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The Relationship between R&D and Competition: Reconciling Theory and Evidence[‡]

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Abstract

The hypothesis of a hump-shaped relationship between innovation and competition due to Aghion, Bloom, Blundell, Griffith, and Howitt (2005), has been tested for different data sets without garnering conclusive support. In this paper we argue that this lack of agreement is because of a difference in approaches to measuring innovation (either in terms of R&D outcomes or by R&D effort). We develop a unified tractable general-equilibrium framework, in which, while R&D outcomes are a hump-shaped function of competition, R&D effort can be observed to be either increasing, decreasing, or hump-shaped. This enables our paper, first, to reconcile the conclusions by Aghion et al. (2005) with more recent results and, second, to inform further attempts to identify the hump-shaped relationship in data.

Key words: Inverted-U (hump-shaped) relationship, research and development, vertical innovation, Cournot-competition.

JEL classification: L13, O31, O41.

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1 Introduction

1.1 Motivation

The nature of the relationship between competition and innovation has been in a focus of economic research since the seminal work of Schumpeter (1943), who argued that competition has a detrimental impact on incentives to innovate, as it reduces innovators' profits.[1] To reconcile Schumpeter's conjecture with contrary empirical evidence,[2] Aghion, Bloom, Blundell, Griffith, and Howitt (2005) developed and tested empirically the theory of a hump-shaped (inverse-U) relationship between the two variables.

The understanding of a relationship between competition and innovation is confounded, however, by the fact that the theory due to Aghion et al. (2005) has not received conclusive support from other empirical studies. Tingvall and Poldahl (2006) show the relationship between competition and innovation to be negative (and thereby the hump-shaped pattern to be absent) in a dataset of Swedish firms when competition is proxied by the Lerner index;[3] Askenazy, Cahn, and Irac (2013), using a panel of French firms, demonstrated that the relationship is inverse.[4]

This paper argues that the inconsistency between the conclusions by Aghion et al. (2005) on one hand, and Tingvall and Poldahl (2006) and Askenazy et al. (2013) on the other, is due to the difference in the papers' approaches to measuring innovation. In particular, while the empirical conclusions in Aghion et al. (2005) are based on proxying innovation by flows of patents (which is R&D out-

[1] This idea is reflected in the classic industrial organisation models of product differentiation due to Dixit and Stiglitz (1977) and Salop (1979), and the first-generation models of endogenous growth (e.g., Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992)).

[2] See, e.g., Nickell (1996), Blundell, Griffith, and van Reenen (1999).

[3] The hump-shaped pattern is present in the results by Tingvall and Poldahl (2006) for competition's being proxied by the Herfindahl index, but this conclusion is sensitive with respect to the econometric techniques employed

[4] Askenazy et al. (2013) show that the relationship becomes hump-shaped for a subsample of large firms.

come (R&D accomplishment)[5]), the other two papers measure innovation by R&D spending (i.e. R&D effort).

We introduce a unifying theoretical framework for the contradictory results by Aghion et al. (2005), Tingvall and Poldahl (2006) and Askenazy et al. (2013), which allows R&D effort and R&D spending to behave differently as functions of competition. In addition to bringing together the aforementioned papers' conclusions, stressing the distinction between the behaviour of R&D effort and R&D has the merit of bringing further methodological awareness (with regards to approaches to proxying innovation) to the efforts to confirm the hypothesis of the hump-shaped relationship between competition and innovation.

1.2 Description of the Framework

In our model, R&D accomplishment does not necessarily have to be an increasing function of R&D effort, and, in particular, can retain its hump-shapedness even when R&D effort becomes observably monotone. This result is achieved by constructing a tractable general equilibrium model in which the final consumable good is produced using a multitude of intermediate inputs produced in imperfectly competitive industries, each of which is populated by a mass of homogeneous Cournot-competing firms engaging simultaneously in R&D in order to incrementally increase their total factor productivity. We show that even if each firm's R&D effort is not a hump-shaped function of competition (in the case of our model, it can be an increasing, or a decreasing, or a hump-shaped function of competition), the relationship between competition and ensuing R&D outcomes can still exhibit the hump-shaped pattern.

The hump-shaped pattern in R&D outcomes in our model obtains as a result of interaction between two forces: the escape-costs effect and the depletion ('fishing-out') effect. The former is prompted by the upward pressure of competition (which we proxy with the number of firms per industry) on firms' effective marginal costs, which induces them to invest more in R&D to counteract this increase, and higher R&D effort leads to better R&D outcomes. On the other

[5] The latter term is used by Griliches (1998).

hand, we assume the aggregate industry level of R&D effort to have a negative impact on individual R&D outcomes:[6] in our model R&D effort is exerted simultaneously by all firms in every industry, and '... when many firms are undertaking R&D, ... they are likely to try similar ideas; thus there will be some amount of external diminishing returns', – put differently, in the situation of simultaneous engagement in R&D firms are 'fishing out of the same pond' (Acemoglu, 2009, p. 473).[7]

In our framework, R&D effort is a hump-shaped function of competition as well, but can potentially be observed as a monotonic function because of the following logic. By contrast with R&D outcomes, R&D effort is affected not by two, but by four forces: two direct (the escape-costs effect and division effect[8]) and two indirect (the indirect escape-costs effect and indirect depletion effect).[9] Thus, since the two aspects of innovation, as functions of competition, are affected by different sets of forces, their turning points do not have to coincide. If, in addition, the measure of competition can take values from a limited interval, and R&D effort's turning point is outside this interval (whereas that of R&D outcomes is within it), then R&D effort and R&D outcomes will be observed as, respectively, a monotonic and a hump-shaped function, thus producing a situation of discrepancy in their behaviour.

[6]Note that the negative impact of aggregate R&D on individual R&D outcomes does not rule out the possibility of knowledge spillovers: while those imply the presence of intertemporal dimension (i.e., benefiting from someone else's *previous* research), our setting describes firms' conducting R&D simultaneously, which results in duplication of their effort.

[7]The logic of this assumption follows the one employed in the premises of the model in (Acemoglu, 2009, Sec. 14.3) and further generalised in Acemoglu and Cao (2015).

[8]The division effect inhibits R&D effort and arises from the fact that, as the number of firms per industry increases, each one of them can attract a smaller share of the economy's resources, thus diminishing each firm's scale of production, which acts as the base over which costs of R&D are spread (the R&D cost-spreading effect, as introduced and termed in Cohen and Klepper (1996)).

[9]The two indirect effects emanate from R&D outcomes, which themselves linearly enter a firm's output function, which in turn is linearly related to R&D effort (i.e., the indirect effects shape R&D effort through R&D outcomes).

1.3 Related Literature

Our paper is related to several branches of literature. First and foremost, similarly to Aghion et al. (2005), d'Aspremont, Dos Santos Ferreira, and Gérard-Varet (2010), Askenazy et al. (2013), Onori (2015), we study the phenomenon of a hump-shaped relationship between competition and innovation in the context of a general equilibrium model.

We contribute to this body of literature by stressing the difference in the behaviour of R&D effort and R&D outcomes with regards to competition intensity, through demonstrating that the latter can be a hump-shaped function of competition even though the former is not – a distinction that is not made in any of the aforementioned papers, which prevents them from accommodating the inconsistency of the results by Aghion et al. (2005) on one hand, and Tingvall and Poldahl (2006) and Askenazy et al. (2013) on the other. In addition, in our model we suggest a novel mechanism producing the hump-shaped pattern, which comprises the superposition of the costs-escape effect and the depletion effect, as well as the division effect – in the case of R&D effort.

Our paper is also related to a few other strands of research. First of all, by highlighting the differences between the behaviour of R&D effort and that of R&D outcomes on the theoretical level, our model is related to the works by Link (1980) and Pohlmeier (1992), where those differences are underscored in the context of empirical IO studies.

Furthermore, our model can be thought of as a general-equilibrium generalisation of the stylised model by Cohen and Klepper (1996), which allows it, similarly to the authors' model, to match a number of stylised regularities of R&D, including:

1. R&D expenditure increases in a firm's output size (Cohen and Klepper, 1996, Stylised fact 2, p. 928);

2. The elasticity of R&D expenditure with respect to the volume of a firm's output is unity (Cohen and Klepper, 1996, Stylised fact 3, p. 929);

3. R&D productivity (defined as the ratio of R&D accomplishment to R&D effort) decreases with a firm's size (Cohen and Klepper, 1996, Stylised fact 4,

p. 930): in our model, the resulting increase in productivity is independent of the firm's output, whereas R&D effort is linear in the latter, so that in the long-run their ratio diminishes to zero.

1.4 Structure of the Paper

The rest of this paper is structured as follows: in Section 2 the model is presented and solved: Sections 2.1 and 2.2 introduce the model's equations; in Section 2.3 we describe the equilibrium conditions in intermediate-good markets, solve for firms' optimal decisions on production and R&D investment, and establish the conditions under which the hump-shaped patterns in R&D effort and R&D accomplishment emerge; Section 2.4 enquires into the behaviour of the aggregate household to pin down the economy's behaviour in the long-run. Section 3 is devoted to investigating the conditions under which the situations of discrepancy in the behaviour of R&D effort and R&D outcomes occur, and checking the empirical compatibility of the model's predictions. In particular, given that our results depend crucially on the value of the elasticity of substitution between inputs, we start with assessing the range of its values compatible with our model's setting (Section 3.1), after which we proceed to deriving the discrepancy conditions in Section 3.2 and discussing their empirical plausibility in Section 3.3. The last section concludes.

2 The Model

From a mathematical standpoint, our paper's key intuition is as follows. Suppose that R&D effort γ is a function $g(m;\chi)$ of competition (as measured by some parameter $m \in [\underline{m};\bar{m}]$) and a set of other parameters χ. The standard logic, which implicitly underlies the shift from R&D outcomes to R&D effort in Tingvall and Poldahl (2006) and Askenazy et al. (2013), suggests that if q stands for R&D outcomes, it is technologically related to γ through an increasing function $R(\gamma;\theta)$ (where θ are other affecting parameters). Thus, the signs of m's effects on q ($\frac{\partial q}{\partial m} = \frac{\partial R}{\partial \gamma} \cdot \frac{\partial g}{\partial m}$) and on γ ($\frac{\partial \gamma}{\partial m} = \frac{\partial g}{\partial m}$) have to coincide, which allows one to treat γ and q equivalently. By contrast, R&D outcomes in our model are related to R&D effort via function $R'(\gamma; m; \theta)$, which increases in γ and *directly* depends on competition. In this case, the signs of competition's effects on the two aspects of innovation are determined by expressions: $\frac{\partial q}{\partial m} = \frac{\partial R'}{\partial \gamma} \cdot \frac{\partial g}{\partial m} + \frac{\partial R'}{\partial m} \gtreqless 0$ and $\frac{\partial \gamma}{\partial m} = \frac{\partial g}{\partial m} \gtreqless 0$, and no longer have to coincide. In particular, when $\frac{\partial \gamma}{\partial m}$ maintains the same sign for any $m \in [\underline{m}; \bar{m}]$, whereas that of $\frac{\partial q}{\partial m}$ changes from positive to negative at some point $\hat{m}_q \in [\underline{m}; \bar{m}]$, a situation of discrepancy in the observed behaviour of γ and q occurs.[10] In this section, we present the above logic through a richer, micro-founded model, whose conclusions lend themselves to quantitative assessment (see Section 3.3).

The model describes an economy where the final good is produced competitively using inputs supplied by industries populated by homogeneous Cournot-competitive firms. Each firm engages in production and R&D (which leads to a decrease in a firm's marginal costs). Both R&D effort (R&D spending in the model) and R&D outcomes (a drop in marginal costs) are shown to be hump-shaped functions of competition in an industry, whose peaks do not generally coincide. This can bring about a situation of discrepancy in the observed behaviour of R&D effort and R&D outcomes, exact conditions for which are formally investigated in Section 3.

2.1 Aggregate Production

Suppose that final output is produced competitively with the CES technology using intermediate inputs provided by the unit mass of identical industries

$$Y(t) = \left(\int_0^1 y(i;t)^{\frac{\xi-1}{\xi}} di \right)^{\frac{\xi}{\xi-1}} = y(t) \tag{1}$$

$y(i;t) = y(t)$ is the i-th industry's output at t, and $\xi > 0$ is the elasticity of substitution between inputs. The second expression is implied by industries' homogeneity. We assume the final good to be the numeraire.

Each industry is populated by $m(i) = m$ homogeneous Cournot-competing

[10] The author wishes to thank David Ulph for this interpretation.

firms,[11] so that an industry's output equals

$$y(i;t) = \int_{0}^{m(i)} \tilde{y}(i;j;t)\,dj = m\tilde{y}(t) \tag{2}$$

Throughout the paper, we use tildes to denote firm-specific variables.

In what follows, we use m as a measure of competition (similarly to Onori (2015)): although the number of firms in an industry is not among standard measures of competition, the model's m maps monotonically into those – in particular, the Herfindahl[12] and Lerner[13] indices.[14] Since we are interested in investigating the relationship between competition and different aspects of R&D, we consider the latter in a strictly oligopolistic environment, so that $m \geqslant \underline{m} \equiv 2$ throughout the paper, similarly to existing theoretical literature (Aghion et al. (2005), d'Aspremont et al. (2010), Askenazy et al. (2013), Onori (2015)).

2.2 Individual Firms

Each firm seeks to maximise its profits by choosing the volume of output and the amount of R&D effort $\tilde{\gamma}(i;j;t) = \tilde{\gamma}(t)$

$$\tilde{\pi}(i;j;t) = \Big(p(i;t) - \tilde{\psi}(i;j;t)\Big)\,\tilde{y}(i;j;t) - \tilde{\gamma}(i;j;t) \tag{3}$$

[11]The main reason we use the Cournot-competition mechanism in our model is because it establishes a negative relationship between the number of firms per industry m and their mark-ups (see (12) and the following discussion), which in turn gives rise to the escape-costs effect introduced below. Alternatively, one could use a more direct approach suggested in Galí (1994) and Galí (1995), and later used in Comin and Gertler (2006), wherein the elasticity of substitution $\xi(m)$ is posited to be an increasing function of the number of firms (in particular, Comin and Gertler (2006) use the following functional form: $\xi(m) = \frac{1+Dm^{\chi}}{Dm^{\chi}}$, so that the mark-up is a constant elasticity function of m: $\mu(m) = Dm^{\chi}$).

[12]The Herfindahl index is calculated as $I_H = \sum_{i=1}^{m} \delta_i^2$, where δ_i is the market share of the i-th firm. Since all firms are homogeneous, the model's Herfindahl index equals $\frac{1}{m}$.

[13]A firm's Lerner index equals $I_L = \frac{p-\psi}{p}$ (the notations are taken from the model). As follows from equation (12), the model's Lerner index for each firm equals $\frac{1}{\xi m}$.

[14]For a detailed discussion, see (Martin, 2002, pp. 335–338) and references cited therein.

where $\tilde{\psi}(i;j;t)$ is the average/marginal cost of producing the i-th intermediate good by the j-th firm.[15]

A firm produces its output using a Cobb-Douglas technology of unitary homogeneity by employing capital ($\tilde{k}(i;j;t)$) and labour ($\tilde{l}(i;j;t)$), both provided by the economy's households in competitive markets

$$\begin{aligned}\tilde{y}(i;j;t) &= \tilde{q}(i;j;t)\,Q(t)\,F\Big(\tilde{k}(i;j;t)\,;\tilde{l}(i;j;t)\Big) = \\ &= \tilde{q}(i;j;t)\,Q(t)\,\tilde{k}(i;j;t)^{\,\nu}\,\tilde{l}(i;j;t)^{\,1-\nu}\end{aligned} \tag{4}$$

where $Q(t)$ is the economy-wide total factor productivity (TFP) level, and $\tilde{q}(i;j;t)$ is its increase for the j-th firm, generated by its individual R&D effort by means of the following technology

$$\begin{aligned}\tilde{q}(i;j_0;t) &= \eta\left(\frac{\tilde{\gamma}(i;j_0;t)}{Q(t)\,F\Big(\tilde{k}(i;j;t)\,;\tilde{l}(i;j;t)\Big)}\right)^{\alpha}\times \\ &\times\left(\int_0^{m(i)}\frac{\tilde{\gamma}(i;j;t)}{Q(t)\,F\Big(\tilde{k}(i;j;t)\,;\tilde{l}(i;j;t)\Big)}dj\right)^{-\beta}, \\ &\alpha>\beta>0\end{aligned} \tag{5}$$

$$\tilde{q}(t) = \eta m^{-\beta}\left(\frac{\tilde{\gamma}(t)}{Q(t)\,F\Big(\frac{K(t)}{Nm};\frac{L(t)}{Nm}\Big)}\right)^{\alpha-\beta} \tag{6}$$

where α and $-\beta$ are, respectively, the elasticity of $\tilde{q}$ with respect to a firm's own R&D effort, and the elasticity of $\tilde{q}$ with respect to aggregate R&D effort in the industry. In what follows, we interpret $\tilde{q}(i;j;t)$ as the R&D outcome being brought about by R&D effort $\tilde{\gamma}(i;j;t)$. We assume that firms do not take into account their impact on aggregate within-industry R&D effort $\int_0^{m(i)}\tilde{\gamma}(i;j;t)\,dj$. Expression (6) is derived from (5) using the homogeneity of firms. Note that (6) represents a particular form of $R'(\gamma;m;\theta)$ from the stylised model discussed in the Introduction: $\tilde{q}(t)=\eta\tilde{\gamma}(t)^{\,\alpha-\beta}m^{\alpha-2\beta}\,(Q(t)\,F(K(t)\,;L(t)))^{-(\alpha-\beta)}$.

In order to bound $\tilde{q}$ from below we assume that zero R&D effort does not affect the current level of productivity, so that the latter's incremental multi-

[15] We slightly abuse notation by equating average costs to marginal costs, but this claim is valid in this instance, since firms in our model use a linearly homogeneous technology.

plier $\tilde{q}(i;j;t)$ equals one: $\tilde{\gamma}(i;j;t) = 0 \Rightarrow \tilde{q}(i;j;t) = 1$, i.e. a firm's productivity does not change. In what follows, we assume that η is suitably high, so that the situation when innovation is absent, never occurs.

Dividing every instance of $\tilde{\gamma}$ by $F\left(\tilde{k}(i;j;t)\,;\tilde{l}(i;j;t)\right)$ in (5) results in $\tilde{q}$'s being independent of the amount of production factors employed by a firm, which we motivate by the absence of a statistically significant relationship between the average amount of production factors employed by a firm, and the TFP growth rate, as observed in US data (see Figures 6a, 6b, 7).

We follow existing literature (see, e.g., (Acemoglu, 2009, Section 14.3), Acemoglu and Cao (2015)) in assuming that ideas are fished out: in terms of our model, β is strictly positive, so that it reflects the inhibiting effect of aggregate research effort on individual R&D productivity within a firm. Thus, β can be interpreted as the contamination parameter, which captures the depletion of the stock of available ideas as those are being searched for simultaneously by a multitude of firms (as put in (Acemoglu, 2009, p. 472), 'fishing from the same pond'). Note that, in spite of the contamination component being present in (5), $\tilde{q}(t)$ is still an increasing function of individual research effort $\tilde{\gamma}(i;j;t)$ for every fixed level of competition m (see (6)), which is achieved by setting $\alpha > \beta$.

Similarly to Aghion and Howitt (1992), Howitt (1999), d'Aspremont et al. (2010), the economy-wide TFP level $Q(t)$ grows as a by-product of the individual research activity. In particular, we model the growth rate of $Q(t)$ as the logarithm of the average of individual $\tilde{q}(i;j;t)$-s across the economy

$$g_Q(t) \equiv \frac{\dot{Q}(t)}{Q(t)} = \ln\left(\int_0^1 \int_0^{m(i)} \frac{\tilde{q}(i;j;t)}{m(i)} dj\, di\right) = \ln\left(\tilde{q}(t)\right) \tag{7}$$

We opt for the logarithmic function in (7) primarily because the equation's corresponding discrete-time (i.e., observable) version takes the natural form $Q_{t+1} = \left(\frac{1}{N}\int_0^N \int_0^{m(i)} \frac{\tilde{q}(i;j;t)}{m(i)} dj\, di\right) Q(t)$,[16] which makes our results comparable to pieces of empirical evidence used later in the paper (see Section 3.3). Equation (7) completes the introduction of the model's production side.

[16]See footnote 30.

2.3 Industry Equilibrium

Before specifying the behaviour of intermediate firms, we will derive the demand function for an intermediate input from profit maximisation in the the final-good sector

$$p(i;t) = p(t) = \frac{\partial Y(t)}{\partial y(i;t)} = Y(t)^{\frac{1}{\xi}} y(t)^{-\frac{1}{\xi}} \tag{8}$$

Since $Y(t)$ is a linear homogeneous function of $y(i;t)$, the Euler theorem can be brought to bear to obtain the expression: $Y(t) = \int_0^1 \frac{\partial Y(t)}{\partial y(i;t)} y(i;t)\, di = p(t)\, y(t)$. Combining the last equation with the assumption of firms' homogeneity pins down the price of an intermediate good (in terms of the final good's price)

$$\begin{aligned} Y(t) = y(t) = p(t)\, y(t) \Leftrightarrow \\ \Leftrightarrow p(t) = p = 1 \end{aligned} \tag{9}$$

Given the constancy of each intermediate input's price, we can solve a firm's problem. We shall start with rewriting the cost function. Since the production function is Cobb-Douglas, a firm's cost function takes the form[17]

$$\tilde{\psi}(i;j;t) = \frac{1}{\tilde{q}(i;j;t)\, Q(t)} \left(\frac{R(t)}{\nu}\right)^{\nu} \left(\frac{w(t)}{1-\nu}\right)^{1-\nu} \equiv \frac{\psi(t)}{\tilde{q}(i;j;t)} \tag{10}$$

where $w(t)$ and $R(t)$ are the factor prices of labour and capital, respectively. Given (10), the profit function can be rewritten as follows

$$\tilde{\pi}(i;j;t) = p(i;t)\, \tilde{y}(i;j;t) - \frac{\psi(t)}{\tilde{q}(i;j;t)} \tilde{y}(i;j;t) - \tilde{\gamma}(i;j;t) \tag{11}$$

Since all firms are homogeneous, maximising (13) with respect to $\tilde{y}$ yields the standard result for the price charged in each industry

$$p^*(t) = \frac{\xi m}{\xi m - 1} \cdot \frac{\psi(t)}{\tilde{q}(t)} \Leftrightarrow \psi(t) = \frac{\xi m - 1}{\xi m} \tilde{q}(t) \tag{12}$$

where the second expression in (12) follows from (9). Given the first expression in (12), $p^*(t)$ can be rewritten as $p^*(t) = (1+\mu)\, \frac{\psi(t)}{\tilde{q}(t)}$, where $\mu = \frac{1}{\xi m - 1}$ is the price mark-up, whose inverse relationship with the mass of firms in an industry m is

[17] See (Mas-Colell, Whinston, and Greene, 1995, p. 142).

an implication of Cournot-competition between firms. Given the functional form of μ and our premise that $m \geqslant 2$, we restrict ξ to be strictly greater than $\frac{1}{2}$.

Since the factor markets are perfectly competitive, the problem of maximising $\tilde{\pi}$ with respect to $\tilde{y}$ is equivalent to maximising it with respect to amounts of capital and labour employed (without taking into consideration the impact of $\tilde{k}(t)$ and $\tilde{l}(t)$ on $\tilde{q}(t)$), which allows one to express the economy's equilibrium factor prices

$$\max_{\tilde{y}} \left\{ p(i;t)\, \tilde{y}(i;j;t) - \frac{\psi(t)}{\tilde{q}(i;j;t)} \tilde{y}(i;j;t) - \tilde{\gamma}(i;j;t) \right\} \Leftrightarrow \tag{13}$$

$$\begin{aligned} \Leftrightarrow \max_{\tilde{k};\tilde{l}} \Big\{ & p(i;t)\, \tilde{q}(i;j;t)\, Q(t)\, \tilde{k}(i;j;t)^{\,\nu} \tilde{l}(i;j;t)^{\,1-\nu} - \\ & - R(t)\, \tilde{k}(i;j;t) - w(t)\, \tilde{l}(i;j;t) - \tilde{\gamma}(i;j;t) \Big\} \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{\partial \tilde{\pi}}{\partial \tilde{k}} = 0 \Leftrightarrow R(t) &= \frac{\xi m - 1}{\xi m}\, p(t)\, \tilde{q}(t)\, Q(t)\, F'_{\tilde{k}}\Big(\tilde{k}(t)\,; \tilde{l}(t)\Big) = \\ &= \nu \frac{\xi m - 1}{\xi m} N^{\frac{1}{\xi - 1}} \tilde{q}(t)\, Q(t) \left(\frac{L(t)}{K(t)} \right)^{1-\nu} = \frac{\xi m - 1}{\xi m} MPK(t) \end{aligned} \tag{15}$$

$$\begin{aligned} \frac{\partial \tilde{\pi}}{\partial \tilde{l}} = 0 \Leftrightarrow w(t) &= \frac{\xi m - 1}{\xi m}\, p(t)\, \tilde{q}(t)\, Q(t)\, F'_{\tilde{l}}\Big(\tilde{k}(t)\,; \tilde{l}(t)\Big) = \\ &= (1-\nu) \frac{\xi m - 1}{\xi m} N^{\frac{1}{\xi - 1}} \tilde{q}(t)\, Q(t) \left(\frac{K(t)}{L(t)} \right)^{\nu} = \frac{\xi m - 1}{\xi m} MPL(t) \end{aligned} \tag{16}$$

The term $\frac{\xi m - 1}{\xi m} = \frac{1}{1+\mu}$ reflects the distortive impact of monopoly power on factor prices in the economy: the greater it is (or, put equivalently, the higher oligopolists' mark-ups are) the more pronounced the deviation of $R(t)$ and $w(t)$ becomes from the marginal products of capital and labour ($MPK(t)$ and $MPL(t)$, respectively), determining factor prices in a competitive economy.

Turning to characterising firms' decisions on R&D investment, differentiating (13) with respect to $\tilde{\gamma}$ leads to the expression

$$\alpha \frac{\tilde{y}(i;j_0;t)}{\tilde{\gamma}(i;j_0;t)} = \frac{\tilde{q}(\tilde{\gamma}(i;j_0;t))}{\psi(t)} \Leftrightarrow \tilde{\gamma}(i;j_0;t) = \alpha \tilde{y}(i;j_0;t)\, \frac{\psi(t)}{\tilde{q}(\tilde{\gamma}(i;j_0;t))} \tag{17}$$

$$\tilde{\gamma}^*(t) = \alpha\, \tilde{y}(t)\, \frac{\xi m - 1}{\xi m} \tag{18}$$

where expression (18) follows from (9). A noteworthy feature of equation (18) is that the elasticity of $\tilde{\gamma}^*(t)$ with respect to $\tilde{y}(t)$ is unity, which matches a widely recognised stylised fact on R&D.[18] This result can be interpreted along the lines of reasoning advanced by Cohen and Klepper (1996): as a firm's output increases, so does, for two reasons, the total effect of R&D. Firstly, with a larger scale of production, the costs of R&D can be spread across a larger level of output; secondly, the drop in production costs resulting from R&D, applies to a larger number of items produced, thus increasing a firm's gains, and thereby encouraging further R&D effort.

Plugging (18) into (6) yields the final expression for $\tilde{q}^*$

$$\tilde{q}^*(t) = \tilde{q}^* = E\left(\frac{\xi m - 1}{\xi m^{\frac{\alpha}{\alpha-\beta}}}\right)^{\frac{\alpha-\beta}{1-(\alpha-\beta)}}, \; E \equiv \left(\alpha^{\alpha-\beta}\eta\right)^{\frac{1}{1-(\alpha-\beta)}} \tag{19}$$

Together, equations (15), (16), (18) and (19) pin down the equilibrium level of profits accruing to a firm

$$\tilde{\pi}^*(t) = \frac{\mu-\alpha}{m}\,\frac{\xi m - 1}{\xi m}\,\tilde{q}^* Q(t)\, K(t)^{\nu} L(t)^{1-\nu} = \frac{\mu-\alpha}{m}\,\frac{\xi m - 1}{\xi m}\, Y(t) \tag{20}$$

Note that the optimal solution for $\tilde{\pi}(t)$ can potentially be negative if term $\mu - \alpha$ is so. This property comes from the fact that R&D effort enters the profit function in the fixed-cost form. In what follows, we require that the term be non-negative, which sets the upper bound on the number of firms per industry

$$\alpha \leqslant \mu = \frac{1}{\xi m - 1} \Leftrightarrow m \leqslant \bar{m} = \frac{1+\alpha}{\alpha\xi} \tag{21}$$

As the final step in deriving a firm's optimal solution, combining (18) and (19) allows one to express $\tilde{\gamma}^*(t)$ as a function of the economy's production factors. By plugging (19) into (18) and using $\tilde{y}(t) = \frac{\tilde{q}(t)\,Q(t)}{m} F(K(t)\,; L(t))$, we have

$$\tilde{\gamma}^*(t) = \alpha E\left(\frac{\xi m - 1}{\xi m^{2-\alpha+2\beta}}\right)^{\frac{1}{1-(\alpha-\beta)}} Q(t)\, F(K(t)\,; L(t)) \tag{22}$$

[18] See (Cohen and Klepper, 1996, p. 929).

One can show that both $\tilde{q}^*$ and $\tilde{\gamma}^*(t)$ (once the dynamics of $Q(t)$, $K(t)$ and $L(t)$ is controlled for)[19] may be related to the degree of competition in the hump-shaped fashion – the following Observations specify the conditions under which this obtains.

Observation 2.3.1. *Let $\hat{m}_q = \frac{\alpha}{\beta\xi}$ and $\alpha < 1+\beta$. Then $\tilde{q}^*$ is increasing $\forall m < \hat{m}_q$ and is decreasing otherwise, so that $\hat{m}_q$ is $\tilde{q}^*(t)$'s global maximum. Therefore, when $\alpha > 2\beta\xi \Leftrightarrow \hat{m}_q > 2$ and $\hat{m}_q < \bar{m}$, the hump-shaped pattern in the relationship between competition and innovation outcomes becomes observable.*

Proof. Follows from calculating $\frac{d\tilde{q}}{dm}$ and applying the method of intervals. ■

The hump-shapedness of $\tilde{q}^*$ is achieved through the superposition of two forces: on one hand, if we assume for a moment that $\beta = 0$, $\tilde{q}^*$ becomes an increasing function of the mass of firms in an industry: as follows from (9), each industry's price is fixed, which, together with a firm's pricing rule (12), has the general-equilibrium implication that firms' effective marginal costs $\frac{\psi(t)}{\tilde{q}^*}$ increase in the degree of competition, which prompts them to invest more in R&D as an attempt to drive effective costs down through increasing their productivity (the escape-costs effect).[20] Finally, greater R&D effort translates into better R&D outcomes. On the other hand, the effect's impact is counteracted by the 'fishing-out' effect discussed above. As suggested by Observation 2.3.1, the escape-costs effect prevails for lower values of m (below $\hat{m}_q$), whereas the opposite is true for $m \geqslant \hat{m}_q$.

Note that, as suggested by the formula for $\hat{m}_q$, the relative strength of the escape-costs effect decreases in the elasticity of substitution ξ: if it is low, then the mark-up wedge μ between costs and prices is further from zero, and hence an increase in m has a greater impact on μ, prompting firms to invest more in

[19]Controlling for industry-specific and time-specific effects is a standard feature of empirical analyses in the field – see Aghion et al. (2005), Tingvall and Poldahl (2006), Askenazy et al. (2013).

[20]Naturally, greater costs create stronger incentives to innovate in our model: if, for the sake of the argument, a firm's costs amount to £100, then doubling its productivity level reduces effective costs by £50. By contrast, if the costs' level is £2, then the impact a two-fold increase in $\tilde{q}$ saves a firm only £1 per unit of output.

R&D. In addition, rather naturally, the markedness of the hump-shaped pattern increases in α, since higher productivity of innovation induces more R&D effort.

Observation 2.3.2. *Let* $\hat{m}_\gamma = \frac{2-\alpha+2\beta}{1-\alpha+2\beta}\frac{1}{\xi}$*. If* $\alpha < 1+2\beta$*, then* $\tilde{\gamma}^*(t)$ *is increasing* $\forall m < \hat{m}_\gamma$ *and is decreasing otherwise, so that* $\hat{m}_\gamma$ *is* $\tilde{\gamma}^*(t)$*'s global maximum. Therefore, when* $2 < \hat{m}_\gamma < \bar{m}$*, the hump-shaped pattern in the relationship between competition and innovation effort becomes observable. Otherwise when* $\alpha > 1+2\beta$*,* $\tilde{\gamma}^*(t)$ *is an increasing function of* m*.*

Proof. Follows from calculating $\frac{d\tilde{\gamma}^*(t)}{dm}$ and applying the method of intervals. ■

In the case of the hump-shaped pattern in R&D effort, the escape-costs effect is present in $\tilde{\gamma}^*(t)$ both directly, as reflected by term $\frac{\psi(t)}{\tilde{q}}$ in (17) (and equivalently, term $\frac{\xi m-1}{\xi m}$ in (18)) and indirectly, as encapsulated in productivity term $\tilde{q}^*$ entering $\tilde{y}(t) = \tilde{q}^* F\left(\frac{K(t)}{m}; \frac{L(t)}{m}\right)$. In addition, the presence of $\tilde{q}^*$ also serves as a channel for the indirect depletion effect, which is further reinforced by the factors division effect: a larger number of producers m entails that each one of them can attract a smaller share of the economy's capital and labour, which in turn reduces a firm's scale of production and, by that means, shrinks its opportunities to spread R&D costs across their output.

Comparing the mechanics of the hump-shaped patterns in $\tilde{\gamma}^*(t)$ and $\tilde{q}^*$ suggests that since in the case of the latter the positive (escape-costs) and negative (depletion) effect are enhanced (through the indirect escape-costs effect and a combination of the indirect depletion effect and the factor division effect, respectively), depending on which of them is reinforced more strongly, we may expect either of the situations $\hat{m}_q \geqslant \hat{m}_\gamma$ and $\hat{m}_\gamma \geqslant \hat{m}_q$ to occur. This gives rise to the possibility of the discrepancy in the behaviour of the two functions when the turning point of one of them is outside the range $[\underline{m}; \bar{m}]$. The situations of particular interest for us are those when the hump-shapedness in R&D outcomes is observable, while that in R&D effort is not. We set a detailed discussion of these conditions aside until Section 3, after we specify the steady-state dynamics of the model's economy, for which we turn to enquiring into the behaviour of the aggregate household.

2.4 Representative Household and Long-Run Equilibrium

Moving on to the consumption side of the economy, we model it as the aggregate household comprising $L(t) = L_0 e^{nt}$ individuals, with a CRRA-type instantaneous utility function, so that its lifetime utility equals

$$U = \int_0^{+\infty} e^{-\rho t} \, \frac{c(t)^{\,1-\theta} - 1}{1-\theta} L(t) \, dt \tag{23}$$

where $c(t)$ is consumption per capita, $C(t) = c(t)\,L(t)$ is total consumption, and $\rho > n$ is the consumers' time-discount factor.

The household's members are assumed to own together all firms and production factors (capital and labour) in the economy, so that their income is composed of firms' profits $\tilde{\pi}(t)\,Nm$ and total factor payments ($w(t)\,L(t)$ – for labour and $(R(t) - \delta)\,K(t)$ – for capital, where δ is the rate of capital depreciation).

The household splits its assets between consumption and investment, taking firms' profits and factor prices as given, which gives rise to the standard intertemporal budget constraint

$$\dot{K}(t) = (R(t) - \delta)\,K(t) + w(t)\,L(t) + \tilde{\pi}(t)\,m - C(t) \tag{24}$$

Equation (24) can be transformed using (15), (16) and (20)

$$\dot{K}(t) = \frac{\xi m\,(1-\alpha) + \alpha}{\xi m} \, \tilde{q}^* Q(t)\,K(t)^{\,\nu} L(t)^{\,1-\nu} - C(t) \tag{25}$$

Maximising (23) with respect to (25) constitutes a canonical Ramsey-Cass-Koopmans dynamic optimisation problem with a Cobb-Douglas production function and CRRA preferences. Therefore, we can argue that the model exhibits saddle-path convergence to the unique steady state, in which the economy's wage rate and per capita variables (namely output, capital and consumption) grow at the rate $\frac{\ln \tilde{q}^*}{1-\nu}$.[21] In addition, the ratio $\frac{K(t)}{Q(t)^{\frac{1}{1-\nu}} L(t)}$ equals fixed number k^{SS}

[21]Naturally, the last conclusion suggests that the economy's total output, capital and consumption grow in the long-run at the rate of $\frac{\tilde{q}^*}{1-\nu} + n$. Therefore, the growth rates of both aggregate and per-capita quantities in the economy inherit the hump-shapedness properties of $\tilde{q}^*$.

determined by the steady-state Euler equation[22]

$$R^{SS} = \nu \tilde{q}^* \frac{\xi m - 1}{\xi m} \left(k^{SS}\right)^{\nu - 1} = \rho + \delta + \frac{\theta \ln \tilde{q}^*}{1 - \nu} \tag{26}$$

$$k^{SS} = \left(\frac{\nu N^{\frac{1}{\xi - 1}} \frac{\xi m - 1}{\xi m} \tilde{q}^*}{\rho + \delta + \frac{\theta \ln \tilde{q}^*}{1 - \nu}} \right)^{\frac{1}{1 - \nu}} \tag{27}$$

Equations (26) and (27) complete specifying the solution of the model by pinning down its long-run dynamics. In the next section, we turn to investigating the situations of discrepancy in the shapes of functional relationships between $\tilde{q}^*$ and $\tilde{\gamma}^*(t)$ on one hand, and m on the other.

3 Quantitative Assessment of the Model

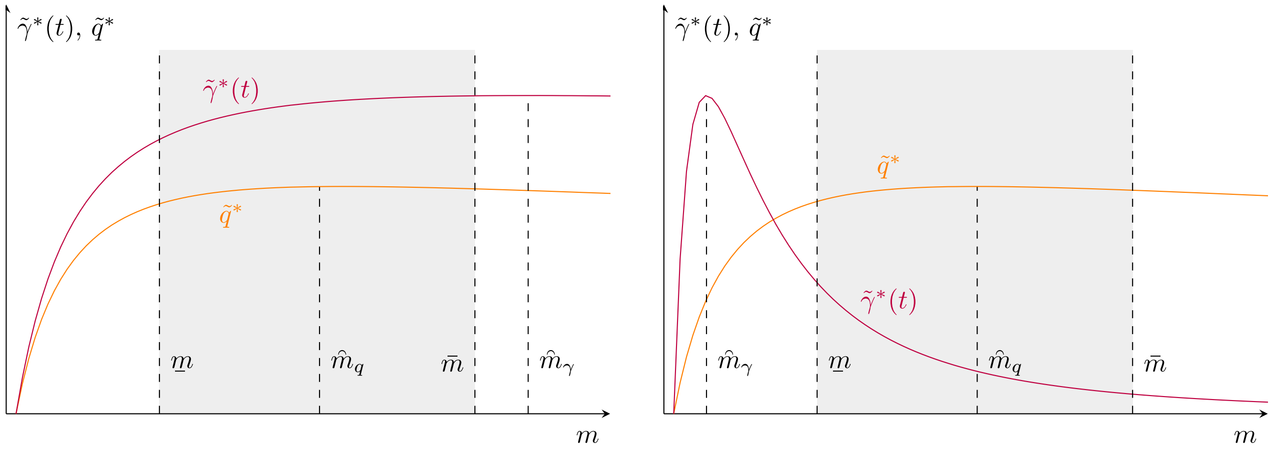
Despite the fact that both $\tilde{\gamma}^*(t)$ and $\tilde{q}^*$ are hump-shaped with respect to the degree of competition m, a situation of discrepancy in the behaviour of these functions can occur if the maximum point of one of them lies outside the interval $[\underline{m}; \bar{m}]$. Given the motivation of this paper, we are interested in looking into the conditions under which $\tilde{q}^*$ is hump-shaped, whereas $\tilde{\gamma}^*(t)$ is not (which, given that $\tilde{\gamma}^*(t)$ is a hump-shaped function, suggests that it has to be observably monotone). This can be the case if the turning point of $\tilde{q}^*$ (i.e. $\hat{m}_q$) lies within the interval of m's permissible values $[\underline{m}; \bar{m}]$, while $\tilde{\gamma}^*(t)$ (i.e. $\hat{m}_\gamma$) is outside it, which is described by either of the two conditions as follows

$$\underline{m} < \hat{m}_q < \bar{m} \leqslant \hat{m}_\gamma \tag{28}$$

$$\hat{m}_\gamma \leqslant \underline{m} < \hat{m}_q < \bar{m} \tag{29}$$

Equations (28) and (29) describe the cases when $\tilde{\gamma}^*(t)$ increases (respectively, decreases) for any $m \in [\underline{m}; \bar{m}]$, while $\tilde{q}^*$ retains its hump-shapedness (see

[22]For a detailed derivation and discussion of the Ramsey-Cass-Koopmans model's properties see, e.g., (Aghion and Howitt, 1998, Section 1.2), (Barro and Sala-i-Martin, 2004, Chapter 2), (Acemoglu, 2009, Chapter 8).



(a) Discrepancy case №1, eq. (28).

(b) Discrepancy case №2, eq. (29).

Figure 1: The cases of discrepancy in the behaviour of $\tilde{\gamma}^*(t)$ and $\tilde{q}^*$.

Figures 1a and 1b for the illustration). For the sake of brevity, we refer to these situations as HS-I (hump-shaped, increasing; corresponds to Aghion et al. (2005) vs. Askenazy et al. (2013)) and HS-D (hump-shaped, decreasing; corresponds to Aghion et al. (2005) vs. Tingvall and Poldahl (2006)) discrepancies, respectively.[23]

Given that $\hat{m}_q$, $\bar{m}$, $\hat{m}_\gamma$ depend on three parameters (namely α, β, ξ), we are going to simplify enquiring into conditions (28), (29) by restricting the range of ξ's values to those implied by existing literature, and thus concentrating on mapping out the relationships between α and β, which underlie (28) and (29). Unfortunately, our task is hampered by the incompatibility of our model (where each industry is populated with a multitude of firms) with existing estimates in macro literature (see, e.g. Smets and Wouters (2003), Christiano, Eichenbaum, and Evans (2005), Primiceri, Schaumburg, and Tambalotti (2006), Broda and Weinstein (2006), Broda and Weinstein (2010)), where the estimates of ξ are

[23]Unfortunately, our model cannot account for the fragility of the hump-shaped pattern, as reported in Tingvall and Poldahl (2006), which comes from the presence of the hump-shaped pattern when competition is measured by the Herfindahl index – we do not think of it as a major drawback though, since, first of all, the estimated hump-shaped pattern is itself fragile with respect to the choice of econometric procedure, as discussed above; secondly, the use of the Herfindahl index is questioned in, e.g., Aghion et al. (2005) as not reflecting the true size of a market for exporting firms. For these reasons, we focus on the negative result in Tingvall and Poldahl (2006), where innovation is negatively related to competition (as measured by PCM).

obtained under the assumption of each industry's being monopolised. This consideration motivates the next section, where we derive, in a stylised fashion, the ranges of ξ's values compatible with our model's setting.

3.1 Preliminary Considerations – the Value of the Elasticity of Substitution

Our approach to assessing the value of ξ is akin to the line of argumentation used in Atkeson and Burstein (2008): we are going to equate the model's mark-up μ to mark-up values inferred from existing data and backup ξ from them, for which, given μ's functional form, we need to gauge the number of firms in an industry m first.

We would like to precede the assessment of m with a discussion of what real-life concepts may match our model's notion of industry. Clearly, it cannot be equated to the industry in the sense of a unit in an industrial classification table: our assumption of Cournot-competition on the intra-industry level implies that each firm takes into account how its decisions affect the whole industry, which clearly implies that, while determining the value of m, we need to take into account the 'compactness' of the corresponding group of firms. In the spirit of the models due to Hotelling (1929) and Salop (1979), we focus on interpreting this 'compactness' in spatial terms, which is why we assess the value of m as the average number of firms[24] per 'industrial-table' industry *per* local economy unit. The last consideration has led us to using US business data, because (primarily for the purposes of labour market research) the US territory has been split into 709 the so-called commuting zones,[25] which are interpreted as local economy units (see, e.g., (Killian and Hady, 1988, pp. 3–5), Tolbert and Sizer (1996), (Walden, 2008, Ch. 5)).

In addition, in order to reflect the homogeneity of each industry's product, in our evaluation of m we use the number of six-digit NAICS industries, which

[24]We take the latest available data on the total number of firms (for year 2013) from the Statistics of US Businesses and Business Dynamics Statistics databases.

[25]Data source: U.S. Commuting Zones and Labour Market Areas: Documentation.

Table 1: The ranges of ξ's values for a selection of countries (mark-up estimates from (Oliveira Martins et al., 1996, Tables 1, 2)).

Country	$m = 7$			$m = 10$		
	$\underline{\xi}$	ξ_{med}	$\bar{\xi}$	$\underline{\xi}$	ξ_{med}	$\bar{\xi}$
France	0.350	0.937	3.714	0.245	0.656	2.6
Sweden	0.475	1.036	2.184	0.333	0.725	1.529
USA	0.407	1.571	4.905	0.285	1.100	3.433

constitute the most detailed level of the US industrial classification and, hence, are expected to be comprised of the most substitutable products.[26] Resorting to data from the Statistics of US Businesses database yields the total of 978 six-digit industries.

Depending on whether the figures on the total number of US firms are taken from the Statistics of US Businesses database or the Business Dynamics Statistics database,[27] the resulting number of firms per model's industry m equals either 7 or 10.

We recover the range of ξ's values by drawing upon the body of literature on mark-up estimation (see, e.g., Oliveira Martins et al. (1996), Klette (1999), De Loecker and Warzynski (2012)). In particular, we use mark-up estimates for samples of industries in France and Sweden from Oliveira Martins et al. (1996), which, when coupled with the definition of μ, generate the estimates for ξ ranging from 0.245 to 3.714 (for France) and from 0.333 to 2.184 (for Sweden, see Table 1).

[26] As an example, mayonnaise and ketchup are likely to be less substitutable than two ketchup brands. The assumption of higher substitutability is in line with existing evidence – see Broda and Weinstein (2006), Broda and Weinstein (2010).

[27] The discrepancy in numbers partly occurs because firms entering the Business Dynamics Statistics database are those active during the pay period which covers the 12^{th} of March, whereas firms in the other database are active at some point in a year.

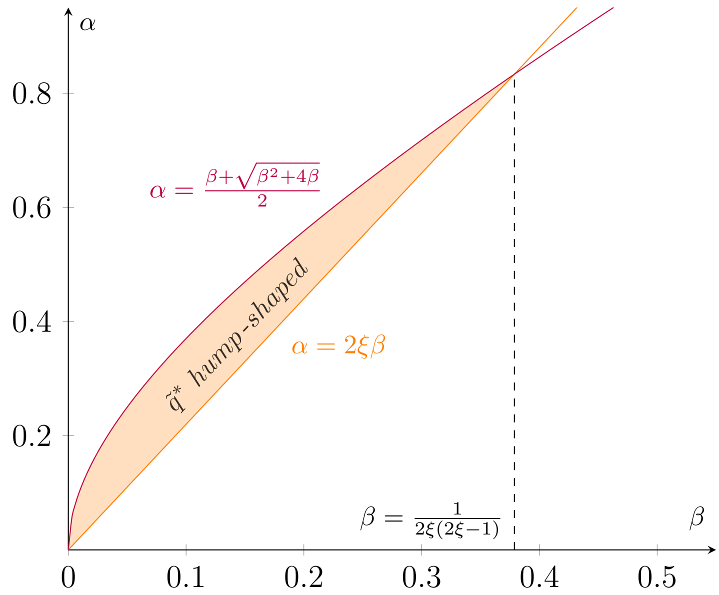


Figure 2: The hump-shapedness conditions for $\tilde{q}^*$ (for $\xi = 1.1$).

3.2 Theoretical Considerations

First of all, in the discussion to follow we assume that there are possibilities for firms to make profits, viz. $\underline{m} < \bar{m}$, or, equivalently

$$2 < \frac{1+\xi}{\alpha\xi} \Leftrightarrow \alpha < \frac{1}{2\xi - 1} \tag{30}$$

In the case of both HS-D and HS-I discrepancy we require the following condition to hold: $\underline{m} < \hat{m}_q < \bar{m}$. Given Observation 2.3.1, this implies the following system of inequalities

$$\begin{cases} \alpha & > & 2\beta\xi \\ \frac{\alpha}{\beta\xi} & < & \frac{1+\alpha}{\alpha\xi} \end{cases} \Leftrightarrow \begin{cases} \alpha & > & 2\beta\xi \\ \alpha^2 & < & \beta\alpha + \beta \end{cases} \tag{31}$$

Note that system (31) implies that for its solution $2 < \frac{\alpha}{\beta\xi} <$ – i.e. condition (30) is satisfied automatically. Solving (31) for α yields the result

$$\alpha \in \left(2\beta\xi; \frac{\beta + \sqrt{\beta^2 + 4\beta}}{2}\right) \tag{32}$$

A graphic representation of condition (32) is shown in Figure 2. Naturally, the range of α's suitable values is nonempty whenever $2\beta\xi < \frac{\beta+\sqrt{\beta^2+4\beta}}{2}$, which is the case when the following condition holds

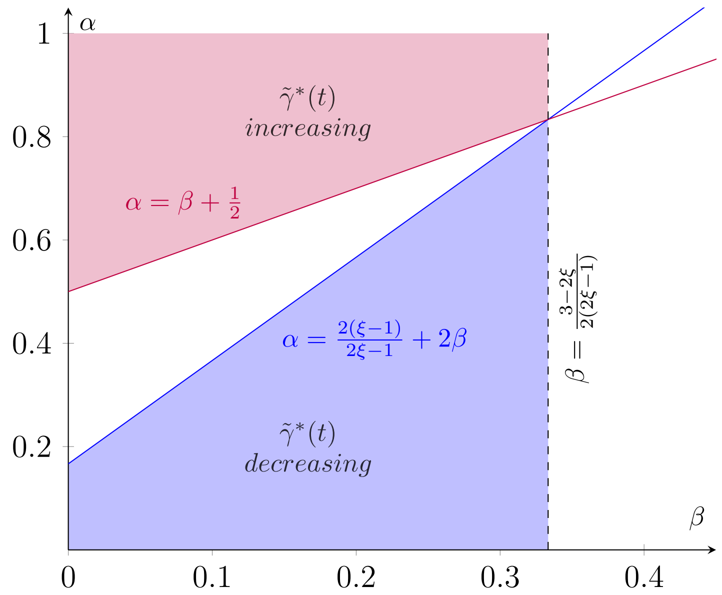


Figure 3: The unobserved hump-shapedness conditions for $\tilde{\gamma}^*(t)$ (for $\xi = 1.1$). The intersection point and values of β above it correspond to the situation in which condition (37) is violated.

$$\beta < \frac{1}{2\xi\,(2\xi - 1)} \tag{33}$$

Expression (33) imposes the upper limit on the value of 'fishing-out' coefficient β, above which the level of α required to generate a detectable hump-shaped pattern, exceeds the mark-up. A noteworthy feature of the conditions obtained is that, since the derivative of the upper limit in (32) increases to infinity for $\beta \to 0$, a suitable pair of α (in the interval specified in (32)) and β (satisfying (33)) can be chosen for any given value of ξ – put differently, for any level of ξ the shaded area in Figure 2 is nonempty.

Turning to the conditions of observed monotonicity of R&D effort, it is decreasing in m if the following condition holds

$$\frac{2 - \alpha + 2\beta}{1 - \alpha + 2\beta}\,\frac{1}{\xi} \leqslant 2 \Leftrightarrow \alpha\,(2\xi - 1) \leqslant 2\,(\xi - 1) + 2\beta\,(2\xi - 1) \tag{34}$$

Depending on whether ξ is greater than $1/2$ or not, the final condition for α takes either of two forms: $\alpha \leqslant \frac{2(\xi-1)}{2\xi-1} + 2\beta$ for $\xi > 1/2$, and $\alpha \geqslant \frac{2(\xi-1)}{2\xi-1} + 2\beta$ for $\xi < 1/2$. We can omit the second inequality however, as the corresponding value of $\hat{m}_\gamma$ is outside $\tilde{\gamma}^*(t)$'s domain. Thus, a decreasing pattern in $\tilde{\gamma}^*(t)$ can occur only if $\xi > 1/2$, and the final expression for the corresponding condition is

$$\alpha \leqslant \frac{2(\xi-1)}{2\xi-1} + 2\beta,\ \xi > 1/2 \tag{35}$$

Naturally, if α is too large (in the sense of (35)), the productiveness of R&D is sufficiently high to induce further R&D effort and reinforce the escape-costs effect to the degree when $\hat{m}_\gamma$ is pushed further right into interval $[\underline{m};\bar{m}]$, which makes $\tilde{\gamma}^*(t)$ observably hump-shaped.

An increasing pattern in R&D effort obtains if

$$\frac{2-\alpha+2\beta}{1-\alpha+2\beta}\,\frac{1}{\xi} \geqslant \frac{1+\alpha}{\alpha}\,\frac{1}{\xi} \Leftrightarrow \alpha - \beta \geqslant \tfrac{1}{2} \tag{36}$$

Note that unlike all previous conditions, equation (36) imposes a constraint on α not only in relative, but also in absolute terms: since $\beta > 0$, α cannot be smaller than $1/2$. The last consideration, in conjuncture with the stipulation that $\underline{m} < \bar{m}$, imposes the upper limit on ξ (in the form of a necessary condition) – even though ξ does not enter (36) directly. Given that $\frac{1+\alpha}{\alpha\xi}$ is a decreasing function of α, replacing it with $\beta + \frac{1}{2}$ yields the inequality $\frac{1+\beta+\frac{1}{2}}{\left(\beta+\frac{1}{2}\right)\xi} = \frac{3+2\beta}{(1+2\beta)\xi} \geqslant \frac{1+\alpha}{\alpha\xi}$, which suggests the following necessary condition for ξ

$$\frac{3+2\beta}{(1+2\beta)\,\xi} > 2 \Leftrightarrow \xi < \frac{3+2\beta}{2\,(1+2\beta)} < \frac{3}{2} \tag{37}$$

As the final step in this section, let us specify the conditions for the presence of HS-I and HS-D discrepancies. As to the former, it occurs in the event of (at least partial) overlap of the orange area in Figure 2 and the blue area in Figure 3, which is the case when $2\xi\beta \leqslant \frac{2(\xi-1)}{2\xi-1} + 2\beta$. Depending on whether $\xi \geqslant 1$ or otherwise, the last expression becomes either $\beta \leqslant \frac{1}{2\xi-1}$ or $\beta \geqslant \frac{1}{2\xi-1}$, respectively. The latter condition can be omitted though, as it implies that $\alpha > 2\xi\beta = \frac{2\xi}{2\xi-1} > \frac{1}{2\xi-1}$ – the latter result is incompatible with the requirements that $\tilde{q}^*$'s hump-shapedness is observable: $\alpha < \frac{1}{\xi\hat{m}_q-1} < \frac{1}{\xi\underline{m}-1} = \frac{1}{2\xi-1}$. Thus we can argue that for HS-D discrepancy to occur, the elasticity of substitution has to exceed one, and β has to be smaller than $\frac{1}{2\xi-1}$. Given our assumption that $\xi > 1/2$, the condition obtained for β is weaker than that required for the observability of $\tilde{q}^*$'s hump-shapedness (33): $\tilde{q}^* - \textit{hump-shaped} \Rightarrow \beta < \frac{1}{2\xi(2\xi-1)} < \frac{1}{2\xi-1}$, which suggests that whenever $\tilde{q}^*$ is hump-shaped and $\xi \geqslant 1$, HS-D discrepancy is observable

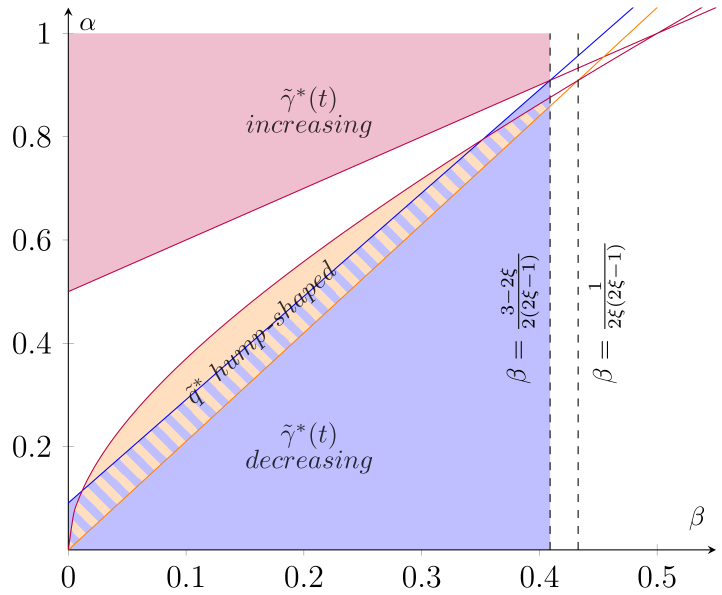


Figure 4: The HS-D discrepancy region (the hatched area) for $\xi = 1.05$.

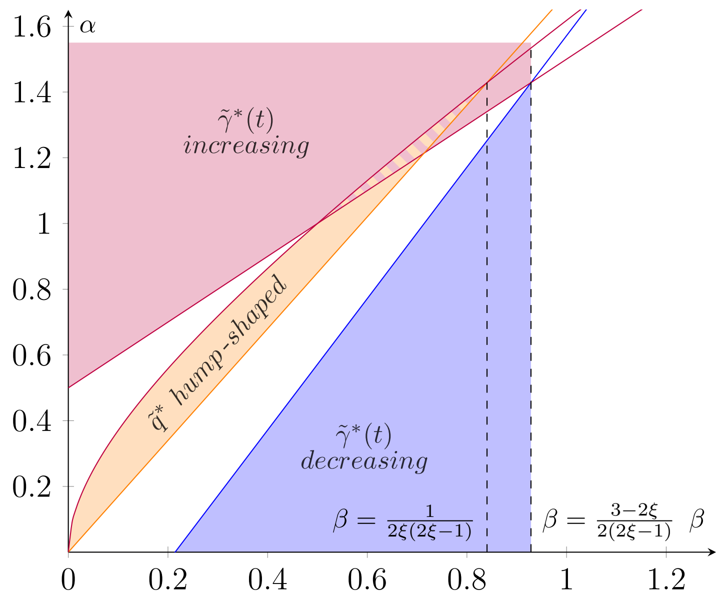


Figure 5: The HS-D discrepancy region (the hatched area) for $\xi = 0.85$.

$$\left.\begin{array}{c} \tilde{q}^* - \textit{hump-shaped} \\ \xi \geqslant 1 \end{array}\right\} \Rightarrow \textit{HS-D discrepancy} \tag{38}$$

In light of the conclusions by Tingvall and Poldahl (2006), we expect condition $\xi \geqslant 1$ to hold for Sweden, and, as suggested by our estimates in Table 1, it is indeed satisfied for the median estimate of ξ when $m = 7$.

As regards HS-I discrepancy, we require, in graphic terms, that the orange area in Figure 2 overlap with the red area in Figure 3, which is the case when

$$\beta + \frac{1}{2} \leqslant \frac{\beta + \sqrt{\beta^2 + 4\beta}}{2} \Leftrightarrow \beta + 1 \leqslant \sqrt{\beta^2 + 4\beta}$$

$$\beta^2 + 2\beta + 1 \leqslant \beta^2 + 4\beta$$

$$\beta \geqslant \frac{1}{2} \tag{39}$$

Given that we consider the situation of HS-I discrepancy, condition (37) of $\tilde{q}^*$'s hump-shapedness has to hold, so that

$$\xi < 1 \tag{40}$$

Note that condition (40) is satisfied for the assessed median values of ξ for France (see Table 1), where, given the results by Askenazy et al. (2013), we expect HS-I discrepancy to occur.

Putting together conditions (39) and (40) alongside the stipulation that $\tilde{q}^*$ is hump-shaped, yields the final result

$$\left.\begin{array}{c} \tilde{q}^* - \textit{hump-shaped} \\ \beta \geqslant \frac{1}{2} \\ \xi < 1 \end{array}\right\} \Rightarrow \textit{HS-I discrepancy} \tag{41}$$

Combining conditions (38) and (41) suggests that, conditional on $\tilde{q}^*$ being observably hump-shaped, $\tilde{\gamma}^*(t)$ is so as well (and, thus, no discrepancy occurs), if $\xi < 1$ and $\beta \leqslant \frac{1}{2}$.

Having specified the analytical conditions for both HS-D and HS-I discrepancy, we would naturally like to check whether the restrictions imposed on α, β and ξ are consistent with available pieces of empirical evidence – we address this question in greater detail in the next section.

3.3 Empirical Plausibility of the Results

In order to check the validity of our theoretical conclusions, we need to relate α and β to a variable, whose range of values can be inferred from existing empirical literature, and check whether the constraints we impose on the parameters, are compatible with that range. Our candidate to that end is the parameter known in the empirical literature as ‘the return on R&D’[28] ζ which is estimated using the following equation[29]

$$\ln\left(\frac{\tilde{y}_{t+1}}{\tilde{y}_t}\right) = b_0 + b_1 \ln\left(\frac{\tilde{k}_{t+1}}{\tilde{k}_t}\right) + b_2 \ln\left(\frac{\tilde{l}_{t+1}}{\tilde{l}_t}\right) + \zeta\frac{\tilde{\gamma}_t}{\tilde{y}_t} + u_t \tag{42}$$

$$E_t\left\{\ln\left(\frac{\tilde{y}_{t+1}}{\tilde{y}_t}\right) - b_1 \ln\left(\frac{\tilde{k}_{t+1}}{\tilde{k}_t}\right) - b_2 \ln\left(\frac{\tilde{l}_{t+1}}{\tilde{l}_t}\right)\right\} = b_0 + \zeta\frac{\tilde{\gamma}_t}{\tilde{y}_t} \tag{43}$$

where u_t is a temporal sequence of independent identically distributed random variables. Given the model's parameterisation and the absence of stochasticity in it, $b_1 = \nu$, $b_2 = 1 - \nu$, which reduces equation (43) to

$$\ln\frac{Q_{t+1}}{Q_t} = b_0 + \zeta\frac{\tilde{\gamma}_t}{\tilde{y}_t} \tag{44}$$

Given that discrete data for Q_t is generated by $Q(t)$ (i.e., $Q_t = Q(t)$ at any discrete point of time t), we have that $Q_{t+1} = \tilde{q}^* Q_t$.[30] Using the definition of $\tilde{y}(t)$, equation (19) can be transformed as follows: $\tilde{q}^{*\,1-(\alpha-\beta)} = \left(\frac{\tilde{\gamma}^*(t)}{\tilde{y}(t)}\right)^{\alpha-\beta} m^{-\beta}$. Plugging these results into (44) establishes the connection between α and β on one hand, and ζ on the other

$$\frac{\alpha-\beta}{1-(\alpha-\beta)}\frac{\tilde{\gamma}^*(t)}{\tilde{y}(t)} - \frac{\beta}{1-(\alpha-\beta)}m = b_0 + \zeta\frac{\tilde{\gamma}^*(t)}{\tilde{y}(t)} \tag{45}$$

As follows from (45), $\frac{\alpha-\beta}{1-(\alpha-\beta)} = \zeta \Leftrightarrow \alpha - \beta = \frac{\zeta}{1+\zeta}$, which allows us to take difference $\alpha - \beta$ to data.

[28] We use the quotation marks here, as ζ can be interpreted as the return on R&D investment if R&D effort is assumed to stack up in the form of research capital, which in our model is not present directly (although one can potentially interpret total factor productivity $Q(t)$ in this vein) – see (Mairesse and Sassenou, 1991, pp. 2–3) for a detailed discussion.

[29] (Mairesse and Sassenou, 1991, p. 3).

[30] To see that, consider two functions $A_t : A_{t+1} = \lambda A_t \Leftrightarrow A_t = \lambda^t A_0$ and $A'(t) : \dot{A}'(t) = \omega A'(t) \Leftrightarrow A'(t) = A'_0 e^{\omega t}$. Discrete data for A_t is generated by $A'(t)$ if $A_0 = A'_0$ and $e^{\omega} = \lambda$. With regards to Q_t, this implies $Q_{t+1} = e^{\ln \tilde{q}^*} Q_t = \tilde{q}^* Q_t$.

In a comprehensive survey of ζ's estimates, Hall, Mairesse, and Mohnen (2010) list a sufficiently wide range of values for France from 16% to 128%;[31] as regards Sweden, we use the estimate of 50.7% from Griffith, Redding, and Van Reenen (2004).[32] These values can support empirically the situations of both HS-D (Aghion et al. (2005) vs. Tingvall and Poldahl (2006)) and HS-I (Aghion et al. (2005) vs. Askenazy et al. (2013)) discrepancy. Starting with the former, condition (38) stipulates that $\tilde{q}^*$ be hump-shaped, and that the elasticity of substitution be no smaller than one. As regards the former, given (45), it is satisfied for the values of α such that

$$\begin{cases} \frac{\alpha}{\left(\alpha-\frac{\zeta}{1+\zeta}\right)\xi} > 2 \\ \alpha < \frac{\alpha-\frac{\zeta}{1+\zeta}+\sqrt{\left(\alpha-\frac{\zeta}{1+\zeta}\right)^2+4\left(\alpha-\frac{\zeta}{1+\zeta}\right)}}{2} \end{cases} \Leftrightarrow \zeta < \alpha < \frac{\zeta}{1+\zeta} \cdot \frac{2\xi}{2\xi-1} \tag{46}$$

The set of α's values is non-empty (or, equivalently, (46) is compatible with condition (30) for the non-negativity of firms' profits) when

$$\zeta < \frac{\zeta}{1+\zeta} \cdot \frac{2\xi}{2\xi-1} \Leftrightarrow \zeta < \frac{1}{2\xi-1} \tag{47}$$

The range of ξ's values prescribed by condition (47) covers the empirical estimate of ζ for Sweden 50.7%, when $0.507 < \frac{1}{2\xi-1} \Leftrightarrow \xi < \bar{\xi} \approx 1.486$. Given that $\xi > 1$ (as stated in (38)), the last consideration suggests the range of ξ's values of $(1; 1.486)$, which fits our estimate of ξ's median value for Sweden (for $m = 7$), thus suggesting that the situation of HS-D discrepancy is compatible with the pieces of empirical evidence presented.

As to the HS-I discrepancy, we still require $\tilde{q}^*$ to be hump-shaped, so that condition (47) holds. In addition, given (41), $\alpha - \beta = \frac{\zeta}{1+\zeta} > \frac{1}{2} \Leftrightarrow \zeta > 1$. Thus, the range of ζ's suitable values is $\left(1; \frac{1}{2\xi-1}\right)$, which, given (41), is non-empty: $\xi < 1 \Rightarrow \frac{1}{2\xi-1} > \frac{1}{2-1} = 1$. Thereby, so long as $\xi < 1$, the range obtained overlaps with the upper tail of the interval of empirical estimates $[16\%; 128\%]$,

[31] See (Hall et al., 2010, Tables 2–5) for the full list of estimates for different countries.

[32] We use the authors' estimate of return on innovation net of technology transfer contribution – see (Griffith et al., 2004, Table 3).

which confirms the plausibility of the conditions for the HS-I discrepancy as well. This inference concludes the section.

Conclusion

In this paper we have explored theoretically the possibility of discrepancy in the behaviour of R&D effort and R&D outcomes (R&D accomplishment) as functions of competition.

We have shown that, since in the context of their relationship with competition, R&D outcomes and R&D effort are affected by non-identical sets of factors (viz. escape-costs effect and the depletion ('fishing-out') effect for the former, and direct and indirect escape-costs effects, indirect depletion effect and the division effect for the latter), the two can exhibit different kinds of detectable behaviour: even though both functions are hump-shaped with respect to the degree of competition m, the turning point of R&D effort can reside outside the permissible range of m's values, which makes it observably increasing or observably decreasing function, thus, coupled with the hump-shapedness of R&D outcomes, producing observed discrepancy in the behaviour of the two aspects in innovation with respect to the degree of competition. Conditions imposed on our model's parameters in order to generate the discrepancy, seem to comply with existing ranges of their empirical counterparts' estimates.

One merit of our approach is that it reconciles the contradictory conclusions drawn in Aghion et al. (2005) on one hand (the presence of the hump-shaped pattern in the relationship between innovation (measured in terms of R&D outcomes) and competition) and in Tingvall and Poldahl (2006), Askenazy et al. (2013) on the other (rejection of the hump-shaped pattern hypothesis in the relationship between innovation, as proxied by R&D effort, and competition).

We hope that our illustration of the possibility that the two aspects of innovation cannot necessarily be equated to each other (in terms of their relationships with competition), will inform further attempts to confirm empirically the hypothesis of the hump-shaped pattern.

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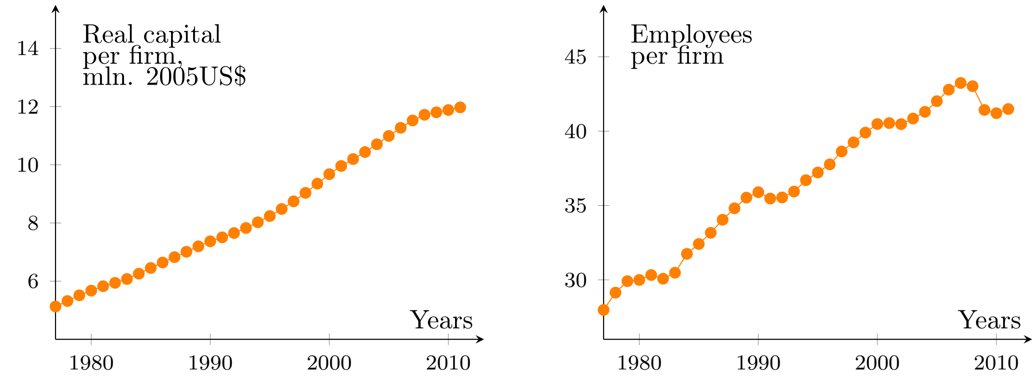
Appendix A Auxiliary Graphs



(a) Average real capital per firm.

(b) Average workers per firm.

Figure 6: Average quantities of production factors employed, US data.
Data source: Feenstra et al. (2015).

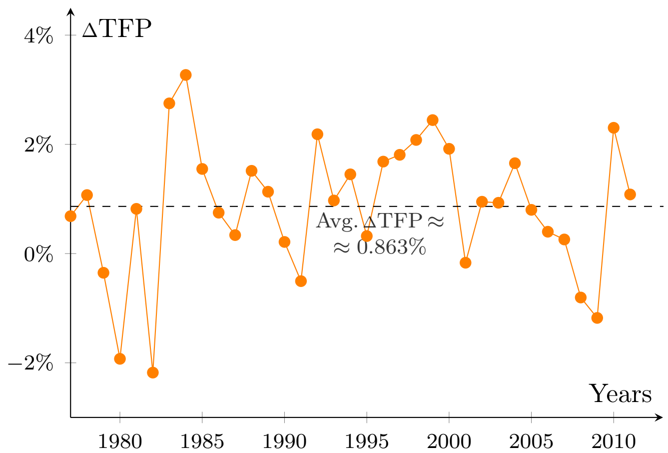


Figure 7: Percentage changes in TFP levels, US data.
Data sources: Feenstra et al. (2015), Jarmin and Miranda (2002)