Frege's enterprise.

Thesis Outline

Acknowledgments p.5

Introduction p.7
§0.. p.7
§1 Determining logicality.. p.8
§2 Semantics and elucidation p.11
§3 Outline of dissertation p.13

Chapter 1: Foundationalism and Inference p.15
§0 .... p.15
§1. Foundationalism... p.15
§1.1 The heritability strategy p.15
§1.2 Evidence for foundationalism p.17
§1.3 Are axioms determined or chosen? p.18
§1.4 Self-evidence p.19
§1.5 The role of pragmatic considerations p.22
§1.6 Burge on Frege's rationalism.. p.23
§2. Axioms vs inference rules p.24
§2.1... p.24
§2.2 Logical and non-logical inference rules p.24
§2.3 The nature of the inference-rule/axiom distinction p.26
§2.4 Laws used as inference rules p.27
§3. Conclusion p.29

Chapter 2: Heuristic Indicators of Logicality: Generality and Undeniability p.31
§0.. p.31
§1. Logic, truth, and normativity.. p.31
§2. Introduction to the heuristic indicators p.34
§3. Heuristic indicator 1: generality p.35
§3.1 Types of generality p.35
§3.2 Universal applicability vs topic-neutrality.. p.36
§3.3 Constraints associated with universal applicability p.37
§3.4 Universal applicability and logic as all-encompassing p.39
§3.5 Logic as maximally general p.40
§4. Heuristic indicators 2: conceptual undeniability.. p.44
§5. How the heuristic indicators relate to Frege's case for logicality.................... p.45
§5.1 Some possible accounts p.45
§5.2 The no-method account p.46
§5.3 The heuristic indicator account p.47
§5.4 The demarcation account p.49
§5.5 The EA account, and its relation to the demarcation account....................... p.50
§6. Conclusion p.52

Chapter 3: The EA Interpretation I: Inference and Understanding p.53
§0.. p.53
§1. Truth, judgement, and understanding p.53
§1.1... p.53
§1.2 Our implicit grasp of truth p.54
§1.3 Our implicit grasp of truth-values p.55
§1.4 From truth to judgement p.56
§1.5 From judgement to understanding... p.57
§2. Understanding, compositionality, and inference....................................... p.60
§2.1 Stroud on inference p.60
§2.2 Compositionality p.62
§2.3 Inferential constraints upon understanding p.64
§2.4 Which inferences are requisite inferences? p.65
§2.5 How INF+ explains the heuristic indicators p.69
3. Conclusion.. p.70

Chapter 4: The EA Interpretation II: Frege's Arguments for Basic Laws...... p.71
§0.. p.71
§1.The data: how Frege discusses his laws.. p.71
§1.1 Introductions of symbols and functions p.71
§1.2 Arguments for basic laws.. p.72
§1.3 Arguments for inference rules p.73
§1.4 Two questions p.74
§2. The E.A. interpretation's view of the data p.74
§2.1 Laws I-IV p.74
§2.2 Laws V and VI p.76
§3. Some clarifications of the E.A. interpretation p.80
§3.1 The role of semantics in the E.A. interpretation............................. p.80
§3.2 The role of sense in *Grundgesetze*... p.82
§3.3 Differences from Burge's view p.83
§4. Replies to some objections p.85
§4.1 Is this circular, and if so, should that bother us?............................ p.85
§4.2 Actual vs hypothetical inferences... p.86
§4.3 Overgeneration... p.88
§5. Other readings of Frege's arguments.. p.90
§5.1 Weiner on Frege's arguments p.90
§5.2 Against Weiner's reading.. p.90
§5.3 Heck's reading and the nature of metalanguage p.91
§6. Conclusion p.94

Chapter 5: The Elucidatory and Correctness Interpretations p.95
§0.. p.95
§1. The elucidatory interpretation.. p.96
§1.1 Lack of a meta-perspective... p.96
§1.2 Elucidation p.98
§1.3 Talking about concepts, functions, and objects p.100
§1.4 Ricketts on Frege on truth p.103
§1.5 Against Ricketts's account of Fregean truth p.105
§1.6 Against the elucidatory reading of §29-31....................................... p.106
§2. The correctness interpretation p.108
§2.1 The correctness requirement.. p.108
§2.2 Frege's critique of the formalists p.109
§2.3 Heck and Stanley on Frege's argument p.110
§3. Conclusion p.112

Conclusion p.113

Bibliography p.115

Acknowledgements

First and foremost, I'd like to acknowledge the very large role of my primary supervisor, Prof. Peter Milne, for tirelessly picking through a great many drafts of various chapters and subsections of the present work, and engaging with me in countless discussions of interpretive issues, discussions to which the present work owes a great deal. He pointed me to a great many sources which exerted a decisive influence on the shape of my work, and also instilled upon me an awareness of the ways in which Frege was subtly unclear about certain matters, an awareness which I hope has resulted in a more realistic picture of Frege's views on logicality and semantics.

Many thanks as well to my secondary supervisor, Prof. Peter Sullivan. Discussions with him frequently marked a turning point in how I treated certain issues, and his probing questions always made me think very hard about precisely what I was claiming, and whether or not what I was claiming successfully addressed the most significant questions that could be asked about Frege's perspective.

Thanks go to Ravi Thakral, Caroline Touborg, Femke van Coile, and to Drs. Sebastian Becker, Noah Friedman-Biglin, and Matt McKeever for reading through penultimate drafts of various chapters, and helping me catch residual errors. I'm very grateful to my reviewers, Prof. Michael Beaney and Dr. Philip Ebert, for their extensive and helpful comments during the viva. I'd also like to acknowledge the helpful feedback I received on portions of my thesis in the St Andrews Friday seminars for philosophy research students, as well as comments received at the Arché work-in-progress seminar, in the Arché research groups "Modals Modality and Meaning" and "History and Philosophy of Logic and Mathematics", and especially at the Early Analytic Group at Stirling in 2016.

Thanks are owed for the tremendous amount of moral support I received from friends and family over the past four years, which made the process far more pleasant than it might otherwise have been—to Caroline for our frequent walks and associated discussions, to Dr. James Page for his advice, to Professor Stephen Read for his support and presence, to Michael Traynor, Femke van Coile, and Fenner Tanswell for much-needed mutual commiserations, to the support of my fellow philosophy postgrads, and to Aikman's for providing the setting at which a considerable amount of this support took place. Thanks for the encouragement of my family, especially my Uncle, Peter Vatne, who provided support, helped me travel to and from airports, and got me out of St Andrews when I needed it most, and to my parents, Drs. Diane Vatne and Martin Yates, for their love and support.

Introduction
§0

Well over a century after Frege's first logical tract was published, his writings continue to be a source of inspiration for philosophers of logic, maths, and language, and are regularly being interpreted and employed in novel ways and contexts. This shouldn't surprise us, for although logic has gone through many decades of quite dramatic development since Frege's time, it nevertheless remains true that his originality, achievements, and insights into logic are difficult to exaggerate. He showed how to employ second-order principles to provide a justification of mathematical induction, and demonstrated how basic arithmetical principles could be derived from Hume's principle (Heck 2012, 114-173). But these mathematical results, impressive though they are, pale in comparison to those achievements which are apt to be less immediately striking to the modern reader, simply because these insights have become such an essential and fundamental part of our common logical toolkit. Chief among those achievements are Frege's quantifier-variable notation, which provided a leap in expressive power sufficient to allow the perspicuous expression of concepts and arguments found in mathematics itself, and Frege's development of a symbolic system in which the validity or invalidity of any proof within it is an entirely formal and mechanically checkable matter.

To list Frege's accomplishments in this piecemeal way risks, however, missing out on one of the features of Frege's work that most contributes to its enduring appeal. And that is that Frege's various logical achievements were all intimately tied together, products of his dogged commitment to an obscure technical mission—to demonstrate that arithmetic reduces to pure logic. His goal constrains the means—a mechanically checkable system is needed in order to rule out surreptitious reliance on intuition, and his function-argument analysis of propositions, and the resulting quantifier-variable notation based upon it, is necessary for allowing the expression of arithmetical concepts in what Frege would have considered purely logical language.

Why would Frege embark on this technical project in the first place? Presumably because, if successful, showing arithmetical propositions to be logical ones would go some way to explain why arithmetic appears to be a priori, undeniable, and generally applicable. Frege was especially concerned to account for the generality of arithmetical reasoning. Any sorts of objects whatsoever may be counted, not merely those which happen to figure in spatio-temporal relations, and this calls out for explanation. It struck Frege as unlikely that anything like Kantian intuition could serve as such an explanation. Such intuition is, by definition, limited only to that which is intuitable, so any such account would either have to leave unexplained how it is that arithmetic applies to that which is thinkable but not intuitable, or else would have to artificially limit arithmetic's range of applicability[1].

Almost all interpreters of Frege agree that he saw himself as trying to reduce arithmetic to pure logic[2]. Consensus doesn't extend much further than this, however—there's considerable difference of opinion over just what Frege took such a reduction to involve. Although the exegetical questions surrounding it are myriad, we can sort most questions concerning Frege's logicist reduction into three broad categories:

1. Questions concerning the reductions themselves
2. Questions concerning the foundations
3. Questions concerning semantics and meta-perspectives.

---

[1] This is, I take it, the main *reason* why Frege wanted to dispense with intuition. There is also a more historical explanation—Frege was a mathematician, and mathematicians at the time in which Frege was writing were increasingly concerned with how to prove even apparently obvious truths without appeal to intuition.

[2] One possible exception is Joan Weiner—she says that "antecedent to Frege's introduction of his definitions, the concept of number is not fixed" (2010, 41). Even she grants that conceptual content, as Frege characterizes it in *Begriffsschrift* (content which is significance for inference) must be preserved by definitions (Weiner 2010, 37).

Questions in the first category involve Frege's views on conceptual analysis—what Frege derives from his basic laws are not informal arithmetical truths, but truths expressed by complex logical formulas, the relevance of which for arithmetic is not immediately obvious. What relation do these ordinary truths bear to their arcane analyses? To pose the question in Fregean terms, did he intend his intricate analyses of cardinal numbers and other notions to preserve sense, to preserve reference, or is a Fregean analysans connected to its analysandum in some subtler way? Questions in the second category take us to the other end of the explanatory chain—they concern what we are reducing arithmetic *to*. What general criteria enable us to determine whether a law is logical? How should we employ these criteria? Do we require a positive characterization of logicality, or can one rest content with a more piecemeal approach?

The third and final category contains more general questions, the answers to which have implications concerning the sorts of tools Frege could utilize in answering questions of the first and second type. These questions concern the necessity and possibility of a meta-perspective, and whether Frege would have been willing to employ semantic methods for certain purposes. Could Frege give semantic justifications of his logical laws? Could he be said to have a semantic conception of logical consequence? Can he have an overarching conception of what logic amounts to, or is this precluded by other theoretical commitments of his?

Although I will have a fair amount to say about the status of non-basic logical truths (I will discuss, for instance, whether logical truths expressed by very complex formulas should be considered undeniable) I will not have a great deal to say about conceptual analysis. To be sure, questions in the first category are essential for understanding and evaluating Frege's philosophy of mathematics. Logicism is no good if we have no idea how arithmetical truths as ordinarily understood relate to Frege's complex analysans—absent some connection, Frege has only established his logicist thesis for thoughts expressed by rather arcane formulas, rather than for the arithmetical statements about whose nature we were originally curious. But the purpose of this thesis lies not in contributing to the revival of logicism, but in showing that even if Fregean logicism is, in the end, something which admits of no easy fix, this shortcoming should not lead us to impugn Frege's philosophy of logic. Accordingly, the bulk of my attention concerns questions of the 2nd and 3rd categories. In this thesis, the focus is on Frege's basic laws—how he selected them, how he thought we knew them, and what grounds he took himself to have given for our taking them to be logical. One finds that it's impossible to progress very far with these questions without turning to general issues concerning whether, and how, Frege employed semantics methods.

§1. Determining logicality

Let's briefly put aside questions concerning whether, and in what sense, Frege used semantic tools in carrying out his logicist project, and concentrate instead, for the moment, solely on the question of the logical or non-logical nature of his basic laws. If Frege intended to justify his logicism, it goes without saying that he must have taken himself to provide some sort of evidence that arithmetical claims are justified solely on the basis of pure logic. His model for doing so is a foundationalist one—we start by proving everything that can be proven, following the chain of justification backwards until we arrive at the ultimate ground upon which the propositions rest. This foundationalism of Frege's has, I think, been underplayed by some, due no doubt to the difficulty in seeing what it is that makes primitive laws genuinely primitive. For now, it suffices to say that Frege did take his foundationalism seriously, and that while we have a certain amount of leeway in our selection of axioms, our choice is still limited to objectively primitive truths (I take Tyler Burge to have argued convincingly for this point—I return to it in the relevant part of Chapter 1).

After proving all that is provable, we then have to examine the place at which the chain of justification for the proposition in question terminates, and somehow determine whether or not this ultimate ground is logical. It is here that matters get difficult, for Frege himself said relatively little about how we should go about making this determination. He wasn't (and, given his commitments, he couldn't have been) completely silent on the matter though—we do find in Frege's works certain necessary conditions which he believed basic laws must meet if they are to be logical. Throughout the present work, I shall refer to the necessary conditions for logicality that Frege explicitly discusses as his "heuristic indicators" for logicality. The intention is to highlight the fact these indicators provide no very *sharp* way of demarcating logical from non-logical truths—they're little more than a rules of thumb which Frege uses to make a prima facie case for the logical or non-logical nature of certain laws. What's needed is a careful account of the role that these conditions play in Frege's epistemology of logic, and how they relate to those places in which Frege, however briefly, discusses and motivates his basic laws. In particular, we must ask whether these necessary conditions were *all* that Frege had at his disposal, or whether his practice suggests he thought a more determinate case could be made for the logicality of his basic laws.

What are these heuristic indicators? Some necessary conditions for logicality are not *distinctively* logical—for instance, Frege thought basic logical-laws must be self-evident, but required geometrical axioms to be so as well, even he took the latter to be non-logical. Other conditions seem more distinctly logical. Firstly, it's universally agreed that Frege took logic to be *general* in some sense. Of course, there are many different respects in which a thought or an inference rule can be general. It can be *topic neutral*, dealing with no specific subject matter. It can be *maximally general*, containing only terms which must figure in any formal presentation of any given science. It can be *universally applicable* to all subject-matter. Although Frege did not, by the time of *Grundgesetze*, take logic to be topic neutral, he thought maximal generality and apparent universal applicability served, in certain circumstances, as heuristic indicators for the logical nature of a law.

Secondly, Frege thought that logical laws were *undeniable*. Like generality, this indicator admits of multiple readings. We might think of undeniability in instrumental terms—we *shouldn't* deny any law of logic *if truth is our goal*. Alternatively, we might think of undeniability in stricter terms—we *cannot* effect a cognitive denial of a law of logic, because of the inextricable connection between logical laws and thinking. I opt for the latter reading, on the grounds that the instrumental reading has a sort of circularity which would prevent it from being a useful heuristic indicator.

There are other conditions one might think function as heuristic indicators of logicality, though their precise role is more unclear. Frege often has recourse to certain broadly pragmatic considerations in his epistemology of logic—for instance, he uses considerations of fruitfulness and simplicity as a guide to selecting his basic laws. He also occasionally seems to take the fact that a principle is implicit in the practice of logicians and mathematicians as some sort of indirect support for it being logical (as he arguably does in support of the logical nature of V, in §147, vol. II, of *Grundgesetze*). Despite initial appearances, I don't think that pragmatic elements are used as indicators of logical nature in the same way as undeniability and generality—I argue that such considerations as fruitfulness, simplicity, and implicit presence in practice instead play a role in determining which laws count as more basic than others.

So we have at least two heuristic indicators for logicality—generality and conceptual undeniability. Frege uses these, in places such as §14 of *Grundlagen*, to make a preliminary case for the logicality of arithmetic. There's much to say about how Frege understood these conditions, and how he applied them, but let's put these aside for the moment in order to consider the overall role of the indicators. After we've told a story about how Frege understood and applied them, is there anything left to do in explaining the case Frege made for the logical nature of his axioms? May we rest content with a rather piecemeal approach to logicality, or must we say something about what binds the heuristic indicators together? I shall argue that we must, if we are to do justice to the epistemic motivations of Frege's project, look for a deeper conception of the logical, implicit in Frege, from which the indicators follow. Recall that part of why Frege wants to reduce arithmetic to logic is to account for the special nature of arithmetical propositions—he wants to *explain* the general and certain nature of such statements in terms of their justification on the basis of purely logical principles. But for this to truly be an *explanation*, it's not enough to show that the derivational basis of arithmetic is general and undeniable, and promptly conclude that this basis is logical. We must also know *something* about what it means to say that logicality is what generality and undeniability are indicators *of*—otherwise, the claim that arithmetic reduces to logic will be an uninteresting one.

If Frege allowed no deeper case to be made for why we're right to take a law to be logical, then it becomes mysterious why he should have cared so much about his logicism in the first place. Logicality would become a sort of unnecessary middle-man between the heuristic indicators and the special features of arithmetic that he wished to account for. If, to show that a arithmetical truths are a priori, it was sufficient to show first that certain primitive truths are general and undeniable, and that arithmetical truths are derivable from these primitive truths, what would we gain by saying that generality and undeniability are heuristic indicators of *logicality*? We could explain the special features of arithmetic directly in terms of generality and conceptual undeniability, and the question of whether the primitive axioms are logical could drop out of the picture altogether.

There are two aims we might see Frege as working towards—trying to demonstrate conclusively *that* arithmetic is general, a priori, etc, or trying to *explain why* it has these properties. We need no general conception of logic to carry out the first aim—to be sure, for instance, that arithmetic is general in some important sense, it suffices to show that the arithmetical truth is justified on the basis of principles the generality of which is more clear and conclusive (and that inference rules preserve the appropriate notion of generality). Nothing need be said about logicality—to show that arithmetic has property P, we just show it rests on principles which more obviously possess property P. In doing this, however, we fail to give an account of *why* the more fundamental principles are general, and thus fail to give a deeper explanation of *why* it is that arithmetical truths are general.

Thus, if we're to make a deeper case for the nature of arithmetic, Frege had to have a general conception of logic which underlies his heuristic indicators. This isn't to say that he thought we could *define* logical truth—I argue that he thought that such a definition was impossible. But lack of a definition of logicality, or even a lack of a very sharp criterion for it, is *not* the same as the lack of a conception of the sort called for. Although his comments are scant, Frege does, especially in introductions to his various logical treatises, tell us what he thinks about the relation between truth, logicality, and normativity. So, after identifying the heuristic indicators and explaining how Frege applies them, the next step in my project is to identify the conception of logic which these general comments reflect.

I maintain that Frege conceived of basic logical laws as epistemically analytic—he thought that a full understanding of them sufficed for recognition of their truth. One of the goals of the current work is to flesh out just how this conception of logic is tied to how understanding works, and to present grounds for thinking such a conception is implicit in Frege's view of logic. There are at least three things which a conception of logical laws as epistemically analytic can nicely account for:

1. It makes sense of the way in which Frege discusses and motivates his logical laws.
2. It best explains some of his various platitudes about how logic, truth, and judgement connect.
3. It directly motivates Frege's heuristic indicators.

I think that attributing to Frege the claim that epistemic analyticity is a necessary (though perhaps not sufficient—more of this anon) goes some way to solving this puzzle. For in the version of epistemic analyticity I argue for, it's implicit in Frege's way of arguing for basic laws that he took understanding to be importantly connected with logically valid inference—for a given thought, no one who isn't competent with certain simple inferences in which that thought figures as a premise or a conclusion can be said to have fully grasped that thought. If this is so, it explains the sense in which logic is normative—if we fail to infer logically enough, we fail to understand, and thus fail to genuinely infer at all.

§2. Semantics and elucidation

I said that there are three classes of questions we might ask about Frege's logicism—what must be preserved in reductions, whether and how we can determine the logicality of basic laws, and questions concerning the possibility, for Frege, of meta-perspectives, meta-theory, and semantics. I've said quite a bit about the second class of questions, but it should be noted that it connects interestingly with questions in the third class. One of the most salient methods for sorting logical from non-logical primitive truths is to exploit semantic concepts—some of the most plausible candidate explanations for why we're right in taking certain thoughts to be logical ineliminably reference the semantic properties of those formulas that express them. If Frege thought semantic reasoning couldn't be put to serious and substantial use, this would have implications for ways of demarcating the logical which are available to him.

In both the *Begriffsschrift* and *Grundgesetze*, Frege provides what look for all the world like arguments for his basic laws. Many of these arguments, even when they talk of functions instead of directly of symbols, appear semantic in form—that is, they bear a striking resemblance to precisely the sort of semantic arguments that we'd use now-a-days to argue for the truth of a formula in the predicate calculus. For instance, in the early parts of *Grundgesetze der Arithmetic*, Frege provides truth-conditions for his propositional connectives and quantifiers ($\S5 - \S13$), and then uses these conditions to informally argue for the truth of several of his basic laws ($\S18, \S20, \S25$), as well as to argue that the transitions between formula which he allows are truth preserving ($\S14 - \S17$). One of the goals of my project is to show that the conception of logic I attribute to Frege motivates a reading of logical laws which provides an interpretive middle way between two common interpretations of these arguments.

The first interpretation of Frege's arguments for his basic laws, which counts Richard Heck and Jason Stanley among its proponents, sees the primary purpose of Frege's arguments to be the establishment of the truth of formulas (as opposed to the thoughts which they express), and the truth-preserving nature of his inference rules. I shall call this the *correctness interpretation*[3]. The problem with this interpretation is *not* that it sees Frege as doing semantics of a sort—he certainly did, $\S10$ and $\S31$ being the most salient examples. Rather, the issue is the role that this interpretation (particularly Heck's version) supposes semantics plays in Frege's view of logic. Heck makes two claims. Firstly, according to Heck, the *primary* purpose of Frege's $\S5 - \S25$ arguments in *Grundgesetze* was demonstrating correctness, and what's more, Frege was *committed* to doing so, if his own position was to avoid the criticisms he himself puts forth against the formalists' theories of arithmetic, namely that the formalists err in setting down rules for transitions among strings of symbols without regard to the meaning of those symbols.

Opposing the correctness interpretation, we have what I shall call the *elucidatory interpretation* of Frege, of which Thomas Ricketts and Joan Weiner are two influential proponents. According to the elucidatory interpretation, the relevant passages of *Grundgesetze* were *not*, despite appearances, intended to provide arguments for basic laws, nor should they be read as a genuine employment of semantic techniques to argue for the correctness of symbolic rules or axioms. Instead, Frege used these arguments, and the semantic notions which appear within them, in a purely *elucidatory* capacity—to help familiarize readers with his system, after which such loose talk could simply be dropped, having no official place within the system itself. Weiner develops this point further—Frege's arguments help us to gain fluency in the concept-script, and once we've attained the requisite understanding, we can just *see* that the axioms of Frege's system are logical, and any deeper justification for their logicality is neither possible nor necessary.

[3] I use "correctness" instead of "soundness" in order to dispense with the model-theoretic connotations of the latter. Correctness should be understood as the claim that all of the axioms, qua sentences, express *true* propositions, and that all inferences preserve *truth*. Soundness need not imply correctness—an intuitionist can easily recognize that all formulas of propositional intuitionistic logic are sound relative to classical semantics, but this fact doesn't establish the correctness of these formulas.

Neither interpretation is quite right[4]. The elucidatory reading doesn't do the introductory sections of *Grundgesetze* justice—the details of Frege's finely crafted argumentative techniques found there call out for explanation, which the elucidatory interpretation is ill-placed to provide. On the other hand, the correctness interpretation, while rightfully acknowledging the innovative use of semantic notions in Frege's work, nevertheless misses out on an epistemic dimension of Frege's philosophy, and in doing so misses out on the role of self-evidence in his philosophy, as well as the distinctive way in which Frege conceives of conceptual undeniability.

My own position, the E.A. interpretation ("E.A." for "epistemic analyticity"), is that it is in Frege's arguments for the truth of his basic laws that our grounds for taking these truths to be logical are provided. I also maintain that articulating these grounds requires an ineliminable use of semantic talk—if semantic talk is forced to play a merely elucidatory role, then there is no room for a Fregean explanation of why we take some laws and not others to be logical, and his logicism is rendered uninteresting (for the reasons sketched above, and to be developed in §3 of chapter 2). And contra Heck, I argue that using these semantic tools to prove the correctness of his system is not the primary aim of Frege's arguments for his basic laws. Instead, what Frege hopes to accomplish with his arguments for logical laws is eliciting our reflective appreciation of their self-evidence. That is, Frege leads us to see that his laws are true by getting us to think through how they are composed, and this is not primarily to get us to recognize *that* they are true, but for us to gain a reflective appreciation for *why* we are justified in taking them to be true purely on the basis of understanding. Frege's aim is to elicit a reflective awareness of the fact that a complete grasp of his basic laws is sufficient for a recognition of their truth—once we appreciate *that fact*, we also thereby appreciate that the law is one to which the two heuristic indicators, generality and undeniability, apply.

What we have here is a nice interplay between, on the one hand, some deep conceptual issues concerning how Frege conceived of his project and the nature of logic, and, on the other, a narrower exegetical puzzle concerning what Frege's primary purpose was in his arguments for basic laws. This interplay is illuminating for several reasons. Firstly, the appropriateness of attributing to Frege a conception of how we should go about determining logicality must, in absence of specific comments of his evincing such a conception, be tested by seeing how it makes sense of the way in which Frege actually goes about motivating his logical laws. Secondly, looking at the narrower exegetical questions concerning how Frege presents his foundations can provide a focus that allows one to make fine distinctions between superficially similar but importantly different accounts of what Frege's exposition on his logic aimed to achieve. This is useful, because interpreting Frege in a manner which avoids attributing to him an anachronistic use of semantic concepts while still denying that his view at all precludes semantic theorizing involves walking a rather fine line. Focusing on the purpose of Frege's discussion of his basic laws can help to make fine distinctions between general accounts of Frege's project. The question of whether Frege's arguments are for the truth of sentences, or for the logicality of thoughts, or whether they instead play a purely elucidatory role can have implications for more general interpretive issues.

---

[4] I hasten add that my own interpretation is probably closer to that of Heck and Stanley than to that of Ricketts, Weiner, and Goldfarb. As I shall explain, there are important differences between us, but I find very much to agree with in what Heck says about the structure of some of Frege's arguments and about Frege's approach to truth, and shall draw on him in support of some of my points. His correctness interpretation, as a reading of Frege's basic laws, should also be distinguished from his quite separate claim that we should see Frege's exploration of semantic notions as inching towards an inchoate strategy of using semantics to separate logical from non-logical terminology, which would allow us to one characterize logical truths as maximally general. This later view, while tentative, has much to recommend itself—I discuss it in chapter 2.

§3. Outline of dissertation

In chapter 1, I do some groundwork, explaining my stance on certain interpretive issues and how they relate to the topic at hand. I start with a discussion of Frege's foundationalism, arguing that it's an element of Frege's philosophy which endured throughout his career, and thus a matter which any interpretation of him must do justice too. I discuss the special properties which Frege thought must be possessed by the axioms, such as self-evidence and unprovability. I argue that both of these properties should be understood in an objective, system independent sense, and I say a bit about how Frege uses considerations of simplicity and fruitfulness to identify which laws are objectively unprovable. I then switch gears, and talk about how Frege saw the relation between axioms and inference rules, with the aim of highlighting the special relation logic holds to inference. The main conclusion of this subsection will be that Frege thought the distinction between basic logical truths and logical inference rules had no extra-systemic significance.

In chapter 2, I focus on generality and undeniability, and the relation that they bear to the case Frege made for the logical nature of his basic laws. After sorting through some of his platitudes concerning the connection between logic, truth, and normativity, I say a bit about explaining how generality and undeniability are used as a fallible rule-of-thumb in making preliminary judgements concerning whether a given subject-matter is logical or not. I then run through the various senses in which one might take logic to be general and undeniable, settling upon a few which I take to be most central to Frege's view of logic. My position is that Frege was able to give, and was committed to giving, a deeper case for the logicality of his primitive truths than merely noting that these truth are general and undeniable. In the final subsection, I run through some different accounts of what case Frege made for the logicality of his basic laws, in order to outline some of their shortcomings, and set the stage for how my own account fills in the gaps.

In chapter 3 and 4, I flesh out the E.A. interpretation, my own account of the case for logicality of basic laws which I argue is implicit in Frege's treatment of them. I begin in chapter 3 by looking at Frege's enigmatic quotes on how logical laws unfold the content of the word "true" and argue that these remarks motivate a view of truth as implicit in judgement and understanding. I then present an account of how inference connects to understanding—we must be competent in performing inferences involving a thought if we can be said to understand that thought. One virtue of this account is that if it's implicit in Frege's perspective, it explains why he took logic to be general and undeniable. In chapter 4, I explain how my account makes sense of the way in which Frege goes about arguing for his basic laws. Particular attention is paid to what we should say about law V—I argue that Frege's error in adopting law V doesn't point to anything deeply problematic about Frege's method of determining logicality, but rather shows that he ought to have been more careful regarding when his stipulations properly secure reference. After providing some clarifications of my own view, and responses to several possible objections to it, I conclude with a summary of the elucidatory and correctness interpretations views of the nature and role of Frege's arguments for his basic laws.

In the final chapter, I critically examine the elucidatory interpretation of Thomas Ricketts, Joan Weiner, and Warren Goldfarb, and the correctness interpretation of Richard Heck and Jason Stanley. I start by discussing elucidations, and then address and refute two of the main arguments for the elucidatory interpretation—Weiner's argument that difficulties in talking about concepts imply that much of Frege's talk about his basic laws, including his arguments for them, couldn't be stated, and Ricketts's argument that Frege's views concerning the way in which truth and judgement relate imply that we can make no use of a truth-predicate in semantics. I then turn to arguments against the correctness interpretation. While I'm entirely in agreement with Heck and Stanley that nothing about Frege's perspective precludes the use of semantic reasoning, I differ from them on the *purpose* of such semantic reasoning. Frege was not only, or even primarily, interested in establishing the correctness of his system—his comments can instead be seen as an attempt to get us to see these laws are logical.

Chapter 1: Foundationalism and Inference

§0

Before discussing the nature and use of Frege's heuristic indicators (chapter 2) and the conception of logic which underlies it (chapter 3), it's first necessary to go over some conceptual groundwork. Frege had an interesting and cohesive strategy for establishing his logicism, and this rests on certain background assumptions which we must be clear about before we can properly parse the more contentious and conceptually demanding elements of his treatment of logicality. The first is Frege's foundationalism. Since this view has since fallen out of fashion, at least in a mathematical context, its role in Frege's treatment of logic isn't always sufficiently emphasized. Although I argue that many distinctive elements of his picture could be maintained even if his foundationalism is rejected, it was still a fairly central commitment which he took seriously. As we'll see, Frege was a particularly canny foundationalist—although self-evidence has an important role in his treatment of foundations, he does not let things rest with *mere* obviousness. Next, I then shift my focus to the relation which Frege's logical foundations bear to his inference rules, arguing that Frege thought the distinction had no deep extra-systemic significance. This will be of relevance later when I flesh out the details of my epistemic analyticity (E.A.) interpretation.

§1. Foundationalism

§1.1 The heritability strategy

There are certain core elements of Frege's views which, while uncontentiously attributable to him, are well worth repeating, as any effective interpretation must give an account of them. Frege was a logicist—he wanted to show that arithmetic was logical. He took the logicality of arithmetical propositions to have prima facie plausibility—it seems that anything at all may be counted, and that attempting to think the opposite of an arithmetical law results in complete confusion (1884, §14). But this is not *conclusive*—the fact that arithmetic concerns numbers might be thought to bar it from being completely general. Similarly, some story ought to be told about why the apparent undeniability of arithmetical laws isn't just a sign of a failure of imagination on our part (after all, it took a long time before mathematicians found out that the denial of the parallel postulate is perfectly coherent). Providing more conclusive grounds for the generality and rational undeniability of arithmetical statements requires showing that arithmetic is justified on the basis of pure logic. Demonstrating this justification requires proof, and such proof must be gapless in order to ensure that intuition doesn't sneak in unnoticed.

Thus, Frege's strategy for establishing his logicism is genetic in flavor—propositions inherit their logicality from their basis of justification (1884, §3). A word on such inheritability: Frege fashioned his deductive systems so that in order to show a given proposition P has characteristic C, it's sufficient[1] to show that, for some set of propositions Γ:

i. P can be deduced from Γ in the deductive system.
ii. All propositions in Γ have characteristic C.
iii. Each inference rule preserves C—that is, characteristic C is inheritable from the inference rules.

The nature of Frege's deductive system assures us that we won't always have to go through all three steps. In setting up his system, Frege must have had some reason to think that his axioms and inference rules were logical. But this logicality is a precondition for the ability of his symbolic apparatus to be able

[1] Or *ought* have been sufficient—strictly speaking, it isn't, because Frege's deductive system in *Grundgesetze* is contradictory, and hence trivial.

to rigorously determine a given proposition's justificatory basis *at all*—if inference rules instead smuggled in something like Kantian intuition, then P would not be shown to rely *solely* on the premises Γ. That is, it's *built into* Frege's system that his inference rules must preserve logicality. Although Frege's inference rules need to be logical, once he's demonstrated this logicality he need not go through the *extra* step of showing they preserve various characteristics. Consider, for example, Frege's attitude towards axiomatizing geometry—the content of geometry is contained in geometrical axioms, and logical inference rules then draw out this content. No additional question need arise for whether logical inference rules also preserve geometricality—it's taken for granted that they must, because logic, being wholly general, will not contribute any specific content of its own when drawing consequences from the axioms.

What do (i-iii) have to do with *foundationalism*, though? Thus far, I've said a bit about how we demonstrate logicality by justifying a truth P whose logicality is in question on the basis of a collection Γ of truths, the nature of which is more easily discernable. This story didn't require that I say anything about this justificatory basis being *unprovable*—it's consistent with what I've said thus far that all propositions in Γ be provable and thus non-basic. Wouldn't it be rather mysterious and remarkable if, when we go about proving things to determine whether they are logical, our answer always happens to come at just the moment we hit justificatory bedrock? But this seems to be just the line Frege adopts. Of course, he sometimes seems to *characterize* the logicality of propositions in terms of its *ultimate* justificatory basis (1884, §3). But this alone does nothing to dispel the impression of odd coincidence—for why should we bother characterizing logicality in a way that presupposes foundationalism? Even on Frege's view, we'll have to say *something* about why basic laws are logical—once we know what story must be told there, why expect that this story can *only* be told for the foundations? Need we follow the chain of proof back so far in order to determine whether a proposition is logical, and if we needn't, why suppose that the chain of proof terminates at all?

I don't think there are any grounds for supposing Frege was committed to us *never* being able to directly argue for the logicality of truths without first reducing them to basic truths. Consider the argument which he uses to justify $\vdash a \supset (b \supset a)$—he gets us to see that it could be false only if, *per impossibile*, the antecedent would be both True and False (Frege 1893, §18). This style of argument familiar enough to anyone who's ever seen a truth-table, and there's no reason to suppose Frege wouldn't be willing to give a similar argument for a more complicated formula such as '$\big((a \supset b) \supset a\big) \supset a$', showing how its denial leads swiftly to an absurdity. But there are several reasons for Frege to have maintained a more foundational model nevertheless. Firstly, we won't generally be able to give such direct arguments for the logicality of sentences containing terms which need definition. A denial of arithmetical principles may *seem* absurd, but until Frege has given his definitions of cardinal numbers, he has no hope of demonstrating that such a denial implies a contradiction. Secondly, the fact that Frege was a *Euclidian* foundationalist may have pushed him toward thinking that any thought which can be argued for in a manner similar to Axiom 1 is automatically a good candidate for a basic truth, for such a truth will be self-evident. Thirdly, even if we *could* directly argue for the logicality of, say, individual arithmetical truths, we clearly cannot argue piecemeal if our aim is to show that *all* arithmetical truths are logical, since there are infinitely many—we must show that principles upon which all these truths rest are logical[2]. Lastly, Frege had, apart from his logicism, other motivations for taking a foundationalist view of justification—he thinks that the simplicity by the provision of a small set of axioms encapsulating the content[3] of a given science is something worth pursuing for its own sake (1884, §2).

[2] And, as I shall argue shortly, Frege may have taken the fact that a principle allows us to derive a great many truths as grounds that this principle is basic.

[3] Recognizing, of course, that the case is a bit different with logic than with geometry—with the latter, we have a specific geometrical content, the consequences of which are drawn out by logical inference rules, whereas in the former there is no *specific* content to be drawn out, since logic is general. This was pointed out to me by Peter Sullivan.

§1.2 Evidence for foundationalism

Whatever we think of the philosophical cogency of Frege's foundationalism, there's no denying that he was a foundationalist about mathematical and logical knowledge from the beginning to the end of his career. More specifically, he was a *Euclidean Foundationalist* about justification—he believed that thoughts stand in an objective justificatory relation, and thus that those laws which constitute the foundations needed to have certain special epistemic properties. This much is generally acknowledged, but there's much debate over how central a commitment this is, and how Frege's view of justification constrains his choice of axioms. Some, such as Tyler Burge, discuss the view in considerable detail. Richard Heck, on the other hand, doesn't think that we can make good interpretive sense of a Fregean notion of objective basicness required of axioms for two reasons—firstly, Frege was quite aware that there are multiple ways to select axioms to lie at the basis of a deductive system, and, secondly, he says very little about what criterion we should use to determine what counts as a basic truth, a law not justifiable on the basis of others (2012, 37). Given this attitude, I'll take a moment to argue that Euclidean Foundationalism was an abiding commitment that lasted throughout Frege's career. This will establish that no interpretation of Frege's *Grundgesetze* can be complete if it fails to properly integrate Frege's distinctive views on the structure of justification.

As early as the *Begriffsschrift*, Frege insists:

> It seems natural to derive the more complex of these judgements from simpler ones, not in order to make them more certain, which would be unnecessary in most cases, but in order to make manifest the relations of the judgements to one another (Frege 1879, §13)

At issue here is which propositions should lie at the base of Frege's deductive system. Which considerations ought to constrain our choice of axioms? Frege's answer is that more complex (*zusammengesetzteren*) judgements ought to be derived from simpler (*einfacheren)* ones. For this constraint to make sense, we must suppose that the complexity of a proposition is, for Frege, an extra-systemic notion, a property that the proposition has independently of particular axiomatic systems in which it might figure. For suppose that "simpler" meant something purely intra-systemic, such as the shortest derivational distance between the axioms and the proposition in question. Under such a reading, Frege's statement above would be inexplicably banal—it becomes a demand that our derivation of a proposition P not be shorter than any of its sub-derivations. It's clearly more charitable to read Frege as making an objective, justificatory relation manifest.

Similar comments appear in §3 of *Grundlagen*, where Frege discusses the a priori and analyticity in foundationalist terms:

> When a proposition is called a posteriori or analytic in my sense…it is a judgement about the ultimate ground upon which rests the justification for holding it to be true. (Frege 1884, §3)
>
> But if, on the contrary, its proof can be derived exclusively from general laws, which themselves neither need nor admit of proof, then the truth is a priori (Frege 1884, §3).

Again, Frege speaks of proofs as bottoming out in statements with some particular characteristic—in this case, unprovability. Frege must take "provable" in a restrictive and justificatory sense, as distinct from mere derivability—to prove a proposition P from a collection of propositions Γ is for P to be justified on the basis of Γ. This is clear from context—his comment above would be pointless if he meant to identify unprovability with some purely intra-systemic notion, such as axiomhood. If we are working within a particular axiomatic system, and take the "unprovability" of P to mean that P admits of a one-line derivation (namely, just citing the principle as an axiom), then clearly saying that axioms are unprovable tells us nothing—that's just what axioms *are*. We must suppose instead that there is an objective, system-independent component to unprovability.

The comments of Frege's cited above were, of course, written many years before Frege's introduction of the sense-reference distinction. But despite the changes which resulted from the introduction of that distinction, his confidence in the foundational structure of logical truths appears to have persisted. In *Grundgesetze*:

> It cannot be required that everything be proven, as this is impossible; but it can be demanded that all propositions appealed to without proof are explicitly declared as such, so that it can be clearly recognized on what the whole structure rests. One must strive to reduce the number of these fundamental laws as far as possible by proving everything that is provable. (Frege 1893, vi).

The reasoning used above applies here as well—we must suppose that Frege means something extra-systemic by "provable". This passage is significant because it makes it clear that unprovability is a criterion which constrains one's choice of axiom—we go about proving things, and only once we reach bedrock do we find truths worthy of being taken as axioms. Frege is quite explicit about this in his two articles written in response to his correspondence with David Hilbert:

> Traditionally, what is called an axiom is a thought whose truth is certain without, however, being provable by a chain of inferences. The laws of logic, too, are of this nature. (1903b, 319; CP 273)
>
> Axioms differ from theorems in that they are unprovable (1906a, 296; CP 295)

And in his later "Logic in Mathematics":

> Science demands that we prove whatever is susceptible of proof and that we do not rest until we come up against something unprovable. It must endeavor to make the circle of unprovable *primitive truths* as small as possible, for the whole of mathematics is contained in these primitive truths as in a kernel." (Frege 1914, 221/FR 310)

Many of the exegetical debates which revolve around Frege concern which commitments he kept—how different his pre and post-1891 notions of content were, whether *Grundgesetze* marks an abandonment of the context principle, and so on. The evidence above serves to show that regardless of where one comes down on these other debates, one must acknowledge Frege's foundationalist stance as a commitment that lasted throughout his career—it shows up in his early, middle, and later writings, often in introductory passages explaining how he conceives of his technical work. Thus, it is a commitment which we must take seriously if we are to do justice to Frege's picture.

§1.3 Are axioms determined or chosen?

Un-provability is evidently an objective, extra-systemic notion for Frege—logical truths occur in a rational order of justification, connected by inferences, with more basic truths at the bottom, and which justify the truths further up the tree. But he clearly doesn't think that this rational ordering *determines* his choice of axioms, since he was well aware that there are different combinations of laws which have equal merits (1879, §13). Nor did Frege maintain that what propositions are taken as basic is an entirely system relative matter—he critiques, in *Grundlagen*, proposed axiomatic theories on the grounds of their having taken as basic arithmetical propositions which actually admit of further justification. If the basicness of propositions or concepts were system relative, then one could not use the notion to criticize a given systematization of mathematics. In short: in rationally reconstructing Frege's view, we must seek an interpretation which refrains from wholly identifying logic as a whole with a particular formalization thereof, but which nevertheless places certain epistemic constraints on what propositions we may take as axioms.

Burge offers an interpretive solution which allows us to navigate this exegetical dilemma—for Frege, the basic laws deductively over-determine the collection of logical truths (Burge 1998, 320-1). That is, all axioms must be basic laws, but not all basic laws need be axioms—in setting up an axiomatic system, we choose, among the basic laws, a few which appear to be sufficient to generate all logical truths, and which appear to be mutually independent. Frege's system is *faithful* to an independently existing justificatory structure, in the sense that the derivation relation is monotonic—if we derive 'B' from 'A', then the thought expressed by 'B' must not be *more* basic than the thought expressed by 'A'. 'B' can, however, be *as* basic as 'A'—this will particularly be the case when 'A' is a basic law which was selected as an axiom, whereas 'B' is a basic law which was not.

But surely we should allow that sometimes B can be proved from A even when B is logically simple and A is not? But this is to equivocate upon the term "prove". In contemporary parlance, to rightly say that we have "proved B from A", it's sufficient[4] that B be a model-theoretic or deductive consequence of A, where these relations tell us nothing about any deep justificatory priority amongst the content they relate. In this sense of proof, if A is a basic logical law (or indeed, any logical truth whatsoever), $X \vDash A$ whatever X may be. But '$\vDash$' in the modern sense does not denote derivability in Frege's sense—as seen in §13, quoted above, Frege proves more complex laws from simpler ones, in order to map out the relations which hold between thoughts (1879, §13). When Frege speaks of proof in this sense, he is not speaking of a consequence relation in which every logical truth follows from every other, but of something more fine-grained. In Frege's system, syntactic derivability between sentences must respect independently existing justificatory relations among thoughts which these sentences express if this derivation is to be a genuine *proof*.

§1.4 Self-evidence

Frege's conception of the foundational structure of mathematical justification is not unique, but his take on it is. It's instructive to compare his unique spin with the sort of abductive foundationalism held by Russell. Russell does not require that his basic propositions have any special character—in particular, they needn't be obvious. In *Principia Mathematica*, he says:

> That the axiom of reducibility is self-evident is a proposition which can hardly be maintained. But in fact self-evidence is never more than a part of the reason for accepting an axiom, and is never indispensable. The reason for accepting an axiom, as for accepting any other proposition, is always largely inductive, namely that many propositions which are nearly indubitable can be deduced from it, and that no equally plausible way is known by which these propositions could be true if the axiom were false, and nothing which is probably false can be deduced from it. (Russell 1910, 62)

For Russell, all we require of axioms is that they generate the body of mathematics ordinarily accepted as true, and no falsehoods besides. In particular, these axioms needn't be obvious or self-evident—thus, Russell is a foundationalist, but not a *Euclidean* foundationalist. Euclidean foundationalism demands more of axioms than mere success in generating a body of truths—they must, in addition to this, have certain epistemic properties. As for what these properties are, Frege says notoriously little. Several criteria show up in his writings, however; axioms must be:

i. True (Frege 1903b,319; CP 273)[5]
ii. Unprovable (1884, §3;1903b, 319; CP 273)

[4] Though not necessary—see (Tanswell 2015) for a helpful discussion of why we'd be amiss to take all mathematical proofs as reducible to formal ones.
[5] Although Frege's mostly concerned with geometrical axioms here, he makes it clear that he intends much of what he says to hold try for logical axioms as well (1903b, 319; CP 273).

iii. Certain (*selbstverständlich*) (1903b, 319;CP 273; Burge 1998, 342)
iv. Independently recognizable as true (1899-1906, 168)

Frege's comments on how one goes about selecting axioms are as cursory as his provision of criteria for them. We must select a collection of principles which are *fruitful*, which generate the mathematical truths which we intended to capture, and which are independent of one another (1893, vi). But this cannot be the *whole* story, or else there would be no need to harp upon unprovability (which, as we saw, is objective). In *Grundlagen,* for instance, his grounds for denying that arithmetical identities are basic is not that we cannot derive other sorts of arithmetical facts from these identities, but because they do not have the requisite certainty (*einleuchtend*) (1884, §5) [6].

According to Burge's gloss on Frege's foundationalism, axioms are objectively basic truths which are selected for their sufficiency in certain derivational purposes, and for their mutual independence. In that case, we should see the question of how to select axioms as requiring an answer on how we discover which laws are basic. If we accept this gloss, we should see the criteria for axiom-hood listed above—truth, unprovability, certainty, and independent recognizability of truth—as necessary conditions for a given law being basic. It now remains to look at what these criteria amount to, and how they connect.

Since Frege's attitude towards certainty is quite important, it's worth sorting out some terminological points. "Certain" and "self-evident" are variously used to translate either of two distinct German words—*selbstverständlich* and *einleuchtend*. There's some disagreement among translators over whether Frege uses these words to mark an important distinction. Philip Ebert and Marcus Rossberg argue that *selbstverständlich* (when used in a technical sense) is best translated as self-evident, whereas *einleuchtend* is instead used to signify that a claim is easily understood or accepted (2013, xxii). Burge doesn't take there to be any systematic difference between the two terms, save for the fact that *selbstverständlich* is never used with "mentalistic overtones". Frege thought that all axioms needed to be *selbstverständlich* (Burge 1998, 327) and explicitly held both geometrical and logical axioms to be *einleuchtend* as well (1903, 253; Burge 1998, 346)[7]. I shall reserve "self-evident" for Frege's technical usage, the special epistemic qualities he believes must be had by axioms, and will use "obvious" to denote the less technical and more subjective certainty.

Burge makes an important point about how Frege understands self-evidence. Frege took self-evidence to involve an idealized component—something can be self-evident, but not self-evident *to us* (1998, 354). Rather, self-evidence is something like being recognizable as true to anyone with the requisite level of understanding of the proposition. If this is right, then a law's being independently recognizable as true doesn't mark a separate necessary condition for basicness. Another consequence of this understanding of self-evidence is that it isn't automatic—recognizing a self-evident truth as true might take a bit of conceptual legwork, and presuppose some inferential abilities. As we shall see, this makes perfect sense of the way in which Frege motivates his basic logical laws. He gets us to see they are self-evident by helping sharpen our understanding of the articulate internal structure of the basic laws in question[8].

[6] His argument there is illuminating. First, he points out that those identities involving many digits, such as $135664 + 37863 = 173527$ are not plausibly self-evident, as it's the kind of thing we *argue* for (by some algorithm or other for addition). Second, he thinks that "it is awkward to make a fundamental distinction between small and large numbers, especially as it would scarcely be possible to draw any sharp boundary between them"(1884, §5). So, if identities involving large numbers are not self-evident, neither are those involving small numbers. Clearly, this argument only works if we cannot take self-evidence as a matter of degree.
[7] See (Burge 1998, 346-7) for a very helpful list of the places in which Frege says that axioms are *einleuchtend* or *selbstverständlich.*
[8] The obvious worry here is how this could help distinguish between geometrical and logical axioms, if both are self-evident. I'll discuss with this problem in some detail in §3.1 of chapter 4. The short answer is that there's

Frege's foundationalism is not only distinct from that of his contemporaries by being Euclidean—if Burge is correct, his spin on self-evidence also marks an improvement over more naïve versions of Euclideanism that place too firm an emphasis on obviousness, an emphasis which leaves the naïve version open to charges of brute dogmatism. Against dogmatism, Frege explicitly tells us that:

> In mathematics we must never rest content with the fact that something is obvious or that we are convinced of something, but we must strive to obtain a clear insight into the network of inferences that support our conviction (Frege 1914, 221/FR311).

While Frege thought obviousness was a necessary condition for axiom-hood, we cannot rest content with *mere* obviousness, because this doesn't give us any insight into the *source* of such certainty. Frege is quite clear that obviousness is not a distinctive mark of basic logical truths—we must always ask whether the obviousness of a thought ought to be attributed to logicality, or rather to some sort of mathematical intuition (1884, §90). He says in a letter to Huntington:

> I have set myself the goal of basing arithmetic on logic alone…for this it is again necessary to produce for each proof a chain of inferences without gaps, so that every transition proceeds according to a known logical law. Thus nothing must be left to mere obviousness, for its nature and laws are unknown. One could then never be certain that this evidence was purely logical. (Frege 1980, 57, substituting "obvious" for "self-evident" in the translation of "Einleuchten").

What is this unknown nature, and these laws which govern obviousness? If Burge is correct in maintaining that *selbstverständlich* , unlike *einleuchtend* and its grammatical variants, is never used with mentalistic overtones (1998, 346), then the laws Frege had in mind may well have been psychological ones. If this is so, then Frege's admonitions against relying on obviousness are part and parcel with his rejection of psychologism. What it is that leads us to a subjective sense of certainty, or to label a truth as easily understood and accepted, is an empirical matter. This is the main difference between obviousness and proper self-evidence—the latter is an objective matter, and devoid of psychologistic considerations. It might take some time, and some conceptual legwork, for us to recognize a law as self-evident, whereas it makes no sense for something to be obvious but not obvious *to us*.

If obviousness has mentalistic overtones, then how, without falling into psychologism, can Frege require that axioms be obvious? Firstly, note that Frege never uses obviousness as *grounds* for taking a law to be true, or for taking it to be logical—Frege has a keen awareness that what's obvious at first may turn out to be false (see Burge 1998, 338-9 for a helpful discussion). The role of obviousness in axiomhood is much clearer if we suppose that it importantly connects to self-evidence. Though something can be self-evident without being obvious to us, it may still be that self-evident truths are at least *potentially* obvious to us—once we do the necessary conceptual legwork to appreciate them as self-evident, the truths in question will strike us as clearly true. Because of the legwork involved, the truths can never be called *unmittelbar klar*, or immediately clear[9]. But it will be beyond a reasonable doubt to anyone who's understood it.

This fits nicely with Burge's picture of how basic laws and axioms relate—just as self-evidence entails potential obviousness, to be a basic law is to be among the collection of objectively unprovable truths from which one can select ones axioms. That obviousness is a necessary condition of axiom-hood fits perfectly with self-evidence being a necessary condition for being a basic law. If obviousness is the

---

something crucially semantic about Frege's arguments for his basic laws, and a reflective appreciation of the implicitly semantic dimension of his arguments is part of what leads us to recognize these laws as logical.

[9] Except, perhaps, in rare cases—the law of self-identity is immediately clear if anything is.

end result of seeing for oneself that something is self-evident, then it's a necessary condition for axiom-hood without being any sort of independent grounds for taking that axiom to be true. And to say obviousness can arise in this way is not to say that it *only* arises in this way—something can strike us as obvious because of incidental facts about our psychology[10].

To say that self-evidence is objective, and devoid of psychologistic considerations, is not to say that it's a purely logical matter. Consider the following remarks of Frege's on the epistemological status of the foundations:

> Now the grounds which justify the recognition of a truth often reside in other truths which have already been recognized. But if there are any truths recognized by us at all, this cannot be the only form that justification takes. There must be judgements whose justification rests on something else, if they stand in need of justification at all.
>
> And this is where epistemology comes in. Logic is concerned only with those grounds of judgement which are truths. To make a judgement because we are cognizant of other truths as providing a justification for it is known us inferring. There are laws governing this kind of justification, and to set up these laws of valid inference is the goal of logic. (Frege 1879-1891, 17).

Self-evidence is not a sort of inferential support—when one shows a thought to be self-evident, this doesn't involve justifying it on the basis of some other thought. Rather, self-evidence is an epistemological notion—it has to do with thought, but not with the contingent peculiarities of particular thinkers (as Frege may have put it, "of the mind, not of minds" (1918a, 74; CP 342)). In chapters 3 and 4, I'll argue that self-evidence, at least in the case of logic, amounts to epistemic analyticity, and I'll explain how the resultant picture avoids falling into psychologism.

§1.5 The role of pragmatic considerations

Now that I've said so much about obviousness and self-evidence, a bit needs to be said about unprovability. Frege thought axioms needed to be unprovable, and I've argued that, for this to be a helpful criterion for axiom-hood, Frege takes such unprovability to be objective and extra-systemic. How, then, do we determine which truths are objectively unprovable, especially if we're to avoid putting undue emphasis on mere obviousness? One option would be to suppose that, somehow, self-evidence—considered as potential obviousness, as the truth of a statement being independently recognizable—informs what we take to be unprovable. But then it's unclear why Frege, in his short list of necessary conditions for axioms, would list unprovability alongside truth and self-evidence as a separate condition (Frege 1903b,319;CP 273).

I think that it's here—and here alone—that broadly pragmatic considerations must enter into Frege's account—it's all that's left. We develop a systematization of logic which is fruitful, simple, and elegant, and whatever is apt to lie at the base of such systematizations is unprovable. Frege seems open to such considerations. They already play an important role in his evaluation of definitions (Frege 1880-81, 33). He says that how "simple and precise everything is made" should give weight to his conception of logic and the role of truth-values within it (1893, x), says as a point in favor of his introduction of value-ranges of functions gives him "far greater flexibility" (1893, ix). And tying unprovability, and hence basicness, to pragmatic considerations isn't an implausible view. After all, it's such considerations that lead mathematicians to adopt the axiom of choice, and the assumption of the existence of infinite sets, as basic

[10] I should emphasize, however, that as plausible as this account of relation between obviousness and self-evidence is, the account is not crucial for any of the main claims to follow. In particular, one might take obviousness as some sort of fallible indicator of logicality, as Robin Jeshion does (2001, 960-73).

assumptions when axiomatizing set theory. Of course, using such considerations in axiom selection hardly commits one to foundationalism. But if one *did* want to be a modern foundationalist, it's difficult to see how considering the elegance and simplicity of systematizations of subject-matter could be eschewed as part of what guides our judgements concerning what's basic. And the use of such considerations fits nicely with the Fregean view that our choice of axioms is constrained but not determined. Nothing *forces* one to take choice as an axiom rather than one of its equivalents (the principle of well-ordering, or Zorn's lemma) or rather than adopting a weaker version (the axiom of dependent choice, or countable choice) together with some principle which can, in conjunction with the weaker version, prove the axiom of choice[11].

§1.6 Burge on Frege's rationalism

Burge takes Frege's commitment to foundationalism, and his canny approach to objective self-evidence, to be evidence of an abiding rationalist tendency underlying Frege's works (1998, 354). Frege saw the a priori sciences as finely structured and deeply rational. Thoughts timelessly stand in an objective justificatory ordering, and this deep structure is slowly uncovered by logicians and mathematicians (1990, 245). According to Burge, what's unique about Frege's rationalism is not his lack of reliance on mere obviousness, but the role of pragmatic considerations, and logical and mathematical theory building, in helping us uncover this rational order.

While I agree with Burge's picture in broad outlines, a difference between us, which will become increasingly apparent as I further clarify my E.A .interpretation in §3 of chapter 4, is that I take Frege's underlying rationalist tendencies to be separable from the case he makes for the logicality of his axioms. These two currents of Frege's thought are not separable for Burge—he thinks that seeing a law as logical will involve clarifying its sense, and that to grasp sense just is to have insight into the deep justificatory structure in which thoughts figure. In his 1990 article "Sense and Linguistic Meaning", Burge argues that thought and sense may be *very* far removed from the structure of sentences (1990, 245). He motivates this claim by attending to those places in which Frege talks about scientists of different languages and eras investigating the same thoughts, and the places where (most notably in Frege's 1914 "Logic in Mathematics") Frege suggests that grasp of sense admits of degrees (Burge 1990, 259). Burge conceives of theory-building (1990, 256) and pragmatic considerations (1990, 263) as helping us gain a more refined grasp of sense.

My E.A. interpretation maintains the link between theory building, pragmatic considerations, and the objective justificatory ordering—as I just explained in §1.5, such considerations are all that one has available for identifying which laws are unprovable, at least for anyone who, like Frege, puts little emphasis on obviousness. I'm not, however, committed to the claim that sense is something far removed from language, something quite separate from linguistic meaning. Burge gives excellent support for his reading from the texts in which he examines. But *Grundgesetze* is not one of these texts—although Burge heavily references it in his 1998 article on Frege's foundationalism, his 1990 article on sense refers to *Grundgesetze* only once, when he takes the last paragraph of Frege's foreword as evidence of the role of pragmatic considerations (1990, 263). If I'm right, this is not just because Frege says fairly little about sense in *Grundgesetze*—it's because in that work, Frege's conception of sense is more closely tied to language than it is in some of his unpublished works, such as "Logic in Mathematics". Or, to hedge a bit, the case Frege makes for the logicality of sentences is tied quite tightly to reflectively appreciating what subsentential components contribute to the truth and falsity of sentences in which they figure—perhaps we may bracket whether this linguistic meaning should be identified with Fregean sense. More on this chapter 3 (§2.2) and chapter 4 (§3.2).

[11] Of course, now, model-theoretic proofs of independence results is a tool we now have at our disposal which Frege did not.

§2. Axioms vs inference rules

§2.1

The foundationalist picture that I've laid out so far contains much that is not specific to logic—Frege was also a foundationalist about at least some non-logical subject matter, most notably geometry. It's a bit unclear just how much of what he commits himself to in the logical case carries over to his treatment of other axiomatic systems—in particular, it's not obvious whether Frege thought that our choice of axioms for a given non-logical subject matter is (as it is for logical axioms) constrained without being determined, and whether the set of basic truths in a given non-logical discipline must, unlike the logical case, be independent of one another. However, aside from these potential divergences, it's plain that Frege's foundationalism isn't uniquely logical—even his *Euclidean* foundationalism extends to geometry, since he requires geometrical axioms to be true, certain, and unprovable (1903b, 319; CP 273).

But although logic is an axiomatic discipline (though an abstract one), it is not *just another* science: these superficial foundationalist similarities aside, there's much which is deeply distinctive about Frege's approach to logical laws. Firstly, although logical axioms, like geometrical ones, must be self-evident, this self-evidence comes from different sources—in the case of geometrical axioms, the certainty comes from our spatial intuition, whereas the self-evidence possessed by logical laws is a sort of epistemic analyticity (I shall argue this point in chapter 3 and 4). In the present section, I focus on another facet of Frege's view of logic which will come up again in chapter 4 when we look more closely at the way in which Frege justifies his laws—Frege didn't think that there was any extra-systemic distinction between basic logical laws and inference rules. I'll also say a bit about how Frege's way of treating basic logical laws suggests that he saw logic as intimately tied to inference and justification.

§2.2 Logical and non-logical inference rules

Let's return, for the moment, to consider the case of geometry. According to Frege, if we are to present geometry in its most rigorous form, we first lay out axioms, which encapsulate, in just a few formulas, the content of the subject. We then draw this content out through the use of *logical* laws:

> An inference simply does not belong to the realm of signs; rather, it is the pronouncement of a judgement made in accordance with *logical laws* on the basis of previously passed judgements. (Frege 1906a, 387;CP 318; italics my emphasis)

But why logical laws? Is there any problem with allowing for non-logical inferences, so long as one knows what empirical or intuitive content they bring to the thoughts upon which they act? Elsewhere (and much earlier), Frege suggests there's something problematic with such inferences:

> One may not appeal to intuition as a means of proof; for it is a law of scientific economy to use no more devices than necessary (Frege 1880-81, 32).

In Frege's axiomatic systems which formalize rules for correct proof, we shouldn't incorporate anything intuitive as an inference rule because such rules are, he thought, dispensable—we can always instead regard these "inferences" as general geometric *truths*, partially generalized formulas whose justification rests on intuition, and from which we draw consequences via logical means.

We must distinguish between two positions Frege could be taking here**:**

a) An axiomatic system with non-logical inference rules is defective in some way (un-useful, un-economic, or whatever else)
b) There are *no such things* as non-logical inference rules.

Which of a) and b) do the passages above point to? Frege's argument from "scientific economy" might strike one as a straightforward desire not to multiply varieties inferences beyond necessity—when it comes to postulating non-logical modes of inference, we mustn't because we needn't. This would suggest Frege held b). However, in *Grundgesetze,* he says it was due to "scientific parsimony" that he adopted modus ponens as the sole inference rule in *Begriffsschrift,* and suggests he would have done the same in *Grundgesetze* were it not for the practical consideration of keeping his proofs to a reasonable length (1893, §14). In *Begriffsschift,* Frege says explicitly that he recognizes many forms of inference, but takes only modus ponens as a rule in his system (1879, §13). So it's clear that considerations of parsimony don't show (or provide evidence for the claim that), that modus ponens is the sole method of inference. If the scientific parsimony here is the same sort Frege appeals when keeping non-logical inference rules out of his formal systems (and there's no obvious reason to suppose it isn't the same consideration in both cases), this points towards Frege adopting a) but not b).

However, there still remains an important difference between the logical and non-logical cases. It is true that we can eschew *any* given logical inference, opting instead to adopt an axiom that lets us do the same work—for example, in both *Begriffsschrift* and *Grundgesetze*, Frege adopts $\vdash \forall \mathfrak{a} f(\mathfrak{a}) \supset f(a)$ instead of taking universal instantiation to be a primitive law[12]. But the key difference from the logical case is that *no* worthwhile axiomatic system of the sort Frege was interested in could have *no* logical inferences—we can have a system which lacks *any particular one* of them, but can't have a system which lacks *all* of them. Frege is interested in showing that arithmetical truths, which are to be theorems of his system, are logical, and we can't show *any* theorems to be logical in a system with no logical inference rules. For if the system only has non-logical inference rules, we'll never be sure whether the truth of the theorem is based only upon that from which it's inferred, or whether it instead relies on some intuitive or empirical content brought in by the inference rules in question. It would bring in far too much guesswork to just scrutinize an intuitive inference from A to B, and try to determine whether B could have been established independently of the intuitive bit. The whole point of Frege's axiomatizations are to enable us to perspicuously lay out the justificatory relations which hold between thoughts. In a system with no logical inference rules, we'd be utterly unable to draw out the consequences of our axioms—even after deriving A from B with the allowed transition rules, we'd never be sure that A rested *solely* on B[13].

So, even if Frege had only a) in mind when he insisted on keeping non-logical inference rules out of the axiomatizations he provided, logic is more tightly tied to justification than non-logical subject matter[14]. This is why, for Frege's purposes, we can't have an axiom system without any logical inferences, whereas we can (and, insofar as we're following the dictates of scientific economy, must) do

---

[12] Of course, Frege characterizes his logical connectives differently in these two works, but the axiom in question plays essentially the same role in both.

[13] Of course, Frege was quite unaware that purging his system of intuitive inference rules left untouched a very large problem for establishing logicism. Frege thought that the validity of inferences in his system should be mechanically checkable—if they weren't, then a certain amount of guess-work and rules-of-thumb would enter into judgements about the validity of derivations, preventing us from establishing the logical nature of a truth conclusively. But on the reasonable assumption that the technical notion of decidability is the appropriate formal correlate of the informal notion of mechanical checkability, Frege's logicism can never succeed in its most general form. The set of arithmetical truths is undecidable—accordingly, any system which allows one to derive all such truths must have either a non-recursively-enumerable set of axioms, or a non-mechanically-checkable collection of inference rules. If there are infinitely many arithmetical principles which may be *directly* established as logical, and taken as axioms, then there's the possibility of establishing the logicality of ever larger bits of arithmetic. But neither Frege nor anyone else for that matter, can ever show that *all* arithmetical truths are logical, at least as long as mechanical checkability is a constraint.

[14] Although we should note that A entailing B is only a *necessary*, and not a *sufficient* condition for A justifying B—for sufficiency, we also need A to be more fundamental than B in the objective justificatory order in which thought stand.

without intuitive or empirically based inference rules. Logical laws are the lens through which we may peek at the justificatory relations in which thoughts stand, and this is something *only* logical laws are well-placed to do, by virtue of their complete generality.

§2.3 The nature of the inference-rule/axiom distinction

The very fact that Frege takes logic to be so substantial, and spends so much time trying to establish that arithmetical truths which appeared so irreducible nevertheless rest on a purely general and logical basis, is apt to draw our attention from what would have struck Frege as obvious. Logic centrally concerns *inference*, the judgement of a thought as true on the basis of others. Its tight association with inference is something that sets it apart from other sciences, even other abstract sciences.

Frege was well aware that there can be a trade-off between the number of axioms and the number of inference rules (1879, §13)—we can have a system with just a few axioms, and many inference rules[15], or vice versa. Within any particular axiomatic system, we must observe a strict separation between axioms and inference rules—he says of such rules "These rules and the laws whose transforms they are cannot be expressed in the ideography because they form its basis" (Frege 1879, §13). This might lead one to think Frege thought that this distinction is a deep one. However, Peter Sullivan argues forcefully that Frege did not take there to be any deep, extra-systemic significance to the distinction between logical laws and inference rules (Sullivan 2004, 673-4). Firstly, there's the matter of what Frege says—he tells us quite explicitly that

> …the truth contained in some other kind of inference can be stated in one judgement, of the form: If M holds and if N holds, then Λ also holds, or in signs $\vdash N \supset (M \supset \Lambda)$ (Frege 1879, §6)

And again that

> I sought as far as possible to translate into formulae everything that could also be expressed verbally as a rule of inference, so as not to make use of the same thing in different forms. Because modes of inference must be expressed verbally, I used only a single one by giving as formulae what could otherwise have also been introduced as modes of inference. (Frege 1880/81, 37).

Since, for reasons familiar enough from Carroll's paradox (Carroll 1895), there's no formula we could add to the system which would supplant the role of rules for permissible transitions between thoughts, Frege is quite justified in demanding we have separate and strictly delineated inference rules. However, as we can see here, he's quite explicit that the same general content can be expressed either as a formula, or be taken as an inference rule.

This is more than an idle observation—as we shall see, the inferential character of logical laws has consequences for how we understand the way in which Frege goes about convincing his readers that his axioms are logical. My account, which will be explained at length in chapter 3, works by tying Frege's grounds for taking laws to be logical to our competence in inference. If we can be said to understand a basic logical law, then we must be competent in performing certain inferences in which that law and its subcomponents occur. Reflecting on these inferences is sufficient for recognizing the truth of the basic law. If Frege did indeed think that the distinction between inference-rules and basic logical truths had no extra-systemic significance, this makes the crucial role of inference in my account of Frege's epistemology of basic logical laws more plausible.

[15] The possibility that we could make do with *only* inference rules, and no logical axioms, does not seem to have occurred to him—in any case, systems of natural deduction don't fit well with Frege's insistence that premises of inference be genuinely held as true, rather than merely assumed.

§2.4 Laws used as inference rules

In addition to fitting well with what I shall argue is the best account of Frege's epistemology of logic, the fact that Frege evidently didn't take the basic law/inference rule distinction to have any extra-systemic significance also explains something about the way in which Frege selects and uses his axioms in *Grungesetze*. All of Frege's basic laws are easily recognizable as expressing, in conditional form, the import of inference rules.

I. $\vdash a \supset (b \supset a)$
II.
   a. $\vdash \forall \mathfrak{a} f(\mathfrak{a}) \supset f(a)$
   b. $\forall \mathfrak{f} M_\beta\big(\mathfrak{f}(\beta)\big) \supset M_\beta\big(f(\beta)\big)$
III. $\vdash g(a = b) \supset g\Big(\forall \mathfrak{f}\big(\mathfrak{f}(a) \supset \mathfrak{f}(b)\big)\Big)$
IV. $\vdash \neg(-a = \neg b) \supset (-a = -b)$
V. $\vdash \big(\acute{\varepsilon} f(\varepsilon) = \acute{\alpha} g(\alpha)\big) = \big(\forall \mathfrak{a} f(\mathfrak{a}) = g(\mathfrak{a})\big)$
VI. $\vdash a = \backslash \acute{\varepsilon}(a = \varepsilon)$

Laws IIa and IIb are the expression, in the form of a formula, of universal generalization. Laws I, III, and IV are all maximally generalized conditionals as well, corresponding to inferences rules which license transitions from instantiated instances of the antecedent to instantiated instances of the consequent. Law V captures, in a formula, the import of a rule which allows one to move from talk of generalities to talk of extensions and vice versa. The inference rule which least obviously fits this pattern is Law VI, but even this allows one to transition from thoughts in which the sense of an expression of the form '$\backslash\acute{\varepsilon}(\xi = \varepsilon)$' to thoughts in which the sense of an expression of ξ occurs.

Of course, *any* thought whatsoever could be thought of "corresponding to an inference rule" in the very weak sense that it lets you derive things you couldn't before. Take a truth like 'all dogs are mammals' for instance—we could think of it as corresponding to an inference rule which lets us infer (among other things) that a given x is a mammal from the premise that this x is a dog[16]. And any identity statement $a = b$ will license transitions from thoughts in which the sense of 'a' occurs to thoughts where that sense is substituted with the sense of 'b'. Accordingly, my present point is *not* to argue that Frege's basic logical laws have some special inferential character that non-basic logical principles, or even non-logical truths, do not have. Instead, it's a more modest point that there is an interesting commonality between Frege's axioms which points to something important about Frege's perspective—though logical laws are truths in their own right, they are never *just* truths—they correspond to licenses to infer.

Laws II and V don't just correspond to inference rules—they *clearly* do so[17]. In *Grundgesetze*, Frege seems to take this as (weak) evidence that we may take law V as a genuine basic logical law. He says that logicians have law V in mind whenever they speak of extensions, even if they don't make it explicit (Frege1893, vii). In the second volume, he insists that the law is implicit in mathematical practice as well (we use it whenever we think of sets, classes, or manifolds), and suggests that this shows that law V is a basic law, rather than a flawed exercise in stipulation (Frege 1903a, §147). For reasons I laid out in the previous subsection, I think that the role of such considerations is more circumscribed than Burge takes them to be—pragmatic considerations of simplicity and fruitfulness help us to identify which laws are basic, but play no part in the most faithful reconstruction of the core case Frege makes for their logicality. Nevertheless, the fact that Frege even raised the consideration in the first place points to how tightly he takes logical laws to be connected with inference.

---

[16] Others have treated generalizations like "all dogs are mammals" as licenses to infer—Gilbert Ryle does so, calling such generalizations "inference tickets" (2009, 105).
[17] Of course, BLV, used as an inference rule, is invalid, though Frege didn't notice it at the time.

Frege's axioms don't merely *appear* inferential in character—he *uses* them in a manner similar to inference rules. First, a bit should be said about Frege's notation for chains of inferences. When he moves from one or more formulas to another, he introduces a transition sign, with the premise(s) above, and the conclusion below—(______) for modus ponens, ($- - - - -$) for hypothetical syllogism, etc. The two-dimensional diagrammatic nature of Frege's concept script mean that it's not as concise as the one-dimensional formulas which are now commonplace. Accordingly, Frege introduces a convention to save space—whenever he infers from two or more premises to a conclusion, one of the premises is placed directly above the transition sign, whereas the other premises are given labels, and indexed by placing these labels in parenthesis directly to the right of the transition sign. So, instead of writing out each of the propositions as follows:

$$\frac{\begin{array}{c}\vdash \Delta \supset \Gamma \\ \vdash \Delta\end{array}}{\vdash \Gamma}$$

Frege instead gives '$\vdash \Delta \supset \Gamma$' the label '$\alpha$' and '$\vdash \Delta$' the label '$\beta$', and writes either

$$(\beta) :: \frac{\vdash \Delta \supset \Gamma}{\vdash \Gamma}$$

or

$$(\alpha) : \frac{\vdash \Delta}{\vdash \Gamma}$$

This is largely how Frege employs his basic laws—they are indexed propositions, allowing one to reason from a single premise to a conclusion, with the other premises notationally suppressed. In the third and final section of Frege's "Exposition of the concept-script" (1893, §§47-52), Frege derives some simple variants of his basic laws, which he labels 'Ia-g', 'IIIa-i', 'IVa-e', 'Va-b', and 'Va-b'. For instance,

$$\vdash \forall \mathfrak{a}[f(\mathfrak{a}) = g(\mathfrak{a})] \supset F\big(\acute{\varepsilon}f(\varepsilon)\big) = F\big(\acute{\alpha}g(\alpha)\big)$$

and

$$\vdash \acute{\varepsilon}f(\varepsilon) = \acute{\alpha}g(\alpha) \supset f(a) = g(a)$$

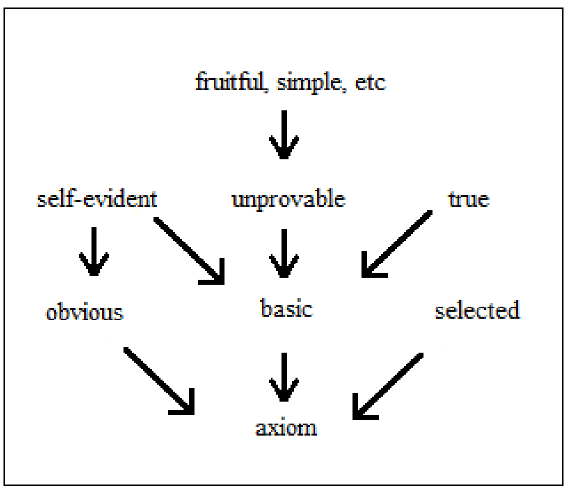
are labeled 'Va' and 'Vb' respectively. These derived laws then surface repeatedly as indexed premises.

The point here is not that Frege isn't, within his particular formalization of logic, careful about the distinction between axioms and inference rules—he is. It is not his practice of indexing per se which I take to be indicative of my claim about the axioms/inference rule distinction having no extra-systemic significance. Frege's formulas take up more space than familiar formulas, so it makes sense that he'd look for a way to make his derivations more concise. Instead, the evidence is that instead of just taking his axioms as a starting point, building upon them, and referring back to them only occasionally, he indexes his axioms, and their twenty-four variants derived in §§49-52 *constantly*. In both volumes of *Grundgesetze*, every two or three transition signs is indexed to one of these laws. To take one example, in §176 of volume II, thirty different transition signs contain indices, and of these, twenty-four refer to the axioms or one of their variants—IIa alone is indexed five times. The remaining six indices are to numbered theorems, all of which are derived in §176 itself. This pattern is typical—Frege slowly builds upon numbered theorems, but refers repeatedly back to his axiom and their variants. In addition to being the ultimate ground upon which his theorems are justified, his axioms are used like inference rules—they are the glue which holds his proofs together.[18]

I've mentioned above that Frege's basic laws are maximally general—this sets them apart from some non-basic logical laws, which might contain singular terms for specific objects. What's particularly of interest is the *sort* of generality that is in play in basic laws. Frege has two devices for expressing generality. The concavity marker, which binds gothic-letter variables, allows him to limit the scope of quantification, and to express multiply general statements. The roman-letter markers, on the other hand, take widest possible scope—Frege even says that their scope extends between formulas, so to speak. In this way, they function as free-variables. Interestingly, Frege never says that $\Phi(a)$ denotes a truth-value (Cook 2013, A-14) —instead, he makes it clear when it's correct to affix the judgement stroke to it. In any case, the presence of the roman-letter notation in all of Frege's basic laws gives them a schematic character which points towards their inferential nature.

§3. Conclusion

Let's stop a moment to take stock of the claims I've made, and how they fit together. Frege was a foundationalist, who thought that investigations in the a priori sciences bottom out in objectively basic truths. In the following diagram, an arrow from X to Y indicates that being an X is a necessary, but not sufficient, condition for being a Y:



Frege took such basicness to be necessary, but not sufficient, for axiomhood—we *select*, from among the basic laws, a subcollection of them which is mutually independent and which lets us derive those truths, the logical nature of which we wish to demonstrate. For a law to be basic (and hence to be an axiom) it must be true, unprovable, and self-evident. To be self-evident, in the sense of being independently recognizable as true, is to be potentially obvious. If we are to select it as an axiom, it must be *actually* obvious to us. For a law to be unprovable, it's necessary that it pass muster with respect to certain pragmatic considerations—that it is a starting point in a systemization which is fruitful, simple, elegant, etc.

There are a few important things to note. Firstly, I should reiterate that none of the connections in the diagram above are logic-specific—they all hold for geometrical axioms, as well as any other a priori sciences which admit of axiomatization. The real work in determining whether a given basic law is

[18] Thanks to Peter Milne for bringing up this reason in discussion, and for drawing my attention to the significance of Frege's use of indices.

logical will involve determining the nature of the self-evidence that law possesses—if its self-evidence is rooted in intuition, it's non-logical, but if it's instead rooted in something deeper and purely general, it's logical. The second point to mark is the way in which self-evidence is related to unprovability, and hence to pragmatic considerations. As I noted above, one of the chief ways in which my account differs from that of Burge is that he maintains, and I deny, that pragmatic considerations play any role in getting us to appreciate or identify a law as self-evident. Insofar as there's a connection, it's in the other direction—if a truth is obvious, that can be a pragmatic consideration which informs our decision of which principles to take as basic.

It is significant that Frege's understanding of self-evidence, and hence of logicality, is independent of unprovability, and hence of pragmatic considerations. For although Frege's Euclidean foundationalism is a constant thread which is to be found in all stages of his philosophical development, he does little to motivate it. His foundationalism can partially be explained by it being very much in vogue in the mathematical milieu in which he was writing, but this explanation only goes so far, since Frege's variation is appreciably different from that of his contemporaries. His Euclidean foundationalism was the framework in which he investigated the logicality of various truths, so any account of his logicism needs to explain its role. But when we *evaluate* Frege's logicism, his foundationalism may be one of the most likely elements of his picture we reject (though his distinctive views on self-evidence have much to recommend themselves)[19]. One of the aims of this chapter, in addition to clarifying the setting in which Frege approached questions of logicality, is to establish that, in the end, Frege's method of determining logicality will be independent of his foundationalism—we can reject the latter without rejecting the former.

[19] See Stewart Shapiro's 2009 article "We Hold These Truths to be Self-Evident: But What Do We Mean By That?" for a sustained critique of foundationalism which is very much applicable to Frege's version. In a nut shell, Shapiro argues that it beggars belief that the objective justificatory ordering in which truths stand should line up so nicely with the orderings suggested by pragmatic considerations. Why not just do away with this ordering entirely and rest content with the pragmatic considerations alone? As discussed above, Burge argues that Frege's view of logic shows his underlying rationalism. But as convincing as Burge's arguments are for this claim, it tells us little about *why* Frege adopted this rationalism. My view shows that we can accept Frege's view of logic even if we don't ourselves share his rationalist inclinations.

Chapter 2: Heuristic Indicators of Logicality: Generality and Undeniability

§0

In chapter 1, I covered some crucial preliminaries which will remain very much in the background in subsequent discussion. Frege took logical truths to be substantial and contentful, and arranged in an objective justificatory order which we gain access to through logical theory-building and the concomitant pragmatic considerations. Once this structure is clear to us, we may, using Frege's concept-script, reduce the question of the logicality of theorems to the logicality of those axioms which express basic logical laws. The goal is now to investigate how Frege determines whether a given basic law is logical. This investigation is complicated by the fact that Frege says relatively little about how one should make such a determination, and he nowhere gives an explicit, sharp criterion for when a basic law is logical. This lacuna has led some, most notably proponents of the elucidatory interpretation, to suppose that Frege had no sharp criterion for logicality, and, in fact, no general conception at all of what logicality amounts to.

Lacking a sharp criterion for logicality is not, however, the same as having no conception of the logical at all. Frege's works contain some general comments on the relation between truth, normativity, and logical consequence. Much can also be gleaned from Frege's tirades against psychologism. His comments concerning logicality, cursory though they are, are found throughout Frege's writings, most notably in the prefaces to his major logical works. In addition to these broader comments, Frege clearly articulates at least two necessary conditions for logicality. Firstly, logical laws must be general (1879, 5; 1884, §14; 1893, xv). Secondly, they cannot be denied in conceptual thought (1884, §14; 1893, xvii).

I take the necessary conditions Frege mentions to reflect an employment, on his part, of what I shall call "heuristic indicators" of logicality. The idea is that Frege had a list of necessary conditions which, when jointly possessed by a law, provide evidence that the law is logical. It does not, however, provide *conclusive* evidence, hence the need to prove laws from their ultimate basis. I'll argue that, in order to be employed effectively, Frege's heuristic indicators must not be a *mere* list of necessary conditions, but must also signify a deeper conception of the logical. I will argue that this conception is *implicit* in Frege's work, because it motivates Frege's indicators, explains the way in which he treats his foundations, and makes sense of Frege's various platitudes concerning truth, logic, and normativity.

This chapter is structured as follows. After some preliminary stage-setting in §1, I give, in §2, two examples where Frege uses generality and undeniability as heuristic indicators. I'll discuss the sense in which logic is general in §3, and do the same for undeniability in §4. In §5, I'll present and compare some different accounts of how Frege argues for logicality, with the aim of showing that we must give an deeper explanation for why Frege took his basic laws to be general and undeniable.

§1. Logic, truth, and normativity

Before discussing the indicators, however, let's take a moment to examine Frege's broader comments on the nature of logic, where he sets out the relation between logic, truth, and normativity. His extended discussions of logic, and its connection with truth, appear in three places[1]—in the introduction to *Grundgesetze*, in a 1918 article called "Thought", and in an unpublished work called "Logic" which was written sometime between 1879 and 1891.

[1] This list is not, of course, intended to be exhaustive—it merely marks the discussions which I take to be the most explicit, extended, and significant.

In the foreword to *Grundgesetze*, Frege spends a huge amount of time—12 out of 22 pages—railing against psychologism. Although he was, on occasion, prone to excess in his criticisms[2], the prominent position of the discussion ought to tell us something about its importance to Frege. The point which he repeatedly harps upon is that psychologism neglects the proper relationship between logic and truth. Psychological laws are merely descriptive, describing the causal origin of our beliefs, rather than our justification for holding them—as Frege puts it, they are the laws of taking to be true, rather than of *being* true (1893, xv). What is also clear is that there is a sense in which psychologism misses out on the *normative* dimension of logic. Logic concerns the laws with which we *ought* to follow insofar as we care about truth. We can learn arbitrarily much about how we come to hold a belief, or how we *happen* to move from one belief to another, but there will still remain the question of whether we were *right* in our belief, or whether our inferential practices are *correct*. Thus, the first thing to take from Frege's opposition to psychologism is that logic's concern with truth is intimately bound up with its normative force—truth points the way for logic as the good does for ethics, as Frege puts it (1918a, 58/FR 325).

Like ethics, logic is normative. But whereas actions (or perhaps intentions) are good or bad, it is *thoughts* which are true or false. *Any* thought[3] is evaluable in this way, not only logical truths:

> The ambiguity of the word "law" here is fatal. In one sense it says what is, in the other it prescribes what ought to be. Only in the latter sense can the logical laws be called laws of thought, in so far as they legislate how one ought to think. Every law stating what is the case can be conceived as prescriptive, one should think in accordance with it, and in that sense it is accordingly a law of thought. This holds for geometrical and physical laws no less than the logical. The latter better deserve the title "laws of thought" only if thereby it is supposed to be said that they are the most general laws, prescribing how to think wherever there is thinking at all. (Frege1893, xv)

The upshot of this is that although logic is normative, logical laws don't have distinctive normative *content* over and above that possessed by non-logical truths.

I think that there's another important element to notice in this passage, however, concerning the connection between judgement, truth, and normativity. Frege is clear that thoughts are the primary bearers of truth, and sentences only derivatively so. But normativity does not enter into the picture with thoughts alone—although thoughts can be true or false, thoughts considered in isolation cannot be right or wrong. We need a connection between thoughts and reasoners, and judgement is just such a connection. To judge that Φ is to recognize that Φ is true (1918a, 62/FR 329). A thought alone cannot miss its mark, but *we* can, when we aim for truth and miss by putting something forward as true which is false. This connection comes into sharper focus when we consider assertions, the verbal manifestation of judgements. Any time we assert something, we put it forward as *true*. And since truth, as the goal of logic, is intimately connected to normativity, this is why any law may be seen as prescribing that we think in accordance with it—whenever we assert a law, we simultaneously put it forward as something one ought to think in accordance with.

[2] As Tappenden humorously puts it, Frege occasionally repeats points with "with the relentless outrage of a cranky great-uncle who ruins every family gathering" (Tappenden 2000, 226)

[3] That we may ask of any thought whether or not it is true or false shouldn't be taken here to exclude the possibility of truth-value gaps. In unpublished work, Frege excludes this possibility by saying sentences containing a referentless term express no genuine thoughts at all, but "mock thoughts" (1897a, 141/FR 229-30). This runs against what he says elsewhere though—in "On Sinn and Bedeutung" he's quite happy to say that "Odysseus was set ashore at Ithaca while sound asleep" has a sense (Frege 1892a, 32/FR 157). We needn't worry about this issue too much, as it's a presupposition of language that all names refer (1892a, 31/FR 156), and in a rigorous science, this presupposition will be met (1891, 19/FR 141).

To know that logic is how one ought to reason insofar as one cares about truth is to have said relatively little, however. The most we can draw from this is that Frege would have assented to certain principles which have no categorical force, but only hold if and insofar as attaining truth is our goal. If attaining truth is our goal, then:

1) We ought to hold as true only thoughts which are true.
2) If a thought φ is a logical consequence of a collection of thoughts Γ, then we ought to not refrain from affirming φ while also affirming each of Γ.

The second principle can be understood as a deontic operator taking wide scope over a conditional. It can be rephrased as follows:

2) If thoughts Γ jointly entail φ, then Ought(if one affirms each of Γ then one affirms φ).

In other words, we ought to avoid the case where the conditional in the scope of the operator has a true antecedent and a false consequent.

I shall refer to these as the *minimal connection* between logic and normativity. 1) and 2) might look quite demanding at first—after all, it might be very hard to tell if a principle is true, and some entailments are extremely difficult to see. However, this demandingness is defanged by the laws merely hypothetical character: it's uncontroversial that a greater degree of compliance with the principles above will help one avoid believing inconsistencies and falsehoods. When properly understood, the minimal connection is as unhelpful it is uncontentious—1) and 2) become the kind of bland statements which are consistent with a wide variety of different conceptions of logic, and which could be held by proponents of wildly varying logics. As such, the comments discussed so far don't reveal anything distinctive about Frege's notion of logical consequence[4].

The question is, then, whether Frege thought there was a tighter connection between logic and truth than is provided by the minimal connection. On the one hand, Frege takes logical laws to be intimately connected with truth. On the other hand, the minimal connection, as discussed, seems to allow *all* true thoughts an equal claim to being called "laws of truth". If the minimal connection is the only connection between logic and truth, then greater generality really is the *only* difference between logical and non-logical truths. We can't point to necessity as a special feature of logical truths, for necessity and possibility hardly figure in Frege's works—he says that to be necessarily true just is to be logically true, and that to be possible is to be epistemically possible (1879, §4).

The passages which most strongly suggest a connection between logic and truth deeper than the minimal connection are those containing his peculiar suggestion that logic is the unfolding of the content of "truth". These, I argue, reveal a more specific and distinctive conception of logicality.

> …I assign to logic the task of discovering the laws of truth, not the laws of taking things to be true or of thinking. <u>The *Bedeutung* of the word 'true' [*wahr*] is spelled out in the laws of truth</u>. (Frege 1918a, 59; 326)

> <u>It would not perhaps be beside the mark to say that the laws of logic are nothing other than an unfolding of the content of the word 'true'</u>. Anyone who has failed to grasp the meaning of this word—what marks it off from others—cannot attain any clear idea of what the task of logic is. (Frege 1879-91, 3)

[4] This isn't to say that the bland platitudes listed are entirely presuppositionless. Some anti-realists will demand clarification concerning what notion of truth is at play. Tim Maudlin demurs in a quite different way—his solution to the liar paradox involves the appropriateness of asserting the liar sentence, even though in doing so, one asserts not a truth, but an ungrounded sentence (Maudlin 2008, 193).

Frege makes this comment twice, a few decades apart, so it's more than a passing comment—the idea stuck with him. In the next chapter, I'll argue that these passages suggest that logically valid inference are implicit in our grasp of thoughts—logic is intimately connected to understanding in such a way that if we fail to infer logically, we fail to understand sentences and grasp the thoughts they express.

I've just discussed the question of whether Frege was committed only to the minimal connection or had a more subtle conception of logic in mind. The reason I raise this question here, in advance of my own answer described in chapter 3, is that a parallel questions arise when we ask in what sense Frege took logic to be undeniable. There's minimal and relatively non-committal senses in which logic can be undeniable, and I argue that Frege must have taken logic to be undeniable in a deeper sense, if he was to have made effective use of it as a heuristic indicator.

§2. Introduction to the heuristic indicators

Frege took logic to be general and undeniable, and paired these conditions when making a prima facie case for the logicality of arithmetic. One of the most notable places where Frege puts forward this argument is in *Grundlagen*:

> For purposes of conceptual thought we can always assume the contrary of some one or other of the geometrical axioms, without involving ourselves in any self-contradictions when we proceed to our deductions, despite the conflict between our assumptions and our intuition. The fact that this is possible shows that the axioms of geometry are independent of one another and of the primitive laws of logic, and consequently are synthetic. Can the same be said of the fundamental propositions of the science of number? Here, we have only to try denying any one of them, and complete confusion ensues. Even to think at all seems no longer possible. The basis of arithmetic lies deeper, it seems, than that of any of the empirical sciences, and even than that of geometry. The truths of arithmetic govern all that is numerable. This is the widest domain of all; for to it belongs not only the actual, not only the intuitable, but everything thinkable. Should not the laws of number, then, be connected very intimately with the laws of thought? (Frege 1884, §14)

Frege ends §14 with this question—he does not suppose himself to have established that arithmetic is logical, but evidently takes himself to have rendered it likely enough to make it worthy of more detailed investigation. Undeniability can give us a negative result which is a bit more solid—though the apparently undeniability of arithmetic is only prima facie evidence that arithmetic is logical, the deniability of geometrical truth is concrete evidence that it's not logical.

In "Über formale Theorien der Arithmetik", an article published a year later, Frege again pairs generality and undeniability, using them again as heuristic indicators.

> Herewith arithmetic is placed in direct contrast with geometry, which, as surely no mathematician will doubt, requires certain axioms peculiar to it where the contrary of these axioms—considered from a purely logical point of view— is just as possible, i.e. is without contradiction. Of all the reasons that speak in favour of this view, I here want to adduce only one based on the extensive applicability of mathematical doctrines. As a matter of fact, we can count just about everything that can be an object of thought: the ideal as well as the real, concepts as well as objects, temporal as well as spatial entities, events as well as bodies, methods as well as theorems; even numbers can in their turn be counted. What is required is really no more than a certain sharpness of delimitation, a certain logical completeness. From this we may undoubtedly gather at least this much, that the basic propositions on which arithmetic is based cannot apply merely to a limited area whose peculiarities they express in the way in which the axioms of geometry express the peculiarities of what is spatial; rather, these basic propositions must extend

> to everything that can be thought. And surely we are justified in ascribing such extremely general propositions to logic. (Frege 1885a, 94-5/CP 112)

What's particularly helpful about the 1885 quote is that it suggests that the generality and undeniability of logical truths are transmitted via logical consequence—if they weren't, we couldn't conclude from the fact that a principle is general that its ultimate ground of proof is general as well. Frege emphasizes this fact elsewhere within the same article:

> Therefore if arithmetic is to be independent of all particular properties of things, this must also hold true of its building blocks: they must be of a purely logical nature. (Frege 1885a, 96/CP 114)

All of this just makes a tentative case for the logicality of arithmetic, however—if Frege thought the fact that everything can be counted was *conclusive* evidence for logicism, he could have saved paper and ended *Grundlagen* at §14. A conclusive case will require Frege make use of the heritability strategy, which I discussed in §1.1 of the previous chapter, reducing questions concerning the logicality of theorems to questions concerning the logicality of the basic laws from which they can be proven. The open question is whether, upon reaching the basic laws, it's enough to see that they're general and undeniable, or whether some deeper case must be made for them being logical. After examine the heuristic indicators in turn, I'll argue that Frege was committed to making a deeper case.

§3. Heuristic indicator 1: generality

§3.1 Types of generality

In the Preface to *Begriffsschrift*, Frege tells us that logic is general:

> The most reliable way of carrying out a proof, obviously, is to follow pure logic, a way that, disregarding the particular characteristics of objects, depends solely on those laws upon which all knowledge rests. (Frege 1879, p.5)

But what is it for logic to be general? There are several senses[5], some very close together, of what one might mean:

1. <u>Universal applicability:</u> Logical laws apply to any subject matter whatsoever.
2. <u>Logic as all-encompassing:</u> All thinking, without exception, is subject to logical laws.
3. <u>Topic-neutrality:</u> Logic has no objects specific to it—it treats of all domains equally
4. <u>Maximal Generality:</u> Logical laws are not especially metalinguistic, but are the truths of the most general science.
    a. <u>Vocabulary version:</u> Primitive logical laws contain only vocabulary not specific to any discipline, vocabulary which appears in any discipline whatsoever.
    b. <u>Syntactic version:</u> Primitive logical laws contain no generalizable but ungeneralized positions.
5. <u>Logic as a universal language:</u> There are no thoughts inexpressible in some extension of Frege's concept-script.

This list is not intended to be exhaustive or mutually exclusive. Logic as a universal language, for instance, is perhaps best thought of not as a distinct conception of the generality of logic, but rather an exegetical thesis concerning what Frege took to follow from his treatment of logic as maximally general

[5] Proops 2007, though primarily aimed at arguing that Russell did not hold logic to be universal in any interesting sense, provides a useful list of several different types of generality. Though I put this list to a very different purpose here, the method of categorization I adopt here is based on that of Proops.

and all-encompassing—I postpone discussion of it until §1 of chapter 5. What I argue for in this subsection is that while Frege did not take logic to be topic neutral, he thought that it was universally applicable, and that his commitment to it being so is rooted in a more fundamental commitment to it being all-encompassing. I'll also argue that maximal generality has been misunderstood and overemphasized—Frege's basic logical laws simply aren't maximally general in the syntactic sense.

§3.2 Universal applicability vs topic-neutrality

The first sense in which logic could be general is if it's *universally applicable*—logical laws may be applied to any subject matter whatsoever. In *Grundlagen* quote mentioned at the start of this chapter, Frege takes the fact anything thinkable is numerable as prima facie evidence for the logicality of arithmetic (Frege 1884, §14). Frege was quite impressed by this property of arithmetic, and his arguments against other theories hinge, in part, upon the fact that they'd prevent arithmetic from being purely general—if arithmetic has an empirical basis, its applicability would be restricted only to the sensible (1884, §24), and if it has its basis in spatial intuition it would be applicable only to the intuitable (Frege 1884, §14).

That logical laws apply to everything must not be confused with topic-neutrality, the idea that logic treats of everything equally, or is not *about* anything in particular. Although Frege held that logic was universally applicable, he also denied that logic has nothing peculiar to it—he says that

> Just as the concept *point* belongs to geometry, so logic, too, has its own concepts and relations; and it is only in virtue of this that it can have a content…To logic, for example, there belong the following negation, identity, subsumption, subordination of concepts (Frege 1906a, 428/CP 338).

Elsewhere, he talks about how the ultimate building blocks of a discipline "contain, as it were in a nutshell, its whole contents" and says

> …if arithmetic is to be independent of all particular properties of things, this must also hold true of its building blocks: The must be of a purely logical nature (Frege 1885a, 96/CP 114)

Frege evidently sees no conflict between logic being general, and nevertheless being specially related to such building blocks. So, logic is topic-specific insofar as it deals with the functions designated by the negation sign, the identity sign, etc. This is not uncontroversial however, due to a perennial problem—it seems that Frege cannot make sense of talk *about* any functions at all, not just logical ones. This has led proponents of the elucidatory interpretation to deny that we should take quotes like the one above ontologically seriously (Weiner 1995).

There is, however, a more direct route to seeing that logic isn't topic-neutral—Frege clearly took there to be logical *objects*. In *Grundlagen*, Frege introduces extensions (*Umfange*) into his ontology (Frege 1884, §68), and defines numbers in terms of them. Frege thought that every concept had an associated extension, and that two concepts have the same extension just in case they both hold of exactly the same objects. Frege later generalizes this idea by associating value-ranges (*Werthverläufe*) with every function. There's much debate to be had over just how Frege construed such objects—whether he has a robust conception of an aspatial, atemporal third realm (Burge 1992) or whether we'd instead be amiss to read Frege as a platonist in the ordinary sense (Weiner 1990) [6]. But regardless of where we come down on the *nature* of extensions and value-ranges, it isn't up for dispute whether Frege took them to be logical objects—he explicitly refers to them as such in a July 28, 1902 letter to Russell (1980, 141).

[6] Weiner reads Frege a platonist, but not a platonist of the usual sort. She says that the world is not articulated independently of the structure we impose upon it (Weiner 1990, 202-4), and she extends this view to the abstract as well: "…discovering what is 'there' in the 'realm of the abstract' amounts to discovering what meets the descriptions that interest us" (1990, 203).

Of course, these objects still have, in a certain qualified sense, general import. In every domain of inquiry, every sentence, without exception, must contain concept-words—no word sequence consisting solely of singular terms strung together could ever express a thought. Since, on Frege's view, every concept whatsoever has an extension, every domain of inquiry has extensions indirectly associated with it, namely extensions of concepts specific to that domain. But this sort of generality won't amount to topic-neutrality—pure logic adjudicates the existence of extensions in a way it does for no other sort of object.

§3.3 Constraints associated with universal applicability

Frege clearly took logic to be universally applicable, and this has consequences for how he treats his concept-script. Firstly, Frege intended for his first-order quantifiers to range over absolutely everything, i.e. over all objects (concepts, are not *things*)[7]. This is born out in Frege's general perspective on objecthood—he evidently sees no logical significance in the distinction between actual and non-actual objects, or between entities, such as numbers, which exist necessarily and those which do not. Secondly, Frege placed no restrictions on which objects may be taken as values of object-variables. He was quite explicit about this point—in "Function and Concept", he says:

> We see here we have undertaken to extend the application of the term in the other direction, viz. as regards what can occur as an argument. Not merely numbers, but objects in general, are now admissible; and here persons must assuredly be counted as objects. The two truth-values have already been introduced as possible values of a function; we must go further and admit objects without restriction as values of functions. (Frege 1891, 17/FR 140)

Frege then proceeds to argue from the claim that functions can take anything as an argument to the claim that the function must be *determined* for all arguments—for example, '$\odot + 1$' must take a value even if '$\odot$' is 'the sun'[8]. Virtually no contemporary logician agrees with Frege in this regard. Even Weiner, who is sympathetic to Frege's insistence that predicates don't refer if they aren't sharply defined for all inputs[9], doesn't go so far as to say that formally rigorous mathematics ought to make no use of partial functions. Lucky for those of us who are broadly sympathetic to Frege's logical view, one can distinguish between two threads here—his insistence that any object can be taken as the value of an object variable, and his claim that rigor requires us to define functions for all input. If we demur from accepting the latter claim, then the first is harmless enough—we just won't have any general assurance that

---

[7] There's some debate over whether Frege construed first-order quantification objectually or substitutionally. His discussion seems substitutional in *Begriffsschrift* (1879, §9), though there's reason to think his exposition here is a bit muddled (Stevenson 1973, 207-11). In *Grundgesetze*, his discussion of the concavity notation for first-order quantification is objectual(1893, §8; §20), whereas his discussion of higher-order quantification is sometimes objectual (1893, §24) and sometimes substitutional (1893, §20; Heck 2012, 53). Heck makes a compelling case that in the absence of any independent reason for taking Frege to think every object and function has a name, we should take him to be objectual about quantification—some of his exposition, such as his conditions for reference in §29, appear more substitutional than they really are. This is because, instead of the Tarskian variable assignments developed decades later, Frege's alternative is use upper-case Greek letters as metatheoretic auxiliary names which can temporarily stand for an object with no name in the object-language (Heck 2012, 56-8).

[8] His insistence in this regard is *strong*—in *Grundlagen*, he took such considerations as grounds for rejecting particular analyses of arithmetical statements. He takes such undefined-ness to be grounds for rejecting certain definitions of number—one of his complaints of the adjectival strategy for analyzing ascriptions of cardinality is that it leaves certain identity statements indeterminate. (1884, §56; §63).

[9] Weiner gives a helpful example—health researchers used the term "obesity" and regarded the sentence "obesity increases risk of heart disease" as true, despite the fact that this isn't a precise term, and the accepted BMI threshold used to define it has changed over the decades. She's careful to distinguish between regarding something as true, and a sentence actually *being* true, however—the former is a sort of pragmatic proto-supervaluationist affair, whereas the latter notion does, in fact, strictly require that every sub-sentential component of the sentence whose truth value is in question have a fixed referent (2010, 46-7)

substituting proper names for variables will never result in truth-value gaps. But it's still instructive to look at what might have led Frege to take on such a peculiar commitment.

Why suppose functions should be defined for all input? First of all, the referent of a function name is a function, a way of pairing arguments with values. So, to determine a function name's referent, it must be determined what input/output pairings belong *and which do not* belong in the function's value-range[10], in an exhaustive way—for any arbitrary (1^{st} level) function sign 'f', if there exists even a single (referring) singular term 'a' such that the complex term '$f(a)$' has no reference, then 'f' has no determinate value-range, and hence no reference. And worse—it follows from Frege's compositional conception of meaning, that *any* expression in which the non-referring functional symbol 'f' is a constituent will fail to refer as well (regardless of whether or not that expression also contains 'a'). For example, if 'The Sun $+1$' fails to refer, so does the first level function symbol '$\odot + 1$', and so does the expression '$1 + 1$' (since this latter term contains a referent-less function symbol). The two-place first-level function symbol '$\odot + \circledcirc$' would fail to refer for similar reasons, with the result that *every* complex name in which the addition symbol was a constituent would fail to refer. But recall that for Frege, propositions are just names of truth values. Thus, *every* arithmetical expression involving addition fails to take on a truth-value. Fail to determine the value of an arithmetical function for a single object, and the whole of arithmetic comes crashing down!

If a subsentential term fails to refer, this will always result in truth-value gaps—in "On Sinn and Bedeutung", Frege says—"But since it is doubtful whether the name 'Odysseus', occurring therein, has a *Bedeutung*, it is also doubtful whether the whole sentence does" (1892a, 32/FR 157). Somewhat more prosaically, "It is the striving for truth that drives us always to advance from the sense to the *Bedeutung*"(1892a, 33/FR 17) Thus, Frege's motivation for the determinateness of value-ranges is a consequence of his compositional account of meaning, together with his desire to assure us that his system does not result in truth-value gaps. He says:

> It seems to be demanded by scientific rigour that we ensure that an expression never becomes *Bedeutungslos*; we must see to it that we never perform calculations with empty signs in the belief that we are dealing with objects. (Frege 1891, 19/FR 141)

He reiterates this in *Grundgesetze* that "Correctly formed names must always refer to something"(1893, §28). And of course, this is precisely what Frege sets out to show in §29-§32 of *Grundgesetze*—he explicitly concludes in §32 that his proof establishes that every sign in his language has a referent, and hence that every formula expresses a thought. Immediately preceding and succeeding the proof are discussions of the correct formation of names and correct principles of definition—these show, in conjunction with his proof, that all of his formulas making use of his definitions are contentful as well.

Another reason Frege gives his for claim that functions must be defined for all input is as follows:

> In other words: as regards concepts we have a requirement of sharp delimitation; if this were not satisfied it would be impossible to set forth logical laws about them (Frege 1891, 20/FR 141)

Whence this impossibility? The law of the excluded middle is one such law—if a concept F wasn't determined for some object Δ, it wouldn't be true that $F(\Delta) \vee \neg F(\Delta)$. In several places, he simply cites

[10] The referents of function and concept terms are *not* value-ranges, although complete knowledge of the later exhausts knowledge of the former. Functions are "unsaturated", whereas extensions are not—they are saturated objects. I speak of value-ranges here for one of the reasons Frege himself did—we cannot, strictly speaking, talk *about* the referent of a function symbol, because it does not make logical sense for a term for something unsaturated to appear in the subject position of a sentence. I talk about functions by talking about value-ranges—in particular, to exist, functions must have precise value-ranges, so pointing out that it is not determinate whether an object belongs to a value-range suffices to show that the associated functional symbol is referentless.

L.E.M. for his requirement functions be everywhere defined (1897-8, 1; 1906b; 212/FR 298). And this point generalizes—*any* logical law expressed by a formula containing '$F(\Delta)$' will fail to come out true. This problem affects inference as well—since second-order quantifiers range over all concepts, we can logically infer statements with truth value gaps from logical truths. From

'$\vdash \forall \mathfrak{f} \forall \mathfrak{a}(\mathfrak{f}\mathfrak{a} \supset \mathfrak{f}\mathfrak{a})$'

which is derivable from Frege's basic laws, we can derive

'$\vdash F(\Delta) \supset F(\Delta)$'

which is not a truth—if '$F\Delta$' has no reference, neither does '$F(\Delta) \supset F(\Delta)$'. If we could logically infer non-truths from truths, logical laws couldn't be laws of truth even in the weak sense which motivates the minimal connection.

§3.4 Universal applicability and logic as all-encompassing

So, Frege took logic to be universally applicable, and this leads him to reject restrictions on the range of variables, and to insist on sharp delimitation of concepts. Why, though, did Frege take logic to be universally applicable in this way? Why not suppose it applies only to some restricted subject matter, or even take a more pluralistic perspective, where the sorts of inferences which we take to be valid varies depending on which sorts of objects we're considering? The short answer is that Frege seems to take it as constitutive of what we mean by logic that it apply to everything—if we had inference rules that were only valid with respect to thoughts treating of a specific subject matter, then these rules can't rightly be called logical.

But there's a more interesting possibility—that universal applicability can be traced to the inextricable connection logic has with truth. Frege insists that the laws of logic are the laws of being true (1893, xv). He also says that part of what he considers distinctive about his conception of logic is, along with the priority he gives to truth, his immediate introduction of thoughts as those entities to which truth and falsity primarily apply (1919, 273/FR 362). This suggests that as far as logic is concerned, all thoughts are on par, insofar as they are the primary bearers of the truth and falsity with which logic is concerned[11]. And since we can have thoughts concerning any subject matter whatsoever, all our reasoning concerning any subject matter whatsoever is subject to the laws of logic. This brings out something important about the generality of logic—its generality is not exhausted by the bare fact that logical laws applies to all subject matter, or that variables have to range over all objects. More fundamental is that logic is *all-encompassing*—it gives us rules to which *all* reasoning is subject.

Though plausibly Fregean on its face, what is meant by all reasoning being subject to the same rules will take some care to disambiguate. It can't be that we *explicitly* recognize that our judgement of one thought on the basis of another is in accordance with a logical law—presumably, humans were reasoning before *modus ponens* was explicitly recognized. Nor can it be that all of our inferences are, or should be, implicitly based on one of some small set of allowed inference rules—this would make Frege's choice of inference rules charmed, when in fact he was well aware that there could be a tradeoff between axioms and inference rules (Sullivan 2004, 673; Frege 1880/81, 37). Instead, for logic to be all-encompassing is

[11] There's an important caveat here. Frege thought sentences fail to refer to truth values if they contain a term with no referent (1892a, 32/FR 157) and that premises must always be true (1918b, 145/FR 348), meaning that logical laws don't apply at all to thoughts expressed by sentences with referentless terms. Frege himself draws this conclusion—he saying says that if concepts are not defined for all input (which would prevent concept-words from having a determinate referent) "it would be impossible to set forth logical laws about them" (1891, 20/FR 141).

for all of our judgements to be subject to evaluation by logical laws. It is via, and *only* via, logical laws that we know when one assertion stands in opposition with another, for instance. By showing what follows from an assertion, logical principles allow us to tease out what has been said—to make a judgement is to take a stand on the way things are, and logical laws tell us what stand has been taken.

If logic is all-encompassing, this implies that logic is universally applicable. Note, however, that one could take logic as universally applicable without taking it to be all-encompassing. The sort of pluralism propounded by J.C. Beall and Greg Restall serves as an example. A truth is logical iff it is true in all cases, and ψ follows from a set of premises Γ iff ψ is true for every case for which every premise in Γ is true (Beall and Restall 2000, 476). But "case" is ambiguous—we might think of it as Tarskian models, or as constructions, or something else, and different choice will give us different logics (2000, 478). Since the logic we employ doesn't, on this picture, vary with subject matter, logic is universally applicable. But no single logic is all-encompassing, in the sense of being a universal standard.

§3.5 Logic as maximally general

To say that logic is maximally general is to say that logical laws are like those of science, but more general. This is the sort of generality most emphasized by certain proponents of the elucidatory interpretation, such as Thomas Ricketts (1996, 123; 2010, 151) and Warren Goldfarb (2010, 68). Goldfarb explains maximal generality by way of contrast with the "schematic conception" of logic that we inherit from Tarski and Quine. Although the specifics of the formulation vary depending on the author, the schematic conception is that logic is about logical schemata. We take fully contentful formulas, and then replace their non-logical terminology with multiply interpretable schematic letters. Logical laws are statements *about* such schemata, and permissible transitions between them. Logical truths are quite different from logical laws—they're particular sentences in the object language, which are schematizable by formulas which are true under every interpretation (Goldfarb 2010, 66).

For Frege, on the other hand, logical laws are not about schemata, but about the world—the formulas in his concept-script contain no uninterpreted components (Goldfarb 2010, 67). As Goldfarb sees it, the difference between Frege's conception and the schematic conception of logic is one of level—in the latter conception, logic is metalinguistic—"the claims of logic are claims about schema or about sentences", whereas for Frege, "logic sits squarely at the object level, issuing laws that are simply statements about the world"(Goldfarb 2010, 69).

Here, Goldfarb's committing himself to a claim which is crucially connected to maximal generality:

> Content claim: There is nothing especially metalinguistic about the content of logical laws.

The content claim is implied by maximal generality, under the very plausible assumption that names of signs and formulas wouldn't figure in statements of a maximally general science. But one could hold the content claim without holding logic to be maximally general.

I think it's fairly clear that there's something crucially correct about the content claim, which forms a central part of Ricketts's and Goldfarb's conceptions of the maximal generality of logic. Frege's concept-script allows him to express laws without the use of schematic letters, or special devices such as propositional quantifiers. In *Grundgesetze*, Frege has two devices for expressing generality—his concavity symbol (which I'll represent as '∀'), and his roman letter notation[12]. It is with the latter that he expresses his logical laws, such as in:

[12] Both the concavity symbol and the roman letter notation express generality, but are put to different purposes. The concavity symbol is introduced in order to allow us to make distinctions of scope, which in turn allows for

Law I: $\vdash a \supset (b \supset a)$ (§18)

According to Frege's comments on his roman letter notation, a formula like the one above is correct if the function named by '$\xi \supset (\zeta \supset \xi)$' is the true for every possible pair of arguments (1893, §17; Cook 2013, A13-14). By Frege's rule for eliminating roman letters (1893, §9), we can substitute 'a' and 'b' with any proper names Γ and Δ to derive

'$\vdash \Gamma \supset (\Delta \supset \Gamma)$'

from law I (1893, §9)[13].This, together with his characterization of propositions as proper names of truth-values, means that we can derive instances of laws *within* Frege's system, rather than thinking of such laws as schematic patterns described metalinguistically.

However, the content claim alone cannot constitute maximal generality. Goldfarb says

> Logical laws are as descriptive as physical laws, but they are more general. Indeed, they are supremely general; for, aside from variables, all that figure in them are the all sign, the conditional and other signs which are not special to any discipline, but which figure in discourse on any topic whatsoever. (Goldfarb 2010, 68)

Thomas Ricketts takes an identical line here:

> Logical laws are maximally general truths – generalizations whose statement requires only that topic universal vocabulary required to express the results of any science, e.g. an expression for negation. Logical laws are then on the same level as the laws of the various special sciences. The relation of logic to other sciences is that of a more abstract, less detailed science to a more detailed one. (Ricketts 2010, 151)

These comments give us a slightly sharper conception of what maximal generality amounts to:

Vocabulary version of maximal generality: Primitive logical laws contain only vocabulary not specific to any discipline, vocabulary which appears in any discipline whatsoever.

Most of Frege's basic laws certainly *seem* maximally general—negation, quantifiers, conditionals, etc are all plausible candidates for vocabulary shared by rigorous concept-script formalizations of any given science. The main obstacle for the vocabulary version is that Frege's basic laws include elements, like his operator $\acute{\varepsilon}f(\varepsilon)$ from functions to value-ranges, for which it's far from obvious that they'll appear in a formalization of any given science. I'll call worries of this sort—the issue of how to identify the special

---

statements with nested general terms. The generality of roman letter markers, on the other hand, is always read as having the widest scope possible—indeed, the scope even extends *between* premises (Frege 1893, §17; Heck 2012, 61). For instance, to infer '$\vdash \forall\mathfrak{a}(F\mathfrak{a} \supset H\mathfrak{a})$' from '$\vdash \forall\mathfrak{a}(F\mathfrak{a} \supset G\mathfrak{a})$' and '$\vdash \forall\mathfrak{a}(G\mathfrak{a} \supset H\mathfrak{a})$', we first infer '$\vdash Fa \supset Ga$' and '$\vdash Ga \supset Ha$', and then derive '$\vdash Fa \supset Ha$', before generalizing again. The subderivation is only permissible because the roman letter notation licenses us to recognize 'a' as always picking out the same thing in both '$Fa \supset Ga$' and '$Ga \supset Ha$'.

[13] Of course, Frege needs to state inference rules, in addition to his basic laws. These are specified syntactically. But this doesn't pull against Frege's view of logic as contentful object-level statements about the world, because inference rules, considered solely as specifications of permissible transitions between formulas, aren't logical principles. What's logical is the principle which underlies it—inference rule qua license for permissible transitions between thoughts. It's an inference rule in this latter sense which is not different in kind from Frege's basic laws, as argued in §2.3 of the previous chapter.

vocabulary of which maximally general expressions are composed—the *demarcation problem* for maximal generality.

One might hope to sidestep the demarcation problem altogether in the following way—by avoiding the vocabulary version of maximal generality altogether, and giving maximal generality an alternative version gloss:

> Syntactic version: Primitive logical laws contain no generalizable but ungeneralized positions.

This sort of maximal generality is the one discussed in (Proops 2007, 10-14)[14]. A position is ungeneralizable if generalizing further would result in nonsense. No one supposes that *all* logical truths are maximally general in the syntactic sense—'1=1' is a theorem of Frege's system, but clearly contains a generalized but generalizable position, as '$a = a$', which is perfectly grammatical. But one might put more hope in Frege's *basic* laws being maximally general in the syntactic sense. Consider the degenerate case of I:

$$a \supset a$$

If we try to generalize this expression further by quantifying over the position occupied by '⊃', we get:

$$\forall \mathfrak{f}\, \mathfrak{f}(a)$$

Recall that Frege's higher-order quantifiers range over *all functions*, not just over *concepts*. So'$\vdash \forall \mathfrak{f}\, \mathfrak{f}(a)$' doesn't express the falsehood that all concepts hold of all objects—what it expresses is something weird.

What appears at first to be such a promising way of characterizing maximal generality while avoiding the demarcation problem can be seen as unworkable, when we recall certain facets of Frege's symbolism in *Grundgesetze*. It's certainly *weird* to say that $-\forall \mathfrak{f}\, \mathfrak{f}(a)$ is true/false. But Frege's system is already weird—we can infer '$\vdash 2 \supset 2$' from '$\vdash a \supset a$' via a single application of roman letter elimination, so we *can* judge $\vdash 2 \supset 2$. And what's more, strange things like '$-\forall \mathfrak{f}\, \mathfrak{f}(a)$' *do* express thoughts. For '$\vdash \forall \mathfrak{f}\, \mathfrak{f}(a)$' is correct iff, for every argument Δ which the function $\forall \mathfrak{f}\, \mathfrak{f}(\xi)$ takes as input, $\forall \mathfrak{f}\, \mathfrak{f}(\Delta)$ is the True (1893, §17; Cook 2013, A13-14). And $\forall \mathfrak{f}\, \mathfrak{f}(\Delta)$ is the True iff for every function $F(\xi)$, $F(\Delta)$ is the True. Since $\Delta \neq \Delta$ isn't the true, $\forall \mathfrak{f}\, \mathfrak{f}(\Delta)$ isn't the true, and we can't write '$\vdash \forall \mathfrak{f}\, \mathfrak{f}(a)$'—not because '$\forall \mathfrak{f}\, \mathfrak{f}(a)$' is nonsense, but because it's false. Even though '$-\forall \mathfrak{f}\, \mathfrak{f}(a)$' might look at first like it expresses no thought, we can think of it as saying something like "every function outputs the true for every input".

And it won't do to modify the syntactic version, saying that it's impermissible to generalize over logical terminology. Firstly, this brings the demarcation problem back into the mix. Secondly, it *is* permissible. Law V can be generalized from

$$\acute{\varepsilon} f(\varepsilon) = \acute{\varepsilon} g(\varepsilon) = \forall x\big(f(x) = g(x)\big)$$

to the falsehood

$$-\forall \mu_\beta\big[\mu_\beta f(\beta) = \mu_\beta g(\beta) = \forall x\big(f(x) = g(x)\big)\big]$$

[14] Again, Proops's main concern is Russell, not Frege—he's arguing against a claim by Peter Sullivan that Russell took logic to be maximally general in the syntactic sense. However, it's instructive to see why one can't attribute the syntactic version of maximal generality to Frege—it means that the problems with the vocabulary version are problems we're stuck with, if maximality plays a central role in Frege's conception of logic. Confusingly, Richard Heck calls the sort of maximal generality which Goldfarb and Ricketts expound the "syntactic interpretation of Frege's conception of logic", though he immediately proceeds to explain that the syntactic interpretation isn't really syntactic (2012, 35).

And Law VI can be generalized from

$$a =\backslash\acute{\varepsilon}(a = \varepsilon)$$

to the falsehood

$$-\forall\mathfrak{f}\big[a = \mathfrak{f}\big(\acute{\varepsilon}(a = \varepsilon)\big)\big]$$

As tempting as the prospect is, Frege's basic laws in *Grundgesetze* just aren't maximally general in the syntactic sense[15]. This conception is *far* more promising when we look at *Begriffsschrift*, when Frege hadn't yet taken on his strange commitment that sentences name truth-values, objects which may (and must) be arguments for functions. If Frege dispensed with that commitment, laws I-IV would be maximally general in the syntactic sense—generalizing upon them would yield expressions like '$\vdash \forall\mathfrak{f}\ \mathfrak{f}(a)$', which could no longer be characterized as saying that all functions yield the True for every input. However, laws V and VI would still be generalizable. It seems that the syntactic notion of maximal generality is unworkable if we want logical objects.

Something important to note about maximal generality is that regardless of which version of it we adopt, it can never be a *sufficient* condition for logicality (although it may be a sufficient condition for the logicality of *basic* laws). Richard Heck argues decisively for this point with a pair of clever examples.

'$\exists x \exists y (x \neq y)$'

is maximally general, and a logical truth as well, since we can prove it from existentially quantifying over '1' and '2' in the logical truth '$1 \neq 2$'. But we don't know '$\exists x \exists y (x \neq y)$' is logical *because* it's maximally general—after all, even if one allowed for no logical objects, we could know on empirical grounds that more than one thing exists. And what's more,

[15] One might try to deny that expressions like '$\forall\mathfrak{f}\big[a = \mathfrak{f}\big(\acute{\varepsilon}(a = \varepsilon)\big)\big]$' are well-formed. In (1893, §30), Frege says that there are two ways of forming names. The first is filling an argument-place in a function-name with an argument of the same type as the argument place—this can give us an object name or a function name, depending on whether it's a unary or binary function-name whose argument place the argument is filling. The second way is taking a previously formed name, and omitting from it some or all occurrences of a particular *proper name*, in order to form a *first-level function name*. Frege says *all* names are thus formed. But we could only form '$\forall\mathfrak{f}\big[a = \mathfrak{f}\big(\acute{\varepsilon}(a = \varepsilon)\big)\big]$' by omitting a *function* name, forming the name of a *second-level* function, which we then put in the argument place of '$\forall\mathfrak{f}\ \mu_\beta\ \mathfrak{f}(\beta)$'. By §30, we just can't form things like '$\forall\mathfrak{f}\big[a = \mathfrak{f}\big(\acute{\varepsilon}(a = \varepsilon)\big)\big]$'.

The response is that §30 simply clashes with Frege's actual practice in forming names. Consider *any* formula formed by putting a complex second-level function name into the argument place of the second-order quantifier '$\forall\mathfrak{f}\ \mu_\beta\ \mathfrak{f}(\beta)$'. *Grundgesetze* is littered with examples. To choose just one, I'll consider a relative of Frege's law III (1893, §20):

'$\vdash g(a = b) \supset g\Big(\forall\mathfrak{f}\big(\mathfrak{f}(a) \supset \mathfrak{f}(b)\big)\Big)$'

By successive applications of roman-letter elimination, we can replace the 'a's and 'b's with some name Δ, and '$\forall\mathfrak{f}\big(\mathfrak{f}(\Delta) \supset \mathfrak{f}(\Delta)\big)$' will be a subformula of the result. As required, '$\forall\mathfrak{f}\big(\mathfrak{f}(\Delta) \supset \mathfrak{f}(\Delta)\big)$' could only have been formed by putting a complex second-level function name '$\Phi(\Delta) \supset \Phi(\Delta)$' into the argument place of the second-order quantifier '$\forall\mathfrak{f}\ \mu_\beta\ \mathfrak{f}(\beta)$'. But by §30, '$\Phi(\Delta) \supset \Phi(\Delta)$' *can't* be well-formed! It can't be the result of omitting two occurrences of a first-level function from a proper name, since we can only form first-level function names through omission of proper names. And we can't form it from '$\Phi(\Delta)$' and '$\xi \supset \xi$', because '$\Phi(\Delta)$' doesn't fit the argument places—'$\xi \supset \xi$' takes proper names as arguments, and '$\Phi(\Delta)$' is not a proper name. So we *must* allow other means for forming names, and once we do, we see that generalizations of Frege's basic laws like the ones I provided are perfectly legitimate expressions.

‘$\exists x \forall F\big(x \neq \acute{\varepsilon}F(\varepsilon)\big)$’

which asserts the existence of an object which is not an value-range, is not a logical truth[16], even though it’s at least as maximally general as law V is (2012, 36).

I’ll have a bit more to say about maximal generality, and the demarcation problem, in §5.4 below.

§4. Heuristic indicator 2: conceptual undeniability

I’ve argued that Frege took logic to be general—it is universally applicable and all-encompassing. I’ve also shown how these two sorts of generality are, for Frege at least, intertwined—logic is universally applicable because it’s all-encompassing. In this chapter, I will examine the sense in Frege took basic logical laws to be undeniable, and will explain how it ties together, in a natural way, with the claim that logic must apply to all thinking.

The laws of logic are prescriptive, in a sense—in Frege’s parlance, they are the laws of being true rather than the laws of our taking to be true. However, as discussed earlier, he’s quite clear that this prescriptive nature alone isn’t what set logical laws apart—*any* true descriptive law, even contingent physical laws, can be read as demanding that one thinks in accordance with them (1893, xv). But Frege also suggests that logical truths are those the denial of which results in complete confusion—as we’ve seen, in *Grundlagen*, Frege says that the fact that we can assume the contrary of various geometrical axioms shows us that these axioms are independent of the primitive laws of logic (1884, §14). Frege’s discussion in *Grundgesetze* is more tentative—he says that:

> Stepping outside logic, one can say: our nature and external circumstances force us to judge, and when we judge we cannot discard this law—of identity, for example—but have to acknowledge it if we do not want to lead our thinking into confusion and in the end abandon judgement altogether. I neither want to dispute nor endorse this opinion, but merely not that what we have here is not a logical conclusion. What is offered here is not a ground of being true but of our taking to be true. (Frege 1893, xvii)

At first blush, it looks like Frege restates the same position on the undeniability of logical laws which he held in *Grundlagen*, but then refuses to endorse it, which might be taken to indicate a change in view. Matters aren’t so straightforward, however, as the passage is ambiguously worded. In particular, it’s unclear whether the claim that he’s declining to endorse is 1) that we cannot abandon basic logical principles like the law of identity without abandoning judgement, or 2) that our “nature and external circumstances” have anything to do with this fact. I read the passage as affirming undeniability, but (rightly) refraining from endorsing the psychologistic claim that our nature leads us to endorse certain laws. A point in favor of my reading is that this passage follows a protracted criticism of psychologism.

By taking laws to be undeniable, Frege does not of course mean that beings cannot *verbally* deny a logical law, or even that they cannot think of the logical law that it is the sort of thing that one ought not to hold:

> This impossibility, to which we are subject, of rejecting the law does not prevent us from supposing beings who do so; but it does prevent us from supposing that such beings do so rightly; and it prevents us, moreover, from doubting whether it is we or they who are right (Frege 1893, xvii)

[16] At least on the plausible assumptions that a) value-ranges are the only logical objects Frege recognizes and b) whether or not there exists a non-logical object is not itself a logical matter.

Frege doesn't mean that laws are undeniable *tout court*, but rather that they are *rationally* undeniable—denying them reveals a failure of some sort on the part of the denier. But what sort of failure? Consider someone who denied:

> Basic Law I: $\vdash a \supset (b \supset a)$

One weak sense in which the denier has failed is simply that they're *incorrect*. Basic Law I is true, so in denying it, they violate the "minimal connection" mentioned earlier—the weak bridge principle we ought to refrain from denying truths. If logic is how one ought to reason insofar as one cares about truth, then there's a sense in which Basic Law I is rationally undeniable. But surely *this* can't be all there is to undeniability, since all truths, not just logical ones, are undeniable in this sense. Plus, clearly it's not generally the case that thinking at all is no longer possible when any old truth is denied! Accordingly, if the undeniability of logical law only amounts to upholding the minimal connection, then Frege couldn't use undeniability to make a case for the prima-facie logicality of arithmetical truths, as he does in §14 of *Grundlagen*. Thus, the cognitive failing possessed by someone denying a logical law must be something subtler than mere mistaken belief.

I interpret the sort of undeniability found in Frege's works as a more substantial form of undeniability, which I shall call *conceptual undeniability:*

> Conceptual undeniability: A law ψ is conceptually undeniable if anyone who professes to deny it must have some defect in their grasp of that law.

This is just a first-approximation—there's quite a lot of detail which needs to be filled concerning just what the defect in question is supposed to be. In §2.5 of chapter 3, I'll argue that the defect in grasping the law is concomitant with a misunderstanding of the formula which expresses it. That is, the undeniability of logical laws is explained in terms of their epistemic analyticity. If this is so, it gives a sharper picture of the sense in which one who professes to deny a law is subject to a confusion. If someone denies ψ, then they misunderstand it. If understanding ψ is necessary for judging that the thought which it expresses to be true or false (and clearly we need to take some care clarifying and arguing for this claim), then the reasoner in question has not *cognitively* denied ψ, though they may of course have effected a verbal denial.

Another perk of conceptual undeniability is the way in which it ties in with the all-encompassing nature of logic. Logical laws apply to all thought partly because they are general principles by which one may securely pass from truth to truth, but that's not the *only* reason. If one reasons too illogically—if one is not competent in employing ψ in enough simple inferences—then one simply fails to properly grasp the thoughts with which one is reasoning. Logical laws are what one must follow insofar as one opts in for reasoning at all—one who is utterly incompetent with logical inference just can't be said to be inferring. Thus, logical laws apply to all thinking without exception—logic is all-encompassing. More details to come in the chapter 3.

§5. How the heuristic indicators relate to Frege's case for logicality

§5.1 Some possible accounts

Here's where we're at thus far. In the previous chapter, I established the importance of foundationalism for Frege, and talked about how he reduces questions concerning the logicality of arithmetical theorems to questions concerning the logicality of the basic laws from which they may be proven. In this chapter, I've looked at two necessary conditions for logicality which Frege uses in arguing that arithmetical principles are very likely logical, and have said a bit about what these principles involve. Now, I'll look at whether, for Frege, making a more conclusive case for the logicality of an axiom just involves applying his heuristic indicators to *basic* laws, or whether instead a deeper case can be made for their logicality.

Here's a list possible accounts of Frege's way of sorting logical from non-logical laws:

i. No-method account: Frege gives us nothing principled to say about why a basic law should or should not be taken as logical.
ii. Heuristic Indicator account: A basic law is logical if and only if it's general and undeniable.
iii. Demarcation account: A basic law is logical if it's maximally general in the sense of the vocabulary version—it's a truth expressed with special vocabulary.
iv. E.A. account: Determining that a basic law is logical requires appreciating its epistemic analyticity.

This list is just a rough framework for categorizing accounts of Frege's case for logicality. Some categories contain rather different conceptions—there are very different ways in which one might try to demarcate which terms can appear in maximally general expressions. Also, one shouldn't see the options on this list as mutually exclusive—(iii) could be seen as a specific version of (ii). Also, these accounts shouldn't be understood as attributing to Frege any very *sharp* means of determining logicality—one could hold (ii) or (iv), etc, without attributing to Frege anything like a totally clear and unambiguous method for sorting laws.

§5.2 The no-method account

As mentioned previously, Frege's explicit comments concerning how we determine logicality are few and far between. This might lead one to conclude that Frege just didn't *have* any way of making this determination, even implicitly. There are two broad version of this view, depending on how principled supposes this lack of a method is. Proponents of the elucidatory interpretation argue that Frege had philosophical commitments which precluded him from having a general conception of what being logical amounts to—a Thomas Ricketts puts it, "Frege has only a retail conception of logic, not a wholesale one"(1996, 124). Ricketts takes this to follow from his reading of Frege's arguments on truth—he thinks that Frege's perspective precludes the use of a truth-predicate, and that this in turn implies that there can be no genuine theorizing about logic (1996, 136). On the other hand, one might suppose Frege lacked a method of determining logicality for far less principled reasons—either because he thought it wasn't an important question, or because he thought such a method would be unnecessary, because the status of a basic law would be immediately clear to anyone who properly understood it.

As for the unprincipled version, it simply beggars belief that Frege just didn't think questions concerning logicality of basic laws were important, since questions concerning the status of theorems reduce to questions concerning the status of the unprovable laws upon which they rest, and determining the status of arithmetical theorems is the whole point of logicism. And even if a general conception of logic which motivates a method for determining logicality was unavailable to him, it's difficult to see what could justify confidence that it will just be *obvious* whether a given basic law is logical, especially if, as seems plausible, the doubts concerning law V which Frege anticipates in his foreword to *Grundgesetze* concern not the truth, but the logicality of the principle (Frege 1893, vii). I share a concern raised by Heck—if there's *nothing* we can say for or against the logicality of a principle, then there's a danger that logicism becomes a merely verbal doctrine, a suggestion that we call certain laws "logical" (2012, 34).

I don't want to attribute the "no-method view" to proponents of the elucidatory interpretation. If one supposes Frege had no means of determining logicality, and thought it would just be obvious, the best way to make this view plausible might be to connect it to the view that Frege had no general conception of logic. But one can hold the latter view without holding the former—one can deny that Frege had a general conception of logic, but still think there's some useful conceptual legwork that could be done to jolt us into recognition of a law's logicality. Accordingly, I think that the heuristic indicator method and the syntactic criterion provide the most charitable reconstructions of the position held by proponents of the elucidatory interpretation. The sort of logicism allowed by the no-method view would be easier to establish, but rather less interesting than the genuine article.

§5.3 The heuristic indicator account

According to what I'm calling the heuristic indicator view, we simply see whether the heuristic indicators of logicality—generality and undeniability—are present in the basic laws. This view is held most explicitly by Joan Weiner:

> To see how this might work, consider an example: the claim that every object is identical to itself. Since its truth is self-evident, it satisfies the first eligibility requirement for primitive laws. Supposing this to be a primitive law, is it analytic? In *Grundlagen*, Frege mentions two features of analytic truths. One is maximal generality…Another is that we cannot deny them in conceptual thought….The law that every object is identical to itself exemplifies both of these features. First, this law surely tells us, not just about every actual (spatio-temporal) object or every intuitable object, but about *every* object. Second, it seems we cannot deny it without involving ourselves in contradictions. Given these criteria, the law in question is analytic. (Weiner 2010, 34-5)

Weiner explicitly lists generality and undeniability as necessary conditions of logicality. Note that by "maximal generality", she just means logical laws tells us something about all objects, rather than just about spatio-temporal or intuitable ones—this is closer to logic as universally applicable and all-encompassing than it is to maximal generality in the narrower sense discussed in §3.5 above.

Applying the heuristic indicators to basic laws is supposed to be more conclusive than the preliminary arguments Frege provides in §14 of *Grundlagen* and in "Über formale Theorien der Arithmetik", because it's supposed to be more *obvious* that the heuristic indicators apply to basic laws than it is that they apply to theorems:

> On Frege's understanding of primitiveness, primitive truths must be unprovable and the status of primitive truths (i.e., whether they are analytic or synthetic, a priori or a posteriori) must be obvious. But, except for instances of logical laws, which are provable, no truths of arithmetic are obviously logical. (Weiner 1990, 42)

For example, arithmetical principles *seem* general insofar as everything can be counted, but aren't obviously general, since they appear to be concerned specifically with numbers. The claim that every object is identical to itself, on the other hand, is obviously a general claim[17]. (Weiner 2010, 35).

This is only a *method* for determining logicality in the broadest sense of the word "method". Due to the role of obviousness, it provides no very sharp criteria for when a basic law should be counted as logical. Weiner crucially assimilates questions concerning the source of the evidence of logical laws to questions concerning their ultimate ground of justification (1990, 71-2). Since, as she repeatedly emphasizes, basic laws, by virtue of their unprovability, admit of no justification, questions concerning the source of their evidence are unanswerable (1990, 72-3). So, no explanation for why we're justified in taking these laws to be logical can rest on an explanation (such as epistemic analyticity, my own favored account) for why primitive logical laws are self-evident—noting the presence or absence of the heuristic

[17] The reflexivity of identity is taken as a logical axiom in *Begriffsschrift* (1879, §21) but not in *Grundgesetze* (1893, §47). This doesn't mean Frege changed his mind about whether the law was basic—as argued in §1.3 of the previous chapter, Frege thought that all axioms were basic laws, but not all basic laws are axioms.

indicators is all we have at our disposal. Nevertheless, the connection with the heuristic indicators gives one *something* to go on—it's not as if "logical" is a complete black box, an ascription we give to primitive laws with no conception of what we're ascribing to them.

There are a number of worries concerning the heuristic indicator strategy. The first is that the strategy above puts too much emphasis on obviousness—as I argued in chapter 1, one of the ways in which Frege's approach to the foundations is distinctive is the extent to which he eschews any special appeals to obviousness. He insists that inferences not appeal to obviousness, since its nature and laws are unknown (Frege 1902, 57)—in particular, psychological facts likely play some role in determining what seems obvious to us. This comment does not appear in a context in which he is discussing his foundations, but it would be strange if he eschewed any role for psychologistic considerations in inferring, but then appealed to them when it came to determining logicality. And Weiner is elsewhere explicitly opposed to psychologistic considerations when it comes to dealing with Frege's foundations (1990, 61). In any case, her example of the reflexivity of identity is cherry-picked, because it's one of the rare examples of a principle for which it *is* immediately obvious that it's both general and conceptually undeniable. For Frege's basic laws in *Grundgesetze*, it takes a bit of conceptual legwork and thinking through their content to see them as true, and thus presumably to see them as logical.

Weiner is quite aware that that our recognition of the logicality of primitive truths is not invariably immediate. She thinks that elucidations can play the role of getting us sufficiently acquainted with Frege's language, enabling us to grasp primitive laws distinctly enough to see that they're true –presumably, we then recognize them a logical as well, since Weiner allows for no deeper case to be made for the logicality of these laws (2005, 340-1). However, even with this clarification, there are additional problems with the heuristic indicator method. If the only evidence that we have that a law is logical is the fact that it's general and conceptually undeniable, it's unclear why Frege needs his logicism at all. The interest of the logicist thesis is proportional to how substantial a claim one makes when one says that a principle is logical. Here are two things one might want logicism to accomplish—to explain the source of the a priority of arithmetic, and to show that there's a deep and interesting distinction between arithmetic and geometry. Demonstrating that arithmetic can be proven from general and conceptually undeniable basic laws meets both these goals quite independently of whether we take them as signs that these laws are *logical*—showing that arithmetic is conceptually undeniable goes some way to explaining the source of its a priority, and both conceptual undeniability and generality serve to show arithmetical principles are importantly different from geometric ones. So why do questions of logicality need to enter the picture at all? The pressing worry here is that heuristic indicator strategy makes logicism a less important, and less interesting, claim.

The view I present in chapters 3 and 4 is fashioned to get around these two problems. If we take logical laws as epistemically analytic, the resultant account is not over reliant upon obviousness. Also, the epistemic analyticity of logic allows one to explain why logical principles are general and conceptually undeniable. The result is that we're left with a deeper explanation of logicality—generality and undeniability aren't quite separate conditions, but have their root in a common source. And this conception of why we're justified in taking Frege's basic laws as logical is not selected *merely* because it motivates the heuristic indicators—it also makes sense of Frege's comments concerning truth, and gives us a connection between logic and normativity which is stronger than the minimal connection mentioned above.

§5.4 The demarcation account

The demarcation account of Frege's way of determining logicality gives the vocabulary version of maximal generality pride of place. For those who take logic to be maximally general in this sense, it's a necessary condition for a basic law to be logical that expressions for them only contain certain general vocabulary. Recall that the *demarcation problem* is determining just what this class of terms is.

Not every proponent of the demarcation account holds that Frege can say anything principled about how to make this determination. Although he takes logic to be maximally general, Thomas Ricketts also denies that Frege has any general, overarching conception of logic (1996, 124). Goldfarb also explicitly rejects that Frege's commitment to the maximal generality of basic logical laws could be sharpened into a more exact criterion for logicality (2010, 72-4). No general solution is available for the demarcation problem—all that's left is to hope that judgements concerning logicality will be sufficiently unobjectionable in individual cases[18]. This makes Goldfarb and Ricketts's version of the demarcation account similar to Weiner's heuristic indicator account, which isn't surprising, considering that all three writers are proponents of the elucidatory view of Frege, and thus maintain that Frege could make no serious use of any of the metatheoretical tools one might employ to give a sharper conception of logicality. The difference from Weiner is that Goldfarb and Ricketts put most emphasis upon *one* of the two heuristic indicators—we prove theorems from basic laws, eyeball whether the basic laws are maximally general, and on that basis reach a decision regarding their logicality. This runs into some of the same worries as Weiner's heuristic indicator account—it seems to leave far too much up to guesswork. If Goldfarb, Ricketts, and Weiner are correct that Frege's views on truth preclude any semantic theorizing, then such guesswork may be unavoidable. We should, however, prefer a more fleshed out account if one is possible—in chapter 4 I'll argue that it *is* possible.

If the demarcation problem is one which can be intelligibly posed and approached, two strategies for resolving it suggest themselves. The first is a broadly pragmatic strategy. Ricketts says maximally general truths contain only "universal vocabulary required to express the results of any science", and Goldfarb says something similar (Ricketts 2010, 151; Goldfarb 2010, 68). It's fairly obvious that to describe any laws of nature, we require quantifiers and conditionals. It's not so obvious that we need to use Frege's value-range operator in any perspicuous representation of the thoughts of any given science. The only way to find out is to go about formalizing sciences in Frege's concept-script—those expressions we find indispensable in all such formalizations are the "universal vocabulary" of which expressions of basic logical truths are formed. In particular, if numbers can only be defined in terms of extensions, and numbers prove indispensable to any formalization of any science, then extensions are suitably universal. The resultant view would be a bit like that of W.V.O. Quine's, only where indispensability is a sign of the special nature of a class of terms, rather than of the existence of a class of objects (Quine 1981).

The chief worry for this strategy is that it's not clear that such indispensability considerations are implicit in Frege. Granted, he insists that law V is implicit in the work of mathematicians and logicians (1893, vii; 1903a, §147; 1893, §9), but we find no justification for the claim that names of value-ranges must appear in the formalization of *any* science. In *Begriffsschrift*, Frege boldly claims that his concept-script "borders all other" formalizations of particular sciences (1879, 6-7), but he does little to justify this claim (plausible though it may be). Even if Burge (1990; 1998) is correct about there being pragmatic considerations which run throughout Frege's work, indispensability is not one such pragmatic consideration.

[18] Goldfarb suggests that there still remains an experimental question of whether or not our basic laws "suffice to derive all the particular results that we have set ourselves to derive" (201, 72-3), and cites a comment of Frege's that if we happen upon a truth ψ which seems unprovable, this might lead us to acknowledge a new inference (the alternatives would be see ψ as non-logical or just false) (1897b, 363/CP 235).

Heck tentatively suggests a very different solution to the demarcation problem. He thinks maximally general truths should be ones which contain only logical terminology, and thinks it quite unlikely that Frege could have provided any principled distinction between logical and non-logical terminology without making use of semantics (Heck 2012, 36). Elsewhere, he goes into more detail. His "hunch, and it is just a hunch" is that one motivation for Frege investigating semantic notions is that he was developing a conception of logic in which they would play a fundamental role (2010, 357). He sees Frege's articles in the early 1890's as arguing that semantic notions are central to any account of understanding, judgement, and assertion. So, showing that negation, conditionals, and quantifiers are explicable in terms of these semantic notions would show that they're "available to anyone able to think and reason", and "implicit in our capacity for thought" (2010, 357).

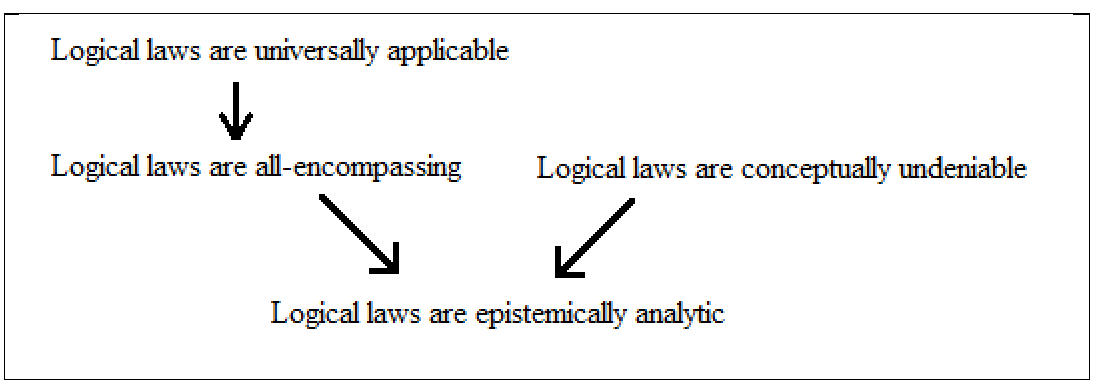
This tentative view which Heck sketches is incomplete as it stands. Firstly, as Heck himself acknowledges, his view wouldn't help explain why Frege took law V to be logical, since "the notion of a value-range does not seem to be fundamental to thought in this way"(2010, 357). The second problem is something that is shared by all versions of the demarcation account—it does not give conceptual undeniability its due. Heck confesses that he has no account to give of those places, such as the foreword of *Grundgesetze*, where Frege says we must acknowledge logical laws as true, less we fall into confusion, and abandon judgement altogether (Frege 1893, xvii; Heck 2012, 38).

The E.A. view handily fills these two lacunae.

§5.5 The E.A. account, and its relation to the demarcation account

According to the E.A. account, Frege took epistemic analyticity to be a necessary condition for a basic law to be logical. In his arguments for his basic laws, he's aiming to elicit a reflective awareness of the fact that understanding one of his basic laws is sufficient for recognizing it as true. We can't conceptually deny a logical law, because to do so would show we've misunderstood it. The logicality of law V can be explained as well—Frege thought that anyone who denies it must have misunderstood what the stipulation governing his smooth-breathing sign demands[19]. It will fall to the next two chapters to describe and argue for this view in detail, and provide the many needed qualifications—at present, I will limit myself to outlining the ways in which it may relate to the demarcation account, which I take to be the most plausible alternative to the E.A. account.

One of the aims of the E.A. account is to explain Frege's two heuristic indicators. In the diagram below, $X \rightarrow Y$ means that I explain or gloss X in terms of Y



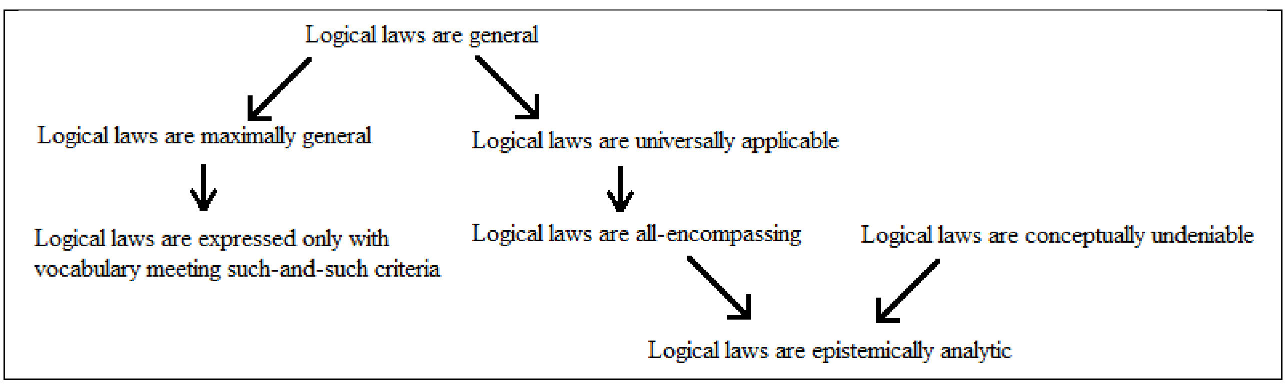
[19] Clearly something went wrong with law V—his stipulation governing law V is crucially different from his stipulations governing his other logical laws.

My account glosses generality in terms of universal applicability, and explains universal applicability in terms of logic's nature as all-encompassing. Logic as all-encompasssing and logic as conceptually undeniable are then connected, motivated, and explained via my claim that epistemic analyticity is a necessary condition for the logicality of a basic law.

A decision must then be made as to whether epistemic analyticity is a *sufficient* condition for the logicality of a basic law. Here are two reasons why one who agrees that epistemic analyticity is a necessary condition for logicality might deny that it's a sufficient condition. The first worry is that taking it to be a sufficient condition over-generates—sentences like "all bachelors are unmarried" and "I am here now" have been taken to be epistemically analytic, and they're far cry from anything Frege explicitly labels as logical. I'll address this worry in §4.3 of chapter 4. Another (and very much related) worry is that the E.A. account alone does not give maximal generality its due.

This second worry could be countered by arguing that maximal generality just isn't as important to Frege as one might have thought. We can see this by seeing how little we give up by de-emphasizing it. Firstly, to deny the importance of maximal generality isn't to deny one of its main consequences, the content claim—we can still grant that logical laws sit squarely at object level. Secondly, we needn't give up the claim that Frege thought some concepts and relations were proper to logic—he clearly thinks that there are (1906a, 428/CP 338;1885a, 96/CP 114), but that's different from characterizing basic logical laws as those basic laws expressible by formulas containing only expressions for such relations. Finally, denying the importance of maximal generality isn't even to deny that Frege took his basic logical laws to be maximally general. In §1.5 of the previous chapter, I argued that pragmatic considerations of fruitfulness, simplicity, elegance, etc are important elements of determining which principles are basic. Such considerations will frequently—though not invariably—place maximally general truth at the beginning of the deductive chain. Insofar as is possible, it's more systematic, simple, and elegant to have maximally general basic laws from which one can derive more particular principles—this does not mean that these laws are selected *because* they're maximally general.

If none of his strikes the reader as convincing, the most plausible alternative is a hybrid account which combines the best features of the demarcation and E.A. accounts. The result would look as follows:



While it might be extensionally adequate to characterize basic logical laws as basic maximally general truths, extensional adequacy does not make for a full explanation—augmenting the demarcation account with the E.A. account results in a view which explains the conceptual undeniability of logic, while also preventing overgeneration of truths labeled as logical.

One worry about the hybrid account is that it looks quite disjunctive. In particular, why should we expect maximally general basic truths to be epistemically analytic? This question does not admit of a fully satisfactory answer, largely due to the absence, in Frege's writings, of a worked-out solution to the demarcation problem. However, if Richard Heck is correct in supposing that Frege was tentatively inching towards a semantic solution to this problem, there are some very suggestive (if highly speculative) reasons for supposing that the E.A. account and Heck's version of the demarcation account are far more deeply intertwined than is first apparent. After my general arguments for the E.A. account in chapter 3, in chapter 4 I lay out some clarifications to the effect that a reflective awareness of epistemic analyticity involves explicit appreciation of the role stipulations play in securing our understanding of the basic laws. In Heck's speculative Fregean solution to the demarcation problem, logical terms are those explicable in terms of semantic concepts implicitly grasped by anyone who can think and reason. Depending on how this view is precisified, we may expect the demarcation account and the E.A. account to converge and connect in those places where acceptable stipulations—those which, in terms of appropriately semantic notions, properly secure a reference for, and our understanding of, the terms they elucidate—are distinguished from unacceptable ones.

Unfortunately, from an interpretive standpoint, one cannot, aside from these tantalizing possibilities, say more about how the demarcation and E.A. accounts connect. There's no shortage of ways in which they might be organically connected, but as far as accounting for Frege's treatment of logicality is concerned, it would be foolhardy to see him as committed to any very specific such connection, given how little Frege says on the matter. Rational reconstruction is one thing, and puppetry is another. What's clear enough, however, is that what the E.A. account and Heck's spin on the demarcation account have in common is an appreciation of the role of semantics in Frege, and a confidence that the lack of a fully explicit and fleshed out account of logicality is not a sign that he thought that questions concerning the demarcation of logical laws, notions, and terminology were unintelligible, but instead due to the fact that he was wrestling with extraordinarily difficult problems.

§6. Conclusion

To sum up: Frege thought generality and undeniability were heuristic indicators for logicality. They're a rule of thumb which allows us to make preliminary judgements concerning the logicality of statements, pending deeper investigation. The role of the heuristic indicators becomes less clear once we reach the foundations—I've argued that we should see Frege as doing more than just seeing whether the basic laws are general and undeniable, and concluding that they, and all theorems from which they may be derived, are logical. It would be preferable to have an account that ties together generality and undeniability, and shows how they're rooted in a common source. The E.A. interpretation promises to be such an account, and it is to this account which I now turn.

Chapter 3: The E.A. Interpretation I: Inference and Understanding

§0

In chapter 2, I argued that Frege took logical truths to be general and undeniable, and showed how he used these as heuristic indicators to make a case for the prima facie logicality of arithmetical truths (and for the non-logicality of geometrical truths). I then argued for two important claims. Firstly, these indicators deeply connected, and this connection needs explaining. Secondly, to make a more conclusive case for the logicality of a principle, Frege needs more than the mere application of his indicators—he needs a deeper explanation for why one should take his axioms to be logical. The purpose of this chapter will be to flesh out a conception of logic which is plausibly attributable to Frege and which is also sufficient to motivate Frege's claim that logic is all-encompassing and conceptually undeniable. In the chapter 4, I'll show how this conception allows for a deeper explanation for why Frege takes some truths but not others to be logical. According to the reconstruction I'm pushing, Frege took basic logical laws to be epistemically analytic, and took there to be inferential constraints upon understanding: if we're not competent in making certain inferences involving a sentence, we don't understand that sentence.

The structure of this chapter is as follows. In §1, I'll establish the claim that, for Frege, logical laws are implicit in our grasp of thoughts and in our understanding of formulas. To do this, I'll argue that logical laws are implicit in our grasp of truth-values, and that this implies that these laws are implicit in judgement, and finally that this in turn implies that they're implicit in understanding itself. In §2, I'll lay out a particular picture of how logical laws connect to understanding— implicitly appreciating what the referents of expressions contribute to the truth-value of a given sentence in which those expressions occur must involve competence in using this sentence, and its subexpressions, in certain logically valid inferences. I'll then show how this picture explains the generality and undeniability of logic.

§1. Truth, judgement, and understanding

§1.1

In §1 of chapter 2, I spent some time discussing the relation between logic, truth, and normativity—the upshot is that we need to account for the sense in which Frege took logic to be normative. It's obvious that he thought his logical laws were *at least* principles which allow us us avoid falsehood and inconsistency, if this is our goal—it was this non-categorical normative role which I called the *minimal connection.* But, as argued in the aforementioned section of chapter 2, the connection between logic and normativity should be deeper than this, as the minimal connection doesn't give us something *distinctive* about logic—as Frege himself notes, *any* true thought, logical or not, is such that one ought to think in accordance with it (1893, xv). A connected issue is my argument for conceptual undeniability—if the minimal connection was all there was to undeniability, Frege's use of such undeniability as a heuristic indicator of logicality wouldn't have made sense. Previously, I touched briefly on the suggestion that giving an account of the way in which logical laws connect to judgement and understanding will provide a sense, over and above the minimal connection, in which logic can be said to be normative. Before moving on to the way in which such a picture motivates the indicators, I shall now spend some time arguing in more detail that Frege thought logical laws are deeply connected to understanding.

§1.2 Our implicit grasp of truth

Let's return to the quotes of Frege's, discussed previously in §1 of chapter 2, which most strongly suggest this deeper, more interesting connection between logic and normativity. In his 1918 article "Thought", Frege says:

> The *Bedeutung* of the word 'true' [*wahr*] is spelled out in the laws of truth. (Frege 1918a, 59; FR 326)

The surrounding context doesn't do a great deal to shed light on why this quote is so distinctively interesting—it follows after an opening paragraph where Frege, after a few characteristically harsh words about psychologism, says that it falls to logic to discover the laws of truth. All of this is quite consistent with the minimal connection. However, a similar quote appears several decades earlier. In 'Logic', a fragment of an unfinished textbook written sometime between 1879 and 1891, Frege says:

> It would not perhaps be beside the mark to say that the laws of logic are nothing other than an unfolding of the content of the word 'true'. Anyone who has failed to grasp the meaning of this word—what marks it off from others—cannot attain to any clear idea of what the task of logic is (Frege 1879-1891, 3)

The context of this quote is more helpful. In the immediately preceding paragraphs, Frege says:

> Now the grounds which justify the recognition of a truth often reside in other truths which have already been recognized. But if there are any truths recognized by us at all, this cannot be the only form that justification takes. There must be judgements whose justification rests on something else, if they stand in need of justification at all.
>
> And this is where epistemology comes in. Logic is concerned only with those grounds of judgement which are truths. (Frege 1879-1891, 3)

Frege's concern here is what we ought to say when the chain of justification comes to an end—we thus ought to expect the succeeding comment about logic unfolding the meaning of "true" to bear on the present question of what we ought to say about the logicality of primitive truths.

Since Frege repeats, decades apart, this claim that logical laws unfold the content of "true", it appears to be an idea of some importance. But what relation between logical laws and truth do these informal turns of phrase point towards? Logical laws certainly don't *define* truth—Frege repeatedly emphasizes truth's indefinability (1897a, 126; 1979, 174; 1918a, 60/FR 327). Nor are such laws *about* truth, at least not explicitly—as Joan Weiner has emphasized, not one of the primitive logical laws Frege gives us in either *Begriffsschrift* or *Grundgesetze* contains a truth predicate or operator[1]. In the 1918 quote about logical laws spelling out the meaning of truth, Frege uses the term "Bedeutung", but it's quite unlikely that he meant this term to be understood in its technical sense in the passage in question[2].

[1] We would be amiss to take Frege's horizontal stroke as a truth-predicate—it maps objects to truth-values, whereas Frege is quite clear that genuine truth predicates would predicate truth of thoughts. Such predicates do so either directly, or else indirectly, as we do when we call a verbal or written sentence true or false.

[2] If "is true" is a predicate of thoughts, then the *Bedeutung* of "is true" is a function $T(x)$ which maps all true thoughts to the True, and everything else to the False. One possible (but very bad) reading of what Frege means by saying that logical laws "spell out" or "unfold" the content of "is true" is that such laws, in leading from truth to truth, help us to discover new truths, and hence to know more about what thoughts $T(x)$ maps to the True. But if

One illuminating possibility is that Frege thought logical laws to spell out the *sense* of "true". Of course, Frege does explicitly say in several places that "is true" adds nothing to thoughts of which it is predicated, and this might lead us to conclude that "truth" is somehow empty or that the content it possesses is somehow insubstantial. But for Frege, this conclusion would get things very much the wrong way around, as he explicitly holds that predicating truth is always involved in predicating anything whatsoever (1897a, 128). The impression one gets is that truth is quite substantial, but that its simplicity and primitiveness prevents us from specifying the content of "is true" explicitly. Perhaps logical laws somehow implicitly illuminate the word's sense—one possibility is that Frege took logical laws to be integral to any account of what it is to implicitly grasp the sense of "true".

As tempting as one may find the possibility that the quotes in question concern the sense of "true", this interpretation faces several complications. Firstly, it's unclear whether Frege had developed the sense/reference distinction by the time his quote about laws unfolding meaning appears in *Logic*. The distinction first appears in print in 1891, but may well have been developed a few years earlier, whereas *Logic* was written as early as 1879 or as late as 1891. If he hadn't hit on the technical notion of sense before *Logic* was written, he clearly didn't intend to say there that logical laws unfold the sense of "is true". And even if he had developed the sense/reference distinction by then, it's clear that the quote on unfolding the meaning of the word "true" is meant only informally—if he *had* intended the comment to concern sense specifically, we would have expected Frege to just *say so* rather than resorting to a potentially misleading turn of phrase. But even if Frege didn't have a technical notion of content in mind when he spoke of content of "true", much of the above discussion still stands—logical laws tell us something about truth, and since truth is undefinable, just what truth consists in can't be given explicitly. All that remains, then, is for logical laws to bring to the surface what is *implicit* in our grasp of truth.

§1.3 Our implicit grasp of truth-values

This conclusion, that logical laws unfold what is implicit in our grasp of truth, would certainly fit well with the previous observation that logical laws contain no explicit truth predicates. Taking this line also keeps the account from putting too much emphasis on Frege's use of truth as a predicate. Frege harbored doubts about whether "is true" is a genuine property (1918a, 61-2/FR 328-9). In fact, Frege evidently took truth-values to be conceptually prior to truth-predicates. He suggests that the relation between a thought and a truth-value is that of sense to a referent, rather than of subject to predicate (1892a, 34/FR 158). Elsewhere, reflecting upon how truth contributes nothing to the sense of whole sentences in which it occurs *as a predicate,* he says:

> But it is precisely for this reason that this word seems fitted to indicate the essence of logic. Because of the particular sense that it carried any other adjective would be less suitable for this purpose. So the word 'true' seems to make the impossible possible: it allows what corresponds to the assertoric force to assume the form of a contribution to the thought. And although this attempt miscarries, or rather through the very fact that it miscarries, it indicates what is characteristic of logic. (Frege 1915, 252)

Truth-predicates point at the essence of logic, but truth-predicates "miscarry" in their attempt to indicate assertoric force (just what one should make of this "miscarrying" will be discussed in §1.5 of chapter 5). Truth-values, on the other hand, are peculiarly fundamental for Frege. One way to see this is by attending to the tight connection that holds between truth-values and some of the most fundamental notions Frege

---

this is what Frege meant, then *any* science, insofar as it helps us arrive at new truths, tells us about the *Bedeutung* of "is true".

uses in expounding his conception of logic. Concepts are quite central to Frege's picture of logic[3], and from 1891 onwards he characterizes them in terms of truth-values—he first introduces functions as unsaturated entities formed by uniformly omitting certain (perhaps recurring) expressions in a formula, and then characterizes concepts as just those functions which yield a truth-value regardless of what values are given as input. This isn't intended as a definition of what it is to be a concept—Frege explicitly held concept-hood to be logically simple and indefinable (1892b, 193/FR 182). Nevertheless, the fact that concepts are tied to truth-values in this way gives them pride of place (indeed, Frege's peculiar claim that sentences refer to truth-values falls out of this characterization of concepts).

This connection between truth-values and concepts also gives us another way to see that Frege must have taken truth-values as explanatorily prior to truth-predicates. Suppose for the moment that Frege's view allowed for a truth-predicate at all[4]. Predicates stand for unsaturated referents—in this case, they'll designate concepts that take thoughts (or sentences, though Frege was adamant that thoughts are the primary truth-bearers) as arguments, and output *truth-values*. The meanings of truth-predicates are just a special case of concepts, which are, as we saw, characterized in terms of truth-values.

If truth-values are more fundamental than truth-predicates (if Frege's view has a place for the latter at all), this raises the possibility that what it is for logical laws to make explicit what is implicit in our grasp of truth is for them to spell out what is implicit in our grasp of truth-*values*. To see what it is to have an implicit grasp of such values, we must turn to judgement[5].

§1.4 From truth to judgement

Like concept-hood, Frege takes judgement to be a notion which, while indefinable, is nevertheless capable of characterization in terms of truth-values. Frege says that to judge is to advance from a thought to a truth-value (1892a, 35/FR 159). Frege also insists that truth-values are *implicitly* recognized by anyone who judges (Frege 1892a, 34/FR 158). Since truth-values are apparently more central to his picture of truth than truth-predicates, and we get at truth-values through judgement, this suggests that the act of judgement, on Frege's view, ought to be explanatorily prior to his truth-predicate (if he allowed for such a predicate). Putting these considerations together with the conclusion of the previous paragraph, we arrive at the following connection: we implicitly grasp the content of "true" via our competence with judgement. If this is so, then to grasp the meaning of "true" need not necessarily involve employing the *word* "true" correctly, but *judging* correctly—to judge, for example the truth of A and the truth of B on the basis of A and B, where such judgement involves no use of the word "true".

In the picture given here, our grasp of truth *consists*, at least partially, in competence with judgement—when I say it is through competence in judgement that we come to grasp the content of

[3] There is, of course, unclarity in just what it means to say that concepts are central to Frege's view—Ricketts and Weiner would opine that the concept/object distinction, while important, plays a rhetorical role in getting us to understand how his language functions, and doesn't have any deep extra-linguistic significance. But whether we take this line or not, Frege took it as fairly fundamental. When, after discovering that the inconsistency of *Grundgesetze* admitted of no easy fix, Frege took stock of what he took to be his most central insights he says his achievements are almost all tied up with "a concept construed as a function…unsaturatedness both in the case of concepts and functions. The true nature of concept and function recognized" (1906, 184).

[4] There's some question whether or not his perspective meant he could make no serious of a truth-predicate. This is another issue I'll get into in more detail in §1.5 of chapter 5.

[5] Of course, Frege only started writing about truth-values around the same time that his sense/reference distinction appeared in print. Thus, just as it's not clear Frege could have intended (1879-1891, 3) to concern the technical notion of sense, it's for the same reason unclear whether he could have intended it to concern the truth-values specifically. Accordingly, my claim is not a narrow exegetical one of what Frege literally meant by his informal bit about laws unfolding content—it's instead a broader claim about the conception of the relation between truth, judgement, and logical laws that this quote points to.

"true", this shouldn't be taken to mean that the competence is some sort of proximate cause of the grasp in question. It should also be clear that this claim does not amount to an attempt to give our grasp of truth some sort of purely behavioral grounding, at least if one keeps in mind that our practices of judgement are not mere behaviors—they are behaviors with *content*, the aim of which is truth. The content is, for Frege, what is of primary interest.

§1.5 From judgement to understanding

To explain how it is that logical laws enter the picture, and to bolster the claim that our implicit grasp of truth consists in competence with judgement, we must connect judgement with understanding—it will then be the task of §2 below to explain why it is that logical laws must form an integral part of any Fregean account of understanding, hence of judgement, and hence of our grasp of truth.

What's understood—sentences, thoughts, or both? One answer is that only sentences are understood—thoughts are not *understood*, but *grasped*. But perhaps this answer makes too much of a verbal distinction—surely one could just as well say that to understand a thought *just is* to grasp it. Frege himself suggests that his primary reason for preferring to speak of "grasping" content is just to emphasize that the content is already there, independently of us, to be grasped (1893, 24). To avoid possible confusion, I'll reserve "understanding" for sentences and "grasping" for thoughts.

Getting clear about the distinction on the outset is important here, and allows one to distinguish between two different possible versions of my account:

> Linguistic version: Frege's arguments for his basic laws aim to elicit a reflective awareness of how a *full understanding of the formulas expressing them* is sufficient for a recognition of their truth.
>
> Non-linguistic version: Frege's arguments for his basic laws aim to elicit a reflective awareness of how a *complete grasp of them* is sufficient for a recognition of their truth.

It also splits the question of how judgement relates to understanding into three questions:

1) How does the grasp of thoughts relate to the understanding of sentences?
2) How does judgement connect to understanding of sentences?
3) How does judgement connect to grasping of thoughts?

For 1), one plausible connection is:

> Claim (1a): If we understand a sentence Φ, we thereby grasp the thought Φ expresses

One possible hitch for (1a) concerns indexical language. Suppose that Φ is a sentence which contains indexical: "I am related to Queen Elizabeth" for example. Assume that Φ expresses the same thought in all contexts of utterance and that sense uniquely determines reference. It follows that Φ must have a fixed and context-invariant truth-value. But it doesn't: Φ clearly takes on different truth-values depending on which speaker utters it. Denying that Frege held (1a) is one common strategy for solving this puzzle: understanding Φ means that we grasp the linguistic meaning of Φ, but not that we grasp the sense which Φ expresses—for that, we'd need to know the context of utterance, and use that information together with the linguistic meaning to determine the sense and the truth-value of Φ.

This argument against (1a) is not conclusive, however. Robert May has put pressure on the assumption that Frege took every sense to uniquely determine a referent. Instead, Frege believed that a sense uniquely determines a referent *only* in those cases where the sense is a mode of presentation of that referent. May argues that Frege did not think of the senses of indexical terms as modes of presentation of referents, and thus wasn't committed to denying (1a) (2006, 488-489). In any case, it's unclear whether Frege thought indexicals should appear in a proper scientific language. His formal languages in *Begriffsschrift* and *Grundgesetze* contain no indexical terms. Thus, even if (1a) turns out not to hold in its full generality, it holds in the cases we care about—if we understand a formula written in the concept-script, then we grasp the thought which that formula expresses.

If (1a) holds then there's admittedly a tension with everyday attributions of understanding. For there are times when we'd ordinarily say that A understands a very complex formula Φ without having fully parsed the whole compositional structure of the formula in question, especially if the formula is presented in an abbreviated form by making use of definitions. If, as seems plausible, a finely individuated notion of sense means that one must parse the compositional structure of a thought in order to grasp it, then A understands Φ without grasping the thought it expresses. However, one doesn't get the impression that Frege is much concerned with such everyday standards of understanding. And if Frege *did* care about our everyday standards for when one understands a formula, that wouldn't count against claim 1a. It would just show that Frege occasionally inclines towards a more course-grained notion of sense—he speaks of scientists in different languages and ages grasping the same thought, for instance (Frege 1914, 216-7).

What about the *other* direction? Does grasping a thought imply that one understands a sentence which expresses it?

> <u>Claim (1b):</u> We cannot grasp a thought without there being a sentence expressing it which we understand.

As I intend to characterize (1b), its truth doesn't entail that there are no inexpressible thoughts—it instead amounts to the claim that all thoughts we grasp need be grasped through language, so inexpressible thoughts couldn't be thought by anyone. So does Frege hold (1b)? A notable quote in "Thought" suggests that he does—he says that:

> The thought, itself imperceptible by the senses, gets clothed in the perceptible garb of a sentence, and thereby we are enabled to grasp it. (1918a, 61/FR 328)

Intriguing, but not as conclusive, is a quote from 1923, where Frege wrote that language functions as "a bridge from the perceptible to the imperceptible", allowing us to come into cognitive contact with thoughts (Frege 1923-26, 259). However, immediately preceding the "bridge" quote, he says "As a vehicle for the expression of thoughts, language must model itself upon what happens at the level of thought" (1923-26, 529). And this points to a strain of thought which points *against* (1b). This strain is also found in his earliest logical *Begriffsschift*, where he says:

> If it is one of the tasks of philosophy to break the domination of the word over the human spirit by laying bare the misconceptions that through the use of language often almost unavoidably arise concerning the relations between concepts and by freeing thought from that with which only the means of expression of ordinary language, constituted as they are, saddle it, then my ideography, further developed for these purposes, can become a useful tool for the philosopher. To be sure, it too will fail to reproduce ideas in a pure form, and this is probably inevitable when ideas are represented by concrete means…(Frege 1879, 7).

This passage raises a puzzle which pushes against (1b)—if we had no grasp of thought that wasn't already through the medium of language, how could we ever come to realize, in the first place, the way in which ordinary language misleads us? For that, we'd need to have some purchase on the structure of thought even prior to having seen thought displayed perspicuously in a concept-script. But in favor of (1b), it's totally unclear how *else*, if not through language, one might come to grasp a thought, imperceptible and causally inefficacious as they are.

I do not propose to solve here whether Frege ultimately held (1b). However, the question can be bracketed for my purposes. I aim to provide an account of how Frege treats his basic laws and why, and these laws are all expressed in Frege's symbolism—indeed, he writes them all out for us in (1893, §47)! We may similarly put aside questions over whether Frege held:

> Claim (1c): We can judge as true thoughts which we haven't grasped through the medium of language.

For in *Grundgesetze* (as well as in *Begriffsschrift*), Frege's basic laws are all prefixed with the judgement stroke. They are all grasped through a language, namely his concept-script.

We can now move on to questions 2) and 3)—how it is that judgement connects to understanding of sentences and grasping of thoughts. Some facts about these connections should be quite obvious:

> Claim (2a): We can understand a sentence Φ without judging the thought expressed by Φ to be true

> Claim (3a): We can grasp a thought Φ without judging that thought Φ to be true

This is uncontroversial—Frege clearly thinks we can entertain a content without taking it to have any particular truth-value (1879, §2; 1893, §5). What about the converse claims?

> Claim (2b): We can only judge that Φ if we understand a formula "Φ" which expresses it.

> Claim (3b): We can only judge that Φ if we grasp the thought that Φ.

If (1c) holds, (2b) is false. Here's a variant which is compatible with (1c):

> Claim (2b*): We can only *assert* that Φ if we understand a formula which expresses it.

(3b) is true because judgement, as Frege understands it, can involve nothing like blind affirmation. To see why, let Φ be the first thought expressed in the work "On the Non-Existent" by Gorgias the sophist (although several paraphrases of the work exist, the original is lost). Φ can be true. We can even judge

> The first thought expressed in the lost work "On the Non-Existent" by Gorgias is true

as true, perhaps because we have a misplaced trust in the merits of sophism[6]. However, as a simple argument shows, we cannot judge that Φ. For suppose Φ turns out to be the thought that all grass is red. For reasons unrelated to Greek philosophy, Bill knows that grass is not red, and thus judges that ¬Φ. But since he takes Gorgias to be extremely honest and accurate, Bill judges that the first declarative sentence in "On Non-Existence" is true, even though he has no idea what that sentence is. If this were tantamount to judging Φ, then Bill is judging both Φ and ¬Φ, something Frege would have taken to be impossible (I'll discuss this impossibility further below). Thus, to judge that Φ, we need to know what Φ actually *is*.

[6] Frege thought that we could only judge truths, so, strictly speaking, we couldn't judge " 'A' is true" unless it really was. But as I discuss in §4.2 of chapter 4, I'm not trying to square my account with this quixotic commitment of his—indeed, as we shall see, to make sense of his logical practice, we must suppose he allowed inferences from false, and even unasserted, premises.

A similar argument shows that when we assert something, this assertion must always involve an understanding of the sentence, written or spoken, used to make the assertion. For suppose we asserted Φ by speaking/writing "Φ", without understanding what "Φ" meant. Assertion is a manifestation of judgement (1918a, 62/FR 329), so asserting "Φ" would be a case of blind affirmation, contradicting the fact that judgement can involve no such affirmation. Thus, Frege would affirm 2b* as well as 3b.

By 3b, to make a judgement, one must grasp the thought being judged. In all of the cases we're considering, these thoughts are grasped through the medium of language, expressed as they are by formulas in Frege's concept-script. By 2b*, these formulas must be understood if we're to judge as true the thoughts which they express. Since judgement is, in such cases, crucially tied to understanding, showing that logical laws are implicit in understanding will show how these laws are implicit in judgement in these cases.

To sum up: To understand the sense in which logical laws unfold the meaning of "true", we must understand what it is to implicitly understand the meaning of "true". To understand this, we must understand how logical laws are implicit in judgement, and to understand this, we need to understand how logical laws are implicit in our grasp of thoughts, and in our understanding of formulas expressing these thoughts. The hope is that once we have a picture of how logical laws and understanding connect, we will be properly positioned to provide a deeper explanation for why Frege took his axioms to be logical, in turn also enabling us to explain and connect the generality and undeniability of logic.

§2. Understanding, compositionality, and inference

§2.1 Stroud on inference

To set the stage for my own account of the connection between logical laws and understanding which I maintain is implicit in Frege, I'll now say a bit about some thoughts of Barry Stroud's which inspired it, and will say a bit about the parallels between Stroud's concerns and Fregean commitments. In a 1979 paper, "Inference, Belief, and Understanding", Stroud looks at Lewis Carroll's puzzle, and tries to see what moral should be drawn from it. The paradox in brief: Achilles and the tortoise are having a conversation about the following propositions:

A: Things that are equal to the same are equal to each other
B: The two sides of this Triangle are things that are equal to the same
Z: The two sides of this Triangle are equal to each other

The tortoise accepts A and B, and asks Achilles to force him to accept Z on their basis. Achilles offers the following conditional:

C: If A and B, then Z

The tortoise happily accepts the conditional, but maintains that he can accept A, B, and C without being forced to accept Z. Achilles suggests:

D: If A, B, and C are true, then Z

The narrator then leaves for the bank, but the regress clearly continues in the same way—no extra proposition we could add to the premises can *force* one who accepts all of the premises to accept the conclusion.

Stroud's first point is that this puzzle doesn't tell us anything about logic and validity[7]—A and B *do* imply Z, without any extra premises necessary, and that's all there is to it (1979, 179). Instead, Stroud thinks Carroll's story poses two parallel puzzles. The first puzzle is—what should one make of the "must" in "If you believe A and B, you must believe Z" (1979, 182)? It's not a necessity—though A and B imply Z, believing A and B don't obviously necessitate believing Z. One suggestion which Stroud considers, due to Judith Jarvis Thompson , is to explain the "must" by pointing out that anyone who believes A and B while rejecting Z must believe some falsehood. Stroud points out that this does little more than restate the fact that $A, B \vDash Z$ (in any case, the tortoise does not reject Z—he suspends judgement) (1979, 183). Thompson's suggestion would give the normativity in question only a hypothetical grip—the tortoise, accepting A and B, must accept Z *if* he wants to avoid falsehood. The first challenge, as Stroud sees it, is to give an account of logical normativity that amounts to more than merely rehashing the logical connection between A,B, and Z.

Here, Stroud touches upon, in a different context, the same questions concerning the normativity of logic which I raised for Frege's view of logic. Frege took logic to be normative in some important sense. I argued in §1 of chapter 2 that the sort of normativity at play needs to be stronger than the *minimal connection*, which is that we *should* follow truth-preserving logical laws if our aim is to securely pass from truth to truth.. While undoubtedly true, this minimal connection wouldn't point to anything distinctive about logic. Thompson's suggestion amounts to the minimal connection—as I do, Stroud supposes there must be more to it than that.

`The second puzzle Stroud takes to be raised by Carroll's paradox is: what is it to believe Q on the basis of P? Is it to believe that if P then Q, or that P entails Q, or to believe some other R, where R says that P and Q stand in some appropriate relation? No. As the tortoise would tell us, for any such R, one can believe P and R without believing Q. If one then came to believe Q for reasons independent of P and R, then one would believe Q, P, and R, but still wouldn't believe Q *on the basis of* P—so, a belief in R can't be what it is to believe Q on the basis of P. Put another way, attributing to a reasoner a belief that a certain relation holds between P and Q just pushes the question of what it is to believe Q on the basis of P back to the question of what it is to believe Q on the basis of P and R. The puzzle seems quite general—for any premises and conclusion, believing the conclusion on the basis of the premises can't just be a matter of having acknowledged that some relation holds between them. If our minds are like Achilles' notebook, a mere list of accepted propositions, then there's no proposition we could add to that list whose addition would constitute our accepting one proposition on the basis of another (Stroud 1979, 186-8).

Stroud's solution is this. Believing is never *just* a matter of having a proposition imprinted on some mental notebook. What the notebook picture of belief leaves out is understanding:

> Believing something involves understanding it, and that in turn appears to involve seeing some of its connections with other things one understands, or at least having the capacity to see and accept those connections in appropriate circumstances. (Stroud 1979, 189).

That the tortoise accepts P, and accepts that if P then Q, but doesn't accept Q, counts against the tortoise understanding the conditional in question. It's not a *conclusive* case against his understanding it—if the tortoise recognizes enough other inferential connections in which the conditional figures, he can still be rightly said to understand it. This also goes some way to explaining the normative force of logic—given a proposition, one *must* be competent in making *some—enough*—inferences to and from it, for if one didn't, one couldn't be said understand the proposition in question.

[7] Though Stroud doesn't discuss it, there is one point concerning consequence (albeit not a very deep point) one can take from Carroll's puzzle—that relative to a certain formal system, there must be a clear separation between propositions and inference rules.

Stroud isn't aiming to give necessary and sufficient conditions for understanding. Nor am I, and Frege certainly isn't. Nor is Stroud aiming to give a full account of logical normativity, or of what it is for one sentence to be believed on the basis of another. What he's saying is that a necessary component of any such account is that it addresses the special way in which inference is connected to understanding—understanding involves seeing inferential connections. All of this is directly relevant to the exegetical puzzle of how Frege might have thought logical laws connect to understanding—if understanding involves seeing inferential connections, it gives a definite sense in which logical laws are implicit in understanding. If we replace "belief" with "judgement", Stroud's picture is close to the one which I intend to attribute to Frege—judgement requires understanding (or grasp of a thought), and we don't understand a sentence/grasp a thought if we're not competent in employing it in certain simple inferences.

Another important thing to note about Stroud's account is that even as a merely necessary condition, it gives no very sharp criterion for when one has failed to understand an inference. Though we can say that one who sees *no* inferential connections involving a sentence ψ doesn't understand ψ, there's no fine line that must be crossed in order for understanding to obtain (1979, 191). It's not that for some n, anyone incompetent in identifying less than n different inferential relationships involving ψ fails to understand ψ. Nor is Stroud committed to the claim that there is some special circumscribed set of inference rules (introduction and elimination rules taking us to and from ψ, for instance) an incompetence with which is a *conclusive* sign of failure to understand. Nevertheless, it's consistent with Stroud's view that some inferences are more central to understanding than others—perhaps incompetence with modus ponens counts more heavily in favor of a misunderstanding of ψ having occurred than incompetence with some more convoluted inference rule.

This might give one pause. Though Frege says fairly little about understanding of sentences, one might worry about attributing to Frege this fuzziness of what inferences are centrally tied to understanding in this way. My response is that such fuzziness is already a part of Frege's picture for reasons quite independent of the current account—there's a certain amount of indeterminacy surrounding the way in which Frege thinks we ought to identify which laws are primitive. I'll return to deal with this concern in more detail in §2.4 below.

§2.2 Compositionality

The account, given in this section, of the way in which logical laws connect to understanding is intended as a likely story, a rational reconstruction of how Frege may have conceived of the connection between logical laws, inference, and understanding. While I'll give a few references in support of certain points, the focus for the time-being is on getting the ideas themselves clear—the fuller justification for attributing the resultant picture to Frege can only come from seeing how neatly it accounts for the way in which he argues for his heuristic indicators, and how it sheds light on the way in which he argues for his axioms.

I claim that the sense in which Frege thought logical laws were implicit in understanding is deeply tied to compositionality:

> COMP: Understanding a formula involves understanding its constituent parts and seeing how these parts are put together.

The content of a formula is a function of its semantically simple components together with the structure in which these components are placed. Frege doesn't explicitly state compositionality often, but when he does it figures prominently. In *Grundgesetze,* Frege says

> Now, the simple or complex names of which the name of a truth-value consists contribute to expressing the thought, and this contribution of the individual name is its *sense*. If a name is a part of the name of a truth-value, then the sense of the former name is part of the sense of the later name. (1893, §32)

Frege was also explicit about compositionality in a course on his concept-script which he gave in the summer semester of 1913. In Carnap's notes on these lectures, the second line he reports Frege as writing down was:

> The sense of a part of the sentence is part of the sense of the sentence (Carnap 2004, 87)

So Frege clearly took his concept-script to be compositional. This doesn't commit us to as much as one might think, however—in particular, compositionality leaves it open whether the content arrived at by composing senses is *unique*, or whether instead two formulas with quite different structures may nevertheless express the same content[8]. It's consistent with compositionality, for instance, to think Frege took both sides of law V to express the same sense. What it does rule out is that Frege took there to be any *other* factors determining the meaning of formulas in his concept-script, such as a time or source of inscription, or other such contextual elements. For Frege, thoughts contain within themselves all information needed to fix their truth-value:

> All determinations of the place, the time, and the like belong to the thought whose truth is in point; its truth itself is independent of place or time (1893, xvi-ii).[9]

He reiterates this elsewhere (Frege 1897a, 135; 148). He doesn't take sentences of *natural language* to be eternally true in the same way—"I am hungry" will express different thoughts in different mouths. But his concept-script is intended to be a *lingua characterica* in which "the content is to be rendered more exactly than is done by verbal language" (1880/1, 12-13). And Frege's formulas contain no terms which require the sentences in which they occur to be supplemented by contextual factors.

There are other ways a language could fail to be compositional—for example, if substitutivity fails for co-referential terms which occur in the scope of propositional attitudes. Frege tries to get around this by taking the referent of words in such context (the "indirect referent") to be what would normally be the sense of the same word—the word takes on a new sense, (the "indirect sense") as well (1892a, 28/FR 153-4). Whether this solution saves compositionality depends partly on whether we count words occurring in such contexts as the same word or not[10]. It's unclear if Frege's solution is a workable one. If it is, language is safely compositional. But even if Frege's solution isn't workable, this shouldn't worry us here, for no such embedded occurrences occur in the formal work we're considering.

The §32 quote given above suggests that as Frege was writing *Grundgesetze*, he would have conceived of COMP thusly—to understand a sentence requires grasping what the referents of its sub-sentential expressions contribute to the truth or falsity of the whole sentence. For simplicity, in what follows, I'll assume that this contribution *just is* the sense of the sentence, and thus that to grasp sense just

[8] At least at one point in his career, Frege commits himself to the view that quite different senses express the same thought—in "Function and Concept", he says that '$x^2 - 4x = x(x-4)$' and '$\acute{\varepsilon}(\varepsilon^2 - 4\varepsilon) = \acute{\alpha}\big(\alpha(\alpha-4)\big)$' express the same sense in a different way (1891, 10-11/FR 136). Much later, in "Compound Thoughts", he says that 'A and B' and 'B and A' is "a case where two linguistically different expressions correspond to the same sense", and that the same holds for 'A and A' and 'A' (1923-6, 39; CP 393). This runs against what he says elsewhere, however: in Grundgesetze, Frege tells us that "If a name is a part of the name of a truth-value, then the sense of the former name is a part of the thought expressed by the latter" (1893, §32).
[9] Note, however, my caveats in §1.5, where I discuss indexicals.
[10] This thought is due to discussion with Peter Milne.

is to appreciate what referents of expressions contribute to truth-values of sentences in which the expression occurs. For those who find this assumption objectionable (Burge 1990 raises some pressing puzzles for this view), in what follows one may simply replace 'sense' with '**sense**', the later taken as a stipulated term of art taken to mean what I'm claiming 'sense' in fact does mean. If this substitution is made, I claim that all of what is said here still stands.

It's our implicit grasp of sense (as compositionally construed) which logical laws make explicit—such a grasp presupposes certain inferential abilities, and just what is presupposed in seeing how the truth-value of a sentence depends on the referents of its constituent parts is spelled out in logical laws. Broadly speaking, my suggestion is that Frege implicitly held, for grasp of sense, views similar to that which Stroud holds for understanding—we cannot be said to implicitly grasp the way in which referents contribute to truth-values of expressions if we are incompetent in making any inferences involving it. To the extent that grasping a thought involves understanding a sentence expressing it (caveats of §1.5 above aside), Frege was similarly Stroudish about understanding.

§2.3 Inferential constraints upon understanding

Since Frege said relatively little about how understanding and inference connect, the account I give must be, at best, a rational reconstruction. Here's a plausible constraint which any account must meet:

> INF: If one is not competent in using a sentence ψ and its subexpressions in certain inferences, then one fails to implicitly grasp the sense of ψ.

For brevity, I introduce the following term of art:

> Requisite inferences: The requisite inferences of ψ are all inferences such that competance with those inferences is a necessary condition for grasping the sense of ψ.

The requisite inferences of ψ will include certain inferences to and from ψ, as well as to and from subexpressions of ψ. INF provides the key to explaining how Frege understood the case which he makes for the epistemic analyticity of his axioms. For if implicitly grasping the content of a sentence requires competence with certain inferences, this raises the possibility that *explicit* reflection on the content of ψ can help one identify the requisite inferences of ψ. We can then explain Frege's arguments for his basic laws as the activity of eliciting such an explicit reflection.

Before looking at requisite inferences, and the consequences of INF, in more detail, a few clarificatory comments are in order. Firstly, INF is a necessary condition for, rather than an analysis of, understanding. If someone is *also* incompetent in avoiding as many *wrong* inferences regarding ψ as well, we wouldn't rightly attribute understanding to them. And in any case, someone would undoubtedly need to do more than verbally manifest competence with inferences between certain thoughts to convince us she understands 'dog', and thus sentences in which 'dog' occurs[11]. This should not bother us—the present

[11] In Jaroslav Peregrin's version of inferentialism, for instance, one understands an expression ϵ only when one is competent with certain inferences in which ϵ and its subexpressions (if it has any) occur, and *also* when one is competent in transitioning from circumstances to sentences involving ϵ, and from sentences containing ϵ to actions (Peregrin 2014, 25). (Actually, Peregrin is careful to say he cares more about competence in approving or disapproving of *others* making such transitions (2014, 8-10), but I leave this aside for the sake of simplicity)

For instance, if one understands the name "dog" if one is competent in making inferences to and from sentences in which "dog" occurs, in saying things like "There's a dog!" when in a dog's presence, and in pointing to a dog when someone says "where is the dog?". It would be extremely implausible to foist anything so detailed on Frege, but this purpose is not to develop a full-fledged Fregean theory of understanding, but to say enough about what it involves to explain how Frege makes his case for logicality, and how he motivates his two heuristic indicators.

Secondly, despite talk of competence with inference, none of this should be thought of as giving logic any sort of behavioral or linguistic grounding—such competence is what leads us to rightly attribute understanding to the competent, but it has no bearing on whether these inferences are *correct*. Nor does INF give judgement such grounding—if judgement could be explained entirely in terms of inferential behavior, this would run against Frege's insistence that judgement is something "peculiar and incomparable" (1892a, 35/FR 159). In fact, even if I'm right in attributing INF to Frege, it's not obvious that Frege would even be committed to a behavioral account of *understanding*. INF is a necessary condition for when someone implicitly grasps what a referent contributes to the truth-values of expressions in which it occurs, but it can be left open just how tightly this condition is tied to grasp of sense—the present account does not commit itself to the claim that Frege would have taken INF to be a full analysis of what it is to have such a grasp.

§2.4 Which inferences are requisite inferences?

Now for some details of INF. Firstly, it's important to distinguish between competence and performance when it comes to making correct inferences. For any given inference, even ones which seem very central to the meaning of ψ, a thinker might understand ψ perfectly well, but fail to use ψ in the requisite inference in question. This could be for any number of reasons—perhaps the thinker is told that ψ, but finds ψ to be about a topic of no interest to her, and puts it out of her mind prior to performing any inferences concerning ψ or its subcomponents. Competence is also consistent with one-off errors—even very effective reasoners might fail to spot an obvious inference from time to time. If an extremely fatigued but otherwise excellent logic student momentarily fails to see that ψ and $\psi \supset \varphi$ imply φ, then even if this is a requisite inference of $\psi \supset \varphi$, it would seem odd to say that this is strong evidence that she's failed to understand "$\psi \supset \varphi$", unless she systematically fails in employing *modus ponens* or, in an individual case, fails to see that the entailment holds despite sustained reflection.

The obvious question is now: *which* inferences involving ψ and its subcomponents did Frege take to be requisite inferences of ψ? As this account is already a rational reconstruction, just which inferences these will be must be left a bit open-ended, but there are some useful points which can made. The first point is that there's a question about whether the class of requisite inferences *should* be limited—perhaps *every* inference in which ψ or its sub-formula occurs as a premise or conclusion counts as a requisite inference. What if Frege held:

> <u>INF+</u>: If one is not competent in performing *every* valid inference in which a sentence ψ and its subexpressions figure, then one fails to implicitly grasp the sense of ψ.

Put another way, INF+ is the claim that every inference involving ψ and it's subformulas is a requisite inference.

There's more to be said in favor of INF+ than one might think. Firstly, in accordance with the distinction between competence and performance introduced above, it's consistent with being competent in inferring $\Gamma \vDash \Delta$ that one never do so, because one is never placed in the appropriate circumstance. An illustrative example: let PA be the conjunction of second-order axioms of Peano arithmetic, and let E be an appropriately formalized version of Euclid's theorem, the statement that there are infinitely many

account illustrates how very much more would need to be added before we have what looks like an even remotely plausible *sufficient* condition for understanding.

primes. It's known that PA $\vdash$ E (Kleene 1952, §40). It clearly seems that one can understand PA without inferring E. But that doesn't mean we are incompetent in inferring PA $\vdash$ E—it might only be in very rare circumstances that this competence is manifested. For PA $\vdash$ E, this circumstance might be one in which we're given ample time to reflect upon a sequence of simpler inferences leading from the premise to the conclusion. If someone in such a circumstance could not then judge E on the basis of PA, then we've grounds to think they misunderstand PA. Secondly, it's plausible that Frege might have taken at least some non-basic logical truths to be conceptually undeniable. Finally, if epistemic analyticity is what undergirds conceptual undeniability, then having more stringent necessary conditions of understanding, where one must be competent with *any* inference involving ψ or its subformulas, is the more natural option.

Some hedging is appropriate at this point, however. It's consistent with the account I'm providing that Frege did *not* accept INF+, but instead took only competence with particularly perspicuous inferences to be requisite for understanding. The picture would then be this: Frege's arguments for his basic laws utilize only requisite inferences of that law or its contradictory—if we didn't, upon reflection, accept these arguments, then we simply don't understand the law in question. While I think that INF+ offers a simpler and more plausible view of inference than the view that requisite inferences of ψ form a special subclass of all inferences in which ψ or its subcomponents occur, it's unclear that Frege held INF+.

Accordingly, the rational reconstruction I develop assumes INF, but not INF+. The inferences which Frege uses in his arguments for his basic laws do appear quite brief and perspicuous—they're requisite inferences if any are. Rejecting INF+ leaves us with the problem of determining which valid inferences concerning ψ are requisite inferences of ψ, but I shall argue below that this is not a *new* problem, but ties in closely to a matter Frege would have needed to deal with anyway—how to discern which laws are primitive. And if Frege *did* hold INF+, then, nothing in the present account is overturned—if *all* inferences are requisite inferences, then the ones which my account claims Frege took to be tied to understanding certainly are.

So, assuming that Frege rejected INF+, which inferences are requisite inferences? Putting COMP and INF together yields a specific constraint upon requisite inferences. As I've characterized it, INF allows for requisite inferences of ψ in which ψ figures neither as the conclusion nor as a premise—the premise and conclusions may instead be sub-formulas of ψ. The leading insight here is that if understanding is compositional, then understanding ψ requires that we understand each closed formula which occurs as a subexpression in ψ. Thus, the compositionality of understanding, together with INF, implies:

> <u>Compositional constraint</u>: All requisite inferences of sentential sub-expressions of ψ are also requisite inferences of ψ.

But this constraint only tells us one way of identifying new requisite inferences of a formula, given previously identified requsite inferences of its subexpressions—it doesn't tell us how the requisite inferences of the subexpressions in question are determined in the first place. We can, of course, engage in some plausible speculation about what would have counted as a requisite inference for Frege—instances of modus ponens ought to be a requisite inference for conditional statements if anything is. More generally, instances of introduction and elimination rules for the main connectives of sub-formulas of ψ are particularly plausible candidates for requisite inferences—if we aren't competent with such rules governing a connective '*', we don't understand formulas in which this formula figures as a main connective[12]. However, Frege was not Gentzen, and there's no obvious reason to suppose he would have

[12] If we accept the compositional constraint, then incompetence with *-I and *-E also indicates a failure to understand any formula ψ in which '*' occurs. For some '*' will be the main connective of some such subexpression ψ_{sub} of ψ, and all requisite inferences of ψ_{sub} are requisite inferences of ψ.

thought that introduction and elimination rules were the *only* requisite inferences. Perhaps he would have seen $\neg(\psi \vee \varphi) \vdash \neg\psi$ as a requisite inference of"$\neg(\psi \vee \varphi)$", for example.

I've suggested some candidates for requisite inferences, but have as of yet said nothing about what they have in common that allow us to know that they're requisite inferences. One usually has more reason to call another's understanding into question the more obvious the inference that this other failed to make. If this is right, then requisite inferences for ψ should be inferences which are brief, or perspicuous—they're a small step, one in which the conclusion is sufficiently close to the premises. But this is very vague. And there's a deeper question of whether Frege even could have had a story, consistent with his other commitments, of what determines the class of requisite inferences for a given ψ. It's doubtful that the notion of a conclusion being sufficiently close to its premises could be explicated in a way that makes no reference to reasoners and their behavior—as inferences which could be made in a reasonable amount of time by a reasonable agent, for instance. It's one thing to say that the satisfaction of certain conditions of inferential competence is necessary for understanding the sentences one uses, but quite another to say that the class of inferences in question should be characterized in terms of behavior and reasonability—if it should, then what we're doing when we ask whether someone meets certain necessary inferential conditions for understanding isn't really *logic*.

One response to this worry is just to conclude that we cannot see INF as implicit in Frege's view without also supposing he was committed to INF+. I take this to be the most philosophically attractive and charitable option. However, the cogent philosophical worry raised above isn't a *decisive* reason for attributing INF+ to Frege. This is because this concern about Frege's view is one which we should already have for independent reasons. In §1of chapter 1, I discussed how it is that we determine whether a given logical law is basic. I suggested, in §1.5 of chapter 1, that it is here that pragmatic considerations such as simplicity, elegance, and fruitfulness will enter in. If this is so, then there's great deal of indeterminacy concerning what counts as basic, and our determinations of basicness hardly seem to rest on purely logical considerations.

The concern about how to demarcate the collection of basic laws isn't just a concern parallel to the one of how to demarcate requisite inferences—there's reason to suspect it's the *same* concern. Recall my contention, in §2 of chapter 1, that there is, for Frege, no extra-systemic distinction between axioms and inference rules. This raises an interesting possibility that Frege could have taken an inference to be a requisite inference if and only if the truth contained in that inference is a basic logical law.[13] If this is so,

[13] Speaking of "the truth contained in an inference" might sound awkward, but it's Frege's own turn of phrase (Frege 1879, §6).

One might worry that by saying "basic *logical* laws", this gloss on requisite inferences would make Frege's account circular—our explanation for why we're right in taking a basic logical law as logical would be given in terms of requisite inferences, but requisite inferences would be explained in terms of basic logical laws. This should not worry us. Firstly, Frege was not trying provide arguments which a non-classical logician would find compelling, but was trying to explain, from the standpoint of a classical logician, why we're right in taking certain laws to be logical. Secondly, this explanation won't be circular, so long as one properly recognizes the important distinction between grounds for a law being true and grounds for our taking a law to be true. I return to concerns of circularity in §4.1 of chapter 4, and provide further arguments that such worries are misplaced.

Recall my contention in §1.3 of chapter 1 that the collection of basic laws overdetermines the set of all logical facts—there are more objectively basic laws than are needed as axioms. Thus, Frege's inference rules can correspond to basic laws even if the basic laws they correspond to aren't actually taken as axioms by Frege. For example, the fact that '$\vdash \big(a \wedge (a \supset b)\big) \supset b$', the truth contained in the inference rule of *modus ponens*, is not one of Frege's axioms should not bother us—that $\vdash (a \wedge a \supset b) \supset b$ is not an axiom doesn't mean it's not a basic law.

then the indeterminacy of what counts as a basic law and of what counts as a requisite inference is just the same indeterminacy. Even if the problem of identifying which inferences are requisite inferences is a problem, it's not a *new* problem for Frege's view, and hence not a consideration which would count against attributing INF to Frege.

One concern with explaining requisite inferences in terms of basic laws is that it marks a change in the role of pragmatic considerations in explanations for why we take a law to be logical. A law is basic if it's true, unprovable, and self-evident. If the law is basic, then whether it's logical will then depend on the nature of its self-evidence. If what I've argued in this chapter is correct, then self-evidence, in the case of logic, amounts to epistemic analyticity. In §§1.5-6 of chapter 1, I claimed that pragmatic considerations play a role in determining whether a law is unprovable, but played no role in determining whether a law is self-evident. But if epistemic analyticity, and hence logical self-evidence, is cached out in terms of requisite inferences, and we maintain that requisite inferences are just those which reflect basic laws, then pragmatic considerations have a crucial role to play in a full account of why a truth is self-evident. Thus, the options are to either grant that pragmatic considerations play a deeper role than I suggested in §§1.5-6 of chapter 1, or just to attribute INF+ to Frege. As I've already noted, I prefer the second option, though both alternatives are workable interpretations of Frege.

The picture we've arrived at is that for different sentences ψ, there will be different associated requisite inferences, enough of which one must be competent in making in order for us to rightly attribute understanding of ψ. In the picture given here, requisite inferences of any given ψ are all valid logical inferences[14]. This gives us a deeper sense in which logic can be said to be laws of thought—logical laws are, in part, laws constraining the legitimate attribution of understanding. Of course, logical laws also allow us to spell out truths, and to identify inferential relations between thoughts. But insofar as an axiomatic theory of logic, such as Frege formulates in *Grundgesetze* (and in *Begriffsschift* prior to that) churns out logically valid inferences which could be requisite inferences of some ψ, they spell out the laws of understanding, and hence of judgement, and hence of truth.

To sum up: logical laws tie closely with our implicit appreciation of what the referents of sub-sentential expressions in a sentence ψ contribute to the truth or falsity of ψ. By COMP, understanding ψ involves this implicit appreciation. Since understanding is, for Frege, necessary for judgement, logical laws are tied to judgement—if we are not competent in inferring rightly, we do not understand, and if we do not understand, we do not judge. If all this is right, then there are two senses in which all judgement is subject to logical laws. Everyone already recognizes that Frege thought logical laws are a criterion for evaluation of judgements in the sense that they tell us when a thought A judged to be true on the basis of B really does follow from B. But if the reasoning above is correct, there is another sense in which logical laws are a criterion for the evaluation of judgements. They concern not only truth (by revealing justificatory relations between thoughts), but also understanding—indeed, they are only well-placed to govern the latter because of their ties to the former.

[14] As mentioned previously, some contemporary views which touch on similar territory, such as that of Peregrin's, take non-logical inferences to be tied to understanding (Peregrin 2014, 25). But there's no reason whatsoever to suppose that such a view is implicit in anything Frege does. My inferential picture of understanding rests in part on my interpretation of the claim that *logical* laws unfold the meaning of "true"—we have grounds to take Frege to have implicitly taken logically-valid inference rules to be tied to understanding, but no such grounds for thinking he thought non-logical inference rules were similarly relevant. In any case, as discussed in §2.2 of chapter 1, there's even some doubt as to whether Frege thought there were *any* acceptable but non-logical inference rules.

§2.5 How INF explains the heuristic indicators.

If I'm right in attributing this picture to Frege, the necessary conditions for logicality fall right out as a result. In §3.4 of chapter 2, I argued that the universal applicability of logic was rooted in Frege's view of logic as all-encompassing—logic provides criteria for the evaluation of judgements. I've outlined two ways in which *all* judgements are subject to logical laws, and for all reasoning to be so subject just is for logic to be all-encompassing. Logic is the gold standard for the evaluation of all our reasoning in a deep sense—we must be competent in making classically valid inferences in the appropriate circumstances if we are to count as reasoners at all. Since logical laws form a constraint upon understanding, and sentences of any special science may be understood, the laws governing the correctness of our reasoning do not vary with the subject matter in question.

Before showing how this picture ties in with conceptual undeniability, first a brief word on the contradictory of a formula ψ. Note that the contradictory of a formula ψ is *not* generally $\neg\psi$, since ψ may contain roman-letter notation, which functions as a universal generalization of widest possible scope. The contradictory '$\Phi(a)$' isn't '$\neg\Phi(a)$', since '$\neg\Phi(a)$' is interderivable with '$\forall\mathfrak{a}\big(\neg\Phi(\mathfrak{a})\big)$', which denotes the True only if *no* object falls under the function denoted by '$\Phi(\xi)$'[15]. Since $\vdash\Phi(a)$ means that $\Phi(\xi)$ is the True for *every* argument α, the contradictory of $\Phi(\xi)$ should be such that it is False if and only if $\Phi(\xi)$ denotes the False for *some* argument α. Clearly, $\neg\forall\mathfrak{a}\Phi(\mathfrak{a})$ fills this role. The point generalizes to formulas containing more than one roman letter: the contradictory of ψ will, in general, be $\neg\forall\mathfrak{a}\forall\mathfrak{b}\ldots\psi$, the negation of the universal (concavity) closure of ψ.

Some terminology to shorten the exposition: Let

$\sim\psi$

be the contradictory of ψ. In other words:

$\sim\psi \coloneqq \neg\forall\mathfrak{a}\forall\mathfrak{b}\ldots\psi$

To see how the picture ties in with the second condition for logicality, recall that a law is called 'conceptually undeniable', as I'm using the phrase, if a denial of it always involves some fatal defect of *understanding*. With the connection between inference and understanding in hand, we're now well-placed to say just what Frege would have thought this defect was. Consider any logical truth ψ. Frege allows for no special speech act of denial—he suggests that to deny ψ just is to affirm $\sim\psi$ (1918b, 154/FR 357-8). So consider the requisite inferences of $\sim\psi$—that is, the inferences anyone who could be said to understand '$\sim\psi$' must be competent with. Will an inference [$\sim\psi\vdash\bot$] (where $\bot$ is some contradiction) be one such inference? If Frege accepted INF+, then obviously the answer is yes—if *every* inference in which '$\sim\psi$'or its subexpressions figure as a premise or a conclusion will be a requisite inference of '$\sim\psi$', of course [$\sim\psi\vdash\bot$] will be as well. If Frege rejected INF+, then whether [$\sim\psi\vdash\bot$] is a requisite inference of '$\sim\psi$'will depend on the ψ in question—if ψ is some very difficult to prove arithmetical truth, incompetence in inferring [$\sim\psi\vdash\bot$] hardly counts against our understanding $\neg\forall\mathfrak{a}\forall\mathfrak{b}\ldots\psi$. But Frege's axioms are not such difficult to prove truths—they are *primitive*, and thus objectively unprovable. Thus, for a Fregean axiom A, it will be plausible that [$\sim A\vdash\bot$] is a requisite inference of $\sim A$. Put another way, that's part of what it *is* to be an axiom—the simplicity and self-evidence required of axioms mean that they are such that, if they're logical, a contradiction could be quickly shown to follow from their denial.

Anyone who denies A thereby judges that $\sim A$—judgement requires understanding, and in this case, understanding requires competence in making the inference $\sim A\vdash\bot$. So, anyone who denies A must be

[15] Frege introduces his first-order quantifier objectually (1893, §8)

competent in affirming, on the basis of this denial, a contradiction, say, $Q \wedge \sim Q$. But $[Q \wedge \sim Q \vdash Q]$ and $[Q \wedge \sim Q \vdash \sim Q]$ are both obvious candidates for being requisite inferences of $Q \wedge \sim Q$, so someone who affirms a contradiction will be competent in inferring Q and $\sim Q$. And Frege would have thought this affirmation impossible. As I mentioned above, denial is not, for Frege, a mode of judgement quite distinct from affirmation—he opts instead to say that there is just one sort of judgment, and that affirming $\sim A$ can do the work done by denying A (1918b, 154/FR 357-8). There's no such thing as at once affirming and denying A, so there is, for Frege, *no such act* as genuinely judging A and $\sim A$. Since a genuine denial of A would require us to do the impossible, Frege must have thought this denial was itself impossible. We must, as Frege sees it, suppose that anyone appearing to deny A is merely performing a verbal denial, not a genuine, cognitive denial—A is conceptually undeniable.

The impossibility of asserting both Q and $\sim Q$ is not a claim Frege justifies, and I think it likely that he wouldn't have thought it admits of any further justification. We shouldn't see his commitment here as resting upon some *prior* commitment to the impossibility of both asserting and rejecting A since, again, Frege ultimately characterizes rejection in terms of asserting the contradictory (1918b, 154/FR 357-8). He also assumes, without argument, that truth and falsity are exhaustive and distinct as truth-values (1892a, 34/FR 157-8). Additional evidence that Frege takes this as fundamental can be found in the *Begriffsschrift*—in §15, in the course of arguing for his law $\vdash a \supset (b \supset a)$, Frege says, without any further support, that we can't at once affirm and deny a. If we don't (and we shouldn't) read very much into the fact that Frege, in §5, characterizes his conditional stroke in terms of affirmation conditions here, and accept (and we should) that he's ultimately supplying truth-conditions in that section, then §15 is additional evidence that the impossibility of asserting Q and $\sim Q$ is bedrock, something Frege takes entirely for granted[16].

§3. Conclusion

I've shown that Frege took logical laws to be implicit in our grasp of truth-values, in our judgement, in our grasp of thoughts, and in our understanding of formulas. It's implicit because there are inferential constraints upon understanding. If we aren't competent with logically valid inferences, we simply cannot understand Frege's logical formulas—including, crucially, the formulas which express Frege's basic laws. I've also shown how this picture underlies Frege's conception of logical laws as all-encompassing and conceptually undeniable. Of course, far more than that is needed to support my claim that this view is implicit in Frege. The full support will come from seeing the way in which a view of logical laws as epistemically analytic fits neatly with the way in which Frege goes about arguing for his basic logical laws, and for how it coheres will with his Euclidean foundationalism, and his distinctive take on self-evidence. In the next chapter, I'll make this case, and will show how my view fares better than its competitors in providing a reading of the introductory sections of Frege's *Grundgesetze*.

[16] In fact, Frege may just not have seen a very deep distinction between the law of non-contradiction and the exclusivity of truth-values. Rather than arguing for, say, the law of excluded middle in terms of his semantics, it seems to be the other way around—he argues that functions need to be defined for all input or else we couldn't set forth laws like L.E.M. (Frege 1891, 20/FR 141). So, the exclusivity of truth-values might be similarly tied to his adoption of L.N.C. He also doesn't see a deep distinction between different ways of affirming a proposition— '*P*' and 'the thought that *P* is true' have the same sense (1892a, 34/FR 158), and he generally isn't careful about the distinction between truth predicates ("the thought that *P* is true") and ("it's true that P") (1914, 233-4). This, when put together with the fact that negation does the job of denial, makes it plausible that Frege wouldn't have seen a deep distinction between different ways of denying a proposition, in which case the exclusivity of truth-values amounts to the same as L.N.C.

Chapter 4: The E.A. Interpretation II: Frege's Arguments for Basic Laws

§0

In chapter 3, I presented the E.A. interpretation—that Frege took logical laws to be implicit in understanding, and that this motivates a view where there are inferential constraints upon understanding—if we aren't competent in using a formula appropriately in logically valid inferences, then we don't understand that formula, or grasp the thought it represents. In this chapter, I argue that this picture explains how it is that Frege made a case for the logicality of his basic laws—his arguments for his basic logical laws elicit a reflective awareness of the way in which a full grasp of these laws is sufficient for recognition of their truth. I argue that this interpretation fares better than rival interpretations by both making sense of the way in which Frege presents his symbolism and argues for his basic laws, while also explaining how Frege does so without contravening his Euclidean foundationalism.

First, I'll present some textual data from *Grundgesetze*, highlighting some observations which different interpretations of these passages hinge upon. I'll then present how my own E.A. reading accounts for Frege's arguments for laws I-IV. Next, I'll focus on laws V and VI, showing that law V doesn't represent a problem for what Frege *did* do to establish logicality, but instead shows that he wasn't sufficiently careful about when stipulations successfully confer referents upon the terms involved. I provide some clarifications concerning the role of semantics and sense in my view, and I then respond to some possible objections— the charge that Frege's way of demonstrating the logical nature of his axioms is problematically circular, the charge that it's incompatible with Frege's view that premises must be true, the charge that Frege's account would label non-logical truths as logical. Finally, I'll look at rival readings of what Frege hopes to accomplish with his arguments for basic laws—against the elucidatory interpretation, I argue that these arguments must be semantic in a sense, and against the correctness interpretation, I argue that Frege takes logical laws to justify claims about semantic concepts, rather than the other way around.

§1. The data: how Frege discusses his laws

In this section, I will examine Frege's discussion of his symbolic system in the first volume of *Grundgesetze*—his introduction of his primitive terminology, his arguments for his basic laws, and his discussion of inference rules. I'll highlight some particularities of Frege's discussion of his system, as many of the arguments for the elucidatory and correctness interpretations will hinge on these details. For the moment, I'll be restricting my focus to that part of *Grundgesetze I* called "I. Exposition of the concept script"—particularly, §§$1-25$ on his primitive signs (I'll have more to say about §§$26-33$ below).

§1.1 Introductions of symbols and functions

In Volume I of *Grundgesetze*, Frege, after some preliminary comments on judgment and the unsaturated nature of functions, introduces his horizontal stroke and negation signs:

> <u>Horizontal Stroke:</u> "$-\Delta$ is the True when Δ is the True, and, is the False if Δ is not the True"(§5).
>
> <u>Conditional stroke:</u> "Next, in order to be able to designate the subordination of concepts and other important relations, I introduce the function with two arguments $\vdash \zeta \supset \xi$ by means of the specification that its value shall be the False if the True is taken as the ζ-argument, while any object that is not the True is taken as ξ-argument; that in all other cases the value of the function shall be called the True. (§12).

Although these two passages are similar to the standard truth-table explanations of propositional operators[1], neither makes explicit use of semantic ascent. Neither §5 nor §12 contain names for sentences or thoughts, nor do they speak of reference—strictly speaking, these are explicit introductions only of functions, not of the signs that denote these functions. The same is true of Frege's introduction of his negation stroke (§6) and his backslash operator (§11). But Frege's introductory passages are far from consistent on this point, and in the next two introductions, it's explicit that he's laying down the reference of a sign, though the stipulations are not uniformly ascended:

> Identity: '$\Gamma = \Delta$' refers to the True, if Γ is the same as Δ; in all other cases it is to refer to the False (§7)
>
> First-level Universal Quantifiers: "Let '$\forall\mathfrak{a}\Phi(\mathfrak{a})$' refer to the True if the value of the function $\Phi(\xi)$ is the True for every argument, and otherwise the False" (§8)[2]

Frege's introduction of his '$\acute{\varepsilon}\Phi(\varepsilon)$' notation for extensions in (§9) is similarly mixed—he says that '$\acute{\varepsilon}\Phi(\varepsilon)$' refers to the value range of the function $\Phi(\xi)$.[3]

§1.2 Arguments for basic laws

Frege then goes on give what are, prima facie, arguments for his basic laws. Here are two examples:

> Law I: "According to §12, $\Gamma \supset (\Delta \supset \Gamma)$ would be the False only if Γ and Δ were the True while Γ was not the True. This is impossible. Accordingly, $\vdash a \supset (b \supset a)$" (§18)
>
> Law II: "$\forall\mathfrak{a}\Phi(\mathfrak{a})$ is the True only if the value of the corresponding function $\Phi(\xi)$ likewise has to be the True. So, $\Phi(\Gamma)$ likewise has to be the true. From this it follows that $\forall\mathfrak{a}\Phi(\mathfrak{a}) \supset \Phi(\Gamma)$ is always the true, whatever function with one argument $\Phi(\xi)$ may be." (§20)

In these arguments, semantic ascent is entirely absent—there is talk only of functions, not of symbols, and no explicit mention is made of reference. This shouldn't be seen as willful abstention on Frege's part, however, as the surrounding discussion is shot through with semantic talk. §17, for example, contains an argument that '$\forall\mathfrak{a}\big(\Gamma \supset \Phi(\mathfrak{a})\big)$' always refers to the same value as '$\Gamma \supset \forall\mathfrak{a}\Phi(\mathfrak{a})$', for any proper name '$\Gamma$' (only closed sentences are names of Truth-values for Frege, so nothing becomes unbound). This argument bobs back and forth between talk of functions and of signs—in part, he says:

> …Compare this with '$\Gamma \supset \forall\mathfrak{a}\Phi(\mathfrak{a})$'. The latter formula refers to the false if Γ is the True and $\forall\mathfrak{a}\Phi(\mathfrak{a})$ is the False. But this is the case if for some argument the value of the function $-\Phi(\xi)$ is the False. In all other cases $\Gamma \supset \forall\mathfrak{a}\Phi(\mathfrak{a})$ is the True…(Frege 1893, §17)

Frege has six basic laws. His arguments for laws I- IV make no explicit use of semantic ascent. His arguments for laws IIa and IIb, which deal with quantification over objects and functions respectively,

[1] This similarity, though undeniable, mustn't be exaggerated. Because these functions are total, taking any object whatsoever as input, the functions defined are distinct from the truth functions with which we are so familiar. The approach is not entirely unique to Frege—Russell, at least at one point, had a similar approach—when discussing his conditional function, he says that " 'p implies q' is a relation which holds between any two entities p and q unless p is true and q is not true" (Russell 1906, 161-2), and explicitly allows that p and q may take on values which are not propositions.

[2] Frege says that this explanation is, as it stands, incomplete, since we need to specify precisely the function which is in question, and give some rules concerning how we ought to replace bound letters with free variables (§8).

[3] This stipulation is incomplete as well, both because Frege must explain which '$\Phi(\xi)$' corresponds to '$\acute{\varepsilon}\Phi(\varepsilon)$', and because the stipulation fails to fix the truth-value of identity statements between objects given to us as extensions, and those which aren't, such as truth-values (see §10). More on this anon.

aren't explicitly semantic, but are immediately preceded and succeeded by clarifications of terminological points. Law VI, which governs the back-slash operator, is explicitly semantic—Frege briefly notes that the truth of the law follows from the reference of '$\backslash\xi$'. By way of justification of law V, Frege just says, citing §3 and §9, that an identification of value-ranges can be converted into the generalization of an equality and vice versa (§20). Below, I'll argue that what Frege does to justify law V here is no different than what he does to justify law VI—both case are just citations of stipulations, and the problem with law V is that the stipulation for '$\acute{\varepsilon}\Phi(\varepsilon)$', unlike that for '$\backslash\xi$', fails to secure a reference for the expression in question.

§1.3 Arguments for inference rules

Frege is similarly sporadic in his use of semantic ascent in his arguments for his inference rules. Most of his rules follow a pattern—when stating the rules themselves, he speaks in terms of which propositions, i.e. contentful formulas, can be inferred from which others, and when he argues for these rules, his arguments speak only of functions rather than signs. For example:

> Modus ponens: "From the propositions '$\vdash \Delta \supset \Gamma$ and '$\vdash \Delta$' one can infer: '$\vdash \Gamma$'; for if Γ were not the True, then, since Δ is the True, $\Delta \supset \Gamma$ would be the false"(§14).

The same pattern holds for his treatment of generalized hypothetical syllogism (§15), generalized contraposition (§15), and generalized dilemma (§16).[4] He motivates these laws with familiar reasoning about truth-values his functions take when given certain arguments, and these arguments only use the phrase "*das Wahre ist*" ("is the True/False") rather than "*bedeutet das Wahre*" ("designates the True") (Weiner 2005, 328). Explicit semantic ascent is similarly absent in his treatment of concavity introduction, though, as I mentioned above when discussing law IIa, much of the subsequent clarification and discussion in §17 moves between talk of symbols referring to the True, and functions being identical to the True when taking on certain arguments. To sum up: Frege statements of his inference rules are largely syntactic[5], and his arguments for these rules, when he gives them, generally contain no semantic ascent.

[4] These inference rules read as follows:

Generalized hypothetical syllogism: From "$\vdash \{\Theta_1, \ldots, \Theta_n\} \supset \Delta_i$" and "$\vdash \{\Delta_1, \ldots \Delta_i, \ldots, \Delta_n\} \supset \Gamma$" we may infer "$\vdash \{\Delta_1, \ldots \Delta_{i-1}, \Theta_1, \ldots, \Theta_n, \Delta_{i+1}, \ldots, \Delta_n\} \supset \Gamma$"

Generalized contraposition: We may infer "$\vdash \{\Delta_1, \ldots, \neg\Gamma, \ldots, \Delta_n\} \supset \neg\Delta_i$" from "$\vdash \{\Delta_1, \ldots, \Delta_n\} \supset \Gamma$" and vice versa.

Generalized dilemma: From "$\vdash \{\Gamma_1, \ldots, \Gamma_n, \Theta\} \supset \Delta$" and "$\vdash \{\Omega_1, \ldots, \Omega_n, \neg\Theta\} \supset \Delta$" we may infer "$\vdash \{\Gamma_1, \ldots, \Gamma_n, \Omega_1, \ldots, \Omega_n\} \supset \Delta$"

See (Cook 2013, A19-A24) for a helpful overview of each of Frege's inference rules.

[5] In Frege's §48 summary of 17 distinct rules, 15 are expressed purely syntactically. The first exception is his statement of the rule 1, concerning amalgamation of horizontals—in his statement of it, he speaks *both* of the function $-\xi$ and of the horizontal stroke qua sign. The second exception is 9, concerning the replacement of Roman letters. Frege says that we may replace all occurrences of a Roman object-letter by the same proper name or Roman object-marker (and may likewise replace Roman function-letters with function names and markers), and his clarification of what is meant by "same" here speaks of functions directly—"It is to be observed that the argument is not part of the function, and that hence a change of argument-sign does not alter the function name".

§1.4 Two questions

What's Frege up to in these passages? In one sense, the answer is dead obvious: Frege is introducing us to his symbolic language. The section under consideration is, after all, called "Exposition of the concept-script"! But the deeper question is, of course, just how much his introduction to his script aims to accomplish, and what sort of tools Frege had at his disposal to accomplish them.

I want to pay particular attention to the role of Frege's apparent arguments for his basic laws. It's important that two questions be kept separate. The first question is whether Frege's apparent arguments for his basic laws are genuine arguments, or instead merely apparently so. Both Heck and Stanley's correctness reading and my own E.A. reading take the arguments to be genuine, while the elucidatory reading of Ricketts, Goldfarb, and Weiner does not. The correctness and E.A. readings come apart in how they respond to another question—if we grant that Frege was giving genuine arguments, what did he hope to *accomplish* with them? This second question admits of three possible answers:

1) Frege's arguments justify basic laws qua thoughts.
2) Frege's arguments justify the truth of formulas which express basic laws, rather than the laws themselves.
3) Frege's arguments somehow establish that his basic laws are logical.

1) is a non-starter if we acknowledge the importance to Frege of his Euclidian foundationalism—for a logical law to be basic *just is* for there to be no more basic logical principles from which it can be proven. 2) is the answer given by proponents of the correctness reading, and 3) is the one given by the E.A. interpretation. Note, however, that 2) and 3) are not mutually exclusive. Does this mean that the correctness and E.A. readings are compatible? Yes and no. It's compatible with the account I'm putting forward that Frege had all the semantic tools at his disposal for a correctness proof, and that Frege uses them to justify the formulas expressing his basic laws. Where my account differs is in claiming:

i) Frege's purpose in his arguments is not exhausted by 2).

ii) Frege's philosophical goals did not require him to prove correctness.

My argument for ii) will be laid out in §2 of chapter 5, where I address Heck and Stanley's arguments that Frege owed us a correctness proof, on pain of being subject to certain criticisms which Frege himself puts against his contemporaries. In the next section, I'll try to support i), arguing that Frege is trying to elicit a reflective awareness of how a full grasp of his basic laws is sufficient for recognizing them as true.

§2. The E.A. interpretation's view of the data

§2.1 Laws I-IV

In chapter 3, I presented a picture of how Frege conceived of the relation between logic, judgement, and truth, and have explained how this picture motivates generality and undeniability as necessary conditions of logicality. I will now return to the question of what sort of evidence Frege gave us for taking his axioms to be logical. Here are two different claims about what's required to recognize a primitive law as logical:

> <u>UL:</u> Fully understanding a primitive law is sufficient to recognize it as logical.
>
> <u>RL:</u> Recognizing a primitive law as logical requires a reflective appreciation of how our recognition of the law's truth follows from our understanding of that law.

According to UL, once we fully understand a primitive law, we either recognize it as logical, or not—there is nothing more to be said, and no deeper explanation available for why we take it as logical. The heuristic indicator account of Joan Weiner, discussed in §5.3 of chapter 2, is committed to UL (2005, 340; 2010, 57-8) as is Thomas Ricketts (1996, 135) among others. The heuristic-indicators are what gives UL a faint plausibility. It's not as if we understand a proposition, and then feel an inexplicable urge to label it as 'logical' or 'non-logical', with no understanding of what such a label entails—rather, in understanding the proposition we see that it has the sort of generality and undeniability required of logical laws. Weiner, in particular, explicitly acknowledges that appreciating the generality and undeniability of a principle is part of seeing that it's logical (2010, 57-8). RL, on the other hand, ties in with the account of the relation between logical laws and understanding that I attributed to Frege in chapter 3. This is why it's a deeper case for logicality than UL is—RL it's part of an account that picks out an underlying unity in the generality and undeniability of logic.

The early sections of *Grundgesetze* (Frege 1893, §§1-25) include an introduction of Frege's basic terminology, and a series of brief arguments for his axioms. This introduction to his symbolic system clearly serves several goals. For starters, he wanted to give us a firm grip on the syntax of his language. Another aim is elucidation—Frege is quite explicit that not everything can be defined, but that he must nevertheless get us to understand his new primitive terminology (1893, iii-iv). But while attributing to Frege elucidatory aims is sufficient to account for his treatment of his logical vocabulary (including much of the careful discussion of his notation for quantification), it doesn't explain as well what he was aiming at when arguing for his logical laws. My claim is that these arguments also serve the purpose[6] of eliciting a reflective appreciation for how our understanding of the axioms suffices for a recognition of their truth.

Consider the following argument:

> Law I: According to §12, $\Gamma \supset (\Delta \supset \Gamma)$ would be the False only if Γ and Δ were the True while Γ was not the True. This is impossible. Therefore, $\vdash a \supset (b \supset a)$. (Frege 1893, §18).

According to the E.A. interpretation, Frege takes us through the content of the axiom by getting us to reflect upon what the conditions under which the axiom in question would be false, until we realize there *are* no such conditions—no object could ever be both the True and not the True. A modicum of reflection upon the way in which referents[7] contribute to the truth-value of the whole sentence is sufficient to see that it's true—the passage from this recognition of truth to a recognition of logicality comes when, upon reflection, we realize that a full understanding of the sentence, of the sort elicited by Frege's argument together with his explanation of his symbolism, suffices for this recognition of truth. Put in terms of the inferential conception of understanding motivated in the chapter 3, the fact that a supposition that law I is False leads so quickly to an absurdity supports the claim that the inference from the contradictory of law I to a contradiction is a requisite inference—since anyone who denies law I must be subject to some misunderstanding, an understanding of that law is sufficient grounds for affirming it.

Frege's treatment of law IV works the same way—he shows that its denial leads to a contradiction, thereby getting us to recognize that it has the special sort of conceptual undeniability, and hence epistemic analyticity, which suffices for logicality. His arguments for laws II and III (1893, §20) don't proceed by arguing from a denial to a contradiction, but nevertheless get us to see that these axioms are epistemically analytic by exploiting our understanding of the symbols involved. For instance, for law IIa,

[6] Some (notably Heck 2010) have argued that part of Frege's purpose was to establish that his axioms are true and his inference rules truth-preserving. Though I contest this elsewhere, nothing in the current account is incompatible with this reading, as long as it is allowed that Frege's arguments *also* aim to get us to recognize his laws as logical.
[7] One might object by pointing out that in Frege's arguments for his axioms, he most often speaks about when Δ 'is the true' not when 'Δ' 'refers to the true'. I deal with this objection in §5.2 below.

$\vdash \forall \mathfrak{a} f(\mathfrak{a}) \supset f(a)$, Frege assumes the antecedent, and then shows that the consequent follows. This argument, together with his explanation of his notation for quantification, ought to convince us that $[\forall \mathfrak{a} f(\mathfrak{a}) \vdash f(a)]$ is a requisite inference of $\forall \mathfrak{a} f(\mathfrak{a})$—anyone incompetent with this inference fails to implicitly grasp the sense of the notation for first-order quantification, and anyone who *is* competent with this inference will immediately see that law IIa is true. A recognition of this epistemic analyticity is sufficient for a recognition of the laws logicality[8].

If this is correct, Frege is not a simple dogmatist with respect to the logicality of laws I-IV—he aims to give us objectively good reasons for taking our laws to be logical. He does so by means of RL—recognizing them as logical requires that he gets us to think through the content of the laws in question, with the purpose of eliciting a reflective awareness of the fact that doing so is sufficient for a recognition of their truth. He is not (and needn't be) systematic in how he elicits this awareness. For laws I and IV, Frege gets us to see that it's impossible to hold them as false, since doing so would result in a contradiction (which is unjudgeable, as anyone who understood and affirmed a contradiction would need to affirm both conjuncts, which Frege thought impossible). For laws IIa, IIb, and III, he reasons from their semantics to their truth more directly. In both cases, INF helps shed light on how Frege accomplishes this, by connecting understanding with inference.

§2.2 Laws V and VI

Frege's treatment of laws V and VI don't fit into this general pattern—instead of taking us through their content at length, he just cites the passages where the signs which figure in them are introduced. Reflection on the way in which Frege deals with law V reveals the limitations of his approach to convincing us of the logicality of his axioms—his problem isn't the length of his argument for it, which may be seen as a degenerate case of the sort of grounds given for the logicality of his other axioms. Instead, it reveals something crucial about what it takes to make a case for logicality—the oft-mentioned "reflective awareness" of how understanding such a basic law is sufficient for a recognition of its truth must either include, or be augmented with, an appreciation of the conditions under which stipulations successfully secure a reference for terms they involve. Frege's case for law V fails because the stipulations governing law V fail to secure reference.

Frege's final two laws in *Grundgesetze* are:

Law V: $\vdash \big(\acute{\varepsilon} f(\varepsilon) = \acute{\alpha} g(\alpha)\big) = \forall \mathfrak{a}\big(f(\mathfrak{a}) = g(\mathfrak{a})\big)$ (1893, §20)

and

Law VI: $\vdash a \supset \backslash \acute{\varepsilon}(a = \varepsilon)$ (1893, §18)

Whereas for laws I-IV, Frege spends some time taking us through their content, for laws V and VI Frege simply cites the passages in which the relevant logical vocabulary (§3 and §9 for '$\acute{\varepsilon}\phi(\varepsilon)$', and §11 for '$\backslash\xi$') is introduced and discussed. A word first about law VI: this law is (rightly) not regarded as a flaw in Frege's system—it amounts to little more than a formal restatement of the stipulation governing '$\backslash\xi$' given in §11. Frege breaks the stipulation into two cases (1893, §11):

[8] Or is, at the very least, a crucial necessary condition for recognizing logicality. In §5.5 of Chapter 2, I discuss whether the view closest to Frege's was the epistemic analyticity interpretation augmented with an account of how we ought to sort logical from non-logical vocabulary. If this hybrid view is right, then recognizing a formula as epistemically analytic is insufficient to recognize it as logical. I concluded that although some of what Frege says could be plausibly taken to foreshadow the hybrid view, Frege says too little to warrant attributing such a view to him.

1. If Γ is the name of a value-range of a concept that holds of only one object a, then '$\backslash\Gamma$' refers to a

2. Otherwise, '$\backslash\Gamma$' refers to Γ.

Frege's trivial argument in §18 for law VI is:

From the reference of the function name $\backslash\xi$ (§11), $a \supset\backslash\acute{\varepsilon}(a = \varepsilon)$ (1893, §18):

This is a simple, degenerate case of Frege's general strategy of eliciting in us a reflective awareness of the epistemic analyticity of his axioms. The reflective component enters in with his referral to §11. Since §11 secures the reference of '$\backslash\xi$', it secures its sense as well. So, reflecting on §11 gets us to reflect on the sense of VI. Thus, in getting us to reflect both on the sense of '$\backslash\xi$' and of the formula '$a \supset\backslash\acute{\varepsilon}(a = \varepsilon)$', §18 gets us to reflect on how a grasp of the sense of '$a \supset\backslash\acute{\varepsilon}(a = \varepsilon)$' is sufficient for a recognition of its truth, as is required for logicality. Frege in effect provides a trivial, immediate argument for epistemic analyticity of VI.

Can we really see law V as another degenerate case, especially since it too seems, at first glance, like little more than a more formal version of Frege's informal stipulation governing smooth-breathing? I think we can, and we should see it as such. For the moment, imagine that we're quite convinced that Frege's stipulation governing for '$\acute{\varepsilon}\phi(\varepsilon)$' is appropriate and successful. Then the fact that Frege's argument for law V is not as long or involved as his arguments for I-IV is not a crucially important difference—we just don't *need* as long an argument to see that understanding law V suffices for a recognition of its truth. Nor can we say that the difference between law V and the rest is that he doesn't show that a denial of law V results swiftly in a contradiction. His argument for laws II and III don't proceed by showing their denial results in a contradiction, though we could easily provide an argument with that form. And if we're not worried about the appropriateness of Frege's stipulation, his argument for law V could be (unnecessarily) reworked into a similar form as follows: Suppose we deny law V, and hold it as false. By the stipulation governing '$\acute{\varepsilon}\phi(\varepsilon)$', law V is true. But it can't be both true and false—contradiction! Therefore law V is true.

Since Frege's argument for law V is just the same—a mere citation of a stipulation—as it is for law VI (which no one takes to be problematic), the problem cannot be with the simplicity of the argument he gives for it, but must be a difference between the stipulations governing the functions '$\backslash x$' and '$\acute{\varepsilon}\Phi(\varepsilon)$'. The difference is that while Frege's stipulation governing '$\backslash x$' secures that name's referentiality, his stipulations governing the function name '$\acute{\varepsilon}\Phi\varepsilon$' isn't similarly successful. According to Frege (1893, §29):

1) A singular-term 'α' refers if every referring function-name '$f(x)$' yields a referring expression when 'α' is taken as an argument.

2) A first-level function name '$f(x)$' refers if every result of putting in some referring singular-term 'α' for 'x' yields a referring name '$f(\alpha)$'.

3) A second-level function name '$\mu_x\Phi(x)$' refers if every result of putting in some referring first-level function-name 'f(x)' yields a referring expression

It's easy to see that the first-level function name '$\backslash x$' refers, since, as mentioned above, there are only two cases to consider, and reference is secured in both cases[9]. Frege's criteria for reference do not, however, settle whether the second-level function '$\acute{\varepsilon}\Phi(\varepsilon)$' refers. By 3), to know whether '$\acute{\varepsilon}\Phi(\varepsilon)$' refers, we need to know whether the result of substituting a referring '$f(x)$' for '$\Phi(x)$' yields a referring name '$\acute{\varepsilon}f(\varepsilon)$'. But by 1), to know whether '$\acute{\varepsilon}f(\varepsilon)$' refers, we need to know whether the result of substituting it for 'x' in referring functions '$\Psi(x)$' yields a referring function '$\Psi\big(\acute{\varepsilon}f(\varepsilon)\big)$'. If '$\Psi$' is a primitive function-name, our job is easy—it's easy to see from the stipulations governing such function-names that '$-\acute{\varepsilon}f(\varepsilon)$', '$\neg\acute{\varepsilon}f(\varepsilon)$', '$\backslash\acute{\varepsilon}f(\varepsilon)$', etc. all refer. Frege's mistake was to suppose that the claim that

a) '$\Psi\big(\acute{\varepsilon}f(\varepsilon)\big)$' refers for any *primitive* '$\Psi(x)$'

implies that

b) '$\Psi\big(\acute{\varepsilon}f(\varepsilon)\big)$' refers for *all* '$\Psi(x)$'.

It was natural enough to suppose that b) could be established via induction on the number of second-order quantifiers in '$\Psi(x)$', using a) as a base case. But second-order quantifiers bring in a whole new order of difficulty—we've arrived at what Dummett called Frege's 'amazing insouciance concerning the second-order quantifier' (1991, 218). For according to Frege's criteria for the referentiality of second-level function names (1893, §29), a '$\Psi_n(\beta)$' containing n second-order quantifiers will refer if every result of replacing the newly bound second-order variable with a first-order functional expression in turn refers, and these first-order functional expressions might contain unboundedly many second-order quantifiers, something not covered by the induction assumption. Because of such circularity, any attempt to show that the function-name '$\acute{\varepsilon}\Phi(\varepsilon)$' has a referent (including Frege's own attempt in §31), is bound to fail.

Failure of a proof of reference is not a proof of lack of reference. However, we know from Russell's paradox that Frege's seemingly innocent claim in §3—that he uses 'f and g have the same value-range' and 'f and g take the same values for the same arguments' as co-referential—is in fact, false. He *doesn't* use them as co-referential, because one *can't*. Unless Frege thought something false could serve as the basis of a stipulation, §3 and §9 do not suffice to secure a reference for '$\big(\acute{\varepsilon}f(\varepsilon) = \acute{\alpha}g(\alpha)\big)$'. Since '$\big(\acute{\varepsilon}f(\varepsilon) = \acute{\alpha}g(\alpha)\big)$' does not refer and '$\xi = \zeta$' clearly refers, it must be that '$\acute{\varepsilon}\Phi(\varepsilon)$' does not refer—there is no such function from concepts to value-ranges.

Since '$\acute{\varepsilon}\Phi(\varepsilon)$' fails to refer, so does any expression in which it occurs—including law V. So, does '$\acute{\varepsilon}\Phi(\varepsilon)$' have a sense? Frege does allow for referentless terms to have sense—he claims that "Odysseus" has a sense, but no referent (1892a, 32-3/FR 157). But there are several reasons to doubt he would have been willing to pull the same trick with '$\acute{\varepsilon}\Phi(\varepsilon)$'. "Odysseus" is a term in natural language, and '$\acute{\varepsilon}\Phi(\varepsilon)$' is a term in a formal language, and we can't assume that what holds for languages of the former sort hold for the latter—Frege is quite explicit that "scientific rigor" requires we make sure we are never reasoning with referentless signs (1891, 19-20/FR 140-1) and that reference failure is a defect of unrigorous symbolic systems (1892a, 40/FR 163). Secondly, it's often been pointed out that Frege was broadly descriptivist about terms like "Odysseus" (1892a, 27/FR 153), whereas '$\acute{\varepsilon}\Phi(\varepsilon)$' has a totally different character—it's a stipulated primitive term. Finally, in *Grundgesetze,* Frege thinks if we show that a term has a reference, that's *all* that's needed to show that the term has a sense as well—after having shown (or so he thought) in §31 that all names have a referent, he immediately concludes on that basis in §32 that every name has a sense. In the case of syntactically complex terms, such as contentful sentences, there's

[9] We needn't worry about the possibility that '$\backslash\xi$' might be given a non-referring name as an argument—for '$\backslash\xi$' to refer, we require only that '$\backslash\Gamma$' refer for any *referring* name Γ. In particular, if some or all smooth-breathing terms turn out not to have referents, this won't affect the referentiality of '$\backslash\xi$'.

reason to suppose he took some of that syntactic structure to reflect the structure of the sense in question, but this can't be the case with '$\acute{\varepsilon}\Phi(\varepsilon)$', since this function-name is primitive, and syntactically simple. Thus, unlike 'Odysseus', where we can cash out the sense in terms of descriptive content, it's unclear what the sense of '$\acute{\varepsilon}\Phi(\varepsilon)$' could be, other than the way it picks out its referent—if that's the only option, then where the referent of '$\acute{\varepsilon}\Phi(\varepsilon)$' is lacking, sense is lacking as well.

Under the plausible assumption that a sentence cannot express a thought if one of its constituent terms is senseless, law V does not, despite initial appearances, express a thought[10]. This is where Frege's argument for the logicality of law V went awry—he couldn't show that a recognition of its truth follows from our grasp of its sense, because, unbeknownst to him, he didn't *have* such a grasp, for there was no determinate sense to *be* grasped. However, this claim is clearly a contentious one, so suppose instead that I'm incorrect in thinking that Frege's perspective implies that '$\acute{\varepsilon}\Phi(\varepsilon)$' has no sense and law V expresses no thought. Still, '$\acute{\varepsilon}\Phi(\varepsilon)$' and law V have no **sense** where (as suggested in §2.2 of chapter 3) we stipulate that 'the **sense** of ψ' means whatever the referents of the sub-sentential expressions of ψ contribute to the truth or falsity of ψ. Frege's argument for law V goes wrong because he didn't elicit a reflective awareness of how a grasp of the **sense** of law V is sufficient for a recognition of its truth—he couldn't have, because law V *has* no **sense**. Even if it turns out that Frege, in the end, thought that sense was more course-grained than **sense**, what's said in this account should still stand.

The upshot is that Frege's problems with law V don't show something wrong with what he *did* do to show his laws were logical—taking us through their meanings to get us to see how understanding them is sufficient for recognizing them as true. Rather, the problem lies with Frege's failure to show that his stipulations secure referents for all of his primitive symbols. Here are two ways in which successful demonstrations of the referentiality of primitive terms correlate to the case Frege makes for logicality of laws in which those terms figure:

i. <u>Success presupposed:</u> That all terms in a formula refer is something which must be presupposed before questions concerning the logicality of the thought expressed by that formula can be legitimately posed.

ii. <u>Proof of success required:</u> Showing that the stipulations secure reference for a term is *part of* the case for the logicality of any formula in which that term figures.

For most of Frege's terms—his horizontal, negation, and conditional strokes, for instance—it's quite obvious that the stipulations governing them secure reference, according to the §29 conditions for referentiality. Frege tried (and failed) in §§29-31 to show that *all* of his symbols refer, but the placement of §31 makes it unlikely that he saw this as a part of showing a law is logical. Here, and in his discussion of '$\acute{\varepsilon}\Phi(\varepsilon)$' in §10, it's likely he was more concerned with avoiding the danger of reasoning with referentless symbols, which scientific rigor demands we take pains to avoid for independent reasons. This, together with his 1892 claim that the referentiality of terms we use is generally something we just presuppose (1892a, 31-2/FR 156) suggests Frege would have opted for i)—given that our stipulations *in fact* secure reference, in order to prove logicality we require just that one shows that grasping what these referents contribute to the truth of the law is sufficient to recognize the law as true.

[10] Of course, after his flawed §31 proof that every name in his language has a referent, he concludes on this basis that every sentence in his language expresses a thought (1893, §32). But this doesn't suggest that he was at all worried about whether law V expresses a thought—he thought it was an unproblematic assumption, as seen by the fact that he maintains on several occasions that mathematicians reason in accordance with it (1893, vii; §9; 1903a, §147).

But if Frege took i), then, unless we read him as some sort of externalist, he must be a fallibilist about whether his arguments for logicality succeed—even if we *think* we've recognized a law as epistemically analytic, we might later realize that the law is no law at all, due to a failure of reference of terms figuring in its expression. Not much rides on whether Frege's strategy is i) or ii). Determining the conditions under which stipulations secure reference is a trickier matter than Frege appreciated. Regardless of whether we think of this determination as being part of the case for logicality or not, it's something which needs more careful treatment than Frege gives it.

§3. Some clarifications of the E.A. interpretation

§3.1 The role of semantics in the E.A. interpretation

Now to clear up a potential ambiguity in my account. The ambiguity concerns the role of semantics in the case Frege makes for logicality. Distinguish between the following two questions:

a) Is ψ a logical law?
b) What are we doing when we demonstrate that ψ is a logical law?

Frege doesn't say much explicitly about these two questions, but, as I argued, must have had something to say about why we're justified in taking his basic laws to be logical if logicism is to amount to an interesting thesis. So, he clearly needed a way of answering a), given a basic law ψ. My suggestion has been that he does so by determining whether ψ is epistemically analytic. Frege provided no explicit answer for b)—it's been my contention that he should have, and that my account makes explicit a method already implicit in Frege's practice. One definitely needs to explicitly talk about semantics in order to answer b)—for a formula to be epistemically analytic is for it to be justifiably recognizable as true for anyone who understands that formula, and giving a Fregean account of understanding requires one talk of reference. What's not so obvious is whether one must *also* explicitly talk about semantics in order to answer a). Distinguish between the following two versions of the E.A. interpretation:

> <u>Version 1 (V1):</u> To be justified in taking ψ to be logical, it's necessary to have explicit knowledge of the semantics of 'ψ', the concept-script expression of ψ. In particular, we explicitly recognize that the truth of 'ψ', the concept-script expression of ψ, follows from the stipulations governing the primitive terms in 'ψ'.
>
> <u>Version 2 (V2):</u> To be justified in taking ψ to be logical, it's *not* necessary one have explicit knowledge about the semantics of ψ. It *is* necessary that one be taken through the content of ψ, but one needn't *know* that this is what's happening.

Semantics figures in the story I tell about why we're justified in taking a basic law to be logical. The open question is whether semantics figures in the justification itself, as well as in an account of it.

There are at least two major reasons to favor V1 over V2. Firstly, when I say that one purpose of Frege's arguments is to elicit a *reflective awareness* of how the understanding of certain formulas is sufficient for recognizing those formulas as true, this statement isn't much help unless one can specify the sort of reflective awareness in question. In particular, it needs to be given a gloss that prevents my account from collapsing back into that of the elucidatory interpretation. Both the E.A. and elucidatory interpretations agree that Frege thought understanding a basic law is sufficient for recognizing it as true—where we differ is that I take the case for logicality to involve an explicit recognition of this fact, whereas proponents of the elucidatory interpretation take understanding a formula alone to be sufficient grounds for recognizing it as logical. V1 makes the distinction clear—what one is reflectively aware *of* when attending to Frege's arguments are the semantic properties of the formulas which express his basic laws.

Secondly, V1 is what allows Frege's method of determining logicality to sort logical truths from geometrical truths, which Frege, following Kant, took to be synthetic a priori. Consider the following quote:

> I can only say: so long as I understand the words 'straight line', 'parallel' and 'intersect' as I do, I cannot but accept the parallels axiom. If someone else does not accept it, I can only assume that he understands these words differently. Their sense is indissolubly bound up with the axiom of parallels. (Frege 1914, 247)

This quote, provided purely for illustrative purposes, is quite at odds with what Frege says elsewhere. This isn't so surprising—"Logic in Mathematics" is an unpublished work in which sense is treated rather differently than it is in Frege's published writings.[11] Frege's quite explicit elsewhere—(1885a, 94, CP 112; 1884, §14) to choose two examples—that the sorts of confusion that result from denying logical and non-logical laws are importantly different. And Frege is elsewhere explicit that the difference concerns sense:

> The assertion of a thought which contradicts a logical law can indeed appear, if not nonsensical, then at least absurd; for the truth of a logical law is immediately evident of itself, from the sense of its expression. (Frege 1923-6, 50/CP 405)

I think Frege's 1914 quote can be used to illustrate an ambiguity in the notion of epistemic analyticity. And this is that, depending on how it's stated, it can apply to synthetic a priori truths. Compare:

> <u>Weak Epistemic Analyticity:</u> ψ is weakly epistemically analytic if understanding ψ is sufficient for being justified in taking ψ as true.
>
> <u>Strong Epistemic Analyticity:</u> ψ is strongly epistemically analytic if understanding ψ is sufficient for being justified in taking ψ to be true, and this justification is rooted in that understanding alone.[12]

Weak epistemic analyticity doesn't help one distinguish between the logical and the non-logical. Geometrical truths are weakly epistemically analytic, because understanding ψ them is sufficient condition for recognizing ψ as true on the basis of spatial intuition. If we're to be Kantians about geometry, then, at least for creatures like us, who bring spatial intuition to the table when enquiring about the truth of, say, the axiom of parallels, then upon understanding this axiom, there's no separate act of attentiveness which must be accomplished in order for such intuition to take effect, and hence for us recognize the axiom as true. That is, it's not that one must first understand the axiom then sit cross-legged in a quiet place to consult one's intuition—intuition is, and must be, something already employed. After all, if the recognition, on the basis of intuition, of a geometrical truth ψ was something that happens only at some point *after* we fully understand ψ, then the process of determining whether a given axiom is logical would be more straightforward—find a neophyte mathematics student who's never encountered the axiom, get them to understand it, and then ask them whether they recognized it as true immediately afterwards, or only after some additional reflecting, during which their intuition plays a role. Geometrical

[11] For one, in this work Frege seems to take sense to be something *deep*. Competent users of natural language might only partially grasp the sense of a term such as "number"—Frege uses the analogy of seeing it as if through a mist (1914, 217). This perspective, while occasionally alluded to (Frege thinks that the same thought is grasped and investigated by different thinkers, of different languages, in different ages), is hardly central in *Grundgesetze*. Another anomaly is Frege's insistence here that analysis articulates sense (1914, 210-1), where it's difficult to see how this could be consistent with the examples of analysis found in his other works (Blanchette 2012, 79-82).

[12] This distinction of mine is similar to Jeshion's distinction between propositions which are self-evident through intuition vs. those which are logically self-evident (2001, 959)

truths will fail, however, in being strongly epistemically analytic—we cannot, by reasoning about the semantics of ψ alone, show that 'ψ' designates the True.

Here's a quick summary of what the case for logicality will look like for each version of the E.A. interpretation:

Version 1:

i. The truth of 'ψ', the concept-script expression of ψ, follows quickly from the stipulations governing the primitive terms in 'ψ'.
ii. So, understanding 'ψ' is sufficient for recognizing 'ψ' as true, and this recognition of truth is rooted solely in conceptual understanding.
iii. So, 'ψ' is strongly epistemically analytic as is required for ψ to be logical

Version 2: The person investigating the logicality of ψ is taken through the content of 'ψ', a concept-script expression of it. They needn't know that this is what's happening. They think as follows:

i. I came to fully understand 'ψ', and thereby recognized it as true
ii. So, understanding 'ψ' is sufficient for recognizing 'ψ' as true
iii. So, 'ψ' is weakly epistemically analytic, as is required for ψ to be logical

Version 1 is the one to which I commit myself. Note that strong epistemic analyticity does not amount to a reintroduction of the notion of metaphysical analyticity. ψ is metaphysically analytic if it is true *because* of meaning—if, as Paul Boghossian puts it, if "in some appropriate sense, it *owes its truth-value completely to its meaning*, and not at all to 'the facts'" (1997, 334). Metaphysical analyticity is distinct from strong epistemic analyticity, so long as we acknowledge the *very* crucial distinction between reasons for *being* true, and reasons for *our taking* something to be true (Burge 1992, 314-5). To ask why ψ is true is to ask a question about the objective justificatory ordering in which it stands—if ψ is non-basic, its truth is grounded in the more fundamental truths in which it's is grounded, and if it is basic, then we've hit bedrock. When we ask why *we* should take it to be true is to ask a different question—if ψ is non-basic, our reason for taking it to be true will be *the fact that* it was proven from a more fundamental truth (a proof of a thought from thoughts which may be demonstrated via a derivation of formulas from formulas), and if ψ is basic, our reason for taking it to be true will involve ineliminable reference to the semantics of a concept-script formula which perspicuously represents it.[13]

§3.2 The role of sense in *Grundgesetze*

In §2.2 of chapter 3, I claimed that my position could be held even if Frege did not, in fact, hold the following:

> Linguistic sense (LI): The sense of a term is just whatever that term contributes to the truth or falsity of sentences in which it occurs.

Although my position is independent of the truth of this claim, this conception of sense is clearly a natural fit with the present view. We could then paraphrase my interpretation as follows—to convince us that his basic laws are logical, Frege elicits our recognition of their epistemic analyticity by clarifying their sense. That is, his arguments get us to reflect upon the sense of the propositions which express his basic laws. I do not propose to mount a defense of LI here—not just because it's unnecessary for the success of my interpretation to do so, but also because I think it evident that Frege's view of sense shifted throughout his career. However, I do want to refute one of the primary obstacles to the sense-clarification

[13] In addition to the Burge passage cited, see (Sullivan 2004, 735-741) for an illuminating discussion, to which the present use of the distinction owes much.

gloss of my interpretation—that this can't be what he's up to in his arguments for his basic laws, because *Grundgesetze* says fairly little about sense.

An abiding element in Frege's writings is his repeated admonitions against the danger of reasoning with empty symbols. The system found in *Grundgesetze* is a *calculus ratiocinator*, with mechanically checkable deductions, but there is no *point* to these deductions if not all symbols involved are properly endowed with content—we would find ourselves playing a mere game with words (1903a, §§89-95). This gives rise to Frege's requirement that every symbol, and combination of symbols, in his formal system be meaningful. In "Function and Concept", he says it is "demanded by scientific rigor" that we have provisos which prevent the formation of meaningless compound expressions (Frege 1891, 19-20/FR 140-1). Frege makes it clear elsewhere that this requirement is in place for both sense and reference—proper names and concepts must have both, if we are to make use of them in a rigorous science (1892-95, 124-5).

These comments were all written within a few years of the publication of *Grundgesetze*, so Frege would have had the requirement concerning the sense and reference of his symbols firmly in mind while writing his Magnum Opus. Strange, then, that *Grundgesetze* says fairly little about sense, while he says a great deal directly about reference, such as when he provides stipulations governing the reference of complex formula containing his various function symbols. But, of course, this puzzle is no real puzzle, because for Frege, "it is via a sense, and only via a sense that a proper name is related to an object." (1892-95, 124-5). We cannot fail to involve ourselves with sense—whenever we say something about the reference of a name, simple or complex, this will tell us something about sense, even if only that the name has one and that it determines that referent. Thus, although virtually all of Frege's explicit discussion of meaning in *Grundgesetze* concerns reference, sense is always in the background. For instance, in §32, after giving his (flawed) proof that every name has a reference, he immediately concludes that every sentence expresses a thought.

In Frege's discussion of definitions and basic terms found in his 1914 "Logic and Mathematics", he says that

> The effect of the logical analysis of which we spoke will then be precisely this—to articulate the sense clearly. Work of this kind is very useful; it does not, however, form part of the construction of the system, but must take place beforehand. Before the work of construction is begun, the building stones have to be carefully prepared so us to be usable; i.e. the words, signs, expressions, which are to be used, must have a clear sense, so far as a sense is not to be conferred on them in the system itself by means of a constructive definition"(Frege 1914, 211)

There are two things to take from all this. Firstly, the fact that Frege was so concerned with articulating sense, and with assuring us that all formulas in his system expressed distinct thoughts, lends plausibility to the E.A. interpretation—we would expect him to be clarifying sense, since he says elsewhere that this is a requirement which must be met by systems of logic. The second thing to take from this is that the relative lack of *direct* discussion of the sense counts not at all against the sense-clarification version of the E.A. interpretation. In Frege's way of seeing things, we involve ourselves with sense any time we speak of the referent.

§3.3 Differences from Burge's view

It's also important to distinguish the version of the E.A. interpretation being expounded here from its closest competitor—the interpretation of Tyler Burge, from which the current account draws much inspiration. I take Burge to have argued decisively for the following points—firstly, that Frege conceived of logical truths as standing in a system-independent rational order with objectively basic and unprovable truths at the bottom, and secondly that in the introduction to *Grundgesetze*, Frege is eliciting the self-evidence of basic-laws, rather than justifying them, or arguing for the truth of formula as part of a correctness proof. What my account adds to this (in addition to arguing for the conclusion from a different angle) is that I take the self-evidence had by logical laws to stem from their epistemic analyticity, and that I emphasis the role semantics plays in getting us to recognize epistemic analyticity, and hence in getting us to see that laws have the sort of self-evidence required for logicality.

It is, of course, quite consistent for Burge to maintain that the process of eliciting self-evidence is done by drawing attention to the semantics of formulas. The bigger difference between our views is the role played by pragmatic considerations in eliciting the self-evidence of basic laws. Burge argues for a connection between what he calls "pragmatic considerations" and the recognition of self-evidence (Burge 1998), and in an earlier article (Burge 1990), argues for a similar connection between pragmatic considerations and sense-clarification (as he uses the term, "pragmatic considerations" encompasses general theoretical considerations of fruitfulness, independence, and simplicity of axioms—I shall follow this usage here). His reasoning, in brief, is as follows—self-evidence of a proposition is more objective than mere obviousness, and rather more akin to being independently recognizable as true by anyone who fully understands it (1998, 324). As Burge understands it, this full understanding may, however require one to first appreciate certain inferential connections (Burge 1998, 341). Where this conception sharply differs from my own account is that for Burge, logical theory-building helps us appreciate these inferential connections. Insofar as pragmatic considerations are part and parcel with such theory building, pragmatic considerations help us identify inferential connections, and determine which truths are self-evident (Burge 1998, 340-341). Burge also thinks that pragmatic considerations can play the role of a sort of secondary justification—even if a certain basic logical law is not quite self-evident to *us*, we can take the simplicity and fruitfulness of that law as a weaker sort of evidence that the law would be self-evident to a reasoner whose powers exceed our own.

The view I put forth here is committed to no such connection between pragmatic considerations and self-evidence. While I take pragmatic considerations play a crucial role in Frege's selection of axioms and inference rules, and in his identification of laws as objectively basic and unprovable, they play no role in eliciting our recognition of self-evidence. I grant it's plausible that gaining the requisite level of understanding to recognize self-evidence requires appreciating some of the inferential connections in which that thought figures—this figures in my account with the claim INF. It's also plausible that logical theory-building requires pragmatic considerations. But it's *far* less plausible that identifying inferential connections requires those elements of theory-building in which the pragmatic considerations come into play. Think of it this way—in Frege's view, axioms must be objectively basic, and unprovable. And it's totally unclear how one could identify truths as such, unless by pragmatic considerations, since Frege is adamant that we must not rely on *mere* obviousness. But one could be in perfect agreement with Frege about what inferential connections exist between thoughts, while leaving the question of which laws are objectively basic out of the picture entirely. Our axioms and basic inference rules help generate more inference rules (or more exactly, conditionalized versions thereof on which we can use modus ponens), but for identifying inferential connections, it is of no consequence which laws we take to be objectively basic. If the role of pragmatic considerations is in identifying such basicness, then it has no role in helping us see inferential connections, and thus no role in eliciting the self-evidence of basic laws by clarifying their sense.

§4. Replies to some objections

§4.1 Is this circular, and if so, should that bother us?

It's worth taking a moment to discuss whether this picture is circular and, if it is, whether this should bother us. Frege certainly gives no grounds for adopting classical logic that would convince someone not antecedently sympathetic to its dictates—no proponent of, say, paraconsistent or relevance logic is going to be much tempted to abandon their view in light of Frege's arguments. For instance, Frege gives no further grounds for his claim that one may not assert both P and $\neg P$—he simply takes it as basic. What's more, if my interpretation is correct, classical logic is *built in* to Frege's conception of understanding—it forms a constraint not just on when judgements are correct, but on what it *is* to judge, insofar as judging requires understanding. I think that this points to a monolithic view of logic. It's a consequence of the present interpretation that for Frege, there is no stepping outside of logic, no external vantage point from which we may justify its laws. We can now see the exact sense in which it's true—it's not that we cannot have an external viewpoint on a language, or that semantic theorizing is impossible. Rather, it is that logic is part of our reasoning in a deeper way than merely being the laws in which one must follow if one is to attain truth—logic is something any thinker *must* bring to the table if they are to see others as reasoners at all.

There's a more specific charge of circularity which really would be damaging. While Frege didn't set out to do anything like defending classical logic from alternatives (indeed, in his own time, rigorous alternatives were few), he does attempt, from within the viewpoint of classical logic, to give us objectively good *reasons* for taking certain truths to be logical. Some principles are logical because they can be deduced from others which are logical—it is this that Frege aims at in proving arithmetic from his axioms. But once we hit upon the primitive truths, Frege's account really *would* be viciously circular if, when asked why a supposedly primitive law A is logical, the answer is that some other law B is logical—we should then be in danger of an endless loop, or at least a regress, as we constantly explain logicality of some principles in terms of the logicality of others.

Are Frege's arguments circular in this way? Consider law I of *Grundgesetze*. It would be circular if Frege argued thusly:

If we deny law I, we would affirm a contradiction.
<u>But the law of non-contradiction is a *logical* law, and thus undeniable.</u>
∴ So law I is undeniable.

If *this* had been his argument, we should then have to give an account about why LNC is logical. The response is simply that Frege argues nothing of the sort—in the reconstruction I'm pushing, the impossibility of affirming a contradiction is explained not in terms of the logicality of the law of non-contradiction, but in terms of the impossibility of simultaneously affirming P and $\neg P$. [14]

Another way to fall into vicious circularity would be if, in order to establish the logicality of an axiom A, Frege's case rested on the following premise:

<u>Premise (P):</u> $\perp$ follows *logically* from the contradictory of A.

[14] As I mentioned in §2.5 of chapter 3, this impossibility is particularly fundamental for Frege—he makes no attempt to justify this claim, nor would there be anything he could appeal to for support. Recall also my argument in a footnote in the same section that Frege wouldn't have seen a deep difference between L.N.C. and the exclusivity of truth-values.

For if the claim that A is logical requires argument, so does (P). How would one establish (P)? Frege thought derivations in his concept-script were the best way of showing that an inference is logical, by showing that it makes use of the premise together with logical laws (let $B_1, B_2, \dots B_n$ be some such laws), rather than some unnoticed intuitions. But if that's how he argued for (P), he'd be relying on the claims $B_1, B_2, \dots B_n$ (which may or may not be include the claim A among them) are logical. This isn't quite a circle yet, but since the logicality of these laws in turn needs support, we'd soon find ourselves in a vicious circle, or regress, as before. So supposed instead Frege argued for (P) in some *other* way. If we're required to show that *this* new argument is in turn a logical one, another regress threatens—and if we are *not* required to show the argument for (P) is logical, then it becomes unclear why we needed to argue for (P) in the first place—why not just argue from $\neg A$ to $\perp$, without also trying to show that this argument is *logical*? But on the account I'm putting forward, that's just what Frege's doing—his case for the logicality of A does not rest on (P), for while $\perp$ must *actually* follow logically from $\neg A$, Frege need make no use of this logicality as a *premise*. He again avoids vicious circularity and regress in his case for the logical nature of his basic laws.

§4.2 Actual vs hypothetical inferences

A more minor, though not unimportant, objection concerns a peculiar constraint Frege believed thoughts needed to satisfy in order to be used as a premise in any inference. Frege makes the following claim:

> JP: Only thoughts which we judge can be premises.

He first states JP explicitly in his "Foundations of Geometry II" (1906a 387/CP 318), and repeats the claim in later works, and in several letters[15]. JP is incompatible with the reading I give of the grounds for undeniability which I take to be implicit in Frege's arguments for his basic laws. This leaves us with three options—reject my reading, modify my reading, or retain my reading and maintain that Frege's commitment is the product of some confusion. I'll be taking the third option, on the grounds that JP conflicts with Frege's practice in any case.

Recall that in Frege's argument for law I, $\vdash a \supset (b \supset a)$, he considers what would have to be the case if a and b were replaced by a Γ and Δ such that $\Gamma \supset (\Delta \supset \Gamma)$ is the False. He gets us to see that in such a case, Γ would have to be both the True and not the True, which is impossible, and he concludes that $\vdash a \supset (b \supset a)$. I parsed this in terms of requisite inferences—I take it that the inference from the contradictory of law I, $\sim\big(a \supset (b \supset a)\big)$, to a contradiction is a requisite inference of "$\sim\big(a \supset (b \supset a)\big)$", so that anyone who denied law I (hence affirming $\sim\big(a \supset (b \supset a)\big)$) and failed to infer a contradiction would have their understanding of law I called heavily into question. But there's a problem—for any basic logical truth 'δ', how can I say, for any Ψ, that $\sim\delta \vdash \Psi$ is a requisite inference of 'δ'? For according to JP, one who performs the inference $\sim\delta \vdash \Psi$ would need to judge that $\sim\delta$. Judging that $\sim\delta$ is tantamount to denying that δ. But, as discussed previously, basic logical truths δ are conceptually undeniable!

There are a few ways one can respond to this. The first is to point out that I characterized requisite inferences in terms of competence—the view I've sketched does not require that one who understands the contradictory $\sim\psi$ *actually* perform the impossible inference $\sim\psi \vdash \perp$. Competence in inferring means that one could be relied on to infer if one were in the appropriate circumstances—in the case considered here, there *are* no such appropriate circumstances, since ψ is undeniable.

[15] For a helpful summary of places where Frege reiterates his commitment, see (Currie 1987, 56).

There are several issues with this response. Firstly, it's a bit odd to think of someone as being competent in doing something impossible. It would be strange on my part to claim competence in squaring the circle or writing out the general quintic equation, as these things can't be done. It may be that some sense could be made of this if we take competence to be competence in reasoning with forms—perhaps we can be said to be competent with the inference $\sim\psi \vdash \perp$ because we are competent with making those *sorts* of inferences. But even if something along these lines were workable, it wouldn't be enough to salvage any merit for the competence response—for in order to see that ψ is undeniable, one must actually make the inference in question.

Another option would be to separate two slightly different ways in which the notion of "requisite inferences used in reconstructing Frege's argument for the logicality of law I:

> Actual Requisite (AR): Frege thought $\sim\psi \vdash \perp$ is a requisite inference of $\sim\psi$.
>
> Hypothetical Requisite (HR): Frege only thought that *if* we could judge $\sim\psi$, then $\sim\psi \vdash \perp$ would be a requisite inference of ψ. But since $\sim\psi$ is unjudgeable, it has no requisite inferences.

If we see Frege as doing HR rather than AR, we can see what he's doing as compatible with the impossibility of judging $\sim\psi$. The problem isn't gone entirely—surely we must *understand* $\sim\psi$, even if we can't *judge* it—and if we understand it, it should have requisite inferences, of which $\sim\psi \vdash \perp$ is a plausible candidate. But we can then appeal, as before, to the non-restrictive notion of competence at play. There will still be a worry about whether we can make good sense of being competent in an impossible act, but the second worry drops away. If Frege held HR, then to see that ψ is undeniable, we needn't actually *make* the inference $\sim\psi \vdash \perp$—it suffices that we appreciate that we *would* have to make this inference if we could judge $\sim\psi$ (which we can't).

But this is no good—in addition to being left with the oddness of a notion of competence in activities we could never perform, seeing Frege as doing HR rather than AR means that we'd need to make sense of impossible counterfactuals. Even if we can make sense of what would be the case if something impossible were the case, any such account certainly seems out of place in Frege exegesis! In offering a rational reconstruction of an account determining logicality which is implicit in Frege's practice, surely it's too much to read a theory of impossible counterfactuals into his work.

If JP was tantamount to the claim that we can only infer from things which we take to be true, then perhaps some sense can be made of Frege's position—to infer just is to take something to be true on the basis of something you also take to be true. This conception might fit in well with a conception of inference governing what one may permissibly assert on the basis of what else. But Frege's claim is even stronger than this—with regards to JP, at least, Frege clearly takes judgement to be factive. He says quite explicitly that we can't infer anything from false thoughts (1918b, 145/FR 348) , and says the same in an earlier correspondence, namely an undated letter to Philip Jourdain (Frege 1980, 79) and in a 1917 letter to Hugo Dingler (Frege 1980, 16).

And this position is a deeply peculiar one—it's certainly prima facie implausible that there should be no inferential connections between false propositions[16]. And if we're to see Frege's arguments for basic laws in *Grundgesetze* as genuine examples of argumentation (which the proponents of the elucidatory interpretation will not grant—I'll deal with that in §1 of the next chapter) then the most natural reading is that he just *is* making inferences with false premises in these arguments, as part of a reductio aiming to

[16] Gregory Currie gives a thorough discussion of why Frege might have been tempted to hold this view, concluding that "The theory of inference embodies his preference for a certain epistemological ideal: knowledge as a deductive system grounded in self-evident axioms" (1987, 64). As plausible as this explanation is of why Frege held that only true thoughts can be judged and used as premises, it does little to make the claim more plausible.

show that these premises should be denied. Frege occasionally opines that since an unjudged thought ψ can't serve as a premise, when we want to infer $\psi \vdash \phi$, we must instead argue directly for thoughts of the form $\psi \supset \phi$. But this isn't what Frege actually *does*—in his argument for law IIa, he argues from $\forall \mathfrak{a} \Phi(\mathfrak{a})$ being the True to $\Phi(\xi)$ being the True, and concludes that $\vdash \forall \mathfrak{a} f(\mathfrak{a}) \supset f(a)$, even though he can't judge that $\forall \mathfrak{a} \Phi(\mathfrak{a})$. Gregory Currie (1987, 56) points out that Frege, in the previously cited letter to Dingler (1980, 17), says that we may transition from:

$$\frac{\begin{array}{c} 2 < 1 \\ \text{If something is smaller than 1, it's greater than 2} \end{array}}{\therefore 2 < 2}$$

Frege is at pains to say that we are deriving this in a "purely formal way", and that this isn't a genuine inference. But it shows that we must have grounds for allowing something very *like* inference which can relate false propositions. And there's no cogent reason that this inference-like relation need be thought of as purely syntactic.

With this in mind, I propose we take the final option—accept that the account I give is inconsistent with JP, and bracket this commitment to JP. While this isn't a line to be taken lightly, no interpretation dealing with material of this complexity can hope to be consistent with *all* of the commitments of the source being interpreted. In particular, there's a tradeoff between explaining how particular explicit commitments are motivated, and explaining the conception which most naturally underlies the argumentative practice (in this case, the mathematical practice) of the author in question. As we shall see in the next chapter, the elucidatory interpretation allows one to retain some theoretical positions of Frege (it's certainly consistent with JP) but fares worse in accounting for the specifics of Frege's argumentative practice.

§4.3 Overgeneration

The final worry which I want to address is the concern that the method of sorting logical from non-logical truths which I attribute to Frege overgenerates—that it labels all logical truths as such, but leads to some non-logical truths being incorrectly categorized as well. Note that this is only a problem if we take epistemic analyticity to be a *sufficient* condition for a basic law to be logical. In §5.5 of chapter 2, I raised the possibility of merging the E.A. interpretation with Heck's version of the demarcation account—we could take epistemic analyticity to be a necessary condition for logicality, but demand that these truths be maximally general as well, and this would defuse the counterexamples given below. I'll argue below that worries about overgeneration don't have as much bite as one might think, but keep in mind that not a great deal hangs on this—my primary contention is that getting us to see that a law is logical inextricably involves getting us to recognize it as epistemically analytic, and this is perfectly consistent with supposing that the fleshed out view of logicality which fits closest with Frege's comments and commitments will be one which solves the demarcation problem.

There are *many* sorts of truths which have a fair claim to be epistemically analytic, and only some of these seem plausibly logical. Here are some truths which one might take to be epistemically analytic:

1) If the Shard is in London, then the Shard is in London
2) 7=2+5
3) Hume's Principle: The number of Fs equals the number of Gs if the Fs and the Gs can be put into one-to-one correspondence.
4) I am here
5) All bachelors are unmarried
6) Whatever is colored is extended (Boghossian 1997, 338)
7) Whatever is red all over is not blue (1997, 338)

1)-3) are plausibly epistemically analytic. And Frege would categorize them as logical—they're all derivable from what he regarded as basic, objectively unprovable, self-evidently analytic truths. What about 4)-7)? In contemporary logic some proponents of two-dimensional semantics, take 4) to be a logical truth, but 5)-7) certainly don't look like prime candidates for logical truths. And it's no good to object by pointing out that these truths might well not be basic—although it's not necessary for non-basic laws to be epistemically analytic in order to be logical, there are no good grounds for holding epistemic analyticity to be sufficient for logicality in the case of basic laws *alone*. So, do epistemically analytic truths which are non-logical show a flaw in Frege's method of identifying logical truths?

There are at least three different responses one can give to this over-generation concern. The first response is to deny that any sentences in the the list above are logical truths because they contain terms which won't figure in a rigorous scientific language. Frege thought all terms in a scientific language must have a sharply determined reference (1891, 19-20/FR 140-1) and also that natural language words with borders that are even slightly vague are referentless (Frege 1885b, 159-60; 1892-95, 133/FR 178; Burge 2005, 246). For example, if "is red" is slightly vague and not defined for some input then 7) is referentless, thus not true, and thus certainly not a candidate for being a logical law. It's also unclear whether Frege thought that indexicals should appear in a scientific language. And generally speaking, Frege doesn't seem directly concerned with natural languages when he raises questions concerning their logicality, so it may be that questions concerning the logicality of thoughts could only be legitimately raised for thoughts couched in a scientific language. However, it's not clear 4)-6) in the list above contain vague terms. So, let's assume that everything in the list above could be legitimately expressed in an extension of Frege's concept script—if this assumption is unwarranted, then it just gives us fewer cases to consider, and makes the job of refuting concerns about over-generation all the easier.

The second response would be to highlight the distinction between strong and weak epistemic analyticity. Again, a sentence is weakly epistemically analytic if it just so happens that understanding it is always sufficient for our recognizing it as true, whereas strong epistemic analyticity only holds if that recognition of truth is due entirely to understanding, with no admixture of intuition. Although it depends partly on how the notion of a recognition being due to understanding is fleshed out, this distinction may be sufficient to take care of some of the examples above, if these examples turn out to be epistemically analytic *only* in the weak sense.

Finally, one could just bite the bullet, and maintain that Frege would have, upon reflection, been willing to admit truths like the ones above as logical. It's clear enough that 4)-7) can't be derived from Frege's axioms, but this doesn't conclusively show that they're non-logical, for Frege did not intend for his logical system in *Grundgesetze* to be complete[17], and Frege elsewhere is open to the possibility of us identifying new unprovable logical truths (1897b, 363/CP 235). That is, we just don't see logical truths like "all bachelors are unmarried" in Frege's writings, because Frege's purposes didn't require him to be working in systems which contained the terms which would allow him to formulate such truths.

[17] Law VI, $\vdash a = \backslash\acute{\varepsilon}(a = \varepsilon)$ (1893, §18), only captures part of the stipulation governing '$\backslash\xi$' which Frege gives in §11, the case where ξ is substituted with the name of the extension of a concept holding of a single object. But Frege says $\backslash\xi = \xi$ when ξ is not substituted with the name of such an extension—were Frege's deductive system not unintentionally trivial due to law V, he would have had no way to derive

$$\vdash \neg\exists F(\exists x \forall y(Fy \supset y = x) \land \xi = \acute{\varepsilon}F\varepsilon) \supset \xi = \backslash\xi$$

§5. Other readings of Frege's arguments

Now that the clarifications and disambiguations of, and some objections to, my own account have been covered, I'll argue how the E.A. interpretation fits better with the way in which Frege argues for his basic laws than either the elucidatory or correctness interpretations. I'll restrict my focus here to relatively narrow considerations concerning how Frege states his arguments, considerations which the elucidatory and correctness interpretations use as evidence for their views. I'll postpone until the next chapter a discussion of the deeper philosophical motivation for these views.

§5.1 Weiner on Frege's arguments

Joan Weiner's article "Semantic Descent" sets out to establish a key component of the elucidatory interpretation—Frege didn't engage in genuine metatheory. While she readily admits that Frege introduces, discusses, and argues about his concept-script, her main contention is that Frege doesn't engage in metatheory in the sense of justifying his laws by employing a truth-predicate. Part of her argument rests upon a parallel she argues holds between Frege's treatment of truth, and his treatment of concepthood (Weiner 2005, 322-333)—I'll delay discussion of this argument until §1.3 of the chapter 5.

Weiner's second argument is that a close examination of the passages where Frege introduces and argues for his symbolism reveals he does not use a truth-predicate. She says that one mustn't think of Frege's horizontal stroke as a truth predicate, even though '$-\Gamma$' denotes the True for every true sentence we substitute in for Γ (2005, 328-332). For the function denoted by '$-$' is a function from objects to truth-values. Since sentences, for Frege, refer to truth-values, rather than the thoughts which determine those values, '$-(1 = 1)$' can't really be said to predicate anything of either '$1 = 1$' or of the thought it expresses. For the same reason, the natural language expression "is the True" (*das Wahre ist*) can't be considered a truth-predicate, since, again, what is or isn't the true are the truth-values denoted by sentences, not the thoughts those sentences express.

Weiner points out that Frege's argument for law I only makes use of "is the true" rather than "denotes the True" (*bedeute das Wahre*) (2005, 336) and that the same is true for his arguments for his inference rule (2005, 337). Her impression:

> It may seem surprising that Frege avoids using a truth predicate in his discussions of basic logical laws and rules. But it is, I think, no accident that he did. (Weiner 2005, 346)

She maintains that the reason Frege eschewed a truth-predicate is that his task was to prove arithmetical truths which were plausibly but not obviously logical from truths whose logicality is obvious. If we used truth-predicates for these basic logical truths, then this would give the impression that logical laws, rather than being completely general (and hence logical), concern a specific concept, namely truth.

§5.2 Against Weiner's reading

The first problem with Weiner's arguments is that it simply falls short as a description of the arguments. As I showed above, Frege drifts back and forth between semantic ascent and descent. While his arguments for basic law I, and for his inference rules, are largely devoid of talk of denotation[18], this isn't the case with his arguments for some of his basic laws. Frege's discussion directly preceding and succeeding laws IIa and IIb is shot through with semantic talk, and his argument for law VI, insofar as he provides one at all, is *purely* semantic—he says that the truth of law VI follows "from the reference of the

[18] It should be noted that his arguments for inference rules are occasionally metatheoretic insofar as they clarify points of syntax—his arguments for generalized contraposition (1893, §15) and generalized dilemma (§16) contain syntactic asides.

function name $\backslash\xi$" (1893, §18). Since Weiner speculates that one of Frege's reasons for eschewing a truth predicate is to avoid the impression that logical laws specifically concern truth, then surely, by the same token, he ought not to have said that VI follows from the referent of a sign, in order to avoid the impression that logical laws are specifically concerned with signs and reference, as opposed to being maximally general truths about the world.

In any case, even those passages which only make use of *"das Wahre is"* must be semantic in *some* sense. As Heck points out, that Frege puts explanations which contain no explicit semantic ascent to semantic use is evidenced by the fact that claims which *are* expressed using semantic ascent are argued for on the basis of claims which are not. As noted above, Frege's introduction of the horizontal function contains explicit no semantic ascent. However, in a footnote, Frege says that his stipulation (*Festsetzung*) shows that '$-\Delta$' refers whenever 'Δ' does, so his introduction must also serve as an introduction of the symbol (Frege 1893, §5; Heck 2012, 39). If introductions of functions can have semantic import even when containing no explicit semantic ascent, clearly we cannot so simply infer, for Frege's arguments for his basic laws, from lack of semantic ascent to lack of semantic intent.

Heck also notes that Frege's arguments in §30 and §31 provide additional reason to suppose Frege's introductions of his symbolism are semantic. In §29, Frege gives recursive clauses for what it means for a symbol to have a reference. These clauses are quite explicitly semantic, and in §30 and §31 are used in an intricate argument that every sentence in *Begriffsschrift* has a unique reference—obviously a semantic claim. Frege's proof strategy is to first show that any name formed out of referring names must itself refer (§30) and then to show that his symbols for his function symbols refer (§31)[19], giving a base case which, with the inductive step in §30, shows that all names properly formed from referring names themselves refer. His proof of his base case must rely on implicitly rely on those passages where Frege introduces his logical functions.

One thing to take from this is that explicit semantic ascent is not necessary in order for a sentence to stipulate a meaning for a symbol is to be used—even when no component of the sentence in question refers to words or symbols, we can conclude things about language from the fact the sentence itself is asserted. If you ask what '$\sqrt{x}$' means, and I reply that $\sqrt{x}^2 = x$, I've told you something about the meaning of '$\sqrt{x}$', despite the fact that my answer never mentions (though it uses) the name for this sign—it would be pedantic to insist that the response be rephrased if it's to count as a response at all. Likewise, Frege's discussion of his functions are (with the exception of his discussion of $\acute{\varepsilon}\Phi(\varepsilon)$) successful in effecting stipulations concerning symbols, even when they contain no explicitly semantic notions. These arguments serve to show that Frege's stipulations serve a semantic role, even when they don't contain explicit semantic ascent. This doesn't, in and of itself, conclusively establish that Frege's arguments for his basic laws have a semantic character as well, though it makes such a conclusion more plausible.

§5.3 Heck's reading and the nature of metalanguage

Heck and Stanley take the primary aim of Frege's arguments to be to demonstrate that his laws and rules, qua formulas and transitions between formulas, are true and truth-preserving. Accordingly, they conclude that Frege's arguments are always semantic in spirit insofar as they are crafted (they suppose) to demonstrate the adequacy of Frege's symbolic system (Heck 2010, 360). It should be obvious from §5.2 above that I'm much more in agreement with this reading of Frege's arguments than I am with that of Weiner. There are, however, some important points of difference.

I fully agree with Heck and Stanley that Frege was engaged in semantics of a sort, and that his semantic talk isn't merely elucidatory. I also agree that Frege was not attempting anything like formal

[19] Recall that Frege's script in *Grundgesetze* contains no primitive terms for objects.

semantics, and that sometimes the way in which he speaks of his symbolism is muddled. However, I differ from Heck on the nature of that muddle, and its significance. Heck says that Frege wasn't as careful about use/mention as he ought to have been (2010, 43). This isn't quite right—Frege's occasional waffling between meta-theoretic variables for objects and for object names aside, he was generally quite careful about the distinction between signs and what they denote. He never, for any singular term "*A*" referring to a truth-value (and hence expressing a true thought), makes the mistake of saying that *A* refers to the true, or that "*A*" is the true. And if we take to heart some of the points I raised in the end of my above criticism of Weiner's reading, Frege, in the informal context in question, just didn't *need* to utilize systematic semantic ascent in order to provide arguments which are implicitly semantic.

This is a relatively minor point. Of more significance is the fact that Frege is less clear than we now are about the distinction between object-language and meta-language. This is *not* to say that he never takes the language itself as an object of study—§§29-31 is a clear example of a meta-theoretic proof of a property (referentiality) of expressions in Frege's language, and Frege's critique of the formalists clearly shows he recognized that one could regard his language as a purely syntactic object, though his comments on the matter suggest he saw little point in doing so (1903a, §90). But there's no indication that he thought that it was *essential* that such discussion take place in a language distinct from the object-language.

Chapter 7 of Patricia Blanchette's book *Frege's Conception of Logic* contains a very helpful discussion of the sorts of worries one might have about the availability of metatheory for Frege. First, she addresses and dismisses concerns that the all-encompassing and universally applicable nature of logic poses any problem for the possibility of metatheory (so understood as a justification of a symbolic system, not as a justification of basic logical laws qua thoughts). A distinct worry stems from what Blanchette calls the "exclusivist position":

> Exclusivist position: One is an exclusivist about a formal system if derivations within it "offer the only way of presenting compelling or scientifically acceptable arguments" (Blanchette 2012, 158)

Thomas Ricketts adheres to the exclusivist position (1997, 172,179). Compare this to the type of generality I mentioned briefly in §3.1 of chapter 2:

> Logic as a universal language: There are no thoughts inexpressible in some extension of Frege's concept-script.

Logic is a universal language if we *can* express any given content within some suitable extension of the concept script—the exclusivist adds to this position that we *must*. Blanchette points out that neither the universality nor the exclusivity of the concept-script necessarily prevents *all* meta-theory—without some additional argument to the contrary, there's no obvious reason one can't write metatheoretical arguments within the concept-script itself[20]. However, certain metatheoretical results lose their force. If we accept the exclusivity position, then even if arguments for the correctness of Frege's system may be formalizable, they'll be circular—we'll be arguing for the truth of laws and the truth-preserving nature of inference rules within a system in which those laws already figure (Blanchette 2012, 159).

It would be simply mad to hold that Frege adhered to the exclusivity thesis in the *very* strict sense of holding that everything must be expressible in Frege's particular notation. If Frege cared for the

[20] Ricketts, Goldfarb, and Weiner attempt to provide such arguments, most notably those hinging on the difficulty of talking about functions, and those hinging on a particular reading of Frege's regress argument about truth. I'll turn to these in the next chapter. Also, Blanchette points out that the exclusivity position brings with it a whole different sort of problem—languages which can express enough of their own semantics are inconsistent (Blanchette 2012, 160). I won't say much about this worry—suffice to say that it's not a problem Frege was concerned with.

convenience of the typesetter even less than he evidently did, he could have replaced all of his conditional strokes with zigzags, and his concavity strokes with a depictions of hands gingerly holding the relevant variables—no one can seriously supposes that working within resulting symbolism would mark a crucial difference, even though it might make derivations less easy on the eyes. What must be meant is that all rigorously presented content must be presented in a script structurally identical to the concept-script Frege in fact provides us with.

I don't want to commit myself to the exclusivity position. Though some, such as Weiner, maintain that Frege thought that sentences in natural language can never have the same sense as a concept-script formula[21], Frege often talks about how mathematicians and scientists in different languages and ages investigate the same content with varying degrees of clarity (See Burge 1990, 252-7 for a helpful summary). However, one needn't hold the full exclusivity position to hold the following weaker version:

> Weak exclusivity position: Logic is a universal language, and content is always expressed more clearly and perspicuously in Frege's concept-script, or suitable structural variants.

I think that weak exclusivity is plausibly attributable to Frege—weak exclusivity needn't deny that natural language gets at content, just that it doesn't display the content as perspicuously as it might. A corollary of the weak exclusivity position is that the *only* reasons one might have for not expressing the content of a sentence in a formal language are pragmatic reasons—because writing everything out like that would be too difficult, or too hard for the reader to digest, or because one wants to convey some effect or other through pragmatic means.

The crux of my disagreement with the correctness interpretation is this—it necessarily places too much importance in the claim that Frege, in his arguments for the correctness of his axioms and inference rules, had to be expressing his arguments in a meta-language, or at least something structurally quite distinct from the object language. After all, if Frege's arguments were not so expressed, they would be circular—he'd be justifying the soundness of axioms and inference rules within a system already making use of both. *This* is an assumption that isn't plausibly attributable to Frege—it would mean that sometimes we *must* resort to something other than the concept-script, for reasons *other* than pragmatic considerations of expressive convenience.

This is a point at which one might part ways with me. If one isn't convinced that Frege held the weak exclusivity position, or the corollary that natural language is only ever to be preferred for pragmatic reasons, then one can reject this assumption, and Heck and Stanley's correctness interpretation of Frege's arguments is totally unproblematic. There are two things to be said about this. The first is that if one takes this route, one need reject no part of my reconstruction of the case for logicality implicit in Frege's arguments—it might be that his arguments do double-duty, establishing both the correctness and logicality of his basic laws. Although the correctness interpretation is potentially problematic, and open to some objections which the E.A. account is not, it is consistent with the E.A. account. The second thing to note is that the E.A. account is quite consistent with the weak exclusivity position (at least if certain arguments from the elucidationists can be met). There's nothing circular about arguing, within Frege's system, for the logicality of the axioms of that very system. This is because arguments for logicality do not require, as a premise, the claim that any particular bit of Frege's system is logical—it was this point I raised in §3.1 above.

Before I move on, I want to address one more potential worry. The weak exclusivity position vitiates certain metatheory—soundness arguments are expressible, but carry no justificatory force due to their

[21] This follows from her contention that no sentences in natural language have truth-values—if sense determines reference, then these sentences can't have the same sense as concept-script formulas which *do* have truth-values (Weiner 2010, 44).

circularity. The worry is that weak exclusivity *also* makes arguments for referentiality circular, even though such arguments evidently play an important role in Frege's view of logic and language. The worry is thus: that an argument that a primitive symbol— the conditional, for example—refers carries no weight if that argument itself must expressible using that very conditional.

First of all, note that it's not like we're in a *better* position than if we insisted that arguments for referentiality must be expressed in a metalanguage quite distinct from the object language the symbols of which we're proving referentiality for. For the terms in the metalanguage are as (or more, if the metalanguage is a natural language) likely to lack reference as the object language we're investigating! Nevertheless, the argument has some bite. Unless we assume Frege's just warranted in presupposing that symbols figuring in formal versions of arguments refer (in which case, it's strange that he'd need to prove referentiality at all)—Frege is in no position to prove the referentiality of all of his terms. But this shouldn't worry us, because this doesn't preclude him from proving the referentiality of a sign τ_1 on the assumption that some other terms $\tau_1, \tau_2, \ldots$ refer. And this is just what Frege does. He argues that the referentiality of his complex signs follows from the referentiality of his simple signs (1893, §30). As far as his simple signs go, the bulk of his effort is spent showing that '$\acute{\varepsilon}\Phi(\varepsilon)$' refers. He doesn't really argue that his negation and horizontal strokes refer—he just says it "follows immediately from our explanations" (1893, §31). His argument for the referentiality of his conditional-stroke and identity sign is similarly brief. Unless it's absolutely essential that his (flawed) argument for the referentiality of '$\acute{\varepsilon}\Phi(\varepsilon)$' use, not mention, that sign, his argument is not problematically circular (though it will fail to go through for other reasons).

§6. Conclusion

There's much to be said for the E.A. account. In §2.5 of the previous chapter, I showed how it ties in with Frege's conception of the generality and undeniability of logic and how it fits with his view of the connection between logic, truth, and judgement, making for a more charitable reconstruction of his epistemology of logic. Now, I've shown how the E.A. interpretation accounts for the way in which Frege argues for his basic laws, and how it shows the case he makes for logicality. Frege isn't reduced to guesswork, to rule-of-thumb judgements concerning whether certain primitive laws seem general and undeniable, and hence likely logical. Though he attempts nothing like an argument for his principles which would convince the non-classical logician, he can, on his own terms, give a deeper explanation for why he had good reason for taking his axioms to be logical—we can be led to see that understanding them is sufficient results in a recognition of their truth.

I've also shown that the E.A. interpretation is superior to the correctness interpretation in at least one respect—it's compatible with the weak exclusivity position, the thesis that even if natural language sentences express content, formal versions are always preferable if all one is concerned about is giving a precise and perspicuous articulation of content. But this compatibility will fail if I can't meet certain arguments from Goldfarb, Ricketts, and Weiner to the effect that Frege could make no justificatory use of semantic notions in his arguments for basic laws, and that accordingly, Frege's arguments must be *merely* elucidatory. Heck and Stanley's argument that Frege owed us a correctness proof must also be dealt with. It is to these concerns I now turn.

Chapter 5: The Elucidatory and Correctness Interpretations

§0

The elucidatory interpretation of Ricketts, Weiner, and Goldfarb is that Frege couldn't have provided genuine arguments for his basic laws, as his commitments elsewhere allow for no meta-perspective from which one may intelligibly discuss and justify the laws of logic. Accordingly, his arguments are mere *elucidations*. On the other hand, Heck and Stanley, in their correctness interpretation, argue that Frege not only *could* justify his basic laws, but he was *committed* to doing so. I've already put forward some arguments against these two interpretations of Frege's arguments for his basic laws. In §5.3 of chapter 2, I argued that one must give a deeper case for logicality than merely noting that the basic laws seem obviously general and undeniable. In §5.2 of chapter 4, I argued, contra Weiner, that the way in which Frege states his arguments for his basic laws doesn't provide evidence for the elucidatory interpretation. In the §5.3 of the same chapter, I argued that the correctness interpretation is at odds with the weak exclusivity position, a principle to which Frege likely adhered. But proponents of the elucidatory and correctness interpretations do more to motivate their views than argue that it makes sense of the way in which Frege states his arguments for his basic laws. Thomas Ricketts argues that Frege's perspective on truth precludes the use of a truth-predicate which could be employed in semantic reasoning. Weiner focuses on considerations raised by Frege's infamous "concept horse" problem to argue that many of Frege's arguments are anomalous. As for the correctness interpretation, Heck argues if Frege didn't justify his own basic laws, he'd be subject to the same criticisms which he himself puts forwards against his formalist contemporaries.

Much of what's at issue in the arguments for the elucidatory and correctness interpretations hinges upon the status of Frege's discussion of his symbolism and of his arguments for basic laws. For Ricketts, Weiner, and Goldfarb, all of this is purely elucidatory. For Heck, on the other hand, some of Frege's introductory exposition is essentially meta-theoretic—in Frege's arguments for his basic laws he's justifying laws in the object-language in terms of claims in the meta-language. My own position is that there's reason for Frege to have supposed that his arguments could in principle have been formalized within a suitable extensions of his concept-script.

The structure of this chapter is as follows. I start by tackling the elucidatory interpretation. After getting clearer about just what elucidations are, and the role they play, I address the two chief arguments for the elucidatory view. First, I address Weiner's argument concerning the impossibility of taking Frege's talk of functions and concepts at face-value. In response to this point, I argue that at least some of Frege's talk about functions and concepts can be sensibly formalized, and in showing this I make plausible the compatibility of the E.A. interpretation with the weak exclusivity thesis. I then address Rickett's arguments about Fregean truth-predicates, and conclude my discussion of the elucidatory interpretation by arguing that it can't make good sense of Frege's arguments in §29-§31. I then turn to the correctness interpretation. For the correctness interpretation, I argue that nothing in the passages which Heck and Stanley cites in support of their view favor their interpretation over my own. This, together with the plausible compatibility of the weak exclusivity with the E.A. interpretation (and its certain incompatibity with the correctness interpretation) means that the balance is tipped in favor of my own view (at least if Frege did, in fact, hold the weak exclusivity thesis).

§1. The elucidatory interpretation

§1.1 Lack of a meta-perspective

The elucidatory reading[1] is part and parcel with a general trend in Frege scholarship, one which was sparked by Jean van Heijenoort in his short article "Logic as calculus and logic as language". There, van Heijenoort urges that some early logicians, Frege and Russell chief among them, differ starkly from contemporary logicians in the way they viewed logical inquiry[2]. For Frege, the story goes, there was no meta-perspective—no vantage point outside of logic from which one may ask questions about logic itself:

> Another important consequence of the universality of logic is that nothing can be, or has to be, said outside of the system. And in fact, Frege never raises any meta-systematic question (consistency, independence of axioms, completeness). Frege is indeed fully aware that any formal system requires rules that are not expressed in the system; but these rules are void of any intuitive logic…
>
> …Since logic is a language, that language has to be learned. Like many languages in many circumstances, the language has to be learned by suggestions and clues. Frege repeatedly states, when introducing his system, that he is giving 'hints' to the reader, that the reader has to meet him halfway and should not begrudge him a share of 'good will'. The problem is to bring the reader to 'catch on'; he has to get into the language. (van Heijenoort 1967, 326).

This same no-meta-perspective view is found in various articles of Thomas Ricketts:

> …there can be, in a sense, no genuine theorizing about logic. There is only theorizing within logic—the proof of derived logical laws from basic logical laws and the application of logic in formal proofs within the framework of the Begriffsschrift to the laws and facts uncovered by the special sciences… (Ricketts 1996, 136-7)

van Heijenoort argues for the no-meta-perspective view by giving textual evidence that Frege held logic to be *universal* in certain key respects, and then insisting that meta-theory is therefore precluded. For Frege, in order for logic to be a *calculus ratiocinator*, a codification of the laws of reasoning, it had to also be a *lingua characterica*, which allows for perspicuously articulated content (1880-1, 12). This is accomplished with Frege's introduction of his function/argument analysis of propositions, and the quantifier-variable notation which is based upon it. van Heijenort says that this is the first sense in which logic may be said to be universal (1967, 324-5). The second sense of universality is that once Frege understood the vast leap in expressive power which nested quantification provides, it became plausible for him that *any* claims whatsoever might be expressible in this notation, provided that one is first given a suitable addition of non-logical vocabulary (1879, p.7). Frege accordingly took variables to be absolutely unrestricted—nothing else about the logic need change upon the introduction of new constants, since the referents of these constants were already included in the universe. Logic is, as Frege said, a language which "borders all the others" (1879, p. 7). To sum up, according to van Heijenoort, Frege took logic to be universal in the sense of being a *universal language*, as well as in the previously discussed senses of being universally applicable and all-encompassing.

[1] Van Heijenoort says that Frege and Russell conceive of logic as *universal* (1967, 324). I avoid using the term "universalist" to characterize the interpretive trend found in van Heijenoort, Ricketts, Weiner, and others because of its ambiguity—as I showed in §3.1 of chapter 2, there are a great many senses in which logic can be taken to be universal.

[2] See Peter Milne 2008, especially section 6 and 7, for a detailed discussion of why certain metatheoretical proofs eluded Russell—Milne argues that it is not because Russell couldn't, consistent with his commitments, have recognized such proofs as correct.

What does it mean to say that logic is a universal language? Logic is clearly not a language in the syntactic sense of "language", as Frege was consistent in demanding that formulas be interpreted. Nor can everything be expressed entirely in terms of symbols with which Frege provides us, in either *Begriffsschrift* or *Grundgesetze*—these languages contain no terms for non-logical concepts of the various special sciences. I think there are two related claims one might be making when one says that logic is a universal language:

a) Formalizability claim: All thoughts pertaining to each special science can in principle be formalized within Frege's concept-script, once it's suitably augmented with non-logical terminology.
b) Maximal perspicuity claim: Formulas in Frege's concept-script provide a maximally perspicuous representation of those thoughts which those formulas express.

Something should be said about how these two claims relate to the weak exclusivity position:

> Weak exclusivity position: Logic is a universal language, and content is always expressed more clearly and perspicuously in Frege's concept-script (or suitable structural variants) once it's suitably augmented with non-logical terminology.

I take weak exclusivity to imply the formalizability claim. It doesn't imply the maximal perspicuity claim—to say that formal languages are always *more* clear and perspicuous than natural language doesn't imply that they're *maximally* clear and perspicuous.

There's a great deal to be said for the formalizability claim. Even if Heck is correct in saying that Frege treats quantification objectually, and that there's no reason to suppose that Frege thought every object in the domain had a name (2012, 56-9), this doesn't conflict with the claim that for any given object, we *could* have a name for it, allowing us to express thoughts about this object. And the formalizability claim also fits well with Frege's comments in *Begriffsschrift* about how his concept-script can be gradually expanded to express thoughts in geometry, physics, or any other given science—formalization is "conquered by gradual advance" (1879, 6-7). The maximal perspicuity would need a bit more justification—there's some reason to think that even his formal language may have been an imperfect guide to the structure of thoughts[3].

In any case, even if Frege *did* hold a) and b), van Heijenoort leaps from the universality of logic to the no-meta-perspective view all too quickly—he simply lists the latter as one of the consequences of the former. This won't do—it's certainly not prima facie plausible that a greater level of expressiveness, or a fixed and maximal range of variables, would preclude metatheory—as Aldo Antonelli and Robert May point out (2000, 253), if logic can apply to anything, why not logic itself? Even if there's something right in the claim that Frege's conception of logic as universally expressive marks something philosophically important and distinctive about his perspective, this must be distinguished sharply from the question of whether Frege could or couldn't make sense of talk *about* logic.

[3] In "Function and Concept", Frege says that '$\acute{\varepsilon}(\varepsilon^2 - 4\varepsilon) = \acute{\alpha}\big(\alpha(\alpha - 4)\big)$' and '$x^2 - 4x = x(x - 4$' express the same thought, but in a different way (1891, 10-11/FR 136) despite their very different grammatical structure. And in "Compound Thoughts", Frege says that '$\neg(\neg A \wedge \neg A)$', '$\neg\neg A$', and '$A$' all express the same thought (1923-6, 49/CP 405). See also (Burge 1998, 335-6) There's much to be said about whether Frege maintained these commitments, but it does show that while he sometimes seemed to treat sense as something close to linguistic meaning, other instincts occasionally pulled him in the opposite direction.

§1.2 Elucidation

Frege's most extensive discussion of elucidation is found in a 1906 article "On the Foundations of Geometry: Second Series".

> We must admit logically primitive elements that are indefinable…Since definitions are not possible for primitive elements, something else must enter in. I call it **elucidation**. It is this, therefore, that serves the purpose of mutual understanding among investigators, as well as of the communication of the science to others. We may relegate it to a propaedeutic. It has no place in the system of a science; in the latter, no conclusions are based on it. Someone who pursued research only by himself would not need it. The purpose of explications is a pragmatic one; and once it is achieved, we must be satisfied with them. And here we must be able to count on a little goodwill and cooperative understanding, even guessing…[Frege1906a, 301/CP 300-1, substituting "elucidation" for "explication" in translation of "Erläuterung"]

The elucidatory reading claims that Frege's earlier arguments for basic laws in the first volume of *Grundgesetze*, as well as Frege's informal exposition of and commentary upon his script elsewhere, are elucidatory in this sense. Ricketts characterizes elucidations as follows

> In addition to scientific statements formalizable in the Begriffsschrift, there are what Frege calls elucidations. These are the things one might say to convey some notion that cannot be defined in other more basic terms. Some elucidatory remarks, having facilitated communication, may themselves be straightforwardly stated in the Begriffsschrift. Other elucidations, like those employing analogies, will not be thus formable. Frege's logical innovations and notational novelties require extensive elucidation. Some of his rhetoric—including, I believe, much of what we tend to think of as Frege's semantics—is not stateable within the framework of the Begriffsschrift. Frege's Universalist conception of logic gives it an anomalous status. (Ricketts 1996, 127-8)

The important point to bear in mind is that elucidations are something of a mixed bag. In particular, some, but not all, elucidations are *anomalous*, meaning that they don't express something that is formalizable in the concept-script. If we accept the formalizability claim, it will follow from the fact that an elucidation is anomalous that it expresses no thought at all, and serves only the pragmatic function of getting us to recognize something. Weiner is also careful in noting that not all elucidations need be anomalous (2010, 59). I argued in the previous chapter (§5.2) that Frege's arguments for his basic laws are importantly semantic, and that they need to be (§3.1)—the question, then, is whether these arguments in particular are anomalous, or whether instead they express content which could in principle be formalized.

The first volume of *Grundgesetze* is a very different context from Frege's 1906 discussion—it was written 14 years earlier and deals with purely *logical* laws and inference rules. Frege's 1906 comments on elucidation occur in the course of his critique of Hilbert's axiomatization of geometry by means of schematic postulates, postulates which Hilbert believes define the non-logical (for Frege) subject matter. Frege claims that we cannot take these postulates to be "definitions" in the traditional sense of the word (1906, 383-4/CP 315), nor can we take them to be elucidations in Frege's more specialized sense (1906a, 302/CP 301). It's clear from context that it's the meanings of geometrical terms such as "point" and "line" that are under discussion. These two terms, as Frege himself notes, have a long history of previous use—they aren't terms of art being introduced to a formal system (1906a, 302/CP 301). Frege's concept script expressions on the other hand *are* terms of art—his "negation" and "conditional" strokes differ from their natural language variants in being defined for all input, not just truth-values, and form a part of a new, artificial, and two-dimensional notation. Thus, Frege's discussion of his horizontal stroke must do *more* than his discussion of "point"—he needs to give his symbols meanings, and he accomplishes this via his stipulations (*Festsetzungen*)[4]. This isn't to say that the functions his primitive logical terminology denote are conventionally constructed, or selected merely because they're of interest to us—he says elsewhere that logic has its own building blocks (Frege 1885a, 96/CP 114). Negation, identity, and concept-relations such as subsumption and subordination of concepts are proper to logic (1906a, 428/CP 338). But Frege *must* be giving the meaning to the signs for these functions—his stipulations are elucidatory insofar as they get readers on the same page, but play a role over and above that which would be played by elucidations for terms already in use.

The term "Erläuterung" appears only very occasionally in *Grundgesetze*—only three times in the whole first volume. The first time is in a footnote, where he says that his comments concerning the use of the place-holder 'ξ' aren't intended as stipulations, for they "never occur in the concept-script developments" and he uses them only "in the exposition of the concept-script and in elucidation" (1893, §1). The other two times Frege uses the term, it's with reference to his informal discussion intended to clarify his definitions, not with reference to stipulations governing the use of logically simple terminology[5]. Proponents of the elucidatory interpretation shouldn't be faulted for thinking that Frege's 1906 remarks shed light on the early parts of *Grundgesetze* however—the latter contains a passage which clearly parallels the former:

> It will not always be possible to give a regular definition of everything, simply because our ambition has to be to reduce matters to what is logically simple, and this as such allows of no proper definition. In such a case, I have to make due with gesturing at what I mean. (Frege 1893, 4)

What Frege's 1893 and 1906 comments on elucidations have in common is that no conclusions are based on them (1893, §35; 1906a, 301/CP 300-1). They form no part of his formal proofs—they're merely comments to help us better grasp those proofs. It makes perfect sense that Frege would be careful here. After all, the entire *point* of Frege's concept-script is to make sure that nothing sneaks into his formal proofs unnoticed—he has a way of meticulously tracking what content goes into the justification of any conclusion. If Frege had *also* thought that we had to carefully account for just what sort of content enters into his explanation of his basic terminology, then the best way to go about this would be to formalize his stipulations. But this would need to be done in another formal language, which would in turn need to be presented and elucidated—we'd have a vicious regress on our hands.

In fact, one might charge the E.A. interpretation, as I've construed it, as falling into just such a regress, a worry I touched on in §4.3 of chapter 4. I argued in §3.1 of the same chapter that to recognize a law as logical, we must recognize it as *strongly* epistemically analytic. That is, we must have a reflective awareness of how a full understanding of it leads to recognition of its truth, where this reflective awareness involves an explicit knowledge that the recognition of truth is rooted in conceptual understanding alone, and not in intuition—recognizing this will require that we think through the consequences of stipulations. If this recognition required us to formalize Frege's stipulations, in order to guarantee that they contained *only* intuitive content, then we'd be off on a regress.

The response is, again, just to deny that there's any need to formalize stipulations in order for the reflective awareness in question to be elicited. While it's problematic if something intuitive sneaks into *proofs*, it's harmless if the thoughts expressed by stipulations (and I shall argue shortly that stipulations can, indeed, express thoughts) end up having some non-logical content. For the thought "my recognition of the truth of this principle follows from my understanding of symbols expressing it" need not itself be a

[4] Note that, contra Hilbert, Frege insists we pin down the interpretation of "point" before we use it in axioms. See (Blanchette 2012, 108-131) for a helpful discussion.
[5] "Erklärung" appears far more often, but this is used in the more generic sense of "explanation"(Ebert/Rossberg 2013, xix).

logical truth in order to play a role in our recognition of a truth as logical. It's perfectly legitimate if something non-logical to enter into our recognition of this truth—for us to recognize a law as logical, it's necessary, for all logical simples which figure in expressions of such a law, that the stipulations secures reference for such simples, and that we recognize this fact, and that we recognize the truth as epistemically analytic. But it's unnecessary for our recognition of these facts to stem from a purely logical source. That is, recognition of logicality need not itself be a logical matter. And this is what we'd expect, given Frege's comments on how "something else must enter in" when we reach the foundations (1879-91, 3). Simply put—there's no reason to expect, or to require, our belief in the logicality of a law to itself be a logical principle. "Basic Law I is a logical law" need not itself be a logical truth[6].

§1.3 Talking about concepts, functions, and objects

So far in §1, I've said little that proponents of the elucidatory interpretation need disagree with. Frege's discussion in *Grundgesetze* is undoubtedly elucidatory, in some sense. He intends to familiarize us with his concept-script (as he'd better—his first 53 sections do, after all, fall under the heading "Exposition of the concept script"), and none of the material used in this discussion is used as a premise in deriving theorems within the system. I think that Weiner, in particular, is quite right to emphasize the heterogeneity of elucidations (2010, 60-1). Despite Frege's 1906 comments, there's evidence that he didn't take elucidations to be restricted to helping minds meet with respect to the meaning of primitive terms—as mentioned before, two of the three occurrences of the term "Erläuterung" refer to his informal comments on definitions (1893, §§34-5).

Proponents of the elucidatory interpretation want to claim more than this, however—though they grant that not all of Frege's elucidations are paradoxical, they maintain that much of Frege's semantic talk is anomalous. That is, some elucidations are not only unformalized, but unformalizable—by the formalizability claim, such elucidations express no content at all. One line of argument here relies on the notoriously thorny problems surrounding Frege's discussion of his concept/object distinction, a discussion which, by Frege's own lights, can't be taken at face-value. Frege's discussion of these problems, laid out in "On Concept and Object", also contain some discussion clearly reminiscent of Frege's 1893 and 1906 discussions of elucidation. He says:

> Kerry contests what he calls my definition of 'concept'. I would remark, in the first place, that my explanation is not meant as a proper definition. One cannot require that everything be defined, any more than one can require that a chemist decompose every substance. What is simple cannot be decomposed, and what is logically simple cannot have a proper definition…On the introduction of a name for something logically simple, a definition is not possible. There is nothing for it but to lead the reader or hearer, by means of hints, to understand the words as intended (1892b, 193/FR 182).

Frege's not defining concept-hood—he's providing hints. What significant here is that it's clear that at least some talk of concepts *couldn't* be posed in more precise terms.

> I admit that there is a quite peculiar obstacle in the way of an understanding with my reader. By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought; I mention an object, when what I intend is a concept. I fully realize that in such cases I was relying upon a reader who would be ready to meet me halfway—who does not begrudge a pinch of salt. (1892a, 204/FR192)

[6] It might seem odd to say that our evidence for claims of logicality needn't itself be logical. But this is a strangeness he's already committed to—in 1906, he floats the possibility of a new science which would allow him to prove the independence of thoughts, and says that this science would have its own basic assumptions as much as geometry does (Frege 425-6, CP 33). The independence of thoughts is hardly contingent, yet it isn't a purely logical matter.

Weiner argues that these are elucidations, and ones which *are* paradoxical. She argues that there's no more legitimate means of expression by which Frege could state in a logically perspicuous manner what he intends when he says that a function-name denotes a concept rather than an object, or when he says that "all whales are mammals" gives us a relation of subordination between the concept "whale" and the concept "mammal" (1989, 115-120). She then emphasizes that this claim "will infect all discussions in which Frege's notion of concept plays a role" (1989, 120), and concludes that a great deal of Frege's philosophical exposition is anomalous, and merely elucidatory.

I don't propose here to offer any general account of what Frege did, or ought to, have said about his concept horse problem. I only wish to point out that these difficulties need render neither Frege's stipulations nor his justifications of logical laws as anomalous. Even if they have no role *within* Frege's axiomatization of arithmetic, it would have plausible for him to think that his "hints" pointed at thoughts which could have been formulated within his concept-script, suitably augmented with additional primitive symbols. This possibility has been obscured by a conflation between two distinct problems:

> <u>The Hard Problem:</u> How to say that there are both concepts and objects?
>
> <u>The Easier Problem:</u> How to say things about functions? More specifically, how to say that a name denotes a function, and make the sort of semantic talk which appears in Frege's arguments legitimate?

The hard problem is just unsolvable—if we accept Frege's insistence that a concept and an object can never be referred to by names of the same grammatical type, it's simply impossible to set out any contrast between concepts and objects[7]. The distinction between concept and object is totally unlike the distinction between abstract and concrete objects. Unless we wish to interpret Frege as committing himself to truths which are completely inexpressible[8], there's simply no thought which "no concepts are objects" could be sensibly intended to express. This problem also extends to any talk which is *implicitly* contrastive—when we say "numbers are objects", what one wants to get at can't really be expressed just by adding a predicate for "ξ is an object"—what one really intends to say is that numbers are objects *rather than concepts*, and this is something which can't be expressed, no matter how much Frege's concept-script is extended with primitive vocabulary.

This problem does not, however, extend to *all* talk about functions. Frege's language allows us to quantify over, and predicate things of, functions with ease. When we say that the concept *whale* is subordinate to the concept *mammal*, this misses what is more legitimately expressed by "All whales are mammals"[9]. I don't think there's any reason to suppose that Frege denied that some crucial bits of his semantic talk were formalizable in principle, and thus hint at legitimate content. This is important, because it allows me to explain what "reflective awareness" amounts to when I claim that Frege's arguments for his basic laws aim to elicit a reflective awareness of how an understanding of his basic laws suffices for a recognition of their truth—in §3.1 of chapter 4 I clarified that reflective awareness amounts to recognizing that we can argue from the stipulations governing terms which figure in the expression of

[7] Thomas Ricketts has a helpful discussion about how it's the *contrast* which is the problem, rather than talk about concept per se, and how this isn't something we can get around by introducing a second-level predicate for concepthood (2010, 179-185).

[8] In "Thought" Frege suggests that "everyone is presented to himself in a special and primitive way, in which he is presented to someone else" (1918a, 66;FR 333)—therefore, one grasp a thought about oneself which no one else can grasp. This is quite different than "no concept is an object"—if, *per impossibile*, this sentence expressed a thought, it would be a thought *no one* could grasp.

[9] Although we're still saying something essentially contrastive, and hence inexpressible, if our intention is to emphasize that "All whales are mammals" expresses a relation between concepts, *rather* than between objects.

these laws to the claim that these expressions denote the true[10]. This recognition is of arguments between thoughts, and such arguments need not be taken as anomalous, and are in principle formalizable.

Much of what Frege says in his informal semantic arguments could be expressed within his concept-script, augmented with non-logical terms for the relation of designation. Suppose we add:

Formal expression	**Stipulation**
$b(\xi, \zeta)$	'$b(\Delta, \Gamma)$' refers to the True if: • Δ is a proper name • Δ refers to Γ Otherwise, '$b(\Delta, \Gamma)$' refers to the False.
$B_x\big(\xi, \Phi(x)\big)$	'$B_x\big(\Delta, \mathrm{F}(x)\big)$' refers to the True if • Δ is a first-level function name • Δ refers to $F(\xi)$ Otherwise, '$B(\Delta, \mathrm{F})$' refers to the the False.
$B'_{\mathfrak{f}}\left(\xi, \mu_\beta \mathfrak{f}(\beta)\right)$	'$B'_{\mathfrak{f}}\left(\Delta, \eta_\beta \mathfrak{f}(\beta)\right)$' refers to the True if • Δ is a second-level function name • Δ refers to $\eta_\beta \Phi(\beta)$ Otherwise, '$B'_{\mathfrak{f}}\left(\xi, \mu_\beta \mathfrak{f}(\beta)\right)$' refers to the False.

I claim that these functions are entirely legitimate. Of course, the stipulations for them miss their mark—"Δ refers to $F(\xi)$" is ungrammatical. But this difficulty is already present in Frege's *Grundgesetze* stipulations governing his primitive logical terminology, and this doesn't present any difficulty in securing a meaning for his novel signs. And what's more, Frege *himself* seems open to the possibility of augumenting his system with primitive, higher-order predicates. In a 1902 letter to Russell, Frege notes that "$\Phi(\xi)$ is a concept" is ungrammatical, for the ξ in 'ξ is a concept' can be filled by a proper name, and '$\Phi(\xi)$' is not a proper name. He suggests that we can get around this with an appropriate second-level function[11] $C_\varepsilon \Phi(\varepsilon)$ which predicates concepthood of $\Phi(\varepsilon)$ (Frege 1980, 136), so he was evidently open to the possibility of primitive higher-level concepts. Also, formulas containing the function names I introduced above will be perfectly grammatical—to form "$B_x(\text{"}\zeta{=}\zeta\text{"}, x = x)$" (which says properly what "'$\zeta = \zeta$' denotes the function $\zeta = \zeta$" only hints at), we take the function name "$B_x\big(\xi, \Phi(x)\big)$", and substitute for "$\xi$" the proper-name ⌈"$\zeta = \zeta$"⌉, and for "$\Phi(x)$" the function name "$\zeta = \zeta$", which is perfectly in line with Frege's §30 methods of name-formation.

There are a few more things to note about the primitive signs which I suggest above. Firstly, the presence of primitive signs for reference relations would not commit Frege to any particular doctrine concerning reference—in particular, supposing he could have allowed for such signs does not conflict with the rejection of a full-blooded correspondence theoretic picture of language. Secondly, it shouldn't bother us that there would be different reference relations for each logical type referred to, and that we're

[10] Actually, what we're aware of needs to be stated slightly more carefully—since expressions of basic laws contain roman letters, we can't really say that they denote the true. Rather, we recognize from their stipulations that every result of substituting the roman letters for names of the appropriate types will result in a name of the True (and note, again, Heck's claim that this doesn't amount to a substitutional treatment of roman-letter quantification, as Frege evidently includes in this not just names in the language, but names we could add to the language for any given object (2012, 56-9)).

[11] The notation Frege himself suggests is indistinguishable from his smooth-breathing notation, save for a reversal of the direction of the accent mark over 'ε'.

barred from saying just what it is that all of these reference relations have in common. The goal here isn't to solve the hard problem—some of Frege's semantic talk will remain anomalous. Thirdly, there is no regress here—I'm not claiming Frege thought we needed to formalize his stipulations and arguments for his basic laws, but just that it must be possible to do so in principle. Finally, Frege need not commit himself to the claim that terms like the ones I've suggested above, are part of his primitive *logical* terminology. This should not worry us: as mentioned above, while it's very important for Frege's project that he give us evidence for taking his laws to be logical (less his logicism be an uninteresting, or worse merely verbal, claim), there's no reason to suppose that this evidence need itself consist of logical truths.

It would be a chore to formalize Frege's arguments for his basic laws with the use of new terminology of this sort. The important point is that there's no reason that such formalization should have struck Frege as impossible in principle. If this is right, then even if Frege is committed to the claim that content is in principle formalizable in a suitable extension of the concept-script, he could nevertheless have taken his arguments for his basic laws to hint at perfectly legitimate content. Thus, the version of the E.A. interpretation, argued for in §3.1 previous chapter, which holds that recognizing a law as logical requires us to understand how the truth of the sentence expressing the basic law follows from the stipulations governing its constituent terms, does not require us to think the unthinkable—it's a perfectly legitimate way of recognizing a law as logical.

§1.4 Ricketts on Frege on truth

Another one of the chief arguments for the elucidatory interpretation , which finds its fullest expression in Thomas Ricketts's 1996 "Logic and Truth in Frege" rests upon a particular reading of Frege's comments concerning truth. Ricketts examines these comments, and attributes to Frege the following three claims:

- <u>Non-property claim:</u> Truth is not a property (1996, 132)
- <u>Anomaly claim:</u> A truth predicate which could be made use of in semantic arguments is unavailable—accordingly, his discussions which contain truth-predicates can't be taken at face-value (1996, 128; 135-6)

I shall argue that while Frege was simply undecided about whether truth is a property, he nevertheless thought that truth predicates were available, and places where such predicates figure in semantic arguments are non-anomalous.

Ricketts's attributions to Frege of the two claims above rests on his reading of Frege's regress argument in "Thought", a work which contains Frege's most extensive published comments on the nature of truth. In the course of critiquing the correspondence theory, Frege says:

> But could we not maintain that there is truth when there is correspondence in a certain respect? But which respect? For in that case what ought we to do so as to decide whether something is true? We should have to inquire whether it is *true* that an idea and a reality, say, correspond in the specified respect. And then we should be confronted by a question of the same kind, and the game could begin again. (Frege 1918a, 60/FR 327)

His conclusion is that "the content of the word 'true' is sui generis and indefinable" (1918a, 60/FR 327). He intends this argument to be quite general—if there were *any* characteristic τ which all true propositions had in common, then to deciding whether thought ψ was *true* requires:

<u>Step *a*</u>: deciding whether $\boldsymbol{\psi}$ **has characteristic** $\boldsymbol{\tau}$, which requires:

<u>Step *b*</u>: deciding whether **it is *true*** $\boldsymbol{\psi}$ **has characteristic** $\boldsymbol{\tau}$**.**

Call the highlighted bit ψ'. Completing step b requires:

<u>Step a':</u> deciding whether $\boldsymbol{\psi'}$ **has characteristic** $\boldsymbol{\tau}$**,** which requires:

<u>Step b':</u> decide whether **it is *true* that** $\boldsymbol{\psi'}$ **has characteristic** $\boldsymbol{\tau}$

⋮

The regress is clear.

Although Frege's explicit conclusion of his argument is that truth is sui generis and indefinable, Ricketts's reconstruction of the argument has Frege arguing for the "stronger conclusion that truth is not a property at all". He interprets Frege's argument in the following way: if truth were a property, every assertoric sentence would need to contain a truth predicate, in order to "make a predication implicit in every assertion explicit" (1996, 134) (the implicit component being, of course, the property had by all true propositions). The result of making the truth predicate explicit is itself another assertoric sentence, containing *another* truth predicate which must be made explicit, and so on *ad infinitum*. So, truth cannot be a property. We must, then, hold the non-property claim if we want to avoid the regress.

Ricketts then argues that the non-property claim implies the anomaly claim:

> The word 'true' is an attempt to formulate as a predicate the import of asserting something, by splitting off 'true' from 'recognize-as-true' and treating 'true' as simply a predicate of what is judged" (Ricketts 1996, 134)

Such an attempt always miscarries. As Frege says in "My Basic Logical Insights"

> So the word 'true' seems to make the impossible possible: it allows what corresponds to the assertoric force to assume the form of a contribution to the thought. And although this attempt miscarries, or rather through the very fact that it miscarries, it indicates what is characteristic of logic. (Frege 1915, 272, FR 323)

According to Ricketts, the fact that truth miscarries means that truth-predicates have no place within the concept-script. Thus, Frege's semantic arguments are "not stateable within the framework of the begriffsschrift" and Frege's "monolithic view of truth" gives much of his semantic talk "an anomalous status" (1996, 127-8). Ricketts says that while it is tempting to use a truth-predicate to give informal soundness arguments for Frege's rules in the concept-script, the necessity of a truth-predicate in such explanations is only due to the imprecision of natural language—it does not give "uniform and perspicuous expression to the topic-universal notions of logic" and if it did, "then either there would be no need to formulate logical principles…or logical principles could without ado be read off rom language"(1996, 135). Ricketts concludes that "Within logic, no truth-predicate is required, only an indication of asserting force" and that "the relationship between a thought and its truth-value is not describable in a sentence" (1996, 135-6).

Elsewhere, Ricketts is clear that "true" is not unique among terms which, as he sees it, ought to fall away once a completely perspicuous language is provided. Ricketts argues elsewhere that Frege's perspective on truth carries over to his perspective on reference. He reasons as follows—we identify objects by means of equations, so when asking about the referent of, to use his example, '2', we may ask what to put in the blank for "the object designated by '2'=___" to come out true. The answer is, of course, always that we should put '2' in the blank, since '2' refers to 2. Ricketts then says,

> Finally, once '2' has been recognized in sentences as a designating proper name, we can inquire about the object designated by the name '2' by inquiring after the truth of thoughts expressed by equations of the form "2 = __". Talk of designation drops away. (Ricketts 2010, 195)

All of this is reasonable enough, save for the last sentence. It's of course true that '2' designates 2, and also true that, for Frege, we express what something is primarily through the use of identities (hence Frege's preoccupation with the Julius Caesar problem). The most important issue, however, is whether "the object designated by '2'" has the same *sense* as '2'—if not, then it's simply misleading to say that talk of designation falls away, as "designates" expresses something which could not be expressed otherwise than by a relational predicate between a name and an object. Frege nowhere explicitly states such a sense equivalence, and for Ricketts to assume that it holds would be to beg the question in favor of 'designation' being a deflated notion. Frege thought "It's true that A" and "A" had the same sense, but here, truth is applied to thoughts, not sentences.

What Ricketts concludes from this discussion mirrors what he says about Frege's attitude towards "true":

> It belongs to the preliminaries that establish a notation for the full, unambiguous and perspicuous expression of scientific knowledge. Once this notation is in place, talk of designation is not required for the communication of any proper scientific knowledge, since 'identifications' of particular objects can be expressed without semantic ascent (Ricketts 2010, 197).

§1.5 Against Ricketts's account of Fregean truth

The first thing to note is that Frege himself does not conclude from his regress argument that truth isn't a property. His immediate conclusion is that truth is indefinable (1918a, 60/FR 327). Jason Stanley provides what I take to be a more plausible reconstruction of Frege's argument, a reconstruction which emphasizes the conclusion which Frege actually gives us. Frege thinks that truth and assertion are connected in such a way that asserting "it is true that ψ" just is to assert "ψ", since both express the same thought. The only way that truth could satisfy this connection is if it's sui generis—if for ψ to be true was for ψ to have some characteristic τ, then "ψ" and "it is true that ψ" would have quite different content, meaning that determining whether or not ψ would require we first do the *distinct* task of determining whether ψ is true, leading to a regress. If truth is sui generis, on the other hand, this distinctness drops away, and the regress is avoided (Stanley 1996, 52).

This doesn't settle, however, whether Ricketts is right in attributing to Frege the non-property and anomaly claims. Did Frege take truth to be a property? When he addresses the problem, he strikes a rather tentative note:

> All the same it is something worth thinking about that we cannot recognize a property of a thing without at the same time finding the thought *this thing has this property* to be true. So, with every property of a thing there is tied up a property of a thought, namely truth. It is also worth noticing that the sentence 'I smell the scent of violets' has just the same content as the sentence 'it is true that I smell the scent of violets'. So it seems, then, that nothing is added to the thought by my ascribing to it the property of truth. And yet is it not a great result when the scientist after much hesitation and laborious researches can finally say 'my conjecture is true'? The *Bedeutung* of the word 'true' seems to be altogether *sui generis*. May we not be dealing here with something which cannot be called a property in the ordinary sense at all? In spite of this doubt, I will begin by expressing myself in accordance with ordinary usage, as if truth were a property, until some more appropriate way of speaking is found. (Frege 1918a, 61-2/FR 328-9).

Frege flip-flops here—truth is a property of thought, but it contributes nothing to thought, and might not be a property at all in the ordinary sense. I suspect that this is a sign that Frege was genuinely puzzled about whether truth was a property. This suspicion is borne out by how often Frege seems to have changed his mind on the matter—first he says that the relation of a thought to the true is that of sense to reference rather than that of subject to predicate (1892a, 34/FR 158), only to say five years later (and *directly* after giving another version of the regress argument) of truth that "predicating it is always included in predicating anything whatever" and that truth predicates apply to thoughts (1897a, 140-1/FR 228-9). In "Logic in Mathematics", Frege says quite explicitly that "truth is not a property of sentences, or thoughts" (1914, 234), only to strike a more tentative note just four years later, as seen in the quote above.

I do not wish to dispute Ricketts's view that truth is deeply tied to assertion—Frege does, after all, characterize assertion as a manifestation of the acknowledgement of the truth of a thought (1918a, 62/FR 329). It is this same connection Frege emphasizes with his frequent use of the example of the actor on a stage—

> The truth claim arises in each case from the form of the assertoric sentence, and when the latter lacks its usual force, e.g. in the mouth of an actor upon the stage, even the sentence 'the thought that 5 is a prime number is true' contains only a thought, and indeed the same thought as the simple '5 is a prime number'. It follows that the relation of the thought to the True may not be compared with that of subject to predicate (Frege 1892a, 34/FR 158).

That the relation of a thought the True is not that of predication does *not* preclude the use of a truth predicate. It only shows that any such truth predicates do not exhaust the role of truth— truth predicates cannot give a sentence assertoric force where such force is lacking to begin with. This point is repeated *much* more frequently than his regress argument[12]. Accordingly, there's reason to think that Frege took the connection between truth and assertoric force to be more fundamental than applications of truth predicates to thoughts. But to say that truth is not *fundamentally* a property of thoughts is not to say that there can be no property had by all and only true thoughts, or that there cannot be a predicate which expresses this property.

In any case, the question of whether or not truth is a property of thoughts is distinct from that of whether there's a concept which holds of all and only true formulas of Frege's concept-script. *This* question is one which Frege says very little about—he insists that truth applies primarily to thoughts, rather than to sentences (1897, 140-1/FR 229; 1918a, 60/FR 327), but this doesn't bar him from recognizing *some* predicate applies only to sentences which designate the true. Heck thinks that "designates the True" is just such a predicate (2012, 48), and I fully agree. In the absence of explicit discussion on Frege's part about truth-predicates of sentences, the only consideration to decide the matter is to look at what's implicit in his practice—and there, "designates the true" appears poised to do just the sort of work one would expect of a truth-predicate of sentences. Acknowledging that "designates the True" can play an important role in semantics needn't conflict at all with the claim that 'true' is not *primarily* ascribed to sentences, nor with the claim that truth is not *fundamentally* a property of thoughts.

§1.6 Against the elucidatory reading of §§29-31

Above, I've tried to counter what I take to be the two most central strategies for supporting the elucidatory strategy. In addition to this, there's a more direct argument against the elucidatory interpretation—the implausibility of any such reading of Frege's arguments in §§29-31. These arguments are quite explicitly semantic. They start from obviously semantic premises—Frege's §29 conditions for

[12] Frege's point about how an actor on a stage can utter any sentence without assertoric force, even if that sentence contains a truth predicate, is repeated in (1892a, 33/FR 157), (1903b, 371; 281) ,(1906b, 211/FR 297), (1915, 271/FR 323) ,(1914, 233-4) , and (1918a, 63/FR 330). His regress argument appears only in (1897a, 140-1/FR 228-9) and (1918a, 60/FR 327).

when names refer. The conclusion these arguments aim to establish is explicitly semantic as well—Frege claims to have shown that every name in his language refers. None of this is new information for the elucidationists, of course. But the primary reason that the elucidatory interpretation of these passages beggars belief is this that Frege's arguments here are *intricate*—they form an elaborate inductive argument. In §30, Frege shows that all names built out of referring names must themselves refer, and in §31, he tries to show that his simple names refer, something which itself seems to involve an implicit inductive argument on the complexity of first-level functions into which names of value-ranges could be embedded (Dummett 1991, 215-6; Heck 2012, 79-80). Indeed, Frege's argument was intricate enough that he erred in the details—the proof must be flawed, since the consistency of his system is a corollary of its conclusion.

The elucidationists are committed to denying that §31 is a proof, even an attempted one, since it makes reference to semantic notions, and they denies that Frege is entitled to a meta-semantic perspective of any sort. Ricketts even goes so far as to claim that Frege's view of judgement implies that that he can't ask whether a statement of language expresses a thought (1985, 8)! But §31 is, of course, aimed at establishing just that—the only corollary Frege draws from his proof is that every sentence in his script expresses a thought (Frege 1893, §32). However, Ricketts elsewhere makes it clear that he thinks that there is, in Frege's writings, a deep tension between, on the one hand, the special connection between logic, truth and, on the other hand, the way in which Frege introduces and discusses his system (Ricketts 1985, 3). So it *could* be that Frege is just confused on the matter—that his comments have anomalous status, but that this is something which he himself fails to explicitly recognize. But the principle of charity counts heavily against such a variation on Ricketts's theme—for Frege to have failed to notice the anomalous status of some of his arguments would have been a massive oversight on his part. If Frege's apparent correctness arguments were anomalous, we must suppose he was cognizant of this fact.

Joan Weiner, on the other hand, explicitly insists that Frege intended §§29-31 to be elucidatory. She acknowledges that these passages seem to conflict her reading, since

> on this interpretation, the question of what a term means is unanswerable…it should not be possible to *prove* the primitive terms denote something. (Weiner 1989, 124)

She provides two arguments to try to establish that Frege is not, despite his explicit statements to the contrary, trying to establish that every term has a reference (Weiner 1989, 124-6). The first is that Frege's "proof" is no proof at all, since in *Grundgesetze*, Frege refers to his formal derivations as "proofs", and his §§29-31 arguments are not formal derivations. Weiner considers that the reader may accuse her of equivocating upon the word "proof", but she takes Frege to be a reformist who thought the only acceptable mathematical proofs are those couched in his symbolism. The second argument of Weiner's is that she takes §§29-31 fall within the scope of matters which must be addressed in order to meet certain objections, but which "are not required for understanding concept-script positions", and which may be skipped on a first reading (Frege 1893a, xii).

These arguments are utterly unconvincing. The second argument ignores the context—Frege is desperately trying to convince prospective readers that his extremely dense and symbol-heavy tome is worth the effort it takes to work through it—he spends the two paragraphs following the one above presciently lamenting how dim the prospects are that he'll be properly read by his contemporaries. He thought his suggestions would make the book easier to process. Also, just a few sentences after the passage about how §§29-31 may be skipped on the first reading, Frege say that "the principle that all correctly formed signs out to refer to something" is "a principle essential for full rigour" (Frege 1893a, xii). And as for Weiner's reading of Frege as a reformist, it's very tempting to turn her modus ponens into a modus tollens, and take the presence, in Frege's work, of proofs like the one in §§29-31 as strong evidence against the particular sort of reformism she attributes to Frege.

For Weiner, the statements §31 must be anomalous. Anomalous statements do not establish conclusions, but rather provide hints which serve as cognitive aids as the reader tries to accomplish a certain task, such as grasping the symbolism, or getting the feel for the role of an primitive term, or grasping the self-evidence of an axiom once its subcomponents are articulated. What pragmatic role could the proof in §31 serve? The only purpose the *apparent* argument *could* serve can only be fulfilled if it's *actually* an argument—establishing its conclusion, that every term in the language has a reference. If this is Frege's purpose, it can only be accomplished by genuine argument. If §§29-31 has some other purpose, then one can only wonder why Frege uses such complex means to accomplish it.

§2. The correctness interpretation

§2.1 The correctness requirement

The way in which Richard Heck and Jason Stanley view Frege's aims in his exposition of the concept-script could not be more different than the view taken by proponents of the elucidatory interpretation. They fully acknowledge the deep and non-anomalous role which semantic reasoning plays in these sections. Stanley, citing work by Tappenden and Heck, briefly argues that Frege was quite familiar with proto-model-theoretic techniques (1996, 64-5). Heck goes as far as to argue that elements of Frege's §§29-31 proof anticipate Tarski's theory of truth (2012, 52). I will not adjudicate here on the merit of these specific claims—suffice to say that I'm largely in sympathy with their claims, as should be obvious from my use of some of their arguments in my case against the elucidationists.

The disagreement between myself and them is more narrow. While both Heck and Stanley readily grant the possibility that that some contemporary techniques might not have the same relevance for Frege as they do for us (Stanley 1996, 65-6; Heck 2012, 27), correctness argument are not among them—proofs of correctness have a vital role for Frege. They make the following claim:

> Correctness Requirement: Frege thought that it was possible to prove that his axioms were true and that his inference rules were truth-preserving, and he thought that we were *required* to prove this.

My worry about the correctness requirement is quite distinct from the sort of concerns raised by the elucidationists. If we accept, contra Rickett, Weiner, and Goldfarb, the claim that it's perfectly possible to express semantic claims, there's no reason to suppose a passage which expresses a soundness argument need be anomalous—it can express, or at least hint at, perfectly legitimate content. The worry, as expressed in §5.3 of the previous chapter, is instead that such argument will prove circular. If natural language can properly serve as a meta-language in which to express the correctness proof, then there's no issue. But given the lack of appropriate second-level predicates in English and German, it's unclear whether correctness arguments can be properly expressed in these natural languages. And even if they can be so expressed in natural language, the *necessity* of doing so conflicts with the claim that logic is a universal language in which all content can be clearly expressed, and which is always to be preferred *except* for pragmatic reasons. Expressing arguments outside of the concept-script is usually desirable, and sometimes essential, but because arguments are more readable, or because we want to make use of poetic license—it's quite another matter to say that there are arguments the force of which rely essentially on their being expressed in something other than concept-script.

Heck and Stanley provide strong arguments for the correctness requirement, however. I'll attempt to address these arguments below, but before I do so, there are two essential points to keep in mind. The first point is that I *don't* deny that Frege argues for the truth of his sentences and the truth-preserving nature of his inference rules. Rather, I deny that this is the *point* of these arguments—the point is, instead, that once we reflect on these arguments, we recognize the associated axioms as epistemically analytic[13]. The second point is that the arguments below are inessential to my project—the E.A. and correctness interpretations are consistent with one another, and if my claim below fails to go through, the natural alternative is just to maintain that Frege's argument for his basic laws establish both the epistemic analyticity *and* truth of formulas.

§2.2 Frege's critique of the formalists

The strongest argument for the correctness requirement hinges on Heck and Stanley's reading of Frege's critique of formalist theories of arithmetic, found in its most detailed form in §§86-137 of volume II of *Grundgesetze*. Frege's prime opponents in these passages are E. Heine and J. Thomae. Since what matters here are the lessons Frege draws from his critique, I'll suspend judgement on whether Frege's characterization is a fair one, and will restrict my attention to his own views. Frege draws a distinction between *formal* and *significant* arithmetic, which do and do not, respectively, concern themselves with the reference of symbols we use in arithmetic. He advocates the latter—he focuses on Heine and Thomae's theories of arithmetic as examples of former. As Frege characterizes it, formal arithmetic concerns itself only with the tangible signs themselves rather than with their reference, a move made in order to "safeguard the existence" of numbers by identifying them with something concrete (1903a, §87).

As Frege sees it, the essential flaw in the formalist view is the complete freedom possessed with respect to the laws selected—we "lay them down on our own absolute authority, reserving complete freedom in principle and acknowledging no necessity to justify these rules"(§94). Thomae compares the rules of arithmetic to the rules in a game of chess, an analogy which Frege enthusiastically adopts in his subsequent criticisms. In formal arithmetic, as in chess, signs/pieces are "modified according to certain rules without regard for a sense", whereas significant arithmetic expresses thoughts (1903a §91). That it expresses thoughts is what explains why significant arithmetic is applicable, while formal arithmetic and chess are not:

> Why can no application be made of a configuration of chess pieces? Because, obviously, it does not express a thought…Why can one make applications of arithmetical equations? Solely because they express thoughts. (1903a, §91)

But as soon as we allow that formulas express thoughts, what we take to be permissible transitions between them ceases to be an arbitrary matter:

> if a sense had to be observed, the rules could not be laid down arbitrarily; rather, they would have to be fashioned in such a way that from formulae expressing true thoughts one could only ever derive formulae which also expressed true thoughts. (1903a, §91)

[13] Of course, inference rules can't be epistemically analytic. But they have a parallel property—understanding any premises and conclusion of the appropriate form is sufficient for recognizing the inference from the premises to the conclusion as valid, and this recognition is rooted in conceptual understanding alone. Given my arguments that Frege conceived of no extra-systemic significance to the distinction between axioms and inference-rules, a reflective awareness that an inference rule possesses this property should contribute the same amount of support for the logicality of the corresponding logical law as would a reflective awareness of the epistemic analyticity of the corresponding generalized conditional.

So, we have two connections:

> Applicability biconditional: Formulas are applicable if and only if they express thoughts.
>
> Rule biconditional: Formulas express thoughts if and only if rules governing transition between them must be truth-preserving

As Frege sees it, the formalists content themselves with shuffling symbols according to arbitrary rules and only avoid the hard work of constraining these rules at the cost of shunting the task off to the various sciences applying the rules in question. The rule and applicability biconditionals imply that to delegate the work of laying out truth-preserving rules is to delegate the applicability of arithmetic. But this won't do, for "it is applicability alone which elevates arithmetic above a game to the rank of a science" (1903a, §91). Formal arithmetic is no science at all—to be a science, we need applicability, and to guarantee applicability,

> it is necessary that the formulae express a sense and that the rules find their grounding in the reference of the signs. (Frege 1893, §92)

§2.3 Heck and Stanley on Frege's arguments

Heck takes Frege's comment above to straightforwardly imply the correctness requirement:

> Since Frege is interested in developing a system of significant arithmetic, he in particular owes some account of why the rules of the Begriffsschrift are non-arbitrary, that is, a demonstration that they are truth-preserving (and a similar demonstration that its axioms are true). Unless Frege flagrantly failed to do just what he is criticizing the Formalists for failing to do, he must somewhere have provided such an account. There is no option but to suppose that he does so in Part I of *Grundgesetze* and that the elucidatory demonstrations in particular are intended to show that the rules of the system are truth preserving and that the axioms are true (Heck 2007, 47-48).

So Heck quite explicitly thinks that Frege's criticism of the formalists is one which can be met only by demonstrating the correctness of a formal system for arithmetic, presumably on the grounds that such a correctness proof is what Frege means by "grounding rules in the reference of signs". Stanley arrives at the same conclusion—citing §91, he says

> The role of sense and reference is thus to help justify the axioms and rules of inference of the formal system, and thereby to account for the application of the formal theory of arithmetic (1996, 61)

This conclusion, however, is far too quick. Frege's rule and applicability biconditionals jointly imply that arithmetic is applicable if and only if the rules which govern it are truth-preserving. While it's blindingly obvious that Frege thought inference rules in arithmetic (and, indeed, in any discipline) must be truth-preserving, and plausible that the formal arithmeticians ignore this only at the expense of failing to account for applicability, it does not follow that there's any requirement that one is required to (or can) *prove* that these rules are truth-preserving. It's a point simple enough that it's apt to be overlooked: if requiring arithmetic to be truth-preserving implied that we were required to *demonstrate* this fact, then Heck and Stanley needn't have gone through the trouble to address Frege's arguments against the formalists at all—these arguments are hardly necessary to appreciate the importance of truth-preserving inference rules.

The obvious retort is to insist that Frege wanted to explain the applicability of arithmetic. But while it's essential for Frege that arithmetic be applicable, Heck and Stanley have given no argument that *explaining* this fact was a task he was concerned with. Of course, one of the central purposes of my E.A.

interpretation is to explain why Frege took logical laws to be universally applicable. But the emphasis here is on *universal*—it was important for Frege to account for the universal applicability of arithmetic, because the whole point of his logicism is to account for arithmetic's special status, and its universal applicability is what sets it apart. By contrast, precious little of what Frege says about mere applicability in the sections under attention concerns numbers especially—his arguments for his biconditionals are not at all peculiar to arithmetic, but suffice to show that *any* time we lay down rules arbitrarily, we must be disregarding content, and hence applicability. That is, a formalist physics or chemistry would be just as bad as a formalist theory of arithmetic, and for precisely the same reasons.

To be fair, however, some comments of Frege's suggest he may have found the question important:

> In formal arithmetic, we do not need to justify the rules of the game; we simply lay them down. We do not need to prove that there are numbers with certain properties; we simply introduce the figures for whose deployment we give rules. (1903a, §89)

If Frege means to contrast formal and significant arithmetic here, then a plausible implicature is that significant arithmetic *does* need to justify its own rules. But it's just not clear that doing so would explain the applicability of arithmetic. In Frege's bi-conditionals above, there remains a question of explanatory priority—it seems most natural to hold not only that the contentfulness of arithmetic explains its applicability, but also that this contentfulness explains why rules concerning it must be truth-preserving, rather than the other way around. The most natural way to explain contentfulness of a formula is as follows—show that all of the terms within the formula have a reference, and thereby conclude that the formula expresses a thought. And this, of course, is precisely the argument Frege gives in §§29-31, and precisely the conclusion he draws in §32. It just wouldn't make sense to establish that a formula expresses a thought by showing that certain inference rules to which it I subject are truth-preserving—this gets things backwards. What's more, Frege himself emphasizes the proper explanatory priority

> This goal [of knowledge] requires the rules to be so constituted that, if a new proposition is derived from true propositions in accordance with them, it too is true. Whether the rules are of this kind can, naturally, be judged only after the signs have been given reference; for beforehand we cannot use them to form propositions that express a true thought. (1903a, §104)

Finally, even if we accept that Frege thought we needed to explain the applicability of arithmetic, and that explaining this applicability required demonstrating that our rules are grounded in the reference of signs, it's not obvious that this requires a full correctness proof. Arithmetical statements are, for Frege, emphatically non-basic—we define numbers in terms of value-ranges. Thus, grounding arithmetical inferences in the reference of signs might just amount to grounding statements about number in statements about value-ranges[14].

If Frege required that his arithmetical principles be grounded in the reference of signs, it won't follow that we need to prove the same for his basic laws—we can't assume that what we require of non-basic truths we must also require of basic ones. While Frege was quite concerned with showing that his value-range names refer, there's no evidence that he was similarly concerned with demonstrating the truth of this law. If he was worried about potential charge that he's setting out rules for value-ranges arbitrarily,

[14] Jason Stanley claims that requiring that inferences used in arithmetic are grounded in the reference of signs necessarily involves justifying logical inference rules, since logical inference rules are used in arithmetic (as they are in any science) (1996, 61). However, it's not obvious that these were the rules Frege was concerned in grounding—the illustrative example Frege discusses is the transition from formulas of the form '$\Phi(3+5)$' to formulas of the form '$\Phi(8)$'. It seems to me that we've grounded such inferences in the reference of signs if we give a purely logical proof that $3+5=8$.

we'd expect him to mention it in §§86-137—after all, in the proceeding section, he's not shy about defending himself against the claim that he's engaging in "arbitrary, lawless creation" when he introduces value-ranges.

§3. Conclusion

I've argued for two claim s in this chapter. Firstly, we shouldn't take any of Frege's semantic arguments to be *merely elucidatory*—they are not anomalous, and play a crucial role in perfectly legitimate arguments. Frege's perspectives on the relation between truth and judgement don't preclude the use of truth-predicate in argumentation, and although there's no legitimate way to engage in contrastive talk about concepts and objects, much of his semantic talk, even that which talks about the referents of function names, can in principle be unproblematically rephrased. Secondly, I argued that despite the legitimacy of semantic talk, correctness arguments are problematically circular. This should not bother us, however, because, because Richard Heck and Jason Stanley have failed to convincingly show that Frege's criticisms of his formalist contemporaries imply that he was committed to providing such arguments.

Conclusion

In this dissertation, I've argued that there's a case for logicality implicit in Frege's arguments for his basic laws. Frege argues for their truth, but since they're basic, he can't be giving his laws, qua thoughts, any deeper justification—he must be arguing for their truth qua formulas, or for their logicality, or both. Although I've suggested Frege would have seen something problematically circular about correctness proofs, my primary contention is that whatever other role Frege's arguments for his basic laws may play, one of their main roles is to get us to see that his basic laws are logical. They accomplish this in the following way—they take us through the content of the formulas which express Frege's basic laws, and this gets us to see how the truth of these formulas follows from the stipulations which govern the symbols which occur within them. Once we recognize that, we appreciate that understanding his basic laws is sufficient for recognizing them as true, and that this recognition is rooted in conceptual understanding alone. We thereby recognize Frege's formulas for his basic laws as strongly epistemically analytic, as is required for us to take those basic laws as logical.

To explain why appreciating the presence of simple arguments for the truth of formulas should tell us anything about understanding, I proposed that a certain connection between understanding and inference is implicit in Frege's view. If we're not competent in making certain simple valid inferences to and from 'ψ' we can't be said to understand 'ψ'. It's impossible for judging beings to be uniformly incompetent with all logical inferences—such a being wouldn't genuinely understand any of the thoughts they seem to assert. Since logical laws are implicit in our understanding, they are implicit in assertion, the manifestation of an acknowledgement of truth. Insofar as understanding a sentence is connected with grasping the thoughts those sentences express, logical laws are similarly implicit in judgement. Since truth-values are implicitly recognized by anyone who judges, our competence in logical inference is inextricably tied to our grasp of truth.

If the picture I've laid out is correct, then Frege, far from being barred from giving any grounds for his laws being logical, was able to make a very deep case for the logicality of his basic laws. While he did not, and did not aim to, say anything which would prove compelling to a non-classical logician, he did considerably more than merely prove plausibly general and undeniable arithmetical principles from basic laws which are more obviously general and undeniable—he gave a deeper explanation for why we have reason to take those basic laws to be universally applicable, all-encompassing, and conceptually undeniable. No one can effect a sustained denial of a basic logical law, for anyone who does so must not be competent in seeing that such a denial results in a quick contradiction—their failure to realize this means that they just don't grasp the law in question, and hence can't affect a cognitive denial of it. And the deep connection between inference and understanding means that *all* assertions concerning any subject-matter whatsoever is subject to evaluation by logical laws.

In my E.A. interpretation, our reasons for taking certain basic laws as logical are very deeply involved with our recognition of the semantic properties of concept-script formulas which express them. However, I've been careful to not make it appear that Frege gave logic any sort of grounding in linguistic or inferential behavior. Arithmetical laws are true because of basic logical principles, and these basic laws stand completely on their own, and require no deeper metaphysical grounding—there is no more fundamental fact we can appeal to in order to explain why they are true. But there is still a great deal to be said for how *we* relate to such truths—in particular, why thinkers are right in taking them to be true. The utterly fundamental nature of logical laws does not mean that we need to resort to brute dogmatism as regards their logicality, or that their logicality will be immediately clear to us. We can instead give a deep reason for why we're right in taking them to be logical. The provision of such a reason in no way marks a retreat to psychologism, a view the primary fault of which, Frege thought, is its abandonment of the normative dimension of logic.

One of the main attractions of my view is that it provides a middle way between the correctness and elucidatory interpretations. Unlike the elucidatory interpretation, my view is perfectly compatible with recognizing that Frege's works, *Grundgesetze* especially, contain subtle and finely crafted semantic arguments—we can take at face-value those places in which Frege is clearly proving things about his concept-script. But on the other hand, my view is, I've argued, compatible with the claim that Frege's concept-script forms a universal framework for the perspicuous expression of thoughts on any subject whatsoever. We needn't provide anything like a general solution to the notorious difficulties in expressing the contrast between concepts and object in order to take semantic talk seriously, and as something which is in principle expressible in the concept-script, once the script has been suitably augmented with certain non-logical terminology. Though some of Frege's semantic talk misses its mark, it hints *at* something—such passages are imprecise, but not unavoidably so. The sorts of arguments Frege is well-placed to provide for logicality will not themselves consist solely of logical truths, but they nevertheless consist of legitimate, contentful, non-anomalous reasoning.

If the E.A. interpretation is workable, then a number of things have been accomplished. The role of the heuristic indicators has been clearly explained. Something of the possibilities and limits of semantic theorizing have been demarcated as well—semantic reasoning is legitimate, but there are certain conclusions semantic arguments cannot establish without circularity. But most importantly, the interpretation goes some way to explain highlight the connections which hold between logic, truth, judgement, and understanding, providing a compelling picture of how Frege, even given his many distinctive commitments, was well-situated to provide a deep explanation for why we should take certain truths as logical.

Bibliography

Frege: Primary Sources

- Frege, Gottlob. 1879/1977. "Begriffsschrift." In *From Frege to Gödel: A Sourcebook in Mathematical Logic,* edited by Jean van Heijenoort. Cambridge: University Harvard Press.
- Frege, Gottlob. 1879-1891/1979. "Logic." In *Posthumous Writings*, edited by Hans Hermes, Friedrich Kambartel, and Friedrich Kaulbach. Oxford: Basil Blackwell.
- Frege, Gottlob. 1880-81/1979. "Boole's logical Calculus and the Concept Script." In *Posthumous Writings*, edited by Hans Hermes, Friedrich Kambartel, and Friedrich Kaulbach. Oxford: Basil Blackwell.
- Frege, Gottlob. 1884/1960. *The Foundations of Arithmetic: A Logico-mathematical inquiry into the concept of number.* Translated by J.L. Austin. New York, Harper Brothers.
- Frege, Gottlob. 1885a/1984. "On Formal Theories of Arithmetic." In *Collected Papers on Mathematics, Logic, and Philosophy*, edited by Brian McGuinness. Oxford: Basil Blackwell.
- Frege, Gottlob. 1885b/1984. "The Law of Inertia." In *Collected Papers on Mathematics, Logic, and Philosophy*, edited by Brian McGuinness. Oxford: Basil Blackwell.
- Frege, Gottlob. 1891/1997. "Function and Concept." In *The Frege Reader,* edited by Michael Beaney. Oxford: Basil Blackwell.
- Frege, Gottlob. 1892a/1997. "On Sinn and Bedeutung." In *The Frege Reader*, edited by Michael Beaney. Oxford: Basil Blackwell.
- Frege, Gottlob. 1892b/1997. "On Concept and Object." In *The Frege Reader,* edited by Michael Beaney. Oxford: Basil Blackwell.
- Frege, Gottlob. 1892-1895/1997. "Comments on Sinn and Bedeutung." In *The Frege Reader*, edited by Michael Beaney. Oxford: Basil Blackwell.
- Frege , Gottlob. 1893/2013. *Basic Laws of Arithmetic*. Translated by Philip Ebert and Marcus Rossberg. Oxford: Oxford University Press.
- Frege, Gottlob. 1897a/1997. "Logic." In *The Frege Reader*, edited by Michael Beaney. Oxford: Basil Blackwell.
- Frege, Gottlob. 1897b/1984. "On Mr. Peano's Conceptual Notation and my Own." In *Collected Papers on Mathematics, Logic, and Philosophy*, edited by Brian McGuinness. Oxford: Basil Blackwell.
- Frege, Gottlob. 1897-8/1979. "Argument for my stricter Canons of Definition." In *Posthumous Writings*, edited by Hans Hermes, Friedrich Kambartel, and Friedrich Kaulbach. Oxford: Basil Blackwell.
- Frege, Gottlob. 1899-1906/1979. "On Euclidean Geometry." In *Posthumous Writings*, edited by Hans Hermes, Friedrich Kambartel, and Friedrich Kaulbach. Oxford: Basil Blackwell.
- Frege , Gottlob. 1903a/2013. *Basic Laws of Arithmetic*. Translated by Philip Ebert and Marcus Rossberg. Oxford: Oxford University Press.
- Frege, Gottlob. 1903b/1984. "On the Foundations of Geometry: First Series." In *Collected Papers on Mathematics, Logic, and Philosophy*, edited by Brian McGuinness. Oxford: Basil Blackwell.

- Frege, Gottlob. 1906a/1984. “On the Foundations of Geometry: Second Series.” In *Collected Papers on Mathematics, Logic, and Philosophy*, edited by Brian McGuinness. Oxford: Basil Blackwell.
- Frege, Gottlob. 1906b/1997. “Introduction to Logic.” In *The Frege Reader*, edited by Michael Beaney. Oxford: Basil Blackwell.
- Frege, Gottlob. 1914/1979. “Logic in Mathematics.” In *Posthumous Writings*, edited by Hans Hermes, Friedrich Kambartel, and Friedrich Kaulbach. Oxford: Basil Blackwell.
- Frege, Gottlob. 1915/1997. “My Basic Logical Insights.” In *The Frege Reader*, edited by Michael Beaney. Oxford: Basil Blackwell.
- Frege, Gottlob. 1918a/1997. “Logical Investigations Part I: Thoughts.” In *The Frege Reader*, edited by Michael Beaney. Oxford: Basil Blackwell.
- Frege, Gottlob. 1918b/1997. “Logical Investigations Part II: Negation.” In *The Frege Reader*, edited by Michael Beaney. Oxford: Basil Blackwell.
- Frege, Gottlob. 1919/1997. “Notes for Ludwig Darmstaedter.” In *The Frege Reader*, edited by Michael Beaney. Oxford: Basil Blackwell.
- Frege, Gottlob. 1923-6/1984. “Logical Investigations Part III: Compound Thoughts.” In *Collected Papers on Mathematics, Logic, and Philosophy*, edited by Brian McGuinness. Oxford: Basil Blackwell.
- Frege, Gottlob. 1980. *Frege’s Philosophical and Mathematical Correspondence*. Translated by Hans Kaal. Edited by Gottfried Gabriel, Hans Hermes, Fredrich Kambartel, Christian Thiel, Albert Veraart. Oxford: Basil Blackwell.

Frege Exegesis

- Antonelli, Aldo and Robert May. 2000. “Frege’s New Science.” In *Notre Dame Journal of Formal Logic*. Vol 41, 3, 242-270.
- Blanchette, Patricia. 2012. *Frege’s Conception of Logic.* New York: Oxford University Press.
- Burge, Tyler. 1990/2005. “Frege on Sense and Linguistic Meaning.” In *Truth, Thought, Reason: Essays on Frege*. Oxford: Oxford University Press
- Burge, Tyler. 1992/2005. “Frege on Knowing the Third Realm.” In *Truth, Thought, Reason: Essays on Frege*. Oxford: Oxford University Press
- Burge, Tyler. 1998/2005. “Frege on Knowing the Foundation.” In *Truth, Thought, Reason: Essays on Frege*. Oxford: Oxford University Press
- Currie, Gregory. 1987. “Remarks on Frege’s Conception of Logic.” In *Notre Dame Journal of Formal Logic*, 28, No. 1, 55-68.
- Cook, Roy. 2013 “Appendix: How to Read *Grundgesetze*.” In *Basic Laws of Arithmetic*, Translated by Philip Ebert and Marcus Rossberg. Oxford: Oxford University Press.
- Dummett, Michael. 1991. *Frege: Philosophy of Mathematics*. London: Gerald Duckworth & Co.
- Goldfarb, Warren. 2010. “Frege’s Conception of Logic.” In *The Cambridge Companion to Frege*, edited by Michael Potter and Thomas Ricketts. Cambridge: Cambridge University Press
- Heck, Richard. 2010. “Frege and Semantics.” In *The Cambridge Companion to Frege*, edited by Michael Potter and Thomas Ricketts. Cambridge: Cambridge University Press
- Heck, Richard. 2012. *Reading the Grundgesetze*. Oxford: Oxford University Press
- Jeshion, Robin. 2001. “*Frege’s Notions of Self-evidence*.” In *Mind*, 110, No.440, 937-976.

- May, Robert. 2006. "Frege on Indexicals". In *Philosophical Review,* 115, 487-516.
- Ricketts, Thomas. 1985. "Frege, the Tractatus, and the Logocentric Predicament". In *Noûs*, 19, 3-15.
- Ricketts, Thomas. 1986. "Objectivity and Objecthood: Frege's Metaphysics of Judgment." In *Frege Synthesized: essays on the philosophical and foundational work of Gottlob Frege.* edited by Jaakko Hintikka and Leila Haaparanta. Dordrecht: D. Reidel Publishing Co.
- Ricketts, Thomas. 1996. "Logic and Truth in Frege." In *Proceedings of the Aristotelian Society, Suppliementary Volumes,* 70, 121-140.
- Ricketts, Thomas. 1997. "Frege's 1906 Foray into Metalogic." In *Philosophical Topics,* 25, 169-188.
- Ricketts, Thomas. 2010. "Concepts, objects and the Context Principle." In *The Cambridge Companion to Frege*, edited by Michael Potter and Thomas Ricketts. Cambridge: Cambridge University Press
- Stanley, Jason. 1996. "Truth and metatheory in Frege'. In *Pacific Philosophical Quarterly*, 77, 45-70.
- Stephenson, Leslie. 1973. "Frege's Two Definitions of Quantification." In *The Philosophical Quarterly*, 23, No. 92, 207-223.
- Sullivan, Peter. 2004. "Frege's Logic." In *The Rise of Modern Logic: From Leibniz to Frege (Handbook of the History of Logic)*, edited by Dov M. Gabbay and John Woods. Oxford: Elsevier.
- Tappenden, Jamie. 2000. "Frege on Axioms, Indirect Proof, and Independence Arguments in Geometry: Did Frege Reject Independence Arguments?" In *Notre Dame Journal of Formal Logic,* 41, 271-315.
- Weiner, Joan. 1989. "On Concepts, Hints, and Horses." In *History of Philosophy Quarterly*, 6, No. 1, 115-130.
- Weiner, Joan. 1990. *Frege in Perspective*. Ithaca: Cornell University Press.
- Weiner, Joan. 1995. "Realism Bei Frege; A Reply to Burge." In *Synthese*, 102, No. 3, 363-382.
- Weiner, Joan. 2005. "Semantic Descent." In *Mind*, 454, 321-354.
- Weiner, Joan. 2010. "Understanding Frege's Project". In *The Cambridge Companion to Frege,* edited by Michael Potter and Thomas Ricketts. Cambridge: Cambridge University Press

Other Primary sources

- Carnap, Rudolf. 2004. *Frege's Lectures on Logic: Carnap's Student Notes, 1910-1914.* Translated by Eric H. Rick and Stephen Awodey. Chicago; Open Court.
- Russell, Bertrand. 1906. "The Theory of Implication". In *American Journal of Mathematics*, 28, No. 2, 159-202.
- Russell, Bertrand and Alfred N. Whitehead. 1910. *Principia Mathematica: Volume I*. Cambridge: Cambridge University Press

Other Exegesis

- Milne, Peter. 1999. "Tarski, Truth, and Model Theory." In *Proceedings of the Aristotelian Society,* 99, 141-167.
- Milne, Peter. 2008. "Russell's Completeness Proof." In *History and Philosophy of Logic*, 29, No 1, 31-62.
- Proops, Ian. 2007. "Russell and the Universalist Conception of Logic." In *Nous*, 41, 1-32.

Non-exegetical

- Boghossian, Paul. 1997. “Analyticity.” In *A Companion to the Philosophy of Language*, edited by Bob Hale and Crispin Wright. Oxford: Blackwell
- Carroll, Lewis. 1895. “What the Tortoise said to Achilles.” In *Mind*, 4, 278-80.
- Dummett, Michael. 1973/1978. “The Justification of Deduction.” In *Truth and Other Enigmas*. London: Duckworth.
- Kleene, Stephen. 1952. *Introduction to Metamathematics*. New York: Van Nostrand
- Maudlin, Tim. 2008. “Reducing Revenge to Discomfort”. In *Revenge of the Liar: New Essays on the Paradox*. Oxford: Oxford University Press.
- Peregrin, Jaroslav. 2014. *Inferentialism: Why Rules Matter*. New York: Palgrave Macmillan.
- Quine,W.V.O. 1981. “Things and Their Place in Theories.” In *Theories and Things*, Cambridge, MA: Harvard University Press.
- Ryle, Gilbert. 1940/2009. *The Concept of Mind: 60th Anniversary Addition*. New York: Routledge.
- Beall, J.C. and Greg Restall. 2000. “Logical Pluralism.” In *Australasian Journal of Philosophy*, 78, No. 4, 475-493.
- Shapiro, Stewart. 2009. “We hold these truths to be self-evident: But what do we mean by that?” In *Review of Symbolic Logic*, 2, No 1, 175-207.
- Stroud, Barry. 1979. “Inference, Belief, and Understanding.” In *Mind*, 88/305, 179-196.
- Tanswell, Fenner. 2015. “A Problem for the Dependence of Informal Proofs on Formal Proofs.” In *Philosophia Mathematica*, 24, No 2, 295-310.
- van Heijenoort, Jean. 1967. “Logic as calculus and logic as language”, in *Boston Studies in the Philosophy of Science, vol. III, In Memory of Russell Norwood Hanson*, pp. 440–446, edited by R.S. Cohen and M.W. Wartofsky.