Electoral Systems, Taxation and Immigration Policies: Which System Builds a Wall first?

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Abstract

When exposed to similar migration flows, countries with different institutional systems may respond with different levels of openness. We study in particular the different responses determined by different electoral systems. We find that Winner Take All countries would tend to be more open than countries with PR when all other policies are kept constant, but, crucially, if we consider the endogenous differences in redistribution levels across systems, then the openness ranking may switch.

Keywords: Proportional representation, Median voter, Taxation, Occupational Choice, Migration, Walls.

JEL Classification codes: D72, F22.

1 Introduction

What role do institutions play for the interpretation of the different responses that different countries seem to have to the threat of increasing migration flows? When there is a perception that migrants could be a threat for employment or income levels, politicians’ electoral incentives may push them to display increasing hostility to open borders, but such electoral concerns could have different intensities and/or implications depending on the electoral system. We analyze this question using a political economy model previously used to study the implications of electoral systems for the level of redistribution, with the additional goal of studying the interplay between immigration and redistribution policies.

We use a model of policy making with endogenous occupational choice, an extension of Austen-Smith (2000). In that paper the population size is fixed, while in this paper we assume that entry of immigrants is a constant flow as long as the institutional system is such that leaving the doors

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open is preferred to building a wall by the majority of members of parliament. The main insight of the paper is that the predictions about immigration policies chosen in countries with different electoral systems may be completely reversed when redistribution levels are made endogenous to the electoral system as well. We show that openness would be more likely in winner take all systems than in PR systems for any given same level of redistribution of income, but, once one takes into account the endogenous redistribution levels, the relative openness result switches.

As an intuition, in the absence of endogenous taxation differences, PR is weakly more closed because the average worker’s preferences are the ones that matter (especially in a capital intensive and productive country), while in a WTA system the decisive agent is the median voter in the distribution of preferences over immigration policies, which happens to be an agent with lower talent, deriving relative greater utility from the increased aggregate income from migration. Endogenous taxation is crucial, and can reverse the prediction: PR induces higher taxes, and this can induce the set of native agents who self-select into an employee occupation to be less than 1/2 of the total native population, which implies that the decisive agent cannot remain the average worker, but rather an unemployed individual, whose primary concern are the redistributive benefits. Given that total benefits increase in aggregate income under reasonable conditions, especially at the beginning of the migration flow, PR countries can be more open than WTA countries.

Let us briefly discuss the relationship of this paper to the empirical literature. In our model, immigration affects the native population both through wages and through welfare transfers. The report produced by the National Academy of Sciences, Engineering, and Medicine (2017) contains an extensive review of theoretical and empirical results of the effect of immigration on employment and wages as well as on its fiscal impact. The empirical evidence about the impact of immigration on natives’s wages is mixed. Some papers found that immigration decreases wages in the receiving country (among others, Altonji and Card (1991), Monras (2015), Borjas (2003, 2016)), others that the effect is negligible (among others Card (2001, 2009)) and some others that the effect is positive (Ottaviano and Peri (2012)). Dustman, Frattini, and Preston (2013) estimate the effect of immigration along the distribution of wages and show that the effect is negative for lower parts of the distribution and positive at the top. Important factors affecting the outcome of the analysis are the degree of substitutability between natives and immigrants, the degree of substitutability among different groups of workers and whether the analysis takes a short-run or long run perspective (i.e. whether capital is allowed to adjust to the inflow of migrants or not). Our model assumes perfect substitutability between natives and immigrants, it considers only one labour market and disregards the adjustment of capital. Given these assumptions, the negative effect of immigration on equilibrium wage that our model displays seems in line with empirical findings.

Immigration impacts welfare transfers to natives in two ways. On the one hand, when working,
immigrants increase tax revenues and therefore transfers. On the other hand, they increase the number of people among which tax revenues must be redistributed. Whether immigrants are net contributors or receivers depends on the share of resources they are entitled to receive. This is a key parameter in our analysis (the parameter $\alpha$) and is a key determinant of natives’ attitudes towards immigrants (see e.g. Facchini and Mayda, 2009, and Preston, 2014).

Given that in our model migration is described as a flow that keeps modifying endogenous variables as it continues, the paper offers a stylized dynamics also of natives’ preferences. The economic consequences of immigration can indeed affect the natives’ preference over immigration (see e.g. Scheve and Slaughter, 2001). Barone et al (2016) show that these effects can also affect voting decisions by the native population: immigration leads natives to vote more for center-right parties.

As far as the political economy literature is concerned, to our knowledge we are the first to compare winner take all and proportional representation electoral systems in terms of endogenous immigration policies. We have chosen to use mainly the modeling insights of Austen-Smith (2000) because endogenous occupational choice seems to be an important part of the dynamic phenomenon we wanted to describe. Morelli (2004) displays other important contrasts between winner take all and proportional systems, in terms of party formation and policy outcomes, and hence some more future results could be obtained also from that framework.

The paper is organized as follows: Section 2 describes the model of political and economic choices, namely occupational choice by citizens and the consequent class and party formation that determines, through the political institutions, the taxation and immigration policies. Section 3 describes the equilibrium results when the tax rate is kept equal across countries with different electoral systems. Section 4 displays the results when redistribution levels differ endogenously across systems. Section 5 shows by simulation the reversal result, and section 6 concludes.

2 Model

We consider two countries that are identical in every aspect, except for the electoral system they use (see description below). Both countries have a mass one of native individuals. Moreover, there is a mass one of potential entrants in each country. At the beginning of the game all potential migrants are out, and, if a country leaves the borders open, they enter at a constant rate. Formally, consider any country and let $Q_t \in [0, 1)$ be the share of immigrants that have already entered in the country at time $t$, with $Q_0 = 0$. The assumption of constant flow if borders are kept open implies that $Q_{t+1} = Q_t + \delta$, $\delta > 0$, until either $Q = 1$ is reached or until the country’s government decides to build a wall to stop the flow, whichever comes first.

Each individual (native or immigrant) is characterised by a type $\theta \in (0, \bar{\theta})$. We denote by $g(\cdot)$ the distribution of types in the population of natives. We assume $g(\cdot)$ symmetric, with mean and
median denoted by $\tilde{\theta}$. The set of immigrants entering each of the two countries in each period is
sampled from a distribution $h(\theta)$. Let both $\theta g(\theta)$ and $\theta h(\theta)$ be non decreasing in $\theta$. We will be
more precise about the characteristics of $h(\theta)$ later in the paper.

Individuals can select one of three possible occupations: becoming an employer ($e$), becoming
an employee ($l$) or being unemployed ($d$). An employer of type $\theta$ can employ $L$ units of labor to
produce an amount $F(L, \theta)$ of consumption good, which is assumed to be the only good consumed
in the economy and whose price is normalized to one. The function $F(\cdot, \cdot)$ is at least twice
differentiable, strictly increasing in both arguments, strictly concave in $L$ and strictly convex in $\theta$.
Furthermore, it is also assumed that $\partial^2 F/\partial \theta \partial L > 0$ for all $\theta > 0$.

Letting $w$ be the wage paid for each unit of labor, the employer’s gross income is

$$ y_e(L, w, \theta) = F(L, \theta) - wL. $$

If an individual chooses to become an employee, she inelastically provides $\theta$ units of labor and
receives a gross income

$$ y_l(w, \theta) = \theta w. $$

Both employers and employees pay a cost of working $c > 0$ and their income is taxed at a rate
$\tau \in [0, 1]$. Taxes are redistributed to the whole population in the form of lump-sum transfers. For
any stock $Q_t$ of immigrants having entered the country at a given date $t$, and for any tax level
$\tau$ and wage $w$, let $\lambda_j(\tau, w, Q_t)$ be the set of types choosing occupation $j \in \{e, l, d\}$. The total
aggregate income in the country is

$$ Y(\tau, w, Q_t) = \int_{\lambda_e(\tau, w, Q_t)} y_e(L, w, \theta) [g(\theta) + Q_t h(\theta)] d\theta + \int_{\lambda_l(\tau, w, Q_t)} y_l(L, w, \theta) [g(\theta) + Q_t h(\theta)] d\theta $$

so that tax revenues are $\tau Y(\tau, w, Q_t)$. We assume that no debt can be accumulated and that
each immigrant obtains a fraction $\alpha \in (0, 1)$ of the tax revenues. The remaining amount is
redistributed equally among natives. Let $b_l(\tau, w, Q_t, \alpha) = \alpha \tau Y(\tau, w, Q_t)$ be the benefits received
by each immigrant and $b(\tau, w, Q_t, \alpha) = (1 - \alpha Q_t) \tau Y(\tau, w, Q_t)$ be those received by each native.
The net income $x_j(\cdot, \theta)$ of a native individual of type $\theta$ in occupation $j \in \{e, l, d\}$ is

$$ x_e(L, \tau, w, Q_t, \alpha, \theta) = (1 - \tau) y_e(L, w, \theta) + b(\tau, w, Q_t, \alpha) - c $$

$$ x_l(\tau, w, Q_t, \alpha, \theta) = (1 - \tau) y_l(w, \theta) + b(\tau, w, Q_t, \alpha) - c $$

$$ x_d(\tau, w, Q_t, \alpha, \theta) = b(\tau, w, Q_t, \alpha) $$

The corresponding net incomes for immigrants are obtained by replacing $b(\tau, w, Q_t, \alpha)$ with $b_l(\tau, w, Q_t, \alpha)$
in the expressions above.

For any wage level $w$ and any type $\theta$, let $L(w, \theta)$ denote the amount of labour that maximizes
an employer’s net income. Given the assumptions on the production function, \( L(w, \theta) \) is strictly decreasing in \( w \) and strictly increasing in \( \theta \). Since from now on we will only consider the optimal amount of labour demanded by employers, we will sometimes simplify notation by using \( L \) instead of \( L(w, \theta) \). Definition 1 extends the concept of sorting equilibrium contained in Austen-Smith (2000) (AS henceforth) to our framework.

**Definition 1.** At any fixed tax rate \( \tau \in [0, 1] \) and immigration level \( Q_t \in [0, 1] \), a sorting equilibrium is a wage rate \( w_t = w(\tau, Q_t) \) such that

\[
\int_{\lambda_e(\tau, w_t, Q_t)}^{\lambda_l(\tau, w_t, Q_t)} L(w_t, \theta)[g(\theta) + Q_t h(\theta)]d\theta = \int_{\lambda_l(\tau, w_t, Q_t)}^{\lambda_l(\tau, w_t, Q_t)} \theta[g(\theta) + Q_t h(\theta)]d\theta
\]

and for all \( \theta \in \Theta \), for all \( j, j' \in \{e, l, d\}, \theta \in \lambda_j(\tau, w_t, Q_t) \) implies \( x_j(\cdot, \theta) \geq x_{j'}(\cdot, \theta) \).

By Proposition 1 in AS, a sorting equilibrium always exists and is characterised by pairs of types \( \theta_1^t = \theta_1(\tau, w_t, Q_t) \) and \( \theta_2^t = \theta_2(\tau, w_t, Q_t) \), with \( \theta_1^t < \theta_2^t \), such that

\[
\lambda_d(\tau, w_t, Q_t) = (0, \theta_1^t) \quad \lambda_l(\tau, w_t, Q_t) = [\theta_1^t, \theta_2^t] \quad \lambda_e(\tau, w_t, Q_t) = (\theta_2^t, \bar{\theta})
\]

Type \( \theta_1^t \) is the type who is indifferent between becoming unemployed and working as an employee. Given the definition of net income for the two types,

\[
\theta_1^t = \frac{c}{(1 - \tau)w_t} \tag{2}
\]

Type \( \theta_2^t \) is the type who is indifferent between becoming an employee or an employer and is implicitly defined by

\[
F(L(w_t, \theta_2^t), \theta_2^t) - w_t L(w_t, \theta_2^t) = w_t \theta_2^t \tag{3}
\]

From Definition 1, then, the wage rate \( w_t \) satisfies

\[
\int_{\theta_2^t}^{\bar{\theta}} L(w_t, \theta)[g(\theta) + Q_t h(\theta)]d\theta = \int_{\theta_1^t}^{\theta_2^t} \theta[g(\theta) + Q_t h(\theta)]d\theta \tag{4}
\]

Define

\[
X(\tau, w_t, Q_t) = \int_{\theta_1^t}^{\theta_2^t} \theta h(\theta)d\theta - \int_{\theta_1^t}^{\bar{\theta}} L(w_t, \theta) h(\theta)d\theta
\]

**Assumption 1.** The distribution of immigrant types \( h(\theta) \) is such that \( X(\tau, \tilde{w}, 1) \geq 0 \), where \( \tilde{w} = w(\tau, 1) \).

Since \( X(\tau, w_t, Q_t) \) represents the net supply for labour by immigrants, Assumption 1 means that immigrants always contribute more to the supply side of the labour market.

In each period, each country can decide to stop the inflow of migrants. We will sometimes refer to this decision as building a wall against immigration. We assume that if in period \( t \) the option of
building the wall can win the majority in parliament, a party that supports it will propose a wall bill. With this assumption the analysis simply needs to focus on the time when the possibility of building a wall becomes a winning option.

In one of the two countries, the composition of parliament is determined by a winner take all system. We assume that the majority of parliament members has preferences over immigration that are identical to those of the median voter in the population (in the next section, we show that the median voter is well defined in this framework). In this country, the wall will be build at a given time $t$ if and only if the median type $\theta_t^{mn}$ is in favour of it.

The other country uses a proportional representation system. We assume that there exist three parties, each representing a different occupation. We denote by $\mathcal{E}$ the party of employees, by $\mathcal{L}$ the party of employers and by $\mathcal{D}$ the one of unemployed. Each party wants to maximise the average utility of the native individuals in the occupation it represents. That is,

$$u_\mathcal{E}(\tau, Q_t) = b(\tau, w_t, Q_t, \alpha) + (1 - \tau)\hat{y}_e(L, w_t, Q_t) - c$$

$$u_\mathcal{L}(\tau, Q_t) = b(\tau, w_t, Q_t, \alpha) + (1 - \tau)\hat{\theta}_l(\tau, Q_t)w_t - c$$

$$u_\mathcal{D}(\tau, Q_t) = b(\tau, w_t, Q_t, \alpha) - c$$

where

$$\hat{\theta}_l(\tau, Q_t) = \frac{\int_{\bar{\theta}_l}^{\theta_2} \theta g(\theta)d\theta}{\int_{\bar{\theta}_l}^{\theta_2} g(\theta)d\theta}$$

and

$$\hat{y}_e(L, w_t, Q_t) = \frac{\int_{\bar{\theta}_e}^{\theta_2} y_e(L, w_t, \theta)g(\theta)d\theta}{\int_{\bar{\theta}_e}^{\theta_2} g(\theta)d\theta}$$

Each party’s share of parliament seats corresponds to the share of native individuals in the occupation it represents. When a party has the majority of parliament seats, it unilaterally decides about the construction of the wall. If no party has the majority in parliament, coalition governments will be formed and the wall will be built when at least two parties agree about it.

In what follows, we will refer to the country using the winner take all system as country $W$ and to the one using PR as country $P$.

3 Results for a fixed tax rate

We begin by assuming that the tax rate $\tau$ does not differ across the two countries. Our first goal is to establish the effect of immigration on wages and occupational choices. Under Assumption 1, immigrants contribute more to the supply side of the labour market. Then,

**Lemma 1.** The equilibrium wage rate $w_t$ is differentiable, strictly decreasing and nonlinear in $Q$. 

For any level of immigration $Q_t$, then, $w_t < w_{t+1}$. When wages decrease, being an employee becomes less attractive. Indeed, employees’ gross income is strictly increasing in $w$, while the envelope theorem implies 
\[
\frac{\partial y_e(L, w, \theta)}{\partial w} = -L(w, \theta) < 0.
\]
For both occupations, the magnitude of the effect increases with $\theta$. Since benefits are equally distributed across the population, then, the entrance of migrants modifies optimal labour decisions. More formally, from (2) and (3), one gets $\partial \theta_1^1 / \partial w < 0$ and $\partial \theta_2^2 / \partial w > 0^2$. Then, $\theta_1^1 < \theta_{t+1}^1 < \theta_{t+1}^2 < \theta_t^2$.

In order to avoid trivial cases, in what follows we will maintain the following assumption:

**Assumption 2.** $\theta^2(\tau, \tilde{w}, 1) > 1/2$, where $\tilde{w} = w(\tau, 1)$.

In words, we simply assume that even in the extreme situation of full openness, where all potential migrants enter, the set of endogenous employers can never be an absolute majority of the population. Given that $\theta^2(\tau, w, Q_t)$ decreases as migrants keep entering, this assumption is a sufficient condition to guarantee that the set of employers is never an absolute majority throughout the whole entry process.

### 3.1 Immigration under winner take all

Immigration affects the native population through the wage rate and through benefits. Since the negative effect on employees’ wage rate is increasing in type, whenever an employee is in favour of building the wall, all employees with higher type will be in favour too. On the contrary, if an employee prefers to accept more migrants, all lower-type employees will agree. Furthermore, employees can be in favour of accepting more migrants only if these have a strong positive effect on benefits. Given that unemployed individuals are not affected by a change in wages and employers strictly benefit from it, these types must also be in favour of more migration. Thus, if employees constitute a large share of the population, the median must be a relatively low type of employee. If employees are not the majority in the population, instead, the median type in the distribution of preferences on migration will be an unemployed individual. The intuition is similar to the one just described. An unemployed individual prefers to build a wall if and only if the entrance of new migrants reduces the benefits she receives. This immediately implies that employees will be in favour of the construction of the wall too. If instead benefits increase with the entrance of new migrants, then unemployed individuals and employers will both support open borders.

This intuition is formalised in Lemma 2. Let $\theta^d$ denote a type $\theta \in (0, \theta_t^1)$; denote by $\theta_t^d = \theta^d(Q_t)$ the type $\theta$ such that
\[
\int_0^{\theta_t^d} g(\theta) d\theta = \frac{1}{2} \tag{5}
\]
\(^2\)The first result immediately follows by differentiating (2) with respect to $w$. For the second, we refer to equation (A5) in the proof of Proposition 1 in AS (p. 1258).
if such a type is greater than $\theta^{t+1}_1$ (otherwise $\theta^{t}_l$ does not exist). In words, type $\theta^{t}_l$ is such that the mass of types $\theta \in [\theta^{t}_1, \theta^{t+1}_2]$ that are employees at time $t$ and would remain employees at time $t+1$ is exactly one half of the population. Then,

**Lemma 2.** The median type in the distribution of preferences on immigration $\theta^m_t = \theta^m(Q_t)$ is defined by

$$
\theta^m_t = \begin{cases} 
\theta^t_l & \text{if } \int_{\theta^{t+1}_1}^{\theta^{t+1}_2} g(\theta)\,d\theta \geq \frac{1}{2} \\
\theta^d & \text{otherwise}
\end{cases}
$$

The main implication of Lemma 2 is that, whenever (5) is well defined, the median type in the distribution of preferences over immigration is decreasing in $Q$. Since $\theta^{t+1}_2 < \theta^{t}_1$ for all $t$, the lower bound in (5) must move to the left to guarantee that the equality is satisfied. Then, as long as at any given time $t$

$$
\int_{\theta^{t+1}_1}^{\theta^{t+1}_2} g(\theta)\,d\theta \geq \frac{1}{2}
$$

and $x_t(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta^t_l) \geq x_t(\tau, w_t, Q_t, \alpha, \theta^t_l)$, more migrants will enter the country. As the entry process continues, $\theta^t_l - \theta^t_1$ decreases in $t$. Denote by $t'$ the first time in which $\theta^t_l > \theta^t_1$, i.e., the first time in which the median voter becomes an unemployed. If the entry process has reached this stage, without a wall proposal by any previous median voter, then from $t'$ on, more migrants will be admitted until, at some period $t''$, $b(\tau, w_{t''}, Q_{t''}, \alpha) > b(\tau, w_{t''+1}, Q_{t''+1}, \alpha)$ or $Q_{t''} = 1$. A sufficient condition for the wall to be built before all the immigrants enter the country is

$$
b(\tau, \tilde{w}_s, 1 - \delta, \alpha) > b(\tau, \tilde{w}, 1, \alpha)
$$

where $\tilde{w}_s = w(\tau, 1 - \delta)$.

**Proposition 1.** Assume (7) is satisfied. If $t$ is such that (6) holds, country $W$ will build the wall if and only if

$$
x_t(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta^t_l) \leq x_t(\tau, w_t, Q_t, \alpha, \theta^t_l)
$$

When (6) does not hold, the wall will be built if and only if

$$
x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta^d) \leq x_d(\tau, w_t, Q_t, \alpha, \theta^d)
$$

### 3.2 Immigration under PR

Suppose that, at a given time $t$, employees constitute the majority in the population, i.e.

$$
\int_{\theta^{t+1}_1}^{\theta^{t+1}_2} g(\theta)\,d\theta \geq \frac{1}{2}
$$

When (6) does not hold, the wall will be built if and only if

$$
x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta^d) \leq x_d(\tau, w_t, Q_t, \alpha, \theta^d)
$$
Then, party $\mathcal{L}$ must have the majority of seats in parliament and the wall will be built if and only if $u_{\mathcal{L}}(\tau, Q_t) \geq u_{\mathcal{L}}(\tau, Q_{t+1})$, or equivalently if and only if

$$x_l(\tau, w_t, Q_t, \alpha, \hat{\theta}_l(\tau, Q_t)) \geq x_l(\tau, w_{t+1}, Q_{t+1}, \alpha, \hat{\theta}_l(\tau, Q_{t+1}))$$

By the same reasoning used for winner take all systems, if borders are kept open for sufficiently long, a time $t'$ can be reached when

$$\int_{\theta_1^l}^{\theta_2^l} g(\theta) d\theta < \frac{1}{2}$$

Our next lemma focuses on the construction of the wall in this scenario. Suppose that party $\mathcal{D}$ prefers to build the wall, since

$$b(\tau, w_t, Q_t, \alpha) - b(\tau, w_{t+1}, Q_{t+1}, \alpha) \geq 0$$

If employees’ average net income is decreasing in immigration, party $\mathcal{L}$ will support the construction of the wall too. Given the negative effect on benefits, a sufficient condition for this to hold is that employees’ average gross income $\hat{\theta}_l(\tau, Q_t)w_t$ decreases with $Q$. By reducing the wage rate, immigration affects employees’ average gross income both directly and indirectly. The direct effect is a decrease in aggregate employees’ income, which clearly has a negative impact on the average. The indirect effect arises because immigration shrinks the set of types choosing occupation $l$: high-type employees will become employers, while lower-type employees will switch to unemployment. The first component has a negative effect on the average, the second one affects it positively. If the distribution $g(\theta)$ is uniform, the direct effect on aggregate income always dominates the positive indirect effect, so that employees’ average gross income is always decreasing in $Q$. For other types of distribution, a sufficient condition is

$$\int_{\theta_1^l}^{\theta_2^l} \theta g(\theta) d\theta > \left. \frac{\partial \theta_1^l}{\partial w_t} g(\theta_1^l) \right| w_t - \hat{\theta}_l(\tau, Q_t)w_t$$

(11)

The left-hand-side of (11) measures the decrease in aggregate income due to a decrease in wages. The right-hand-side measures the positive effect on the average arising from a switch in occupation by the types around $\theta_1^l$. When (11) holds, employees’ average net income is decreasing in immigration. Notice that this is a restrictive condition, as it completely disregards the behavior of types around $\theta_1^l$.

**Lemma 3.** Consider a country whose parliament is elected with proportional representation. If, at some period $t$, (10) is not satisfied and (11) holds,

$$u_D(\tau, Q_t) \geq u_D(\tau, Q_{t+1}) \Rightarrow u_L(\tau, Q_t) \geq u_L(\tau, Q_{t+1})$$
Lemma 3 and the discussion above directly imply the following proposition.

**Proposition 2.** Assume (7) and (11) hold. If \( t \) satisfies (10), country \( P \) will build the wall if and only if
\[
x_l(\tau, w_{t+1}, Q_{t+1}, \alpha, \hat{\theta}_l(\tau, Q_{t+1})) < x_l(\tau, w_t, Q_t, \alpha, \hat{\theta}_l(\tau, Q_t))
\]
(12)
When (10) is not satisfied, (9) is a sufficient condition for the wall to be built.

### 3.3 Comparison between the two systems

In this section, we combine Propositions 1 and 2 to compare the degree of openness of the two countries. Suppose that condition (6) holds at some time \( t \). Then, (10) must hold too. By Proposition 1, country \( W \) will build the wall if and only if \( \theta_l \) is in favour of it, i.e. if (8) holds. By Proposition 2, country \( P \) will close its borders if and only if (12) holds. Whenever
\[
\hat{\theta}_l(\tau, Q_{t+1})w_{t+1} - \hat{\theta}_l(\tau, Q_t)w_t \leq \theta_l(w_{t+1} - w_t)
\]
(13)
(12) is implied by (8). Then, if a wall bill is passed in country \( W \), the same bill will be passed in country \( P \) too. Since the opposite implication is not true, there can be levels of immigration at which country \( P \) wants to build the wall, while country \( W \) prefers to admit immigrants for at least another period.

Now suppose that (6) does not hold at some time \( t \), so that country \( W \) will pass a wall bill if and only if (9) holds. Again, country \( P \) will want to build the wall any time country \( W \) does. When (10) holds, (12) is implied by (9). When (10) does not hold, the result directly follows from Proposition 2. Let \( t_W \) denote the smallest \( t \) at which country \( W \) decides to build the wall and denote by \( t_P \) the equivalent date country \( P \). Then

**Proposition 3.** Assume (7) and (11) are satisfied. Then, if (13) holds, \( t_W \geq t_P \).

### 4 Results with different endogenous tax levels

Let us now fix the level of immigration \( Q_t \) and examine the optimal choice of tax \( \tau \) under the two electoral systems.\(^3\) For winner take all, we assume that the implemented tax level is the one preferred by the majority of the population of natives. For PR, we consider a legislative bargaining process à la Baron and Diermeier (cite). Denote by \( \tau_0 \) the given status quo level of taxation. After

\(^3\)The conclusions in this section are practically identical to those discussed by AS. The results in AS rely on single-peakedness of individuals’ preferences over taxation. However, as noted by Morelli and Negri (2017), the argument proving single-peakedness contains a mistake and the property cannot be established. Morelli and Negri (2017) provide an alternative proof of the results, based on the property of single-crossing preferences (Gans and Smart (1996)). This section complements the results in AS and Morelli and Negri (2017) with additional results for PR.
elections, if a party $P \in \{E, L, D\}$ obtains the majority of the votes, it will implement the tax level maximizing $u_P(\tau, Q_t)$. If no party obtains the majority, one party will be selected randomly to propose a tax rate. We assume each party is selected as a proposer with a probability equal to the share of native individuals in the occupation it represents. The proposed tax rate is then put to a vote against the status quo $\tau_0$. If at least another party agrees with the proposal, the new tax rate is implemented. The tax rate remains $\tau_0$ otherwise. Denote by $p(\tau|\tau_0)$ the probability that tax rate $\tau$ is chosen by the legislative bargaining process when $\tau_0$ is the status quo. Our focus is on stable tax rates.\(^4\)

**Definition 2.** A tax rate $\tau$ is stable if $p(\tau|\tau) = 1$.

A stable tax rate is a status quo tax rate that is never changed by the legislative bargaining process and could be interpreted as a long-run tax rate. Focusing on stable tax rates allows to not make the comparison between winner take all and PR dependent on the status quo tax rate.

Before introducing the results for the two systems, notice that

**Lemma 4** (Lemma 1 in AS). For a fixed level of immigration $Q_t$, the equilibrium wage rate $w_t$ is differentiable, strictly increasing and nonlinear in $\tau$.

We refer to AS for the proof of the lemma and simply note that

$$
\frac{\partial w_t}{\partial \tau} = \frac{w_t(\theta^1_t)^2[g(\theta^1_t) + Q_t h(\theta^1_t)]}{(1 - \tau)A(\tau, w_t, Q_t)}
$$

with $A(\tau, w_t, Q_t)$ as defined in (21). Let $\epsilon(\tau)$ and $\tilde{\epsilon}(\tau)$ denote the tax elasticity of the equilibrium wage rate and the tax elasticity of the marginal equilibrium wage rate,

$$
\epsilon(\tau) = \frac{\partial w_t}{\partial \tau} \frac{\tau}{w_t}
$$

$$
\tilde{\epsilon}(\tau) = \frac{\partial^2 w_t}{\partial \tau^2} \frac{\tau}{\partial w_t}
$$

In what follows, we assume

**Assumption 3.**

$$(1 - \tau)\tilde{\epsilon}(\tau) \leq (1 - \tau)\epsilon(\tau) + \tau$$

Assumption 3 is identical to condition (5) in AS\(^5\) and allows us to use some of the results contained in the paper.

---

\(^4\)Our definition of stable tax rate is a simplified version of the PRPE-stable equilibrium in AS (p.1251).

\(^5\)Condition (5) in AS also includes a lower bound for $(1 - \tau)\tilde{\epsilon}(\tau)$. As shown in Morelli and Negri (2017), this lower bound is not necessary.
4.1 Taxation under winner take all

For any given immigration level \(Q_t\), let \(\xi(\tau, Q_t, \theta)\) denote a type \(\theta\)'s maximum consumption level at a given tax rate \(\tau\) and sorting equilibrium \(w_t = w(\tau, Q_t)\). That is

\[
\xi(\tau, Q_t, \theta) = \max_{j \in \{e, d\}} x_j(\cdot, \theta)
\]

Suppose the median type in the distribution \(g(\theta)\), \(\tilde{\theta}\), is an employee, i.e. \(\theta_1 \leq \tilde{\theta} \leq \theta_2\).

**Lemma 5.** For any two tax levels \(\tau, \tau'\) such that \(\tau < \tau'\),

1. \(\xi(\tau, Q_t, \tilde{\theta}) \geq \xi(\tau', Q_t, \tilde{\theta}) \Rightarrow \xi(\tau, Q_t, \theta) \geq \xi(\tau', Q_t, \theta)\) for all \(\theta > \tilde{\theta}\)
2. \(\xi(\tau', Q_t, \tilde{\theta}) \geq \xi(\tau, Q_t, \tilde{\theta}) \Rightarrow \xi(\tau', Q_t, \theta) \geq \xi(\tau, Q_t, \theta)\) for all \(\theta < \tilde{\theta}\)

Lemma 5 proves that individuals' preferences over taxation satisfy a weak version of the single-crossing condition (Gans Smart (1996)). The result was proven by Morelli and Negri (2017) and the proof we provide in the appendix is just an adaptation of the proof to our framework. A direct implication of the lemma is that \(\tilde{\theta}\) is the median type in the distribution of preferences over taxation. Let \(\tau^W\) be the tax level implemented by country \(W\). Then

\[
\tau^W = \arg \max_{\tau} (1 - \tau) \tilde{\theta} w_t + b(\tau, w_t, Q_t, \alpha) - c
\]

4.2 Taxation under PR

In order to identify the stable tax rate, we first need to understand parties' behavior in the legislative bargaining process. In the following lemma, we show that parties' preferences over taxation also satisfy a weak version of the single-crossing condition. More precisely, the lemma shows that party \(L\) is the median party. The first part of the lemma is a direct consequence of Lemma 2 and Lemma 5 in AS. Lemma 2 states that, under Assumption 3, benefits (and therefore \(u_D(\tau, Q_t)\)) are strictly concave in \(\tau\), with interior arg max. Define

\[
V(\tau) = 1 - \frac{(1 - \tau) \partial w_t}{w_t} \partial \tau
\]  

(14)

AS shows that \(V(\tau) > 0\).\(^6\) Lemma 5 in AS states that, when

\[
\frac{\partial^2 \tilde{\theta}_l(\tau, Q_t)}{\partial \tau^2} \geq \frac{\partial^2 \tilde{\theta}_l(\tau, Q_t)}{\partial \tau^2} \left[ 1 + \frac{1 - \tau}{V(\tau)} \right]
\]  

(15)

\(^6\)This is shown in the proof of Lemma 2 in AS. Using the formula for \(\partial w_t / \partial \tau\), one gets

\[
V(\tau) = 1 - \frac{(\tilde{\theta}_1^t)^2[g(\tilde{\theta}_1^t) + Q_t h(\tilde{\theta}_1^t)]}{A(\tau, w_t, Q_t)}
\]

and since \(A(\tau, w_t, Q_t) > (\tilde{\theta}_1^t)^2[g(\tilde{\theta}_1^t) + Q_t h(\tilde{\theta}_1^t)]\), \(V(\tau) > 0\).
party $\mathcal{L}$'s utility is strictly quasiconcave in $\tau$. Furthermore, denoting by $\tau_P$ the maximizer of $u_P(\tau, Q_t)$, the lemma proves that $\tau_L < \tau_D$. This immediately implies that for all $\tau' > \tau$, if party $\mathcal{L}$ prefers $\tau'$ to $\tau$, then party $\mathcal{D}$ also prefers $\tau'$, which is the first statement in our Lemma 6. The second statement in our lemma states that, when party $\mathcal{L}$ prefers a lower tax rate, party $\mathcal{E}$ must prefer lower taxes too. A sufficient condition for this to hold is that average employers’ income is decreasing in $\tau$. Higher tax rates imply higher wages. On the one hand, this decreases the income of every employer, therefore decreasing the average. On the other hand, it induces low-type employers to become employees, therefore increasing the average. The net effect is negative when

$$\int_{\theta^L_1}^{\theta_2} L(w_t, \theta) g(\theta) d\theta > \frac{\partial \theta^2}{\partial w} g(\theta^L_1) [y_e(L, w_t, Q_t) - y_e(L, w_t, \theta^L_1)]$$

(16)

The left-hand-side of (16) measures the total increase in the cost of labor due to an increase in wages. The right-hand-side corresponds to the increase in the average employers’ income due to the endogenous occupational decisions.

**Lemma 6.** If (15) and (16) hold,

1. $u_L(\tau, Q_t) \leq u_L(\tau', Q_t) \Rightarrow u_D(\tau, Q_t) \leq u_D(\tau', Q_t)$

2. $u_L(\tau, Q_t) \geq u_L(\tau', Q_t) \Rightarrow u_E(\tau, Q_t) \geq u_E(\tau', Q_t)$

for all $\tau < \tau'$.

Lemma 6 directly implies the following proposition.

**Proposition 4.** If no party has the absolute majority of seats in parliament and (15) and (16) hold, the unique stable tax rate is

$$\tau_L = \arg \max_{\tau} u_P(\tau, Q_t)$$

Let $\tau_P$ be the tax rate in country $P$. Then, $\tau_P = \tau_L$.

### 4.3 Immigration decisions under different endogenous tax rates

From now on, we assume that conditions (15) and (16) in Proposition 4 are satisfied. One of the most important results in AS, which holds in our model too, is the following conclusion about the tax rates in the two countries:

**Proposition 5** (Proposition 6 in AS). There exists a cost of working $\bar{c}$ such that, for all $c \leq \bar{c}$, $\tau_P > \tau_W$.

We refer to AS for the proof.

Proposition 5 becomes very important for our purposes when combined with the following lemma
Lemma 7. The share of employees in the native population is decreasing in $\tau$,
\[
\frac{\partial}{\partial \tau} \left[ \int_{\theta_1}^{\theta_2} g(\theta) d\theta \right] < 0
\]

Assume by contradiction that the share of employees in the native population was increasing in $\tau$. Then the labor supply would increase with $\tau$ too. Since $\partial w / \partial \tau > 0$, the increase in labor supply must be associated with an even larger increase in labor demand. However, $\partial \theta_l^2 / \partial w > 0$ implies that labor demand must decrease. Then, the share of employees in the population must be decreasing in $\tau$.

Consider time $t = 0$ before the inflow of migrants begins and let $\tau_0^W$ and $\tau_0^P$ denote the tax levels implemented by the two countries at this date. By Proposition 5 and Lemma 7, the share of natives choosing occupation $l$ in country $P$ is strictly lower than the one in country $W$. In particular, it is possible to have
\[
\int_{\theta_1(\tau_0^W)}^{\theta_2(\tau_0^W)} g(\theta) d\theta < \frac{1}{2} \leq \int_{\theta_1(\tau_0^W)}^{\theta_2(\tau_0^W)} g(\theta) d\theta \tag{17}
\]
where $\theta_l(W) \equiv \theta_l(\tau, w, 0)$ and $\theta_l^2(\tau) \equiv \theta^2(\tau, w, 0)$. When (17) holds, Propositions 1 and 2 (under (7) and (11)) imply that country $W$ and country $P$ will close their borders when (8) and (9) hold, respectively. For a given tax level $\tau$ and level of immigration $Q_t$, define the maximum share of resources $\alpha$ that types $\theta^l$ and $\theta^d$ are willing to transfer to immigrants to keep the borders open as
\[
\alpha_l^W(\tau, Q_t) \equiv \frac{\tau[Y(\tau, w_{t+1}, Q_{t+1}) - Y(\tau, w_t, Q_t)] - (1 - \tau)\theta_l^1(w_t - w_{t+1})}{\tau[Q_{t+1}Y(\tau, w_{t+1}, Q_{t+1}) - Q_tY(\tau, w_t, Q_t)]} \tag{18}
\]
and
\[
\alpha_d(\tau, Q_t) \equiv \frac{Y(\tau, w_{t+1}, Q_{t+1}) - Y(\tau, w_t, Q_t)}{Q_{t+1}Y(\tau, w_{t+1}, Q_{t+1}) - Q_tY(\tau, w_t, Q_t)} \tag{19}
\]
Notice that, since wage is decreasing in $Q$, $\alpha_l^W(\tau, Q_t) < \alpha_d(\tau, 0)$ for all $\tau$. These definitions imply that
\[
x_l(\tau_l^W, w_1, \delta, \alpha, \theta_0^l) \geq x_l(\tau_l^W, w_0, 0, \alpha, \theta_0^l)
\]
for all $\alpha \leq \alpha_l^W(\tau_l^W, 0)$ and
\[
x_d(\tau_d^P, w_1, \delta, \alpha, \theta_d) \geq x_d(\tau_d^P, w_0, 0, \alpha, \theta_d)
\]
for all $\alpha \leq \alpha_d(\tau_d^P, 0)$. Let
\[
\eta(\tau, Q_t) \equiv \frac{dY(\tau, w_t, Q_t)}{d\tau} \frac{\tau}{Y(\tau, w_t, Q_t)}
\]
\[ \eta(\tau, Q_t) \equiv \frac{d^2Y(\tau, w_t, Q_t)}{dQ_t d\tau} \frac{\tau}{dY(\tau, w_t, Q_t)/dQ} \]

be the tax elasticity of aggregate income and the tax elasticity of the marginal effect of immigration on aggregate income, respectively. Then,

**Lemma 8.** If

\[ \dot{\eta}(\tau, Q_t) > \eta(\tau, Q_t) \]  

(20)

then

\[ \frac{\partial \alpha_d(\tau, Q_t)}{\partial \tau} > 0 \]

Given that \( \eta(\tau, Q_t) < 0 \), condition (20) is equivalent to requiring that higher taxes do not reduce too much the positive effect that immigration has on aggregate income. Lemma 8 implies that, at time \( t = 0 \), \( \alpha_d(\tau^W_0, 0) < \alpha_d(\tau^P_0, 0) \). Then,

**Proposition 6.** Assume (7), (11) and (17). Then, \( \dot{\eta}(\tau^W_0, 0) > \eta(\tau^W_0, 0) \) is a sufficient condition to have

\[ x_d(\tau^P_0, w_1, \delta, \alpha, \theta^d) \leq x_d(\tau^P_0, w_0, 0, \alpha, \theta^d) \Rightarrow x_d(\tau^W_0, w_1, \delta, \alpha, \theta^l_0) \leq x_d(\tau^W_0, w_0, 0, \alpha, \theta^l_0) \]

for all \( \alpha \leq \tilde{\alpha}(\tau^P_0, 0) \).

In words, Proposition 6 states that, at the beginning of the immigration process, country \( P \) can be relatively more open than country \( W \). Country \( P \) will keep its borders open for all \( \alpha \leq \alpha_d(\tau^P_0, 0) \). When \( \alpha \leq \alpha_d(\tau^W_0, 0) \), country \( W \) might or might not build the wall, depending on how strongly immigration affects wages. When \( \alpha_d(\tau^W_0, 0) < \alpha \leq \alpha_d(\tau^P_0, 0) \), the country will for sure close its borders.

Clearly, Proposition 6 does not imply that countries using a PR system are always more open than those using winner take all. However, it has an important implication on the analysis of migration policies. The main conclusion of Section 3 (Proposition 3) was that, \textit{ceteris paribus}, winner take all systems are relatively more open than PR. The results in this section show that the \textit{ceteris paribus} assumption is not innocuous. When the tax levels are determined endogenously, countries using PR systems can be strictly more open to immigration than countries using winner take all systems. In the next section, we simulate the model and provide a graphical representation of how different redistribution levels affect the openness level of the two countries.

5 Simulations

The best way to graphically compare the results in Sections 3 and 4 is to focus on the maximum share of resources \( \alpha \) that a country is willing to transfer to immigrants to be willing to keep its
borders open. For any tax level $\tau$ and immigration level $Q_t$, let such values be denoted by $\alpha^W(\tau, Q_t)$ and $\alpha^P(\tau, Q_t)$, for country $W$ and country $P$, respectively. The values of $\alpha^W(\tau, Q_t)$ and $\alpha^P(\tau, Q_t)$ depend on the share of employees in the country. By Proposition 1,

$$\alpha^W(\tau, Q_t) = \begin{cases} 
\alpha^W_l(\tau, Q_t) & \text{if (6) holds} \\
\alpha_d(\tau, Q_t) & \text{otherwise}
\end{cases}$$

where $\alpha^W_l(\tau, Q_t)$ and $\alpha_d(\tau, Q_t)$ are as defined in (18) and (19), respectively. By Proposition 2,

$$\alpha^P(\tau, Q_t) = \begin{cases} 
\alpha^P_l(\tau, Q_t) & \text{if (10) holds} \\
\alpha_d(\tau, Q_t) & \text{otherwise}
\end{cases}$$

with

$$\alpha^P_l(\tau, Q_t) \equiv \frac{\tau[Y(\tau, w_{t+1}, Q_{t+1}) - Y(\tau, w_t, Q_t)] - (1 - \tau)\left[\hat{\theta}_l(\tau, Q_t)w_t - \hat{\theta}_l(\tau, Q_{t+1})w_{t+1}\right]}{\tau[Q_{t+1}Y(\tau, w_{t+1}, Q_{t+1}) - Q_tY(\tau, w_t, Q_t)]}$$

Our analysis in Section 3 predicts that, when the tax level is the same in the two countries, country $W$ will never build the wall before country $P$. An equivalent way to phrase Proposition 3 is

$$\alpha^W(\tau, Q_t) \geq \alpha^P(\tau, Q_t)$$

for all $Q_t$. If the actual value of $\alpha$ is greater than $\alpha^W(\tau, Q_t)$, both countries will build the wall; if it is smaller than $\alpha^P(\tau, Q_t)$, they will both keep their borders open; Whenever $\alpha^W(\tau, Q_t) > \alpha > \alpha^P(\tau, Q_t)$, country $P$ will build the wall, while country $W$ will let immigrants enter for at least another period. In Proposition 6, we showed that this conclusion might not hold when the difference in redistribution levels in the two countries is taken into account. That is,

$$\alpha^W(\tau^W, Q_t) \leq \alpha^P(\tau^P, Q_t)$$

when $Q_t$ is low enough. In this section, we simulate the values of $\alpha^W(\tau^W_0, Q_t)$, $\alpha^P(\tau^W_0, Q_t)$ and $\alpha^P(\tau^P_0, Q_t)$ as a function of $Q_t$. To perform the simulations, we make the following assumptions on the primitives of the model. We set

$$F(L, \theta) = L^{1/2}\theta^2$$

and assume natives’ and immigrants’ types to be uniformly distributed on $(0, 1)$ and $(0, 0.9)$, respectively. Finally, we set $c = 0.05$. Given the assumptions on the production function,

$$L(w_t, \theta) = \frac{\theta^4}{4w^2_t}$$

16
Substituting for \( L(w_t, \theta) \) in the profit function and using (3), we find

\[
\theta^2_t = (2w_t)^{2/3}
\]

The equilibrium wage can then be explicitly derived from (4). The optimal tax levels \( \tau^W_0 \) and \( \tau^P_0 \) are found by maximizing \( x_l(\tau, w_0, 0, \alpha, \theta^l_0) \) and \( u_L(\tau, 0) \) with respect to \( \tau \). They are \( \tau^W_0 \approx 0.5 \) and \( \tau^P_0 \approx 0.6 \). Given the tax levels and the other parameters of the model, one can check that

\[
X(\tau^W_0, w(\tau^W_0, 1), 1) \approx 0.098 \quad X(\tau^P_0, w(\tau^P_0, 1), 1) \approx 0.095
\]

so that Assumption 1 is satisfied. Figure 1 shows the results of the simulations. The discontinuities in the graphs of \( \alpha^W(\tau^W_0, Q_t) \) and \( \alpha^P(\tau^W_0, Q_t) \) happen at the values of \( Q \) such that (6) and (10) hold with the equal sign. These are the values of \( Q \) such that the median type switches from \( \theta^l \) to \( \theta_d \) in country \( W \) and the median party changes from \( L \) to \( D \) in country \( P \). The graph of \( \alpha^P(\tau^W_0, Q_t) \) is continuous in \( Q \) because the initial share of employees in country \( P \) at tax level \( \tau^P_0 \) is already lower than one-half (0.45).

As predicted by the theory, when the tax level is fixed at \( \tau^W_0 \) in both countries, country \( W \) is always weakly more open than country \( P \), i.e. \( \alpha^W(\tau^W_0, Q_t) \geq \alpha^P(\tau^W_0, Q_t) \) for all \( Q_t \). When the tax level in country \( P \) is also determined endogenously, the opposite conclusion holds when the number of migrants entering the country is small enough. More precisely, \( \alpha^W(\tau^W_0, Q_t) \leq \alpha^P(\tau^P_0, Q_t) \), for all \( Q < 1.06 \). It is interesting to notice that, given our assumptions on the primitives of the model,

\[
\tilde{\eta}(\tau^W_0, Q_t) < \eta(\tau^W_0, Q_t)
\]

for all \( Q_t \), proving that the condition stated in Proposition 6 is indeed only sufficient.

6 Conclusions and Future Research

We have shown that different electoral systems may induce countries to choose different immigration policies, and that the predictions depend crucially on the implications that electoral systems have also for the determination of redistribution policies. We have conducted the analysis keeping constant and equal the supply of migrants across countries, because our focus was exclusively on the demand side.

In future research we plan to complement these results with a supply or selection analysis, and we plan to answer a number of important questions:

- first, it can be shown that borders remaining open is the more politically feasible the more selection is possible in terms of enfranchisement, i.e., giving the right to vote to agents with \( \theta \)

---

\( ^7 \)Even though Assumption 3 is not satisfied, both \( x_l(\tau, w_0, 0, \alpha, \theta^l_0) \) and \( u_L(\tau, 0) \) can be shown to be strictly concave in \( \tau \).
above a certain threshold actually helps the possibility of endogenous open borders, especially in WTA systems.\footnote{The comparison in terms of enfranchisement between the two systems can be done in terms of the $\theta$ above which the majority of parliamentarians is in favor of having them vote.}

- Second, an interesting question could be the attractiveness of different immigration policies across systems, in the sense that one system could favor changes in welfare extensions or enfranchisement rules whereas the other could be more likely to build the wall or choose selection policies at the entry point.

- Third, we plan to address endogenous selection of types on the supply side: for similar economic structure and perspectives in two countries, migrants would prefer one to the other if the conditions on institutional insurance or expectations of integration (or even voting) differ substantially. PR, having higher wages and taxes, could induce negative selection, in the sense that the most talented individuals could prefer to supply themselves to WTA countries. The conjecture is that such selection effects may make it comparatively more likely that borders would be closed first in PR systems.

Can a destination ranking be sustainable and under what conditions? In a world of equal growth rate across destination countries, it seems likely that the expected payoffs of migrants should equalize across destinations, and hence there should be a frontier of immigration policies. For example two countries offer the same expected utility to migrants of a given type if either all variables are the same or one has higher $\alpha$ but the other has more generous enfranchisement.

Answering all these questions will further increase the heuristic power of the model we have chosen to propose for the study of immigration policies, which is an increasingly important topic.
in political economy.
References


Appendix

Proof of Lemma 1. By Proposition 1 in AS, \( w(\tau, Q_t) \) is unique and implicitly defined by (4). Differentiating the condition with respect to \( Q \), we get

\[
\frac{\partial w_t}{\partial Q} = -\frac{w_t X(\tau, w_t, Q_t)}{A(\tau, w_t, Q_t)} < 0.
\]

with

\[
A(\tau, w_t, Q_t) = F(L(w_t, \theta_t^2), \theta_t^2)[g(\theta_t^2) + Q_t h(\theta_t^2)] \frac{\partial \theta_t^2}{\partial w} + (\theta_t^1)^2[g(\theta_t^1) + Q_t h(\theta_t^1)]
\]

\[= \int_{\theta_t^1}^{\theta_t^2} L_w(w_t, \theta) [g(\theta) + Q_t h(\theta)] d\theta \tag{21}\]

where we used (3) to obtain the first term in \( A(\tau, w_t, Q_t) \) and we subsituted for \( \partial \theta_t^2 / \partial w \) using (2) in the second term. Using (3), we get \( \frac{\partial \theta_t^2}{\partial w} > 0 \) (see Footnote 3) and since labour demand is decreasing in \( w \), we get \( A(\tau, w_t, Q_t) > 0 \). \( \square \)

Proof of Lemma 2. Suppose

\[\int_{\theta_t^1}^{\theta_t^2} g(\theta) d\theta \geq \frac{1}{2}\]

and let \( \theta_t^l \) satisfy (5). Since \( w_t > w_{t+1} \), the function

\[x_t(\tau, w_t, Q_t, \alpha, \theta) - x_t(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) = (1 - \tau) \theta(w_t - w_{t+1}) + b(\tau, w_t, Q_t, \alpha) - b(\tau, w_{t+1}, Q_{t+1}, \alpha) \tag{22}\]

must be increasing in \( \theta \). Then, \( x_t(\tau, w_t, Q_t, \alpha, \theta^l_t)x_t(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta^l_t) \geq 0 \), implies \( x_t(\tau, w_t, Q_t, \alpha, \theta) - x_t(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) > 0 \) for all \( \theta \in (\theta_t^l, \theta_t^{l+1}) \). By the definition of \( \theta_t^l \), these voters constitute the majority in the native population.

Now let \( x_t(\tau, w_t, Q_t, \alpha, \theta^l_t) - x_t(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta^l_t) \leq 0 \). By the same reasoning, \( x_t(\tau, w_t, Q_t, \alpha, \theta) - x_t(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) < 0 \) for all \( \theta \in (\theta_t^l, \theta_t^{l+1}) \). Furthermore, since the first term in (22) is positive, it must be that \( b(\tau, w_{t+1}, Q_{t+1}, \alpha) > b(\tau, w_t, Q_t, \alpha) \). This immediately implies \( x_d(\tau, w_t, Q_t, \alpha, \theta) - x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) < 0 \) for all \( \theta \in (0, \theta_t^l) \). Since

\[
\frac{\partial y_e(L, w, \theta)}{\partial w} = -L(w, \theta) < 0
\]

for all \( w \), we have that

\[
x_e(L, \tau, w_t, Q_t, \alpha, \theta^l_t) - x_e(L, \tau, w_{t+1}, Q_{t+1}, \alpha, \theta^l_t)
\]

\[= (1 - \tau)[y_e(L, w_t, \theta) - y_e(L, w_{t+1}, \theta^l_t)] + b(\tau, w_t, Q_t, \alpha) - b(\tau, w_{t+1}, Q_{t+1}, \alpha) < 0 \tag{23}\]
for all $\theta \in (\theta_l^2, \theta_l^1)$). Now consider all $\theta \in (\theta_l^1, \theta_{l+1}^1)$. For all these types
\[(1 - \tau)\theta w_t - c < (1 - \tau)(w_t - w_{t+1}) \leq b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha)\]
which directly implies $x_l(\tau, w_t, Q_t, \alpha, \theta) - x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) < 0$. Finally, consider any $\theta \in (\theta_{l+1}^2, \theta_{l+1}^1]$. For these types
\[(1 - \tau)[w_t \theta - y_e(L, w_{t+1}, \theta)] < (1 - \tau)(w_t - w_{t+1}) \leq b(\tau, w_{t+1}, Q_{t+1}, \alpha) - b(\tau, w_t, Q_t, \alpha)\]
so that $x_l(\tau, w_t, Q_t, \alpha, \theta) - x_e(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) < 0$. Combining everything, we have that if $\theta_l^1$ prefers $Q_{t+1}$ to $Q_t$, then all $\theta \notin (\theta_l^1, \theta_{l+1}^2)$ also prefer $Q_{t+1}$ to $Q_t$. As before, by the way $\theta_L^1$ was defined, these people constitute the majority in the population of natives.

Now suppose
\[
\int_{\theta_{l+1}^1}^{\theta_{l+1}^2} g(\theta)d\theta < \frac{1}{2} \quad \int_{\theta_{l+1}^2}^{\theta} g(\theta)d\theta < \frac{1}{2}
\]
Whenever $x_d(\tau, w_t, Q_t, \alpha, \theta) - x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) = b(\tau, w_t, Q_t, \alpha) - b(\tau, w_{t+1}, Q_{t+1}, \alpha) > 0$, then
\[x_l(\tau, w_t, Q_t, \alpha, \theta) - x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) = (1 - \tau)\theta w_t - c + b(\tau, w_t, Q_t, \alpha) - b(\tau, w_{t+1}, Q_{t+1}, \alpha) > 0\]
for all $\theta \in (\theta_l^1, \theta_{l+1}^1)$. Furthermore, by (22), $x_l(\tau, w_t, Q_t, \alpha, \theta) - x_l(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) > 0$ for all $\theta \in (\theta_{l+1}^1, \theta_{l+1}^2)$. Thus, if an unemployed individual prefers $Q_t$ to $Q_{t+1}$, all types $\theta \notin [\theta_{l+1}^2, \theta]$ also prefer $Q_t$. By the second inequality in (25), these types constitute the majority. Whenever $x_d(\tau, w_t, Q_t, \alpha, \theta) - x_d(\tau, w_{t+1}, Q_{t+1}, \alpha, \theta) \leq 0$, so that unemployed individuals prefer $Q_{t+1}$ to $Q_t$, then by (23) and (24) all types $\theta \in [\theta_{l+1}^2, \theta]$ also prefer $Q_{t+1}$. By the first inequality in (25), $Q_{t+1}$ will have the support of the majority of the native population.

\[\square\]

Proof of Lemma 3. Suppose $u_D(Q_t) - u_D(Q_{t+1}) = b(\tau, w_t, Q_t, \alpha) - b(\tau, w_{t+1}, Q_{t+1}, \alpha) \geq 0$, so that party $D$ is in favour of building the wall. The party will be able to form a coalition with party $L$ if and only if
\[u_L(Q_t) - u_L(Q_{t+1}) = b(\tau, w_t, Q_t, \alpha) - b(\tau, w_{t+1}, Q_{t+1}, \alpha) + (1 - \tau)\hat{\theta}_l(\tau, Q_t)w_t - \hat{\theta}_l(\tau, Q_{t+1})w_{t+1} \geq 0\]
A sufficient condition for this to hold is
\[
\frac{d\hat{\theta}_l^l(\tau, Q_t)w_t}{dQ} = w_t \frac{d\hat{\theta}_l^l(\tau, Q_t)}{dQ} + \hat{\theta}_l^l(\tau, Q_t)\frac{\partial w_t}{\partial Q} < 0
\]
Using the definition of $\hat{\theta}_l(\tau, Q_t)$, we get

$$\frac{d\hat{\theta}_l(\tau, Q_t)}{dQ} = \left( \int_{\theta^1_l}^{\theta^2_l} g(\theta) d\theta \right)^{-2} \frac{\partial w_t}{\partial Q} \left[ \frac{\partial^2 g(\theta^2_l)}{\partial \theta^2_l \partial \theta^2_l} \int_{\theta^1_l}^{\theta^2_l} (\theta^1_l - \theta) g(\theta) d\theta - \frac{\partial g(\theta^1_l)}{\partial \theta^1_l} \int_{\theta^1_l}^{\theta^2_l} (\theta^1_l - \theta) g(\theta) d\theta \right]$$

Then,

$$\frac{d\hat{\theta}_l(\tau, Q_t)w_t}{dQ} = \frac{\partial w_t}{\partial Q} \left[ \left( \int_{\theta^1_l}^{\theta^2_l} g(\theta) d\theta \right)^{-1} \left( \frac{\partial^2 g(\theta^2_l)}{\partial \theta^2_l \partial \theta^2_l} \int_{\theta^1_l}^{\theta^2_l} (\theta^2_l - \theta) g(\theta) d\theta - \frac{\partial g(\theta^1_l)}{\partial \theta^1_l} \int_{\theta^1_l}^{\theta^2_l} (\theta^1_l - \theta) g(\theta) d\theta \right) + \int_{\theta^1_l}^{\theta^2_l} \theta g(\theta) d\theta \right] \left( \int_{\theta^1_l}^{\theta^2_l} g(\theta) d\theta \right)^{-1}$$

When (11) holds,

$$\int_{\theta^1_l}^{\theta^2_l} \theta g(\theta) d\theta > \left( \int_{\theta^1_l}^{\theta^2_l} g(\theta) d\theta \right)^{-1} \left( \frac{\partial g(\theta^1_l)}{\partial \theta^1_l} \int_{\theta^1_l}^{\theta^2_l} (\theta^1_l - \theta) g(\theta) d\theta \right)$$

and $d\hat{\theta}_l(\tau, Q_t)w_t/dQ > 0$.

\[\Box\]

Proof of Lemma 5 (from [14]). Let $\theta > \tilde{\theta}$ first. A sufficient condition for 1. to hold is

$$\xi(\tau, Q_t, \theta) - \xi(\tau', Q_t, \theta) \geq \xi(\tau, Q_t, \tilde{\theta}) - \xi(\tau', Q_t, \tilde{\theta})$$

for all $\tau < \tau'$. Rearranging terms, we get

$$\xi(\tau, Q_t, \theta) - \xi(\tau, Q_t, \tilde{\theta}) \geq \xi(\tau', Q_t, \theta) - \xi(\tau', Q_t, \tilde{\theta})$$

Thus, 1. holds if the function $\Delta \xi(\tau) \equiv \xi(\tau, Q_t, \theta) - \xi(\tau, Q_t, \tilde{\theta})$ is decreasing in $\tau$. By assumption, the median type is an employee, so that $\xi(\tau, Q_t, \theta) = x_t(\tau, w, Q_t, \alpha, \theta)$. For all $\theta \in (\bar{\theta}, \theta^2_t)$, $\xi(\tau, Q_t, \theta) = x_t(\tau, w, Q_t, \alpha, \theta)$. Then, $\Delta \xi(\tau) = (1 - \tau)(\theta - \tilde{\theta})w_t$. Deriving it with respect to $\tau$ and rearranging terms, we get

$$\frac{d\Delta \xi(\tau)}{d\tau} = -(\theta - \tilde{\theta})w_t V(\tau)$$

where $V(\tau) > 0$ is as defined in (14). For all $\theta \in [\theta^2_t, \bar{\theta})$, $\xi(\tau, Q_t, \theta) = x_e(L, \tau, w, Q_t, \alpha, \theta)$. Then, $\Delta \xi(\tau) = (1 - \tau)[y_e(L, w_t, \theta) - w_t \tilde{\theta}]$ and

$$\frac{d\Delta \xi(\tau)}{d\tau} = -[y_e(L, w_t, \theta) - w_t \tilde{\theta}] + (1 - \tau) \left[ \frac{\partial y_e(L, w_t, \theta)}{\partial w_t} - \tilde{\theta} \frac{\partial w_t}{\partial \tau} \right]$$

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By the envelope theorem,
\[
\frac{\partial y_e(L, w_t, \theta)}{\partial \tau} = -L(w_t, \theta) \frac{\partial w_t}{\partial \tau}
\]
Then,
\[
\frac{d\Delta \xi(\tau)}{d\tau} = -[y_e(L, w_t, \theta) - w_t \hat{\theta}] - (1 - \tau) \left[ L(w_t, \theta) + \hat{\theta} \right] \frac{\partial w_t}{\partial \tau} < 0
\]
as gross income is increasing in \( \theta \) and wage is increasing in \( \tau \).

By a similar reasoning, a sufficient condition for 2. to hold is \( d\Delta \xi(\tau)/d\tau > 0 \), whenever \( \theta < \hat{\theta}. \)

For all \( \theta \in [\theta_1^l, \hat{\theta}) \), \( \xi(\tau, Q_t, \hat{\theta}) = x_t(\tau, w, Q_t, \alpha, \hat{\theta}) \) and \( \Delta \xi(\tau) = (1 - \tau)(\theta - \hat{\theta})w_t \). For all types \( \theta \in (0, \theta_1^l) \), \( \xi(\tau, \theta) = x_d(\tau, w, Q_t, \alpha, \theta) \) and \( \Delta \xi(\tau) = -(1 - \tau)\hat{\theta}w_t + c. \) In both cases, \( \Delta \xi(\tau) > 0 \) implies \( d\Delta \xi(\tau)/d\tau > 0 \).

Proof of Lemma 6, point 1. Consider the first item in the statement of the lemma. A sufficient condition for it to hold is that
\[
u_e(\tau, Q_t) - u_e(\tau', Q_t) \geq u_L(\tau, Q_t) - u_L(\tau', Q_t)
\]
or, rearranging terms,
\[
u_e(\tau, Q_t) - u_L(\tau, Q_t) \geq u_e(\tau', Q_t) - u_L(\tau', Q_t).
\]
(27)

Substituting for \( u_e(\tau, Q_t) \) and \( u_L(\tau, Q_t) \), (27) becomes
\[
(1 - \tau)[\dot{y}_e(L, w_t, Q_t) - \dot{\theta}_t(\tau, Q_t)w_t] \geq (1 - \tau')[\dot{y}_e(L, w_t', Q_t) - \dot{\theta}_t(\tau', Q_t)w_t']
\]
where \( w_t' \equiv \tau(\tau', Q_t). \) Define the function \( \Delta(\tau) \equiv (1 - \tau)[\dot{y}_e(L, w_t, Q_t) - \dot{\theta}_t(\tau, Q_t)w_t]. \) Then
\[
\frac{\partial \Delta(\tau)}{\partial \tau} = -[\dot{y}_e(L, w_t, Q_t) - \dot{\theta}_t(\tau, Q_t)w_t] + (1 - \tau) \left[ \frac{\partial \dot{y}_e(L, w_t, Q_t)}{\partial \tau} - \frac{\partial \dot{\theta}_t(\tau, Q_t)w_t}{\partial \tau} \right]
\]
The first term in \( \partial \Delta(\tau)/\partial \tau \) is always negative since
\[
\dot{y}_e(L, w_t, Q_t) > y_e(L, w_t, \theta_1^2) = \theta_1^2 w_t > \dot{\theta}_t(\tau, Q_t)w_t
\]
Furthermore,
\[
\frac{\partial \dot{y}_e(L, w_t, Q_t)}{\partial \tau} = - \left( \int_{\theta_1^l}^{\hat{\theta}} g(\theta)d\theta \right)^{-1} \frac{\partial w_t}{\partial \tau} \left\{ \int_{\theta_1^l}^{\hat{\theta}} L(w_t, \theta)g(\theta)d\theta - \frac{\partial \theta_1^2}{\partial w_t} g(\theta_1^2)\dot{y}_e(L, w_t, Q_t) - y_e(L, w_t, \theta_1^2) \right\} < 0
\]
when (16) holds. Finally,
\[
\frac{\partial \hat{\theta}_l(\tau, Q_t) w_t}{\partial \tau} = w_t \frac{\partial \hat{\theta}_l(\tau, Q_t)}{\partial \tau} + \hat{\theta}_l(\tau, Q_t) \frac{\partial w_t}{\partial \tau} > 0
\]
since
\[
\frac{\partial \hat{\theta}_l(\tau, Q_t)}{\partial \tau} = \left( \int_{\theta_1^2}^{\theta_2^2} g(\theta) d\theta \right)^{-1} \left\{ \frac{\partial \theta_1^2}{\partial w} \frac{\partial w}{\partial \tau} g(\theta_1^2) \int_{\theta_1^2}^{\theta_2^2} (\theta_2^2 - \theta) g(\theta) d\theta + \frac{\theta_1^2 g(\theta_1^2)}{1 - \tau} V(\tau) \int_{\theta_1^2}^{\theta_2^2} (\theta - \theta_1^2) g(\theta) d\theta \right\} > 0
\]
with \( V(\tau) \) as defined in (14). Combining everything, we get \( \partial \Delta(\tau)/\partial \tau < 0 \), implying that (27) holds.

Proof of Proposition 4. Suppose \( \tau_L \) is the status quo tax rate. By Lemma 6, a coalition of two parties always prefers \( \tau_L \) to any other proposed tax rate \( \tau \): when \( \tau < \tau_L \), the coalition includes parties \( L \) and \( D \); when \( \tau > \tau_L \), it includes parties \( L \) and \( E \). This proves that \( \tau_L \) is stable.

Now consider any other status quo tax rate \( \tau_0 \neq \tau_L \). With some positive probability
\[
\pi_L = \int_{\theta_L^1}^{\theta_L^2} g(\theta) d\theta
\]
party \( L \) will be the proposer and will always be able to form a coalition to replace \( \tau_0 \) with \( \tau_L \). Then, for any \( \tau_0 \neq \tau_L \), \( p(\tau_0|\tau_0) < 1 \). \( \square \)