

1 Comparing Spatial Patterns

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11 **Comparing Spatial Patterns**

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14 **Abstract**

15 The comparison of spatial patterns is a fundamental task in geography and quantitative spatial
16 modelling. With the growth of data being collected with a geospatial element we are witnessing
17 an increased interest in analyses requiring spatial pattern comparisons (e.g., model assessment,
18 change analysis). In this paper we review quantitative techniques for comparing spatial
19 patterns, examining key methodological approaches developed both within and beyond the
20 field of geography. We highlight the key challenges using examples from widely known
21 datasets from the spatial analysis literature. Through these examples we identify a problematic
22 dichotomy between spatial pattern and process – a widespread issue in the age of big geospatial
23 data. Further, we identify the role of complex topology, the interdependence of spatial
24 configuration and composition, and spatial scale as key (research) challenges. Several areas
25 ripe for geographic research are discussed to establish a consolidated research agenda for
26 spatial pattern comparison grounded in quantitative geography. Hierarchical scaling and the
27 modifiable areal unit problem are highlighted as ideas which can be exploited to identify pattern
28 similarities across spatial and temporal scales. Increased use of ‘time-aware’ comparisons of
29 spatial processes are suggested, which properly account for spatial evolution and pattern
30 formation. Simulation-based inference is identified as particularly promising for integrating
31 spatial pattern comparison into existing modelling frameworks. To date, the literature on spatial
32 pattern comparison has been fragmented and we hope this work will provide a basis for others
33 to build on in future studies.

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40 1. Introduction

41 The comparison of maps is a fundamental part of how geographers try to understand
42 the world. Quantifying spatial distributions and patterns, and comparing across regions or over
43 time is central to many types of geographical research and applications. To illustrate, Figure 1
44 presents two maps from the recent Intergovernmental Panel on Climate Change (IPCC)
45 Synthesis Report (IPCC, 2014) which shows recorded surface temperature change (1986-2005)
46 and projected changes (2081-2100). We are confronted with two simultaneous map comparison
47 tasks. First, we make comparisons locally within a map, noticing spatial differentiation within
48 both observed and projected temperature regimes – noting in particular the rapid warming in
49 polar regions. Second, we compare the maps globally, recognizing large magnitude shifts in
50 temperature across most continents in the projected scenario compared to the period of
51 observed temperature changes. In making these interpretations broadly, we mask uncertainties
52 associated with more precise questions of change, such as which populations are likely to be
53 most impacted by increasing temperatures, where should conservation resources be allocated,
54 are countries in the global North more impacted than the global South, were the data collected
55 equally in all regions, and countless other geographic questions. These maps are included in
56 the IPCC report designed as guidelines for policy and decision-makers. The recognition and
57 quantification of spatial change through comparison of spatial patterns, both globally and
58 locally, represents an important and under-recognized research area for geography which we
59 aim to review, critique, and contextualize.

60 <Figure 1 here >

61 Many of the origins for studying changes and differences in spatial patterns arose during
62 geography's *quantitative revolution*. Today the sheer volume of geographically referenced data
63 is providing new opportunities for geographers to compare spatial patterns across space, and
64 time. Recent geographical data-intensive research streams include geocomputation (Openshaw
65 & Abrahart, 1996); geospatial big data (Li et al., 2016), human dynamics (Shaw, Tsou, & Ye,
66 2016), and geographic data mining (Miller & Han, 2009). Long-term archives of satellite
67 imagery, crowdsourced geospatial databases, and open data portals are now being developed
68 and maintained for a variety of subject areas. Despite the amount of data-intensive geographic
69 research taking place, geographers have not consolidated methods and models for performing
70 spatial pattern comparisons which facilitate replication, identification of broader trends and
71 underlying spatial process dynamics, and local anomalies.

72 In this paper we review existing quantitative techniques for comparing spatial patterns
73 and discuss commonly encountered issues. We briefly cover basic concepts and terminology

74 associated with spatial patterns before moving on to a review of selected techniques, and
75 provide illustrative examples that highlight strengths and shortcomings of current methods. We
76 conclude with some thoughts on the research needs for spatial pattern comparison (SPC) in
77 geography today. In doing so we hope to provide some coherency and unity to the SPC research
78 which is fractured across fields, and identify research opportunities for geographers interested
79 in quantitative spatial analysis.

80 **2. Characteristics of Spatial Pattern Comparison Problems**

81 ***2.1 Characteristics of spatial patterns***

82 As geographers, we typically ascribe meaning to spatial patterns as the outcomes of multiple
83 and interacting spatial processes (O'Sullivan & Unwin, 2010). Isolation of processes
84 themselves outside of laboratory or simulation environments is impossible at most
85 geographically-relevant scales, so while detecting change in a spatial pattern can reveal
86 changes in underlying processes, it is not sufficient to reveal those dynamics. Complicating
87 matters, pattern itself acts on and perturbs the processes generating patterns (Turner, 1989).
88 Even the term *spatial pattern* can itself imply multiple and conflicting phenomena. Here we
89 use a definition (see Box 1 for a glossary of terms and definitions that will be used throughout)
90 based on that of Dale (2000), that a spatial pattern is the scale-dependent predictability of the
91 physical arrangement of observations.

92 < Box 1 (Glossary) Here >

93 ***2.2 Spatial representation***

94 Spatial data are abstractions of reality, with 'features' (i.e., the things we demarcate,
95 categorize, and label in the world) represented by *points*, *lines*, *polygons (areas)*, and
96 *continuous spatial lattices (irregular or regular)* in a digital mapped form. These are the core
97 spatial data types available in a geographic information system (GIS) and there exists relatively
98 few other ways to represent spatial phenomenon in a GIS (Roberts & Robertson, 2016). The
99 representation of complex physical and societal characteristics (in spatial data) influences how
100 feature or attribute data can be characterized, visualized and subsequently analysed (Miller &
101 Wentz, 2003). Nearly all examples of SPC that will be discussed in this paper represent what
102 can be termed 'diagonal' comparisons (referring to a matrix of spatial data types; e.g., see
103 O'Sullivan & Unwin, 2010, p. p26), that is, for example, point-point and lattice-lattice
104 comparisons.

105 ***2.3 Statistical properties***

106 Methods for SPC can also be characterised as being *entity-based* or *attribute-based*.
107 Entity-based comparison consider only the *locations* of objects in the two maps, and are strictly

108 limited to spatial comparisons of points, lines, and polygons (areas). Attribute-based
109 comparisons simultaneously consider patterns associated with both location and attributes.
110 With attribute-based comparisons, whether the attribute type is *continuous* or *categorical* also
111 impacts how a spatial comparison is framed. Most attribute-based comparisons are associated
112 with fixed spatial arrangements (i.e., the locations are the same in both maps), but this need not
113 be the case.

114 SPC methods can further be broken down along dimensions of spatial pattern that they
115 compare, whether local or global aspects of pattern, or the abundance (composition) or
116 arrangement (configuration) of mapped values (Figure 2). At the global level, SPC can be
117 undertaken by either computing a single univariate measure, such as Moran's I index of spatial
118 autocorrelation (Cliff & Ord, 1973), individually on each map; or by computing a bivariate
119 measure that simultaneously compares values in two maps. In the first case, only a coarse
120 understanding of spatial pattern change can be inferred, for example a change from complete
121 spatial randomness to spatially clustered. One of the challenges with the former approach, is
122 that both local-to-global scaling and composition vs configuration are highly interdependent
123 concepts, posing challenges for robust statistical significance testing. Some specific measures
124 have been proposed for disentangling global and local spatial structure such as the O_i statistic
125 (Ord & Getis, 2001), however these have not been widely adopted. Similarly, composition and
126 configuration are also interdependent, and several authors have highlight the need to compare
127 measures of spatial configuration only in the context of spatial composition (Cushman,
128 McGarigal, & Neel, 2008; Long, Nelson, & Wulder, 2010; Remmel & Csillag, 2003). In
129 bivariate comparison measures, these issues are partially overcome, as the distributional issues
130 associated with comparison are resolved by reduction of the parameter space to a single metric
131 (e.g., the root mean square error or the Kappa statistic).

132 **2.4 Types of questions**

133 Finally, SPC can be characterised as one of three types of question: *change*, *similarity*,
134 and *association*. Each type of SPC question can be specified in terms of the constraints and
135 variability in space, time, and theme of the patterns being investigated (Sinton 1978). Studies
136 of change involve cases where space and theme are fixed and time varies. The goal of change
137 analysis is often to identify whether change has occurred (globally), where such changes are
138 located (locally), and whether changes represent a significant change (Boots & Csillag, 2006;
139 Remmel & Csillag, 2003). Similarity questions involve situations where space is varied and
140 theme is fixed (time can be fixed or varying). Similarity tasks are prominent in the image
141 retrieval literature, and have been framed as SPC problems in many cases involving satellite

142 imagery (e.g., Yang & Newsam, 2013). Studies of associations involve cases where space is
143 fixed but theme varies (again time can be fixed or varying). Questions concerning spatial
144 associations are common in both the physical and social sciences (e.g., Austin et al., 2005;
145 Jones, Rendell, Pirotta, & Long, 2016).

146 A distinct and popular application for SPC is in spatial model assessment. Models that
147 produce mapped outputs can be compared to reference data to assess model fit, or across model
148 frameworks and parameterizations. Early examples of using SPC for model assessment include
149 Cliff (1970) and Sokal et al. (1983), who both characterized model outputs with spatial
150 autocorrelation statistics. Spatially explicit model assessment is critical as it can reveal patterns
151 in error structures not evident in error statistics (e.g., Plouffe, Robertson, & Chandrapala,
152 2015). SPC for model assessment, has seen more recent interest in the area of categorical spatial
153 data (e.g., land cover maps; Hagen-Zanker & Martens, 2008; Visser & de Nijs, 2006).
154 Examining the spatial pattern of model outputs as a complementary measure of model quality
155 underscores the importance of spatial pattern/process in environmental modelling (see Bennett
156 et al., 2013).

157 <Figure 2 here>

158 **3. General approaches for Quantitative Spatial Pattern Comparison**

159 ***3.1 Visual spatial pattern comparison***

160 The human visual system excels at recognizing shapes and patterns. Our brains are able
161 to process information on shapes and patterns independently from context (Marr, 1985). Thus,
162 to date a large body of work on SPC has involved visual comparisons, and most commonly this
163 involves the presentation of maps side-by-side as a tool for visual SPC (see, for example, Figure
164 1 above). Comparing patterns in maps has led to some significant geographical insights, for
165 instance, Wegener's work on continental drift theory was largely initiated by identifying
166 similar patterns in maps of the coastlines of Africa and South America (Wegener 1966).
167 However, comparing spatial patterns is visually challenging, as the human visual system is not
168 well adapted to judging spatial correspondence between two variables on side-by-side maps
169 and is more sensitive to the color classification scheme (e.g., Lloyd & Steinke, 1977; Steinke
170 & Lloyd, 1983). Moreover, MacEachren (1995, p. 403) suggests that we would expect visual
171 comparisons of maps to be more successful when changes are compositional (e.g., symbol size,
172 color) then with configuration changes (e.g., shape, and orientation).

173 Perception of patterns in maps is a function of our perceptual attention (i.e., where, and
174 for how long we look), also termed *saliency*. When comparing maps, *cosaliency* represents the
175 importance of locations in side-by-side comparisons, and *cosalient* features in map pairs may

176 not correspond to salient features in individual maps (Jacobs, Goldman, & Shechtman, 2010).
177 Within the cartographic literature a number of techniques for enhancing visual map comparison
178 tasks have been developed (See Ch. 9 in MacEachren, 1995 for example). Many newer
179 techniques have abandoned side-by-side comparisons in favor of overlay techniques, and
180 implement tools such as translucents, swiping, and lenses (Lobo, Pietriga, & Appert, 2015).
181 However, it is widely acknowledged that visual SPC is challenging, most notably in assessing
182 changes in spatial configuration, a problem that continues to hinder both visual and quantitative
183 assessments of SPC.

184 ***3.2 Comparing spatial point patterns***

185 Tobler (1965) studied the correspondence between pairwise point patterns of the
186 locations of birth for a sample of married couples in Japan using a comparison measure which
187 he termed the affine correlation statistic. The approach was innovative in terms of its attempt
188 to draw on Pearson's correlation coefficient, but was limited to cases where two patterns had
189 the same number of points and the points are naturally paired. Ecologists were early adopters
190 of statistical methodologies for comparing bivariate point patterns, notably the work of
191 Anderson (1992) who drew on seminal methods from Diggle (2003), to compare one point
192 pattern to another with the bivariate extension of the K-function.

193 In biology, tight coupling of spatial and physical factors that drive the control and
194 function of cellular functions and processes demands rigorous methods to detect differences in
195 pattern. Myers (2012) highlights the increasing need for quantitative approaches for SPC in
196 microscopy resulting from new forms of medical imaging data that are often used to derive
197 spatial point patterns. Many proposed approaches have been tailored to specific biological
198 applications (e.g., Bell & Grunwald, 2004; Burguet & Andrey, 2014) however opportunities
199 exist for generalization to other, more complex, classes of spatial data.

200 ***3.3 Comparing line and polygon data***

201 There are fewer methods available for comparative analysis of line and polygonal
202 pattern data. Within the geographical literature, line and polygonal spatial representations tend
203 to be treated as features rather than patterns, with emphasis of methods on the proximity and
204 orientation relations between pairs of objects, and summarizing such metrics over the dataset
205 provide a measure of global similarity. Maruca and Jacquez (2002) provide polygon
206 comparison statistics called area-based association measures, which essentially quantify the
207 degree of correspondence based on area overlap between two of polygon pattern datasets.
208 Proximity relations for linear data have been explored in the context of spatial data accuracy

209 assessment, such as buffering a reference line and comparing the proportional overlap of a test
210 line (e.g., Goodchild & Hunter, 1997).

211 Graph analysis provides a large body of theory and methods for characterizing
212 networks, which is a common way to represent spatial line data in geographic applications.
213 These network representations can be compared through measures of including centrality,
214 connectivity, degree and others. However, often the geographical context is ignored to focus
215 on topological properties. The uptake of these methods in a spatial context has been greatest in
216 landscape research, examining structural (e.g. configurational) properties of a matrix of habitat
217 patches (i.e., nodes) and their spatial connectivity (i.e., edges) (Bunn, Urban, & Keitt, 2000;
218 Urban & Keitt, 2001; Urban, Minor, Treml, & Schick, 2009). Similarly, landscape ecologists
219 are interested in SPC of maps of land cover types, using indices of diversity, fractal dimension,
220 and shape (e.g., Mladenoff, White, Pastor, & Crow, 1993). However, these comparisons are
221 focused on comparing the measures of pattern in one map to another, rather than on bivariate
222 methods. Robertson et al. (2007) provide a comparison framework adapted from Sadahiro and
223 Umemura (2001) for studying temporal changes in spatial polygons which focuses on problems
224 where the spatial locations of polygons move through time (e.g., a forest fire). Here, changes
225 are characterized as events derived from topological and proximity relations of two polygon
226 patterns.

227 *3.4 Comparing patterns in spatial lattices*

228 There are many more methods for comparing patterns on spatial lattices, and here we
229 refer to the case where the spatial structure of the lattice does not differ between the two maps
230 (e.g., the spatial units are the same). One of the first applied quantitative analyses comparing
231 spatial patterns was that of Robinson and Bryson (1957) who looked at the spatial correlation
232 between precipitation and population in Nebraska by mapping regression residuals to describe
233 the spatial correspondence – a technique which has since been employed for assessing spatial
234 models (e.g., Hengl, Heuvelink, & Stein, 2004). Cliff (1970) looked at the correspondence of
235 Hägerstrand's (1967) innovation diffusion data, comparing empirical data to theoretical
236 simulations, which represents the first example where the spatial autocorrelation of the
237 difference between maps (tested using joint counts based on whether the difference was
238 positive or negative) was used as a measure of spatial correspondence. Spatial autocorrelation
239 analysis of residual differences as proposed by Cliff (1970) remains influential today (e.g.,
240 Wulder, Boots, Seemann, & White, 2004).

241 Several contemporary authors have proposed varied approaches for quantifying spatial
242 associations, predominantly for use with continuous-valued lattice datasets (e.g., attributes in

243 counties). Sokal and Wartenberg (1983) used spatial correlograms (from Moran's I and Geary's
244 C) to characterize similarity in gene-frequency surfaces between simulated populations under
245 an isolation-by-distance model. Hubert et al. (1985) propose a spatial cross-product statistic in
246 an attempt to distinguish spatial pattern similarity from attribute similarity. Specifically, Hubert
247 et al., demonstrate how various map patterns can arise when holding Pearson's correlation
248 coefficient constant, confounding spatial comparison problems. Global statistics (like the
249 correlation coefficient) are insensitive to variations in local spatial patterning. Haining (1991)
250 proposed spatial adjustments for the Pearson and Spearman correlation coefficients by
251 adjusting the significance test of the statistic to account for spatial structure (measured as spatial
252 autocorrelation) present in the data.

253 Cumulative distribution functions have been proposed for SPC problems because they
254 are able to consider the shape of the underlying empirical distributions (Syrjala, 1996). Wong
255 (2001) proposes a local cumulative distribution function as a means to use the widely employed
256 cumulative distribution function in a spatially-local comparison. In analysis of neighbourhoods
257 and their social characteristics, comparisons of both geographic and multivariate demographic
258 characteristics has led to use of self-organizing maps to link social factors and spatial patterns
259 (Spielman & Thill, 2008), and approach which decomposes spatial and thematic properties into
260 separate 'map' spaces, which can then be visualized and explored for patterns.

261 Additional methods for SPC have focused on the development of local forms of spatial
262 analysis (Boots & Okabe, 2007; Fotheringham & Brunson, 1999). Fotheringham et al. (2002)
263 propose a geographically weighted correlation coefficient as a spatially-local tool for studying
264 bivariate associations extending Pearson's correlation coefficient to local analysis. A further
265 extension of the locally weighted correlation coefficient was presented by Lee (2001) which,
266 combines Pearson's R with a bivariate Moran's I into a single statistic that simultaneously
267 considers correlation and autocorrelation. A spatially-local version of the statistic is also
268 presented along with a formal statistical testing framework (Lee, 2001). Robertson et al. (2014)
269 extend an image comparison metric - the structural similarity index (SSIM; Z. Wang, Bovik,
270 Sheikh, & Simoncelli, 2004) - for comparing spatial patterns within a spatial model assessment
271 framework. Further, separation of local patterns into the first order, second order and pattern
272 components provides significant opportunity for studying differences in local spatial patterns.

273 Computer scientists have also been intensively developing methods for image matching
274 and comparison, which is analogous to the comparison of raster data. For example, Scharstein
275 (1994) used an image shifting approach based on localized gradient field to assess how well
276 two image patterns align. Comparative histogram-binning methods such as the Earth-mover's

277 Distance (Rubner, Tomasi, & Guibas, 2000) use multi-dimensional histograms that describe
278 colour and texture and define distance measures in these spaces to characterize similarity of
279 images. Applications of these methods in geographic contexts, most notably for retrieval and
280 characterization of satellite imagery, are increasing (Jasiewicz, Netzel, & Stepinski, 2014;
281 Kranstauber, Smolla, & Safi, 2016; Shao, Zhou, Zhang, & Hou, 2014).

282 The above methods focus predominantly on comparisons of continuous value attribute
283 data on a lattice, but there is a great deal of work on comparing categorical lattices as well. The
284 Kappa statistic tests agreement between lattices relative to what would be expected by chance,
285 and its widespread use in remote sensing is thought to be due to its familiar interpretation, even
286 when its mathematical underpinnings are poorly understood or erroneous (Pontius Jr &
287 Millones, 2011). Hagen-Zanker (2009) extended the statistic to use fuzzy relations between
288 categories and spatial location similarities as a bivariate SPC tool for categorical lattices.
289 Pontius Jr and Millones (2011) argue for a new type of comparison measure that considers both
290 *quantity* and *allocation* disagreement in mapped categories. Pontius Jr and Millones emphasize
291 that a valid metric for spatial comparison should a) avoid compressing the two dimensions of
292 pattern into one metric, and b) characterize disagreement rather than agreement.

293 **4. Issues in SPC analysis**

294 ***4.1 Examples***

295 To demonstrate the challenges associated with SPC we have hand-picked a set of five examples
296 (see Table 1 and Figure 3). For each example, we have chosen a representative and current
297 technique for quantitative SPC associated with each data type. In all cases we have selected a
298 single global statistic for comparison. Through the use of basic comparison statistics as a
299 starting point, we highlight some the challenges associated with SPC analysis.

300 < Table 1 Here >

301 <Figure 3 here>

302 ***4.2. Highlight problems in spatial pattern comparisons***

303 **Problem 1: Pattern vs Process**

304 Perhaps the biggest challenge emerging from the ‘big data’ revolution is that of
305 connecting the analysis of patterns in the data with the underlying processes that we are
306 interested in studying (Miller & Goodchild, 2015). As an example, consider the comparison of
307 the red oak and white oak patterns. The results suggest there is evidence of a relationship
308 between the spatial patterns of the two species, but we do not have any theory to support this
309 at the process level. Perhaps there is inter-species attraction due to seed dispersal, shading
310 characteristics, or interactions with forest disturbance agents (e.g., wildfire, insects). Note also

311 that this is really of question of spatial *interaction* at the process level, which manifests in
312 spatial *similarity* at the pattern level. It is unclear how likely this similarity is under independent
313 spatial processes governing the distribution of red and white oaks. Static spatial patterns are
314 inherently limited in their ability to describe dynamic processes. More data does not necessarily
315 improve this limitation and may in fact add additional noise.

316 Problem 2: Topological Complexity

317 Comparison of topological characteristics of spatial data is typically reduced to
318 comparison of connectivity matrices or graphs. In the example comparing node degree for
319 OSM street networks for Waterloo, Canada and St. Andrews, Scotland. Spatial non-stationarity
320 in road network density in Waterloo was present, whereby node degree of the dense parts of
321 the network in the downtown area more difficult to observe in contrast with the less dense parts
322 of the network in rural outlying areas, which tended to have four-node intersections. The
323 similarity in node degree in the two networks was masked partially by the dis-similarity in
324 network densities. While computing the node-degree values is straightforward, the results here
325 highlight the difficulty in isolating one component of pattern to compare. Typically, the overall
326 comparison of pattern similarity for the HVS is a composite of several dimensions of spatial
327 pattern. Developing metrics or aggregate indicators of similarity of spatial patterns therefore
328 hinges on identifying the key dimensions of pattern for a specific comparison task. Comparing
329 topological properties may be an example where ‘spatial intuition’ and computed values are
330 misaligned, as slight spatial changes can have large impacts on topology (e.g, undershoots in
331 routing problems).

332 Topology is also confounded by spatial representation decisions in maps when
333 visualizing comparisons. The visual assessment of pattern similarity between the mountain
334 pine beetle polygons is certainly impacted by a number of classical cartographic pitfalls, such
335 as a failure to include a reference basemap, map graticule, grid lines, or even a scale bar.
336 However, a much more challenging problem arises when comparing objects that exhibit such
337 a highly complex topology (such as the infestation polygons with irregularly shaped borders,
338 holes, and multiple polygon parts). Had these two infestation polygons exhibited regularly
339 shaped boundaries the comparison process would be easier (both visually, but also
340 computationally). But complex topological shapes, including less binary gradients and
341 boundaries, are the norm in environmental applications (Gustafson, 1998), and are salient in
342 many anthropogenic examples (Batty & Xie, 1994). Thus, characterizing the similarities
343 between complex shapes and patterns in a single (or multiple) index remains an ongoing
344 challenge in SPC.

345 Problem 3: Composition vs Configuration

346 The description of spatial patterns can be decomposed into two unique but
347 interdependent components: *composition* and *configuration* (Boots, 1982, 2003). To
348 generalize, composition refers strictly to the aspatial properties of the elements of a spatial
349 pattern (e.g., the type, number, and statistical properties of *what* is being mapped), while
350 configuration refers to the strictly spatial arrangement of these elements (i.e., the *where*).
351 Consider the Plum Island Ecosystem maps in Figure 2 (g-h). The most basic description of
352 configuration refers to homogeneous (no variation exists) vs. heterogeneous (i.e., the observed
353 pattern varies across space) spatial patterns. In practice, assessing configuration involves
354 quantifying the level and nature of heterogeneity in mapped data and a wide set of terminology
355 and techniques are available. These terms are typically both data and application specific; and
356 can be used differently depending on the context of the analysis.

357 Dependency between composition and configuration of spatial patterns is demonstrated
358 in Figure 4. Previous research has demonstrated that the potential for different spatial
359 configurations to arise is largely dependent on the composition of elements in the map (Rommel
360 & Csillag, 2003; X. Wang & Cumming, 2011). Thus, quantifying SPC is complex due to the
361 potential for changes in configuration to arise solely due to changes in composition (i.e., Figure
362 4), confounding inferences into SPC (Long et al., 2010; Rommel & Csillag, 2003; X. Wang &
363 Cumming, 2011). Indices for SPC must be able to simultaneously consider and disentangle the
364 level of compositional and configurational change to be effective.

365 <Figure 4 here>

366 Problem 4: Spatially Global Indices

367 To date, most approaches for SPC are spatially global, producing a single statistic for
368 the entire study area (indeed all five of the indices we employed fall into this category). With
369 large-area and ‘big’ sources of spatial data, this can be misleading as global statistics fail to
370 adequately capture spatial non-stationarities in observed patterns. However, spatially local
371 analysis of big data also poses challenges since outputs require some interpretation, a non-
372 trivial task with increasingly large datasets. With spatial-temporal local models, more
373 sophisticated geovisual analytics may be required to understand the complex output stemming
374 from local analysis of large mapped datasets (Foley & Demšar, 2012). But relying on visual
375 interpretations can be challenging given the characteristics of many modern large datasets (i.e.,
376 coverage over broad-scales, with fine spatial resolution). Visualization as a tool for SPC (see
377 Section 2.2) can be challenging with large datasets, due to maps being portrayed at a minimum

378 resolution that is beyond our perceptual limits. Geographic knowledge discovery (Miller &
379 Han, 2009) approaches may be suitable for performing SPC in large geographic databases.

380 A further challenge commonly encountered is that a single output statistic may not be
381 sufficient for performing SPC with complex spatial patterns. For example, the negative
382 correlation identified in the Georgia data may not be consistent across the entire state, and a
383 spatially sensitive correlation measure (e.g., see p. 172 in Fotheringham et al., 2002) would
384 shed further insight into the spatial variation in correlation. With increasingly large datasets
385 (big data), moving the analysis scale from the global to the spatially local scale is necessary to
386 capture how spatial pattern comparisons vary across space.

387 **4. Moving the Spatial Pattern Comparison Research Agenda Forward**

388 *4.1 Comparing maps as spatial processes*

389 Csillag and Boots (2005) advocate a process-based framework for comparing spatial
390 patterns and identify two underlying questions that we, as geographers, should be seeking to
391 answer in all SPC related-tasks: 1) Could the observed differences in spatial patterns have
392 arisen purely by chance? and 2) Could the observed spatial patterns have been generated by the
393 same process?. Pearl (2009) makes the case for a clear discrimination between *associative* and
394 *causative* statistical analysis where associative analysis considers any relationship that can be
395 defined by joint distribution of two variables and a causative relationship is one that cannot be
396 defined by the joint distribution alone. With respect to SPC nearly all methods would fall into
397 the former category, whilst Csillag and Boots (2005) make the emphatic case for models that
398 fit squarely into the latter. One of the potential areas where new models are providing avenues
399 for new insight along this causative line of thinking is through the development of complex
400 simulations which can be used to test spatially explicit hypotheses (O'Sullivan & Perry, 2013).

401 Spatial analysis theory considers a map as a single realization of a stochastic spatial
402 process, and thus inference regarding two static maps, if treated independently, yields a sample
403 size of two. Spatial inferences pertain to the underlying process, though the particulars of what
404 a mapped pattern represents have been debated (e.g., Summerfield, 1983). Cressie (1993) cites
405 two basic contexts for doing spatial modelling; when a spatial process has reached temporal
406 equilibrium and its spatial properties describe causative components of that process, and when
407 short-term causal effects are aggregated over a fixed time period and expressed spatially.
408 Comparing spatial patterns disconnected from their generative (temporal) processes incurs a
409 high risk of finding differences resulting from natural variability. Explicit incorporation of *time*
410 into an SPC framework may provide a way to both handle big spatial data and still reason about
411 generating processes. Two ways we may be able to develop this are to 1) develop comparative

412 tools for continuous spatial evolution, and 2) undertake spatial multi-pattern comparisons; both
413 which imply a more explicit treatment of spatial processes.

414 Simulations provide an attractive framework for SPC as they allow experimentation
415 with model parameters, incorporation of nonlinear dynamics and feedbacks, and flexibility in
416 the types of model output (e.g., maps) that are generated. Two dominant approaches to
417 simulating spatial patterns and processes are widely used; individual-based models (IBMs),
418 and spatial-covariance models (SVMs). IBMs provide complete flexibility to specify all
419 important dynamics of the geographical system under investigation, which can then be used to
420 draw patterns from the model. Generating reference distributions for SPC metrics can be part
421 of model sensitivity testing. For example, evaluating the model's sensitivity to parameter
422 uncertainty from the perspective of spatial pattern is an interesting application area for SPC .,
423 Emergent spatial patterns play a central role in developing, parameterizing, and extracting
424 knowledge from IBMs (Grimm et al., 2005), and exemplar spatial patterns for specific
425 processes can be used to find model parameter values through inverse fitting procedures that
426 depend on a pattern comparison metric (Burnham & Anderson, 2002; Wiegand, Revilla, &
427 Knauer, 2004). Such an approach has been recently tested in a more formal framework that
428 provides model selection of parameter values *and* structure by 'approximate Bayes' methods
429 (van der Vaart, Beaumont, Johnston, & Sibly, 2015). The extension of these new approaches
430 for constructing, fitting, and assessing IBMs to incorporate explicitly spatial metrics is an
431 exciting research opportunity for SPC.

432 SVMs instead require specification of the form of spatial pattern that results from the
433 model (or process that generates it), which might more accurately reflect observed data, but
434 tend to have less mechanistic meaning. Rimmel et al. (2002) used a conditional autoregressive
435 (CAR) model to simulate three types of landscapes and to compare landscape pattern indices
436 under each landscape-type scenario. The resulting distributions provide reference for
437 interpreting differences between two LPI values when performing landscape. Long et al. (2012)
438 used simulations from a space-time model that incorporated a similar spatial covariance
439 structure (CAR prior) to model the probability of spread of a binary infection process on a
440 lattice. These types of spatial simulations are now widely employed in model testing,
441 comparison, and evaluation where simulated data is used to compare spatial parameter
442 estimates from different model specifications to a known underlying spatial process (e.g.,
443 Fotheringham & Oshan, 2016). Currently, visual comparisons and aspatial metrics are the de
444 facto standard for SPC in this context (e.g., Wheeler, 2010) however the specification of SPC

445 metrics are equally important in the SVM approach, and could perhaps serve as a point of
446 reference for comparing inferences obtained from different modelling frameworks .

447 **4.2 Spatially local analysis**

448 There is a clear need for robust spatially sensitive metrics, which seems like a surprising
449 thing to be championing given widely available tools for local spatial analysis. However, these
450 tools are largely appropriate only with spatial lattices (regular and irregular) and have failed to
451 be adopted more broadly. In a growing number of applications and decision-making contexts,
452 rigorous definitions of pattern similarity need to be adopted (e.g., Churchill et al., 2013; Sakieh,
453 Amiri, Danekar, Fegghi, & Dezhkam, 2015). When numeric or categorical data are obtained
454 over comparable spatial units and the SPC task pertains to how that data are spatially
455 configured across those units, measures of spatial pattern such as Moran's I, Geary's C, or local
456 variants can be employed. Waller (2014) provides a convincing argument for the need for
457 explicitly *spatial statistical* thinking in approaching analysis of geographical data, citing
458 common research motivations such as assessing fit of spatial models or spatial assessment of
459 statistical performance. Methods for SPC reviewed here can directly contribute to development
460 of a spatial statistical approach to science by providing tools for the robust comparison of
461 spatial patterns.

462 Yet the methods needed to answer comparative questions are often lacking. To
463 demonstrate this, a linear regression performed between '% rural' and '% with a college
464 degree' from the Georgia dataset, and the residuals were retained and shuffled across the
465 counties randomly (Figure 5). Two very different spatial patterns emerge which have identical
466 error statistics (MAE 0, RMSE 4.46). A reasonable question might be to ask whether the
467 differences are due to chance or the result of different underlying spatial processes, or rather,
468 what are the chances of obtaining $I = -0.169$ and $I = 0.274$ from this configuration of spatial
469 units and values, if the underlying processes are the same. Given the exact distribution of
470 Moran's I (Tiefelsdorf & Boots, 1995) we can compute probabilities of observed patterns based
471 on a null hypothesis of no spatial structure, but cannot use these results to compare two patterns
472 directly. Tiefelsdorf (1998) gives a conditional expectation of Moran's which allows
473 comparison of competing spatial process hypotheses as expressed through the spatial weights
474 matrix. Clifford et al. (1989) provide a t-test for comparing spatial structure in the context of a
475 correlation coefficient, yet do not give us a tool to understand if the spatial process giving rise
476 to the two patterns is the same or not.

477 <Figure 5 here>

478 **4.3 Scale and MAUP**

479 Spatial scale is often determined either arbitrarily or by a fixed set of intervals – both
480 in terms of grain and extent. This is critical for SPC because scale is intimately tied to the
481 definition of and observation of spatial patterns (Levin 1992; Dale 2000). The infinite number
482 of scales available for both making observations (i.e. grain) as well as observing patterns (i.e.,
483 extent), and the inter-relatedness of these constructs, makes comparison tasks challenging. Not
484 all scales are created equal for a given problem, and a set of *characteristic scales* are optimal
485 for analysis (Wiens, 1989). Big data provides opportunities for linking spatial processes across
486 scales, especially if data evolve over time. Previously, issues of scale in geographical analysis
487 tended to focus on the modifiable areal unit problem, whereby ‘scale effects’ are assessed by
488 varying aggregation units (e.g., Jelinski, Wu, & Wu, 1996). For SPC problems, variances due
489 to scale may be a critical aspect of pattern-observation and thus comparison. Sémécurbe et al.
490 (2016) provide an example of using multifractal analysis that quantifies MAUP to better
491 understand spatial heterogeneities in population density in France, developing a typology of
492 settlement patterns.

493 Yan and Li (2015) stress the need for both mathematical and psychological
494 justifications in the definition of spatial similarity measures (i.e., linking similarity to the HVS).
495 For the case of automated map generalization, a hierarchical scheme of maps, layers, groups,
496 and objects (i.e., points, lines, areas) is presented which define the fundamental units for which
497 spatial similarity relations are sought. The relations for comparing spatial objects at different
498 scales are distinct from the comparison of patterns. In the Yan and Li system, object properties
499 (size, shape, area etc.) and object group properties (topology, distance, correction etc.) may be
500 a way to integrate dimensions of similarity at the pattern or regional scale.

501 **4.4 Guidance for performing SPC**

502 We provide six simple guidelines for researchers wishing to compare spatial patterns in their
503 own applications.

- 504 1. Visual comparisons are useful – comparing two maps visually is a crucial first step in
505 the exploratory spatial data analysis process.
- 506 2. Quantitative measures are necessary – the subjectivity of the visual comparison process
507 means that any visual comparison should be further explored using a quantitative
508 comparison metric.
- 509 3. Local SPC measures are preferred – global SPC measures are subject to all the issues
510 associated with global spatial analysis procedures (Fotheringham & Brunson, 1999).

- 511 4. Quantifying dissimilarity is better – indices that focus on characterizing differences in
512 patterns, over similarities in pattern are more likely to provide meaningful inferences
513 (Pontius Jr & Millones, 2011).
- 514 5. Consider multiple elements of spatial pattern – spatial patterns have a variety of
515 characteristic components. Comparison measures capable of disentangling different
516 elements of spatial pattern within the SPC context are more informative than summary
517 measures.
- 518 6. Don't forget processes – understanding the linkages between processes and patterns is
519 the most challenging part of spatial analysis. Quantified pattern (dis)similarities may
520 be related to unknown confounding processes.

521

522 **5 Conclusions**

523 SPC is a complex task, which is difficult to automate, has a mixture of computational
524 and psychological components, and is increasingly required as geography and other fields
525 exploit bigger and more varied spatial datasets. Here we have reviewed the literature on SPC
526 that comes from a wide array of disciplines where applied problems have developed specific
527 comparison methods, lacking any coherent conceptual or theoretical framework. Our review
528 has focused on comparing spatial patterns of similar spatial representations (e.g., point-point,
529 lattice-lattice), there are however significant prospects for developing new methods for ‘off-
530 diagonal’ comparisons (e.g., point-polygon, line-lattice etc.). Many of the classical problems
531 of geography such as pattern vs process, scale, MAUP, and topology become exacerbated in
532 SPC. The spatial patterns we observe in maps are determined partially by spatial representation,
533 aspatial characteristics, data collection components, the truly spatial component, and some
534 element of randomness. More research into how these various components interact to create
535 spatial distributions we observe, through simulation and empirical data catalogs, would bolster
536 our ability to develop spatial modeling tools that support SPC.

537

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1 **References**

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238

Box 1: Glossary of terms.

Spatial Pattern Comparison - a numerical assessment of the (dis)similarity between two (or more) mapped datasets.

Spatial Pattern - scale-dependent predictability of the physical arrangement of observations

Spatial Process – model that produces spatial patterns with a known probabilistic function

Global Statistic – summary statistic that quantifies a property of spatial distribution with a single value

Local Statistic - summary statistic that quantifies a property of a spatial distribution at each location and sums to a global statistic

Composition – dimension of a spatial pattern that relates to the abundance of mapped values

Configuration – dimension of a spatial pattern that relates to the arrangement of mapped values

Table 1: Example datasets and methods for exploring issues in spatial pattern comparison analysis.

Data	Source (R Package)	Method (Reference)	Range	Interpretation	Result
Point	Lansing Woods (spatstat; Baddeley & Turner, 2005)	NN Correlation (Stoyan & Stoyan, 1994)	0 – 1	Proportion of NN in other group. 0 = all NN from same group 1 = all NN from other group	0.64
Polyline	Waterloo vs St Andrews (osmar; Eugster & Schlesinger, 2013)	Average Deg. Of Intersections	0 – n	Average number of roads connecting at each intersection (for each of W and StA)	W =2.56 SA = 2.48
Polygon	MPB Infestation (stampr; Long, Robertson, & Nelson, n.d.)	Area of intersection Index (Maruca & Jacquez, 2002)	0 – 1	Proportion of overlap 0 = no overlap 1 = perfect overlap/alignment	0.10
Lattice (categorical)	Plum Island Ecosystem (lulcc; Moulds, Buytaert, & Mijic, 2015)	Kappa Coefficient (K_hat) statistic	0 – 1	% of agreement in categories relative to chance. 0 = same as chance 1 = perfect agreement	0.88
Lattice (continuous)	Georgia Degree vs Rural (spgwr; Bivand, Yu, Nakaya, & Garcia-Lopez, 2015)	Pearson Correlation coefficient	-1 – 1	-1 = perfect negative correlation 0 = no correlation 1 = perfect positive correlation	-0.62

Figures

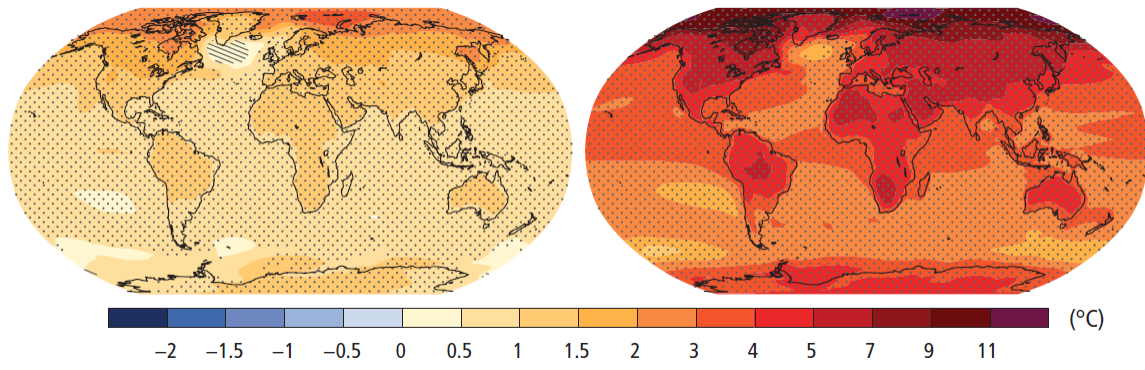


Figure 1: IPCC temperature changes globally, a) recorded observations and b) projections.

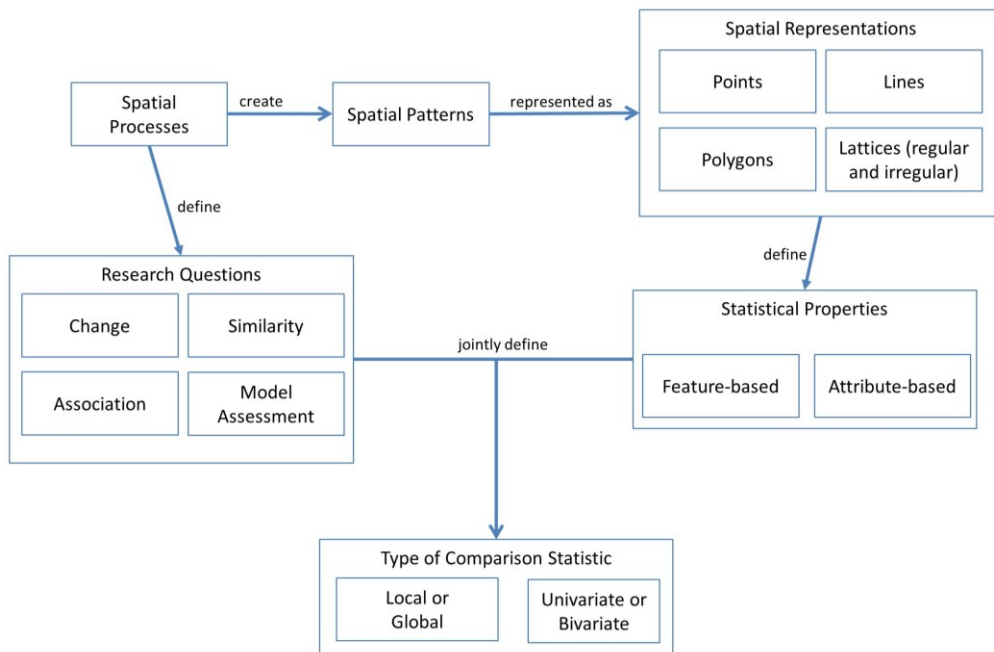


Figure 2: Four dimensions of spatial pattern important for spatial pattern comparison.

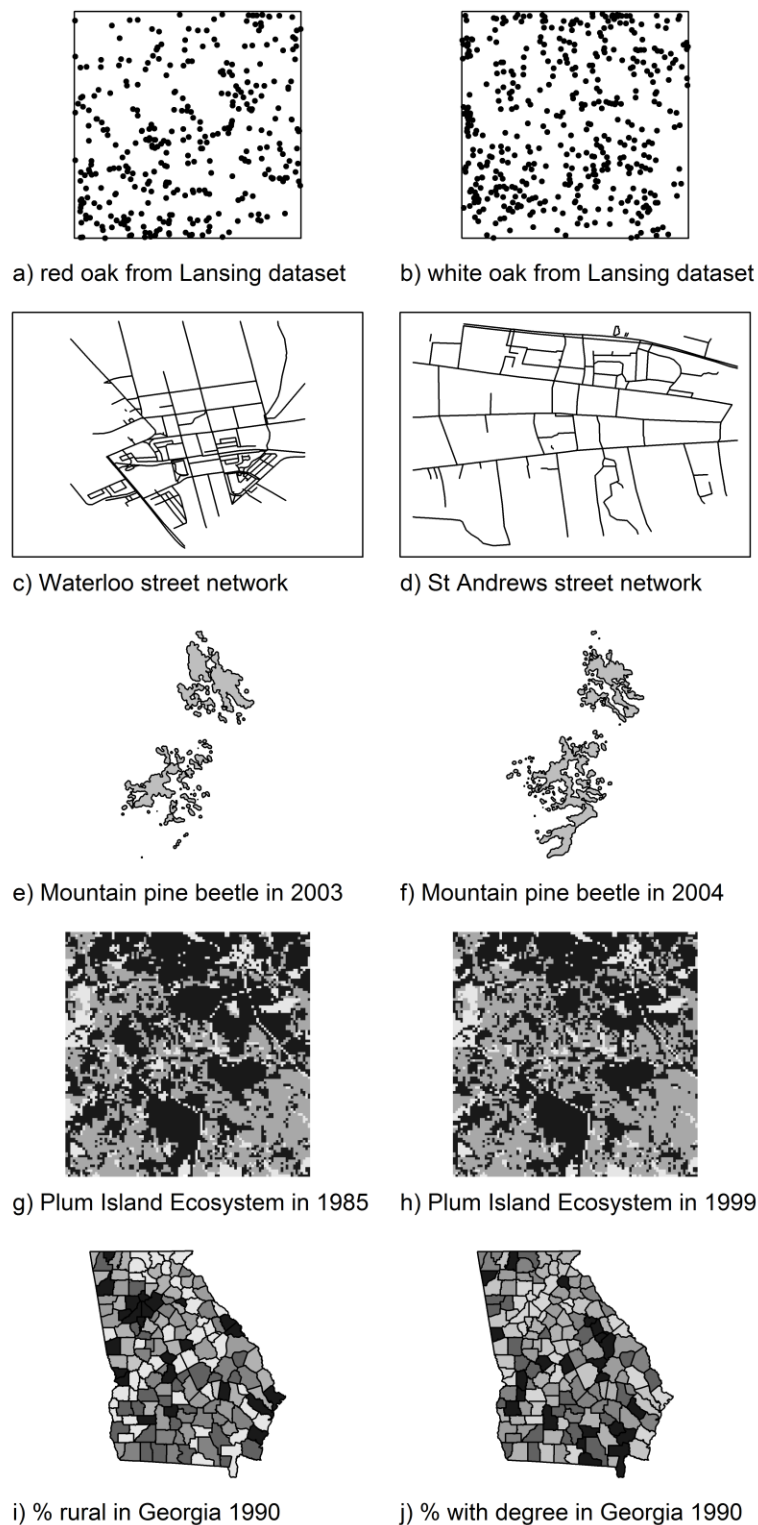


Figure 3: Example datasets used to explore different issues in spatial pattern comparison analysis. All data was sourced from spatial packages in the statistical software R. Data in a) and b) were sourced from the ‘spatstat’ package; c) and d) were sourced from OpenStreetMap using the ‘osmar’ package; e) and f) were sourced from the ‘stampR’ package; g) and h) were sourced from the ‘lulcc’ package; and i) and j) were sourced from the ‘spgwr’ package. Please see the text for appropriate references.

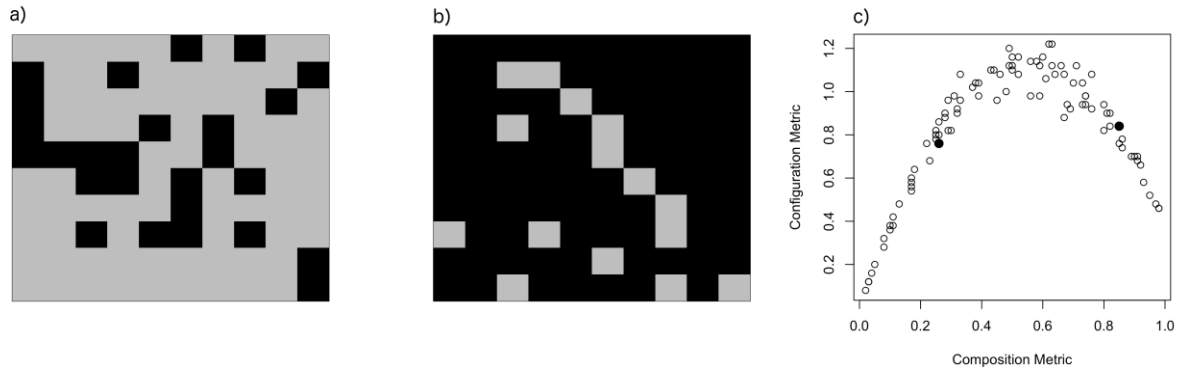
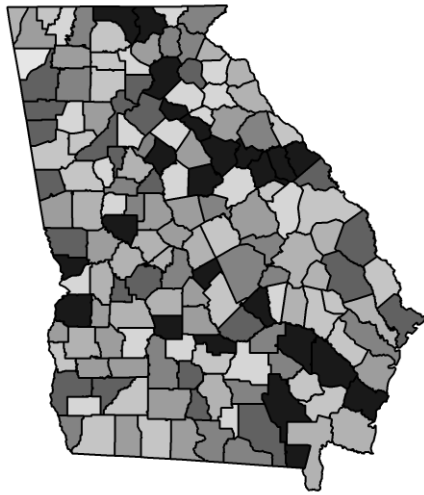
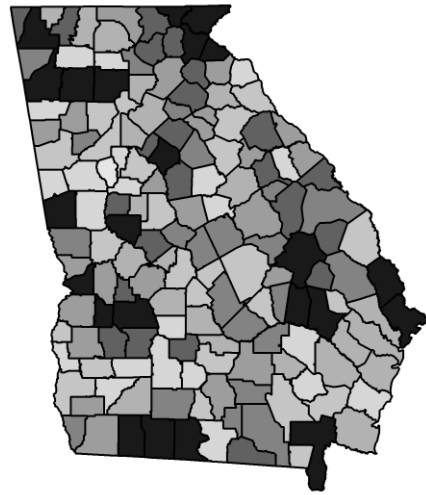


Figure 4: Example of composition and configuration metric dependency on random (uniform) landscapes with a) 30% composition and b) 80% composition. Edge density values for 100 random landscapes are given in c) with the sample landscapes highlighted.



Low SAC: $I = -0.169$



High SAC: $I = 0.274$

Figure 5: Low and high spatial autocorrelations of model residuals with identical values of root-mean squared error.