Analysis of algorithm components and parameters: Some case studies

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Abstract. Modern high-performing algorithms are usually highly parameterised, and can be configured either manually or by an automatic algorithm configurator. The algorithm performance dataset obtained after the configuration step can be used to gain insights into how different algorithm parameters influence algorithm performance. This can be done by a number of analysis methods that exploit the idea of learning prediction models from an algorithm performance dataset and then using them for the data analysis on the importance of variables. In this paper, we demonstrate the complementary usage of three methods along this line, namely forward selection, fANOVA and ablation analysis with surrogates on three case studies, each of which represents some special situations that the analyses can fall into. By these examples, we illustrate how to interpret analysis results and discuss the advantage of combining different analysis methods.

Keywords: forward selection, fANOVA, ablation analysis with surrogates, parameter analysis

1 Introduction

Given a parameterised algorithm with different design choices and a distribution of problem instances to be solved, *algorithm configuration* is the task of choosing a good design and parameter setting (a *configuration*) of the algorithm to be used. This can be done either manually or by an automatic algorithm configurator such as *ParamILS* [11], *SMAC* [8] or *irace* [14]. Besides finding a good algorithm configuration, algorithm developers are usually also interested in understanding how different algorithm components and parameters influence algorithm performance. Various approaches for analysing the importance of algorithm parameters have been proposed, such as [3, 1, 9, 10, 6]. The insights provided by these methods may produce useful knowledge that can be transferred back into the algorithm development process in an iterative manner, leading to higher performing algorithms.

Many analysis methods exploit the idea of learning prediction models from an algorithm performance dataset and then using them for the data analysis on the importance of variables. Among them, there is a group of methods that do not require a specific way of sampling algorithm performance data or particular experimental designs, are able to handle any types of algorithm parameters (including categorical ones), and have been shown to be applicable to cases with a large number of parameters. These include forward selection [9], fANOVA [10] and ablation analysis with surrogates [2]. Due to the general applicability of these methods, they can be used as a next step after the automatic algorithm configuration procedure to give more insights into the decisions of the configurator. More specifically, results of algorithm runs called by the configurator can be given to these analysis methods for building their prediction models.

Unlike the algorithm configuration problem, where the result obtained is a well-performing algorithm configuration, the analysis of algorithm components and parameters does not have a unique output. Different analysis methods can provide results on different perspectives. Forward selection identifies a key subset of algorithm parameters. fANOVA is based on functional analysis of variance [7], where the overall algorithm performance variance is decomposed into components associated with parameter subsets. Ablation analysis [6] addresses the local region between two algorithm configurations, and helps to recognise the contribution of each algorithm component or parameter on performance gain between two configurations under study.

The question of when to apply these methods is related to the kind of information we want to gain for a deeper understanding of how the analysed algorithm works. It might not always be straightforward for an algorithm developer to interpret analysis results and decide what to use for improving his/her algorithm. In the original papers of the three analysis methods [9, 10, 6, 2], interesting example applications in domains of machine learning, propositional satisfiability, mixed-integer programming, answer set programming, and automated planning and scheduling have been discussed. The main aim of this paper is to add to this discussion by illustrating the applications of these analysis methods on a number of case studies, each of which represents a specific situation where the advantage of combining different analyses is illustrated.

The implementation of all three analysis methods is from *PIMP* (`https://github.com/automl/ParameterImportance`), a Python-based package for analysis parameter importance provided by the *ML4AAD Group*. [1]

The paper is organised as follows. The three analysis methods [9, 10, 2] are briefly described in Section 2. The application and combination of the analysis methods on three case studies are then explained in Section 3. Finally, conclusions and future work are given in Section 4.

2 Analysis methods

2.1 fANOVA

The *fANOVA* method [10] is an approach for analysing the importance of algorithm parameters using a random forest prediction model and the *functional analysis of variance* [7]. Given an algorithm performance dataset, fANOVA first

[1] http://www.ml4aad.org/

builds a random forest-based prediction model to predict the average performance of every algorithm configuration over the whole problem instance space. Afterwards, the functional analysis of variance is applied to the prediction model to decompose the overall algorithm performance variance into additive components, each of which corresponds to a subset of the algorithm parameters. The ratio between the variance associated with each component and the overall performance variance is then used as an indicator of the importance of the corresponding algorithm parameter subset. fANOVA also provides some insights on which regions are good and bad (with a degree of uncertainty) for each parameter inside the subset through marginal plots. Given a specific value for each algorithm parameter in the subset, the marginal prediction value is the average performance value of the algorithm over the whole configuration space associated with all parameters not belonging to the subset. A marginal plot shows the mean and the variance of the marginal prediction values given by the random forest's individual trees.

An implementation of fANOVA is provided as a Python package,[2] wrapped by the PIMP package. As a choice of implementation, the package only lists the contribution percentage and shows marginal plots of all single parameters and pairwise interactions. It is also possible to acquire the importance of a specific higher-order interaction given a fixed value for each parameter in the interaction.

2.2 Forward selection

Forward selection is a popular method for selecting key variables in model construction. In [9], it was used to identify a subset of key algorithm parameters. Given a performance dataset of an algorithm, the method first splits this dataset into training and validation sets. Starting from an empty subset of parameters, the method iteratively adds one parameter at a time to the subset, in such a way that the regression model built on the training set using the resulting parameter subset yields the lowest root mean square error (RMSE) on the validation set. Random forest is used as the regression model, since it has been shown to be generally the best for predicting optimization algorithm performance in the literature [12]. Problem instance features can also be added into the analysis by treating them just as algorithm parameters. It should be noted that the resulting selection paths of different runs of this analysis could be different, due to a number of factors: (i) the prediction model's randomness (ii) the availability of correlated variables (iii) different splits of training and validation sets.

2.3 Ablation analysis with surrogates

If an algorithm developer already has a default configuration for their algorithm in mind, and receives a better performing configuration from an automatic algorithm configurator, he/she might wonder which parameters in the default configuration should be modified in order to gain such performance improvement. The

[2] https://github.com/automl/fanova

ablation analysis [6] answers this question by examining performance changes in the local path between the two configurations in the algorithm configuration space. Starting from the default configuration, the method iteratively modifies one parameter at a time from its default value to the value in the tuned configuration in such a way that the resulting configuration gains the largest amount of performance improvement. Important parameters in the local path can then be recognised by the percentage of their contribution to the total performance gains. The analysis can also be done in the reverse direction, in which parameters that yield the smallest performance loss are chosen first. Since results of the reverse path can be different from the original path, doing ablation in both directions was recommended.

The original ablation method [6] performs real algorithm runs during its search. Instead, in this paper, we take the surrogate-based version [2], where algorithm performance is provided by a prediction model. This allows re-using the algorithm performance dataset given by automatic algorithm configuration, hence reducing the computational cost of the ablation analysis significantly.

3 Case studies

3.1 Case study 1: Ant Colony Optimization algorithms for the Travelling Salesman Problem

In this case study, we consider ACOTSP [15], a software package that implements various Ant Colony Optimization algorithms for the symmetric Travelling Salesman Problem. The algorithm has 11 parameters, including two categorical parameters (*algorithm*, *localsearch*), 4 integer parameters (*ants*, *nnls*, *rasrank*, *elitistants*), and 5 continuous parameters (*alpha*, *beta*, *rho*, *dlb*, *q0*). These are configured by the automatic algorithm configuration tool irace [14] with a budget of 5000 algorithm runs. ACOTSP's default configuration and the five best configurations returned by irace are listed in Table 1. The improvement over the default configuration is statistically significant (Wilcoxon signed rank test with a confidence level of 99%). We apply fANOVA and ablation analyses with surrogates on the performance dataset obtained after the configuration step. The analyses aim at explaining the choice of irace on selecting the best configurations.

algorithm	localsearch	alpha	beta	rho	ants	nnls	q0	dlb	rasrank	elitistants
(Best configurations)										
acs	3	2.31	8.77	0.48	34	10	0.45	1		
acs	3	2.95	8.64	0.49	48	14	0.54	1		
acs	3	2.67	7.78	0.46	47	12	0.51	1		
acs	3	2.98	9.58	0.14	31	15	0.81	1		
acs	3	2.44	8.13	0.48	42	13	0.53	1		
(Default configuration)										
mmas	1	1.64	4.65	0.5	50	31		0		

Table 1: The five best configurations returned by irace and the default configuration of ACOTSP

fANOVA analysis. There is a difference between fANOVA and irace (default version) in the way the algorithm performance is measured. The random forest in fANOVA evaluates performance of an algorithm configuration as the mean of performance values across all problem instances. The default setting of irace, on the other hand, uses the Friedman test as the statistical test for identifying bad configurations, which means that the performances of configurations are converted to ranks before being compared. Each strategy has its own advantage. When the ranges of performance values greatly vary among different instances (for example, due to different problem sizes), the Friedman test can avoid the dominance of instances with large performance values. However, there are cases where the magnitude of performance difference is important. For example, when a configuration is slightly better than another one on many instances, but performs dramatically worse on a smaller fraction of instances, the latter configuration may be preferrable. In order to use fANOVA results to explain irace's decisions, performance values should be normalised so that they belong to the same range across different problem instances before they are given to fANOVA. For each problem instance, the normalisation value is calculated by equation 1:

$$normalised\_cost = \frac{cost - min\_cost}{max\_cost - min\_cost} \tag{1}$$

where min_cost and max_cost are the smallest and the largest cost values in the performance dataset for the corresponding problem instance.

Following is fANOVA's partial output on the normalised performance dataset:

```
All single-parameter effects: 87.63%
All pairwise interaction effects: 11.72%
localsearch: 77.85%
rho: 4.86%
... (remaining effects: < 4%)
```

Results indicate that single parameters and pairwise interactions can explain 99.35% of the total performance variance. The categorical parameter *localsearch*, which defines the choice of the local search used inside ACOTSP, is obviously the most important parameter. Its effect clearly dominates all of the others, as it explains 77.85% of the total algorithm performance variance. Its marginal plot shown in Figure 1 points out that 3 is the best value for this parameter, which explains the consistent choice of *localsearch* = 3 in the best configurations returned by irace.

Ablation analysis. Next, we apply the ablation analysis with surrogates on the path from the default configuration to the best one using the same normalised performance dataset. Figure 2 shows the order of parameters chosen and their corresponding percentage of performance gains along the path. Again, the *localsearch* parameter shows a clear dominance over the others, which is in line with fANOVA findings. In this local region between the two configurations, the

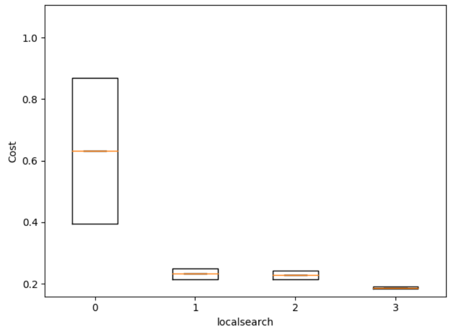


Fig. 1: Marginal plot of parameter *localsearch* given by fANOVA

influence of this parameter is quite strong, as it provides 92% of the improvement gain when switching from the value of 2 (default) to 3.

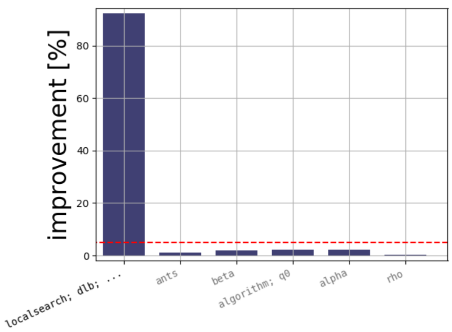


Fig. 2: Ablation analysis (with surrogate) on the path from ACOTSP default configuration to the best configuration returned by irace. The y-axis shows the percentage of performance gain when switching the value of each parameter.

It should be noted that the finding of fANOVA and ablation analysis on the most important parameters is not necessarily consistent in all cases. fANOVA analysis works on the whole configuration space, while ablation analysis focuses on the local path between two specific configurations. Therefore, the dominance of parameter *localsearch* given by both analyses emphasises the importance of this parameter on both the global and the local scale.

In the next case study, we will demonstrate an opposite situation where the analyses become more complicated and the impact of parameters on irace's choices is quite local.

3.2 Case study 2: tuning irace on a configuration benchmark

irace is an automatic algorithm configurator, but it also has its own parameters. In [5], irace is used to configure irace on different configuration benchmarks in a procedure called *meta-tuning*. To avoid confusion, the higher-layer irace is named *meta-irace*. In an algorithm configuration setting, the tuned irace is simply considered as a parametrised optimisation algorithm, and each algorithm configuration benchmark plays the role of a problem instance. In this case study, we consider one of the meta-tuning experiments in [5], where irace is tuned on a configuration benchmark of the mixed-integer programming solver *CPLEX* [13], namely *CPLEX-REG*, with a budget of 5000 irace runs. The tuned irace has 9 parameters, including 3 categorical parameters (*elitist*, *testType*, *softRestart*), 5 integer parameters (*nbIterations*, *minNbSurvival*, *elitistInstances*, *mu*, *firstTest*) and one continuous parameter (*confidence*). The default configuration of irace and the five best configurations returned by meta-irace are shown in Table 2. The improvement given by meta-irace over the default configuration is statistically significant (Wilcoxon signed rank test with a confidence level of 99%).

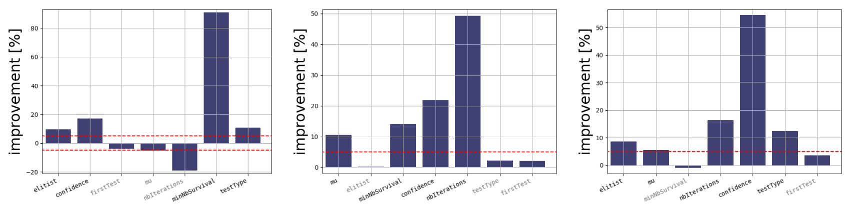
The aim of the analyses in this case study is to understand the choice of the configurator (here, meta-irace) on the best irace's configurations. We will show that the interpretation of the analyses here is more complicated compared to the previous case study. In particular, the ablation analysis in the two directions can provide different results due to the complex interactions between irace's parameters. Moreover, a complementary usage of various fANOVA package's functionalities to gain more insights into the findings given by the ablation analysis is also presented.

nbIterations	minNbSurvival	confidence	elitist	elitistInstances	testType	mu	firstTest	softRestart
(Elite configurations)								
35	2	0.52	false		t-test-holm	4	6	true
29	1	0.55	false		t-test-holm	4	5	true
37	1	0.51	false		t-test-holm	4	5	false
29	1	0.55	false		t-test-holm	3	5	true
12	1	0.77	false		t-test	7	5	true
(Default configuration)								
8	8	0.95	true	1	t-test	5	5	true

Table 2: irace's default configuration and the five best configurations given by meta-irace.

Ablation analysis. Ablation in both directions between two algorithm configurations was recommended [6]. The rationale for this is due to the possibility of interactions between parameters. This case study is a clear example of this situation. We will show that the reverse path can give interesting information that is not really obvious in the original path.

First, we apply the ablation from the default irace configuration to the best one for ten times. One thing we notice from the results is that the orders in which the parameters are chosen as well as their contribution on the performance gain vary quite a bit among different runs. This can be seen in the plots of three example paths shown in Figure 3. Because of those fluctuations, it is difficult to draw any conclusions from the ablation analysis in this direction.

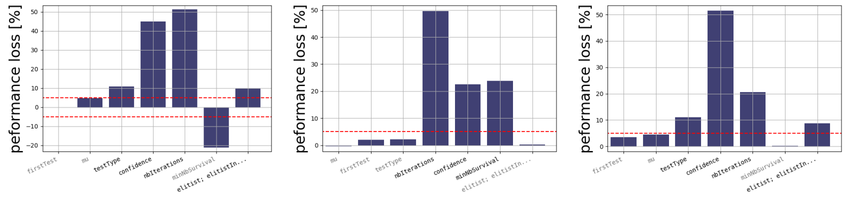


(a) Run 1 (b) Run 2 (c) Run 3

Fig. 3: Three ablation paths from the irace default configuration to the best one returned by meta-irace.

We apply another ten times the ablation, now on the reverse path from the best configuration to the default configuration. At each step, the parameter that introduces the least performance loss when changing its value from the best configuration to the default one is chosen. Figure 4 shows the reverse paths with the same random seeds as in the ones used in Figure 3. We can see that the orders in which the parameters are chosen along the paths are more consistent among different runs. In particular, the four parameters *nbIterations*, *confidence*, *minNbSurvival* and *elitist* are always chosen at the end of the path, which means that changing their values in the best configuration will cause more performance loss than all other parameters. Indeed, most of the performance loss on the paths are due to the three parameters *nbIterations*, *confidence*, *minNbSurvival*. Parameter *elitist* is constantly the last one chosen in the ten paths although it is not explicitly associated with a big loss in performance. This indicates two things: (i) the importance of setting this parameter at the right value for achieving the good performance of the best configuration, as changing its value in the path between the best configuration to the default one will induce larger performance loss than any other parameters, (ii) the strong dependency of this parameter and the others, especially the three parameters *nbIterations*, *confidence*, *minNbSurvival*; or, in other words, the possibly complicated interactions between these parameters in the local region between the two configurations under study.

fANOVA analysis. Next, we use fANOVA analysis to gain additional information on the findings given by the ablation. In particular, we want to see how parameter *elitist* interacts with the others, especially the three parameters *nbIterations*, *confidence*, *minNbSurvival*, and whether their impacts are global.



(a) Run 1 (b) Run 2 (c) Run 3

Fig. 4: Three ablation paths from the best one returned by meta-irace the default irace configuration.

We do two fANOVA analyses, one on the global space, dubbed *global-fANOVA* and one on the local space where the algorithm performance is not worse than the default one, namely *capped-fANOVA*. Parts of them are shown in Table 3.

global-fANOVA	capped-fANOVA
All single-parameter effects: 38.01%	All single-parameter effects: 11.69%
All pairwise interaction effects: 34.31%	All pairwise interaction effects: 28.17%
nbIterations: 11.8%	firstTest x nbIterations: 3.89%
minNbSurvival: 6.22%	minNbSurvival: 3.28%
firstTest: 5.73%	firstTest x minNbSurvival: 2.82%
testType: 5.25%	firstTest: 2.55%
confidence: 5.21%	minNbSurvival x nbIterations: 2.26%
... (remaining effects: <4%)	... (remaining effects: <2%)

Table 3: Partial results of fANOVA analyses on irace's performance data

The differences between the two analyses are that in the local one, single parameter effects become less important, both single parameter and pairwise interaction effects have less contribution to the total performance variance, and their contribution is more widely spread. One possible explanation for these differences is that in the local space, the interactions of algorithm parameters are more complicated. Anyway, one clear thing is that *eliltist* parameter is not among the top list effects in both analyses.

In capped-fANOVA results, the total contribution of all single parameters and pairwise interactions on the overall performance variance is less than 40%, leaving a potential for some important higher-order interaction. We can check if this is the case for any of the interactions between *elitist* and the three parameters *nbIterations*, *confidence*, *minNbSurvival* by giving those four parameters to fANOVA and asking for the percentage of variance explained by all relevant interactions. Results show that the total contribution of all (pairwise and higher-order) interactions between *elitist* and the other three parameters is only 5.18%, and it generally is spread evenly among them. It means that there are no important interactions as expected. Since capped-fANOVA works on the configuration space where performance value is not worse than the default configuration, the observations in this analysis indicate that the importance of *elitist* is even more local, i.e., it only works when not only the three parameters *nbIterations*, *confidence*, *minNbSurvival* are set properly, but also the other ones have to be in the appropriate ranges. In fact, if we look at the values of the other parameters in the default and the best configurations, we can see that they are already quite close to each other. For example, the parameter *firstTest* has the values of 5 and 6 in the default and the best configurations, respectively, while its domain is [4,20]. We can further confirm our conjecture as follows. First, we ask fANOVA for the predicted average cost of the default and the best configurations (here we get 138.92 and 135.81). Next, we modify the value of parameter *firstTest* in the best configuration to a more distant one, say 12. Then we ask fANOVA for the predicted cost of this new configuration (here we get 138.78). This increasing cost implies the importance of having *firstTest* near the local region around 6.

In summary, the conclusion from this fANOVA analysis is that the importance of irace parameters and the choice of the meta-irace are very local, i.e., although the four parameters *elitist*, *nbIterations*, *confidence*, *minNbSurvival* play essential roles in improving the performance between the default and the best configurations, the other parameters also need to be set in the appropriate ranges that are not far from their values indicated in the best configuration. Therefore, if irace's developers want to explain what changes in irace's behaviours (according to its parameters) make the performance improved, they will need to associate those changes with all parameters instead of only a few of them.

3.3 Case study 3: a Large Neighbourhood Search for a Vehicle Routing Problem with Time Windows

The example analyses so far are only on the algorithm configuration space. In this case study, we illustrate the integration of problem instance features into fANOVA analysis, so that we can study not only the impact of algorithm parameters, but also their interactions with instance features.

The algorithm and problem instances considered in this case study are provided in [4]. The algorithm is a Large Neighbhourhood Search (LNS) metaheuristic algorithm for solving a typical Vehicle Routing Problem with Time Windows. The algorithm has 8 parameters, including 2 categorical parameters (*repair*, *destroy*), 2 integer parameters (*random_seed*, *deterministic_parameter*) and 4 continuous parameters (*noise_parameter*, *cooling_rate*, *control_parameter*, *start_temperature*). There are 200 problem instances generated randomly according to 5 problem features, including 3 integer features (*customer_numer*, *customer_demand*, *max_running_time*) and 2 continuous features (*average_service_time*, *average_time_windows*). For each problem instance, 20 random algorithm configurations are randomly generated and tested. In total, the performance dataset contains 4000 data points.

fANOVA analysis. We can add instance features into fANOVA analysis by treating them as algorithm parameters, i.e., each combination of algorithm configuration and instance feature is given to fANOVA as an algorithm configuration. It must be noted that this integration can only be done when the instance features are independent, since this is one of the key assumptions fANOVA makes on its input variables. In this case study, the problem instances are generated based on random values drawn from all instance features in an independent way. A part of fANOVA's output is given in the left column of Table 4.

On original performance data	On normalised performance data
All single-parameter effects: 80.87%	All single-parameter effects: 60.74%
All pairwise interaction effects: 12.23%	All pairwise interaction effects: 15.38%
customer_number: 75.24%	repair: 52.68%
average_service_time x customer_number: 5.24%	destroy x repair: 4.42%
average_service_time: 3.78%	customer_number x repair: 4.19%
... (remaining effects: <2%)	customer_number: 4.0%
	destroy: 3.14%
	... (remaining effects: <3%)

Table 4: Partial results of fANOVA analyses on LNS algorithm performance data

An obvious observation from this result is that only instance features are listed as the most important variables. In particular, the feature *customer_number* has a clear dominant influence, as it explains 75.24% of the total performance variance. Looking at its marginal plot shown in Figure 5, we can see that the average performance values increase according to this feature. This means that the range of the algorithm performance values greatly depends on problem instance size (here it is the number of customers).

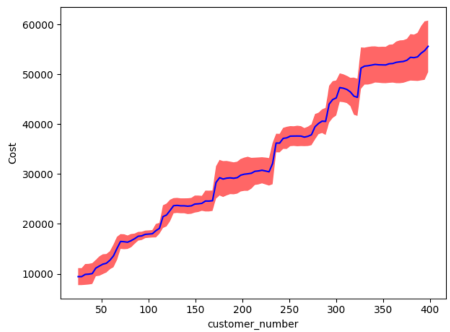
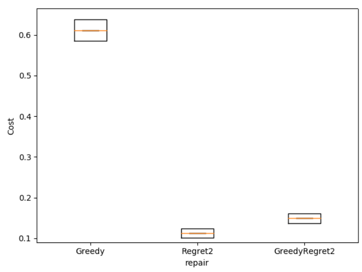
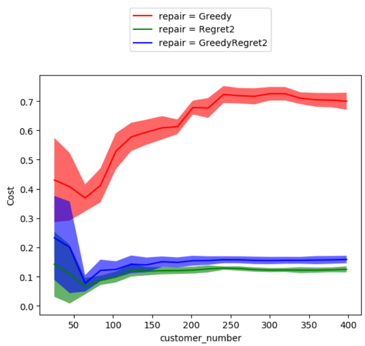


Fig. 5: Marginal plot of instance feature *customer_number* given by fANOVA

In situations like this, depending on the aim of our analysis, normalisation can be necessary. Here we aim to study the algorithm performance without any bias towards particular instances, which is similar to what irace's default setting does as described in section 3.1. Therefore, we re-run the fANOVA analysis, now with normalised performance values. A part of the new results is listed in the right column of Table 4. The impact of *customer_number* is now significantly reduced, so that the influence of algorithm parameters becomes visible. The two categorical algorithm parameters *repair* and *destroy*, which represent the choices of the repair and the destroy operators inside the Large Neighbourhood Search, now involves in the top list of important parameters and pairwise interactions. In particular, the high contribution percentage of parameter *repair* (52.68%) on the total performance variance and its marginal plot (Figure 6a) show that the repair operators Regret2 and Greedy work generally best and worst, respectively, across the whole problem instance space. In addition, we can also see how much influence the choice is according to different problem sizes by looking at the marginal plot of the pairwise interaction *repair* x *customer_number* (Figure 6b): on instances with the number of customers less than 50, the bad performance of Greedy is not as obvious as on instances with larger sizes; for the other two choices, Regret2 and GreedyRegret2, their overall performance difference also gets clearer when the number of customers increases. Other problem instance features do not have important interactions with algorithm parameters.



(a) repair



(b) customer_number x repair

Fig. 6: Marginal plots of the algorithm parameter *repair* and its pairwise interaction with the instance feature *customer_number* given by fANOVA analysis with performance normalisation.

Forward selection. Although the fANOVA analysis provides interesting insights into interactions between algorithm parameters and instance features, such an analysis, in this case study, takes a large amount of time to run (16 hours on a single core Intel CPU 2.4Ghz) [3]. The reason is due to the heavy computation of all pairwise interactions. In this part, we demonstrate a cheaper alternative for this particular case using a combination of fANOVA and forward selection. First, we use fANOVA to calculate the importance of single variables. Next, we use forward selection to identify the key subset of variables, and then use fANOVA to examine interactions of the variables in that set only. In this way, the whole analysis takes less than half an hour.

First, we apply fANOVA on single parameter effects only, which takes less than one minute to finish. Next, we apply forward selection to the normalised performance dataset. Since analysis results might vary among different runs, we repeat it ten times. It takes about 3 minutes in total. The visualisation of one run's result, which is a path of sequentially adding one variable to the model at a time so that the resulting root mean square error on the validation set is minimised, is shown in Figure 7a. We can see that the first three parameters chosen are *repair*, *destroy* and *customer_number*. This happens to be the case for all the ten runs; these three parameters are always chosen first, while the fourth parameter onwards in the path can vary among different runs. This indicates the high relevance of these three parameters on the prediction model.

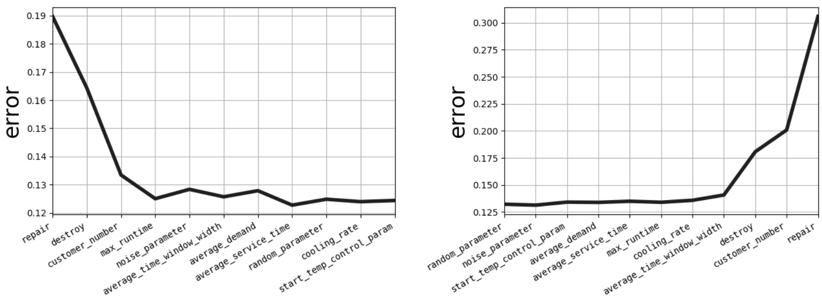
Since these experiments are computationally cheap, we also apply backward elimination, the reverse procedure of forward selection, where starting from the whole variable set, one variable resulting the least decrease in the prediction error is removed sequentially. This may help to take into account dependency between the parameters. These runs take about 5 minutes in total. The chosen path and the error values in one of the ten runs is shown in Figure 7b. The three parameters *repair*, *destroy* and *customer_number* are the last one to be chosen, and this is again consistent among the ten runs. This implies the importance of these parameters' interactions.

Given those indications, we can now focus on analysing the interactions between these three variables using fANOVA. Results of the importance quantification and marginal plots given the three variables *repair*, *destroy* and *customer_number* are obviously the same as the original fANOVA, with an addition of the three-way interaction between those variables. However, since the importance of this interaction is rather small (2.3%), we can simply ignore it. This particular fANOVA analysis takes about two minutes. So the whole analysis only needs 12 minutes.

4 Conclusions and future work

In this paper, we have demonstrated and discussed the complementary usage of three parameter analysis techniques, namely forward selection [9], fANOVA [10] and ablation [6, 2] analyses. This is done using three case studies, each of which

[3] This amount of running time is reported on the Python-based fANOVA package linked by PIMP. The previous fANOVA version, which is Java-based, is faster, although it still needs several hours to finish this analysis.



(a) Forward selection (b) Backward elimination

Fig. 7: Prediction error of the model trained on a subset of variables chosen during one run of forward selection and backward elimination analyses.

represents some special situations that the analyses can fall into. In the first and the second case study, where the aim is to use fANOVA and ablation analyses to understand the choice of the best configuration given by an automatic algorithm configuration tool (here irace), analysis conclusions diverge: in the first one, there is a dominant parameter in both the global configuration space and the local space between the default and best algorithm configurations, so that the algorithm developers can focus on this particular parameter for explaining the improvement over the default configuration; in the second one, the impact of parameters are very local and the algorithm performance improvement gained requires all parameters to be in their reasonable ranges of values, which means that the developers need to look at all parameter values as a whole instead of focusing on a few of them. In the third case study, the integration of problem instance features into fANOVA analysis is illustrated. This shows how the range of algorithm performance values changes according to problem instance sizes; hence, it emphasises the necessity of normalising performance data to avoid bias. The interaction between algorithm parameters and instance features also provides interesting insights into how the impact of important algorithm parameters can vary among different problem sizes. Moreover, a complementary usage of the forward selection analysis that can help to significantly reduce the computation cost of fANOVA analysis in this particular case study is also presented.

In future work, more case studies can be investigated, especially the ones for local search metaheuristics and evolutionary algorithms in solving combinatorial optimisation problems, where the applications of these analysis techniques are still scarce. Moreover, the integration of problem instance features into the combined analyses can be further investigated. As performance of optimisation algorithms might greatly vary according to different types of problem instances, insights acquired by studying both algorithm parameters and problem instance features would provide useful information for algorithm developers.

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