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R&D Cyclicality and Composition Effects: A Unifying Approach[‡]

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Abstract

Existing empirical studies do not concur on whether R&D spending is procyclical or countercyclical: the former hypothesis is supported by studies of aggregate R&D spending, whereas the latter is vindicated by firm-level evidence. In this paper, we reconcile the two facts by advancing a general equilibrium framework, in which, while a single firm's R&D spending profile is countercyclical, aggregate R&D spending is procyclical owing to procyclical fluctuations in the number of R&D performers. Our findings suggest that economic crises might be beneficial for economic performance by fostering individual R&D effort. An advantage of our framework is that it brings together conflicting pieces of empirical evidence, while incorporating and building upon Schumpeter's hypothesis of countercyclical innovation.

Key words: Economic cycles, opportunity cost hypothesis, procyclicality of R&D, countercyclicality of R&D.

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1 Introduction

1.1 Motivation

An influential part of Joseph Schumpeter's legacy is the idea that economic crises allow an economy to restructure itself on a more efficient basis (Schumpeter (1943)). This leads one to think of economic downturns as a way to induce research and development (R&D) activities, thereby suggesting that, in accordance with Schumpeter's view, R&D spending should exhibit countercyclical behaviour. This prediction has been explored extensively on both theoretical[1] and empirical levels. A noteworthy feature of the latter strand of research is the micro/macro dichotomy of the results obtained: macro-data (economy-wide and industry data) based studies[2] show R&D spending to be procyclical, whereas firm-data based results in Aghion, Askenazy, Berman, Cette, and Eymard (2012), Lopéz-García, Motero, and Moral-Benito (2012), Beneito, Rochina-Barrachina, and Sanchis-Llopis (2015) point in the opposite direction.

Aghion, Angeletos, Banerjee, and Manova (2010) accommodate both pro- and countercyclicality of R&D within a single framework by incorporating liquidity constraints: even though firms would like to make their R&D spending profiles countercyclical, they are unable to do so because of insufficient access to loanable funds. In all aforementioned firm-data based papers, taking into account a measure of credit tightness indeed makes R&D spending procyclical.

However, the liquidity constraint approach does not reconcile the aforementioned micro/macro discrepancy in the behaviour of R&D. In addition, and related to the previous point, it leaves unanswered the question why, by contrast with the firm level, economy-wide and industry-wide R&D spending exhibits procyclical properties even without considering credit constraints. Furthermore, the liquidity constraint hypothesis does not shed much light on how countercyclical R&D of unconstrained firms (which make a significant share of observations in the firm-data based papers[3]) transforms into procyclical R&D spending on the

[1] See Section 1.2.

[2] Examples of those are Fatás (2000), Rafferty (2003), Wälde and Woitek (2004), Comin and Gertler (2006), see Table 1 for greater detail.

[3] In particular, financially unconstrained firms make approx. 67%, 77% and 46% of the total

aggregate level of industries and economies.

The reliance on the liquidity constraint hypothesis is undermined by limited industry-level evidence to support it: Ouyang (2011) finds the predictions of the hypothesis (i.e., the reversal of R&D's cyclical behaviour in the presence of credit constraints) to be valid only in the case of demand-driven cyclical fluctuations. Yet, a few manifestations of the opportunity cost theory (e.g. Bental and Peled (1996), Matsuyama (1999, 2001), Francois and Lloyd-Ellis (2003), see discussion below) suggest that the opportunity cost effect can be induced by supply side dynamics as well. The results obtained by Ouyang (2011) further strengthen the idea that the incorporation of liquidity constraints is insufficient for understanding completely the procyclicality of R&D spending on the macro-level.

In this paper, we argue that the difference between macro- and micro-based results is driven by a composition effect: even where an individual firm's R&D spending is countercyclical, aggregate R&D dynamics can be procyclical because of changes in the number of R&D performers (owing to, for example, entry/exit dynamics). We base our conjecture on the combination of the following motivating observations: first of all, it is widely recognised in industrial organisation (IO) literature that the probability of a firm's engaging in R&D depends positively on its size.[4] Naturally, in the situation of a crisis one would expect firms' sizes (as measured by, e.g., sales or employment) to drop, thereby driving down both the volume of each cohort of firms of a given size, and the share of R&D performers within it – together the two observations suggest that during crises a smaller share of a lower number of firms engages in R&D in an industry, thus enabling one to expect that economic cycles produce a procyclical aggregation-based effect on the amount of R&D in an industry, as channelled through procyclical dynamics in the number of R&D-performing firms.

We illustrate our point by presenting a tractable general-equilibrium model, in which the number and size of individual firms are procyclical, while the research intensity for a firm of a given size is countercyclical. Overall, the economic cycle

number of firms in the datasets employed, respectively, by Aghion et al. (2012), Lopéz-García et al. (2012) and Beneito et al. (2015). See (Aghion et al., 2012, pp. 1008–1009), (Lopéz-García et al., 2012, Table 2, p. 32), (Beneito et al., 2015, Table 1, p. 352).

[4] See, e.g., (Cohen and Klepper, 1996, Stylised fact 1, p. 928).

can bring about procyclical fluctuations of R&D on aggregate for industries and the economy.

These results come from introducing a two-level structure in an economy, whereby the final good is produced using industries' outputs, which themselves are aggregated from differentiated products made by competing monopolist firms. Each monopolist engages in two (limitedly substitutable) activities: production and R&D, which are, respectively, procyclical and countercyclical owing to the opportunity cost effect. If, however, substitutability between the two activities is not high enough (i.e., not too large a share of a firm's resources is shifted between production and R&D during an economic cycle), drops in individual R&D spending during upturns are offset by increases in aggregate industry R&D spending resulting from the entry of new firms (the opposite dynamics obtains during downturns).

The rest of the paper is structured as follows. In Section 1.2 we review literature relevant to our research; in Sections 2.1–2.4 the baseline model is introduced (2.1–2.3) and solved (2.4); in Section 2.5 we examine empirical validity of the results obtained. In Section 3 the effect of technology accumulation is introduced and investigated; the last section concludes.

1.2 Related Literature

Our paper is related to the rich literature on cyclicality of innovation. In the empirical dimension, one could list a number of works largely characterised by the macro/micro dichotomy discussed above (see Table 1).[5]

On the theoretical front, given the reconciliatory purpose of our paper, it is related most closely to the work by Aghion et al. (2010), where pro- and countercyclicality of R&D are brought together through the use of the liquidity constraint hypothesis. By contrast with that work, however, not only does our framework accommodate pro- and countercyclicality of R&D, but it also explains

[5] Barlevy (2007) uses firm-level data to show a firm's growth rate of R&D spending to be an increasing function of the industry's growth (i.e. suggesting R&D's being procyclical). One could argue though that an industry's growth, being an industry-wide aggregate indicator, can channel the impact coming from other firms in the industry through, e.g., inter-industrial competition.

Table 1: Empirical findings on the cyclicality of R&D.

Study	Level of Data	Sample	R&D Cyclicality	Salient findings
Fatás (2000)	Country	USA, 1961 – 1996	P/C	Growth rates of total R&D and GDP are positively correlated
Rafferty (2003)	Country	USA, 1953 – 1999	P/C	Real firm-financed R&D and GDP are positively cointegrated
Wälde and Woitek (2004)	Country	G7 countries, 1973 – 2000	P/C	Cyclical components of R&D per capita and GDP per capita are positively correlated
Comin and Gertler (2006)	Country	USA, 1948 – 2001	P/C	Short- and medium-run cyclical components of R&D and GDP are positively correlated
Barlevy (2007)	Industries/ Firms	7 719 firms, USA, 1978 – 2004[6]	P/C	Growth rates of firms' real R&D expenditures and the industry's real output/value added are positively correlated
Aghion et al. (2012)	Firms	≈13 000 firms, France, 1994 – 2004	C/C	R&D[7] is negatively correlated with changes in a firm's sales; the relationship becomes procyclical for financially constrained firms
Lopéz-García et al. (2012)	Firms	3 278, Spain, 1991 – 2010	C/C	
Beneito et al. (2015)	Firms	3 361 firm, Spain, 1990 – 2006	C/C	

the micro/macro dichotomy of R&D cyclical behaviour. Additionally, our results are achieved through the incorporation of a mechanism different from the liquidity constraints – namely, entry/exit dynamics of R&D performers.

[6]Barlevy (2007) also uses two other datasets comprising 3 454 and 6 160 American firms for 1959 – 1999 and 1988 – 2004, respectively.

[7]Aghion et al. (2012) and Beneito et al. (2015) use R&D investment, while Lopéz-García

In addition, our paper is related to the branches of theoretical general-equilibrium literature on both countercyclicality and procyclicality of innovation. With regards to the former, we can mention the works by Bental and Peled (1996), Aghion and Saint-Paul (1998), Matsuyama (1999, 2001) and Francois and Lloyd-Ellis (2003). A common factor behind the countercyclicality of R&D in these papers is the opportunity cost mechanism, explicitly introduced in Aghion and Saint-Paul (1998): when R&D has disruptive impact on production (i.e., engaging in R&D requires channelling some resources from production), it becomes countercyclical, since a firm's costs of diverting funds from production to R&D are procyclical. Similarly to the aforementioned papers, we consider the behaviour of R&D on the firm level in the framework of the opportunity cost theory.[8] Since, however, we are not interested in mechanisms behind economic fluctuations per se, our model does not generate endogenous cycles (as in Bental and Peled (1996), Matsuyama (1999, 2001) and Francois and Lloyd-Ellis (2003)), but rather uses the cyclicality of productivity in production as a modelling 'input', which induces cyclical reallocation of funds between production and R&D.[9]

The procyclicality of R&D is studied by Wälde (2005), Barlevy (2007), Francois and Lloyd-Ellis (2009), Bambi, Gozzi, and Licandro (2014). Our paper contributes to this body of literature by introducing a novel mechanism generating R&D procyclicality on the aggregate level through the composition effect, as embodied in firm entry/exit dynamics. The key difference between the papers above and our work is that R&D procyclicality manifests itself not on the individual firm level, but on that of industries, while coexisting with the countercyclicality firms' R&D. In other words, unlike the other papers, we consider both the procyclicality of aggregate R&D spending and the countercyclicality of individual R&D as two sides of a single phenomenon, with the former acting as an addition on top of the latter.

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et al. (2012) focus on the share of a firm's R&D investment in its total physical and R&D investment.

[8]In particular, by making producers choose between allocating their facilities to production and to R&D, we make the latter 'disruptive', as in the papers discussed in the text.

[9]Aghion and Saint-Paul (1998) employ a technically similar approach by allowing the dynamics of aggregate demand in their model to be driven by a two-state Markov process, which results in cyclical reallocation of funds between production and R&D.

2 The Baseline Model

The model below introduces a three-level economy where the final good is produced by competitive firms using a Codd-Douglas technology to combine labour with the composite of outputs provided by homogeneous competitive industries. Each industry's output is made from products supplied by competing monopolist firms engaging in both production and R&D, of which the latter is countercyclical. The mass of monopolist firms in each industry varies procyclically, and will be shown to act as the driving force of the composition effect behind the procyclicality of aggregate R&D spending.

The model captures a number of stylised facts on innovation within the strands of growth and IO literature

1. Macro facts
   (a) Aggregate output and productivity are procyclical (RBC literature);
   (b) Price mark-ups are countercyclical (see, e.g., Christiano, Eichenbaum, and Evans (2005), Comin and Gertler (2006), Galí, Gertler, and López-Salido (2007), Justiniano, Primiceri, and Tambalotti (2010));
   (c) Net entry of firms is procyclical (see Campbell (1998), Clementi and Palazzo (2016)).
2. IO facts
   (a) A firm's R&D spending increases monotonically in the firm's size (see, for example, (Cohen and Klepper, 1996, Stylised fact 2));
   (b) The elasticity of R&D spending with respect to the firm's size is unity (see, e.g., (Cohen and Klepper, 1996, Stylised fact 3)).

2.1 Aggregate Production

Suppose that the final (consumable) good $Y(t)$ is produced by competitive firms using fixed amount of labour L and the composite capital good aggregated from intermediate inputs supplied by the constant mass N of symmetric industries. The production technology is linear homogeneous and takes the form

$$Y(t) = \tfrac{1}{1-\nu}\left(\int_0^N y(i;t)^{\,1-\nu} di\right) L^{\nu} = \frac{Ny(t)^{\,1-\nu} L^{\nu}}{1-\nu} \tag{1}$$

where $y(i;t)$ is the product of the i-th industry, L is the economy's labour force, and ν is the elasticity of $Y(t)$ with respect to L (and the share of wage income in the economy's output). We assume that all industries are homogeneous, so that $\forall i \; y(i;t) = y(t)$, which gives rise to the last expression in (1). The term $\frac{1}{1-\nu}$ is used for normalisation purposes. The price of the final good is chosen as the numeraire. Time is continuous.

Each industry's output $y(i;t)$ is produced competitively by means of a CES production technology, using intermediate inputs $\tilde{y}(i;j;t)$ supplied by homogeneous monopolist firms

$$y(i;t) = y(t) = \left(\int_0^{m(i;t)} \tilde{y}(i;j;t)^{\,\frac{\xi-1}{\xi}} dj\right)^{\frac{\xi}{\xi-1}} = m(t)^{\frac{\xi}{\xi-1}}\, \tilde{y}(t) \tag{2}$$

where $m(i;t) = m(t)$ is the dynamically changing mass of intermediate producers in the i-th industry (which is the same across all industries), and ξ is the elasticity of substitution between the products of each two producers. In the next two sections, $m(t)$ embodies the composition effect, and it is its procyclical fluctuations that act as the force offsetting the countercyclicality of individual R&D effort (to be introduced in Section 2.2). Throughout the paper, we use tildes to denote firm-specific quantities. We assume that there are no barriers for the entry/exit of firms to industries.

In line with existing empirical evidence,[10] we assume that the elasticity of substitution between firms' products exceeds that between industries' goods

$$\xi > \frac{1}{\nu} \Leftrightarrow \nu\xi > 1 \tag{3}$$

The reason for our choice of assigning production technologies (i.e. aggregation with labour on the economy-wide level and a simple CES aggregator

[10]See, e.g., Broda and Weinstein (2006, 2010).

on the industry level) is that doing it otherwise by applying technology (1) on the industry level, results in ν playing the double role of determining both the elasticity of output with respect to labour and a firm's relative mark-up (ν and $\frac{\nu}{1-\nu}$, respectively), which would bring ambiguity in the quantitative assessment of the model carried out in Section 2.5.

2.2 Individual Firms

Suppose that intermediate inputs are made by firms using production facilities $\tilde{x}(i;j;t) = \tilde{x}(t)$, which can be maintained at the constant marginal costs of ψ. If the facilities are used exclusively for production, a firm's output equals $z(t)\,\tilde{x}(t)$, where $z(t) = \bar{z} + \hat{z}(t) > 0\ \forall t$ is the economy-wide productivity level, which has the fixed component ($\bar{z}$) and the cyclical component with a bounded image ($\hat{z}(t) \in [z_L; z_H]\ \forall t$).[11,12] Following the spirit of real business cycle (RBC) literature (see, e.g., Kydland and Prescott (1982), King, Plosser, and Rebelo (1988)), we assume $z(t)$ to be the source of fluctuations in the model's economy. In what follows, we use $z(t)$ as the cycle indicator, so that some function $B(t)$ is pro-/countercyclical if $(B(t))'_{z(t)} > 0/(B(t))'_{z(t)} < 0$.[13] Although cyclicality is usually understood in terms of a variable's alignment with fluctuations of output, the shift to $z(t)$ in our model is justified by the procyclicality of output in terms of $z(t)$ (see equations (24), (25)).

If a firm allocates part of its facilities $\tilde{\gamma}(i;j;t) = \tilde{\gamma}(t) < \tilde{x}(t)$ to R&D, its production function takes the form

$$\tilde{y}(t) = \left((z(t)\,(\tilde{x}(t) - \tilde{\gamma}(t)))^{\frac{\eta-1}{\eta}} + (\zeta\tilde{\gamma}(t))^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{4}$$

where η is the elasticity of substitution between production and R&D; ζ stands for

[11] We do not specify whether $\hat{z}(t)$ is stochastic (e.g. a Markov stochastic process with two states as in Aghion and Saint-Paul (1998) and Barlevy (2007)) or a deterministic (e.g. trigonometric) function, as it does not affect the model's key conclusions.

[12] Without loss of generality, the cyclical component $\hat{z}(t)$ is stipulated to have zero mean $\lim\limits_{t\to+\infty} \frac{1}{t} E_0 \left(\int_0^t \hat{z}(\tau)\, d\tau \right) = 0 \Leftrightarrow \lim\limits_{t\to+\infty} \frac{1}{t} E_0 \left(\int_0^t z(\tau)\, d\tau \right) = \bar{z}$.

[13] Our approach is similar to the one employed in Aghion et al. (2010) (see (Aghion et al., 2010, p. 252)).

the intrinsic productivity of R&D.[14] Motivated by empirical evidence (see (Griliches, 1998, Ch. 13)) and similarly to other theoretical works in the field (see, e.g., Comin and Gertler (2006), Barlevy (2007)), we keep ζ constant across the cycle. In what follows, our focus is on the situation where $\eta > 1$, so that investment in production facilities and in R&D facilities are gross substitutes, and the predictions of the opportunity cost theory become operational: indeed, as will be shown below, during downturns (viz. when $z(t)$ is low), a firm can use substitutability between research and production to mitigate (at least partly) the impact of a slowdown by reassigning a larger share of its facilities to R&D (the opposite is true for intervals of $z(t)$'s high values).[15]

Expression (4) suggests one to think of the model's R&D as an activity that, only so long as carried out, generates synergy effects with production and has no effect on a firm's future productivity, i.e., it ignores the impact of technology accumulation. In order to focus on our key results though, we leave aside dealing with this consideration until Section 3.

Each firm seeks to maximise its profits by choosing the level of output $\tilde{y}(t)$

[14]By assuming R&D outcomes to be a linear function of R&D spending, we leave outside consideration the stochastic nature of R&D (which is usually modelled as a Poisson process with the arrival rate of $\eta\tilde{\gamma}(t)$ – see, e.g., Grossman and Helpman (1991), Aghion and Howitt (1992)). Our reasoning for this is that the absence of individual stochasticity keeps all monopolist firms homogeneous, thus significantly improving the tractability of our model and keeping it focused on conveying its key message on the role of aggregation in generating procyclical R&D. In addition, the assumption of linear R&D technology can be reconciled with that of stochastic R&D outcomes if each firm is posited to have access to a sufficiently large number of R&D projects, so that the individual uncertainty of each one of them does not affect the dynamics of the firm's aggregate R&D portfolio (because of, for example, the law of large numbers).

[15]Mathematically, expression (4) can be thought of as a generalisation of the approach used in the model due to Aghion et al. (2010) (see equations (2)–(4)) and the stylised model in (Aghion et al., 2012, Sections 2.1, 2.2), wherein the elasticity of substitution between short-run investment and long-run R&D investment is infinity. In our model, however, for the sake of tractability the trade-off between producing and researching is not inter-, but intratemporal, since this paper does not focus on the role of intertemporal factors affecting R&D decisions (i.e. credit constraints).

and the share of facilities devoted to R&D

$$\tilde{\pi}(t) = \tilde{p}(t)\,\tilde{y}(t) - \psi\tilde{x}(t) - \Phi \tag{5}$$

$$\begin{gathered} \max_{\tilde{y}(t),\tilde{\gamma}(t)} \{\tilde{p}(t)\,\tilde{y}(t) - \psi\tilde{x}(t) - \Phi\} \\ 0 \leqslant \tilde{\gamma}(t) < \tilde{x}(t) \end{gathered} \tag{6}$$

where Φ is the fixed cost of staying in an industry, expressed in units of the final good. In addition, when maximising (6), each firm is assumed to ignore its impact on the economy's and an industry's aggregate quantities $Y(t)$ and $y(t)$.

2.3 Households

To close the model, we assume that the representative household of size L supplies inelastically the economy's labour force, and owns collectively all firms in the economy. The household's preferences are characterised by a standard twice differentiable instantaneous utility function: $u(c(t))$, $u'(c(t)) > 0$, $u''(c(t)) < 0$. The household's lifetime utility takes the form

$$U = \int_0^{+\infty} e^{-\rho t} u(c(t))\,dt \tag{7}$$

where ρ is the intertemporal discount factor.

Finally, the household's total income comprises firms' profits and labour income, and, since the economy has no investment goods, is spent exclusively on consumption, which gives rise to the budget constraint

$$C(t) \equiv c(t)\,L = Nm(t)\,\tilde{\pi}(t) + w(t)\,L \tag{8}$$

where $C(t)$ and $c(t)$ are, respectively, total and per capita consumption, and $w(t)$ is the wage rate.

2.4 Solution

We shall start with stating competitive producers' inverse demand functions, which can be derived from the corresponding profit maximisation problems.

In the case of the final good, the functions are

$$p(t) = \left(\frac{L}{y(t)} \right)^{\nu} \tag{9}$$

$$w(t) = \frac{\nu Y(t)}{L} \tag{10}$$

for, respectively, each intermediate good and labour, where $p(t)$ is the price of an industry's output. As regards intra-industry demand functions, those take the form

$$\tilde{p}(t) = \frac{p(t)\, y(t)^{\frac{1}{\xi}}}{\tilde{y}(t)^{\frac{1}{\xi}}} \tag{11}$$

Using (9) and (11) allows one to solve an intermediate producer's problem (6). First of all, the division of facilities between production and R&D can be pinned down by solving the following cost-minimisation problem

$$\begin{gathered} \min_{\tilde{x}(t),\tilde{\gamma}(t)} \{\psi\ \tilde{x}(t)\}\ \ s.t. \\ \left((z(t)\,(\tilde{x}(t) - \tilde{\gamma}(t)))^{\frac{\eta-1}{\eta}} + (\zeta \tilde{\gamma}(t))^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \leqslant \tilde{y}(t) \end{gathered} \tag{12}$$

The optimal allocation of a firm's facilities, as implied by (12), is

$$\tilde{y}(t) = Z(t)\, \tilde{x}^{*}(t) \Leftrightarrow \tilde{x}^{*}(t) = \frac{\tilde{y}(t)}{Z(t)},\ \ Z(t) \equiv \left(\zeta^{\eta-1} + z(t)^{\,\eta-1} \right)^{\frac{1}{\eta-1}} \tag{13}$$

$$\tilde{\gamma}^{*}(t) = \frac{\zeta^{\eta-1}}{\zeta^{\eta-1} + z(t)^{\,\eta-1}} \tilde{x}^{*}(t) = \frac{\zeta^{\eta-1}}{(\zeta^{\eta-1} + z(t)^{\,\eta-1})^{\frac{\eta}{\eta-1}}} \tilde{y}^{*}(t) = \left(\frac{\zeta}{Z(t)} \right)^{\eta-1} \frac{\tilde{y}^{*}(t)}{Z(t)} \tag{14}$$

where, given the absence of technology accumulation, $Z(t)$ is the total productivity of a firm's facilities arising from their optimal allocation across production and R&D. $Z(t)$'s functional form suggests it to be procyclical, which allows it to be used as a measure of cyclicality interchangeably with $z(t)$.

The first feature to note in (14) is that a firm always engages in production and R&D, so that condition $0 < \tilde{\gamma}^{*}(t) < \tilde{x}^{*}(t)$ always holds, as long as $\eta < +\infty$. As suggested by (14), when investing in R&D is a substitute for investing in production facilities (i.e. $\eta > 1$), $\tilde{\gamma}^*(t)$ exhibits countercyclical properties (viz. $(\tilde{\gamma}^*(t))'_{z(t)} < 0$), in accordance with the prescriptions of the opportunity cost theory. Note also that, in line with stylised fact 2.a (see Introduction, p. 7), a firm's R&D spending $\psi\tilde{\gamma}^*(t)$ increases in its scale of production $\tilde{y}(t)$, and the two quantities are proportional to each other (i.e., the elasticity of the former with respect to the latter is unity, as prescribed by stylised fact 2.b, p. 7).

Expression (13) reflects the synergy effect of R&D as embedded in the 'preference for diversity' (or, alternatively, 'taste for variety') feature of the CES production technology (4): as long as substitution between investing in production and R&D is not complete (i.e. $\eta < +\infty$), for any level of R&D productivity ζ and any size of production facilities $\tilde{x}(t)$ optimal engaging in both production and R&D results in a higher level of output than one stemming exclusively from production: $(\zeta^{\eta-1} + z(t)^{\,\eta-1})^{\frac{1}{\eta-1}}\,\tilde{x}(t) > z(t)\,\tilde{x}(t)$. Put differently, optimal engaging in R&D brings about a boost of productivity (as compared to the situation when no R&D is performed) of the size of $(\zeta^{\eta-1} + z(t)^{\,\eta-1})^{\frac{1}{\eta-1}} - z(t) > 0$ for any levels of intrinsic productivity in R&D and production.

A noteworthy property of production technology (4) is that R&D-induced productivity level $Z(t)$ decreases in η, which reflects the fact that as production and R&D become more easily substitutable, the importance of each separate activity diminishes, thereby dragging down the synergy of their joined use.

Plugging (11) and (14) in profit maximisation problem (6) and deriving the FOC pins down firms' prices and volumes

$$\tilde{p}^*(t) = \frac{\xi\psi}{\xi - 1} \cdot \frac{1}{Z(t)} \tag{15}$$

It is worth noting that expression (15) implies a firm's mark-up $\mu(t) = \frac{\tilde{p}^*(t)}{\psi} - 1$ to be countercyclical, which is widely supported by existing macroeconomic literature (see stylised fact 1.b, p. 7).

A firm's sales volume can be derived using (2), (11) and (15)

$$\tilde{y}^*(t) = \frac{L^{\nu\xi} y(t)^{\,1-\nu\xi}}{\tilde{p}^*(t)^{\,\xi}} \tag{16}$$

$$\tilde{y}^*(t) = \frac{Lm(t)^{\,\frac{\xi}{\xi-1}\,\frac{1-\nu\xi}{\nu\xi}}}{\tilde{p}^*(t)^{\,\frac{1}{\nu}}} \tag{17}$$

Free entry to industries entails zero profits for every firm, which enables one to express the number of firms per industry as a function of $z(t)$. To that end, one can calculate a firm's output first

$$\tilde{p}^*(t)\,\tilde{y}^*(t) - \psi\tilde{x}(t) = \Phi$$

$$\tilde{y}^*(t) = \frac{\xi\Phi}{\tilde{p}^*(t)} = \frac{(\xi-1)\,Z(t)\,\Phi}{\psi} \tag{18}$$

Combining (17) and (18) yields the final expression for the equilibrium number of firms per industry $m^*(t)$

$$m^*(t) = \left(\frac{L}{\xi\Phi\,\tilde{p}^*(t)^{\frac{1-\nu}{\nu}}}\right)^{\frac{\xi-1}{\xi}\frac{\nu\xi}{\nu\xi-1}} = \left(\frac{L}{\xi\Phi}\left(\frac{\xi-1}{\xi\psi}Z(t)\right)^{\frac{1-\nu}{\nu}}\right)^{\frac{\nu(\xi-1)}{\nu\xi-1}} \tag{19}$$

Naturally, (19) shows $m^*(t)$ to depend positively on the size of the economy's labour force (which effectively determines the scale of the economy – thereby a larger one has more firms), and to decrease in both cost parameters ψ and Φ. In addition, as expression (19) suggests, firm entry is procyclical: $(m^*(t))'_{z(t)} = (m^*(t))'_{Z(t)} \cdot (Z(t))'_{z(t)} > 0$, in line with existing empirical evidence (see stylised fact 1.c on page 7).

Given (18), one can derive the closed-form expression for $\tilde{\gamma}(t)^*$

$$\tilde{\gamma}^*(t) = \left(\frac{\zeta}{Z(t)}\right)^{\eta-1}\frac{(\xi-1)\,\Phi}{\psi} \tag{20}$$

Finally, given that the cost of maintaining the facilities of the unitary size is ψ units of the final good, a firm's individual R&D spending amounts to

$$\psi\tilde{\gamma}^*(t) = \left(\frac{\zeta}{Z(t)}\right)^{\eta-1}(\xi-1)\,\Phi \tag{21}$$

Note that since $Z(t)$ increases in $z(t)$ and since $\eta > 1$, a firm's R&D spending is countercyclical – in line with the predictions of Schumpeter's hypothesis.

Expression (21), together with (14), suggests a way of understanding how a larger amount of individual R&D can be compatible with a smaller number of R&D performers during a downturn: as the intrinsic productivity in production falls, a firm reacts by shifting part of its facilities to R&D. These actions, while not being able to completely nullify the adverse impact of a slowdown, maximally cushion it, thus putting the firm in the least harmful situation possible during a downturn and thereby reducing the number of firms ceasing to operate.

The behaviour of aggregate R&D spending in an industry $\psi\Gamma^*(i;t) = \psi\Gamma^*(t) \equiv \psi\tilde{\gamma}^*(t)\, m^*(t)$ is described by the expression

$$\psi\Gamma^*(t) = \psi\Gamma Z(t)^{\frac{(1-\nu)(\xi-1)}{\nu\xi-1}-(\eta-1)},$$
$$\Gamma \equiv \frac{\zeta^{\eta-1}\,(\xi-1)\,\Phi L^{\frac{\nu\xi-\nu}{\nu\xi-1}}}{\left(\xi\Phi\left(\frac{\xi-1}{\xi}\psi\right)^{\frac{1}{\nu}-1}\right)^{\frac{\nu\xi-\nu}{\nu\xi-1}}} \tag{22}$$

The pro-/countercyclicality properties of $\Gamma^*(t)$ are determined by whether the power term $\frac{(1-\nu)(\xi-1)}{\nu\xi-1} - (\eta-1)$ is positive (procyclicality) or negative (countercyclicality). Given the motivation of this paper, we are interested in specifying the conditions for the former case

$$\eta - 1 < \frac{(1-\nu)\,(\xi-1)}{\nu\xi-1}$$

$$\eta < \frac{\xi+\nu-2}{\nu\xi-1} \equiv \hat{\eta}_0 \tag{23}$$

Condition (23) constrains the range of η's values from above, so that $\psi\Gamma^*(t)$ is procyclical when $\eta \in (1;\hat{\eta}_0)$. Naturally, if the degree of substitutability between production and R&D is limited, shifts between the two activities during the cycle are less pronounced on the firm level and, hence, are reversed on the industry level by firm entry/exit dynamics (see Figure 1).[16]

To illustrate the last point, suppose to the contrary that $\eta \to +\infty$ (i.e., nearly complete substitution between production and R&D takes place), so that the production technology asymptotically becomes $\lim\limits_{\eta\to+\infty} \tilde{y}(t) = \max\{z(t)\,;\zeta\}\,\tilde{x}(t)$. Suppose that $z(t) > \zeta$ during upturns and vice versa during downturns. In this case, given (20), a firm's R&D spending is going to be $\lim\limits_{\eta\to+\infty} \tilde{\gamma}^*(t) = 0$ and $\lim\limits_{\eta\to+\infty} \tilde{\gamma}^*(t) = \frac{(\xi-1)\Phi}{\psi}$ during upturns and downturns, respectively. Obviously,

[16] Note that condition (23) equally applies to R&D spending in the whole economy, as the latter equals R&D spending within an industry, scaled by N.

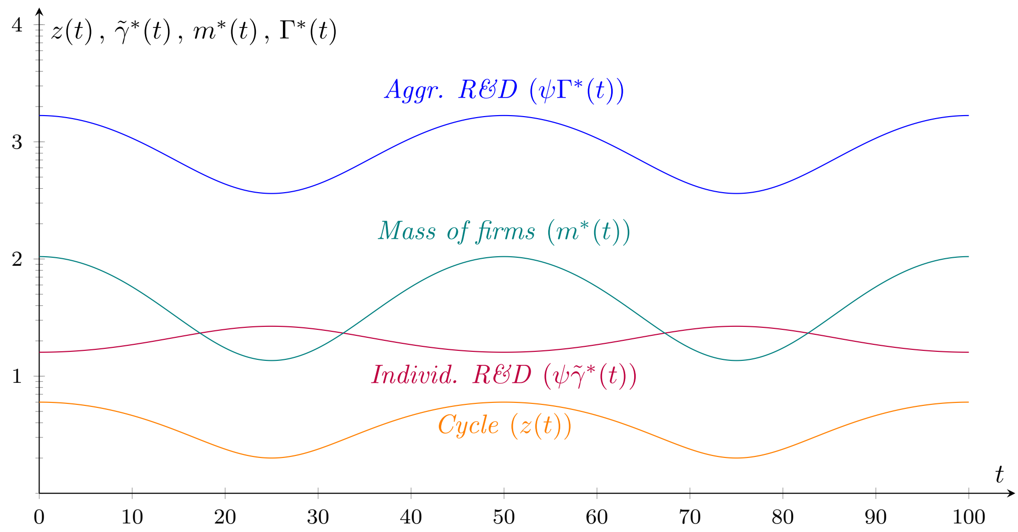

Figure 1: Trajectories of $z(t) = a + b\cos\left(\frac{2\pi t}{\omega}\right)$, $\psi\tilde{\gamma}(t)$, $\psi\Gamma(t)$, $m^*(t)$ on the $\log_{10}$ scale. The values of parameters used are: $a = 4$, $b = 2$, $\omega = 50$, $\eta = 2$, $\xi = 5$, $\nu = \frac{1}{3}$, $\psi = 1$, $\Phi = 10$, $\zeta = 4$, $L = 10$.

for any trajectory of the mass of firms per industry[17] $\{m^*(t)\}_{t=0}^{+\infty}$, an industry's R&D spending is going to be zero during upturns, and positive during downturns, since shifts in $m(t)$ cannot overcome (asymptotically) complete substitution of production facilities for R&D ones on the firm level (see an example in Figure 2).

As the last step in solving the model, one can derive the closed-form expressions for industrial and economy-wide aggregates. The equilibrium output of an industry can be pinned down by combining (18) and (19) with the fact that $y(t) = m(t)^{\frac{\xi}{\xi-1}}\ \tilde{y}(t)$

$$y(t) = \Phi^{-\frac{1}{\nu\xi-1}} \left(\frac{\xi-1}{\xi\psi} Z(t)\right)^{\frac{\xi-1}{\nu\xi-1}} \left(\frac{L}{\xi}\right)^{\frac{\nu\xi}{\nu\xi-1}} \tag{24}$$

Plugging (24) into (1) and (10) yields the closed-form results for $Y(t)$ and $w(t)$

$$Y(t) = \frac{\Phi^{-\frac{1-\nu}{\nu\xi-1}}}{1-\nu} \left(\xi^{-\frac{\xi+\nu-1}{\xi-1}} \left(\frac{\xi-1}{\xi\psi} Z(t)\right)^{1-\nu} L^{\nu}\right)^{\frac{\xi-1}{\nu\xi-1}} \tag{25}$$

[17] One can show that $\lim_{\eta\to+\infty} m^*(t) = \max\{\zeta; z(t)\}^{\frac{1-\nu}{\nu}} \left(\frac{L}{\xi\Phi\left(\frac{\xi}{\xi-1}\psi\right)^{\frac{1-\nu}{\nu}}}\right)^{\frac{\xi-1}{\xi}\frac{\nu\xi}{\nu\xi-1}}$.

[18] The functional form of $z(t)$ and all other parameters' values are as in Figure 1.

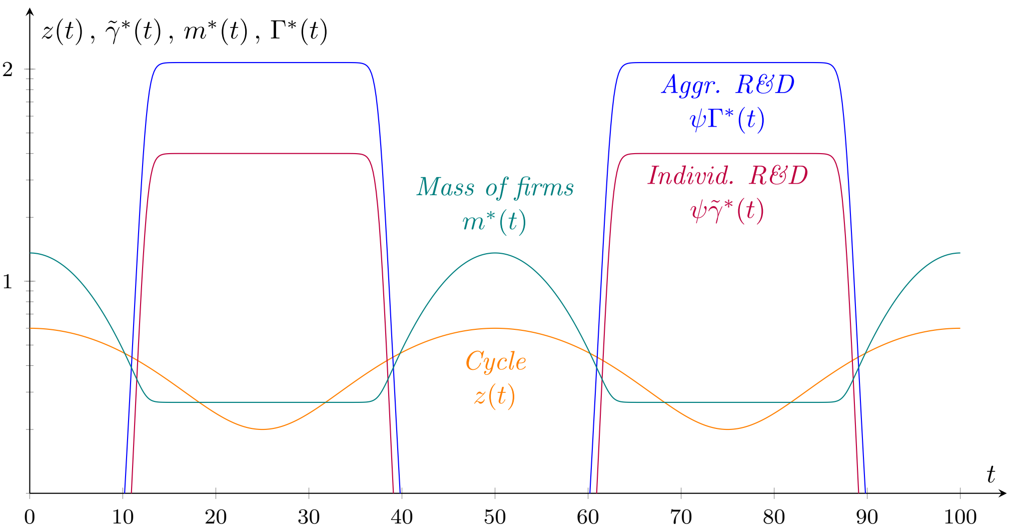

Figure 2: Time trajectories of $z(t)$, $\psi\tilde{\gamma}^*(t)$, $\psi\Gamma^*(t)$, $m^*(t)$ on the $\log_{10}$ scale for high substitutability between production facilities and R&D facilities ($\eta = 40$).[18]

$$w(t) = \frac{\nu}{1-\nu}\left(\frac{L}{\Phi}\right)^{1-\nu}\left(\xi^{-\frac{\xi+\nu-1}{\xi-1}}\left(\frac{\xi-1}{\xi\psi}Z(t)\right)^{1-\nu}\right)^{\frac{\xi-1}{\nu\xi-1}} \tag{26}$$

Equations (24)–(26) formally establish the positivity of the relationships between $z(t)$ on one hand, and $y(t)$, $Y(t)$, $w(t)$ on the other. Finally, given that (because of free entry) firms accrue zero profits, the representative household's income comprises only its labour component $C(t) = w(t)\,L = \nu Y(t)$. This result completes the solution of the model.

2.5 Evaluating the Model

Having solved the model, we conclude its discussion with assessing the empirical plausibility of its key result – aggregated procyclicality condition (23). In particular, our approach splits into two steps: firstly, we retrieve the range of η's values from existing empirical literature, after which we compare it against an estimate of $\hat{\eta}_0$.

The first step can be achieved using the estimates of the econometric model introduced in Aghion et al. (2012) and later employed by Beneito et al. (2015), whereby the natural logarithm of a firm's R&D ($\ln\tilde{\gamma}^*(t) = b_0 - (\eta-1)\ln Z(t)$ in

Table 2: Deduced empirical ranges of η's values.

Study	Country	η's values
Aghion et al. (2012)	France	$[1.032; 1.11]$
Beneito et al. (2015)	Spain	2.055

our model's notations)[19] is regressed, among other variables, on the increment of the natural logarithm of the firm's sales volume ($(\ln \tilde{y}^*(t))'_t = \frac{(\tilde{y}^*(t))'_t}{\tilde{y}^*(t)} = \frac{\dot{Z}(t)}{Z(t)}$ in our model's notations). If $-b_1$ is the coefficient at $(\ln \tilde{y}^*(t))'_t$ in the regression in hand, it determines the marginal effect of $(\ln \tilde{y}^*(t))'_t$ on $\ln \tilde{\gamma}^*(t)$. In our model, this effect can be replicated by differentiating $\ln \tilde{\gamma}^*(t)$, at fixed time t, with respect to an external variation in $(\ln \tilde{y}^*(t))'_t$, denoted as $\vartriangle$

$$(\ln \tilde{\gamma}^*(t))'_\Delta = -(\eta - 1)\left(\int_0^t (\ln \tilde{y}^*(\tau))'_\tau \, d\tau\right)'_\Delta = -(\eta - 1) = -b_1 \qquad (27)$$

Equation (27) allows one to recover the value of η from b_1 as $\eta = 1 + b_1$. Given this result, turning to the empirical studies mentioned above, lands η's value in the intervals listed in Table 2. Overall, the range of η's empirical values is bounded by approximately 2 from above.

Moving on to assessing $\hat{\eta}_0$, following Matsuyama (1999) and Wälde (2005), we interpret the aggregate capital good as a combination of both physical and human capital, which puts the estimate of ν at the approximate level of $1/3$ (see, e.g., Parente and Prescott (2000)). As concerns ξ, its estimates are usually placed in the interval from 3 to 7 (see, e.g., Montgomery and Rossi (1999), Dubé and Puneet (2005), Broda and Weinstein (2006), Broda and Weinstein (2010)). Together, the two estimates suggest that individual countercyclicality of R&D is reversed on the industry level if η belongs to the interval whose lower bound is 1, and whose upper bound $\hat{\eta}$ diminishes from $+\infty$ to 4 as ξ goes from 3 to 7. Regardless of the exact value of ξ, our estimates of η are below $\hat{\eta}_0$, which validates empirical plausibility of condition (23).

[19] In order to guarantee that $\ln \tilde{\gamma}^*(t)$ be well-behaved when R&D is zero, 1 is added to it in the studies cited in the text. We ignore this transformation in our calculations, as in our model $\tilde{\gamma}^*(t)$ is always positive.

3 Extension – Technology Accumulation

3.1 Mechanics of Technology Accumulation

In this section, we extend the baseline model by allowing innovation to have lasting effects on productivity levels. In particular, we assume that a firm's production technology takes the form

$$\tilde{y}(t) = Q(t)\left(\left(z(t)\left(\tilde{x}(t) - \tilde{\gamma}(t)\right)\right)^{\frac{\eta-1}{\eta}} + \left(\zeta\tilde{\gamma}(t)\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{28}$$

where $Q(t)$ is the economy-wide technology level, whose growth is a spillover of individual R&D effort,[20] in the spirit of Romer (1986):

$$g_Q(t) \equiv \frac{\dot{Q}(t)}{Q(t)} = \lambda\left(\frac{\tilde{\gamma}(t)}{Q(t)^{\,1+\chi}}\right) \tag{29}$$

where $\lambda(\cdot)$ is an increasing differentiable function $\lambda'(\cdot) > 0$ of bounded mean oscillation (BMO).[21] Power term $\chi > 0$ connects the dynamics of $Q(t)$ with that of a firm's fixed costs

$$\Phi(t) = \phi Q(t)^{\,\chi} \tag{30}$$

The relationship between $Q(t)$ and $\Phi(t)$, as expressed in (30), is introduced so that we can gain an additional degree of freedom that will be used in the quantitative assessment of the extension in hand, carried out in Section 3.3. We impose the following restriction on the range of χ's values

$$\chi < \frac{1-\nu}{\nu} \tag{31}$$

Condition guarantees that the number of firms $m(t)$, each industry's output $y(t)$ and total output $Y(t)$ increase in time in the long-run.[22]

[20]One can think of $\tilde{\gamma}(t)$ in (29) as the average R&D effort across firms: $\bar{\gamma}(t) = \int_0^N \int_0^{m(i;t)} \frac{\tilde{\gamma}(i;j;t)}{Nm(i;t)} dj di$, which collapses to $\tilde{\gamma}(t)$ given firms' homogeneity.

[21]The fact that $\lambda(\cdot)$ is BMO guarantees the existence of $Q(t)$'s average growth rate (to be derived below, see (40)–(42)).

[22]Formally the results we obtain below (see (34), (36), (37)) suggest that χ's upper bound should be $\min\left\{\xi - 1; \frac{1-\nu}{\nu}\right\}$, but the latter expression collapses to $\frac{1-\nu}{\nu}$ given condition (3).

We divide $\tilde{\gamma}(t)$ by $Q(t)^{1+\chi}$ to reflect the idea that new ideas are harder to obtain as the economy develops and becomes more complex.[23] From the mathematical standpoint, this assumption ascertains that the economy attains a balanced growth path with stationary growth rates. All other equations are as in the baseline model.

As suggested by (28) and (29), engaging in R&D creates two effects: a temporary synergetic one (introduced and discussed in Section 2) and a permanent cost-decreasing one. The latter is channelled through the continuous instantaneous embedding of individual research effort in the aggregate stock of public knowledge – i.e., newly discovered technologies become instantly available for general use, which enhances public knowledge, based upon which further discoveries are made.

3.2 Solution

Going through the same steps as in solving the baseline model, yields the following results

$$\tilde{p}^*(t) = \frac{\xi\psi}{\xi - 1} \cdot \frac{1}{Q(t)\, Z(t)} \tag{32}$$

$$\tilde{y}^*(t) = \frac{\xi - 1}{\psi} \Phi(t)\, Q(t)\, Z(t) \tag{33}$$

$$m^*(t) = \left(\frac{L}{\xi\Phi(t)} \left(\frac{(\xi - 1)\, Q(t)\, Z(t)}{\xi\psi} \right)^{\frac{1-\nu}{\nu}} \right)^{\frac{\nu(\xi-1)}{\nu\xi-1}} \tag{34}$$

$$\tilde{\gamma}^*(t) = \left(\frac{\zeta}{Z(t)} \right)^{\eta-1} \frac{(\xi - 1)\, \Phi(t)\, Q(t)}{\psi} \tag{35}$$

The industry and economy-wide aggregates are

$$y(t) = \Phi(t)^{-\frac{1}{\nu\xi-1}} \left(\frac{\xi - 1}{\xi\psi} Q(t)\, Z(t) \right)^{\frac{\xi-1}{\nu\xi-1}} \left(\frac{L}{\xi} \right)^{\frac{\nu\xi}{\nu\xi-1}} \tag{36}$$

[23] A similar assumption is made in, e.g., Jones (1995), Bental and Peled (1996), Howitt (1999).

$$Y(t) = \frac{\Phi(t)^{-\frac{1-\nu}{\nu\xi-1}}}{1-\nu} \left(\xi^{-\frac{\xi+\nu-1}{\xi-1}} \left(\frac{\xi-1}{\xi\psi} Q(t)\, Z(t) \right)^{1-\nu} L^{\nu} \right)^{\frac{\xi-1}{\nu\xi-1}} \tag{37}$$

The only respect in which the extension's solution (32)–(37) differs from that of the baseline model in Section 2.4, is the temporal variability of firms' fixed costs $\Phi(t)$ and the presence of term $Q(t)$ for the economy's aggregate accumulated technology.

As follows from (32)–(37), by calculating the growth rate of $Q(t)$ one can pin down those of the economy's variables. Combining (29) with (35) suggests that $g_Q(t)$ takes the form

$$g_Q(t) = \lambda \left(\left(\frac{\zeta}{Z(t)} \right)^{\eta-1} \frac{(\xi-1)\,\phi}{\psi} \right) \tag{38}$$

First of all, given that $g_Q(t)$ derives from individual R&D spending $\tilde{\gamma}^*(t)$, it inherits the latter's countercyclical properties. In addition, note that $g_Q(t)$ depends positively on the fixed cost multiplier ϕ, as higher fixed costs lead to a drop in the mass of firms $m^*(t)$ and, in turn, an increase in sales volumes (and, as a result, R&D spending – as follows from stylised fact 2.b) of those remaining in the market. Since $g_Q(t)$ depends on the level of individual R&D effort, it remains unaffected by the dynamics of $m(t)$, and thus reflects only the positive impact of a higher ϕ on individual R&D spending.

Combining (29) with (38) yields the expression for the technology level $Q(t)$

$$Q(t) = Q_0 e^{\int_0^t g_Q(\tau) d\tau} = Q_0 e^{\int_0^t \lambda \left(\left(\frac{\zeta}{Z(\tau)} \right)^{\eta-1} \frac{(\xi-1)\phi}{\psi} \right) d\tau} \tag{39}$$

where Q_0 is the initial technology level. Following Wälde (2005), we treat $Q(t)$ as the product of the trend $\bar{Q}(t)$ and cyclical component $\hat{Q}(t)$[24]

$$\bar{Q}(t) \equiv Q_0 e^{\bar{g}_Q t} \tag{40}$$

$$\hat{Q}(t) \equiv \frac{Q(t)}{\bar{Q}(t)} = e^{\int_0^t \left(\lambda \left(\left(\frac{\zeta}{Z(\tau)} \right)^{\eta-1} \frac{(\xi-1)\phi}{\psi} \right) - \bar{g}_Q \right) d\tau} \tag{41}$$

[24]The existence of $\bar{g}_Q$ follows from $\lambda(\cdot)$'s being a BMO function.

$$\bar{g}_Q \equiv \lim_{t\to+\infty} \frac{1}{t} \int_0^t \frac{\dot{Q}(\tau)}{Q(\tau)} d\tau = \lim_{t\to+\infty} \frac{1}{t} \int_0^t \lambda\left(\left(\frac{\zeta}{Z(\tau)}\right)^{\eta-1} \frac{(\xi-1)\phi}{\psi}\right) d\tau \tag{42}$$

where $\bar{g}_Q$ is the average (or, equivalently, the long-run) growth rate of $Q(t)$. By analogy with $\bar{g}_Q$, the average growth rates of the economy's other level variables can be defined as $\bar{g}_X = \lim_{t\to+\infty} \frac{1}{t}\int_0^t \frac{\dot{X}(\tau)}{X(\tau)} d\tau$, and shown to be as follows

Observation 3.2.1.

$$\bar{g}_{\tilde{y}} = \bar{g}_{\tilde{\gamma}} = (1+\chi)\,\bar{g}_Q \tag{43}$$

$$\bar{g}_m = \frac{\xi-1}{\xi} \frac{\nu\xi}{\nu\xi-1} \left(\frac{1-\nu}{\nu} - \chi\right) \bar{g}_Q \tag{44}$$

$$\bar{g}_y = \frac{\xi}{\xi-1}\bar{g}_m + \bar{g}_{\tilde{y}} = \frac{\xi-\chi-1}{\nu\xi-1}\bar{g}_Q \tag{45}$$

$$\bar{g}_Y = (1-\nu)\,\bar{g}_y = (1-\nu)\,\frac{\xi-\chi-1}{\nu\xi-1}\bar{g}_Q \tag{46}$$

Proof. See Appendix A.1. ■

As regards the cyclical components of the economy's variables, starting with investigating $\hat{Q}(t)$ suggests that it can be potentially unsynchronised with $Z(t)$. To see that (in a heuristic fashion), one can calculate the implicit derivative of $\hat{Q}(t)$ with respect to $Z(t)$

$$\begin{aligned} \frac{d\hat{Q}(t)}{dZ(t)} =& \frac{\left(\lambda\left(\left(\frac{\zeta}{Z(t)}\right)^{\eta-1} \frac{(\xi-1)\phi}{\psi}\right) - \bar{g}_Q\right)\hat{Q}(t)}{\dot{Z}(t)} = 0 \Leftrightarrow \\ \Leftrightarrow& \lambda\left(\left(\frac{\zeta}{Z(t)}\right)^{\eta-1} \frac{(\xi-1)\phi}{\psi}\right) = \bar{g}_Q \end{aligned} \tag{47}$$

As follows from (47), intervals of $\hat{Q}(t)$'s monotonicity in general do not coincide with those of $Z(t)$'s. For the sake of the extension's results' generality and analytical tractability, we rule out this situation by assuming that a downturn in the economy's dynamics starts with a discrete jump in the value of $Z(t)$ such that $Z(t) < \zeta\left(\frac{(\xi-1)\phi}{\psi\lambda^{-1}(\bar{g}_Q)}\right)^{\frac{1}{\eta-1}} \equiv \bar{Z}$, whereas an upturn is initiated by a jump in $Z(t)$ putting its value above $\bar{Z}$, so that the cyclical patterns in $Z(t)$ and $\hat{Q}(t)$ coincide.

As in Wälde (2005), other variables' cyclical components can be defined by replacing $Q(t)$ with $\hat{Q}(t)$ in formulae (32)–(37). In particular, in order to investigate the cyclical behaviour of $\Gamma^*(t) \equiv \tilde{\gamma}^*(t)\, m^*(t)$, one can express its cyclical component as follows

$$\hat{\Gamma}^*(t) = \frac{\zeta^{\eta-1}\,(\xi-1)\,\phi L^{\frac{\nu(\xi-1)}{\nu\xi-1}}}{\left(\xi\phi\left(\frac{\xi-1}{\xi}\psi\right)^{\frac{1}{\nu}-1}\right)^{\frac{\nu(\xi-1)}{\nu\xi-1}}}\hat{Q}(t)^{\frac{(\xi-1)-(1+\chi)(1-\nu)}{\nu\xi-1}} Z(t)^{\frac{(\xi-1)(1-\nu)}{\nu\xi-1}-(\eta-1)} \tag{48}$$

As equation (48) suggests, R&D spending on the industry level is procyclical if

$$\begin{aligned}\frac{\left(\hat{\Gamma}^*(t)\right)'_{z(t)}}{\hat{\Gamma}^*(t)} = \frac{(\xi-1)-(1+\chi)\,(1-\nu)}{\nu\xi-1}\frac{\left(\hat{Q}(t)\right)'_{z(t)}}{\hat{Q}(t)}+ \\ +\left(\frac{(\xi-1)\,(1-\nu)}{\nu\xi-1}-(\eta-1)\right)\frac{(Z(t))'_{z(t)}}{Z(t)} > 0\end{aligned} \tag{49}$$

Although the exact specification of (49) depends on the functional form of $\lambda(\cdot)$, a necessary and a sufficient condition for (49) can be derived even without this piece of information. We shall start with the former. First of all, one may note that term $\frac{\xi-1-(1+\chi)(1-\nu)}{\nu\xi-1}\frac{\left(\hat{Q}(t)\right)'_{z(t)}}{\hat{Q}(t)}$ is negative since $\frac{\left(\hat{Q}(t)\right)'_{z(t)}}{\hat{Q}(t)} < 0$ and $\frac{\xi-1-(1+\chi)(1-\nu)}{\nu\xi-1} > 0$,[25] which implies that $\frac{\left(\hat{\Gamma}^*(t)\right)'_{z(t)}}{\hat{\Gamma}^*(t)} < \left(\frac{(\xi-1)(1-\nu)}{\nu\xi-1}-(\eta-1)\right)\frac{(Z(t))'_{z(t)}}{Z(t)}$, thereby suggesting the necessary condition

$$\left(\frac{(\xi-1)\,(1-\nu)}{\nu\xi-1}-(\eta-1)\right)\frac{(Z(t))'_{z(t)}}{Z(t)} > 0 \Rightarrow \eta < \frac{\xi+\nu-2}{\nu\xi-1} \equiv \hat{\eta}_1^N = \hat{\eta}_0 \tag{50}$$

which coincides with condition (23) obtained for the economy without technology accumulation. This result comes from the fact that condition (50) is derived effectively by omitting term $\frac{\left(\hat{Q}(t)\right)'_{z(t)}}{\hat{Q}(t)}$, through which the impact of technology accumulation is projected, and without which the cyclical behaviour of $\hat{\Gamma}^*(t)$ is affected only by the dynamics of $Z(t)$, as in the baseline model.

[25]The last assertion follows from the assumptions that $\chi < \frac{1-\nu}{\nu}$ and $\nu\xi - 1 > 0 \Leftrightarrow \xi > \frac{1}{\nu}$: $\xi - 1 > \frac{1}{\nu} - 1 = (1-\nu)\left(\frac{1}{\nu} - 1 + 1\right) > (1-\nu)\,(1+\chi) \Rightarrow \xi - 1 - (1-\nu)\,(1+\chi) > 0$.

In order to derive a sufficient condition for (49), we will use the countercyclicality of mark-ups (and, equivalently, the model's prices) to show first that $\frac{\left(\hat{Q}(t)\right)'_{z(t)}}{\hat{Q}(t)} > -\frac{(Z(t))'_{z(t)}}{Z(t)}$. To that end, note that, as the cyclical form of (32) ($\hat{p}^*(t) = \frac{\xi\psi}{\xi-1} \cdot \frac{1}{\hat{Q}(t)Z(t)}$) suggests, since prices are countercyclical, and since their cyclical behaviour is determined by that of $\hat{Q}(t)\,Z(t)$, the latter has to be procyclical. As the product's components fluctuate in the opposite directions – i.e., $\hat{Q}(t)$ is countercyclical, $Z(t)$ is procyclical – for its overall procyclicality to obtain, $Z(t)$'s procyclicality has to dominate $\hat{Q}(t)$'s countercyclicality, viz. $\left(\hat{Q}(t)\,Z(t)\right)'_{z(t)} > 0 \Leftrightarrow \left(\hat{Q}(t)\right)'_{z(t)} Z(t) + (Z(t))'_{z(t)}\,\hat{Q}(t) > 0$, which gives rise to the desired inequality stated above. Combining it with (49) yields the sufficient condition

$$\frac{\left(\hat{\Gamma}^*(t)\right)'_{z(t)}}{\hat{\Gamma}^*(t)} > \left(\frac{(\xi-1)\,(1-\nu)}{\nu\xi-1} - \frac{\xi-1-(1+\chi)\,(1-\nu)}{\nu\xi-1} - (\eta-1)\right)\frac{(Z(t))'_{z(t)}}{Z(t)}$$

$$\left(\frac{(1+\chi)\,(1-\nu)-\nu\,(\xi-1)}{\nu\xi-1} - (\eta-1)\right)\frac{(Z(t))'_{z(t)}}{Z(t)} > 0$$

$$\eta < \frac{\chi\,(1-\nu)}{\nu\xi-1} \equiv \hat{\eta}_1^S \tag{51}$$

Given that condition (51) is sufficient, it is more stringent than the necessary condition (50) – i.e., the upper limit it imposes on η, is lower than that implied by (50)), since $\xi - 1 > (1+\chi)\,(1-\nu)$ (see footnote 25). Such a result comes from the fact that the sufficient condition deals with the case of the highest permissible degree of $\hat{Q}(t)$'s countercyclicality feeding into and reinforcing that of $\tilde{\gamma}(t)$. Since therefore the countercyclicality of $\tilde{\gamma}(t)$ is more pronounced (as compared to the baseline case), it can be offset by the dynamics of $m(t)$, if a smaller share of firms' facilities is shifted between production and R&D, which is controlled by a lower η.

3.3 Evaluating the Extension

Following the logic and structure of Section 2.5, we focus on the range of η's values first. Despite the presence of additional temporal terms $\Phi(t)$ and $Q(t)$ in the expressions for a firm's output (33) and R&D spending (35), one could argue that η's deduced values obtained in Section 2.5 still carry through, as the impact of both of these terms is, in essence, economy-wide and, as a result, would be captured by time-specific fixed effects used by both Aghion et al. (2012) and Beneito et al. (2015) in their estimating procedures.[26] Thus, the relationship between a firm's output fluctuations and R&D, as characterised in the cited studies' results, is driven, in our model's terms, by the interaction between $-(\eta-1)\ln Z(t)$ and $\frac{\dot{Z}(t)}{Z(t)}$, as in Section 2.5.[27]

The remainder of this section focuses on the evaluation of condition (51), for which one first needs to assess the range of χ's values. To that end, we first combine data on the growth rates of U.S. total output ($\bar{g}_Y \approx 0.027$)[28] and those of the number of U.S. firms ($\bar{g}_m \approx 0.011$)[29] during the period from 1977 to 2013, to express that of a firm's production levels $\bar{g}_{\tilde{y}} = \frac{\bar{g}_Y}{1-\nu} - \frac{\xi \bar{g}_m}{\xi-1}$. Parameter χ can be evaluated by assuming a linear relationship between $\bar{g}_{\tilde{y}}$ and $\bar{g}_m$: $\bar{g}_{\tilde{y}} = a\bar{g}_m = \left(\frac{g_{\bar{Y}}/g_{\bar{m}}}{1-\nu} - \frac{\xi}{\xi-1}\right) g_{\bar{m}}$, and then by pinning down χ as a function of $g_{\bar{Y}}/g_{\bar{m}}$, ξ and ν[30]

$$\begin{aligned} \frac{\bar{g}_{\tilde{y}}}{\bar{g}_m} &= \frac{1+\chi}{\frac{\xi-1}{\xi}\frac{\nu\xi}{\nu\xi-1}\left(\frac{1-\nu}{\nu}-\chi\right)} = \frac{g_{\bar{Y}}/g_{\bar{m}}}{1-\nu} - \frac{\xi}{\xi-1} \Leftrightarrow \\ \Leftrightarrow \chi &= \frac{\left(\frac{g_{\bar{Y}}}{g_{\bar{m}}}-1\right)(1-\nu)(\xi-1)}{\frac{g_{\bar{Y}}}{g_{\bar{m}}}\nu(\xi-1)-(1-\nu)} < \frac{1-\nu}{\nu} \;\; \forall \nu,\xi: \;\; \nu\xi > 1 \end{aligned} \tag{52}$$

Combining (51) and (52) casts $\hat{\eta}_1^S$ as a monotonically decreasing function of ξ, which drops from $+\infty$ to 0.685 as ξ goes from 3 to 7. In particular, the estimates of η derived from the results by Aghion et al. (2012) and Beneito et al. (2015) are guaranteed to be accommodated by condition (51) – regardless of $\lambda(\cdot)$'s functional form – for $\xi < 5.57$ and $\xi < 4.5$, respectively.

[26](Aghion et al., 2012, Table 3), (Beneito et al., 2015, Table 2).

[27]We keep our argument in the text more heuristic, with a more formal proof banished to Appendix A.2.

[28]We retrieve the growth rates of $Y(t)$ from data on real output in the U.S. in Feenstra, Inklaar, and Timmer (2015).

[29]Data source: Jarmin and Miranda (2002).

[30]Note that regardless of $\frac{\bar{g}_{\tilde{y}}}{\bar{g}_m}$'s exact value, expression (52) satisfies restriction (31), so long as condition (3) holds.

4 Conclusion

In this paper, we have explored the role of the composition effect, as manifesting itself in fluctuations of the numbers of R&D performers, in reconciling contradictory results in empirical macro- and micro-studies on the cyclicality of R&D spending.

In all three versions of the model introduced in the paper, our results suggest that when the amplitude of shifts between production and R&D, which a firm's resources undergo across an economic cycle, is sufficiently low, the predictions of Schumpeter's hypothesis, while operational on the firm level, are reversed on the industry and the economy-wide level through changes in the numbers of R&D performers, which, by being procyclical, thereby offset countercyclical fluctuations of R&D spending on the individual firm level, and transform them into procyclical macro-oscillations.

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Appendix A Auxiliary Proofs

A.1 Proof of Statement 3.2.1

Before establishing the asserted result, we shall prove the following Lemma

Lemma A.1.1. *From* $\lim\limits_{t\to+\infty} \frac{1}{t} E_0 \left(\int_0^t z(\tau)\, d\tau \right) = \bar{z}$ *follows that* $\lim\limits_{t\to+\infty} \frac{E_0 z(t)}{t} = 0$

Proof. The lemma can be proven by differentiating $\lim\limits_{t\to+\infty} \frac{1}{t} E_0 \left(\int_0^t z(\tau)\, d\tau \right) = \bar{z}$ with respect to t and applying the Leibniz integral rule

$$\frac{d}{dt} \lim_{t\to+\infty} \frac{1}{t} E_0 \left(\int\limits_0^t z(\tau)\, d\tau \right) = \frac{d\bar{z}}{dt}$$

$$\lim_{t\to+\infty} \frac{E_0 z(t)}{t} - \lim_{t\to+\infty} \frac{1}{t} \lim_{t\to+\infty} \left(\frac{1}{t} E_0 \left(\int\limits_0^t z(\tau)\, d\tau \right) \right) = 0$$

$$\lim_{t\to+\infty} \frac{E_0 z(t)}{t} = \lim_{t\to+\infty} \frac{\bar{z}}{t} = 0$$

■

Given formulae (33)–(37), the exact growth rates of the economy's variables take the general form

$$\frac{\dot{X}(t)}{X(t)} = a\frac{\dot{Z}(t)}{Z(t)} + b\frac{\dot{Q}(t)}{Q(t)} \tag{53}$$

Applying the definition of the long-run growth rate to (53) yields the expression

$$\bar{g}_X = a \lim_{t\to+\infty} \frac{1}{t} E_0 \left(\int\limits_0^t \left(\frac{z(\tau)}{Z(\tau)} \right)^{\eta-1} \frac{\dot{z}(\tau)}{z(\tau)} d\tau \right) + b\bar{g}_Q \tag{54}$$

Given that $\eta > 1$ and $z(t) \in [z_L; z_H]$, expression (54) gives rise to the following double inequality

$$\frac{a}{\left(\frac{\zeta}{z_L}\right)^{\eta-1} + 1} \lim_{t\to+\infty} \frac{1}{t} E_0 \left(\int\limits_0^t \frac{\dot{z}(\tau)}{z(\tau)} d\tau \right) \leqslant \bar{g}_X - b\bar{g}_Q \leqslant$$

$$\leqslant \frac{a}{\left(\frac{\zeta}{z_H}\right)^{\eta-1} + 1} \lim_{t\to+\infty} \frac{1}{t} E_0 \left(\int\limits_0^t \frac{\dot{z}(\tau)}{z(\tau)} d\tau \right)$$

$$\frac{a/z_H}{\left(\frac{\zeta}{z_L}\right)^{\eta-1}+1}\lim_{t\to+\infty}\frac{1}{t}E_0\left(\int\limits_0^t \dot{z}(\tau)\,d\tau\right)\leqslant\bar{g}_X-b\bar{g}_Q\leqslant$$
$$\leqslant\frac{a/z_L}{\left(\frac{\zeta}{z_H}\right)^{\eta-1}+1}\lim_{t\to+\infty}\frac{1}{t}E_0\left(\int\limits_0^t \dot{z}(\tau)\,d\tau\right)$$

$$\frac{a/z_H}{\left(\frac{\zeta}{z_L}\right)^{\eta-1}+1}\lim_{t\to+\infty}\frac{E_0 z(t)}{t}\leqslant\bar{g}_X-b\bar{g}_Q\leqslant\frac{a/z_L}{\left(\frac{\zeta}{z_H}\right)^{\eta-1}+1}\lim_{t\to+\infty}\frac{E_0 z(t)}{t}\qquad(55)$$

Combining (55) with Lemma A.1.1 suggests that $0\leqslant\bar{g}_X-b\bar{g}_Q\leqslant 0\Leftrightarrow\bar{g}_X=b\bar{g}_Q$. Applying the last result to formulae (33)–(37) brings about the expressions listed in Observation 3.2.1. ■

A.2 The range of η's empirical values in Extension №1

The natural logarithm of a firm's R&D spending (35) and the time derivative of that of its output (33) and are equal to, respectively, $\ln\tilde{\gamma}^*(t)=b_0^1-(\eta-1)\ln Z(t)+\ln\Phi(t)+\ln Q(t)$ and $\frac{(\tilde{y}^*(t))'_t}{\tilde{y}^*(t)}=\frac{\dot{Z}(t)}{Z(t)}+\frac{\dot{\Phi}(t)}{\Phi(t)}+\frac{\dot{Q}(t)}{Q(t)}$. The former can be transformed as follows

$$\begin{aligned}\ln\tilde{\gamma}(t)=&b_0^1-(\eta-1)\left(\ln Z(t)+\ln\Phi(t)+\ln Q(t)\right)+\eta\left(\ln\Phi(t)+\ln Q(t)\right)=\\=&b_0^1-(\eta-1)\int\limits_0^t\frac{(\tilde{y}(\tau))'_\tau}{\tilde{y}(\tau)}dt+\eta\left(1+\chi\right)\ln Q(t)+\eta\ln\phi\end{aligned}\qquad(56)$$

Note that in (56), the economy's technology level $Q(t)$, being a force affecting the whole economy, is bound to have its impact captured by time-specific fixed effects used in both Aghion et al. (2012) and Beneito et al. (2015). Thereby the impact of $Q(t)$ cannot feed into the estimates of $\frac{(\tilde{y}(t))'_t}{\tilde{y}(t)}$'s effect on $\ln\tilde{\gamma}(t)$, which leaves one with the derived empirical values of η from Section 2.5. ■