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Boundary layer flow beneath an internal solitary wave of elevation

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The wave-induced flow over a fixed bottom boundary beneath an internal solitary wave of elevation propagating in an unsheared, two-layer, stably stratified fluid is investigated experimentally. Measurements of the velocity field close to the bottom boundary are presented to illustrate that in the lower layer the fluid velocity near the bottom reverses direction as the wave decelerates while higher in the water column the fluid velocity is in the same direction as the wave propagation. The observation is similar in nature to that for wave-induced flow beneath a surface solitary wave. Contrary to theoretical predictions for internal solitary waves, no evidence for either boundary layer separation or vortex formation is found beneath the front half of the wave in the adverse pressure gradient region of the flow. © 2010 American Institute of Physics. [doi:10.1063/1.3327289]

I. INTRODUCTION

Internal solitary waves (ISWs)—nonlinear, finite-amplitude waves of permanent form that propagate along density interfaces in stably stratified fluids—are ubiquitous features in the Earth's atmosphere and oceans (for example, see the recent reviews by Ostrovsky and Stepanyants¹ and Helfrich and Melville²). In the ocean, ISWs propagate as waves of depression/elevation when the pycnocline is located nearer to the surface/bed of the ocean than the bed/surface. ISWs of these types can induce significant currents close to the sea bed and are thought to enhance rates of suspension of sedimentary material from the sea floor and across-shore transport of plankton and contaminants.^{3–12} These effects have biological implications for water quality and ocean engineering consequences for long-range acoustic propagation and under water communications with platform semisubmersible vehicles.

The primary interest in this paper lies in the behavior of ISWs of elevation. In this regard, theoretical investigations^{12–16} of the interaction between an ISW of elevation and a flat bottom boundary have shown that the boundary layer can separate in the adverse pressure gradient region in the front part of the wave and vortices may be formed beneath the center of the ISW. Bogucki and Redekopp¹³ and Wang and Redekopp¹⁴ employed as forcing a two-dimensional direct numerical simulation with a weakly nonlinear ISW of elevation, propagating against a linearly sheared flow. They found that boundary layer separation and global instability can occur when the wave amplitude exceeds a threshold level and their simulations predict the generation of coherent vortex structures rising into the water and being advected with the flow. The vortex formation (and shedding) was ascribed by these authors to a manifestation of global instability as seen similarly in the numerical simulations of sepa-

rated channel flow by Pauley *et al.*,¹⁷ Ripley and Pauley,¹⁸ and Alam and Sandham.¹⁹

Stastna and Lamb¹⁵ considered a fully nonlinear ISW of elevation propagating against both an unsheared and a sheared background flow. In the case of an unsheared background flow, the boundary layer remained attached and stable and no vortices were seen. In the sheared case, they found the formation of vortex structures if the opposing background current was of sufficient strength. They concluded that an upstream source of vorticity was vital for vortex shedding to occur and the presence of a separation bubble was not necessarily required. Diamessis and Redekopp¹⁶ recently extended the earlier work of Bogucki and Redekopp¹³ and Wang and Redekopp¹⁴ to ISWs of depression and ISWs of elevation propagating in an unsheared flow. They found separation of the boundary layer in the adverse pressure gradient region and the formation of vortices in both cases. Diamessis and Redekopp¹⁶ suggest that Stastna and Lamb¹⁵ failed to see vortex formation in their work, in the unsheared case, because of the smoothing effect of the high artificial dissipation inherent in their low-order finite difference scheme.

Stastna and Lamb²⁰ have recently added to the debate by extending their work. They found that no instability occurred in the boundary layer, for a wave of fixed amplitude, when the wave-induced Reynolds number of the flow was too low or when the opposing background current was too weak. In particular, they found that an opposing background current was necessary for instability to be seen and in an unsheared flow the boundary layer always remained attached. Stastna and Lamb²⁰ propose that the use of weakly nonlinear waves by Diamessis and Redekopp¹⁶ as an initial state for wave amplitudes beyond those for which weakly nonlinear theory holds is the reason for the discrepancy with their findings. They claim that since the weakly nonlinear theory overestimates the wave propagation speed and underestimates the wave half width, narrower and stronger regions of convergence are generated in the bottom boundary layer leading to overpredictions of instability there.

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In an effort to contribute to the debate, this paper presents experimental findings of the wave-induced velocity field beneath large amplitude ISWs of elevation propagating in an unshered flow. A parameter regime close to that investigated by Diamessis and Redekopp¹⁶ is considered. In all cases it is found that the boundary layer remains attached in the adverse pressure gradient region in the front half of the wave and no vortices are formed. A fuller discussion of the result can be found in Sec. III A.

While the boundary layer dynamics under the front half of the wave did not show anything of great interest, a reverse flow under the rear portion of the wave was observed close to the bed as the wave-induced flow was decelerating. The pressure gradient in this region is favorable so the flow reversal is not expected to be a result of boundary layer separation. It is postulated that the reversal behavior is equivalent to that seen by Khabakhpashev,²¹ Liu *et al.*,²² Lin and Zhang,²³ and Vittori and Blondeaux²⁴ under surface solitary waves. In particular, Liu *et al.*²² presented a theoretical and experimental investigation of the boundary layer flow beneath a surface solitary wave. They show that the horizontal velocity inside the boundary layer is not always in phase with the free stream velocity and, as a result, the horizontal fluid velocity inside the boundary layer reverses direction as the wave decelerates. The flow reversal seen in the experimental data presented here is in very good qualitative agreement with the cases described by Liu *et al.*²² (see Sec. III B for more details).

Carr and Davies²⁵ and Carr *et al.*²⁶ presented experimental evidence of a similar flow reversal at the lower boundary under an ISW of depression. The reverse flow also occurred in the decelerating part of the wave-induced flow, which, for an ISW of depression, is in an adverse pressure gradient region. For this reason, they conjectured that the flow reversal was a result of boundary layer separation due to the adverse pressure gradient as predicted theoretically by Diamessis and Redekopp.¹⁶ Although it occurs in a favorable pressure gradient region, the reverse flow under an ISW of elevation has similar features to those seen under an ISW of depression. Hence, it is possible that the reverse flow under an ISW of depression may, in fact, be the result of a phase lag between the irrotational and rotational velocity components (as in the elevation case) and not as a consequence of the existence of the adverse pressure gradient region. This conjecture is discussed in more detail in Sec. IV.

II. THE EXPERIMENTAL STUDY

A. Model configuration and experimental arrangement

Figure 1 shows a sketch of the experimental arrangement. Within a Cartesian coordinate system (x, y, z) , the x and z directions denote, respectively, the (horizontal) direction of propagation of the wave and the direction antiparallel to the gravitational acceleration vector $\mathbf{g} = (0, 0, -g)$. The origin is chosen such that $z=0$ coincides with the lower solid boundary of the water column. The background stratification consists of upper and lower layers of miscible homogeneous fluid of density ρ_1 and ρ_3 , respectively, and undisturbed thickness h_1 and h_3 , respectively. The pycnocline has an un-

FIG. 1. Schematic diagram of experimental arrangement.

disturbed thickness of h_2 and the density, $\rho(z)$, varies as a linear function of z . An ISW of elevation of amplitude a is generated on the pycnocline and it travels along the interface with celerity c . The flow is two dimensional, with no variation in the cross-flow (y) direction.

The experiments were performed in a wave tank of dimension $6.4 \times 0.4 \times 0.6$ m³ (length, width, and depth, respectively) described in earlier studies.^{25,26} The lower layer was filled first with a prepared solution of brine of prescribed density ρ_3 (typically 1048 kg/m³). The top two layers were then carefully added via a floating sponge arrangement by directly filling with fresh water of density ρ_1 (typically 997 kg/m³). In instances where a very sharp pycnocline was required the experiment was run as quickly as possible after filling. In cases where a relatively thick pycnocline was required, the stratification was either (i) left to diffuse for a period of between 12 and 16 h or (ii) the stratification would be immediately reused after an experiment with a sharp pycnocline had been run (labeled top in Tables I and II). In some instances a third run would be made (labeled topII in Table II).

The profile of the stratification was measured via an array of high precision microconductivity probes.²⁷ The density difference between layers was kept approximately constant ($\approx 5.1\%$) during the study, as was the total fluid depth $H \approx 0.38$ m. Parametric changes to the external conditions were introduced by changing a, h_1, h_2 and h_3 . Due to practical considerations, the densities ρ_1 and ρ_3 varied slightly ($< 0.001\%$) from their prescribed values from run to run but their values were measured before each run was initiated.

The waves were generated using the so-called step pool technique.²⁸ After the main section of the tank had been stratified, a gate G was inserted at the upstream end of the tank at $x=0$. The fluid behind the gate was then carefully mixed and salt was added until the fluid behind the gate had the same density as that in the lower section of the main part of the tank, ρ_3 . The volume V of brine with density ρ_3 was varied by adding solid blocks of polystyrene behind the gate prior to filling the tank. The blocks had the same width and depth as the tank and their thickness was varied to change the free space, V , left between the blocks and the gate. The blocks were fixed in position prior to filling. Once the fluid behind the gate was mixed to the required density, the gate was lifted and removed quickly, thereby generating a wave of elevation that propagated along the density interface into the main section of the tank. The results of previous work^{25,26} on ISWs of depression were used to estimate the gate position, the trapped volume V and the constituent layers to achieve large amplitude ISWs of elevation. A range of

TABLE I. Experimental parameter range for nonbreaking single waves of elevation [case (i)]. h_1, h_2, h_3 are the depths of the bottom, middle, and top layer of fluid, respectively, a is the measured wave amplitude, c is the measured wave speed, and V is the generating volume.

Date	h_3 (m)	h_2 (m)	h_1 (m)	a (m)	c (m/s)	V (l)
260509	0.025	0.058	0.298	0.123 ± 0.002	0.187 ± 0.017	60.8
060808top	0.035	0.085	0.260	0.120 ± 0.002	0.203 ± 0.018	68.4
210509top	0.030	0.080	0.270	0.116 ± 0.002	0.197 ± 0.018	60.8
180609	0.030	0.060	0.290	0.115 ± 0.002	0.190 ± 0.017	57.0
220609	0.025	0.060	0.295	0.115 ± 0.002	0.200 ± 0.018	53.2
040808top	0.028	0.072	0.279	0.114 ± 0.002	0.203 ± 0.018	45.6
170609	0.025	0.060	0.295	0.111 ± 0.002	0.205 ± 0.018	45.6
280109top	0.026	0.064	0.290	0.110 ± 0.002	0.197 ± 0.018	38.0
260509top	0.035	0.080	0.265	0.108 ± 0.002	0.198 ± 0.018	60.8
050808	0.030	0.110	0.239	0.101 ± 0.002	0.200 ± 0.018	45.6
220109top	0.025	0.060	0.295	0.101 ± 0.002	0.185 ± 0.017	34.2

V and background stratification were chosen such that an array of outputs was given. In particular, three wave types were generated: (i) large amplitude nonbreaking ISWs, (ii) a train of two internal waves of elevation, the leading wave having much larger amplitude than the following wave, and (iii) large amplitude ISWs which exhibited a shear instability in the pycnocline. The width to depth ratio of V was arranged such that nearly all of the volume trapped behind the gate went into the volume of the solitary wave in cases (i) and (iii) or the leading wave in case (ii). In such cases, generation is very fast and the leading front of the wave almost instantly takes the form of a classical solitary wave of very large amplitude. A gate position of 0.60 m from the upstream end of the tank was fixed. A rigid lid which covered the majority of the length of the tank was used at the upper boundary. A total of 42 experiments were conducted, from which 34 have been

selected for presentation and discussion. Tables I–III provide an overview of the parameter ranges of the waves generated in cases (i), (ii), and (iii), respectively.

B. Measurement technique

Particle image velocimetry (PIV) was used to visualize and quantify the synoptic velocity field (u, w) in a given two-dimensional (x, z) slice of the flow. To implement PIV, a vertical section in the midplane $y=0$ of the tank was illuminated by a continuous, collimated light sheet from an array of light boxes placed below the (transparent) base of the tank. The light sheet had a thickness of approximately 10 mm and it illuminated two sections of the tank, one upstream centered about $x/H=7.77$ and one downstream centered about $x/H=10.31$. Both illuminated sections were 1.4 m

TABLE II. Experimental parameter range for two consecutive nonbreaking internal waves of elevation [case (ii)]. h_1, h_2, h_3 are the depths of the bottom, middle, and top layer of fluid, respectively, a is the measured wave amplitude of the leading wave, c is the measured wave speed of the leading wave, and V is the generating volume.

Date	h_3 (m)	h_2 (m)	h_1 (m)	a (m)	c (m/s)	V (l)
070808	0.050	0.100	0.230	0.113 ± 0.002	0.184 ± 0.017	68.4
220609	0.025	0.060	0.295	0.115 ± 0.002	0.200 ± 0.018	53.2
180609top	0.040	0.075	0.265	0.106 ± 0.002	0.196 ± 0.018	57.0
220609top	0.040	0.070	0.270	0.105 ± 0.002	0.201 ± 0.018	53.2
170609top	0.035	0.075	0.270	0.103 ± 0.002	0.200 ± 0.018	45.6
070808top	0.060	0.110	0.210	0.097 ± 0.002	0.196 ± 0.018	68.4
180609topII	0.050	0.100	0.230	0.095 ± 0.002	0.201 ± 0.018	57.0
280808top	0.060	0.120	0.200	0.094 ± 0.002	0.168 ± 0.015	79.8
120509top	0.090	0.070	0.220	0.087 ± 0.001	0.129 ± 0.012	91.2
130509	0.085	0.035	0.260	0.084 ± 0.001	0.179 ± 0.016	60.8
130509top	0.095	0.055	0.230	0.078 ± 0.001	0.181 ± 0.016	60.8
070509	0.070	0.035	0.274	0.076 ± 0.001	0.162 ± 0.015	38.0
070509top	0.065	0.060	0.254	0.070 ± 0.001	0.174 ± 0.016	38.0

TABLE III. Experimental parameter range for unstable single waves of elevation [case (iii)]. h_1, h_2, h_3 are the depths of the bottom, middle, and top layer of fluid, respectively, a is the measured wave amplitude, c is the measured wave speed, and V is the generating volume.

Date	h_3 (m)	h_2 (m)	h_1 (m)	a (m)	c (m/s)	V (l)
010708	0.030	0.035	0.314	0.140 ± 0.002	0.198 ± 0.018	91.2
260608	0.030	0.035	0.314	0.138 ± 0.002	0.191 ± 0.017	91.2
270808	0.027	0.047	0.306	0.135 ± 0.002	0.214 ± 0.017	79.8
060808	0.028	0.032	0.320	0.134 ± 0.002	0.203 ± 0.018	68.4
240608	0.030	0.035	0.315	0.133 ± 0.002	0.233 ± 0.021	91.2
280109	0.025	0.035	0.320	0.115 ± 0.002	0.202 ± 0.018	38.0
220109	0.025	0.033	0.322	0.111 ± 0.002	0.199 ± 0.018	34.2
120509	0.076	0.034	0.270	0.103 ± 0.002	0.246 ± 0.022	91.2

long and 0.6 m deep. Motions within this vertical light sheet were viewed and recorded from the side using three fixed digital video cameras set up outside the tank, each had a spatial resolution of 1372×1372 pixels. The lower layer of the illuminated section was seeded with neutrally buoyant, light-reflecting tracer particles of “Pliolite” having diameters in the range 150–300 μm . Two of the digital cameras (one positioned upstream and one downstream) recorded the dynamics close to the bottom boundary. They were positioned level with the bed of the tank to avoid distortion and perspective errors in this portion of the flow field. Both cameras captured an area of the flow field from the base of the tank to a depth of $z/H \approx 0.33$ in the vertical extent and a horizontal length $x/H \approx 0.91$. Highly resolved velocity data close to the bottom boundary were obtained. A third camera was located downstream and used to capture a much larger section of the flow field. The camera was positioned level with the pycnocline to avoid distortion and perspective errors in this portion of the flow field. An area was recorded extending from the base of the tank to a depth of $z/H \approx 0.95$ and over a horizontal extent of length $x/H \approx 2.50$. Measurements of the wave speed c and wave amplitude a were obtained from this field of view. In all cases, the recording systems were stationary with respect to the tank and the ISW traveled through the illuminated measurement window. The horizontal location of the downstream measurement window was carefully chosen such that (i) the wave was fully formed by the time it was observed and (ii) the wave-induced boundary layer dynamics could be observed for as long as possible before the reflected wave from the end of the tank came back into view and interfered with the dynamics of interest.

The resulting video records of the flow field were processed using the software package DIGIFLOW to generate continuous synoptic velocity field data close to the bottom boundary. The development of the interface was monitored using the time series function of DIGIFLOW, by tracking the changes with time of the pixel values in a given column of digitized images extracted from the camera record. The images were analyzed using DIGIFLOW and estimates of the amplitude and the time at which the interface reached maximum displacement were then derived. The amplitude was taken to be the maximum displacement of the upper isopycnal of the pycnocline. This process was repeated at four fixed

locations $x_{1,2,3,4}$ over a known horizontal distance $\Delta x \approx h_1$, enabling an average amplitude a and an estimate of the celerity $c = \Delta x / \Delta t$ of the wave to be made by noting the average time Δt between maximal interface displacement at the four fixed locations $x_{1,2,3,4}$. The variance in Δt over $x_{1,2,3,4}$ produced an error of approximately 9% in measuring c . In general, decay in c was seen between the four measurement locations as the wave propagated along the tank. The error in measuring a was just 1.7%.

III. EXPERIMENTAL RESULTS

A. Boundary layer flow under the front half of the wave

As already discussed in Sec. I, there is debate in the current literature as to whether an ISW of elevation propagating in an unsheared flow can induce boundary layer separation and vortex formation under the front half of the wave in the adverse pressure gradient region where the pressure changes from low (directly under the wave center) to high (upstream of the wave). The flow in the lower layer is in the same direction as wave propagation, so that the induced pressure gradient in the direction of the flow is adverse (see Stastna and Lamb^{15,20} and Diamessis and Redekopp¹⁶). Diamessis and Redekopp¹⁶ predict that boundary layer separation and vortex formation can occur under an ISW of elevation propagating in an unsheared flow in the adverse pressure gradient region. In addition, they show that ISWs of depression can also induce boundary layer separation and vortex formation. In the depression case, the wave-induced flow in the lower layer is in the opposite direction to wave propagation and the pressure gradient is adverse under the rear half of the wave (where the pressure in the direction of the flow changes from low, directly under the center of the wave, to high, downstream of the wave). They found that, for waves of depression, the critical amplitude of the wave (nondimensionalized by the total fluid depth) for instability to occur in the boundary layer, is related to the wave Reynolds number of the flow $\text{Re}_w = c_0 H / \nu$, where c_0 is the linear long wave speed and ν is the kinematic viscosity of water, by the power law best fit

$$(a/H)_{\text{crit}} = 0.5(\text{Re}_w/10^4)^{-0.12}. \quad (1)$$

Carr *et al.*²⁶ confirmed experimentally Diamessis and Redekopp's¹⁶ prediction for waves of depression but found that the weakly nonlinear approach of Diamessis and Redekopp¹⁶ overpredicted $(a/H)_{\text{crit}}$ when compared to the fully nonlinear waves generated in the laboratory. Carr *et al.*²⁶ suggested a more suitable relationship for fully nonlinear waves of depression as

$$(a/H)_{\text{crit}} = 0.5(\text{Re}_w/10^4)^{-0.12} - 0.15. \quad (2)$$

For a wave of elevation, at $\text{Re}_w = 2 \times 10^4$, Diamessis and Redekopp¹⁶ found the critical amplitude to be 0.42, for boundary layer separation and vortex formation to occur under the front half of the wave where the pressure gradient is adverse and the flow is accelerating (in the fixed frame). This is 0.04 units less than that predicted for the depression counterpart by Eq. (1). The gradient of the vertical eigenfunction is stronger in the elevation case than the depression counterpart as the pycnocline is located nearer the bed. This property is thought to be responsible for the reduced value. Diamessis and Redekopp¹⁶ only present one simulation for a wave of elevation but it can be inferred from Eqs. (1) and (2) that a suitable relationship for fully nonlinear IWs of elevation is

$$(a/H)_{\text{crit}} = 0.5(\text{Re}_w/10^4)^{-0.12} - 0.04 - 0.15. \quad (3)$$

Stastna and Lamb¹⁵ present a simulation of an ISW of elevation propagating in an unsheared flow at $\text{Re}_w \approx O(10^7)$ and $a/H = 0.19$ and found that the boundary layer remains attached and no vortices were seen. Equation (3) evaluated at $\text{Re}_w = 10^7$ yields a critical amplitude $(a/H)_{\text{crit}}$ of 0.03, suggesting that the amplitude considered by Stastna and Lamb¹⁵ is supercritical despite the failure to observe any instability in the boundary layer. Note that Eq. (3) is only proposed here for comparative purposes to show the discrepancy in the literature.

In all three cases studied in the present laboratory investigations, the boundary layer remained attached in the front part of the wave and no vortices of any kind were seen. Figure 2 provides a flow regime classification plot of the observations of type (i) waves in terms of the wave Reynolds number and nondimensionalized wave amplitude. For comparative purposes counterpart observations of global instability ($\times$) and no global instability ($\circ$) underneath an ISW of depression from Carr *et al.*²⁶ are plotted alongside the observations of no global instability (vortices) underneath an ISW of elevation of type (i) (*). For a fixed Reynolds number, it can be seen that waves of elevation have been generated at higher amplitudes than the critical amplitude in the depression case yet no instability was seen in the boundary layer. Since waves of elevation are expected to have a lower critical amplitude than their depression counterparts, the amplitudes generated here are expected to be supercritical—yet no instability was seen in the boundary layer.

For completeness and to confirm the findings detailed above, Fig. 3 provides a flow regime classification plot of the observations of type (i), (ii), and (iii) waves plotted alongside those published in Carr *et al.*²⁶ for waves of depression. In

FIG. 2. Observations of global instability ($\times$) and no global instability ($\circ$) underneath an ISW of depression from Carr *et al.* (Ref. 26) compared to observations of no global instability underneath a nonbreaking ISW of elevation (*).

all elevation cases, no instability was seen under the front half of the wave despite the amplitudes observed being significantly higher than the critical amplitudes observed in the depression counterparts. Equation (3) evaluated at $\text{Re}_w = 6.5 \times 10^4$ gives a critical amplitude of $(a/H)_{\text{crit}} = 0.21$. The elevation observations presented in Fig. 3 are at amplitudes higher than this yet no instability was seen. The reason for the discrepancy between these findings and the predictions of Diamessis and Redekopp¹⁶ may be due to the use by Diamessis and Redekopp¹⁶ of weakly nonlinear forcing at wave amplitudes higher than those for which weakly nonlinear theory is strictly valid. They may therefore, be able to generate higher pressure gradients in their simulations than those attainable in the laboratory and, hence, see instability in the boundary layer when it is not seen in the experiments (see the discussion given by Stastna and Lamb²⁰). If this is the case, then it

FIG. 3. Observations of global instability ($\times$) and no global instability ($\circ$) underneath an ISW of depression from Carr *et al.* (Ref. 26) compared to observations of no global instability underneath a nonbreaking ISW of elevation (*), two IWs of elevation ($\diamond$), and an unstable ISW of elevation (+).

FIG. 4. Horizontal velocity u/c vs tc for experiment 210509top, at $x/H=10.0$ and $z/H=0.01$ (+), $z/h=0.04$ (O), and $z/H=0.79$ (*)

is rather surprising that Carr *et al.*²⁶ found qualitative agreement with Diamessis and Redekopp¹⁶ in the depression case. In the elevation case, the adverse pressure gradient region in the front half of the wave is quickly followed by a favorable pressure gradient region in the rear half. In the depression case, however, the adverse pressure gradient region, in the second half of the wave, is allowed to develop unimpeded until the wave-induced horizontal velocity gradient is negligible at which point the boundary layer can reattach. The difference may be that in the depression experiments of Carr *et al.*²⁶ the adverse pressure gradient region was allowed to develop sufficiently so qualitative agreement with Diamessis and Redekopp¹⁶ was found while in the elevation experiments the adverse pressure gradient was suppressed before it was able to develop to a strength comparable to that computed by Diamessis and Redekopp.¹⁶ The predictions of Diamessis and Redekopp¹⁶ for a wave of elevation may therefore, still hold true if the adverse pressure gradient region in the front part of the wave becomes strong enough quickly enough. For the parameter regime considered in the laboratory, however, this was not the case.

B. Boundary layer flow under the rear half of the wave

While no boundary layer separation or vortex formations were seen under the front half of the wave, a reverse flow (i.e., a flow in the opposite direction as the wave propagation) was seen along the bottom boundary aft of the wave crest in the favorable pressure gradient region in the stable solitary wave cases of type (i). To illustrate the flow reversal, Figs. 4 and 5 are presented. Figure 4 is a plot of wave-induced horizontal velocity u , nondimensionalized by the

FIG. 5. Jet depth z/H vs tc for experiment 210509top, at $x/H=10.0$.

measured wave speed c , versus time t multiplied by c for experiment 210509top. The axes are chosen so that comparison can be made with the findings of Liu *et al.*²² The data are presented for a fixed horizontal location, $x/H=10.0$, and the different plots correspond to different vertical elevations above the bed, namely, $z/H=0.01$ (+), $z/h=0.04$ (O), and $z/H=0.79$ (*). The time at which the wave crest is at maximal displacement and hence the induced velocity is at a maximum is chosen to be $tc=0$. For $tc<0$, the wave is accelerating and a strong positive horizontal velocity is induced throughout the water column. In the decelerating phase, $tc>0$, the horizontal velocity decays toward zero throughout the column but for the observation nearest the bed [$z/H=0.01$ (+)] a sign change in u/c can be seen starting at $tc\approx 1.0$. The sign change corresponds to a flow reversal which reaches a local maximum of $u/c\approx -0.08$ at $tc\approx 1.2$ and then slowly decays to zero with time. In Fig. 5 a trace of the thickness of the reverse flow is given for the same experiment and fixed horizontal location as that presented in Fig. 4. The thickness of the reverse flow was measured by tracking the sign of the horizontal velocity. Tracking began at the bed and continued vertically to the point at which the velocity within the vicinity of the boundary changed sign from negative to positive (the wave propagates from the negative x direction to the positive x direction and initially induces a very strong positive horizontal velocity at the lower boundary). This observable was used to define empirically the upper edge of the reverse flow and it is plotted in Fig. 5. It can be seen that the reverse flow appears at $tc\approx 0.8$ and it reaches a maximum depth of $z/H\approx 0.027$ before it begins to decay in vertical extent.

The flow reversal presented here is very similar to that seen under a surface solitary wave in Liu *et al.*²² in particular, Fig. 4 compares favorably with Fig. 10 of their work. Liu *et al.*²² present a nonlinear numerical computation of the horizontal velocity induced in the bottom boundary layer under a surface solitary wave. They also present experimental PIV measurements in the boundary layer and get excellent agreement with theoretical prediction. They find flow reversal in the boundary layer in the decelerating part of the flow which begins at $tc\approx 0.2$ and induces a reverse velocity up to a strength of $u/c\approx -0.15$. The flow reversal in Liu *et al.*²² occurs much sooner after the wave peak than in the ISW case presented in Fig. 5, where it does not appear until $tc\approx 0.8$. The strength of the velocity in the reverse flow in Liu *et al.*²² is also stronger than that seen in Fig. 4. The velocity in the reverse flow presented in Fig. 4, however, is sampled at an elevation above the bed much higher than that in Liu *et al.*²² In Fig. 4 an elevation of $z/H=0.01$ (which is approximately 4 mm in dimensional form) is used, while Liu *et al.*²² present the reverse flow in the boundary layer very close to the bed. This difference in vertical elevation above the bed may explain the discrepancy in the strength of the induced velocity. Note that Fig. 10 in Liu *et al.*²² is for a surface solitary wave of wave height/water depth $=0.2$ while the nondimensional amplitude of the ISW presented in Fig. 4 was $a/H=0.3$.

For waves of type (ii), where two internal waves of elevation were generated, flow reversal was seen throughout the water column in the decelerating part of the waves (rear

FIG. 6. Horizontal velocity u/c vs tc for experiment 170609top, at $x/H = 10.0$ and $z/H=0.01$ (+), $z/h=0.04$ (O), and $z/H=0.79$ (*).

half). The flow reversal always began at the lower boundary, however; hence, a very small reverse jet was seen there before reversal took place throughout the water column. An example to illustrate this is given in Fig. 6 for experiment 170609top. Figure 6 is a plot of wave-induced horizontal velocity (nondimensionalized by the measured wave speed c) versus time t multiplied by c . The data are presented at a fixed horizontal location, $x/H=10.0$ and the different plots correspond to different vertical elevations above the bed, namely, $z/H=0.01$ (+), $z/h=0.04$ (O), and $z/H=0.79$ (*). In the decelerating part of the flow ($tc > 0$) the horizontal velocity decays toward zero and then switches sign. The sign change is seen to occur at the elevation nearest the bed, $z/H=0.01$ (+), before it occurs higher in the water column, $z/h=0.04$ (O) and $z/H=0.79$ (*). This illustrates the very small reverse jet seen at the bed before the whole column reverses direction. For completeness, it is noted here that in the type (iii) cases where the ISW exhibited shear instability in the pycnocline, no reversal was seen close to the bed in the decelerating part of the flow. The shear instability was quite vigorous and disturbed the water column all the way to the bed in the decelerating part. The near bed motion induced by the shear instability disrupted the formation of a reverse flow in these cases.

IV. SUMMARY AND DISCUSSION

The boundary layer induced by an ISW of elevation propagating in an unsheared, stably stratified water configuration has been examined and measured experimentally using the technique of PIV. The undisturbed density field, consisting of a homogeneous top layer and a linearly stratified pycnocline above a homogeneous bottom layer (with the linearly stratified pycnocline located nearer the bottom than the top of the water column) is representative of that observed in coastal oceans. The experiments have demonstrated that the wave-induced boundary layer does not separate in the adverse pressure gradient region ahead of the wave peak. A reverse flow close to the bottom boundary, however, is seen in the decelerating part of the wave after the wave peak. The lack of boundary layer separation and vortex formation in the adverse pressure gradient region is in contrast to the theoretical predictions of Diamessis and Redekopp¹⁶ but in agreement with those of Stastna and Lamb.^{15,20} The discrepancy with Diamessis and Redekopp¹⁶ is thought to be due to the use of a weakly nonlinear theory by those authors for wave

amplitudes beyond those for which weakly nonlinear theory holds. The weakly nonlinear theory is thought to overestimate the wave propagation speed and underestimate the wave half width, leading to narrower and stronger regions of convergence being generated in the bottom boundary layer and overpredictions of instability there.

The flow reversal seen in the bottom boundary in the decelerating part of the wave-induced flow was shown to be similar in nature to that found under surface solitary waves.^{21–23} Liu *et al.*²² suggest that the reversal is a result of a phase lag between the irrotational and rotational velocity components. Carr and Davies²⁵ and Carr *et al.*²⁶ presented experimental evidence of a similar flow reversal at the lower boundary under an ISW of depression in the decelerating part of the wave-induced flow aft of the crest but conjectured that the flow reversal was a result of boundary layer separation in the adverse pressure gradient region. In both instances, however, a reverse flow is seen in the decelerating part of the flow (aft of the wave crest/trough). Hence, it may be that the reverse flow under an ISW of depression is also the result of a phase lag between the irrotational and rotational velocity components and is not the result of the adverse pressure gradient region as previously proposed.^{16,25,26}

On the other hand, the wave-induced flow under ISWs of differing polarity have significant differences and a direct comparison of the two should be made with caution. For example in the elevation case, there is strong shear (i) at the crest of the wave, where the wave-induced velocity changes sign, and (ii) at the lower boundary where there is a no-slip requirement. The velocity gradient is oppositely signed at the crest and the lower boundary, respectively, for example see Moum *et al.*⁶ Fig. 9. Hence, oppositely signed vorticity is injected into the wave core at the crest and at the lower boundary, respectively. This is in contrast to the depression case where the wave core is bounded above by a free surface and below by the wave trough. The free surface does not generate the same kind of high stresses that the lower solid boundary does in the elevation case and, in consequence, input of vorticity into the wave core is significantly different in the two cases. Moreover, the wave-induced shear stress generated at the bottom boundary beneath an ISW of elevation is expected to be higher than in the depression counterpart as the pycnocline is located nearer the bed and so the wave-induced velocity field is stronger.

Note that the flow reversal presented here under ISWs of elevation only reaches approximately half the depth of the flow reversal seen under an ISW of depression (compare Fig. 5 with Fig. 11 of Carr and Davies²⁵ and Fig. 4 of Carr *et al.*²⁶). Carr *et al.*²⁶ and Diamessis and Redekopp¹⁶ showed that there is a critical depth of reverse flow required for global instability to occur and associated vortices to form. No vortices associated with the reverse flow were observed in the elevation experiments but they may be presumed to occur if the reverse flow is strong enough under ISWs of elevation. The wave-induced Reynolds number of the flow and the nondimensionalized amplitude of the wave were optimized in this study. Hence, the strength of the reverse flow was as large as possible for the given experimental configuration and the conjecture above could not be tested.

The laboratory data presented here are characterized by wave Reynolds numbers of $O(10^4)$ while in nature the corresponding Reynolds numbers for, say, ISWs of elevation in coastal waters, are in the range $[10^5, 10^7]$. In the depression counterpart study, see Carr *et al.*,²⁶ it was shown that the critical amplitude for vortex formation to occur was inversely proportional to the wave Reynolds number and when the laboratory study was scaled to oceanic Reynolds numbers, the critical amplitude reduced to a value less than amplitudes commonly observed in the ocean. In this study, a similar range of amplitudes and wave Reynolds numbers was used, yet no vortices were seen and no critical amplitude was established. In the ocean, the effect of a high Reynolds number will be to induce a stronger velocity gradient at the bottom boundary and hence a thinner boundary layer relative to the total fluid depth. This will affect the bottom boundary layer dynamics in the ocean but it is unclear from this experimental study what the effect will be as it was only possible to investigate a limited range of Reynolds numbers. Another important difference between the laboratory study and the ocean is in the bottom boundary condition. In the idealized laboratory configuration, a smooth, flat bottom boundary was utilized and this is generally unrepresentative of oceanic conditions. Future work aims to address the effects of roughness at the bottom boundary and inclination of the bed.

There are limited field observations of the wave-induced flow close to the lower boundary under ISWs of elevation. The most detailed are those of Moum *et al.*⁶ who present observations from portions of the water column extending as low as 1 m above the sea bed. Moum *et al.*⁶ report enhanced regions of optical backscatter beneath ISWs of elevation suggesting regions of turbulence there. However, the region of backscatter is not confined to the lower boundary and indeed Moum *et al.*⁶ report enhanced regions throughout the core of the wave. They show that the waves in their observations have a recirculating trapped core. The trapped core will affect the bottom boundary layer dynamics differently from that of a wave with a laminar core (such as that generated in the laboratory). For the above reasons, direct, detailed, quantitative comparisons between oceanic observations and the present laboratory data are unlikely to be useful.

The reasons for the formation of a reverse flow underneath an ISW have been brought into question. The experimental findings presented here suggest that further theoretical investigation is needed to clarify the present understanding of boundary layer induced flow under an ISW.

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