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Spatial competition and social welfare in the presence of non-monotonic network effects*

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Abstract

We study a spatial duopoly and extend the literature by giving joint consideration to non-monotonic network effects and endogenous firm location decisions. We show that the presence of network effects (capturing, for example, in-store rather than online sales) improves welfare whenever the total market size is not too large. This effect is lost if network effects are specified in a monotonic fashion, in which case isolating consumers from one another always reduces welfare. We also provide a new rationale for a duopoly to be welfare-preferred to monopoly: in large markets, splitting demand between two firms can reduce utility losses due to crowding.

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1 Introduction

When the utility enjoyed by consumers of a particular good depends not only on the attributes of the good in question but also on the extent to which other consumers purchase that good, consumer preferences are said to incorporate *network effects*. Broadly speaking, these network effects may capture two distinct ideas. Firstly, network effects may reflect the degree of crowding associated with larger or smaller groups of consumers gathering in a particular physical retail space, such as a store or restaurant. Alternatively, network effects may capture status considerations. If consumers' preferences display a desire for *conformity*, they enjoy a positive network effect when purchasing from firms that serve a larger total demand. This contrasts with the case in which preferences display *vanity*, in which case purchasing from larger firms tends to reduce consumer utility (see, e.g., Grilo et al., 2001).¹

Conformity and vanity-type effects may be captured simultaneously by specifying a network effect function which is non-monotonic, in the sense that the network effect is positive and increasing in the number of consumers at a given firm up to a certain threshold (reflecting conformity), after which crowding starts to reduce the network effect, eventually turning it negative (reflecting vanity). In terms of physical crowding, such a non-monotonic specification implies a preference on the part of consumers for environments that are moderately crowded rather than being either over- or under-crowded. This captures intuitively the effect likely to be at work in a wide range of “congestible” goods (e.g. telephone systems, restaurants, academic classes) and is also supported by evidence from the marketing literature that consumer preferences with respect to the perceived level of crowding follow an inverse U-shape (see, e.g., Eroglu et al., 2005; Pan and Siemens, 2011). Thus there is a sound empirical basis for moving from a setting of monotonic

¹Leibenstein (1950) alternatively refers to the preference for conformity or vanity as the *bandwagon* or *snob* effect, respectively. While we take them as given throughout this paper, the emergence of conformity- and vanity-type effects is explained by Corneo and Jeanne (1997) in a model where conspicuous consumption serves as a signal of higher income, which enables consumers to achieve higher social status. See also Bernheim (1994) and Pesendorfer (1995).

network effects to one of non-monotonic network effects.²

The key contribution of this paper lies in exploring the impact of such non-monotonic network effects on social welfare when firms sell differentiated products. In so doing, and in contrast to previous work in this area (e.g. Grilo et al., 2001; Lee and Mason, 2001; Häckner and Nyberg, 1996), we allow for firms' product differentiation decisions to be determined endogenously. In particular, we consider the impact of non-monotonic network effects on market equilibrium and social welfare in a two-firm, spatial product differentiation framework. We solve for the subgame perfect Nash equilibrium in a two-stage game in which firms choose product locations on the real line at the first stage and prices at the second stage, analysing Nash deviation incentives at both stages.

Our first welfare result explores the circumstances under which allowing consumers' enjoyment of a good to depend on the volume of consumers patronizing the same firm leads to higher welfare. As we think of preferences as immutable, the case in which consumers ignore the volume of customers at a given firm can be thought of as capturing a situation in which the purchase transaction is conducted at arm's length, for example via the internet, rather than in-store. In this case, consumers are physically isolated from the remaining customers purchasing from a given firm. Our results therefore give an interesting perspective on the welfare impact of e-commerce, a sector which has experienced strong growth in recent years.³

We show that the presence of network effects (capturing in-store sales) increases total welfare whenever the direct utility gained by consumers from the network at each individual firm is positive. Since demand is split equally between firms in equilibrium, and given our non-monotonic specification for the network effect function, this requires the total market size to be sufficiently small. While the network externality generally exerts two

²Existing papers that focus on the case of monotonic network effects include Serfes and Zacharias (2012), Elhadj et al. (2012), and Orsini (2005).

³The share of e-commerce in total US retail sales grew from 8.7% at year-end 2015 to 9.5% at year-end 2016, for example. US Department of Commerce (2017). Note that this setting is more consistent with the view that network effects capture physical crowding as opposed to status effects associated with vanity or conformity. The latter should not depend on whether a purchase is made via the internet or in-store.

effects, one via the purchase price and another via the direct utility gained by consumers as a result of consuming in groups, only the latter is relevant in assessing the impact of network effects on total welfare because the purchase price is simply a transfer from consumers to firms. In our framework, based on non-monotonic network effects, a shift by firms towards online only sales can therefore increase welfare if the total market size is sufficiently large, by eliminating crowding. This effect is lost if network effects are not modelled, or if they are specified in a positive, monotonically increasing fashion (in which case isolating consumers from one another always reduces welfare).

Our second welfare result considers firms' welfare-maximising locations. We argue that, relative to the monotonic case, non-monotonic network effects increase the desirability of splitting the market between two firms rather than letting one firm serve the entire market. The intuition for this result is as follows. With our (empirically more relevant) non-monotonic specification, network effects start to decrease when the total consumer population size rises beyond a particular threshold level. In large markets, splitting demand between two firms therefore not only reduces transportation costs, but also maximises the aggregate network effect (the network externality enjoyed by consumers at either firm under a duopolistic market structure then exceeds that which they would enjoy if demand were concentrated at a single firm). This realistic aspect of the problem is missing if network effects are specified in a monotonic fashion, as in Serfes and Zacharias (2012), Elhadj et al. (2012), and Lambertini and Orsini (2005), amongst others.

In addition to these welfare results, our paper provides a further contribution to the literature which is of a more technical nature. In particular, our derivation of the subgame perfect equilibrium in this two-stage game (on which our welfare analysis builds) pays explicit attention to firms' Nash deviation incentives at the location stage – when network effects are sufficiently strong, firms are shown to face an incentive to move closer to their rival in order to capture the entire demand, thus ruling out the existence of equilibrium. Since, in this setting, profitable deviations cannot arise at the location stage in the absence

of network effects, this result may be of independent interest. Solving for the subgame perfect firm locations in this way also allows us to contrast our results with those obtained in the exogenous location setting of Grilo et al. (2001) and related work.⁴ In particular, unlike in Grilo et al. (2001), vertical product differentiation cannot arise endogenously in equilibrium, and increases in the consumer population size cannot lead to a monopoly outcome when firms choose locations optimally.

The remainder of the paper is organized as follows. Section 2 describes the model set-up. Section 3 derives the market equilibrium and discusses its implications, before Section 4 presents our welfare analysis. Section 5 concludes. All proofs are contained in the Appendix.

2 The Model

Consider two firms, A and B , which produce a homogeneous good and are located at x_A and x_B , where $x_i \in \mathbb{R}$, $i = A, B$. With no loss in generality, we will consider firm A to be the firm which locates to the left of B , so that $x_A < x_B$. Firms sell at mill price p_i , have production costs normalized to zero and choose their locations endogenously.

There is a continuum of consumers of mass n uniformly distributed over the interval $[0, 1]$. We assume that each consumer buys exactly one unit of the product and that, if a consumer purchases from firm $i = A, B$, their (indirect) utility is given by

$$U_i(x) = K - p_i - t(x - x_i)^2 + E(n_i), \quad (1)$$

where K is the gross utility from consumption, $t(x - x_i)^2$ is the total transportation cost given consumer location $x \in [0, 1]$ and transportation cost parameter $t > 0$, and n_i is the mass of consumers at firm $i = A, B$, so that $\sum_i n_i = n$. The last term represents the

⁴Related models in which firms' product differentiation decisions are not modelled endogenously include Lee and Mason (2001) and Häckner and Nyberg (1996).

network effect function. Motivated by the empirical literature, and following Grilo et al. (2001), we consider a non-monotonic network effect function of the form

$$E(n_i) = \alpha n_i - \beta n_i^2, \quad i = A, B, \quad (2)$$

where $\alpha, \beta > 0$. This implies that the network effect is increasing in n_i up to $n_i = \alpha/2\beta$, after which the function becomes downward-sloping. We denote by n_{OL} the “overloading” threshold above which additional crowding generates disutility. That is, for all $n_i > n_{OL}$, $E(n_i) < 0$. It is clear by inspection of (2) that, in this setting,

$$n_{OL} = \frac{\alpha}{2\beta}. \quad (3)$$

Our analysis will make use of several definitions from the literature, which we introduce below. The first definition captures the strength of conformity or vanity effects in consumers’ preferences, relative to a measure of product differentiation.

Definition 1. *Consumer preferences exhibit weak conformity (respectively, strong conformity) if*

$$\alpha n - \beta n^2 < (>) t(x_B - x_A). \quad (4)$$

The second definition describes the relationship between firms’ locations and the form of product differentiation. While our framework is one of horizontal, Hotelling-type differentiation, it can also give rise to outcomes that coincide with common notions of *vertical* product differentiation.⁵ In particular, a vertical differentiation outcome is understood to result when, charging identical prices, one firm or other would capture the entire market. This leads to the following definition.⁶

⁵This is perhaps unsurprising, given that Cremer and Thisse (1991) show in general terms that most horizontal differentiation frameworks can be understood as special cases of vertical differentiation models.

⁶Our notion of vertical vs. horizontal product differentiation follows Grilo et al. (2001). This is in contrast to some other well-known notions of vertical differentiation, such as those in Shaked and Sutton (1982) and Tirole (1988, Sect. 2.1).

Definition 2. *The firms' products are vertically differentiated if their locations satisfy $x_A + x_B \geq 2$ or $x_A + x_B \leq 0$. If $0 < x_A + x_B < 2$, the firms' products are horizontally differentiated.*

Our final definition is fairly standard, and relates to the firms' location decisions.

Definition 3. *Firm A (respectively, B) has a locational advantage if $x_A + x_B > (<)1$. If neither firm enjoys a locational advantage, the firms' locations are symmetric.*

Given uniformly distributed consumers, Definition 3 implies that, in the absence of network effects, the firm with the locational advantage has a larger market share than its rival.

The market is modelled as a two-stage game, in which firms choose their locations in the first stage and prices in the second stage. The equilibrium is derived by backward induction as a subgame perfect Nash equilibrium in pure strategies.

3 Equilibrium and Endogenous Product Differentiation

We first present our main result concerning the subgame perfect Nash equilibrium of the two-stage game, before discussing its implications.

Proposition 1. *There is a unique subgame perfect Nash equilibrium in which the firm locations are $(x_A^*, x_B^*) = (-\frac{1}{4}, \frac{5}{4})$, prices are*

$$p_A^* = p_B^* = \frac{3}{2}t - (\alpha n - \beta n^2) \quad (5)$$

and demand is shared equally between the firms if and only if

$$\alpha n - \beta n^2 < \frac{11}{24}t, \quad (6)$$

and no subgame perfect equilibrium in pure strategies exists otherwise.

Proof. Appendix A. □

The main innovation of this result lies in the careful attention we are obliged to pay to firms' Nash deviation incentives (displacement incentives) to capture the entire market at the location stage. As shown in Appendix A, these incentives vanish in the absence of network effects. Moreover, in considering firms' potential displacement locations, it is crucial to consider the fact that a move by a given firm from its equilibrium location to its optimal displacement location can, in general, shift the market environment from weak to strong conformity or vice versa (see Definition 1).

While these technical aspects are discussed in detail in Appendix A, the main intuition of the result can be described as follows. Rewriting (6) as a condition on the consumer population size as

$$n < \underline{n} \equiv \frac{6\alpha - \sqrt{6}\sqrt{6\alpha^2 - 11\beta t}}{12\beta} \quad \text{or} \quad n > \bar{n} \equiv \frac{6\alpha + \sqrt{6}\sqrt{6\alpha^2 - 11\beta t}}{12\beta},$$

the equilibrium can be illustrated as in Figure 1 below. The parabola represents the network effect function evaluated at n , and the parameter range in which the subgame perfect equilibrium described in Proposition 1 exists is identified by the shaded grey areas.

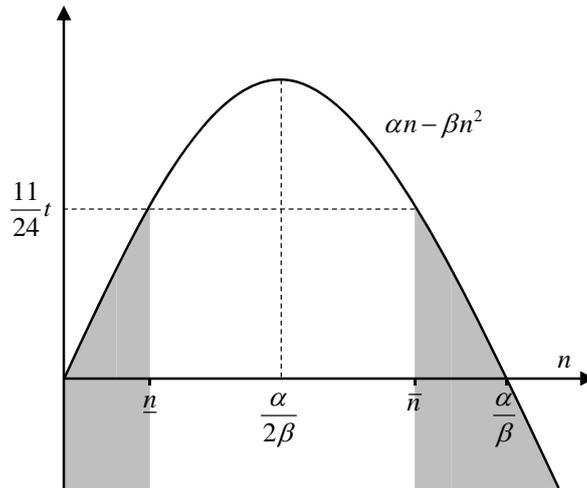


Figure 1: Illustration of Equilibrium.

Clearly, values of n that fall closest to $\frac{\alpha}{2\beta}$ are least likely to be consistent with equilibrium existence. This follows because the incentives for firms to displace their rival at the location stage in order to capture the entire market are greatest in such cases, as then consumers' network externality (and therefore their willingness to pay) is maximised when all consumers purchase from the same firm, be it A or B .

It is also interesting to note that the equilibrium locations are, themselves, unaffected by network effects.⁷ It is the *existence* of the subgame perfect equilibrium that depends on these effects. This result therefore contradicts the premise, implicit in Grilo et al. (2001), that adding network effects allows firms' locations to be varied exogenously. Proposition 1 further allows us to refine three important results from the exogenous location setting of Grilo et al. (2001), as discussed in Corollaries 1-3 below.

Corollary 1. *A subgame perfect equilibrium cannot exist under strong conformity.*

It is clear from (6) that strong conformity is a sufficient (but not necessary) condition for equilibrium not to exist. Under strong conformity (see Definition 1), the only equilibria of the pricing subgame are corner solutions in which one firm or other charges a price of zero (Grilo et al., 2001). These are not supportable as subgame perfect equilibria, since a firm that anticipates zero profits at the pricing stage faces strict incentives to change location in the first stage of the game.⁸

Corollary 2. *Vertical product differentiation cannot arise in equilibrium.*

Since, at the equilibrium location pair in Proposition 1, $x_A^* + x_B^* = 1$, this equilibrium implies horizontally differentiated products (see Definition 2). A vertical differentiation outcome is not feasible here, because neither firm is willing to concede a locational advantage to its rival, so that no asymmetric equilibria survive under the requirement for

⁷Compare, for example, Tabuchi and Thisse (1995).

⁸This point is already made in Grilo et al. (2001), though not in the context of an explicit solution for firms' optimal locations. Our full solution for the subgame perfect equilibrium shows that displacement incentives at the location stage can also rule out the existence of equilibrium when we have weak conformity, provided that network effects are strong enough.

subgame perfection. We can therefore omit the cases of vertical differentiation discussed in Grilo et al. (2001).

Corollary 3. *Increases in the population size cannot lead to monopoly.*

In Grilo et al. (2001), increases in the population size may magnify the locational advantage of a given firm to such a degree that it captures the entire market. Our result shows that no locational advantage will arise endogenously in the first place. In our setting, it is *intermediate* population sizes that are of primary concern, because these can rule out the existence of equilibrium altogether (see Figure 1).

Essentially, the differences between our results and those of Grilo et al. (2001), as emphasised in Corollaries 1-3, are due to the additional degree of freedom those authors enjoy because they treat firm locations as exogenous. This enables them to explore settings of asymmetric product differentiation, despite the fact that firms are (in that setting as in ours) inherently symmetric. Our analysis with endogenous locations focuses attention on the meaningful cases, in which firms choose locations optimally. This, in turn, yields more precise theoretical predictions.⁹

4 Welfare Analysis

The preceding analysis allows us to investigate two interesting welfare questions. Firstly, we ask whether the presence of network effects in consumers' preferences is desirable. That is, we consider whether isolating consumers from one another by conducting sales over the internet rather than via physical sales outlets improves welfare. A second important question concerns the welfare-maximising firm locations, such as would be chosen by a benevolent social planner. We address each question in turn below, relegating proofs to the Appendix.

⁹In future research, it will be very interesting to extend this framework to the case of asymmetric firms.

4.1 Social Welfare in the Presence of Non-Monotonic Network Effects

We first explore (conditional on the equilibrium described in Proposition 1 existing) for which values of the model parameters total welfare is improved, relative to the case in which $\alpha, \beta \rightarrow 0$. Consumer surplus in our framework is given by

$$CS = n \int_0^{\hat{x}} K - p_A - t(x - x_A)^2 + \alpha n_A - \beta n_A^2 dx \\ + n \int_{\hat{x}}^1 K - p_B - t(x - x_B)^2 + \alpha n_B - \beta n_B^2 dx,$$

where \hat{x} denotes the position of the indifferent consumer (so that $n_A = \hat{x}n$ and $n_B = (1 - \hat{x})n$). Defining total welfare as the sum of consumer surplus and industry profits (see Appendix A for details), the following Proposition holds.

Proposition 2. *Provided the equilibrium described in Proposition 1 exists, the presence of network effects increases total welfare if and only if $n < \frac{2\alpha}{\beta}$.*

Proof. Appendix B. □

Intuitively, the welfare impact of network effects may be decomposed into two parts. Firstly, network effects will cause prices to fall (rise) if $n < (>) n_{OL}$, see (3) and (5). If this were the only consequence of network effects, their impact would be welfare-neutral, however, since the purchase price is just a transfer from consumers to firms. The second effect reflects consumers' direct (dis)utility from consuming in groups. Since, in the subgame perfect equilibrium derived in Proposition 1, firms share demand equally, this direct effect is positive if and only if, for $i = A, B$, $n_i < \frac{\alpha}{\beta} \Rightarrow n < \frac{2\alpha}{\beta}$.

Discussion

As motivated in the Introduction, the case in which network effects vanish can be interpreted as a setting in which sales are conducted via the internet rather than in physical

stores. Isolating consumers from the impact of (non-monotonic) network effects is thus welfare-preferred here whenever the total consumer population size is sufficiently large relative to the concavity parameter β .¹⁰ Notice also that, as $\beta \rightarrow 0$ but $\alpha > 0$, capturing a linear network effect function, network effects *always* improve total welfare according to Proposition 2.

This Proposition is best understood when viewing network effects as capturing physical crowding as opposed to status effects associated with vanity or conformity. The latter should not depend on the manner in which the purchase is made. Moreover, this is arguably more in keeping with an interpretation of the spatial differentiation model as reflecting consumers' locations in the space of product attributes, rather than in geographical space – in the former case, the cost of deviating from the consumer's preferred product specification is independent of whether the purchase is made online or in-store. Nonetheless, an interpretation of the model in terms of physical space may also be satisfactory if we think of transportation costs as reflecting *delivery* costs associated with an online purchase and suppose (as appears reasonable) that these delivery costs are proportional to distance.

4.2 Welfare-Maximizing Locations

A second important question concerns firms' welfare-maximising locations. The optimisation problem to be solved by the social planner to this end may be stated as follows:

$$\max_{x_A, x_B, \hat{x}} TW = \max_{x_A, x_B, \hat{x}} \left\{ \begin{array}{l} n \int_0^{\hat{x}} K - p_A - t(x - x_A)^2 + \alpha n_A - \beta n_A^2 dx \\ + n \int_{\hat{x}}^1 K - p_B - t(x - x_B)^2 + \alpha n_B - \beta n_B^2 dx \\ + n_A p_A + n_B p_B \end{array} \right\}.$$

Solving this optimisation problem leads to the following Proposition.

Proposition 3. *Whenever either (i) $0 < n \leq \frac{2\alpha}{3\beta}$ and $t > 8\alpha n - 12\beta n^2$, or (ii) $n > \frac{2\alpha}{3\beta}$*

¹⁰Note that, as we normalise firms' costs to zero in this framework, we abstract from cost advantages that may go along with online only sales. We also leave a fuller discussion of firms' strategic decisions with respect to online only or in-store sales for future work.

holds, the welfare-optimal locations are interior: $(x_A, x_B) = (\frac{1}{4}, \frac{3}{4})$, and consumers are split evenly between the two firms.

Whenever conditions (i) and (ii) are both violated, it is welfare-optimal for a single firm located at the mid-point of the unit interval to serve the entire market.

Proof. Appendix C. □

The intuition for this result goes back to the trade-off between transportation costs and the strength of network effects. When the consumer population size is relatively small ($n \leq \frac{2\alpha}{3\beta}$), splitting the total demand between two firms reduces the magnitude of the network effect enjoyed by a consumer at either of the two firms, relative to what they would have enjoyed were demand concentrated at a single firm (this may be verified graphically by inspection of Figure 1). In this case, there is a cost to splitting demand between two firms, because the aggregate network effect is reduced. To offset this loss in the aggregate network effect enjoyed by consumers, the saving in terms of transportation costs associated with a duopolistic market structure must be sufficiently large. This, in turn, leads to the condition on the transportation cost parameter t provided in part (i) of Proposition 3.

If the population size is sufficiently large ($n > \frac{2\alpha}{3\beta}$), however, the aggregate network effect is maximised by splitting demand between two firms. This follows because, in that case, consumers enjoy a larger network effect at either of the two firms under a duopolistic market structure than they would if demand were concentrated at a single firm (again, this may be verified graphically by inspection of Figure 1). In this case, the socially optimal market structure is unambiguous: both the gain in the aggregate network effect and the transportation cost savings favour a duopolistic as opposed to monopolistic market structure.

Discussion

Proposition 3 highlights an additional rationale for a duopoly to be welfare-preferred to monopoly when network effects are non-monotonic rather than monotonic. Under both specifications for the network effect function, the optimal market structure rests on the trade-off between transportation costs and aggregate network effects. In the monotonic case ($\alpha > 0, \beta \rightarrow 0$), it is optimal to split the market between two firms whenever the aggregate network effect αn is sufficiently weak relative to the transportation cost parameter t . In such cases, the saving in transportation costs relative to monopoly offsets the reduction in the aggregate network effect associated with a duopolistic market structure.¹¹

In our non-monotonic setting, as well as saving on transportation costs, splitting the market between two firms will in fact maximise the aggregate network effect when the population size is sufficiently large (leading to condition (ii) in Proposition 3 above). Thus we can say that, relative to the monotonic network effects case, our non-monotonic specification increases the attractiveness (in welfare terms) of splitting the market between two firms – in large markets, this reduces utility losses due to crowding. This realistic aspect of the problem is missing if network effects are specified in a monotonic fashion.

5 Conclusion

This paper considers a spatial duopoly framework with non-monotonic network effects in which firms choose locations optimally. We derive the subgame perfect Nash equilibrium, which takes the potential for Nash deviations at the location stage explicitly into account. We show, firstly, that network effects will rule out the existence of equilibrium whenever these are sufficiently strong, due to displacement incentives at the location stage. Moreover, “strong conformity” is shown to be a sufficient condition for equilibrium not to

¹¹See also Serfes and Zacharias (2012) for an analysis of the monotonic case when the population size is fixed at unity. Note, however, that the calculations in Serfes and Zacharias (2012) contain an error, which affects their results in quantitative if not qualitative ways. See Appendix D for details.

exist. In contrast to the exogenous location setting of Grilo et al. (2001), we also show that vertical product differentiation cannot arise in equilibrium, and that increases in the population size cannot result in one firm monopolising the industry.

We also examine the conditions under which the presence of network effects improves welfare in this framework. This is particularly relevant given the ongoing trend towards e-commerce, under which consumers are isolated from the effects of physical crowding in stores. In-store sales are shown to improve welfare whenever the direct utility gained by consumers from the network at either firm is positive, which requires the total market size to be sufficiently small. This provides a welfare argument in favour of online only sales when the total market size is sufficiently large, which is absent if network effects are specified in a monotonic fashion. Finally, we study the welfare-maximising firm locations. We find that, relative to monotonic network effect models, the case for a duopolistic market structure is strengthened. For sufficiently large consumer population sizes, splitting demand between two firms not only reduces transportation costs but also maximises the aggregate network effect.

Appendices

A Proof of Proposition 1

We proceed by backward induction.

A.1 Stage 2 – Pricing

Let \hat{x} denote the position of the consumer that is indifferent between buying from firm A and firm B . This \hat{x} is found by requiring the following conditions to hold simultaneously:

$U_A(\hat{x}) = U_B(\hat{x})$, $n_A = \hat{x}n$ and $n_B = (1 - \hat{x})n$.¹² This shows that

$$\hat{x} = \frac{p_B - p_A + t(x_B^2 - x_A^2) - \alpha n + \beta n^2}{2[t(x_B - x_A) - \alpha n + \beta n^2]}. \quad (7)$$

We solve for the firms' optimal prices by substituting (7) into $n_A = \hat{x}n$ and $n_B = (1 - \hat{x})n$ and then maximising each firm's profit function $\Pi_i = p_i n_i$, $i = A, B$, with respect to price. It is straightforward to show that, given firm locations x_A and x_B , the unique (interior) equilibrium prices are given by

$$p_A^* = \frac{t}{3}(x_B - x_A)(2 + x_B + x_A) - (\alpha n - \beta n^2), \quad (8)$$

$$p_B^* = \frac{t}{3}(x_B - x_A)(4 - x_B - x_A) - (\alpha n - \beta n^2). \quad (9)$$

These prices are positive if and only if¹³

$$\alpha n - \beta n^2 < \frac{t}{3}(x_B - x_A) \min \{2 + x_B + x_A, 4 - x_B - x_A\}. \quad (10)$$

Undercutting Incentives

It remains to check whether either firm has an incentive to undercut its rival's price in order to capture the whole market (in what follows, we similarly check for displacement incentives at the location stage). In order to undercut, firm A takes (9) as given and sets undercutting price p_A^c , such that

$$\hat{x}(p_A^c) = \frac{3p_A^c + 2t(x_A - x_B)(2 + x_A + x_B) + 6n(\alpha - \beta n)}{6[n(\alpha - \beta n) + t(x_A - x_B)]} = 1.$$

¹²The latter two conditions draw on properties of the uniform distribution. Recall that we are considering here, with no loss in generality, the case where $x_A < x_B$.

¹³As argued by Grilo et al. (2001), with unrestricted location choices, any firm that anticipates charging a price of zero at the second stage of the game faces strict incentives to change location at the first stage so as to earn positive profits. Therefore such corner solutions at the pricing stage cannot be subgame perfect, and hence we focus on interior solutions.

Firm B , meanwhile, takes (8) as given and sets undercutting price p_B^c , such that

$$\widehat{x}(p_B^c) = \frac{3p_B^c - t(x_B - x_A)(2 + x_A + x_B) + 3t(x_B^2 - x_A^2)}{6[t(x_B - x_A) - n(\alpha - \beta n)]} = 0.$$

Solving in each case for p_i^c and substituting into the expression for undercutting profits $\Pi_i^c = p_i^c \cdot n$ shows that the undercutting profits are symmetric, and equal for firm $i = A, B$ to

$$\Pi_i^c = \frac{2}{3}nt(x_j - x_i)(x_A + x_B - 1), \quad j \neq i.$$

It follows that firm A 's undercutting profits are positive if and only if $x_A + x_B > 1$, while firm B 's undercutting profits are positive if and only if $x_A + x_B < 1$. If $x_A + x_B = 1$, undercutting profits are zero for both firms – with symmetric location choices, undercutting is never profitable.

We now determine when these undercutting profits exceed the firm's equilibrium profits. These equilibrium profits are given by

$$\Pi_A^* = \widehat{x} n p_A^* = \frac{n [t(x_A - x_B)(2 + x_A + x_B) + 3(\alpha n - \beta n^2)]^2}{18 [t(x_B - x_A) - (\alpha n - \beta n^2)]}$$

and

$$\Pi_B^* = (1 - \widehat{x}) n p_B^* = \frac{n [t(x_A - x_B)(x_A + x_B - 4) - 3(\alpha n - \beta n^2)]^2}{18 [t(x_B - x_A) - (\alpha n - \beta n^2)]}.$$

It therefore follows that undercutting is profitable for firm A if and only if

$$\Pi_A^* - \Pi_A^c = \frac{n [t(x_A - x_B)(x_A + x_B - 4) - 3(\alpha n - \beta n^2)]^2}{18 [t(x_B - x_A) - (\alpha n - \beta n^2)]} < 0,$$

and profitable for firm B if and only if

$$\Pi_B^* - \Pi_B^c = \frac{n [t(x_A - x_B)(2 + x_A + x_B) + 3(\alpha n - \beta n^2)]^2}{18 [t(x_B - x_A) - (\alpha n - \beta n^2)]} < 0.$$

In each case, the sign of the expression is determined by the denominator. Incentives for one firm or other to undercut exist whenever $\alpha n - \beta n^2 > t(x_B - x_A)$. Notice, however, that this condition is contradicted by (10), ensuring undercutting is never a profitable strategy at the pricing equilibrium.

A.2 Stage 1 – Locations

Combining the equilibrium prices in (8) and (9) with firms' demands $n_A = \hat{x}n$ and $n_B = (1 - \hat{x})n$ and replacing these in the firms' profit functions $\Pi_i = n_i p_i$, $i = A, B$, yields an expression that depends purely on the firms' locations. It can be easily checked that maximising these and solving for x_A and x_B yields five critical points. Among these candidate equilibria, only $(x_A, x_B) = (-\frac{1}{4}, \frac{5}{4})$ satisfies (10) – the condition for positive prices. At these locations, the firms charge identical prices

$$p_A^* = p_B^* = \frac{3}{2}t - (\alpha n - \beta n^2)$$

and (10) becomes equivalent to weak conformity, namely

$$\alpha n - \beta n^2 < \frac{3}{2}t. \quad (11)$$

Substituting the equilibrium locations and prices into (7), we see that $\hat{x} = 1/2$, so that firms share demand equally in this equilibrium, $n_i = n/2$, $i = A, B$. Equilibrium profits are therefore equal to

$$\Pi_A^* = \Pi_B^* = \frac{n}{2} \left[\frac{3}{2}t - (\alpha n - \beta n^2) \right]. \quad (12)$$

It is straightforward to see that, by (11), these profits are positive.

Given $\alpha, \beta, t, n > 0$, we proceed by checking for this candidate solution that (i) the second-order condition is satisfied, and (ii) there are no incentives to displace the rival by

shifting location in the first stage.

(i) *Second-order Condition*

The second-order condition (SOC) for firm A can be written as

$$\begin{aligned} \frac{\partial^2 \Pi_A}{\partial x_A^2} = & \frac{nt}{9[n(\alpha - \beta n) + t(x_A - x_B)]^3} \times \left\{ -6(\alpha^3 n^3 - \beta^3 n^6) \right. \\ & - t^2(x_A - x_B)^2 [4\beta n^2(x_A + 2x_B - 6) + t(x_A - x_B)(x_A + 3x_B - 8)] \\ & + 2n^2 t(\alpha^2 + \beta^2 n^2 - \beta) \left[x_A(x_A - 10) - 3x_B(x_B - 4) - \frac{1}{2} \right] \\ & \left. + 18\alpha^2 \beta n^4 + 2\alpha n [2t^2(x_A - x_B)^2(x_A + 2x_B - 6) - 9\beta^2 n^4] \right\}. \end{aligned}$$

The second-order condition for B follows in similar form. Both SOC's, evaluated at the candidate equilibrium $(x_A, x_B) = (-\frac{1}{4}, \frac{5}{4})$, are equal to

$$\frac{\partial^2 \Pi_i}{\partial x_i^2} = \frac{nt}{6} \left[\frac{3t}{3t - 2(\alpha n - \beta n^2)} - 4 \right], \quad i = A, B.$$

This is negative when either

$$\alpha n - \beta n^2 < \frac{9}{8}t \quad \text{or} \quad \alpha n - \beta n^2 > \frac{3}{2}t. \quad (13)$$

The second condition can be ignored, as it is contradicted by (11).

(ii) *Displacement Incentives*

We consider Nash deviations in the firms' locations at stage 1 that would allow one firm or other to capture the entire market. Due to symmetry, we can focus, without loss in generality, on the incentives of firm A to relocate at stage 1 in order to capture the entire market.

Firm A will monopolize the market whenever $x_A + x_B > 1$ and

$$\alpha n - \beta n^2 \geq \frac{t}{3}(x_B - x_A)(4 - x_A - x_B), \quad (14)$$

implying that (10) fails and we are in a corner solution in prices.¹⁴ Note that, since $x_A < x_B = \frac{5}{4}$, a necessary condition for there to be any possibility of undercutting is $\alpha n - \beta n^2 > 0$.¹⁵

This leads to the following limit price strategy (see also Grilo et al., 2001):

$$\begin{aligned} p_A^L &= \alpha n - \beta n^2 - t(x_B - x_A)(2 - x_A - x_B), \\ p_B^L &= 0. \end{aligned} \quad (15)$$

To check for profitable displacement opportunities, we proceed as follows. First, we identify the displacement locations for firm A , such that firm B is just driven out of the market. Then, we explore whether there are even more profitable displacement locations for firm A .

The point at which firm B is just driven out of the market corresponds to (14) holding with *equality*. Given $x_B = 5/4$, $\alpha n - \beta n^2 > 0$ guarantees that the quadratic implied by (14) has two real roots, given by

$$x'_A = 2 - \frac{1}{4t} [9t^2 + 48t(\alpha n - \beta n^2)]^{\frac{1}{2}} \quad \text{and} \quad x''_A = 2 + \frac{1}{4t} [9t^2 + 48t(\alpha n - \beta n^2)]^{\frac{1}{2}}, \quad (16)$$

¹⁴We assume, for the moment, that weak conformity also holds at the displacement locations – we return to this point below. The general condition for firm A to monopolize the market may be stated as follows: $U_A(x) \geq U_B(x)$ for all $x \in [0, 1]$ given that $n_A = n$ and $n_B = 0$ (similarly for firm B). As shown in Grilo et al. (2001), the resulting conditions are mutually exclusive with (10) – i.e. in order to capture the entire market, a firm will change location to ensure that (10) fails. Note that, even though corner solutions at the pricing stage are ruled out as candidate subgame perfect equilibria (see footnote 13 above), they still have to be considered as potentially profitable Nash deviations when firms' locations are endogenous.

¹⁵Firm A cannot capture the market by locating further outside the market than B . In this case, the labels of the firms are effectively switched and it is still the firm with the locational advantage that will monopolize the market, if any.

and where $x'_A < \frac{5}{4}$ and $x''_A > \frac{5}{4}$. Therefore, we can rule out x''_A as a feasible displacement location.¹⁶ We know that firm A cannot locate further left than x'_A , as this would violate (14). But, since $x'_A < 5/4$, it is possible that firm A could locate further right than x'_A (that is, closer to firm B), continue charging the limit price given in (15) and make higher profits. It will do so whenever the limit price p_A^L is increasing in x_A , starting from $x_A = x'_A$. From (15),

$$\frac{\partial p_A^L}{\partial x_A} = 2t(1 - x_A).$$

It follows that (for any value of x_B) p_A^L is increasing in x_A if and only if $x_A < 1$, and p_A^L reaches a maximum at $x_A = 1$. Hence, if $x'_A < 1$, firm A will indeed choose to locate further rightwards, at $x_A = 1$. It will never move beyond $x_A = 1$ when $x'_A < 1$, as the limit price is decreasing in x_A beyond that point.

We therefore have two cases to consider. When $1 < x'_A < \frac{5}{4}$, we need to calculate the displacement profits on the basis of (15), $n_A = n$ and $x_A = x'_A$. When $x'_A \leq 1$, we need to calculate the displacement profits on the basis of (15), $n_A = n$ and $x_A = 1$. In each case, we compare the displacement profits with the equilibrium profits of firm A , to determine when displacement is profitable. From (16), $x'_A \leq 1$ if and only if

$$\alpha n - \beta n^2 \geq \frac{7}{48}t.$$

Case 1: $\frac{7}{48}t \leq \alpha n - \beta n^2 \Leftrightarrow x'_A \leq 1$

In this case, firm A 's optimal displacement location is $x_A = 1$. Here the limit price will be (see (15))

$$p_A^L = \alpha n - \beta n^2 + \frac{t}{16},$$

¹⁶See previous footnote.

implying displacement profits of

$$\Pi_A^{dis} = n \left(\alpha n - \beta n^2 + \frac{t}{16} \right).$$

Given the equilibrium profits in (12), the necessary and sufficient condition for there to be no incentives for displacement, that is $\Pi_A^* > \Pi_A^{dis}$, can be stated as

$$\alpha n - \beta n^2 < \frac{11}{24}t. \quad (17)$$

Case 2: $0 < \alpha n - \beta n^2 < \frac{7}{48}t \Leftrightarrow 1 < x'_A < \frac{5}{4}$

In this case, firm A 's optimal displacement location is x'_A . At this location, firm A charges limit price

$$p_A^L = \frac{1}{2} \left\{ [9t^2 + 48t(\alpha n - \beta n^2)]^{1/2} - 4(\alpha n - \beta n^2) - 3t \right\}$$

and earns displacement profits

$$\Pi_A^{dis} = \frac{n}{2} \left\{ [9t^2 + 48t(\alpha n - \beta n^2)]^{1/2} - 4(\alpha n - \beta n^2) - 3t \right\}.$$

Comparing these displacement profits with the equilibrium profits in (12), we see that incentives for firm A to displace its rival exist whenever $\Pi_A^{dis} > \Pi_A^*$, that is when

$$\alpha n - \beta n^2 > \frac{5}{6}t.$$

This is clearly contradicted by the premise of this case, implying that there can be no displacement in this case.

It follows from cases 1 and 2 that there will be no displacement if and only if (17) holds. Stated differently, incentives for displacement at the location stage will rule out

the existence of our subgame perfect equilibrium whenever network effects are sufficiently large and positive. This also implies that displacement incentives at the location stage can never rule out the existence of equilibrium in the absence of network effects ($\alpha = \beta = 0$).

Combining (11), (13) and (17), it is clear that a subgame perfect equilibrium in pure strategies exists if and only if (17) is satisfied.

Note that the above analysis implicitly assumed that weak conformity also holds at the displacement locations. It is straightforward to show that weak conformity holds when firm A deviates to location x'_A . In Case 1 above, moving from x'_A towards $x_A = 1$ may cause weak conformity to fail (this can happen when $\alpha n - \beta n^2$ is sufficiently high). In this case, we appeal to the maximal limit price that firm A can charge under strong conformity, which is identical to that given in (15) (Grilo et al., 2001, Prop. 4). This is still maximised at $x_A = 1$, so that displacement incentives in this scenario are still accurately reflected in Case 1.

Graphically, the different scenarios that can arise are illustrated in Figures 2a-2c below. These illustrate the quadratic on the right-hand side of (14) (which determines when firm A monopolizes the market) and the $t(x_B - x_A)$ line (which determines whether we have weak or strong conformity). In each case, firm B 's location at $x_B = \frac{5}{4}$ is taken as given. The scenarios differ in terms of the value of the network effect function evaluated at n : we consider three values, E_1 , E_2 and E_3 , where $E_1 < E_2 < E_3$. In each case, the circle identifies the displacement location x'_A , given by the intersection between the level of network effects and the parabola, the triangle identifies the final displacement location, should this differ from x'_A , and the square identifies the level of the $t(\frac{5}{4} - x_A)$ line associated with the final displacement location.

In Figure 2a, $x'_A > 1$. Therefore, we are in Case 2 and firm A 's optimal displacement location is x'_A (moving leftwards from this location, towards $x_A = 1$ where the limit price is maximised, violates (14) – the parabola lies above E_1 for all values of x_A to the left of x'_A). This displacement location necessarily implies weak conformity, since the square lies

above E_1 .

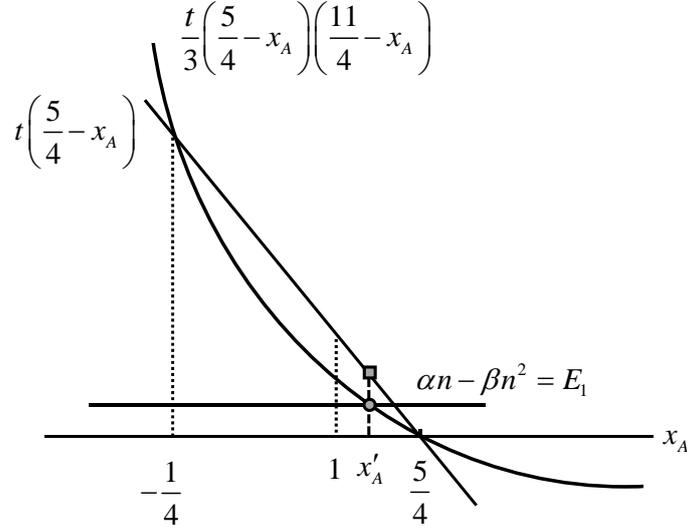


Figure 2a: Optimal Displacement Locations with $\alpha n - \beta n^2 = E_1$.

In Figures 2b and 2c, x'_A lies to the left of $x_A = 1$. Therefore, firm A can increase the limit price it charges while still monopolising the market by moving to $x_A = 1$, indicated, in each case, by the triangle. In other words, we are in Case 1. (This triangle lies below both E_2 and E_3 , so that (14) is satisfied.) The associated level of the $t(\frac{5}{4} - x_A)$ line, identified by the square, lies above E_2 but below E_3 . This implies that, when $\alpha n - \beta n^2 = E_2$, weak conformity holds, even at $x_A = 1$. When $\alpha n - \beta n^2 = E_3$, however, the move to $x_A = 1$ causes weak conformity to fail. In this sub-case, we appeal to the maximal limit price that firm A can charge under strong conformity, which is identical to that given in (15) and is, therefore, still maximised at $x_A = 1$.

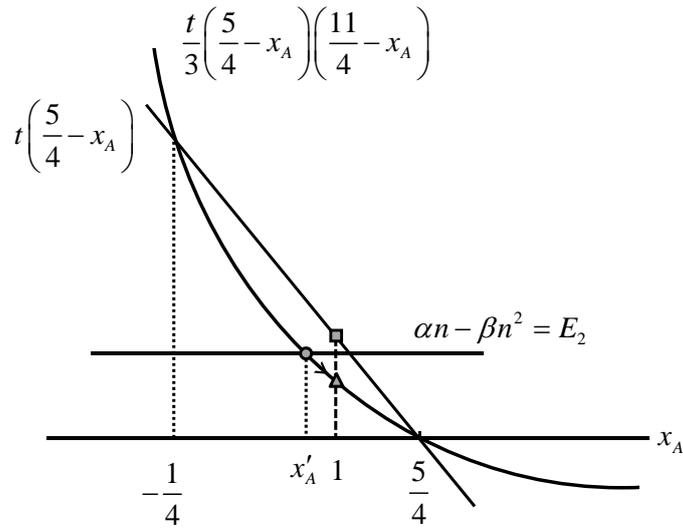


Figure 2b: Optimal Displacement Location with $\alpha n - \beta n^2 = E_2$.

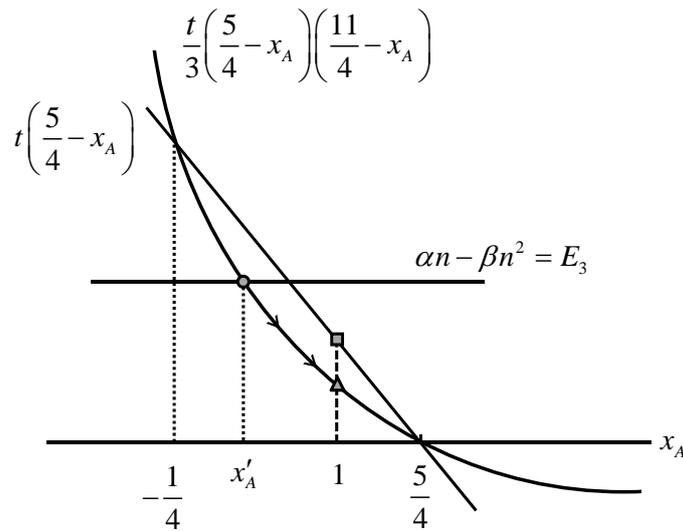


Figure 2c: Optimal Displacement Locations with $\alpha n - \beta n^2 = E_3$.

This completes the proof.

B Proof of Proposition 2

Consumer surplus is equal to

$$CS = n \int_0^{\hat{x}} K - p_A - t(x - x_A)^2 + \alpha n_A - \beta n_A^2 dx \\ + n \int_{\hat{x}}^1 K - p_B - t(x - x_B)^2 + \alpha n_B - \beta n_B^2 dx.$$

This, evaluated at the equilibrium values given in Proposition 1, yields

$$CS = n \left[K - \frac{85}{48}t + \frac{1}{4}(6\alpha n - 5\beta n^2) \right].$$

Based on the equilibrium described in Proposition 1, the firms' equilibrium profits are equal to

$$\Pi_A^* = \Pi_B^* = \frac{n}{2} \left[\frac{3}{2}t - (\alpha n - \beta n^2) \right].$$

Total welfare is then simply equal to $TW = CS + 2\Pi_i$, which is to say

$$TW = n \left[K - \frac{13}{48}t + \frac{1}{4}(2\alpha n - \beta n^2) \right].$$

Since the firms' equilibrium locations are unaffected by network effects, the result then follows immediately.

C Proof of Proposition 3

Following the approach in Serfes and Zacharias (2012), the social planner solves the following optimisation problem:

$$\max_{x_A, x_B, \hat{x}} TW = \max_{x_A, x_B, \hat{x}} \left\{ \begin{array}{l} n \int_0^{\hat{x}} K - p_A - t(x - x_A)^2 + \alpha n_A - \beta n_A^2 dx \\ + n \int_{\hat{x}}^1 K - p_B - t(x - x_B)^2 + \alpha n_B - \beta n_B^2 dx \\ + n_A p_A + n_B p_B \end{array} \right\}.$$

Since $n_A = \hat{x}n$ and $n_B = (1 - \hat{x})n$, this can be simplified to yield

$$\max_{x_A, x_B, \hat{x}} \left\{ \begin{array}{l} n \int_0^{\hat{x}} K - t(x - x_A)^2 + \alpha \hat{x}n - \beta(\hat{x}n)^2 dx \\ + n \int_{\hat{x}}^1 K - t(x - x_B)^2 + \alpha(1 - \hat{x})n - \beta[(1 - \hat{x})n]^2 dx \end{array} \right\}.$$

The unique interior solution to this problem is given by

$$(x_A, x_B, \hat{x}) = \left(\frac{1}{4}, \frac{3}{4}, \frac{1}{2} \right).$$

This solution implies total welfare equal to

$$TW = n \left[K - \frac{t}{48} + \frac{1}{4}(2\alpha n - \beta n^2) \right]. \quad (18)$$

Among the corner solutions in which only one firm serves the market, it is clear that, in order to maximise welfare (minimise transportation costs), this single firm should be located at the mid-point of the market. Without loss in generality, we can represent this corner solution by $x_A = \frac{1}{2}$ and $\hat{x} = 1$. Total welfare in this corner solution is equal to

$$TW = n \left[K - \frac{t}{12} + (\alpha n - \beta n^2) \right]. \quad (19)$$

Comparing (18) and (19), it is straightforward to show that the interior solution dom-

inates the corner solution whenever either

- i. $0 < n \leq \frac{2\alpha}{3\beta}$ and $t > 8\alpha n - 12\beta n^2$, or
- ii. $n > \frac{2\alpha}{3\beta}$,

which completes the proof.

D Supplementary material – Corrected Proposition 2 in Serfes and Zacharias (2012)

Equation (11) in Serfes and Zacharias (2012, p.997) reads as follows:¹⁷

$$\begin{aligned} W &= \int_0^x \alpha z - t(a - z)^2 dz + \int_x^1 \alpha(1 - z) - t(b - z)^2 dz \\ &= \alpha x^2 + t a x^2 - t a^2 x + \frac{\alpha}{2} - \alpha x - \frac{t}{3} + t b - t b x^2 - t b^2 + t b^2, \end{aligned} \quad (20)$$

where x is the position of the indifferent consumer (equivalently, the fraction of the total population that purchases from firm A), $t > 0$ is the per-unit cost of travel, a and b are the respective locations of firms A and B , and α is the intensity of the network effect, given a network effect function of the form $E(n) = \alpha n$.

This equation contains an important conceptual error. Each consumer patronising firm A , say, enjoys a network effect corresponding to that firm's total demand, not the fraction of consumers located between themselves and the endpoint of the line. Equation (20) should therefore be specified (with emphasis added to highlight the difference to the

¹⁷In Serfes and Zacharias (2012), the total population size is fixed at unity. In order to relate this analysis more closely to the original paper, we use the notation of Serfes and Zacharias (2012) here. Note also the typographical error in the second line of this equation – the last term should be $t b^2 x$ rather than $t b^2$.

preceding equation) as

$$\begin{aligned} W' &= \int_0^x \alpha \mathbf{x} - t(a-z)^2 dz + \int_x^1 \alpha(1-\mathbf{x}) - t(b-z)^2 dz \\ &= \alpha - 2\alpha x + 2\alpha x^2 + atx^2 - a^2tx + bt - b^2t + b^2tx - btx^2 - \frac{t}{3}. \end{aligned}$$

Maximizing W' with respect to a , b and x leads to only one interior solution, which coincides with Serfes and Zacharias, namely $(a, b, x) = (\frac{1}{4}, \frac{3}{4}, \frac{1}{2})$. This interior solution is, however, associated with total welfare equal to

$$W' = \frac{\alpha}{2} - \frac{t}{48}, \quad (21)$$

which differs from the welfare level provided in Serfes and Zacharias.

At the corner solution $a = \frac{1}{2}$, $x = 1$, total welfare is equal to

$$W' = \alpha - \frac{t}{12}, \quad (22)$$

which again departs from the result provided in Serfes and Zacharias.

By inspection of (21) and (22), we can conclude that welfare in the interior solution exceeds that under the single-firm outcome if and only if $\alpha \leq \frac{t}{8}$. Thus the condition for the social planner to prefer the interior solution over the single-firm solution in Proposition 2 of Serfes and Zacharias (2012) should be amended from $\alpha \leq \frac{t}{4}$ to $\alpha \leq \frac{t}{8}$. For α between $\frac{t}{8}$ and $\frac{t}{4}$ (as well as for all $\alpha > \frac{t}{4}$), monopoly welfare-dominates a duopolistic market structure in the Serfes and Zacharias (2012) framework.

This result is also confirmed by our Proposition 3 in the case where $\beta \rightarrow 0$ and $n = 1$.

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