THE LESSER NAMES – THE TEACHERS OF THE EDINBURGH MATHEMATICAL SOCIETY AND OTHER ASPECTS OF SCOTTISH MATHEMATICS, 1867-1946

Marit Hartveit

A Thesis Submitted for the Degree of PhD
at the
University of St. Andrews

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The University of St Andrews

2010

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Abstract

The Edinburgh Mathematical Society started out in 1883 as a society with a large proportion of teachers. Today, the member base is mainly academical and there are only a few teachers left. This thesis explores how and when this change came about, and discusses what this meant for the Society.

It argues that the exit of the teachers is related to the rising standard of mathematics, but even more to a change in the Society's printing policy in the 1920s, that turned the Society's Proceedings into a pure research publication and led to the death of the 'teacher journal', the Mathematical Notes. The thesis also argues that this change, drastic as it may seem, does not represent a change in the Society's nature.

For this aim, the role of the teachers within the Society has been studied and compared to that of the academics, from 1883 to 1946. The mathematical contribution of the teachers to the Proceedings is studied in some detail, in particular the papers by John Watt Butters.

A paper in the Mathematical Notes by A. C. Aitken on the Bell numbers is considered in connection with a series of letters on the same topic from 1938–39. These letters, written by Aitken, Sir D'Arcy Thompson, another EMS member, and the Cambridge mathematician G. T. Bennett, explores the relation between the three and gives valuable insight into the status of the Notes.

Finally, the role of the first women in the Society is studied. The first woman joined without any official university education, but had received the necessary mathematical background from her studies under the Edinburgh Association for the University Education of Women. The final chapter is largely an assessment of this Association's mathematical classes.

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First of all, I must thank my supervisors, Professor Edmund F. Robertson and Dr John J. O'Connor. Their encouragement and inspirational attitude has been invaluable and working with them has been a pleasure from the start. I have benefited from their timely words of wisdom on many occasions.

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It must be mentioned that this thesis would not exist had it not been for Professor Audun Holme of the University of Bergen, who made me realise how fascinating the history of mathematics is and who pointed me in the direction of St Andrews.

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This thesis is dedicated to my paternal grandparents, Arnfred and Elsa.

Introduction

What defines a society? Is it its goals and intentions? Its activities? Or is it the people in it? These questions are important ones for the Edinburgh Mathematical Society. The EMS has changed quite a lot since its foundation in 1883, when it was dominated by schoolteachers. This evidently changed at some point, as today's Society is an academical one with very few teachers in it. This thesis explores how this change came about and why, and attempts to answer the question of whether this was as great a change to the Society's nature as it may appear at first glance.

This research project began with a much shorter project undertaken by my supervisors, Professor E. F. Robertson and Dr J. J. O'Connor, in connection with the 125th anniversary of the Edinburgh Mathematical Society. Amongst other things, I was involved in the digitalisation of the Society's Minute books from the ordinary meetings.[1] The strong contrast between the Society in the earlier days and the Society today fascinated me, and made me investigate the matter further. The result is this thesis.

It is not purely the satisfaction of curiosity that makes such a study worthwhile. It was once said to me, in a room full of mathematicians, that every single person in that room was there because he or she had had an inspiring teacher at school; a teacher who was passionate about mathematics and who managed to pass this on to the pupils. Unfortunately, he said, that kind of inspiring teacher appeared to be a dying breed. As will be seen, many of the schoolteachers of 1883 were this passionate about mathematics, and if it really should be the case that they are not so today, then perhaps this can be connected to their exodus from the Society. There will doubtless be many, complex reasons why teachers fail to inspire today, related to educational politics and so on, but one side of it is how attractive the profession appears to those who love mathematics. A graduate who wants to learn more and stay in touch with current mathematics will more often than not seek out greener pastures than can be provided within the school environment. In 1883, it was quite the opposite.

Changing this would doubtless lead to more inspiring teachers, and if the schools and the universities managed to find a common meeting ground, that could be a good start. As it turns out, this is precisely what the Edinburgh Mathematical Society was in the earlier days. Providing such a meeting ground became more difficult as mathematics became more advanced, and the Society was faced with several obstacles, obstacles that

[1] The results of this project are now to be found online at the MacTutor website [67].

they eventually failed to overcome. If one is to succeed at encouraging such contact between the institutions today, it is important to learn what went wrong in the past, and this thesis hopes to explain just that.

Chapter 1 provides the necessary background for the other chapters, as well as presenting aspects of the Society's history that have not been examined before. The chapter describes the circumstances regarding the foundation of the Society, such as who the founders were, and discusses why the Society was formed. The various activities, mainly the meetings, will be explored, before the membership and its development undergo a more thorough study. The occupations of the members will be placed under particular scrutiny. The chapter also considers the organisation of the Society, the Committee and its office-bearers, again focussing on the occupations of the people involved. Most of the remainder of the chapter is then devoted to a detailed study of the Society's two periodicals. The *Proceedings of the Edinburgh Mathematical Society* developed into a research periodical and the *Mathematical Notes* was established to deal with more pedagogical matters. Finally, some remarks will be made on what the nature of the Society truly was in these earliest days. Tables of data that were used for most of the graphs in this chapter will be found in Appendix A.

Chapter 2 will assess the mathematical contributions of the schoolteachers, mainly as papers to the *Proceedings*. This will be done in two different ways. First, all the papers written by teachers are considered as a whole. They will be organised by subject and by type, and the changing trends over time will be observed. After this general treatment, the contributions of one teacher will be studied in much more detail. This is the schoolmaster John Watt Butters who published seven papers and shorter notes between 1889 and 1904. The chapter will address the questions of what the teachers found interesting enough to write on and how much value their papers would hold outside the teaching sphere.

Where chapter 2 can be said to describe papers by teachers in the 'research publication', chapter 3 looks at the other side of the coin, and considers a paper in the Notes by an academic. A. C. Aitken of Edinburgh University published the paper 'A problem in combinations' in 1933. The topic was a particular sequence of numbers that are now known as the Bell Numbers. Dr Aitken would return to this topic six years later, in a series of letters between himself, Sir D'Arcy Thompson, the Professor of Natural History at St Andrews and Dr G. T. Bennett, a mathematician at Cambridge. The trigger for this correspondence was the enumeration of rhyme schemes; Dr Aitken was answering the question of how many different patterns of rhymes one can create with n lines of verse. This correspondence is examined in some detail. In addition to shedding light on the relationship between these three academics, the correspondence also illuminates Aitken's views on the status of the periodical, and shows that the Notes at least on occasion contained material that could arouse great interest in academic circles.

Chapter 4 deals with the events leading to the migration of the EMS into a research society and the exit of the teachers, focussing largely on a debate on the publications that took place between 1926 and 1931. The scene will be set, as it were, with a summary of the situation when this discussion began. The path towards the debate's culmination in 1931 is traced out; then an explanation of the controversy itself and the reasons for it is given.This controversy led to relatively large disruption within the Committee, with the resignation of two of its members, the cessation of the annual Glasgow meetings, and a complete revision of the programme for the following year. The chapter aims to answer two questions. What made the Society turn towards research, and did this mean there could not be a place for the teachers anymore?

Chapter 5 regards the women in the earliest days of the Society. There were not many of them at first, for the good reasons that the Society was aimed at people with university education, and Scottish universities did not admit women until 1893. The first woman to join did so earlier than this, in 1887, and this chapter explains how she received education comparable to university studies. The chapter appeared as a paper in the BSHM Bulletin in 2009.

Sources

Chapters 1, 2, and 4

Very little research has been done on the Society prior to this. Unlike the London Mathematical Society (LMS), that has been the topic of two papers in Historia Mathematica,[2] the Edinburgh society has not undergone any form of thorough study. The Society's President, Professor R. A. Rankin, published a shorter treatment of the Society's history in connection with the centenary [62].

A substantial amount of archival work was required for this thesis. The aforementioned digitalisation of the minute books consisted of summarising every meeting held in the first 64 sessions, from 1883 up to the end of the Second World War. The sum-

[2] See [64] and [63].

maries contain the location of the meeting, the name of the chairman, the papers that were read and any members that were elected. If anything out of the ordinary took place, this has been mentioned as well.

A lot of work needed to be done on the Society's archives. This collection, which is kept at the School of Mathematics at Edinburgh University, had not yet been organised, and no list existed of its contents. This has now to a large extent been remedied, and the process of making the collection more accessible has begun. The full index of all items in this archive is not yet completed, and this work will continue.

Very little exists from the Society's earliest days. Between 1883 and 1921, only the Minute Books from the ordinary meetings, the Register of Members and the Cash book are kept. The Register of Members was the Treasurer's list of current members, and contains the latest addresses and information on payment of subscription fees. The treasurer usually noted whether a former member resigned, was deleted on account of not paying fees, or was deceased.

Some correspondence, mainly in connection with the Committee, is kept from 1921 onwards, and there is more and more of this for later years. The Minute book of the Committee meetings start in 1926. The cover of this book is imprinted with 1 in Roman numerals, which could indicate that earlier meetings were not minuted, but this is not very likely. A much more plausible explanation is that the earlier minutes were lost before this book was bound with this particular imprint.

The Society's periodicals have been used extensively, especially for chapter 2. These are now freely available online ([30] and [31]). An index volume for the first series of the *Proceedings* has also been used. The copy that was used, belonging to the University Library at St Andrews, consists of two indexes bound together. The first covers the first 20 years [48] and the second the remaining 24 [14], finishing in 1926.

The letters referred to in chapter 4 have been included in Appendix C.

Chapter 3

This chapter deals largely with letters, all of which are included in Appendix B. The originals may be found in the D'Arcy Wentworth Thompson-collection at the University Library of St Andrews. Sir D'Arcy kept most, if not all, of the correspondence he received during the 1930s. He did not in general keep his own letters, unless he happened to make drafts. When Thompson's letters to Bennett are held in this collection, this is because they were returned to him on Bennett's death in 1943. A fair few notes in Bennett's hand are attached to Sir D'Arcy's first letter to Bennett, dated 20th of December 1938. These were presumably attached by Bennett himself, as they contain calculations and general notes on the Bell numbers. Thompson's letters to Aitken have not been found, neither in the Thompson collection nor in Aitken's relatively modest collection at the Centre for Research Collections at the University Library of Edinburgh University. No other collection of Aitken's personal papers has been located, and these letters are therefore assumed to be lost. Tables and diagrams have occasionally been simplified slightly for ease of print, but the changes are of a cosmetic nature only.

Chapter 5

Most of the sources for this chapter are to be found in the ELEA/EAUEW-collection labelled 'Gen. 1877' at CRC in Edinburgh. Some of the articles mentioned are stored in a box in the reference section of CRC. The online archive for the Scotsman was also used.

A note on the notation for sessions

The notation 'session 1897/98' can be rather cumbersome and for that reason the notation 'session 1898' has been preferred. 'Session 1898' is therefore defined to be the session beginning in November 1897 and finishing in June 1898. The choice to identity a session by the year it finishes in may look inconvenient, but the other alternative, identifying it by the year it begins in, would produce two sessions 1883, which is undesirable.

Abbreviations

- ELEA / EAUEW — The Edinburgh Ladies Educational Association / the Edinburgh Association for the University Education of Women
- CRC — Centre for Research Collections, The University Library of Edinburgh University
- EMS — The Edinburgh Mathematical Society
- EMS Archives — Archive of the Edinburgh Mathematical Society
- LMS — The London Mathematical Society
- Notes — The *The Edinburgh Mathematical Notes*
- OHE — Other forms of higher education, such as teacher training and technical training outside universities.
- PEMS — The *Proceedings of the Edinburgh Mathematical Society*
- PLMS — The *Proceedings of the London Mathematical Society*
- StASC — The Special Collections at the University Library of the University of St Andrews.

Chapter 1

The Early Days of the EMS

1.1 The Foundation

The Edinburgh Mathematical Society was founded on the 2nd of February 1883, when 53 gentlemen met in the Mathematical Classroom at Edinburgh University. They were there because they had all received a certain circular, proposing the establishment of a mathematical society. This circular had been sent to what was described in the minutes of the meeting as:

> ... gentlemen in Edinburgh, in Cambridge and throughout Scotland generally, whom [the authors] deemed likely to take an interest in such a society.

This circular had three authors, from now on referred to as the Founding Fathers. Two of them were schoolteachers; Alexander Yule Fraser and Andrew Jeffrey Gunion Barclay were both working at George Watson's College in Edinburgh. The third was the physicist Cargill Gilston Knott, who was the assistant of P. G. Tait, the Professor of Natural Philosophy at Edinburgh University. Most of the credit should go to the two teachers, as it was they who conceived of the idea and approached Dr Knott for assistance.

And what exactly did they propose? The circular describes it as such:

> It is proposed to establish, primarily in connection with the University, a Society for the mutual improvement of its members in the Mathematical Sciences, pure and applied.
>
> Amongst the methods by which this object might be attained may be mentioned: Reviews of works both British and Foreign, historical notes, discussion

> of new problems or new solutions, and comparison of the various systems of teaching in different countries, or any other means tending to the promotion of mathematical Education.

The focus would appear to be very much on the benefits to the Society's members; a society was to be created for the society's own sake, even if it was suggested that improving mathematical education would be a part of this. The set of motions that formed the basis for the first constitution was agreed to at this first preliminary meeting, and the phrase on the 'mutual improvement' appears there as well. The constitution says nothing about the aims of the Society, but it will be seen shortly that other records might indicate that the Society's real aim was something slightly different.

At the Society's first ordinary meeting on the 12th of March 1883, Professor George Chrystal lectured on 'Present fields of mathematical research'.[1] No transcript of this talk has been found, but it was referred to in the Scotsman on the following day.[2] According to this notice, Professor Chrystal spoke on the importance of raising the maximum standard in the secondary schools, and the accompanying need for highly-trained schoolmasters. The establishment of a mathematical society such as this would, he believed, be of great importance in this regard. This fits well with the circular, that mentioned the improvement of mathematical education specifically.

It would, however, not be correct to say that Professor Chrystal considered the Society a purely pedagogical entity intended for schoolteachers, as the newspaper article continued:

> Professor Chrystal went on to refer to a wide range of mathematical science at the present day, and the difficulty of keeping abreast of the literature on the subject, and pointed out the advantages to be secured by the members of the subdividing that work and communicating at the meetings the latest views in the different departments.

In other words, Professor Chrystal regarded the Society as a well-suited forum for the diffusion of knowledge, and not just any knowledge. He was here talking about contemporary research, which would obviously benefit more than just the schoolteachers. It can also mean that Professor Chrystal placed more emphasis on current research than the circular would indicate. Although the newspaper cutting does not mention this

[1] Professor Chrystal will be introduced more thoroughly in section 5.2.2.
[2] The Scotsman, Tuesday, 13th March 1883, pg. 4.

specifically, Chrystal also wished for the members to undertake research themselves. As the title indicated, he spent some time talking about the newest research, possibly with the intentions of giving the listeners ideas for possible research topics. He was certainly doing this at the meeting on the 9th of November 1883, where he suggested some geometrical problems he would like to see solved.

One who would agree with him was the Society's second president; the mathematical master Thomas Muir (later Sir Thomas Muir) gave his presidential address in February 1884.[3] The contents of this address imply a different aim of the Society, or at least present an alternative interpretation of the circular.

The talk, aptly named 'On the promotion of research', addressed the situation of mathematics in Scotland. Unfortunately, Mr Muir had little praise to give.[4] His brutally honest assessment of the situation at the Scottish secondary schools and the Scottish universities, which he claimed were little more than schools, forms a very interesting and engaging read, but still more important in this regard is his message on research. When speaking on the great success of the London Mathematical Society and its publications, he said:

> If we only be true to the self-denying aims which our Society started with, each one "bearing and forbearing" lest the Society should suffer, each one steadfastly and unselfishly working for the advancement of his science, then the success of the London Mathematical Society will assuredly be ours. True, no doubt, that it has all the mathematical giants of the nation for members, and that consequently our work must for a considerable time, perhaps for years, be trivial in comparison. But, gentlemen, we must remember that there is an immensity of work to be done for which giants are wholly unnecessary, and that no work is more useful than preparing the way for the giants of the future. [54, pg. 10]

He stressed that he believed every single member of the Society would be able to help advance mathematics, and made suggestions as to how this could be done. Of course, since he needed to draw attention to this, it also meant that this was not being done at the present time, and he hoped the Society would become a breeding ground for

[3]Muir was at the time working at the High School in Glasgow, and had worked as a university assistant before this. He was later to emigrate to South-Africa to become the Superintendent General of Education there. He authored *The History of Determinants*, a rather well-known work in five volumes, and received many honours, in addition to his knighthood in 1910. See [73] for more.

[4]This talk was printed privately and distributed to the members. It is sometimes to be found bound together with the second volume of the Society's *Proceedings*. It can also be viewed online at [54].

research. This was important, he said, because the shortcomings of British mathematics were not caused by a want for great names, of which Britain had comparable numbers, but by a want for lesser ones. It was at the lower levels that Britain was lacking mathematical power.

It is here worth observing that Muir was not only admiring the success of the LMS; he was saying the EMS should aim to become as important as the English society. He considered the LMS to be the role model, especially regarding publications. This will become important in chapter 4.

The 'mutual improvement of its members', an admirable goal as it may be, is not a very self-denying one, so this is presumably not what he was referring to. It is far more likely that he had the more altruistic goal of the 'improvement of mathematics', rather than the improvement of the individual member, in mind. It could be the Society's goal had changed over time, but Muir said it started out this way. It therefore seems more fair to say that the Society was founded with the aims of promoting mathematics in Scotland. The founding fathers most likely found themselves in perfect agreement with this, and it is entirely possible that they had this in mind all along, but merely chose to emphasise the benefits to the members as a recruitment strategy.

As for the intended improvement of mathematical teaching, that was only a first step in the larger scheme of things. There was certainly a need for it. Muir argued that raising the level of the secondary schools was a necessity for raising the level at the universities, but doing so was far from easy. Unlike the elementary schools, the secondary schools had at the time no governing body, and the improvements in a subject were often left to the individual teacher. Some progress towards stronger secondary schools with more academic curricula had been made, but this was before the day of the Scottish Leaving Certificate, which was launched in 1888, and this becoming a requirement for entrance to University (1892).[5] There really was nothing to control the level of mathematics at the secondary schools in 1884.[6]

[5]Professor Chrystal was to be very much involved with the establishment of the Leaving Certificate and it is not unlikely that he found some of his inspiration through the meetings of the EMS. It is also worth mentioning that Muir stressed the need for a proper school-textbook in algebra. This was two years before Chrystal published the first volume of his *Algebra: An Elementary Textbook for the Higher Classes of Secondary Schools and for Colleges*.

[6]More on this can be found in [3].

1.2 The Society's activities

For the first three years, the Society met on the second Friday each month, from November to July.[7] This was changed on the 9th of March 1923, to the first Friday of each month, with the exception of January. On the 10th of July 1885, they decided that the last meeting should be held in June instead of July, possibly because it was very common for city-dwellers of sufficient means to escape the city for the summer months. The April meetings was later cancelled due to the school holidays.[8] Their venues are described thoroughly by Professor Rankin in an article he wrote for the Society's centenary [62], but it bears repeating that the Society began holding meetings in Glasgow from 1901 onwards, eventually as often as twice a year. Meetings would later be held also at other Scottish Universities.

At the Society's ordinary meetings, talks and shorter presentations were given, usually by members of the Society. Eventually, these talks were published in the *Proceedings*. This will be covered by section 1.5. The Society occasionally held discussions, for instance in session 1888 when a prolonged discussion on the teaching of arithmetic covered several meetings. A longer discussion took place on the 12th of May 1899 on the elementary treatment of proportion, initiated by Professor G. A. Gibson of Glasgow University. There were several such discussions on the teaching of mathematics, but other discussions followed later, such as one on the application of mathematics to medical problems (15th of January 1926) and division algebras (13th of January 1928).

The discussions were not without results. The one in 1888 led to a report being sent to Her Majesty's Inspectorate of Schools (HMIS) for Scotland. Similarly, the Society met on the 19th of June 1891 for a special meeting where they considered the draft ordinances of the 'Universities Commission relating to Reputations for Degrees in the Scottish Universities', which also resulted in correspondence being sent, this time to the Universities Commission. The Society also sent recommendations to the various branches of the Secondary Schoolmaster's Association and to the Educational Institute of Scotland [89, 8 March 1895].

The Society was in later years asked to send representatives to the 'National Committee for Mathematics of the Royal Society of London' from 1920 onwards [89, 14 June 1924]. Delegates were also requested from the EMS to various congresses, such as the International Mathematical Congress at Zürich in 1932 [89, 4 June 1932].

[7] An exception was the first session, that only began in February.
[8] This was decided on the 13th of March 1896.

Other arrangements were occasionally made. On Saturday the 15th of June 1889 the members went on a trip to see the construction of the Forth Bridge. At the behest of the Astronomer Royal of Scotland, there was also an excursion to the Observatory on the 29th of March 1901.

The Colloquia

The well-known St Andrews colloquia of the Society began in 1926, but before that, the Society held two colloquia in Edinburgh in 1913 and 1914.[9] The first one was the first of its kind in Britain and was set up largely to explore Professor Whittaker's new Mathematical Laboratory at Edinburgh University.[10] Three courses of five lectures each were given:

- 'The theory of relativity and the new physical ideas of space and time', by A. W. Conway, Professor of Mathematical Physics, University College, Dublin.
- 'Non-Euclidean geometry and the foundations of geometry' by D. M. Y. Sommerville, Lecturer in Mathematics in the University of St Andrews.
- 'Practical harmonic analysis and periodogram analysis; an illustration of mathematical laboratory practice' by E. T. Whittaker, Professor of Mathematics, University of Edinburgh.

The Scotsman contained a report from each of the five days the colloquium lasted. The first of these shows that of the colloquium's 76 members, 20 were university professors or lecturers, and nearly 50 were schoolteachers. These courses, which cannot be said to be on elementary topics, were deliberately set to a level accessible to teachers, and the colloquium was advertised in the *Mathematical Gazette*.[11] Although the St Andrews colloquia would retain this aspect of accessible talks to some degree, the level did eventually surpass that of the teachers, as will be touched upon in a later chapter (see 4.3.6). The second colloquium in Edinburgh was held in connection with the celebrations of the Napier tercentenary. It was most likely rather similar to the first one. It is not known how many of the attendants were teachers, but the courses were on similar lines.

[9] The St Andrews colloquia are discussed further in [62] and [67].
[10] Professor E. T. Whittaker's involvement with the Society will be discussed in more detail in 4.
[11] This was the journal of the English organisation for mathematical teachers, the Mathematical Association.

1.3 Membership

1.3.1 Election

> It is suggested that the Society be formed, in the first instance, of all those who shall give their names on or before February 2, 1883, and who are
>
> 1. present or former students in either of the Advanced Mathematical Classes of Edinburgh University,
> 2. Honours Graduates in any of the British Universities, or
> 3. recognised Teachers of Mathematics;
>
> and that after the above mentioned date, members be nominated and elected by ballot in the usual manner.

This is how the original circular defined the qualified members. At the preliminary meeting, when the Society agreed on what the rules should contain, it was decided that ordinary members should for the time being be defined this way. When the Committee finalised the rules no such restrictions were in place. Perhaps it was assumed that such criteria would rarely be required, as few who did not fulfil them would be interested in joining. Such a rule would effectively have ruled out women, as they did not have entrance to the Scottish Universities at the time. Since there was no such rule, when a woman appeared who had sufficient mathematical background but no University degree, she was allowed to join. This is described in more detail in chapter 5. New members were to be elected, by being proposed (and seconded) at one meeting and balloted for at the next.[12]

The Honorary Members did have some restrictions on them, but no more than being 'persons eminently distinguished in the Mathematical Sciences' [89, 2 Feb 1883]. Ordinary members could be nominated by any member, whereas Honorary Members could only be recommended by the Committee. It was decided, however, that the Committee was to do so having had a candidate suggested by three of the members. Should the Committee not wish to proceed with the proposal, the three members would be allowed to bring it before the Society themselves. At the first meeting, it was agreed that all Professors of pure and applied mathematics of the Scottish Universities were to

[12] The minutes from the meetings do not record rejection of any potential member, but it is of course very likely that they would not wish to minute this.

be invited as Honorary Members.[13] In the end, this was not incorporated into the rules, as it was decided that this should be done at the initiative of the members. Interestingly enough, most of the professors of Mathematics were not honoured this way, but most of the professors of Natural Philosophy were.

The rules governing election remained unchanged for 50 years. The whole constitution was reworked around 1931, forming what will be referred to as the second constitution. This will be examined further in chapter 4. The changes regarding membership concerned fees for the most part, but the 'restriction' that Honorary Members had to be 'persons eminently distinguished' was no longer explicitly stated. The clause requiring nominations for Honorary Memberships to be initiated by three members was also removed.

1.3.2 Subscription fees

The fee was originally set to be 5/−. Two years later, this was raised quite substantially to 7/6 [89, 10 July 1885]. This was presumably a result of the publishing of the *Proceedings*, that they had had no intention of doing when setting the fees first time around. It was raised again in 1919, to 10/− [89, 12 Dec 1919]. The fee included free sets of the *Proceedings* and later the *Mathematical Notes*.

The Society also instituted life memberships. The first to request such a membership was Professor Asutosh Mukhopâdhyây, a Professor of Mathematics from Calcutta, who joined in November 1888. He was granted this, on paying the rather astonishing amount of £10/7/6. When the Society agreed this was 'not to be considered a precedent' [89, 8 June 1888], they were only referring to the fee, not the life-membership. They did acknowledge that such a membership would be very useful for their overseas members. On the 14th of April 1893, they formally instituted a life membership for members outside the UK, available at the payment of £4. A life membership for home members followed in 1915, the fee being £5/5/0, or five guineas, with a reduction of £1/1/0 for every ten year's membership [89, 12 Nov and 10 Dec 1915]. This was changed again in 1927, when it was agreed that the new fee should be £6/6/0, with a £1/1/0 reduction after ten year's membership, and then £1/15/0 for every five years of membership after that [89, 4 Feb 1927].

[13] The motion in question, motion 5, actually says 'the three Scottish Universities'. This is presumably a mistake, as there were four Scottish universities at the time. Motion 4 states that Professor Chrystal and Professor Tait of Edinburgh University shall be elected as Honorary Members, so presumably motion 5 should have said 'the remaining three Scottish Universities.

The second constitution, adopted in 1931, incorporated all these changes, plus a very practical one that a new Society would not have thought of: members whose subscriptions were more than three years in arrears would from now on be considered to have left the Society.

1.3.3 Members by occupation

For the purpose of this thesis, it has been necessary to organise the members by occupation as far as possible. A comprehensive list of all the members for the first 64 sessions has been compiled based on lists of members that were published in the *Proceedings*. These lists include postal addresses for each member, which were often the workplace of the member in question, and often titles.[14] A member has been assumed to be a teacher when a school is given as address. The same procedure cannot be applied as easily to members who gave a university as their addresses, as students were occasionally listed by their university. The various calendars for the Scottish universities, with their lists of staff, have been used instead. The calendars have been consulted for all 64 years. Memberships of professional organisations, such as the Faculty of Actuaries (FFA) have occasionally been used to classify members.

The information produced this way has been added to in several ways. Firstly, the treasurer kept a running Register of Members which, in addition to stating when members left the Society, occasionally provides more information on addresses and occupation than the published lists do. The Society's archives hold some correspondence from members after 1921, and letters have occasionally given a member's occupation. The Oxford Dictionary of National Biography has been used with success in certain, rare cases.[15] Quite a few of the EMS members were also fellows of the Royal Society of Edinburgh. The RSE's compilation of Former Fellows [84] has helped find the occupations of many. On one occasion, one member was identified through an article in The Scotsman.[16]

The members were then divided into the following categories:

- Teachers — Teachers in the Secondary Schools. The larger part of this group was spread throughout Scotland, but a few joined from abroad.[17]

[14]Lists were published for most years, with exceptions in the first 10 and the last 20 years. Lists exist for sessions 1–3, 6, 8, 11–44, 47, 49, 51, 55, 56, 64.

[15]One was Henry Dyer, a Scottish engineer, who was a member of the Society between 1884 and 1887.

[16]This was the builder William Finlayson, as identified by The Scotsman, Saturday 29th January 1921, page 18.

[17]One example is F. E. Edwardes, who was Mathematical Master at Harrow School, Middlesex. He presumably joined

- Academics (Universities) — Academic staff at universities or university colleges. Research students have been counted as academics.

- Other forms of higher education (OHE) — Employees of more technical and professionally oriented institutions of education, such as engineering schools and teacher training. These have not been considered when comparing teachers and academics, as they did not obviously belong in either category.

- Students —There were very few student members, as the Society would hold little interest to students without knowledge of university mathematics. The few students that joined were usually studying for a second degree, for instance doing the mathematical tripos at Cambridge, after completing a Scottish M.A. A few were also students in divinity, or undertaking teacher training.

- Other — Actuaries, clergymen, solicitors and everything else not covered by the above.

- Unknown — Many of these would surely be schoolteachers, but no evidence has been found. A large proportion of these joined for one or two sessions only.

Figure 1.1 contain some erratic dips in membership.[18] This is not because members left in groups, but rather because many of them did not actively leave the Society at all. The Treasurer went through the list of members every now and then and deleted those whose fees were greatly in arrears. This was for instance done around 1893. No one was deleted during the First World War or for some time after (though some resigned) so that when the list was cleansed yet again around 1922, the number of members sank abruptly once more.

There is no explanation to be found in any of the records for why there suddenly was a surge of new members around 1905. There are no minutes preserved from Committee meetings from this period, and very little in forms of other documents. If one is to speculate, it is worth observing that these new members were predominantly teachers. It is possible that the Society was actively trying to recruit them. Other circumstances may have influenced the matter as well. The mathematics curriculum for the secondary schools was undergoing important changes around this time, where the order of Euclid

in order to publish, and five of his papers appears in the *Proceedings*. He was a member in sessions 1909–29. Another is E. S. Awad, a schoolteacher from Egypt, member in sessions 1931–36.

[18] Tables containing the data for these and the remaining graphs are found in Appendix A.

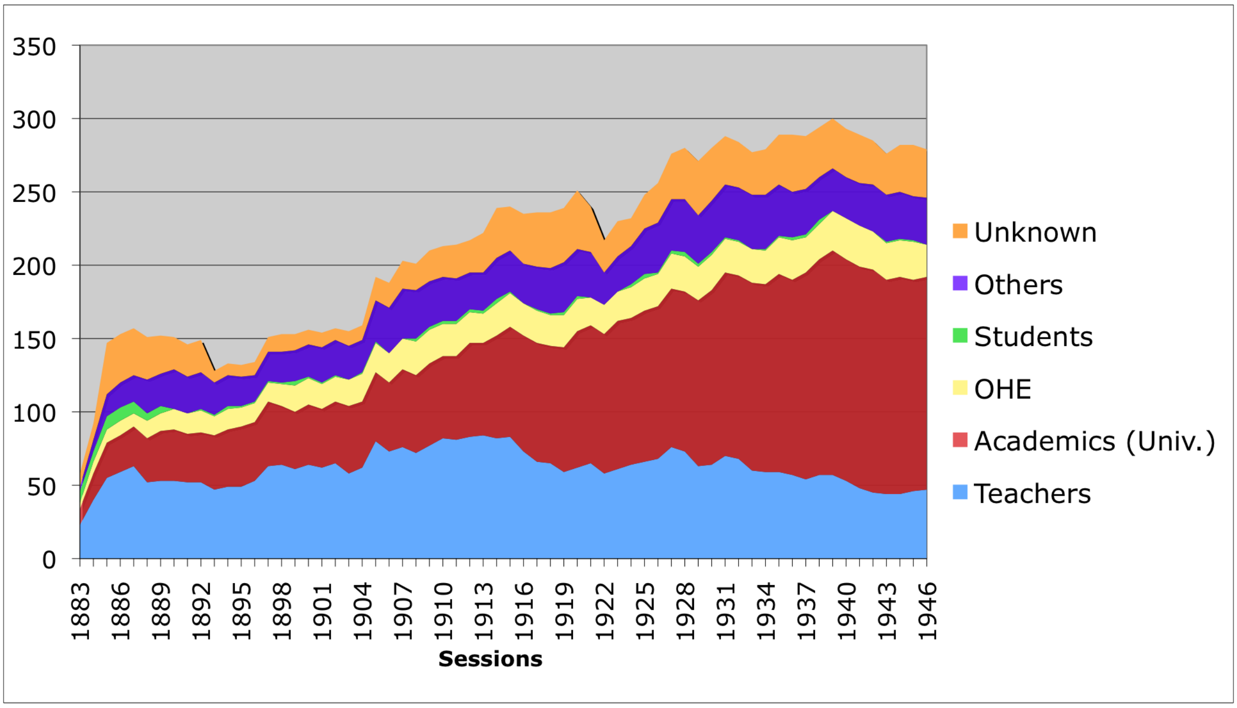


Figure 1.1: Members by occupation (1883–1946)

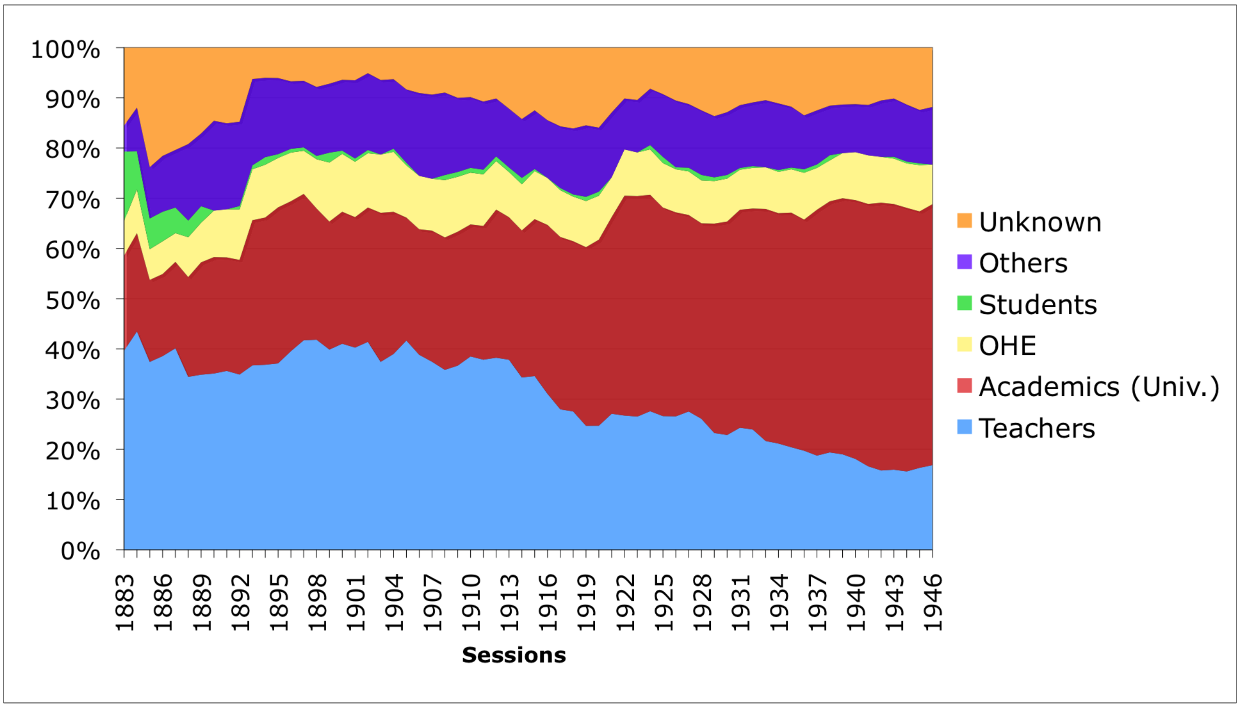


Figure 1.2: Members by occupation — by % (1883–1946)

was abandoned.[19] It is possible that such changes would inspire some teachers to get more involved in mathematics and mathematical societies, either to support the ongoing changes, or to voice their disapproval of them.

Depending on the ‘unknowns’ in the above categorisation, the teachers may have been in the majority among the ordinary members in the early years, but although it is very likely that many of the ‘unknowns’ were teachers, it is very unlikely that all of them were.

1.4 The Committee

The teachers were, however, heavily represented on the Committee. The very first Committee was elected at the preliminary meeting on the 2nd of February 1883. It consisted of a modest 7 members, with three office-bearers occupying four offices. The Mathematical Master (and later Dr) John Sturgeon Mackay was the first president. The post of Vice-President was held by Alexander Macfarlane, at the time University Examiner at Edinburgh University. The founding father C. G. Knott was appointed both secretary and treasurer.[20] The remaining four were ordinary committee members, comprising the two remaining founding fathers, Fraser and Barclay, Robert Edgar Allardice, who was Professor Chrystal’s assistant at Edinburgh University, and William James MacDonald, Mathematical Master at Daniel Stewart’s College. The number of ordinary committee members increased from four to six in the second session [89, 11 July 1884]. This was the only time such a change was reported in the minutes. The total number of committee members, including Office-Bearers, remained around 10 until session 1924 (the 42nd) from then on it fluctuated between 14 and 17. The four Office-Bearers were joined by a fifth in the 42nd session when David Gibb was elected Honorary Librarian.[21]

The first Constitution stated that the Committee should meet once a month, if not more, and that the Honorary Secretary was to be Convener. The second constitution was slightly less ambitious regarding number of meetings; the required number now being four per annum. The secretary was still to be convener. The office-bearers now included the editors of the periodicals, and the number of additional committee

[19] This meant that the geometrical propositions were taught in a different order than in Euclid’s Elements. This change in curriculum is touched upon in a later section. See section 2.3.

[20] This was a joint office, until the two posts were separated on the 10th of July 1885.

[21] Gibb had served as librarian for many years before this, but this was when the post was recognised formally. Before that he had been the convener of a short-lived library committee with three members.

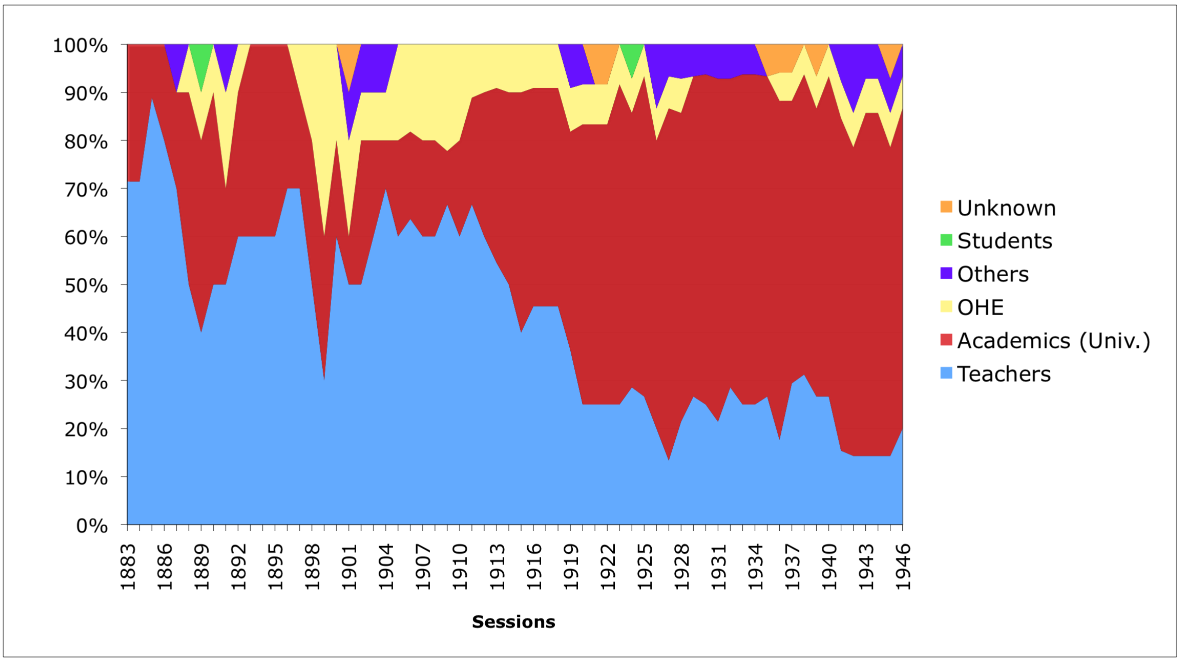


Figure 1.3: Committee members by occupation — by % (1883–1946)

members was capped at eight. The increased number of committee members can explain why the quorum was raised from four to five.

The first committee, with its large proportion of teachers, was not unusual for the first 40 years, as shown in figure 1.3. In fact, the teachers would appear to be rather over-represented for these earlier years, as indeed they were when compared to figure 1.2. The teachers were even more heavily represented when considering the Office-Bearers of the Society. Rankin writes in his article [62] that seven of the first ten presidents were teachers, and it does not stop there; 24 of the first 64 presidents belonged to this profession, though it is worth noting that 21 of these were during the first 37 years.

The graph in figure 1.4 shows the number of teachers and academics in the four offices of President, Vice-President, Honorary Secretary and Honorary Treasurer.[22]

The teachers were clearly a force to be reckoned with in the earlier days, holding 3 or 4 of the posts for no less than 24 of the first 30 years. After the First World War, the teachers and academics appear to have switched places. The same is true for the committee in general (Fig. 1.5), though the switch takes slightly more time to occur

[22] It should be noted that there were only three people in office for the first three years and for the 47th session (1919). This is because of the aforementioned joint office of Secretary and Treasurer for the first three years, and because E. T. Copson was both Secretary and Vice-President in session 47. In these very few cases, the one holding two offices have been counted twice. The office of Honorary Librarian has not been included in this analysis.

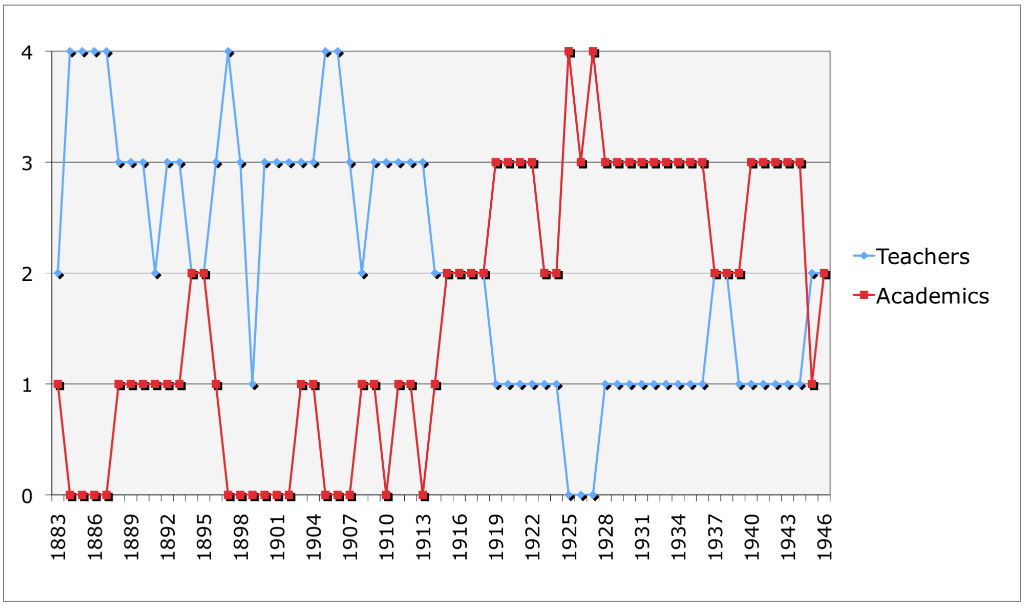


Figure 1.4: Teachers and academics in office

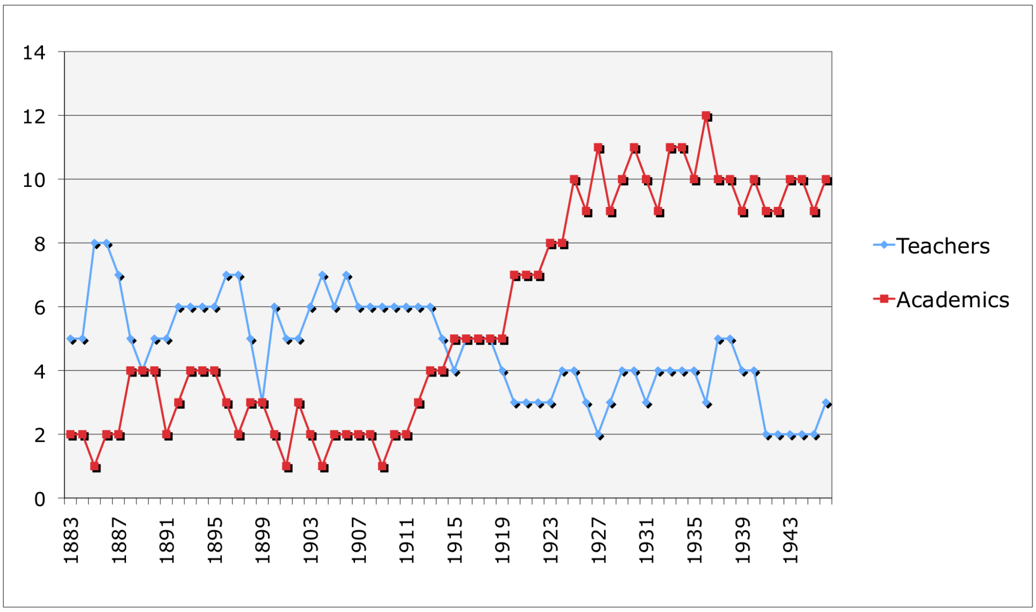


Figure 1.5: Teachers and academics on the committee

there.

It is no wonder that the teachers were over-represented and that the opposite held for the academics, as many of the professors were in the earliest days barred from service. The Honorary Members could not hold any office in the Society, so as long as the professors became honorary members by default, the academic pool remained somewhat small. This is quite possibly why the Society eventually abandoned this custom of honouring the professors this way.

1.5 The Publications

1.5.1 *The Proceedings of the Edinburgh Mathematical Society*

Rankin explains in his article [62] that it was common practice for scientific societies to publish their proceedings. The EMS did not do this at first, simply for want of funds. As Thomas Muir expressed in his Presidential Address, he, and presumably others with him, believed that a publication of some form would play a pivotal role in the Society's success. It is probably through his influence that the *Proceedings* appeared for the first time before the start of the third session. Members were invited to purchase this "first" volume, which was actually called Volume 2, for 2/6. Later that same year, the subscription fees were raised to accommodate publication costs and from then on the members received the *Proceedings* automatically. In 1893, the Society decided to put copies of the *Proceedings* out for sale to the public [89, 10 March 1893].

The reason that the first volume to appear was called Volume 2 was not only to reflect the session it covered, but also because they intended to publish a volume covering their first session. This was done, but not until 1894. For the first 44 sessions, the Society published one volume each year, each volume being issued as a whole. In the 45th session, in 1926, they decided to end this first series of the *Proceedings*. A new one was begun that was arranged somewhat differently. One volume now spanned several years, with seven volumes being published between 1927 and 1946. There were four parts to each volume, the parts issued irregularly, depending on what finances allowed. The time between each issue normally varied between four months to one year. The circumstances regarding this second series, and the other changes it brought about, will be discussed further in chapter 4.

For the first five years of its existence, the *Proceedings* was not given specific editors.[23]

[23] It is not unlikely that the Honorary Secretary filled this role during this period.

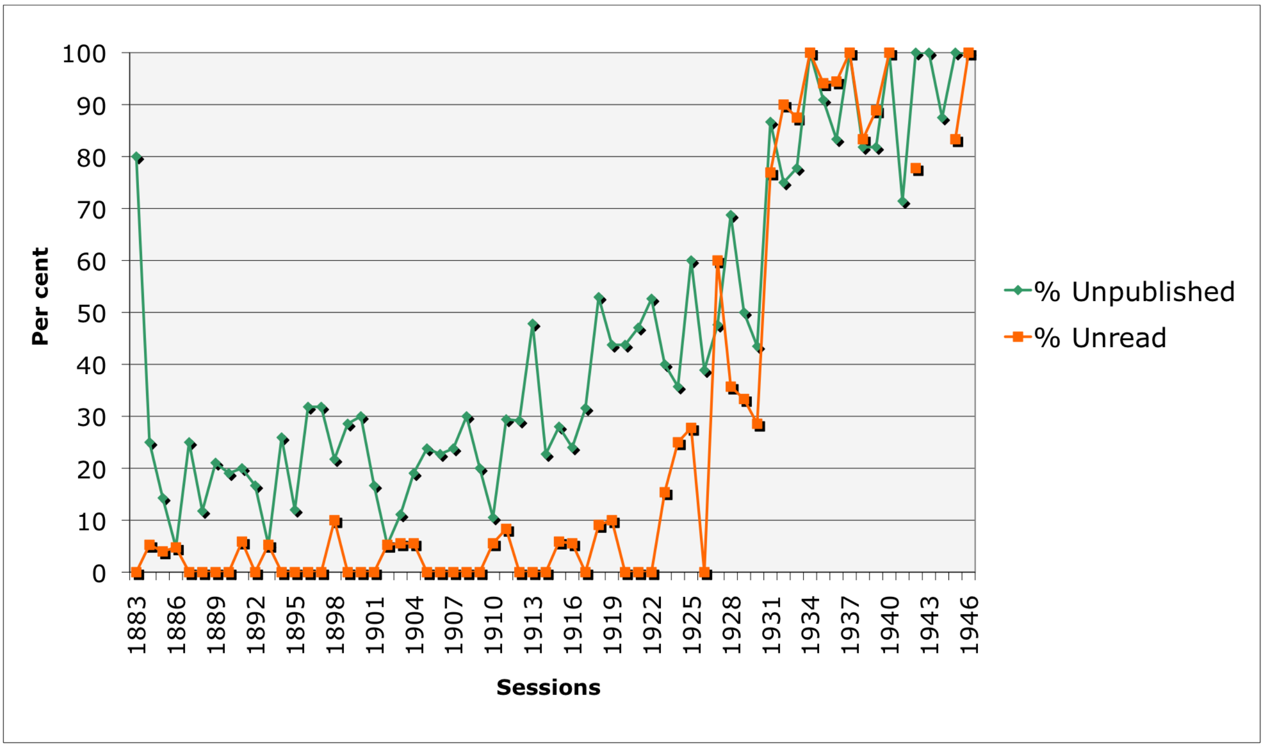


Figure 1.6: Talks unpublished and papers unread

In the seventh session, in November 1888, Robert E. Allardice and William Peddie, both university assistants at Edinburgh University, were appointed to this role. For the following three years, Allardice held the post alone, while also being Vice-President (session 1890) and President (session 1891). There would from then on be between one and three editors, though two was the norm. The full list of editors will be found in Appendix A.

1.5.2 The unread papers and the unpublished papers

The periodical was at first the proceedings in the true sense of the word. For the first 40 years, most of the talks given to the Society appeared in printed form in one way or another.[24] The first Constitution stated that abstracts of all the talks were to be deposited with the secretary 'for preservation'. This was presumably not always done, or done in a way unsuitable for publishing, or there would not be as many unpublished papers for the early years, as shown in figure 1.6. This is especially the case for the

[24]On occasion only the abstract was printed, such as [87].

first volume, with its unusually high percentage of read, but unpublished papers. This is perhaps not to be wondered at, as it took 11 years for it to appear.

The other side of the coin is, of course, the unread papers. These were published but not given as lectures. For the first 40 years, these were relatively few in numbers. The very few that were published were usually short notices, solutions to problems proposed in other papers, and on one occasion, a review.[25] As may be expected, when the percentage of unpublished papers began to rise, so did the percentage of unread papers.

It should here be noted that the very sharp dip in unread papers in session 1926 is unlikely to be entirely correct. It is very unlikely that they would have time to cover as many as five research papers at one meeting, as the minutes claim here. The procedure of reading papers by title was relatively new at the time, so it is far more likely that it was omission to record this properly that caused this dip.[26]

1.5.3 Authors

For the purpose of this work, it is useful to study the occupation of the authors in the *Proceedings*. For the first 40 years, this also serves as a close approximation to a study of the speakers at the Society's meetings. The first figure, showing the first five years, is fairly representative for the late nineteenth century, with a large proportion of papers written by teachers. This changed gradually, evolving into the second graph, which is more representative for 1900–1920. The teachers were still active, but less so than they were. The third graph shows a radically different picture, where the percentage of papers authored by academics has climbed to 80%. The change occurred almost overnight, going from 3 papers written by teachers in 1919 to none in 1920. It was to change even more after 1926; in the following 20 years only one paper was published by a teacher, as far as is known.[27]

1.5.4 Topics

Following the classifications from the indexes makes organising the papers by subject a relatively simple affair for the first series (Fig. 1.9). The indexes for the first series

[25]This was Professor G. A. Gibson's *Revue Semestrielle des Publications Mathmatiques*, which was a review of a journal and appeared in the *Proceedings*, Vol. 11.

[26]If further evidence is necessary, it may be worthwhile pointing out that the secretary was still switching between notations when he noted unread papers in the year before and after, showing that he was still not entirely used to it.

[27]This was J. W. Head's 'The Veronesean of quadrics and associated loci', PEMS Series II, Vol. 5, pp. 14–25.

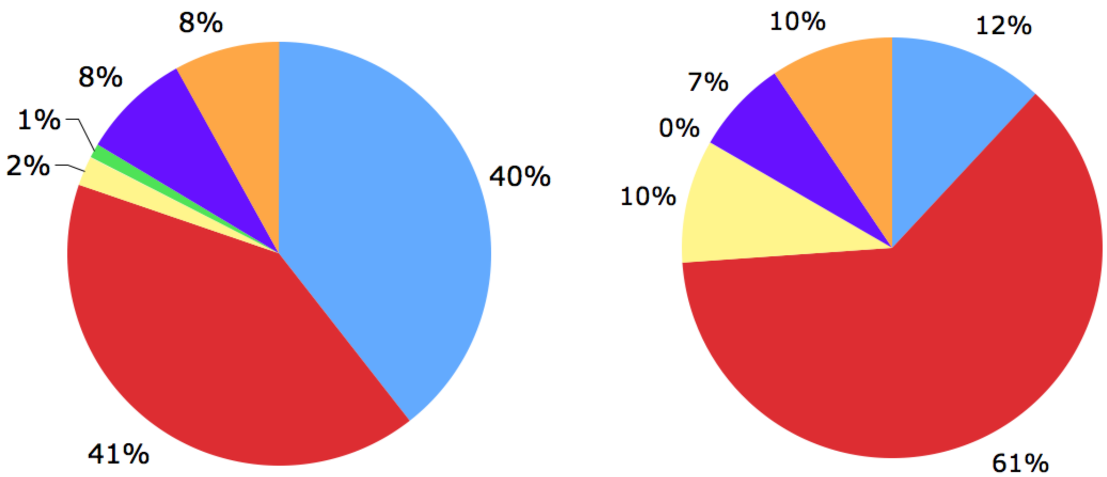


Figure 1.7: Authors in *Proceedings* 1883–1887 and 1918–1922

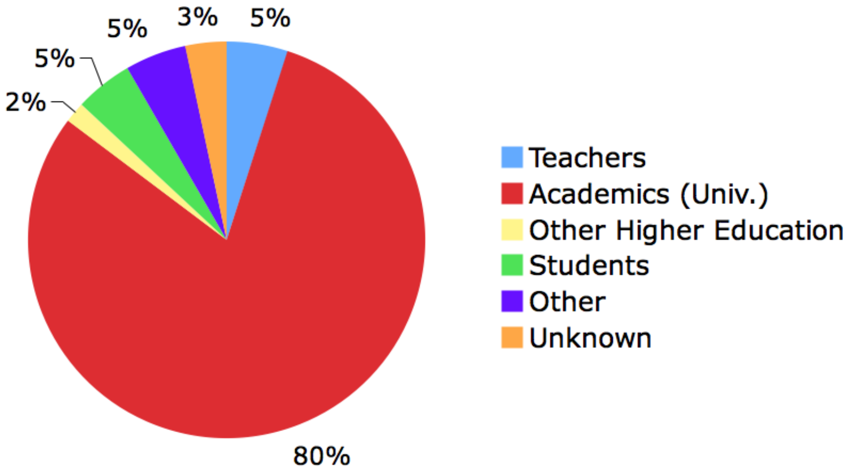


Figure 1.8: Authors in *Proceedings* 1923–1926

organise the papers by subject in two different ways. The first index uses the categories of the *The Bibliographical Repertory of the Mathematical Sciences*. This was a bibliographical catalogue, developed in the late 19th century under the auspices of the Société Mathématique de France.[28] This system consisted of 23 classes, from

> A. Elementary Algebra; theory of algebraical and transcendental equations; Galois groups, rational fractions; interpolation

to

> X. Methods of Calculation; Tables; Nomography; graphical calculation; planimeters; various instruments.

There were various subdivisions of these, marked by lowercase letters and numbers, which are not explained in the index. As the main categories are already more detailed than necessary, the subdivisions have not been considered here. The next 24 volumes are classified by a more modest 17 classes that the compilers developed themselves, with 6 main categories and 16 sub-classes. For this study, the *Bibliographical Repertory* has been converted into the second system, which has then been simplified. Five of the main categories are kept as they are, and the sixth, 'Geometry' has been split into 'Elementary geometry' and 'Advanced geometry'.[29] Of the 749 papers published in the first series, 84 were placed in more than one category. The only anomaly worth noting here is the unusually low number of papers published between 1918 and 1922. This is most likely caused by a paper shortage following the First World War.[30]

As the figure shows, analysis becomes a lot more popular after 1913, and elementary geometry fades out. Papers on history and pedagogics becomes less frequent after 1908; the reason for that being the launch of the *Mathematical Notes*.

[28]It would perhaps have been odd that the Society should use this system for their own journal, had it not been for the fact that the compiler of the index, Dr J. S. Mackay, was heavily involved in the development of this catalogue, being a member of the Permanent International Commission for Mathematical Bibliography. More on this very interesting catalogue can be found in [79].

[29]The category 'Calculus of observations' might need some explanation. It consisted of two sub-classes, which should be more recognisable.

a. Practical analysis, interpolation, mechanical quadrature
b. Statistics, mathematical economics.

[30]This only works as an explanation if the pages per volume went down in this period as well, which it did. The average number of pages per issue for 1913–17 was 156.2, compared to only 84.2 for 1918–1922.

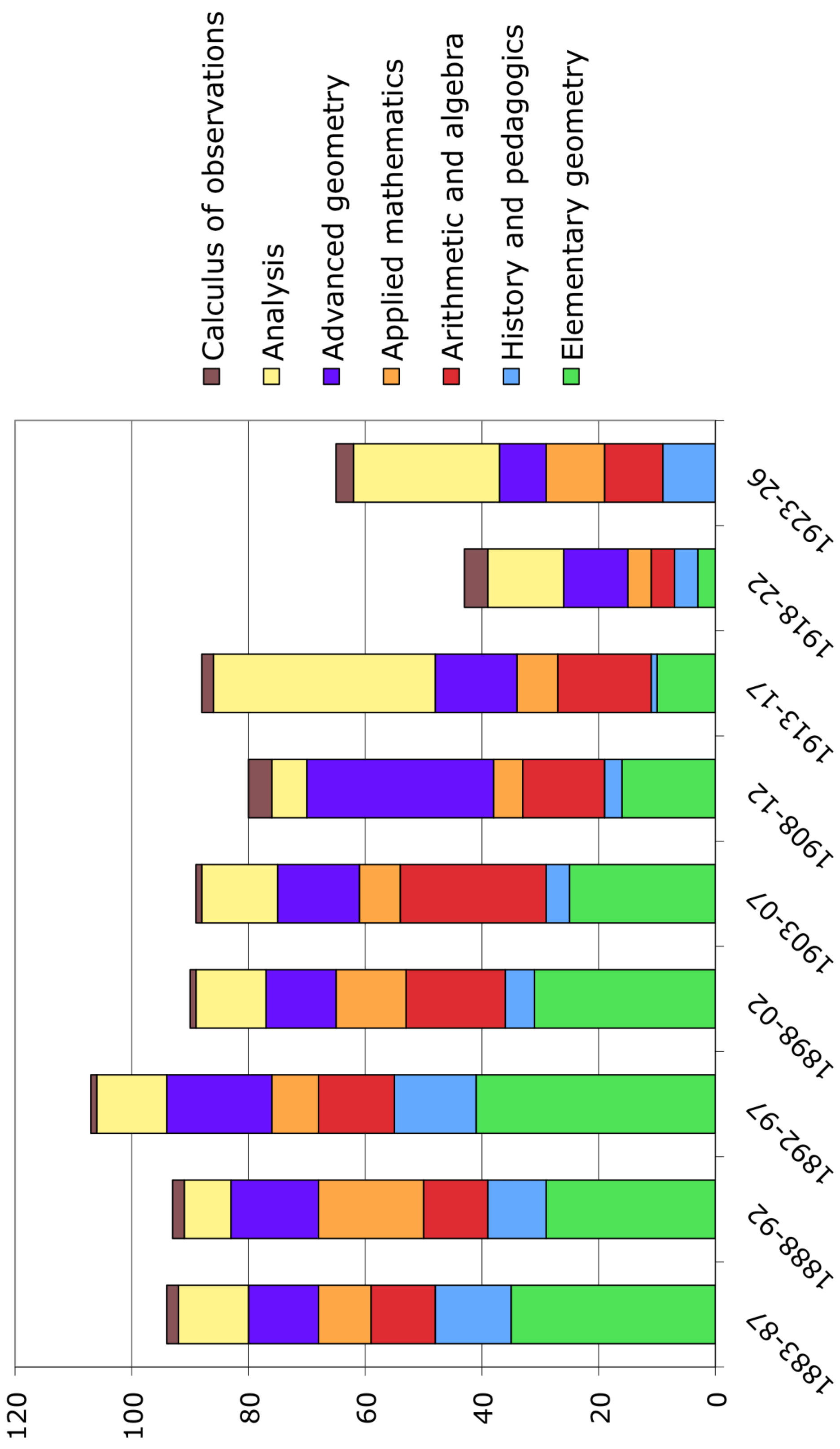


Figure 1.9: Papers by subject, *Proceedings* Series 1

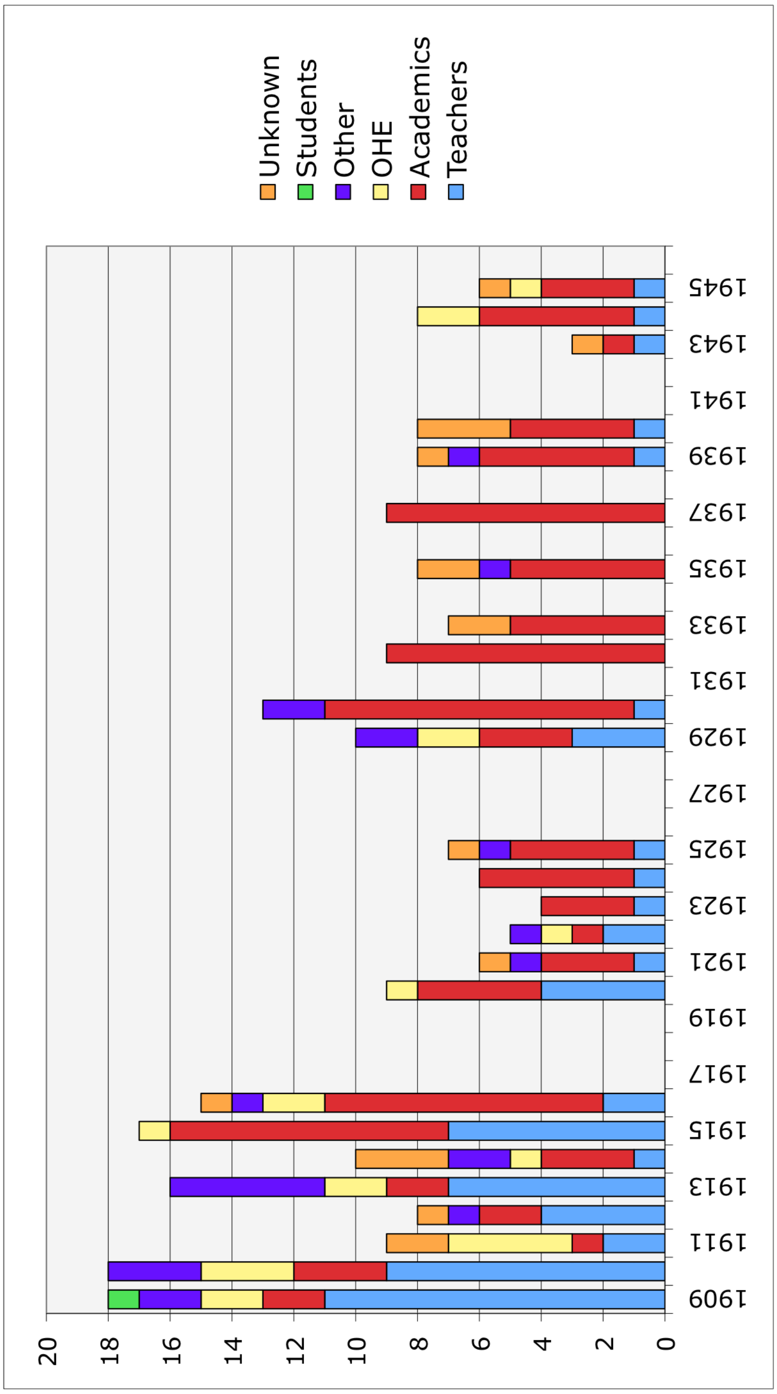


Figure 1.10: Authors in the *Notes* 1909–1946

1.5.5 *The Mathematical Notes*

On the 8th of January 1909, Professor G. A. Gibson suggested that the Committee should consider the possibility of issuing a supplement to the *Proceedings*, dealing with 'work taken up in schools'. The result of this was the *Edinburgh Mathematical Notes*, often called the *Mathematical Notes*, or simply, the *Notes*. It was issued for the first time in April 1909. At first, the publication did exactly as promised, containing papers such as 'Oral and written work in arithmetic' by W. G. Fraser,[31] 'Dynamics as a school subject' by W. Anderson,[32] and 'Multiplication and division of vulgar fractions' by G. Philips.[33] These papers were more elementary in nature than papers that had been appearing in the *Proceedings* for the later years. In fact, although papers of this nature had appeared in the earliest volumes of the *Proceedings*, there had never been nearly as many of them as were now to be found in the *Notes*. This elementary nature was to change, however. Gradually, the publication evolved into something more. More general papers were now to be found here, along with less specialised research papers.[34]

After a rather good start, with 2–3 issues a year, the publishing pattern changed radically. There were two longer breaks where no issues appeared at all, one between 1916 and 1920 and then a second between 1925 and 1929. No separate issue appeared between 1919 and 1923 either. Instead, they were published bound in with the *Proceedings*. This was most likely a measure to save paper, due to the aforementioned paper shortage. This shortage did not seem to set in before 1920, so this alone does not explain the break after First World War. The war brought on financial worries, as will be explained in section 1.5.7. Lack of material would also be a problem, one that would worsen. The reasons for the second break are more complex and will be explored more fully in chapter 4.

As was done for the *Proceedings*, the papers in the *Notes* can be organised by the occupation of the author (Fig. 1.10). The academics take over around 1920, which is exactly what happened for the *Proceedings*.

[31] *The Edinburgh Mathematical Notes*, Vol. 1, pp. 1–3.
[32] *The Edinburgh Mathematical Notes*, Vol. 1, pp. 8–10.
[33] *The Edinburgh Mathematical Notes*, Vol. pp. 23–25.
[34] Unfortunately, the organising of papers by subject in the Index only includes the *Proceedings*. It would be very interesting to see a similar study of the papers in the *Notes*. This would amongst other things show when this change away from strictly school mathematics began.

Table 1.1: Exchanges of *PEMS* and *Notes* by country

USA	13	Belgium	2
USSR	8	India	2
Italy	6	Switzerland	2
Germany	6	Finland	1
Japan	4	Denmark	1
Poland	4	Palestine (now Israel)	1
France	4	Holland	1
Romania	3	Australia	1
Sweden	3	Spain	1
Czechoslovakia	2	Sardinia	1
Yugoslavia	2	South Africa	1
Argentina	2	Canada	1
Austria	2		

1.5.6 Exchanges

Following the example of the London Mathematical Society [63] and others, the Society eventually began exchanging their two publications for other scientific periodicals. J. S. Mackay suggested this on the 13th of November 1891, and also that certain academic libraries should receive them for free. The minutes do not say anything more about this, but the Committee must have been in favour, as this was done. By 1938, the list of exchanges counted 102, including the free gifts.[35]

The exchanges arranged with institutions outside the UK, as seen in table 1.1, shows that the *Proceedings* definitely attracted attention. The minutes from the Committee Meetings from 1927 onwards show that most of these exchanges were arranged by requests to the EMS, and not from it.[36]

1.5.7 Financing

An issue that will become very important in the following chapters is the financing of the *Proceedings*. At first, the *Proceedings* was covered solely through the members' fees. This worked well most of the time for the first 30 years or so. The income from subscription fees and the expenditure on the printing both fluctuated, but on the whole, given the odd donation or two, matters evened out. This was to change rather dramatically, beginning with the First World War. As may be expected, the income

[35] The list actually goes to 103, as it includes the dispatch to the Society's own librarian.

[36] Not all academic libraries were given free copies. They were, however, eventually allowed to join as members, such as the South African Public Library, that joined on the 5th of May, 1939 and the King's College Library, Newcastle-upon-Tyne.

Table 1.2: Income from subscriptions vs. printer's bills, 1922–26

Session	Subscriptions	Printers
1922	£76/12/6	£62/3/6
1923	£82/1/0	£87/4/6
1924	£71/17/6	£251/17/0
1925	£82/5/10	£181/6/11
1926	£90/13/0	£128/12/6

from the subscriptions sank drastically, and the cost of printing rose. The worst year by far of these years was session 1916, where the subscription income amounted to £38/4/6, less than half the amount that it had been in session 1913, and only covering a third of the printer's bill of £115/8/1. The following few years were somewhat easier, though very unstable. This was partly due to the timing of printer's bills. Whatever the cause may have been, no printer's bill appeared for sessions 1917 or 1921, giving unusually high bills in sessions 1919 and 1924.

The subscription fees reached pre-war levels again in session 1920, and the paper shortage kept the printer's bill somewhat lower than before, but not low enough to prevent a deficit. The sale of the *Proceedings* helped offset parts of this, occasionally to a considerable extent, but it was grants and other donations that ultimately saved the day. The financial situation in fact became so dire that the Society had to send out an appeal to various businesses around Scotland, asking for help. They received a total of £13/18/0 this way.

The developments from 1922 to 1926 is shown in table 1.2. Things may have seemed to be on the mend in 1923, only to see a staggering printer's bill in the following session. This did include a delayed bill, but it was essentially the start of a rise in publication costs that the Society would struggle to cope with. The need for grants had never been more urgent.

Fortunately, some were received from various places. The universities would occasionally give financial aid to help publish the papers of their employees. The authors themselves sometimes made contributions and effectively paid for their own publication. Finally, a few grants were acquired from the Carnegie Trust. These latter grants were, however, only available to graduates or lecturers of Scottish Universities, and could only be used to pay the cost of illustrations, which was not often an issue with mathematical papers at the time. These grants were not quite enough, however, and the Society made an application to the Royal Society in 1924 but this was rejected.

1.6 Conclusion

It would now be worthwhile to stop and consider what the Edinburgh Mathematical Society truly was in its earliest years. There is no doubt that it was a society founded largely by schoolteachers, and that it retained a rather large proportion of teacher members throughout its first 44 years of existence. Naturally, where a large number of teachers are gathered, the topic will turn to teaching-related matters every now and then, but did that mean, as was later claimed (see chapter 4), that the Society stood for scholastic mathematics? When considering the Society's goals and aims, and in particular those that were not stated explicitly, but implied by Muir and Professor Chrystal, the answer must surely be no. The real goal of the foundation of the Society was the promotion of mathematics in Scotland, and most importantly, the mathematical research. Improving the mathematics of the schools was a part of this, but that was only a necessary first step. In addition to this, the teachers who signed up on the 2nd of February 1883 did so because they wanted to learn more of their subject, more than they had learnt at university, and certainly more than their work in the schools could teach them. One could in many ways claim that the Society was created to continue educating where the universities left off.

Chapter 2

The Mathematics of the Teachers

2.1 Introduction

For the first part of this chapter, the analysis of all the papers by teachers, a few notes must be made. Only the first 44 volumes, up to 1926, have been considered. As mentioned, only one paper is confirmed to have been written by a teacher between 1926 and 1946. It should also be noted that the first series of the *Proceedings* contains 46 papers (of a total of 670) written by authors of unknown occupation.

Obituaries have been left out. A few other items listed in the indexes have been removed as well, such as 'Correspondence regarding Abel's centenary', which is simply an invitation to a celebration.[1]

2.2 The Papers in the *Proceedings*

2.2.1 The numbers

During the first 44 sessions, 154 papers by 37 teachers were published in the *Proceedings*, giving an average of 4.16 papers per author. This does not differ all that much from the average 4.54 papers of the academics, with 332 papers by 73 authors. The difference lies mainly in that the teachers were more inclined to publish only one paper; 48.6% of them published only one, compared to 34.2% of the academics. This can be explained in part by new teachers who published work undertaken in their student days, shortly after graduating.[2] These figures, however, do not take into account the actual numbers

[1] *PEMS*, Vol. 21, pg. 182–183.

[2] Two examples of this would be William Brash's 'Two general theorems in the differential calculus' in *PEMS*, Vol. 30, pp. 107–116, and G. E. Chappell's 'The properties of a new orthogonal function associated with the confluent

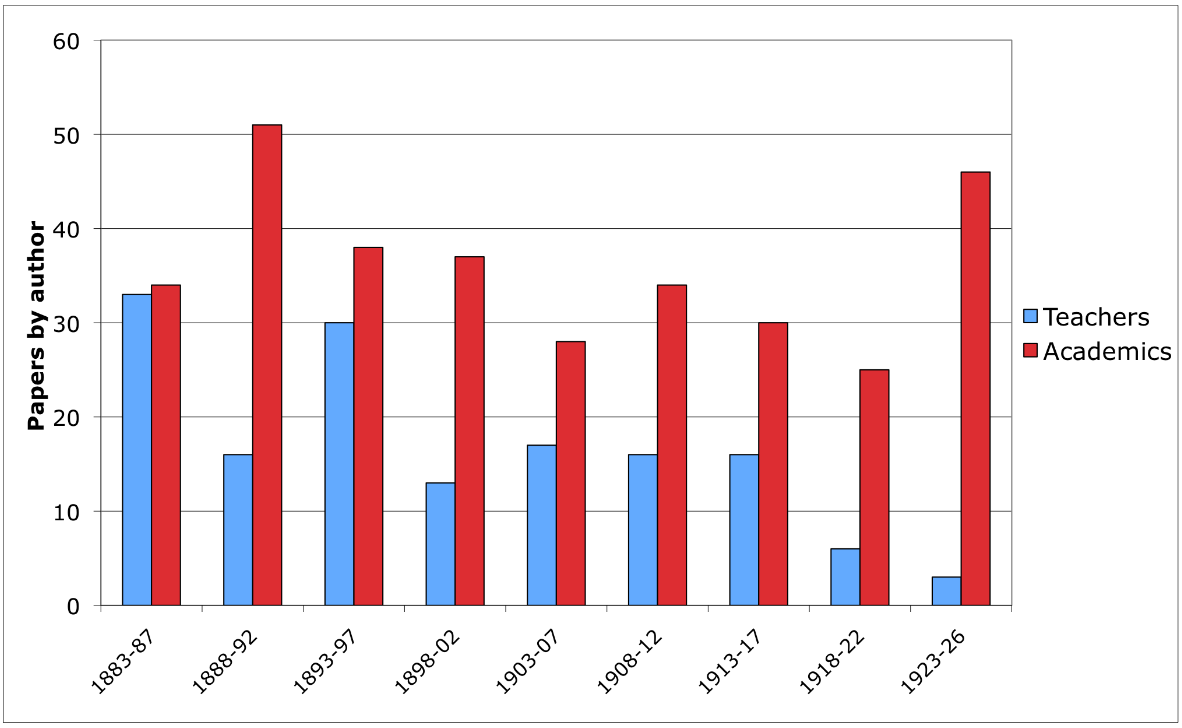


Figure 2.1: Papers by teachers and academics, *Proceedings* Vol. 1–44

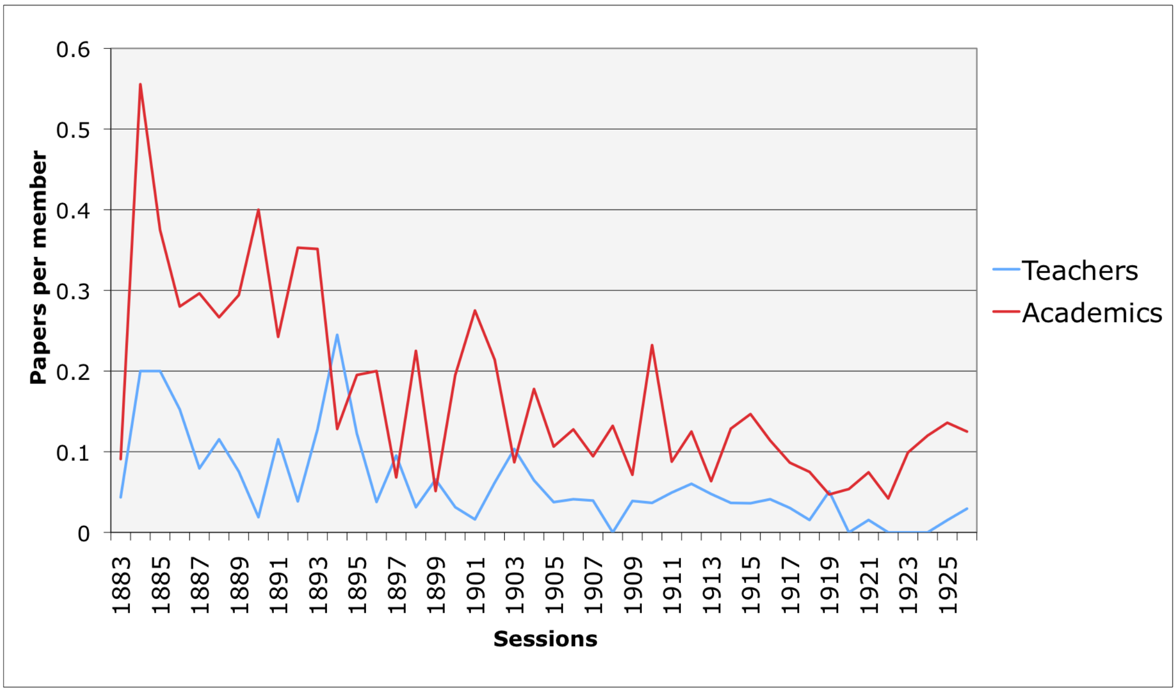


Figure 2.2: Papers per member, teachers and academics, *Proceedings* Vol. 1–44

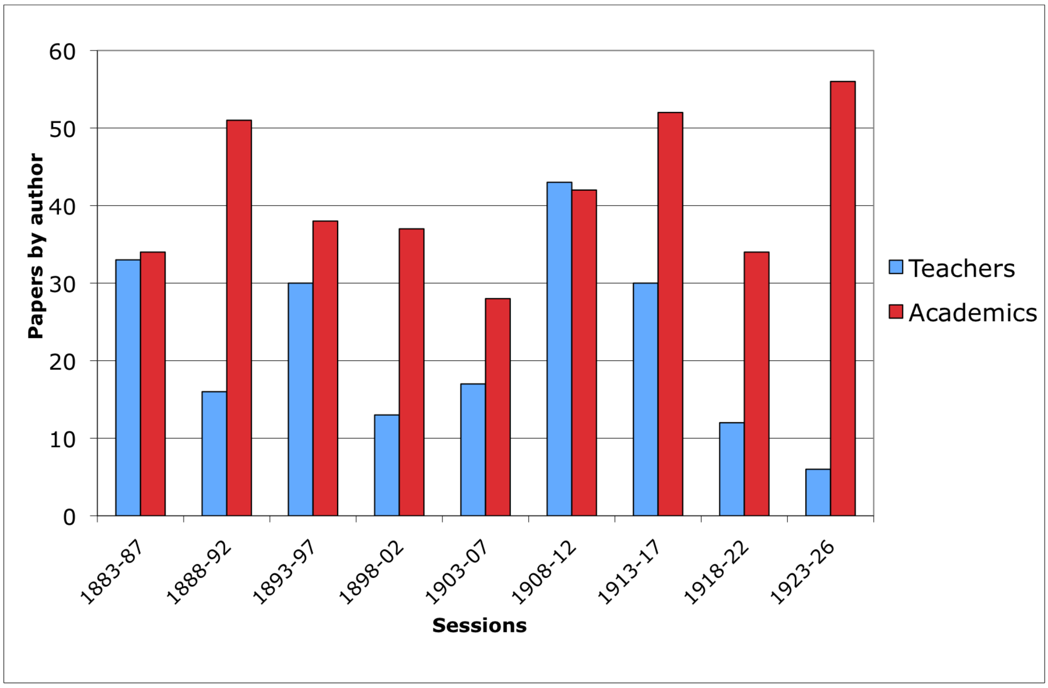


Figure 2.3: Papers by teachers and academics, *PEMS* and *Notes*

of members who were teachers or academics. In figure 2.2 is shown the mean number of papers per member produced annually by each of these groups. It shows that as a group, the academics were substantially more productive than the teachers. The tables of data are to be found in Appendix A.

The teachers wrote on average longer papers than the academics did, though not by much. An overall average for the teachers is 7.9 pages per paper, whereas it is 6.6 for the academics.[3] These numbers do not include the accompanying geometrical figures for papers in the earlier volumes, as these were not included in the papers themselves.

An overview of the total number of papers produced by teachers and academics in five-year intervals is given in figure 2.1. The number of papers by teachers remained relatively stable until it dropped below 10 in the period 1918–22. What this graph does not show quite clearly is that it actually dropped before this, in 1913–14, as shown when these numbers are broken down into years (table 2.1).

It is perhaps not very hard to understand why the teachers became less involved

hypergeometric function' in *PEMS*, Vol. 43 (1925), pp. 117–130.

[3]This average for the teachers does not include a very unusual paper in the first volume, a 125-page long paper by J. S. Mackay. This particular paper, published 11 years after the original talk was given, was greatly expanded, and is twice as long as the second longest paper. It is in this sense far from representative for the average paper, and has therefore been left out. If it were included, the average would be 8.8 pages per paper.

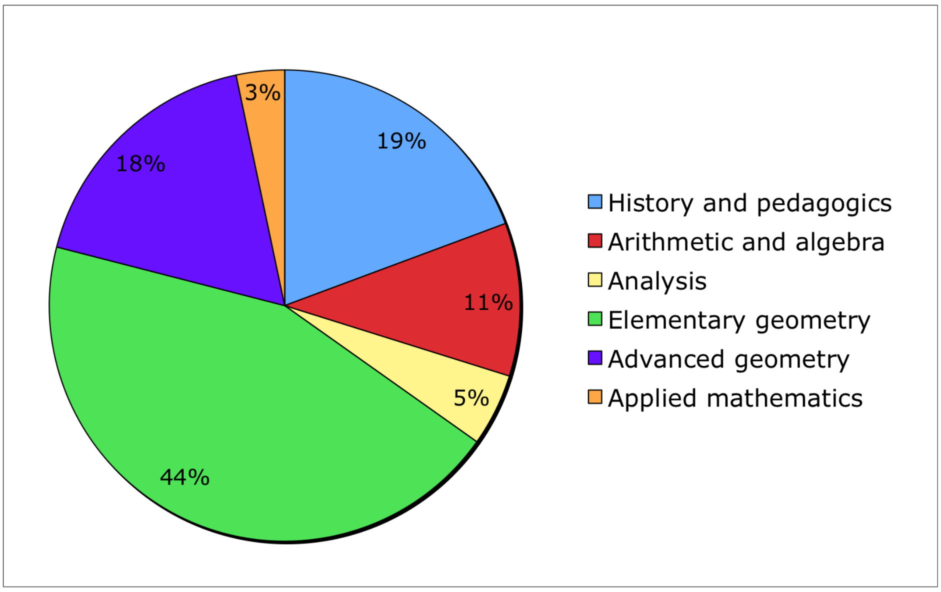


Figure 2.4: Papers by teachers according to subject, Vol. 1–44

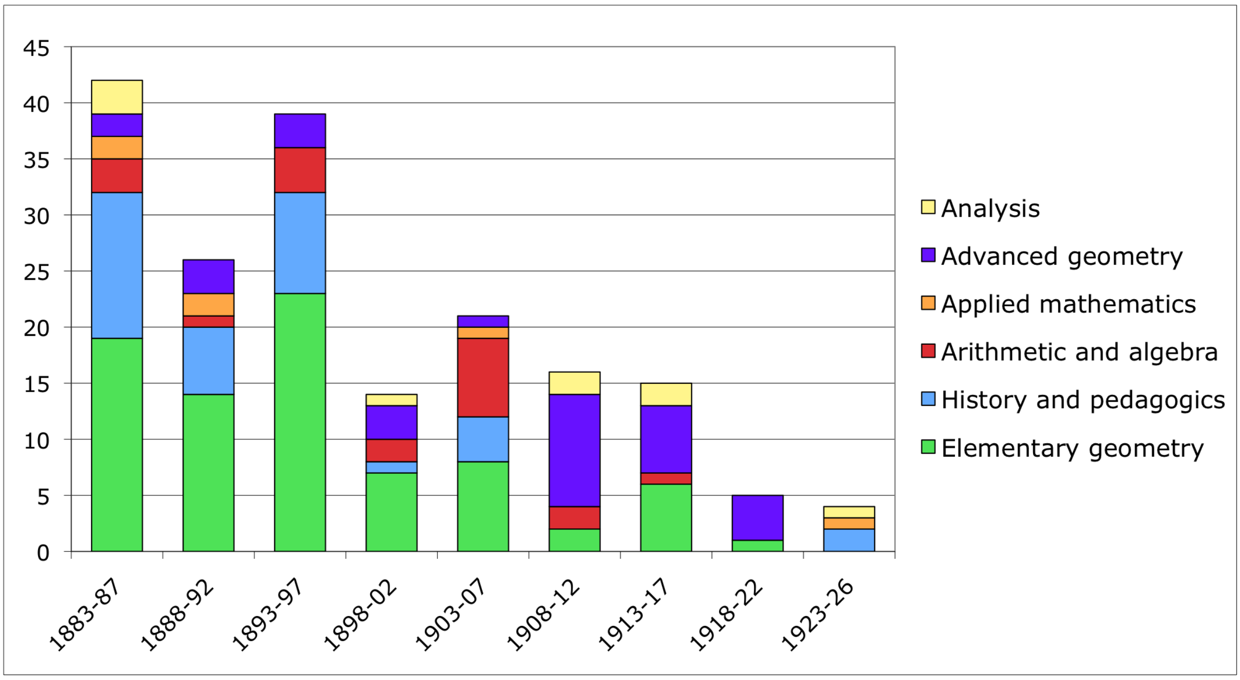


Figure 2.5: Papers by teachers according to subject, 5-year intervals.

in writing papers around 1914. What is more surprising is that these numbers do not drop earlier. If the teachers were really only interested in scholastic matters such as elementary geometry, which was without doubt the teachers' area of expertise, then their papers would have been transferred to the *Notes* when this journal was established in 1909. This did not happen. The figures for the *Proceedings* changed very little when the *Notes* were introduced. In fact, counting papers and notes for both periodicals shows that the introduction of the *Notes* made the teachers publish more (fig. 2.3).

Table 2.1: Papers by teachers

Year	1912	1913	1914	1915	1916	1917	1918
Papers	7	5	3	3	3	2	2

The number of papers by teachers in the *Notes* decline around 1913–14, just as it did for *PEMS*. In 1913, 7 papers in the *Notes* had a teacher author versus 1 in 1914. It rose slightly in 1915, to 4, but it never again reached the heights of the glory days of 1909–1912.

The introduction of the *Notes* may not have had much effect on the number of teachers' papers, but it had other effects, as the next section will show.

2.2.2 The topics of the teachers

The papers by teachers in the *Proceedings* have been organised by topic the same way all the papers were in section 1.5.4 (Fig. 2.4). As one might expect, 'Elementary geometry' is the largest category, followed by 'History and pedagogics' and 'Advanced geometry'.

The choice of topics was not unaffected by changing times, as shown by figure 2.5. 'History and pedagogics' all but vanish from 1908 onwards, coinciding nicely with the birth of the *Notes*. These papers were presumably transferred from one publication to the other. The same must have happened to some extent to papers on elementary geometry, although these did not vanish completely, and their number started to decline before the *Notes* appeared. Papers on advanced geometry appear to more or less take over from the more elementary geometry, at least up until the war.

A similar analysis to this can be done for academics, as shown in figure 2.6. These papers have much more diversity to them, but even here can a relatively large proportion of elementary geometry be found. The academics may have been catering for their audience when writing on this, but it is not unlikely that they did this research for

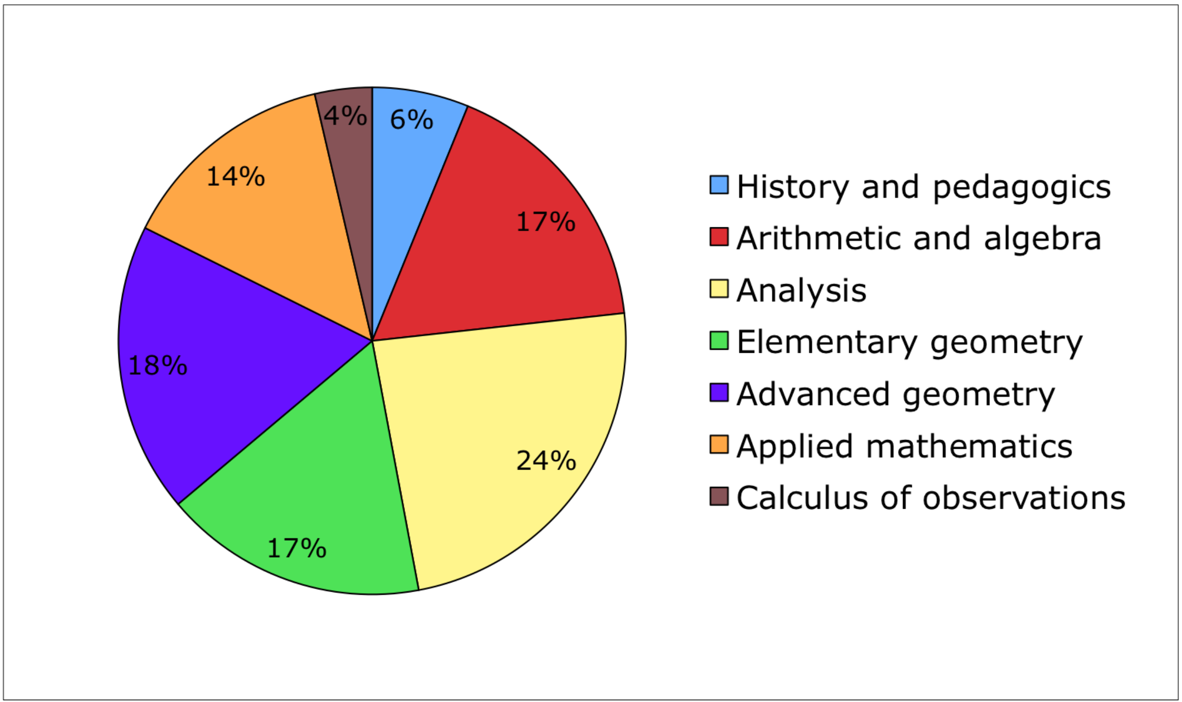


Figure 2.6: Papers by academics according to subject, Vol. 1–44

their own benefit. Euclidean geometry formed a relatively large part of the university curriculum in the earlier days of the Society, which is when most of these geometry-papers were written.

2.2.3 The types of papers

The papers can also be grouped by type. The papers can be divided into four categories.[4]

- Historical — This would be papers on historical matters, such as Mackay's 'Historical notes on a geometrical theorem and its developments (18th century)'.[5]
- Educational— Papers of a pedagogical nature have been put into this category, for instance John Alison's 'Trigonometrical mnemonic'[6]. New proofs, and similar, of very elementary mathematics intended for use in school are also included here, such as Butters's 'Notes on factoring' (See section 2.3.2).

[4]The first two form the first categories on subject, 'History and pedagogics', but they are here separated, as they differ quite a lot as types.
[5]*PEMS*, Vol. 5, pp. 62–78.
[6]*PEMS*, Vol. 4, pg. 88.

- Presentation — The teachers occasionally presented other people's work. Butters presented work by Gauss (See section 2.3.1), and J. S. Mackay wrote the paper 'Similitude and inversion' where he attributed most of the results to others.[7]

- Research — Papers containing new results developed by the author have been placed in this category. This would sometimes be relatively elementary mathematics, but such papers have been classified as research, unless it was obviously intended to be used in teaching. Examples are a paper by A. D. Russell, 'A special case of the dissection of any two triangles into mutually similar pairs of triangles'[8] and 'An analytical treatment of the cam problem' by G. D. C. Stokes.[9] Where nothing else has been specified, the papers have been assumed to contain original content.

It has often been unclear where the line should go between research and educational papers. Attempts have been made to be as fair as possible, but an even more thorough study may end up with a more narrow research category and a correspondingly larger educational one. The chart in figure 2.7 still gives a nice breakdown of the papers into these types. A full 64% being research papers is surprisingly high, even considering that the research would not be the most advanced. The findings from the study of subject is reflected here, especially when these figures are broken down into 5-year intervals (Fig. 2.8). On the introduction of the *Notes*, the educational and historical papers vanish, and the research papers remain relatively stable up until the end of the First World War.

[7] *PEMS*, Vol. 6, pp. 69–87.
[8] *PEMS*, Vol. 18, pp. 28–29.
[9] *PEMS*, Vol. 39, pp. 7-12.

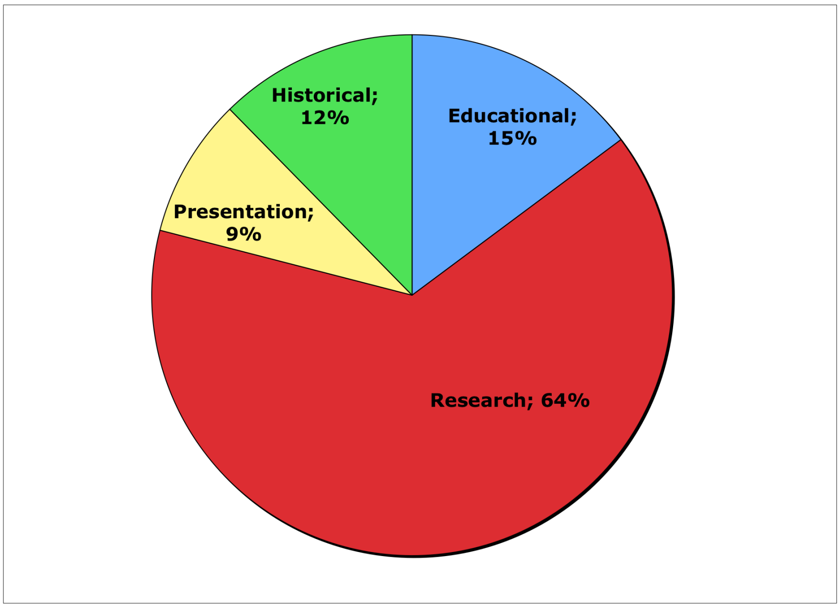


Figure 2.7: Papers by teachers according to type, Vol. 1–44

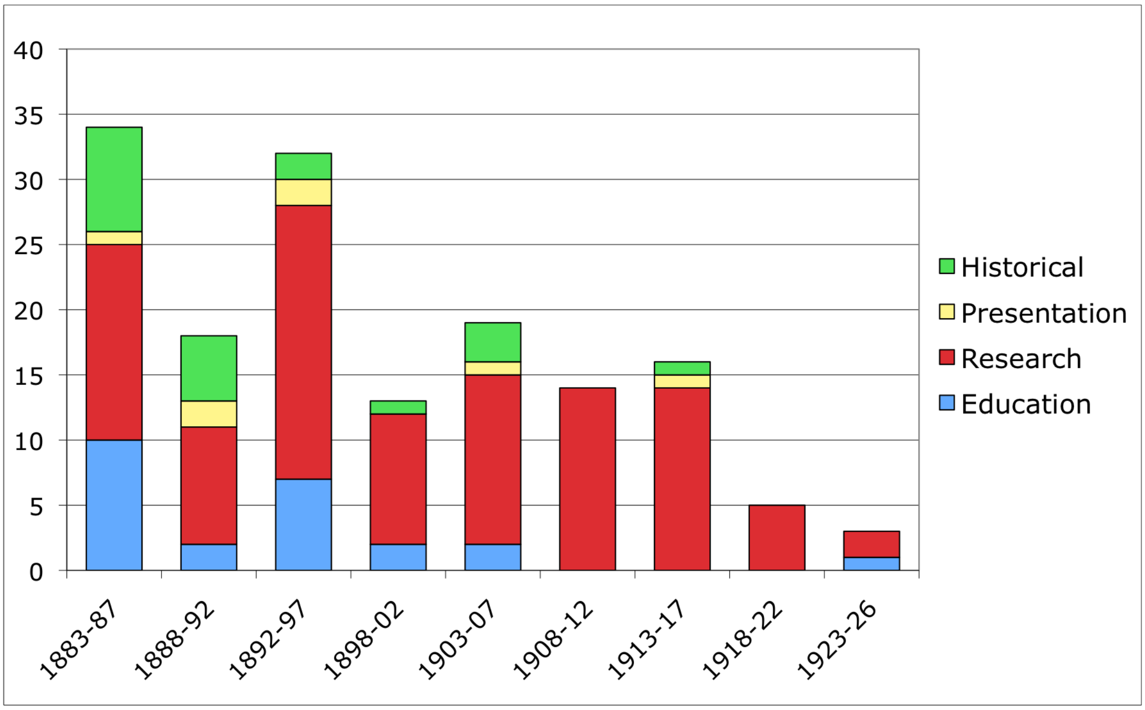


Figure 2.8: Papers by teachers according to type, 5-year intervals

Figure 2.9: John Watt Butters (1863–1946)

2.3 The papers of John Watt Butters

There are many teachers in the society that could have become the subject of this study. The most famous one would be Sir Thomas Muir, the author of the History of Determinants. He published seven notes and articles during his time at the High School in Glasgow. The only problem is that he had already worked as a university assistant before this, and ended up as the Superintendent General of Education in South Africa and later Vice-Chancellor of the University of the Cape. He was an excellent researcher, but as he spent most of his working days at a university, he was perhaps not the best representative for the researching teachers.

One that could have made a better representative was the Society's first president, Dr John Sturgeon Mackay. He spent almost all his career as Mathematical Master at Edinburgh Academy, and was a scholar of renown. He was even awarded an honorary doctorate because of his work, and would so be a suitable candidate. When the choice eventually fell on another, this was largely because Dr Mackay had more than 30 publications in the *Proceedings* alone on his CV, and a thorough study of his works could easily fill a thesis of its own.

John Watt Butters was ideal, with a more manageable seven publications.[10] He was an Edinburgh man, born there on the 10th of August 1863 to John Butters and Isabella Watt. He received his entire education in Edinburgh; his school days were spent at the Old High School and then at George Heriot's School, where he also began his teaching

[10]The biographical information on Butters is found in [74],[32] and [17].

career as a pupil teacher.[11]

In 1882, aged 19, he matriculated at the University of Edinburgh. While studying there, he continued his training as a teacher, at the Established Church Training College. He also held teaching positions at Aberystwyth and at James Gillespie's School, before returning to George Heriot's as mathematical master in 1888. He graduated in 1894, being awarded an M.A. and a B.Sc., with First Class Honours in Mathematics and Natural Philosophy. Before that, he had also qualified for a Certificate of L.A. in October 1886, having passed examinations in Logic, English Literature and Education in April 1884 [102, 1884–85, pg. 119 and 1887–88, pg. 103]. He left Edinburgh in 1899 to become headmaster of Ardrossan Academy. On his retirement in 1928, he returned to his native city, where he died on the 11th of January 1946.

Butters's contribution to the Edinburgh Mathematical Society was substantial. In addition to his publications, he helped prepare the second index [14], and he was also very active in the organisation of the Society, holding several offices.

Sessions	
1890–96	Committee Member
1897–99	Honorary Secretary
1900	Vice-President
1901	President
1902	Committee Member

His last three years of service are particularly noteworthy, as he was then working at Ardrossan Academy. This is located on the west coast of Scotland, which means he had to travel a distance of 80 miles every time he attended a meeting, and he reportedly never missed one [32].

Butters was involved in other organisations as well. He became a fellow of the Royal Society of Edinburgh on the 6th of April 1896. The index *Former Fellows of the Royal Society of Edinburgh* [84] gives his proposers as Peter Guthrie Tait, George Chrystal, John Sturgeon Mackay and David Fowler Lowe, all fellow members of the Edinburgh Mathematical Society. According I. M. H. Etherington in the EMS obituary, Butters was also on the council of the Scottish Geographical Society. The same source explains further that he was a member of the Educational Institute of Scotland and that he was on the Business Committee of the General Council of Edinburgh University.

His other obituarist, J. B. Clark, described his friend's personality in [17].

> Mr Butters was a man of exceptionally wide intellectual interests. He was a

[11] A pupil teacher was a pupil who would teach in the elementary school while receiving secondary education.

> keen nature lover, and the fells of the Lake District, the peaks of Arran, and the bens of the Highlands had, for him, an irresistible attraction. He was, too, a most loyal friend, and all of us who were privileged to know him held him in the highest regard.

2.3.1 'On the solution of the equation $x^p - 1 = 0$ (p being a prime number)'

Vol. 7 (1889) Pp. 10–22 Read: 11/01/1889 Presentation

This paper is, to put it in Butters's own words, 'a sketch of Gauss's treatment of the general equation' (for p prime)[12]

$$x^p - 1 = 0. \tag{2.1}$$

This is a presentation of section VII in *Disquisitiones Arithmeticae*. The idea for this paper came at a meeting in November 1888. Professor J. A. Steggall read a paper entitled 'The value of $\cos 2\pi/17$ expressed in quadratic radicals' [87]. It was then suggested that a sketch such as the one Butters is giving here would be interesting.

Butters is writing this paper in 1889, at which time Gauss's masterpiece has not yet been translated into English, this despite it being published in 1801. Butters mentions a French translation [93], and this is presumed to be the edition he has used for this work.[13] He has consulted other works as well, mainly in French, but two English works are given: Robert Murphy's *A Treatise on the Theory of Algebraical Equations* and Peter Barlow's *An Elementary Investigation on the Theory of Numbers*.[14] Murphy mentions Gauss in his book, but the solution of eq. 2.1 that is presented there is actually due to Lagrange [56, pg. 65]. Barlow does give Gauss's solution, but in a rather simplified way, as this book is aimed at a more general audience. Other authors, apart from the two mentioned by Butters, cover this as well, such as James Ivory in a supplement to the Encyclopædia Britannica in 1824 [44]. A review of the French translation by an unknown author appears in the Monthly Review in 1808 and the treatment found there is very close to the one that Butters is giving here [103]. There is no reason to believe that he is familiar with more works than he mentions, so he presumably believes himself to be the first to give so thorough a presentation of this work of Gauss in English.

[12] This equation is today known as the cyclotomic equation.
[13] Butters refers to this translation as *Recherches Mathématiques*. The correct title is *Recherches Arithmétiques*.
[14] He refers to these books as Murphy's *Theory of Equations* and Barlow's *Theory of Numbers*.

Gauss's work has been translated and presented several times since, for instance by Stuart Hollingdale [41, pp. 416–420], so the aim for the current section is not to present the method, but rather to give a brief outline of it and to comment on Butters's presentation.

The first 11 paragraphs of this paper are devoted to the general equation, and the rest to the special case $p = 17$. As Butters has not got six preceding chapters to back him up, it is necessary for him to include some of Gauss's results on modular arithmetic.[15] Butters follows Gauss closely, but is, as one might expect in a shorter paper, not quite as thorough. Gauss provides more explanations, and gives examples where Butters does not. The proofs are, however, for the most part essentially the same and are covered by Gauss in [92, Art. 335–352]. A few exceptions are pointed out here.

One difference is how the two authors define Gaussian periods. This is a concept much used in subsequent calculations.[16] They are considering the equation[17]

$$\frac{x^p - 1}{x - 1} = x^{p-1} + x^{p-2} + \ldots + x + 1 = 0. \tag{2.2}$$

This equation will have $p - 1$ roots, and if one root r is found, all $p - 1$ roots are found as $r, r^2, r^3, \ldots, r^{p-1}$.

Butters explains that these roots will also be produced by $r^\lambda, r^{\lambda g}, r^{\lambda g^2}, \ldots, r^{\lambda g^{p-2}}$, as long as λ is coprime to p and g is a primitive root[18] of p. Such expressions are difficult to print in the 19th century, so Butters follows Gauss's notation, writing this as $[\lambda], [\lambda g], \ldots, [\lambda g^{p-2}]$. This way, the roots are denoted by the residue classes modulo p. Now, p is prime, so $p - 1$ is not and can be written as some product ef. Gauss now defines (f, λ) to be the sum of the roots $[\lambda], [\lambda g^e], [\lambda g^{2e}], \ldots, [\lambda g^{e(f-1)}]$ (See [92, Art. 343]). He defines the *period* (f, λ) to be the collection of these roots. Although he makes a distinction between the sum and the collection here, he will use the term 'period' to denote the sum where convenient. Butters makes his definition slightly differently. He puts the ef roots in a table, such as this one for $\lambda = 1$.

[15] These are the results found in [92, Art. 39, 45–46, 48–50, 52–55].
[16] The term 'Gaussian period' is a more recent development. Butters refers to them only as 'periods'.
[17] This is now known as an irreducible cyclotomic equation.
[18] A primitive root a of a prime number p has the property that $p - 1$ is the smallest power k making $a^k \equiv 1$.

$[1]$	$[g^e]$	$[g^{2e}]$	$\dots$	$[g^{e(f-1)}]$
$[g]$	$[g^{e+1}]$	$[g^{2e+1}]$	$\dots$	$[g^{e(f-1)+1}]$
$[g^2]$	$[g^{e+2}]$	$[g^{2e+2}]$	$\dots$	$[g^{e(f-1)+2}]$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$[g^{e-1}]$	$[g^{2e-1}]$	$[g^{3e-1}]$	$\dots$	$[g^{ef-1}]$

The form (f, λ) is then defined to be the sum of the roots in a row. Like Gauss, he defines the term 'period' as the collection of roots, but will on occasion use it to denote the sum.[19] Two periods with the same number of elements f are said to be similar.

Butters is not entirely clear on how he lets λ determine which row is to be used. He seems to regard $[\lambda]$ as the first root to be included in the period, and he then includes the $f-1$ next roots on the row. To make this work with his table, he remarks that any row can be extended indefinitely, and that the row will simply repeat itself over and over. Two similar periods that have one root in common are therefore identical, and it becomes clear that a period can be represented by any of elements in the row, not just the first. So, $(f, \lambda) = (f, \lambda g^e) = \ldots = (f, \lambda g^{e(f-1)})$. Therefore, (f, λ) can be read as 'the row containing the root $[\lambda]$'.

If f is composite, say $f = ab$, then each of the e periods can be broken up into smaller periods. The periods contained within (f, λ) would be denoted (b, λ),$(b, \lambda g^e)$, $\ldots$,$(b, \lambda(g^e)^{a-1})$. Each row will then form a new table and this process can be continued as long as there are factors left of f.

Both Butters and Gauss summarise the general method, but only Butters boils it down to seven points of action:

1. Find a primitive root g of the prime number p.

2. Find the residues $\pmod p$ of the series $1, g, g^2, \ldots, g^{p-2}$. These residues will simply be a re-arrangement of $1, 2, \ldots, p-1$.

3. Find the prime factorisation of $p-1$, say $p-1 = abc\ldots k$ (the factors need not be unequal).

4. Put the roots in a table with a rows and $bc\ldots k$ columns, getting a periods. Repeat

[19] Barlow operates with a table similar to this in his book, but he defines the periods first, and then puts the periods in a table.

this process, getting b periods with $c \ldots k$ terms each, for each of the a periods, and so on. Continue until there are no more factors.

5. Form an equation that has the a periods as roots, and find all roots. (See below)
6. Using these a identities, form an equation that has the b periods with $(bc \ldots k, 1)$ terms as roots, and solve. Continue until the values $(k, 1), (k, g), \ldots$ are found.
7. Form an equation that has the elements in $(k, 1)$ as roots. Once one of these roots has been found, all the other roots of eq. (2.2) can be found from the powers of this one root.

This list appears at the end of the general treatment, so he has already explained how to find these equations. He does not quite make his mind up about how to do so. The method he presents in his general treatment actually differs from Gauss's, and he does not point this out. When he later works through the case $p = 17$, he changes to Gauss's method, without having explained it, and without indicating that he is no longer using his own.

They are both looking for an equation A with the a periods as roots, and the question is how to find the coefficients. They have both shown previously that any rational polynomial, in terms of f periods, can be expressed as a linear combination of the e periods $(f, \lambda), (f, \lambda g), \ldots, (f, \lambda g^{e-1})$, with $ef = p - 1$, as before.[20] If $n = (f, \lambda)$, it follows that the rational polynomials $n^2, n^3, \ldots, n^{e-1}$ can be written this way as well. These can be found using a formula for the product of similar periods (See eq. 2.4).

Both Butters and Gauss use these $e - 2$ equations, along with the equation

$$(f, \lambda) + (f, \lambda g) + \ldots + (f, \lambda g^{e-1}) = -1$$

to show that one period can be expressed as a polynomial in another, similar period. As soon as A has been found, this can be used to show how finding one root of A leads to all the other roots.

Now, to find this A, Gauss makes use of the equation[21]

$$x^e - a_1 x^{e-1} + a_2 x^{e-2} - a_3 x^{e-3} + \ldots \pm a_e = 0 \tag{2.3}$$

where a_1 is the sum of the periods, a_2 is the sum of the periods taken two at a time,

[20] The proof in the paper uses $(f, 1), \ldots, (f, g^{e-1})$, but these periods can be represented by $(f, \lambda), (f, \lambda g), \ldots, (f, \lambda g^{e-1})$ instead. Butters refers to these polynomial functions as 'integral' functions.

[21] Gauss phrases it somewhat differently, using A, B, etc, as coefficients.

a_3 the periods taken three at a time, and so on. Finally a_e is the product of all the periods.

Butters uses the expressions for n^i instead. If one computes n^e in addition to the ones above, and eliminate every period but n, what is left is a polynomial in n of degree e. Butters argues that this equation will have all the e periods as roots, and is therefore A.

The case $p = 17$

Like Gauss, Butters now works through the case $x^{17} - 1 = 0$. These seven steps seem to be intended more as general guidelines rather than strict rules, because he does not follow them when working through this special case.[22]

He finds that 3 is a primitive root of 17, and that the residues of the powers of 3 (mod 17) are

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
3^i	3	9	10	13	5	15	11	16	14	8	7	4	12	2	6	1

He now has $16 = 2^4$, $g = 3$, $e = 2$, and hence $f = 8$. The roots can therefore be arranged in two periods, noting that $[3^1] = [3]$, $[3^2] = [9]$, $[3^3] = [10]$ and so on.

$$\begin{array}{lcllcccccccc} n & = & (8,1) & \text{containing} & [1] & [9] & [13] & [15] & [16] & [8] & [4] & [2] \\ n' & = & (8,3) & \text{containing} & [3] & [10] & [5] & [11] & [14] & [7] & [12] & [6] \end{array}$$

If following his own list faithfully, he should now have divided these periods up, but instead he sets out to form an equation that has these two periods as roots. Now using Gauss's method, he needs to find expressions for $n + n'$ and $n'n$. [23] The first is easy, as the equation in (2.2) gives that $n + n' = -1$. For the second coefficient, he makes use of the formula that he has already developed for the product of two similar periods (where $h = g^e$):

$$(f, \lambda)(f, \mu) = (f, \lambda + \mu) + (f, \lambda h + \mu) + (f, \lambda h^2 + \mu) + \ldots + (f, \lambda h^{f-1} + \mu) \qquad (2.4)$$

[22]He begins with a table of residues of the 1–16*th* powers of $1, 2, \ldots, 16 \mod 17$, in order to illustrate the properties of modular arithmetic.

[23]The paper actually says he is computing nn', but it is $n'n$. The result -4 is the same in both cases, but the calculations look slightly different.

Using this, he finds that

$$\begin{aligned}
n'n &= (8,4)+(8,11)+(8,6)+(8,12)+(8,15)+(8,8)+(8,13)+(8,7) \\
&= n+n'+n'+n'+n+n+n+n' \\
&= 4(n+n') \\
&= -4
\end{aligned}$$

The equation needed is therefore

$$n^2+n-4=0. \tag{2.5}$$

Had he used his own method, he would have computed n^2 rather than nn'. This would have given

$$n^2 = 3n+4n'+8$$

which combined with $n+n'=-1$ gives the same equation as (2.5). This equation is perfectly solvable, though Butters does not carry out the calculations for any of the quadratic equations.

He now breaks the two periods up into smaller periods.[24]

$$\begin{array}{lclclcccc}
m_1 &=& (4,1) &:& [1] & [13] & [16] & [4] \\
m_2 &=& (4,9) &:& [9] & [15] & [8] & [2] \\
m_3 &=& (4,3) &:& [3] & [5] & [14] & [12] \\
m_4 &=& (4,10) &:& [10] & [11] & [7] & [6]
\end{array}$$

As the new e is again 2 (as $8 = 2\times 4$), he is looking for quadratic equations, but this time he needs two of them, one for the two periods contained in $(8,1)$ and one for the two in $(8,3)$.

Butters observes that $m_1+m_2=n$. This identity is now known, at least in theory. The other coefficient is then

$$\begin{aligned}
m_1m_2 &= (4,10)+(4,16)+(4,9)+(4,3) \\
&= m_4+m_1+m_2+m_3 \\
&= n+n' \\
&= -1.
\end{aligned}$$

[24]Butters uses m, m', m'', m''' instead of m_i.

The required equation is therefore

$$m^2 - nm - 1 = 0. \tag{2.6}$$

Similarly, the equation for m_3 and m_4 is

$$m^2 - n'm - 1 = 0. \tag{2.7}$$

The periods are now separated even further, giving 8 small periods.

$$\begin{array}{lllllll@{\qquad}lllllll}
l_1 & = & (2,1) & : & [1] & [16] & & l_5 & = & (2,3) & : & [3] & [14] \\
l_2 & = & (2,13) & : & [13] & [4] & & l_6 & = & (2,5) & : & [5] & [12] \\
l_3 & = & (2,9) & : & [9] & [8] & & l_7 & = & (2,10) & : & [10] & [7] \\
l_4 & = & (2,15) & : & [15] & [2] & & l_8 & = & (2,11) & : & [11] & [6]
\end{array}$$

He needs to find an equation with the two periods l_1 and l_2 as roots. The first coefficient is again an easy one, as $l_1 + l_2 = m_1$. The second coefficient is $l_1 l_2 = l_5 + l_6 = m_3$, giving the equation

$$l^2 - m_1 l + m_3 = 0. \tag{2.8}$$

Only one step remains now, which is to form an equation with [1] and [16] as roots. Since $[1] + [16] = l_1$ and $[1] \cdot [16] = [1 \cdot 16] = 1$, it follows that r is a root of

$$r^2 - l_1 r + 1. \tag{2.9}$$

The remaining roots of equation (2.2) can now be found.

This works beautifully for these quadratic equations, but Butters also wishes to illustrate how to deal with those cases that are not as nice and quadratic as this. He therefore shows how to find an equation for the roots contained in the period $(4,1)$. He looks at

$$x^4 - Ax^3 + Bx^2 - Cx + D = 0. \tag{2.10}$$

Let the four roots[25] be denoted x_1, x_2, x_3 and x_4. The first coefficient is readily determined as $A = x_1 + x_2 + x_3 + x_4 = m$. The last is also easy, as $D = x_1x_2x_3x_4 = [1 + 13 + 16 + 4] = [34] = 1$. The sum of the roots taken two at a time, the coefficient B, is

[25]Butters uses x', x'', x''', x''''.

$$\sum_{i \neq j} x_i x_j = [14] + [17] + [5] + [12] + [17] + [3] = 2 + m_2. \tag{2.11}$$

Coefficient C is the sum of the roots taken three at a time:

$$\sum_{i,j,k \text{ unequal}} x_i x_j x_k = [16] + [4] + [1] + [13] = m_1. \tag{2.12}$$

The required equation is therefore

$$x^4 - m_1 x^3 + (2 + m_2)x^2 - m_1 x + 1 \tag{2.13}$$

Butters makes a few remarks after the treatment of the general equation. If $p - 1 = 2^a 3^b 5^c \ldots$, the solution to equation (2.2) can be made to depend on a equations of degree 2, b equations of degree 3, and so on. Butters notes that if the solution is to depend on quadratic equations only, then $p - 1 = 2^a$, and p must be of the form $2^a + 1$. He argues that a cannot have any odd factor (or p will not be prime) so p must have the form $2^{2^m} + 1$. This is a necessary, but not a sufficient, condition for a n-gon to be constructable by means of ruler and compass.

Butters lists a few primes of this form: $3, 5, 17, 257, 65537$. For these, the roots of unity of eq. 2.1 can be found by solving quadratic equations only. As an addendum, he writes that it is known that if an m-gon and an n-gon can be inscribed, so can the mn-gon. A polygon with twice as many sides as an inscribed polygon can also be inscribed this way. He concludes 'Hence an n-gon may be inscribed in a circle if n contains no odd factor except of the form $2^{2^m} + 1$, each such factor prime and not repeated.'

The final section of the paper shows how to inscribe a 17-gon in a given circle, the construction being based on a construction by Serret in his *Algébre Supérieure.* A simpler method of construction that would replace this was later be published by H. W. Richmond, later to become an EMS member, in [65].

2.3.2 'Notes on factoring'

Vol. 12 (1894) Pp. 31–33 Read: 12/01/1894 Educational

The purpose of this paper is to provide a more structured approach to factorising quadratic expressions, intended for beginners. A beginner would solve the monic expression (with p and q integers)

$$x^2 + px + q \tag{2.14}$$

by finding two factors, m and n of q (where $mn = q$) such that $m+n = p$. The difficulty in this would lie in factorising q. The student would either try the pairs of factors in the natural order, beginning with $(1, q)$, then $(2, q/2)$ if possible, and so on; or try pairs of factors at random. Both methods could easily become very time-consuming, especially if the expression turned out not to have any rational factors.

Butters suggests a combination of these two approaches, explaining his method by working through the special case

$$x^2 - 7x - 120. \tag{2.15}$$

If working on the general case (2.14), he would have chosen a pair of factors of $|q|$, m and n, m being the smaller, and compare them to $|p|$. If the sum (or if $q < 0$: the difference $n-m$) was too large, he would try increasing the smaller factor by 1 repeatedly until another factor of $|q|$ was found. This would give a new pair of factors to be tried. If the sum (or difference) was too large still, he would repeat the procedure until he reached $|p|$. If $|p|$ was not reached at all, but passed by, giving a sum (or difference) that was now too small, he would conclude that the expression had no rational factors. Once the factors had been identified, the correct signs would be affixed, depending on the signs of q and p.

If the sum or difference had initially been too small, the smaller factor would have been decreased by 1 in a similar fashion.

For expression (2.15), the first attempt is 10×12. The difference is here too small, so he reduces the smaller factor. He finds that 9 is not a factor of 120, but 8 is, so he tries 8×15. This gives the desired difference, and he arrives at the factorisation $(x + 8)(x - 15)$.

After working through his special case, he presents a shortcut that may be used when the initial choice of factors yields a sum or difference that differs greatly from the one required. Instead of increasing (or reducing) m by 1 at each step, one might try transferring one factor of n to m (or from m to n). This can save time, but he remarks one might also risk skipping the required pair.

Butters leaves a lot unsaid while explaining his method. The special case involves a

p and a q both < 0, and this is the only case he covers. He does not explain how he determines whether to look at sums or differences, or point out that he is working on $|q|$, rather than q. Nor does he explain what happens when a decreasing smaller factor reaches 0, or when an increasing smaller factor ends up exceeding the larger one.[26] As this paper is targeted at teachers, it would not be unreasonable to expect the reader to know these things.

Butters then presents an extension of this method to the general case

$$Ax^2 + Bx + C. \tag{2.16}$$

He attributes this extension to James McKean at Heriot-Watt College in Edinburgh. This extension will here be referred to as McKean's method, though Butters gives it no such name. He refers to his own method for the monic case as 'the method of natural numbers'.

The identity

$$(ax + m)(bx + n) \equiv abx^2 + (an + bm)x + mn \tag{2.17}$$

shows that one might factorise expression (2.16) by finding two factors of AC, an and bm, whose sum is B.

He gives expression (2.17) as a justification for this method, but as before, the theory is explained by use of examples. The extended method follows the monic case closely. Butters uses $|A|$ and $|C|$ as the initial choice of factors of $|AC|$, and proceeds precisely as he would have done in the monic case. The only real difference appears when the factors of AC have been found. The factors of expression 2.16 are then found by removing the greatest common divisor from the two terms in each of the expressions $(abx + an)$ and $(abx + bm)$.

His first example, $18x^2 - 111x + 80$, is dealt with fairly thoroughly, with plenty of explanations. For the second, $12x^2 - 5x - 72$, only the calculations are provided.[27] The latter example is then used to present the theory in a different way. Expression 2.16 can be written as a monic polynomial in Ax multiplied by a constant:

$$Ax^2 + Bx + C \quad \equiv \quad \frac{1}{A}\{(Ax)^2 + B(Ax) + AC)\}$$

[26] The correct conclusion to either scenario would be that the expression has no rational factors.

[27] Later on, he argues that nothing more than the calculations should be required of the student, as factorising should be considered a practical part of algebra.

This can then be solved using the 'method of natural numbers'. If $A = ab$ and $C = mn$, with the required factors an and bm, we get

$$\begin{aligned}\frac{1}{A}\{(Ax)^2 + B(Ax) + AC)\} &\equiv \frac{1}{ab}(abx + an)(abx + bm)\\ &\equiv \frac{1}{a}(abx + an)\frac{1}{b}(abx + bm)\\ &\equiv (bx + n)(ax + m)\end{aligned}$$

Butters argues that this form of the theory provides a much better way for students to deal with equations such as $ax^2 + bx + c = 0$. Instead of having to reduce these to $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$, they can change the variable to ax and factorise a monic polynomial with integer coefficients. This approach will remove the inconvenient fractions, and the students will also get used to the notion of changing variables at an early stage.

2.3.3 'A geometrical proof of certain trigonometrical formulae'

Vol. 15 (1897) pp. 86–89 Read: 12/03/1897 Educational

Butters here presents some alternative geometrical proofs of several trigonometrical formulae. Although he does not say so specifically, this is obviously intended for teaching in schools. Butters explains that most of the direct geometrical proofs already in use are valid only for angles in a certain range.[28] He argues further that the constructions involved are usually applicable to only one set of formulae. A more general method is therefore desirable, and he aims to supply one, using projections of lines. Proofs for all angles involving projections are not unheard of this time. Chrystal states in his *Algebra* that these formulae can be easily be proved by orthogonal projections, but leaves it at that [16, pg. 258]. Such proofs are covered more thoroughly in Pressland's and Tweedie's *Elementary Trigonometry*, though in a different form. They remark that this method 'has long been familiar to the French system of teaching Trigonometry' [101, pg iii].

A few remarks must be made before explaining this method. He appears to consider lines to be directed and infinite; AB is therefore normally read as the line passing through A and then B. An exception is when he projects AB onto another line, in which case he means the directed line segment AB.

[28]See for instance a proof for the addition formula in [91, p. 50].

The easiest way of describing how Butters measures angles between two lines AB and CD is by imagining him moving CD (if necessary), retaining the slope, so that C meets A. The angle is then measured in the counter-clockwise direction with AB as the initial side. The directions of the lines are of importance here. If the first line had been BA, C would have been taken to B instead, so producing the complement of the angle between AB and CD. Butters presumably does not do it quite like this, as he regards lines as infinite and therefore more likely looks at their point of intersection.

The old proof and the new proof

Butters first provides two of the more traditional geometrical proofs that he says led him to the general construction. He shows that

$$\sin 2a = 2\sin a \cos a, \tag{2.18}$$

$$\cos 2a = 1 - 2\sin^2 a. \tag{2.19}$$

As may be expected, these proofs make use of right-angled triangles, and require the angle $2a$ to be acute.[29]

His new method of proof is built on the projection of line segments onto the x- and y-axes. His first step is to establish formulae for these projections, which he does using figure 2.10.

He lets the line segment AB be a chord of a circle of radius r with AB subtending the angle θ. Further, ϕ is the angle between OX and OC, with OC perpendicular to AB. Since the lines OY and AB are respectively perpendicular to the lines OX and OC, ϕ must also denote the angle between OY and AB.[30] Butters reasons that the projection of AB on OY must then be:

$$\begin{aligned} AB\cos\phi &= 2CB\cos\phi \\ &= 2r\sin\left(\frac{1}{2}\theta\right)\cos\phi \end{aligned} \tag{2.20}$$

This formula, he says, can then be used to find projections of line segments on the y-axis. It can be argued that this reasoning is incomplete, as will be done below.

[29] In this first proof he makes use of a line r and a point M without explaining that they are respectively the radius of the circle and the foot of the altitude through the vertex A. These identities are not even marked on the accompanying figure.

[30] This angle can be produced by rotating $\angle XOC$ through one right angle.

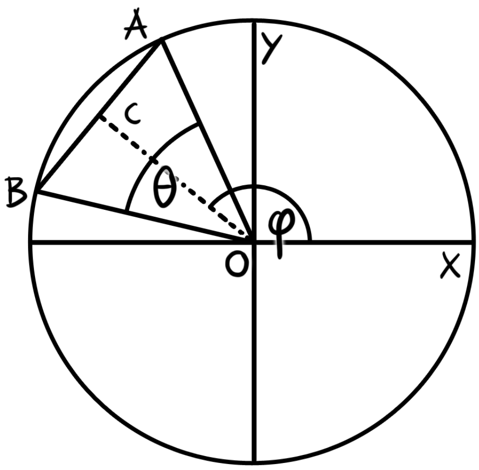


Figure 2.10: Projections on the axes

Similarly for the x-axis, the angle between OX and AB is $\left(\frac{\pi}{2} + \phi\right)$, and the projection of AB on OX must then be

$$\begin{aligned} AB \cos\left(\frac{\pi}{2} + \phi\right) &= 2r \sin\left(\frac{1}{2}\theta\right) \cos\left(\frac{\pi}{2} + \phi\right) \\ &= -2r \sin\left(\frac{1}{2}\theta\right) \sin\phi \end{aligned} \tag{2.21}$$

Another property that is used in the following proofs is that taking a detour while projecting a line segment does not affect the projection. The projection of some AB on, say, the x-axis equals the projection of AP, for some point P, plus the projection of PB. This holds as long as the lines are directed, which Butters considers them to be, giving negative projections. Butters would write such a projection via another point P as 'Project ($AB = AP + PB$) on OX'.

By choosing various expressions for ϕ and θ, he then develops the well-known trigonometrical identities in table 2.2. When projecting on either axis, he writes the projections in two ways, first as the projection of AB and then as a detour via some point P. The related formulae can easily be found, he says, in entirely similar ways, and he indicates how to do so. He supplies all these proofs with figures.

Table 2.2: The trigonometrical identities proved by projections

$\cos 2\alpha = 1 - 2\sin^2 \alpha$	$\theta = 2\alpha, \phi = 0$	(2.22)
$\sin 3\alpha = 3\sin\alpha - 4\sin^3 \alpha$	$\theta = 2\alpha, \phi = \alpha$	(2.23)
$\sin\alpha - \sin\beta = 2\sin\frac{1}{2}(\alpha-\beta)\cos\frac{1}{2}(\alpha+\beta)$	$\theta = \alpha - \beta, \phi = \beta$	(2.24)
$\sin(\alpha-\beta)\sin(\alpha+\beta) = \sin^2\alpha - \sin^2\beta$	$\theta = 2(\alpha-\beta), \phi = 2\beta$	(2.25)
$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$	$\theta = 2\alpha + 2\beta, \phi = \chi$	(2.26)

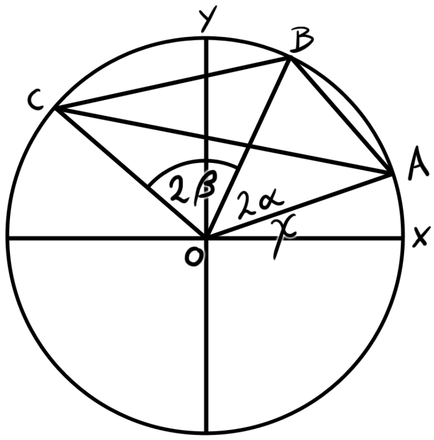


Figure 2.11: The general formula

The general formula and its derivations

The development of the last identity is of particular interest. On projecting ($AC = AB + BC$) on OY (Fig. 2.11), he gets what he calls the general formula:

$$2r\sin(\alpha+\beta)\cos(\alpha+\beta+\chi) = 2r\sin\alpha\cos(\alpha+\chi) + 2r\sin\beta\cos(2\alpha+\beta+\chi). \quad (2.27)$$

The identity in (2.26) is then found by putting $\alpha + \beta + \chi = 0$, giving

$$\begin{aligned}\sin(\alpha+\beta) &= \sin\alpha\cos(-\beta) + \sin\beta\cos\alpha \\ &= \sin\alpha\cos\beta + \sin\beta\cos\alpha\end{aligned}$$

By placing various such restrictions on the relations between the angles, the general formula will produce what Butters calls ‘an endless variety of results’, including the ones he has already developed. He finds by experimenting that it must be possible to express the formula 'in a form involving the three angles symmetrically', and he finds one by putting

$$\begin{aligned}\alpha &= Y - Z,\\ \beta &= Z - X,\\ \chi &= X - Y + Z.\end{aligned}$$

Inserting this into the general formula in (2.27) gives

$$\sin(Y - Z)\cos X + \sin(Z - X)\cos Y + \sin(X - Y)\cos Z = 0. \tag{2.28}$$

This, he claims, can lead to some other well-known results, such as a theorem for a pencil of rays,[31] Ptolemy's Theorem[32] and hence what he calls 'Euler's relation $BC \cdot AD + CA \cdot BD + AB \cdot CD = 0$ for points on a range'.

The final paragraph of the paper shows how the projection of lines on the axes can prove the formula for the sums of sines and cosines with arguments in arithmetic progression (Fig. 2.12). By projecting $(A_0A_1 + A_1A_2 + \ldots + A_{n-1}A_n = A_0A_n)$ on OY, he gets the expression

$$\cos\alpha + \cos(\alpha + \beta) + \ldots + \cos(\alpha + (n-1)\beta) = \frac{\sin\frac{n\beta}{2}\cos(\alpha + \frac{n-1}{2}\beta)}{\sin\frac{\beta}{2}} \tag{2.29}$$

This gives, with m and n integers,

$$\cos\frac{m\pi}{n} + \cos\frac{3m\pi}{n} + \ldots + \cos\frac{(2n-1)m\pi}{n} = 0 \tag{2.30}$$

This happens if $\alpha = \frac{m\pi}{n}$ and $\beta = \frac{2m\pi}{n}$, though Butters does not specify this. This is the projection of a regular n-gon. He remarks that the projection of a regular star polygon,

[31] He gives the following formulae:

$$\begin{aligned}\sin(bc)\cos(ad) + \sin(ca)\cos(bd) + \sin(ab)\cos(cd) &= 0\\ \sin(bc)\sin(ad) + \sin(ca)\sin(bd) + \sin(ab)\sin(cd) &= 0.\end{aligned}$$

[32] This theorem states that if a quadrilateral can be inscribed in a circle, the product of the diagonals equals the sum of the products of the opposite sides. If the four vertices are named in order A, B, C, and D, this can be written as $\overline{AD} \cdot \overline{BC} + \overline{AB} \cdot \overline{CD} = \overline{AC} \cdot \overline{BD}$.

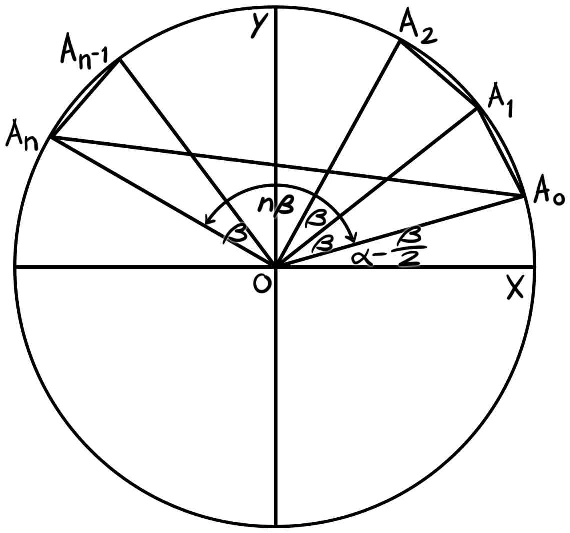


Figure 2.12: Sums of cosines

which he calls a 'regular (crossed) n-gon', would give the same result.

A slight flaw

Butters sets out to show that his formula, in contrast to most other formulae, works for all angles. It is therefore interesting that he avoids pointing out how and where the proofs differ. He does not explain how to deal with the slightly more troublesome angles greater than or equal to π. This is easy enough to do if allowing negative angles, which Butters does. If θ was $\angle BOA$ in figure 2.10, he could work on the negative $\angle AOB$ instead. Extending his geometrical proof in this way is not too complicated, and from a contemporary point of view, it may have been acceptable to leave this out. He can certainly expect the teachers to be familiar with handling negative magnitudes, and he must be doing so. He does not mention that his lines are directed either, he takes it for granted that his readers regard them the same way.[33]

The problem with this is that similar reasoning can be used to extend the more traditional proofs, and according to Butters's contemporaries, the pupils are not familiar with negative geometrical magnitudes. Pressland and Tweedie write in their book from 1899 [101, pg. iii] that such magnitudes are lacking from elementary teaching, and they use this as an argument against the right-angled triangle definitions of trigonometrical

[33]Negative angles and distances are for instance covered in a textbook from 1891 [40], and also in earlier editions of the same book.

ratios. Although Butters's proofs do allow for greater angles than the traditional proofs do, this would still be an issue.

When the point of a paper is to show that one method works where other methods fail, it makes little sense not to point out how. The proofs presented here appear somewhat incomplete. Butters's success at his first goal may therefore be debated, but he certainly succeeds at his second. The method does work for a large variety of formulae, not just one set.

This topic is to be covered twice more in the *Proceedings*, both times by academics. W. E. Philip [59] and D. K. Picken [60] both present proofs by projection of the addition theorem, in 1900 and 1904 respectively. Picken uses a technique similar, but not identical, to Butters's. The difference in style is still obvious. Where Butters take shortcuts, Picken is clear and concise and leaves little room for misunderstandings. Picken defines projections and states clearly any theorem that is to be proved. Philip is closer to Butters in method, but closer to Picken in style.

2.3.4 'Elementary notes'

This paper consists of two short notes; one on the factorisation of polynomial expressions and one on the teaching of geometry. The indexes treat these two as a single paper.

'On the factorisation of a function of n variables'

Vol. 16 (1898) pp. 78–80 Read: 10/06/1898 Research

The first part of the paper gives a further extension of the method of factorisation presented in section 2.3.2. This time Butters deals with polynomial functions of n variables, none having higher degree than 2. Butters calls them simply 'functions'.

Such a function can be written as a trinomial in one of the variables, say x_1, making the coefficients polynomials in the remaining variables. By applying McKean's method from an earlier paper, this can either be factorised or proven not to have any proper factors with rational coefficients.[34] If the coefficient of x_1^2 is $A(x_2, \ldots, x_n)$ and the coefficient of x_1^0 is $C(x_2, \ldots, x_n)$, it is necessary to find two factors of $A(x_2, \ldots, x_n) \times C(x_2, \ldots, x_n)$ that add up to the remaining coefficient. Butters then writes A and C as trinomials in one of the other variables, and tries factorising these, and so on.

[34] Butters does not use the term 'proper', but he disregards the original expression as a factor of itself.

Eventually, polynomials in one variable are reached, which can then be factorised, or proven not to have rational factors.

As before, Butters does not explain this method for the general polynomial. Instead, he works through three examples. The first example is relatively simple.

$$8x^2 + 115xy - 79xz + 52x + 42y^2 - 125yz - 35y + 63z^2 - 13z - 28 \tag{2.31}$$

He writes this as a trinomial in x. The coefficient of x^2 is constant, so Butters proceeds by factorising the third coefficient, writing it as a polynomial in y.

$$42y^2 - (125z + 35)y + (63z^2 - 13z - 28) \tag{2.32}$$

Again, the coefficient of y^2 is constant. The third coefficient is now a quadratic polynomial in z, which he factorises using McKean's method. Working his way back up through the equation gives the factorisation

$$(8x + 3y - 7z - 4)(x + 14y - 9z + 7). \tag{2.33}$$

Butters then shows that this particular example can be factorised more rapidly. As none of the terms in this particular function are of degree higher than 2, the factors must be of the form

$$(ax + by + cz + d)(a'x + b'y + c'z + d'). \tag{2.34}$$

The required coefficients can then be determined 'by inspection'. Butters factorises the terms containing x and y, $8x^2 + 115xy + 42y^2$, and gets $(8x + 3y)(x + 14y)$. This gives the first terms of the required factors in (2.34). Factorising the terms involving y and z, that is $42y^2 - 125yz + 63z^2$, gives the y- and z-terms. Finally, $63z^2 - 13z - 28$ is factorised to add the constant terms. By doing it this way, he has left three terms of (2.31) unused, $-79xz + 52x - 35y$. He remarks that it is necessary to check that the factors found suit these unused ones as well, which they do in this case.

The second example is one that cannot be factorised by inspection:

$$3a^2b^2c - 9a^2bc^2 + a^2b - 3a^2c - 3abc^2 + 2ab^2c + ac + 2bc^2. \tag{2.35}$$

He factorises this by writing the function as a trinomial in b, and proceeding as above, getting[35]

$$(ab - 3ac - c)(3abc + 2b + a). \tag{2.36}$$

The final example shows that this method can be applied in certain cases where some of the variables are of degree higher than 2, such as:

$$a^3c^2 + 2a^3c + a^3 + 4ab^2c + b^3 - b^3c^3. \tag{2.37}$$

If this is written as a trinomial in c, one has to factorise the expressions $(a^3 - b^3)$ and $(a^3 + b^3)$. This can be done fairly easily, as $(a^3 - b^3) \equiv (a - b)(a^2 + ab + b^2)$ and $(a^3 + b^3) \equiv (a + b)(a^2 - ab + b^2)$. This leads to the factorisation

$$(a^2c + a^2 + abc - ab + b^2c + b^2)(ac + a - bc + b). \tag{2.38}$$

'On the use of the term "produced" '

Vol. 16 (1898) pp. 80–81 Read: 10/06/1898 Educational

Butters is here arguing for a change of convention in the teaching of geometry. One of the distinguishing features of modern geometry compared to Euclidean, he says, is that statements are made 'perfectly general', such as 'two straight lines meet in a point'.[36] He remarks that there are two problems with this particular statement; parallel lines do not meet, and two straight lines might not meet unless produced, that is, unless they are extended. From the point of elementary geometry, Butters feels it is best not to incorporate parallel lines in the above statement, but the second problem is avoidable. He suggest adopting the convention that straight lines are infinite, and so need not be produced at all. He believes such a convention will make arguments more comprehensive, and that it will 'also group together cases apparently distinct'. It should be noted that Butters is employing infinite lines in this way in the previous paper, so he is not claiming that the notion is new. The novelty lies in incorporating it in elementary teaching at such an early stage.

From a more practical point of view, Butters believes that changing the convention will also prevent the sometime erroneous uses of the term 'produced'. He gives three

[35] He points out that he could have written it as a polynomial in a or c instead.

[36] This is presumably supposed to be an alternative to the more traditional definition 'If two lines are such that they cannot coincide in any two points without coinciding altogether, each of them is called a straight line.' [47, p. 1].

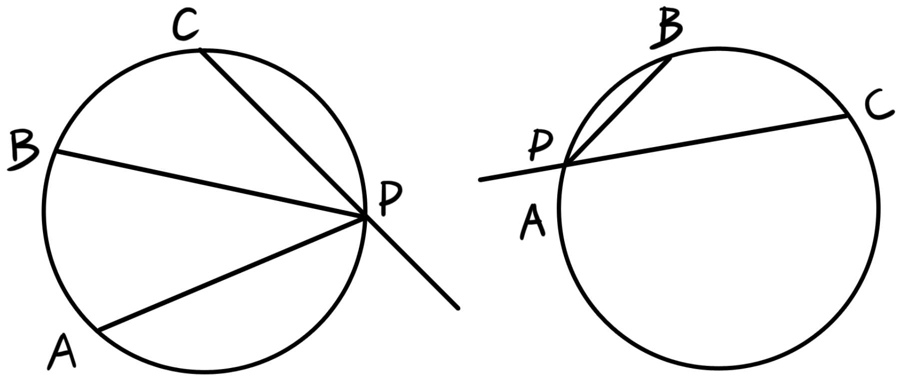


Figure 2.13: Wrong use of the term 'produced'

examples that he finds typical for recent examination papers.

- 'Useless': Butters's first example asks the student to draw a perpendicular from a specific point on a triangle to a specific line 'produced', before proving a proposition regarding a point on this line. Butters does not approve of this use of the term 'produced' here, as it is possible to prove that the perpendicular must meet the line produced and not the line segment itself.
- 'Misleading': The second example also involves drawing perpendiculars to lines 'produced', this time in connection to a quadrilateral. From a point E, the student must draw perpendiculars to a line AK and then to a line KC 'produced'. By the given instructions, it is possible to create a quadrilateral where it is the line AK that has to be produced, and not KC.
- 'Wrong': The third example involves two equal arcs of a circle, AB and BC and a point P on BC. The student is asked to show that BP will bisect the angle contained by AP and CP 'produced'. This is correct if the arcs happen to be minor arcs, but as figure 2.13 shows, it does not hold for major arcs. However, it is true that BP will bisect one of the angles between the two infinite lines passing through AP and CP, regardless of the type of arc.

Butters devotes the final paragraph to a few remarks on the naming of arcs and angles connected with the circle. In this paper, he has named the arcs in the counterclockwise direction, but argues that it might be more useful to change this to clockwise, one of the consequences being that it would be unnecessary to 'distinguish reflex angles, and generality of proof would be gained'.

2.3.5 'Notes on decimal coinage and approximation'

Vol. 20 (1902) pp. 50–61 Read: 09/05/1902 Educational

The introduction to this paper might give the idea that the purpose is to advocate the decimalisation of British currency, but the real aim is to encourage the teaching of decimal arithmetic in schools. Butters begins with a short account of previous attempts at establishing British decimal coinage.[37] He considers some of the benefits of such a system, and emphasises the indirect consequences. Not only might it work as an incentive for decimalising the weights and measures, but it will also favour the decimal fraction, as explained below, above the vulgar fraction. It is this latter thought, which he attributes to De Morgan, that leads him on to the main topic of the paper.

A few remarks are required on the terms he uses. By a 'decimal' he means integers (such as 34), decimal fractions (0.34), and 'mixed' numbers (3.4). Vulgar fractions are simply fractions of the type $\frac{a}{b}$.

The relative ease of calculating with decimals compared to vulgar fractions makes this a very important point for Butters. He refers to his own experience as a teacher and to reports on local examinations that all demonstrate that pupils are not as adept at working with decimals as he believes they should be. One of the reports states that many pupils simply do not know how to work with decimal fractions at all. When asked to do so, they convert them into vulgar fractions before performing the calculations and then convert back to decimals again. Butters argues that this is a major waste of time. When the student is asked to compute an answer to the nearest penny, they often compute to the nearest millionth of a penny instead. He suggests that this can be avoided if the order of teaching is reversed, so that decimal fractions are taught before vulgar fractions. The rest of the paper is devoted to a thorough explanation of the decimal system and how it can be taught.

[37] Amongst other things, he explains that the florin was originally introduced as one tenth of a £, this being imprinted on the coins. In 1893, the text was changed to 'two shillings'.

Decimal arithmetic

Butters illustrates how much simpler the decimal part of the coinage system is compared to the non-decimal part.[38] He then moves on to the basic properties of the decimal system, mentioning how only one denomination is necessary; that of the chosen unit. He defines the decimal point as a point placed to the right of the units figure.[39] He also introduces the terms tens and tenths, hundreds and hundredths, to denote the spaces to the left and right of the decimal point.

Butters main point is that the four rules of addition, subtraction, multiplication and division for integers can easily be extended to include all decimal numbers. This will come as no surprise to modern eyes, but Butters's explanation still holds interest. Addition and subtraction will need no change, as long as one ensures the units figures are arranged in a vertical column. More time is spent on multiplication. In a footnote later in the paper, Butters extends the 'decimal place'-system to the left of the decimal point, operating with decimal place -2 (hundreds), -1 (tens), 0 (units), 1 (tenths), 2 (hundredths) and so on. Using this notation, Butters explanation can be formalised, though he does not do this himself.

If a figure in the multiplier has decimal place m and this figure is multiplied with a figure in the multiplicand with decimal place n, the product will have decimal place $m + n$.[40] This can be illustrated by his example 1.341×43.2:

$$\begin{array}{r@{.}l} 1&341\\ 43&2\\ \hline 53&64\\ 4&023\\ 0&2682\\ \hline 57&9312 \end{array}$$

When multiplying the 4 in 1.341 (decimal place 2) with the 2 in 43.2 (decimal place 1) one gets 8 which must have decimal place $2 + 1 = 3$, as in 0.2682.

Moving on to division, he explains how to place the figures of the quotient when both dividend and divisor are integers. The same procedure is then applied to what he calls mixed numbers. He is not, as one might expect, multiplying the divisor and

[38] He does this by showing how £567 can be paid in four different ways without the digits changing, such as 567 sovereigns, or 56 ten-pound notes and 7 sovereigns, and so on. He then shows how complicated it is to express £5/6/7 in, say, pence only, etc.

[39] He argues that it would be more correct to call it 'the units point' rather than 'the decimal point'.

[40] If the product is greater than 9, then this decimal place is affixed to the 'unit' of the product.

dividend with a power of 10 to make the divisor an integer. Instead, he uses a method derived from his rule for multiplication. Consider the division

$$4.8)1.728.$$

The quotient is then to be placed on the line above this, with the decimal point aligned with that of 1.728. As $172 = 48 \times 3 + 28$, the question is where to place the 3. Butters then says that the 3 should be placed vertically above the product obtained from it and the units figure of the divisor. This can be formalised as for multiplication. If the decimal place of the 4 in the dividend is n and the decimal place of the product 3×4 is m, then the 3 in the quotient will have decimal place $m - n$. In this case $n = 0$ and $m = 1$, so the 3 is placed vertically above the 7.[41]

$$\begin{array}{r} 0.36 \\ 4.8)1.728 \\ 1\,44 \\ 288 \\ 288 \end{array}$$

Butters completes this part of the paper by quoting from a discussion on the teaching of mathematics, voicing the opinion that saving time is more important than ensuring accuracy in the last figure.

Decimal approximation

The next step is then decimal approximation, such as carrying out a division to a certain number of decimal places. Butters believes that most text-books are too thorough on the subject. The most common procedure involves fixing the number of figures to be used as a divisor. He suggest fixing the number of figures in the dividend instead. He works through several examples to show how his method works.

His method can be generalised. If carrying out a division to the mth decimal place, and the leftmost figure in the divisor has decimal place n, you need retain $m+n$ decimal places in the dividend, and as much of the 'divisor as is necessary to go under these figures'. Butters's last example is the division $0.125179 \div 1236$ to 5 decimal places. The

[41] The same result would have been achieved using the 8 instead of the 4 of the divisor. The product of 3 and 8 has decimal place 2. 8 has decimal place 1, so the 3 must have decimal place $2 - 1 = 1$.

leftmost figure of the divisor has decimal place -3, so $5 + (-3) = 2$ decimals of the dividend must be kept:

$$\begin{array}{r} 0.00010 \\ 12\cancel{3}\cancel{6})0.12\cancel{5}\cancel{1}\cancel{7}\cancel{9} \\ 12 \\ 0 \end{array}$$

Butters then shows how this procedure simplifies the arithmetic in expressions such as $(3.962 \times 0.7189) \div 7439.2$. While computing the product in the dividend, several stages may be skipped as only 2 decimals need to be retained.

The remaining parts of the paper is devoted to using decimal approximations while calculating sums of money. He suggests working with a non-existing coin called a 'mil'. This coin was a suggestion made in 1853, and defined to be one thousandth of a pound. This gives 100 mils to a florin, which differs very little from the farthings, that are 96 to a florin. Since the difference is so small, Butters says younger pupils can safely work with pounds, florins and farthings, instead of pounds, florins and mils. For addition, subtraction and division, the inaccuracies will remain fairly small. Extra care must be taken when multiplying. To ensure accuracy, he observes that

$$\begin{array}{rlcllcl} 100 & \text{mils} & = & 1 \text{ florin} & & = & 96 \text{ farthings} \\ 25 & \text{mils} & = & 1 \text{ sixpence} & & = & 24 \text{ farthings} \\ 25 & & = & 1 & + & & 24 \end{array}$$

He explains that

> [T]he number of mils in any sum of money
> = the number of farthings
> + the number of sixpences in the sum.[42]

Expressing a decimal as £. s. d. is easy when reading the money as florins and mils and remembering that 25 mils is 24 farthings. He shows how to write 10/6, 8/4 and 9/ as decimals, stating that parts of a mil may usually be neglected, except when multiplying. As an example, 10/6 is 5 florins and 24 farthings, giving approximately £0.524, and exactly £0.525.

[42]In a footnote to this, he mentions the method of decimal approximation that will be presented in his next paper. It is here explained in a more practical way: 'In these cases the parts of a mil may be obtained as a decimal, by treating the last 2 decimal places (or their excess over 25, 50, 75) as pence, and reducing these to mils for the next two places, and so on'. Seeing as 1 penny is 4 farthings, and that one adds 1 for each 24 farthings, it becomes clear that this is the same method.

He finishes by working through three examples involving currency, with and without decimal approximation, to show how much time, labour and space can be saved.

2.3.6 'On the decimalization of money'

Vol. 21 (1903) pp. 112–115 Read: 09/01/1903 Educational

This is a response to another paper from the *Proceedings*; 'On the decimalization of English money, and some simplifications in long division' [20]. This paper, written by J. D. Hamilton Dickson, is presenting ideas of J. Hamblin Smith. It gives a method of expressing sums of British currency as a decimal of a £ to more than 3 decimal places.[43]

The florins, shillings and sixpence can be accurately expressed with 3 decimal spaces, so the only threat to a nice, clean figure are the pence and farthings, or simply farthings (as pence are readily expressed as such). Hamilton Dickson therefore works on converting n farthings, each being $\frac{1}{960}$ of a £, to decimals. Butters abstracts the method from its application to money. He works on the special case $\frac{17}{96}$, instead of the general, this 'for simplicity'.

> In reducing $\frac{17}{96}$ to a decimal form, if at any stage we multiply the last two digits found (or their excess over 25, 50 or 75) by 4 and increase this product by 1 for each 24 contained in it, we obtain the next two digits.[44]

Hamilton Dickson believes this method to be new, but Butters explains that this is not so and gives references to other works where it can be found.[45] None of these works contain a proof of this method, which is what Butters now wishes to supply.

Since he works on a special case, the proof as given is not perfectly general. His method is fairly easy to extend to the general case $n/96$, where n can be assumed to be any positive number < 96.

[43]A method giving the first three decimals was already well-known. Butters refers to this as the Actuaries' Rule. This rule uses the facts that a florin is £0.1, a sixpence is £0.025 and that a farthing is only slightly more than £0.001. Butters does not explain what this rule is, but he gives references to where it may be found, such as in 'Kershey's edition of *Wingate's Arithmetic*, 1673 (p.191)' and *Cocker's Decimal Arithmetic*, 1685 (although in a form which is not quite accurate)'.

[44]Hamblin Smith's method is somewhat simpler, and produces one new digit at the time. The methods are essentially the same.

[45]He mentions the method himself in a footnote to the previous paper, 'Notes on decimal coinage and approximation'.

A generalisation of Butters's proof

Let $u_1 = n + b_1$, and, for $i > 1$

$$u_i = 4(u_{i-1} - 25b_{i-1}) + b_i \tag{2.39}$$

The b_i is the recurrence relation indicated in the quote above, and counts the number of 24s in $4(u_{i-1} - 25b_{i-1})$, which is the same as the number of 25s in u_i. As Butters is working on a special case, he has no need for such terms as this, and explains it in words instead.

What needs to be proven is that

$$\frac{n}{96} = \frac{u_1}{100} + \frac{u_2}{100^2} + \frac{u_3}{100^3} + \frac{u_4}{100^4} + \ldots \tag{2.40}$$

The u_i can be expressed as follows

$$\begin{aligned} u_i \quad &= 4(u_{i-1} - 25b_{i-1}) + b_i \\ &= 4(4(u_{i-2} - 25b_{i-2}) + b_{i-1} - 25b_{i-1}) + b_i \\ &= 4(\ldots(4(n + b_1 - 25b_1) + b_2 - 25b_2)\ldots) + b_{i-1} - 25b_{i-1}) + b_i \end{aligned} \tag{2.41}$$

Getting rid of the parentheses gives

$$\begin{aligned} &= 4^{i-1}n + 4^{i-1}b_1 - 4^{i-1} \times 25b_1 + 4^{i-2}b_2 - 4^{i-2} \times 25b_2 + \ldots \\ &\quad \ldots + 4b_{i-1} - 4 \times 25b_{i-1} + b_i \\ &= 4^{i-1}n + 4^{i-1}b_{i-1} + 4^{i-2}b_2 + \ldots + 4^2b_{i-2} + 4b_{i-1} + b_i \\ &\quad - 100 \times 4^{i-2}b_1 - 100 \times 4^{i-3}b_2 - \ldots - 100 \times b_{i-1} \\ &= 4^{i-1}n + \left(\sum_{j=1}^{i} 4^{i-j}b_j\right) - 100\left(\sum_{j=1}^{i-1} 4^{(i-1)-j}b_j\right) \end{aligned} \tag{2.42}$$

The sum in (2.40) can now be written as

$$
\begin{aligned}
&\frac{n}{100} + \frac{b_1}{100} \\
&+\frac{4n}{100^2} + \frac{1}{100^2}\left(\sum_{j=1}^{2} 4^{2-j} b_j\right) - \frac{b_1}{100} \\
&+\frac{4^2 n}{100^3} + \frac{1}{100^3}\left(\sum_{j=1}^{3} 4^{2-j} b_j\right) - \frac{1}{100^2}\left(\sum_{j=1}^{2} 4^{2-j} b_j\right) \\
&+\dots \\
&+\frac{4^{i-1} n}{100^i} + \frac{1}{100^i}\left(\sum_{j=1}^{i} 4^{i-j} b_j\right) - \frac{1}{100^{i-1}}\left(\sum_{j=1}^{i-1} 4^{(i-1)-j} b_j\right) \\
&+\frac{4^i n}{100^{i+1}} + \frac{1}{100^{i+1}}\left(\sum_{j=1}^{i+1} 4^{(i+1)-j} b_j\right) - \frac{1}{100^i}\left(\sum_{j=1}^{i} 4^{i-j} b_j\right) \\
&+\dots
\end{aligned}
$$

All the terms involving b_i's will disappear. What is left is

$$\frac{n}{100} + \frac{4n}{100^2} + \frac{4^2 n}{100^3} + \dots + \frac{4^{i-1} n}{100^i} + \dots \tag{2.43}$$

$$= \frac{n}{100}\left(1 + \frac{4}{100} + \frac{4^2}{100^2} + \frac{4^3}{100^3} + \dots\right) \tag{2.44}$$

$$= \frac{n}{100}\left(\frac{1}{1 - \frac{4}{100}}\right) \tag{2.45}$$

$$= \frac{n}{100 - 4} = \frac{n}{96} \tag{2.46}$$

This will hold regardless of the values of the b_i's. The restrictions placed on the b_is simply ensure that none of the u_is exceed 100.

Butters version of this proof is much simpler. He lists the computations for the first five pairs of digits, ending with:

$$0.0000000033 = (4[4\{4(4 \times 17 + 2 - 25 \times 2) + 3 - 25 \times 3\} + 1 - 25 \times 1] + 1)/100^5. \tag{2.47}$$

He adds them all, indicating the remaining terms with '. . . '. Splitting up the parentheses gives the expression in (2.43), for $n = 17$, which he then takes through to (2.46). He gives some justification for why the terms not of the form $\frac{4^{i-1}\times 17}{100^i}$ end up cancelling each other out:

> This is easily seen to be the case when we consider that each addition of 1 makes a 24 into 25; the following subtraction must therefore be of the same number of 25's as we have previously added ones.

This is essentially the same as saying that b_i count both the number of 25s in u_i and the number of 24s in $4(u_{i-1} - 25b_{i-1})$

This method, he states, can be extended to any division $100 - a$, or even $1000 - a$, the latter giving 3 digits at a time.[46] He also shows that a very similar method can be applied to $\frac{17}{104}$, by replacing $u_{i-1} - 25b_{i-1}$ by $25b_{i-1} - u_{i-1}$.

The finishing remark emphasises that the method may be applied to any stage of the process, by using the last two digits found.

2.3.7 'On the use of symmetry in geometry'

Vol. 22 (1904) pp. 54–63 Read: 11/03/1904 Educational

It would not be an exaggeration to say that the foundations of geometry had changed somewhat around 1900. What the press called 'the dethroning of Euclid'[47] was to have the greatest consequence for schoolteachers. Butters is here dealing with one aspect of this. '[T]he order of Euclid has been abandoned', he writes, and with that, the old proofs that are depending on this order have to be abandoned as well.[48] He does not lament this loss much, and not only because such dependency on order is undesirable, but also because the methods used to prove the propositions are simply too many. He writes:

> If we except the method of *reductio ad absurdum*, it is scarcely an exaggeration to say, for example, that in Book I no three propositions are proved by the same method.[49]

[46] When dividing by $100 - a$, one must multiply the last two digits by a and add 1 for each $((100/a) - 1)$ in the product.

[47] See the review by H. E. Piggott of *The Foundations of Euclidean geometry* by H. G. Forder in *The Mathematical Gazette*, Vol. 13, No. 191 (Dec. 1927), pp. 463-466.

[48] These developments are very new at the time Butters is writing this. The strict adherence to the order of Euclid was abandoned in England in the summer of 1903 [85].

[49] *Reductio ad absurdum* means 'proof by contradiction'.

Butters feels this new direction provides an excellent opportunity for developing more user-friendly methods, that are less sensitive to order and have a much wider applicability. He believes symmetry can provide such methods and aims to show how the various forms of symmetry may be used on the propositions in Book I. He does not prove that this works for all propositions, he is only describing and illustrating the methods that may be used. Although he has limited himself to the first book, he remarks that the methods presented here can be applied elsewhere as well, for example to results regarding circles in Books III and IV.

He applies three kinds of symmetry in this paper.

- **Reflection symmetry**: Butters calls this 'axial symmetry', but the modern term is used here. A figure has reflection symmetry if an axis of symmetry can be found, where the one side of the axis is the reflection of the other. Two points, angles or lines that are reflections of each other are said to correspond. A figure may have more than one axis of symmetry.
- **Rotational symmetry (Axial symmetry)**: This is what Butters calls 'central symmetry'. A figure has this kind of symmetry if it looks the same after being rotated 180°, or 'through two right angles', about a centre of symmetry.[50] Two points, angles or lines are said to correspond if they switch places after the rotation.
- **Skew symmetry**: This is derived from reflection symmetry, but now the axis is skewed. In reflection symmetry, lines joining corresponding points are perpendicular to the axis. In skew symmetry, they are not perpendicular, but form some other constant angle with the axis instead. Points, angles or lines correspond if they have the same distance to the axis measured along this non-perpendicular line. This gives what Butters calls 'the theory of skew-symmetrical figures'.

For the first two approaches, reflection symmetry and rotational symmetry, he lists a few of what he calls 'obvious propositions', though the term 'axioms' would have worked equally well. These are given in table 2.3. These two lists are not presented together as they are here, but he remarks that they should be compared to each other. Every statement, he says, made in either of these two symmetries has a corresponding statement in the other, which can be found by interchanging points and lines. Butters

[50] The term 'rotational symmetry' will be used in this treatment, though it is strictly speaking too broad a term. Rotational symmetry involves any angle of rotation, not just 180°. Inconveniently enough, the correct modern term would be 'axial symmetry'. 'Rotational symmetry' is therefore preferred here, to avoid confusion with Butters's use of the other.

	Reflection symmetry	Rotational symmetry
1.	Each point on the axis corresponds to itself, and, conversely, if a point corresponds to itself it is on the axis.	Each line passing through the centre corresponds to itself, and, conversely, if a line corresponds to itself it passes through the centre.
2.	The axis corresponds to itself, as does every line perpendicular to the axis, and, conversely, if a line corresponds to itself it is an axis of symmetry or is perpendicular to an axis of symmetry. (Note: a figure may have more than one axis of symmetry.)	The centre corresponds to itself, and, conversely, if a point corresponds to itself, it is the centre of symmetry.
3.	The line passing through two points corresponds to the line passing through the corresponding points, and, conversely, if two lines correspond, they pass through corresponding points.	The point of intersection of two lines corresponds to the point of intersection of the corresponding lines, and, conversely, if two points correspond, corresponding lines pass through them.
4.	The distance between two points equals the distance between the corresponding points; conversely, if two points on two corresponding lines correspond, then if equal distances be measured from these points in the corresponding directions (both towards or both away from the axis) along these lines, the points thus found are corresponding points.	The angle between two lines is equal to the angle between the corresponding lines, and, conversely, if two lines correspond and at corresponding points on them equal angles are formed in the same direction (both clockwise or both counter-clockwise), then the other arms of these angles are corresponding lines.
5.	The angle made by a pair of lines is equal to the corresponding angle made by the corresponding lines, and conversely. (Note: if we consider the direction of a line towards the axis to be positive in all cases, then corresponding angles are of opposite sense.)	The distance between two points is equal to the distance between the corresponding points, and conversely. (Note: It will be shown that corresponding lines are parallel; corresponding distances are therefore of opposite sense.)
6.	The distance of a line from a point is equal to the distance of the corresponding line from the corresponding points. (This assumes that only one perpendicular can be drawn from a point to a line.)	—

Table 2.3: Basic propositions as given by Butters

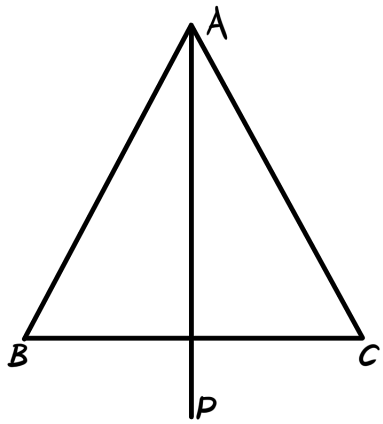


Figure 2.14: Proposition I.5

remarks that this means the pupils can be introduced to the 'concept of Duality' at an early stage. The propositions listed here, Butters says, are sufficient to show that the method can be used on many of Euclid's propositions, though many more such statements can be found.

Reflection symmetry

He gives ten examples of how reflection symmetry can be used, starting with proposition I.5 in Euclid (Fig. 2.14).

I.5 Given $AB = AC$, to prove that $\angle B = \angle C$.

Butters draws the bisector of $\angle A$, calling it the axis of $\angle A$.[51] This is to be used as axis of symmetry. AP corresponds to itself, and the angles $\angle PAB$ and $\angle PAC$ are equal, so AB must correspond to AC. Since A corresponds to itself, and $AB = AC$, it follows that B and C correspond and also that BC and CB correspond. Since the angles $\angle B$ and $\angle C$ are formed by corresponding lines, they must be equal.

The following nine propositions are then proved using reflection symmetry in a similar fashion. With the exception of I.5, the phrasing of the propositions are taken from Mackay's *Elements of Euclid* [47], though the latter three are shortened somewhat.

- **I.6**: *If two angles of a triangle be equal, the sides opposite them shall also be equal.*
- **I.18**: *The greater side of a triangle has the greater angle opposite to it.*

[51] Butters uses the symbol $\widehat{A}$ instead of $\angle A$.

- **I.19**: *The greater angle of a triangle has the greater side opposite to it.*

- **I.20**: *The sum of any two sides of a triangle is greater than the third side.*

- **I.24**: *If two triangles have two sides of the one respectively equal to two sides of the other, but the contained angles unequal, the base of the triangle which has the greater contained angle shall be greater than the base of the other.*

- **I.25**: *If two triangles have two sides of the one respectively equal to two sides of the other, but their bases unequal, the angle contained by the two sides of the triangle which has the greater base shall be greater than the angle contained by the two sides of the other.*

- **I.4**: *If two sides and the contained angle of one triangle be equal to two sides and the contained triangle of another triangle, the two triangles shall be equal in every respect.*

- **I.8**: *If three sides of one triangle be respectively equal to three sides of another triangle, the two triangles shall be equal in every respect.*

- **I.26**: *If two angles and a side in one triangle be respectively equal to two angles and the corresponding side in another triangle, the two triangles shall be equal in every respect.*

After proving the first seven of these propositions, he remarks that they have now been proved independently of the congruence theorems I.4, 8 and 26, this in contrast to how it is done in Euclid. Although these new proofs are fairly independent of each other, they are not completely independent. The proof of I.20 depends on I.19; the proof of I.24 depends on I.20. The proof of I.25 can, like I.24, be made to depend on I.20, but Butters chooses a different approach that is based on I.5 and 6.

For some of these proofs, he has assumed the truth of I.16 (for I.18 and 19) and I.32 (for I.26).[52] These two propositions are left unproved, but he states that they can be proved independently of the propositions he deals with here.

[52] Butters does not say exactly which propositions depend on I.16, but inspection shows them to be I.18 and 19.

Rotational symmetry

Butters then moves on to rotational symmetry and lists the five propositions given in table 2.3. He shows how to use this type of symmetry to prove the following propositions from Euclid:

- **I.15**: *If two straight lines cut one another, the vertically opposite angles shall be equal.*
- **I.27**: *If a straight line cutting two other straight lines make the alternate angles equal to one another, the two straight lines shall be parallel.*
- **I.29**: *If a straight line cut two parallel straight lines, it shall make the alternate angles equal to one another.*[53]
- **Corollary to I.29**: *If two lines are parallel and pass through corresponding points, the lines correspond.*
- **I.33**: *The straight lines which join the ends of two equal and parallel straight lines towards the same parts, are themselves equal and parallel.*
- **I.34**: *A parallelogram has its opposite sides and angles equal, and is bisected by either diagonal.*

The method is here illustrated by his proof for I.27 (Figure 2.15).

It is given that $\angle AGH = \angle DHG$. Let the the mid-point of GH, O, be the centre of symmetry. OG then corresponds to OH. Since they are also equal, G must correspond to H. Since $\angle OGA = \angle OHD$, GA must correspond to HD, and consequently HC corresponds to GB. Prop. 3 then gives that any point of intersection of GA and HC must correspond to any point of intersection of HD and GB. So, if there is *one* point of intersection of AB and CD, then there are *two.* This is impossible for lines that do not pass through the centre, so such corresponding lines cannot have any points of intersection, and they must be parallel. In particular, AB and CD are parallel.

The remaining propositions are proved in similar ways. As was the case for reflection symmetry, the proofs are relatively independent of each other. I.15 and 27 depend only on the given obvious propositions. The remaining three depend only on the conclusion

[53]According to Mackay, this proposition also includes that 'any exterior angle equal to the interior opposite angle on the same side of the cutting line' and 'the two interior angles on the same side of the cutting line equal to two right angles'. Butters does not touch on this.

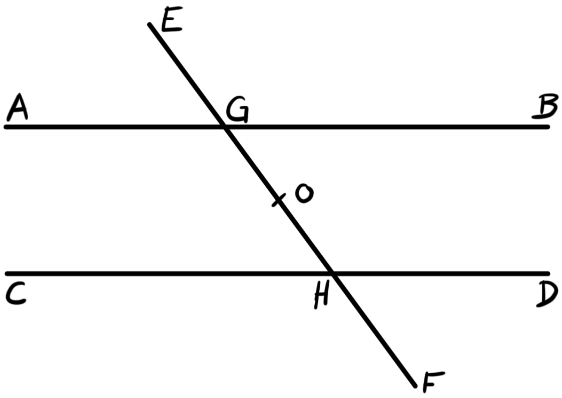


Figure 2.15: Proposition I.27

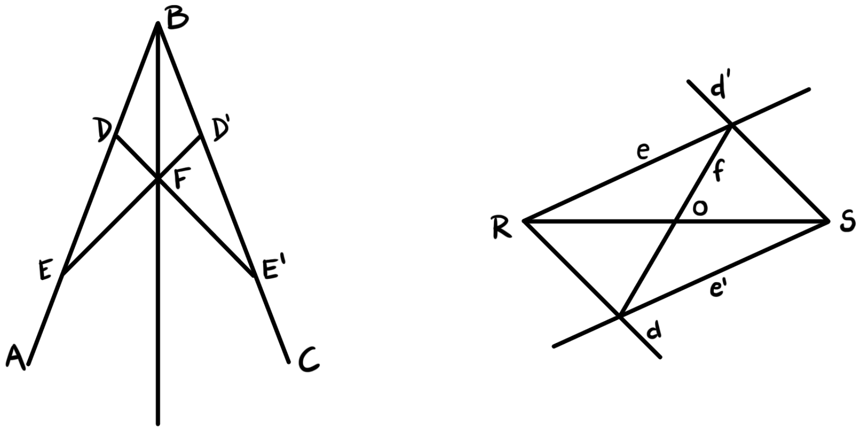


Figure 2.16: Propositions I.9–10

from the proof of I.27 that corresponding lines are parallel. For comparison, a proof of I.29 presented in Mackay's *Elements of Euclid* depends on I.23, I.27, I.15, and I.13 [47, pg. 65–67].

Butters then shows how problems can be solved using reflection and rotational symmetry, by working through I.9 and 10. This also gives an example of how related the two approaches are (Fig. 2.16).

I.9: ***To bisect a given rectilineal angle.***

This problem is particularly well-suited for reflection symmetry, the bisector of the angle being an obvious choice for axis. Butters takes two pairs of corresponding points on the arms of the angle, D, D' and E, E'. ED' and $E'D$ are then corresponding lines. They meet in F, so F must correspond to itself, and hence be on the axis. It follows that $\angle FBE = \angle FBE'$.[54]

I.10: ***To bisect a given finite straight line.***

This is the corresponding problem to I.9 in rotational symmetry, with the mid-point of RS as centre. Butters takes pairs of corresponding, or parallel, lines through R and S, d, d' and e, e', and join their points of intersection. Since these points correspond, the line joining them must pass through the centre. This holds for R and S as well, and it follows that the centre is O, the intersection of f and RS. Corresponding points are equidistant from the centre, so we have $OR = OS$.

Skew symmetry

Butters explains that if corresponding points instead of having equal distance to the centre, are distanced according to some fixed ratio, rotational symmetry turns into the theory of similar figures. This takes him into Book VI of Euclid and hence outside the scope of this paper, so he leaves it at that and turns to the similar result for reflection symmetry. If the lines connecting corresponding points are not perpendicular to the axis, but form some other angle with it, the result is skew-symmetrical figures.

Butters does not give this skew symmetry as extensive a treatment as he gives the other two. A list of 'obvious propositions' is lacking, possibly because he seems to regard skew symmetry as an extension of reflection symmetry, and that such a list would be easy enough for the readers to create themselves. Another explanation may be that he does not feel that the same amount of justification is necessary, as he only uses this to prove two propositions and one example.

It is interesting that he does not, because skew symmetry is actually rather different. When regarded as transformations, the reflection and the rotation of figures are both isometries, and so they preserve distances.[55] The reflection of a figure about a skew axis in the way described here is called a shear transformation. This is an affine transformation, meaning it does not preserve angles or distances, but does preserve area.[56] Butters does not touch on the subject of transformations at all in this paper.

[54]This construction resembles the more traditional proof of I.5.

[55]An isometry is simply a bijective map between two metric spaces that preserves distances.

[56]For more, see Weisstein, Eric W. "Shear." From MathWorld–A Wolfram Web Resource. `http://mathworld.wolfram.com/Shear.html`.

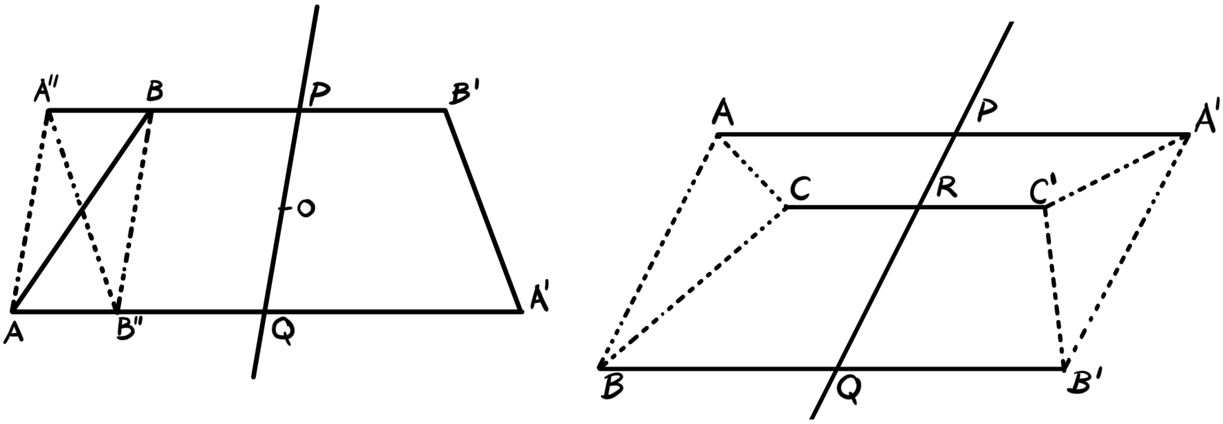


Figure 2.17: Areas in skew symmetry

It is not clear whether he is unaware of this aspect, or if he avoids mentioning it in order to present the matter in a language more suitable for pupils. He does know that corresponding areas in skew symmetry are equal in size, and he sets out to prove this.

In figure 2.17, it is given that $BP = PB'$, $AQ = QA'$, and that AA' and BB' are parallel. Butters first step is to rotate the trapezium $A'B'PQ$ through two right angles about the midpoint O of PQ. Since AA' and BB' are parallel, A' is then taken to A'' lying on the line PB. Similarly, B' is taken to B''. PB must then equal QB'' and $PA'' = QA$. Therefore, both BB'' and $A''A$ are parallel to PQ.

Both triangles ABB'' and $A''B''B$ then equals one half of the parallelogram $AA''BB''$, and so they equal each other. Adding the area of the parallelogram $BB''QP$ to both triangles gives $ABPQ = A''B''QP = A'B'PQ$.

This is then used to prove that two corresponding triangles are equal in area. This is easily done by observing that the area ABC 'is equal to the algebraic sum of the trapezia $ABQP$, $BCRQ$, and $CAPR$'.[57] $A'B'C'$ is then the algebraic sum of corresponding trapezia. Corresponding trapezia are equal, but of opposite signs (when named in the same direction), and this must then be true of triangles 'and hence of figures in general'. This makes it fairly easy to prove Euclid's theorems in areas.

[57]Butters explains that 'those areas being reckoned positive when they lie to the left hand in going round in the order of the letters'. This is presumed to mean that an area is positive when named in a counterclockwise direction, and negative when otherwise. This would match the measuring of angles.

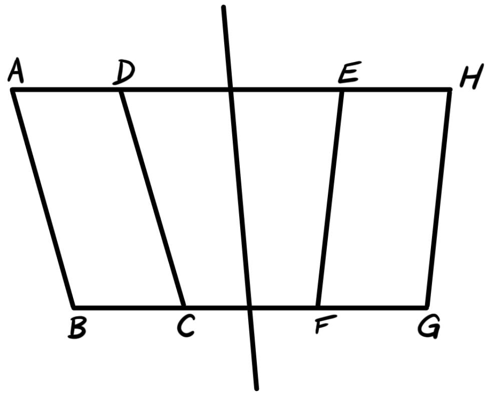


Figure 2.18: Proposition I.36

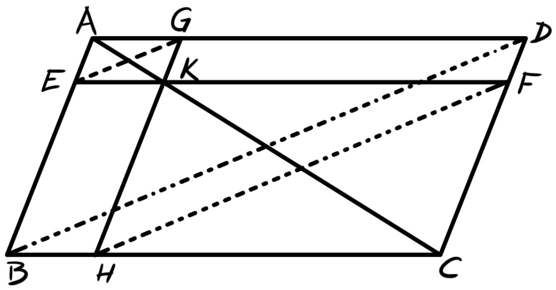


Figure 2.19: Proposition I.43

I.36: ***Parallelograms on equal bases and between the same parallels are equal in area.***

Let the two parallelograms in questions be $ABCD$ and $HGFE$ in figure 2.18. If the line joining the mid-points of DE and CF is used as skew axis, the parallelograms become corresponding areas and must therefore be equal.

I.43: ***The complements of the parallelograms which are about a diagonal of any parallelogram are equal.***

EG, BD, and HF are all diagonals of parallelograms (figure 2.19). They are bisected by AKC and they are parallel.[58] If AC is taken as skew axis, $EBHK$ corresponds to $GDFK$, and the areas must therefore be equal.

[58] These statements are followed by a '(Proof ?)', presumably meaning 'Proof left for reader'.

Butters adds in parentheses that all the main propositions in Book I can be proved by one of these three methods, with the exception of I.47 and its converse. It may be observed, i.e. by proposition I.10, that using symmetry does not necessarily give a more elegant or less time-consuming solution, the point is simply that it can be done. He then provides another example of how skew symmetry works by proving that the medians of a triangle are concurrent.

As a conclusion, he finishes by showing how one may transform a circle into an ellipse using squared paper. He draws a circle, and next to it a line that is not perpendicular to the horizontal lines on the paper. This is to be used as skew axis. For each point of intersection of a horizontal line with the circle, a point is marked on the line on the other side of the axis, and equidistant from it. Proceeding this way, and finally joining all the newly marked points together, will produce an ellipse of area equal to the circle. Other properties, Butters says, can be found just as easily.

2.3.8 Butters's publications

This last paper is typical for Butters in several ways. It fits in nicely with overarching themes, such as generalisation and simplification. He is developing a more general method of proof with a wide applicability, as he is also doing in paper 2.3.3. In doing so, he is attempting to simplify the teaching of the subject. Simplification is also a key goal in paper 2.3.5, where he argues for a simpler way of dealing with sums of money. In this fifth paper, he also touches upon another recurring topic, that of speed. He remarks how much time can be wasted by poor methods, and suggests better ones, as he also does in 2.3.2.

The final paper is typical in choice of readers as well, targeted as it is at schoolteachers. Butters is usually writing with this particular audience in mind; 6 of 7 papers deal with educational matters. He is also expecting his readers to hold such knowledge as would be typical for teachers, such as a solid grounding in Euclidean geometry. Most of Euclid's propositions in 2.3.7 are for instance given by number only, the actual wording being left out. In 2.3.3, he is using what may be presumed to be conventional names for certain lines and points without any explanation, not even marking them on the relevant figure. A teacher would know these things by heart, encountering them on a daily basis.

Butters's choice of audience reveals itself in other ways, too. His papers are interspersed with comments on how best to teach the subject. In his fifth paper (2.3.5), he argues that the conversion between coinage and decimals should never be written down, but should always be taught orally. In the final paper, he suggests teaching reflection symmetry 'experimentally', by folding a piece of paper, letting the fold be the axis, and pricking holes through it. Many more such examples can be found.

Butters can safely make such assumptions on behalf of the teachers, being a teacher himself. These papers can therefore provide some insight into the knowledge of his colleagues. Negative geometrical magnitudes are for instance standard. He also expects them to be familiar with projections. The other side of the coin is perhaps even more interesting. Butters does not expect the teachers to be familiar with modular arithmetic, and it looks as if he does not expect them to be too familiar with decimal arithmetic either. This latter supposition is perhaps more surprising, considering how he refers to school textbooks on decimal arithmetic dating from as early as 1685.[59] It could of course be that he does expect them to know this, and that he is just emphasising which parts are necessary to convey to pupils. However, his treatment of decimal arithmetic seems a little too thorough for that interpretation to hold.

Whether he is writing for teachers only, or simply taking for granted that all non-teachers know the subject equally well is not quite clear. It is clear, however, that the papers would be of less value to an academic. His tendency to leave things unsaid would perhaps not be up to the academic standard, as illustrated by his third paper (2.3.3). As explained in that section, when compared to the academics' treatment of the same topic, Butters's exposition falls short.

It can also be argued that Butters's proofs are a bit unsound. This is particularly relevant for his proof in the sixth paper (2.3.6). In this, he sets out to prove a general theorem, but does so by showing that it holds for a special case only. This procedure gives only a sketch of a proof, albeit an accurate sketch, that can easily be adapted to the general case. He argues that he does so for simplicity, which is plausible enough. Not only is the general case harder to follow, it also places higher demands on the printers, an expensive procedure in 1904.

Although his style may be slightly flawed from an academic point of view, his topics are far from uninteresting. As mentioned, the topic from his third paper is covered by academics, though in different forms, in 1900 [59] and 1904 [60], and by yet again in the American Mathematical Monthly in 1946 [45]. Butters's presentation of Gauss's work would no doubt be of interest as well, considering how the topic was still subject

[59] This would be Cocker's Decimal Arithmetic.

of research as late as 1909 [65].

His final paper stands out in this regard. Parallels can be drawn between his symmetries and the transformations in Klein's Erlangen Programm. This was published in 1872, but was not translated into English until 1892, and its contents were still important topics around the turn of the century. Whether or not Butters is aware of this, it still shows that teachers could benefit directly from learning of contemporary research.

What separates Butters from the academics more than anything else is mode of presentation. The choice of topics is obviously influenced by his profession, but most of his papers would be of interest outside the schools as well.

2.4 Conclusion

Most of what the teachers published qualifies as research, as it was, at least as far as they were aware, original in content. Their topics somewhat reflect the trends in UK at the time, with gradually fewer papers on Euclidean Geometry, and somewhat more analysis. Euclidean Geometry was, and remained, their area of expertise, so it would be only natural to expect a decline in contributions as the current areas of research moved away from this. As they did take an interest in other areas as well, there would be no reason to expect it to stop entirely.

The contributions from the teachers to the *Proceedings* begin declining towards the end of the First World War. If this had been purely a result of the rising levels of mathematics, there would have been no reason for the same to happen in the *Notes*, but it does. It is therefore very likely that this is connected to the First World War. It is for instance not unlikely that the two groups were called in for different kinds of war-service, which would affect them differently. As will be seen in chapter 4, there were other reasons for why this decline became permanent and not just a passing phase.

Chapter 3

The Enumeration of Rhyme Schemes

3.1 Introduction

The current chapter deals with correspondence between A. C. Aitken, Sir D'Arcy Thompson and G. T. Bennett in December 1938 and January 1939. The correspondence begins with the counting of rhyme schemes and quickly leads to Bennett rediscovering certain properties of the Bell numbers, most of which Aitken has already rediscovered in his paper 'On a problem in combinations' from 1933. The chapter will go through the 13 letters and place them into context, especially regarding Aitken's article. Before this can be done, however, a few preliminaries are in order.

3.1.1 Bell numbers and Stirling numbers of the second kind

The Bell numbers are named after Eric Temple Bell, who published systematic studies of them in 1934 [8] and 1938 [9].[1] As noted by him in 1938, the sequence had been discovered and rediscovered many times before this, by many different authors.

This is not a great mystery, as these numbers have a tendency to crop up in a large variety of situations, due to their combinatorial nature. Basically, the *nth* Bell number, denoted B_n, counts the number of ways n distinct objects can be organised in unordered groups, allowing non-empty groups of any size.[2] Two objects, for instance, can be taken

[1] Bell himself called them exponential numbers. Gian-Carlo Rota indicates in an article from 1964 [80] that the term 'Bell numbers' was coined by Becker and Riordan in 1948 [7]. No earlier reference to this name has been found.

[2] These must not be confused with Bernoulli numbers, also denoted B_n.

one at a time, or both together, giving $B_2 = 2$. Three objects, A, B and C, can be organised in five different ways:

$$\begin{aligned}
1 :& (A)\,(B)\,(C) \\
3 :& (AB)\,(C), \quad (AC)\,(B), \quad (BC)\,(A) \\
1 :& (ABC)
\end{aligned}$$

As a part of this problem, one may encounter the Stirling Numbers of the Second Kind, so named after James Stirling.[3] Like the Bell numbers, these have been discovered over and over again, and Stirling was not the first to have written about them. These numbers count the number of ways n objects can be organised in exactly k unordered groups. They are often denoted $S(n, k)$ (though different notation is in use as well). Summing these $S(n, k)$ for all values of k for some n gives B_n. This way, $S(3, 1) = 1$ corresponds to organising 3 elements in one group, which can be done in one way only. The table above gives $S(3, 2) = 3$ and $S(3, 3) = 1$. Summing these give 5.

As many before him, and after, Aitken was astonished by this sequence that he seemed to find everywhere. He wrote a paper on it which was published in the *Notes* in January 1933 [1]. He did not know of Bell's work on these numbers at the time, for which he can be excused as it had not been published yet. Neither of the three gentlemen involved in this series of correspondence were familiar with the previous findings on Bell numbers by other authors, not even on the Stirling numbers.[4]

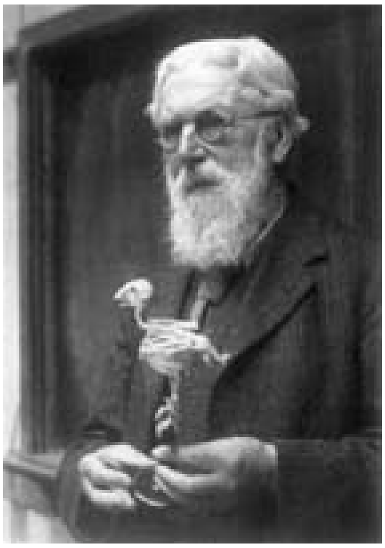
Alexander Craig Aitken

A. C. Aitken, the famous New Zealander and human calculator, spent his entire academic career at the University of Edinburgh. He arrived there from New Zealand in 1923 to study for a PhD under Professor Edmund T. Whittaker. When he submitted his thesis in 1926, it was considered to be of such high quality that he was awarded a D.Sc. instead. His distinctions began even earlier than this, being appointed lecturer at Edinburgh University, and being elected a fellow of the Royal Society of Edinburgh, as early as 1925. He was later to be promoted to Reader in Statistics in 1936 and then Professor of Mathematics in 1946, succeeding Whittaker.

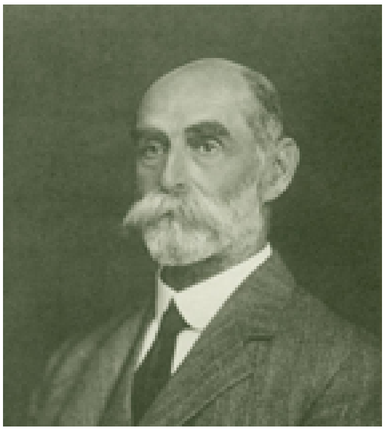
[3] They were at the time more commonly referred to as Stirling Numbers of the Second Species.

[4] Aitken might have learnt of Bell's work by the time of the letters, but this is unlikely, as he never mentions it or uses Bell's names for these numbers. Bell certainly learnt of Aitken's, referring to Aitken's paper in his own from 1938 [9].

A .C. Aitken
(01/04/1895 – 03/11/1967)



D. W. Thompson
(02/05/1860 – 21/06/1948)



G. T. Bennett
(30/06/1868 – 11/10/1943)

P. A. MacMahon
(26/09/1854 – 25/12/1929)

Aitken's research interests lay mainly in statistics, numerical analysis and algebra. This makes sense, considering his astonishing mental arithmetic skills.[5] He could, amongst other things, multiply and divide large numbers on the spot, helped by an equally astonishing memory.

Alec Aitken was very much involved with the Society. He was on the Committee, in one way or another, for nine years from 1926 to 1935. He served as President in the session 1931–32, having been Vice-President for two years prior to this. In 1933, when his paper was published, he was serving his second round as Editor for the *Notes*.[6]

Sir D'Arcy Wentworth Thompson

Almost six years later, in December 1938, Aitken wrote to his friend Sir D'Arcy Thompson on this most curious sequence. Sir D'Arcy was one the most renowned scientists of the time, largely due to his book *On Growth and Form*, He held the chair of Natural History at the University of St Andrews. Thompson was mainly a biologist, but with exceptionally wide-ranging interests. In addition to his hobby as an accomplished Greek scholar, he was also skilled at physics and mathematics. The latter led to him being elected an honorary member of the Edinburgh Mathematical Society. It is possible that he and Aitken got to know each other through the EMS, but more likely they met through the Royal Society of Edinburgh, where they were both fellows.

Their meeting one another resulted in quite a few letters and postcards. The first which has been preserved is from 1934. Aitken's earlier letters show that he admired Thompson greatly.[7] Such admiration, though surely still present, became less palpable over time and their letters grew more frequent. In addition to discussing personal affairs, Aitken often told Thompson of mathematics he was working on, but rarely the research he published. Whether he thought his academic research would be too ad-

[5]These skills are described in detail by I.M.L. Hunter in his paper 'An exceptional talent for calculative thinking', Brit. J. Psychol., Vol. 53, pp. 243–268.

[6]This biographical information on Aitken, except that on his involvement with the EMS Committee, was found in [68].

[7]This can for instance be shown by extracts from two of his letters, one dated 22nd of January 1935.

> It is, of course, no inconvenience whatever to me to have, for company in my room, your trunk and its contents. It serves to remind me, if such reminder were needed, of your virtuosic lecture of last Friday evening, — of which the only disappointing feature was that you had to depart in such haste for your other appointment that you missed the many expressions of praise which you should rightly have heard in person.

Another is dated 19th of May 1937 and regards Thompson's knighthood.

> Let me anticipate the formality of the accolade, and address you by the title which many of us have felt ought long ago to have been yours.

vanced for Thompson to enjoy, or if he simply did not think it would interest him is not clear, but whatever the reason, the mathematics he chose to present to Thompson was usually more readily understood. Thompson encouraged Aitken in writing mathematical letters, and often asked for Aitken's help when he struggled with some mathematical problems of his own.[8]

The Bell numbers greatly intrigued Thompson, so much that he reproduced some of Aitken's notes to his old friend G. T. Bennett.

Geoffrey Thomas Bennett

Before introducing Dr Bennett properly, however, it is necessary to spend a paragraph introducing the mathematician who brought Bennett and Thompson together. Major Percy Alexander MacMahon, a well-respected mathematician of his time, was a sound combinatorialist, specialising in symmetric functions and partitions of numbers. He is important not only because he introduced Bennett and Thompson, but also because the by then late Major MacMahon was mentioned several times in the correspondence. He was an acquaintance of both; he had known Thompson since 1900 or earlier, and it is likely that he met Bennett through the council of the London Mathematical Society, on which they both served (See [33, pg. 134]). Dr Bennett and Professor Thompson were introduced in 1923 via letters, when MacMahon reported to Thompson that Bennett had read *On Growth and Form* and had made several comments. He recommended Thompson contact Bennett, which Thompson did, and this resulted in vast amounts of correspondence and a long-term friendship until Bennett's death in 1943.[9]

The mathematician Geoffrey Thomas Bennett was a fellow of Emmanuel College at Cambridge University, and he spent his entire career there. He had been educated at Cambridge as well, and became Senior Wrangler in the Mathematical Tripos in 1890.[10] MacMahon described Dr Bennett as

> the leading geometrician in this country [33, pg. 190].

While he is not remembered as such today, that is largely because he was not a very productive mathematician. He authored no book, and only a few papers. Most of his time seems to have been spent on his many other interests and, more importantly, on helping his fellow mathematicians and other scientists with their publications. Thomp-

[8] This information is found in [57] and [69].
[9] This information is found in [72] and [33].
[10] Interestingly enough, he was not the best student that year, as that honour went to Philippa Fawcett.

son, for instance, was to benefit greatly from this in connection with the second edition of *On Growth and Form*, which was published in 1942.[11]

Bennett was known for his style of writing, and his letters to Thompson are filled with literary embellishments and ornaments. He did not appear to consider Thompson as an inferior mathematician. Indeed, he asked Thompson to read and give his comments on a geometrical paper that he himself found rather difficult. They wrote on various problems, in biology, mathematics, chemistry, and more, but also on more personal matters, and all in all appeared to be quite close friends.

3.2 Aitken's article

A problem in combinations

The article is divided into eleven parts, each devoted to a different aspect of the Bell numbers. There are two paragraphs numbered 7, making the final paragraph number 10. The numbering from the article has been used for this treatment. Aitken does not give his paragraphs headings. The headings that are used here are therefore new.

He uses the notation $P(n)$ for the *nth* Bell number, instead of the B_n that is used today. This article develops various expressions for $P(n)$, and also other means of producing them.

As the focus is on the letters and not on the article itself, the results that are relevant to the ensuing correspondence have been emphasised here. For the ones that are not, only brief sketches are provided.

3.2.1 Introduction: Grouping individuals

Aitken defines $P(n)$ to be the number of ways one can arrange n individuals into unordered groups, allowing groups of size 1. He will later refer to these as 'Oxford Groups' (See 3.3.9).[12] He gives the example $P(3)$ and shows how 3 people can be organised in groups in $1 + 3 + 1 = 5$ ways, as explained above.

This sum corresponds to the partitions of the integer 3. The first term, 1, corresponds to $1 + 1 + 1$, the second, 3, to $1 + 2$ and the third, 1 again, to simply 3.

[11] This information on Bennett is taken from [76].

[12] This $P(n)$ is not to be confused with the partition function $p(n)$ that counts partitions of the number n. Fortunately, Aitken does not touch on the partition function.

3.2.2 Partitions of numbers

He explores the relation to partitions of numbers further, attributing the notation he uses to MacMahon. The partition

$$\underbrace{a + a + \ldots + a}_{\alpha} + \underbrace{b + b + \ldots + b}_{\beta} + \ldots \tag{3.1}$$

is denoted by $a^\alpha b^\beta \ldots$. The partition $1^1 2^1$ above leads to 3 subgroups, as shown. This 3 can be computed as $3!/(1!2!)$. He states that the number of subgroups corresponding to a particular partition $a^\alpha b^\beta \ldots$ is given by[13]

$$\frac{n!}{a!^\alpha b!^\beta \ldots \alpha!\beta!.} \tag{3.2}$$

Summing this for all partitions of the number n gives

$$P(n) = \sum n!/(a!^\alpha b!^\beta \ldots \alpha!\beta!). \tag{3.3}$$

Aitken remarks that this is 'not a very helpful expression'.

3.2.3 The generating function

His next step is to develop a generating function.

He does so by explaining that the unit groups in a particular partition will be represented by $\sum_0^\infty x^r/r! = e^x$. Similarly, groups of two will be represented by $e^{x^2/2!}$, and groups of r by $e^{x^r/r!}$. He concludes that the generating function for $P(n)$ must be

$$e^{x+x^2/2!+x^3/3!+\ldots} = e^{e^x-1} \tag{3.4}$$

The Bell number $P(n)$ is then the coefficient of $x^n/n!$ in the expansion of this. This can be expressed using Maclaurin's theorem, (with $D = \frac{d}{dx}$)

$$P(n) = [D^n e^{e^x-1}]_{x=0}. \tag{3.5}$$

[13]The powers α, β, etc, are missing from the two next formulae in the article. This is presumably a typographical error, as Aitken is obviously working with the correct versions.

3.2.4 Repeated differentiation

This is related to repeated differentiation of a function of a function. He gives as an example

$$\left.\begin{array}{rl} \frac{d}{dx}f(u) = & f'(u)\frac{du}{dx} \\ (\frac{d}{dx})^2 f(u) = & f''(u)(\frac{du}{dx})^2 + f'(u)\frac{d^2u}{dx^2} \\ (\frac{d}{dx})^3 f(u) = & f'''(u)(\frac{du}{dx})^3 + 3f''(u)\frac{d^2u}{dx^2}\frac{du}{dx} + f'(u)\frac{d^3u}{dx^3} \end{array}\right\} \tag{3.6}$$

The coefficients in the third line, 1, 3 and 1, are identical to the number of subgroups from 3.2.1. By putting $f(u) = e^u$, with $u = e^x$, he shows how to arrive at yet another expression for $P(n)$:

$$P(n) = [e^{-1}(\frac{d}{dx})^n e^{e^x}]_{x=0}. \tag{3.7}$$

3.2.5 Differential equations

Another related result comes from the theory of homogeneous differential equations. Using the operator $x\frac{d}{dx}$, or xD, he gets

$$\begin{array}{rcl} (xD)^2 & = & x^2D^2 + xD \\ (xD)^3 & = & x^3D^3 + 3x^2D^2 + xD. \end{array} \tag{3.8}$$

These have the same coefficients as the set (3.6).[14] This method leads to a slightly different expression for $P(n)$.

$$P(n) = [(xD)^n e^{x-1}]_{x=1} \tag{3.9}$$

This, combined with (3.5), gives an identity Aitken finds most peculiar:

$$[D^n e^{e^x-1}]_{x=0} = [(\overline{x+1}D)^n e^x]_{x=0}. \tag{3.10}$$

3.2.6 Dobiński's result

In this very short paragraph, he remarks that since

$$e^{e^x-1} = e^{-1}(1 + e^x + e^{2x}/2! + e^{3x}/3! + \ldots), \tag{3.11}$$

[14] He spends some time explaining the close connection between this and the previous result. As it is not very relevant for the letters, it is not explained here, but he notes that if $u = e^x$, the relation between the two approaches 'becomes exact'.

$P(n)$ can also be expressed by

$$P(n) = e^{-1} \sum_{s=0}^{\infty} (s^n/s!). \tag{3.12}$$

The only justification he gives for this is that $P(n)$ is the coefficient of $x^n/n!$ in the expansion of the generating function. Bell, and others, refer to this as Dobiński's result of 1877 [9, pg. 540].[15]

3.2.7 Differences of zero

Newton's forward difference formula

This paragraph explains how $P(n)$ is the sum of the divided differences of 0^n, or, how

$$P(n) = \sum_{r=0}^{n} \Delta^r 0^n / r!. \tag{3.13}$$

This formula is reached by forming a difference table from 0^n, 1^n, $\ldots n^n$. Using the Gregory-Newton interpolation formula[16] with $f(x) = x^n$ yields

$$s^n = 0^n + s\Delta 0^n + \frac{s(s-1)}{2!}\Delta^2 0^n + \ldots + \binom{s}{n}\Delta^n 0^n \tag{3.14}$$

This expression for s^n is then substituted into eq. 3.12, which gives

$$P(n) = e^{-1} \sum_{s=0}^{\infty} \left[\sum_{r=0}^{n} \Delta^r 0^n / ((s-r)!r!) \right] \tag{3.15}$$

All that remains is to observe that the summation of expressions like $1/(s-r)!$ gives e in each case. This gives (3.13). Aitken calls this the 'sum of divided differences of 0^n'.[17]

[15] Bell remarks in this article that the result has been ascribed to Euler, but without reference.

[16] This is a finite difference identity, giving an interpolated value of a function, in this case for $f(x) = x^n$ in terms of $f(0)$ and the powers of the forward differences Δ. Since these particular forward differences reach 0 after a certain amount of steps, the interpolated value becomes exact.

[17] He remarks that this was given as an equivalent to eq. 3.12 by Herschel. This is presumed to be the mathematician John Herschel (1792 - 1871).

Subgroups

Aitken now explains that these divided differences also produce the number of subgroups. A table of divided differences is provided for $n = 3$ as follows:[18]

$$\begin{array}{cccc} 0 & & & \\ & 1 & & \\ 1 & & 3 & \\ & 7 & & 1 \\ 8 & & 6 & \\ & 19 & & \\ 27 & & & \end{array} \tag{3.16}$$

The numbers $1, 3, 1$ are precisely the amount of subgroups found in section 3.2.1. The numbers produced this way are actually the Stirling numbers of the second kind, but Aitken does not seem to be aware of Stirling's work.

He does not prove this, but he states that this holds in general and indicates how it can be proved.

3.2.8 Recurrence relation and Aitken's Array

In this paragraph he develops another method for producing Bell numbers, making use of a recurrence relation:

$$P(n+1) = P(n) + nP(n-1) + \binom{n}{2}P(n-2) + \ldots + nP(1) + P(0) \tag{3.17}$$

$$= (P+1)^n \tag{3.18}$$

This holds for the Bell numbers $P(n)$ if exponents are replaced by suffixes after the expansion (and $P(n) = P_n$, making $P = P^1 = P(1)$). The technique of lowering indices this way is today better known as umbral calculus. Aitken says this relation holds 'symbolically', indicating that he would be more inclined to use the alternative term 'Blissard's symbolic method', due to John Blissard.[19] This technique, though regarded

[18]This is the similar to the difference table just mentioned, but the entries are divided. For instance, the entry $(i^n - (i-1)^n) - ((i-1)^n - (i-2)^n)$, in the third column of the difference table, is divided by the difference between the largest and the smallest number: $i - (i-2)$. This way, $(3^3 - 2^3) - (2^3 - 1^3) = 12$ is divided by $3 - 1 = 2$, which gives the 6 in this table.

[19]He may have used the term umbral as well. According to the minutes from the ordinary meeting on the 4th of March 1927, he spoke warmly on the advantages to using an umbral notation for determinants.

with some skepticism, was widely used.[20]

He provides a sketch of a proof of this, by writing:

$$P_{n+1} = e^{-1}D^{n+1}(e^{e^x}) = e^{-1}D^n[e^x \times e^{e^x}] \quad x = 0. \tag{3.19}$$

He states that expanding this derivative using Leibniz's theorem will produce eq. 3.17 above.

In fact, equation (3.17) has the shape of a Gregory-Newton interpolation formula. This, he says, allows us to construct a new difference table from all the $P(n)$, in which $\Delta^n P(1) = P(n)$.

P	Δ	Δ^2	Δ^3	Δ^4	Δ^5
1					
	1				
2		2			
	3		5		
5		7		15	
	10		20		52
15		27		67	
	37		87		
52		114			
	151				
203					

Aitken is not crystal clear when explaining how this construction works, so his explanation has been expanded here. Assume $P(5) = 52$ has been found and has been placed on the diagonal in column Δ^5. Adding 52 and 15 gives 67, which is then added to 20 to produce 87. The process is continued until $151 + 52 = 203$ is reached, and 203 can then be put in a new column Δ^6.

This table is today known as a Bell triangle, or, less commonly, as Aitken's Array. The latter name will be used here.[21]

Aitken himself finds this to be the easiest way to produce the first values of $P(n)$, though he is to change his mind later.

[20] At the time, umbral calculus was 'nothing more than a set of "magic" rules of lowering and raising indices.' See [13, pg. 3].

[21] It is normally constructed as a mirrored image of the table given here. The more common version looks like this.

1			
1	2		
2	3	5	
5	7	10	15
...			

Each cell, except those in the first column, is constructed by adding the cell to the left, and the one directly above this.

3.2.9 Prime number division

After giving the first ten values of $P(n)$, he points out that if the n in $P(n)$ is a prime p greater than 1, then it follows that

$$p|(P(p) - 2). \tag{3.20}$$

He also provides a proof for this, which is not included here, as it is of no relevance for the letters.

3.2.10 Asymptotic expression

As a finishing remark, he mentions that it would be pleasing to find an asymptotic expression to represent $P(n)$ for large values of n. [22]

Aitken was not a combinatorialist in any way, so naturally, his treatment of these numbers is not as advanced as Bell's is, nor would an advanced paper such as Bell's find a suitable home in the *Notes*. The lack of specialist jargon makes Aitken's paper easily accessible, not only to teachers but other academics as well. It gives an interesting overview of the various ways these numbers can be produced.

3.3 Letters

3.3.1 Aitken to Thompson, 6 Dec. 1938

The first letter that has been preserved on this enumeration of rhyme schemes is written by Aitken to Thompson on the 6th of December 1938. It is not the first time they discuss rhymes, though it is the first time they discuss mathematics. A year before this they amused themselves by performing a statistical analysis of some of Virgil's poems. This earlier batch of letters does not involve counting of rhyme schemes, so these letters are only vaguely related. The letters on the rhyme schemes are reproduced in full in Appendix B.

Aitken opens this letter by commending Sir D'Arcy for his pronunciation of the Greek word 'Cosmocapeleion' on the previous night. This is relevant, because Aitken's introduction to the rhyme schemes shows that it is not the beginning of their discussion:

[22] According to Eric Weisstein, a relatively complicated one was found in 1981 by de Bruijin [104].

> As for the numbers which enumerate rhyme schemes. For one line, simply A, 1 member; two lines, AB and AA, 2 members; three lines, ABC, ABA, AAB, ABB, AAA, 5 members; four lines, one finds 15 types; 5 lines, 52, and so on. The sequence is 1, 2, 5, 15, 52, 203, 877, 4140...

This appears to be a response to a query from D'Arcy Thompson, possibly in a letter that has not been kept, or, in light of the opening comment, voiced at their meeting on the day before.[23]

This enumeration of rhyme schemes is completely equivalent to his groups of individuals in his 1933 paper (section 3.2.1), with the people corresponding to lines, and two lines being in the same 'group' if they rhyme. Now would have been a natural moment for Aitken to mention his five-year-old paper, but he does not. Instead, he reproduces five of his results, in a slightly mystifying way.

> These numbers, which I have met elsewhere in mathematics, have most curious properties. Construct from them a table of successive differences, and you find that they are their own extreme differences, as below; from which one can build them by entering each one, as it is found, on the far right and completing by summation a fresh line of the table,[...]

The table is the table of extreme differences from his paper, section 3.2.8, in a slightly amended form. It includes one more Bell number, the seventh, and also arrows that helps explain how the summation works. In his paper, he justifies this using the Gregory-Newton Interpolation formula, but here he gives no explanation.

He continues:

> This is a most rapid way of finding the first dozen or so. But they occur most strangely thus:

$$\begin{aligned}
\frac{1^1}{1!}+\frac{2^1}{2!}+\frac{3^1}{3!}+\ldots\text{ad inf.} &= e\\
\frac{1^2}{1!}+\frac{2^2}{2!}+\frac{3^2}{3!}+\ldots &= 2e\\
\frac{1^3}{1!}+\frac{2^3}{2!}+\frac{3^3}{3!}+\ldots &= 5e\\
\frac{1^4}{1!}+\frac{2^4}{2!}+\frac{3^4}{3!}+\ldots &= 15e
\end{aligned} \tag{3.21}$$

[23]It is not clear what kind of meeting this was, only that it was not a meeting of the EMS. It is possible that they did not actually get to speak to one another on the previous night; D'Arcy Thompson may have been giving a talk with Aitken in the audience. In fact, this is very likely. Aitken's comments on the Greek word would indicate that he has not spoken with Thompson after hearing him pronounce it.

When multiplied by e^{-1}, these are simply the first four terms of Dobiński's result, as explained in section 3.2.6 of the paper. In contrast to his paper, he gives no explanation for why this is so.

He then provides two difference tables, similar to his table of divided differences from section 3.2.7. These tables cover the cases $n = 4, 5$. Where he in the paper divided each column before taking differences to create the next column, he here completes the table without division and then divide the extreme differences by their respective factorials:

> And yet again, they occur in relation to tables of differences of powers in the natural numbers: for example, to obtain the fourth of them, form a table of $4th$ powers:
>
> | x | x^4 | Δ | Δ^2 | Δ^3 | Δ^4 |
> |---|---|---|---|---|---|
> | 0 | 0 | | | | |
> | | | $\underline{1}$ | | | |
> | 1 | 1 | | $\underline{14}$ | | |
> | | | 15 | | $\underline{36}$ | |
> | 2 | 16 | | 50 | | $\underline{24}$ |
> | | | 65 | | 60 | |
> | 3 | 81 | | 110 | | |
> | | | 175 | | | |
> | 4 | 256 | | | | |
>
> Divide the underlined differences by 1!, 2!, 3!, 4! and add. Thus $1+7+6+1 = 15$.

Aitken then explains how these numbers represent subgroups of rhymes:

> The separate numbers in such sums as these appear to enumerate the rhyme-types according to groups. For example 5, the number of 3-line types, appears from the difference-table of cubes as $1+3+1$. Here the first 1 refers to ABC, the 3 to AAB, ABA and ABB, and the last 1 to ABC.[24]

He finishes by giving the generating function from eq. 3.4 in his article:

> In short, they are most fascinating and strange enumerants. Perhaps the most curious property is that if one takes the exponential series,

[24] Aitken is here slightly imprecise. It would be more correct to say that the first 1 refers to AAA and the last 1 to ABC. This is shown by the case $n = 4$, where the $1 + 7 + 6 + 1$, where the 7 corresponds to 2 different rhymes and the 6 to 3 rhymes.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \text{ad inf.}, \tag{3.22}$$

truncates its first term and exponentiates once again, the result is

$$e^{\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots} = e^{e^x-1} = 1 + \frac{x}{1!} + \frac{2x^2}{2!} + \frac{5x^3}{3!} + \frac{15x^4}{4!} \dots \tag{3.23}$$

in which our friends appear as coefficients.

The letter so presents five of the results from his article, though unlike the article, the letter omits every explanation. This is particularly obvious in the case of the generating function, where Aitken even avoids calling it such. His choice of words helps mystify his results even further, with phrases such as ‘But they occur most strangely thus. . . ’ and ‘Perhaps the most curious property is. . . ’. His article shows that he knows more of the underlying theory than the letters let on, and so these results are not as mysterious to him as he might indicate. His reasons for doing so may be that where his paper was trying to explain or to teach, his letter wants to amaze and intrigue. Aitken appears to have set out to answer the rather simple question of how many different types of rhymes-patterns one can create. After answering that, he adds a few of the more interesting properties connected with these numbers, mainly as curiosities, for Sir D’Arcy to enjoy.

3.3.2 Aitken to Thompson, 19 Dec. 1938

Between the previous letter and this, Sir D’Arcy has most likely sent a response to Aitken, that Aitken is now responding to. Thompson has apparently not fully understood the counting of rhymes, as Aitken sees the need to explain further.

The enumeration 15 for 4-line rhyme schemes is correct. We have:

a a a a	a a b b	b c a a
a a a b	a b a b	b a c a
a a b a	a b b a	a b c a
a b a a	a a b c	a b c d
b a a a	a b a c	
	b a a c	

> You probably discounted abcd as having no rhyme at all; for I should have explained that I included the null-rhyme scheme, blank verse, as one possibility.

He turns his attention to another aspect of the Bell numbers

> On consideration I observe that the simplest relation between the successive enumerants 1, 2, 5, 15, 52, 203, 877, ... is provided by the binomial numbers in the Pascal triangle. Thus, prefix a conventional enumerant 1 for the no-line scheme. (Like $a^0 = 1$.) Then

$$\begin{array}{ccc c c}
1 & & 1 & = & 1 \\
1\ \ 1 & & 1 \times 1 + 1 \times 1 & = & 2 \\
1\ \ 2\ \ 1 & & 1 \times 1 + 2 \times 1 + 1 \times 2 & = & 5 \\
1\ \ 3\ \ 3\ \ 1 & & 1 \times 1 + 3 \times 1 + 3 \times 2 + 1 \times 5 & = & 15 \\
1\ \ 4\ \ 6\ \ 4\ \ 1 & & 1 \times 1 + 4 \times 1 + 6 \times 2 + 4 \times 5 + 1 \times 15 & = & 52 \\
1\ \ 5\ \ 10\ \ 10\ \ 5\ \ 1 & & \textit{etc.} & &
\end{array} \tag{3.24}$$

This is the recurrence relation explained in section 3.2.8 of his article, in a slightly different guise. When he states that he 'on consideration' observes that this is 'the simplest relation', he only means that the realisation is new, not the knowledge itself.

His next sentence requires some explanation.

> This is clearly the most rapid way of deriving them in succession.

It is perhaps odd that he should say this. Multiplying integers by binomial coefficients and then adding them would appear to be a more complicated procedure than simply adding the numbers in his table of extreme differences. To everyone else but Aitken, that table would be a lot more efficient. It is tempting to accuse Aitken of having forgotten that not everyone is quite as skilled at mental arithmetic as he is. It is easily believed that this rather more systematic approach would indeed take less time for a calculator such as him, than the more space-consuming table of extreme differences.

His next comment may shed some light on why Professor Thompson chooses to write to his friend Bennett on these particular numbers. Aitken writes, most likely as a direct response to something Thompson mentions in his letter:

> Your island and hedges problem is very interesting. I am greatly reminded of Cayley's paper on enumerating the forms he calls "trees".[25]

The 'island and hedges' problem is presumed to be a particular problem Bennett and Thompson had corresponded on in 1932–33, though it was not so named at the time.[26] Bennett constructed an arithmetical way of presenting and enumerating the different types of islands with maritime counties and three-way junction points. They also referred to this as the problem of 'enumerating the different modes of cellular subdivision'.[27] As he is to write in a following letter, Thompson feels the two problems are somewhat related, so it is no wonder that he also thought of his old friend and wanted to share this new mystery with him.

3.3.3 Thompson to Bennett, 20 Dec. 1938

And mysterious it was to him. It is obvious that Aitken's attempt to enthral Thompson worked. While finishing one of his regular letters to Bennett, Thompson writes:

> Fourthly, (en parenthese), Aitken has been writing me two or three extremely interesting letters on a certain series, which become more and more wonderful the more you look at it.[28] You begin with the idea of rhymes, or rhyme-types.

With	one line, you've	simply A, —	one type.
	two lines	AA or AB,	two ,,
	three ,,	ABC, ABA, AAB, ABB, AAA	five types
	four ,,		fifteen ..
	five		fifty-two..

> etc. The sequence goes on 52, 203, 877, 4140...
>
> As for their properties, beginning gently, with a table of differences, they merely bob up again!

He then reproduces Aitken's table of extreme differences and the table of differences for 0^4. He finishes by mentioning that Aitken has more to say about these numbers.

[25] Cayley wrote several papers on this, for instance 'On the analytical forms called trees', from the American Journal of Mathematics, Vol. 4, No. 1, 1881, pp 266-268. This paper also gives reference to two other papers of his by the same title.

[26] See letters ms26070–76 at StASC.

[27] See footnote 41.

[28] He had at this point received only two letters on the rhyme scheme from Aitken. There are no indications that there may have been a third letter on rhyme schemes that is now lost. Aitken had, however, written another letter to Thompson, also dated the 20th of December, dealing in Fibonacci numbers, which is included in Appendix B.

> But even that's only the beginning of the story! Perhaps you know it all already. — (but perhaps not).

His choice of results to present is interesting. He is leaving out the three results that Aitken himself described as the most interesting ones; the four terms of Dobiński's formula, the generating function, and the use of Pascal's triangle. The omitted results are of a rather different nature than the tables, and could easily seem less appealing or miraculous to a non-expert such as Thompson, simply because they were slightly harder to grasp.

3.3.4 Bennett to Thompson, 27 Dec. 1938

Bennett takes his time while responding, writing about other matters, before turning his attention to the counting of rhymes.

> The number-series 1 2 5 15 52 203 877... is new to me, and I do not q. clearly make out its provenance as counts of rhyme-types. What is the precise definition? I see that the differencing of the powered natural numbers gives the numbers in the way you indicate — but not <u>why</u>. The one thing that I do see is that if the series is u_1, u_2, u_3 $\ldots u_n$,$u_{n+1}\ldots$ & if $u_0 = 1$ is put as antecedent then symbolically
>
> $$u_{n+1} \quad = \quad (u+1)^n \tag{3.25}$$
>
> where in the expanded binomial suffixes replace indices: L.E.[29]
>
> $$\begin{aligned} u_2 &= u_1 + u_0 \\ u_3 &= u_2 + 2u_1 + u_0 \\ u_4 &= u_3 + 3u_2 + 3u_1 + u_0 \\ &\&c. \end{aligned} \tag{3.26}$$
>
> deriving each term from its predecessors.

This is the same result that Aitken arrives at in his article, part 3.2.8, though Bennett is of course not aware of this. Although Aitken did give a version of this to Thompson, Thompson did not include this in his letter to Bennett. As Aitken did, Bennett refers

[29] It is not clear what this abbreviation is intended to mean.

to this technique as ‘symbolic’, instead of ‘umbral calculus’. Bennett, however, goes a little further.

There is also an extension of this formula, in the shape

$$u^{n+1}(u-1)^m = u^m(u+1)^n. \tag{3.27}$$

ex. gr.

$$u_6 - 3u_5 + 3u_4 - u_3 = u_5 + 2u_4 + u_3 \tag{3.28}$$

But you already have the whole “story”, and these fragments are in it?

Bennett’s notes, the ones attached to the letter from Thompson, explain how he may have found this extension of the formula. By using the table of extreme differences, and building it up in two different ways, he finds two expressions for each cell. One is by constructing it as the name indicates, by entering all the numbers in the left column and taking the difference:

$$\begin{array}{llll}
u_1 & & & \\
 & u_2 - u_1 & & \\
u_2 & & u_3 - 2u_2 + u_1 & \\
 & u_3 - u_2 & & u_4 - 3u + 3u_2 - u_1 \\
u_3 & & u_4 - 2u_3 + u_2 & \\
 & u_4 - u_3 & & \\
u_4 & & &
\end{array} \tag{3.29}$$

The second method is Aitken’s description on how to create new numbers in the sequence using this table:

$$\begin{array}{llll}
u_0 & & & \\
 & u_1 & & \\
u_1 + u_0 & & u_2 & \\
 & u_2 + u_1 & & u_3 \\
u_2 + 2u_1 + u_0 & & u_3 + u_2 & \\
 & u_3 + 2u_2 + u_1 & & \\
u_3 + 3u_2 + 3u_1 + u_0 & & &
\end{array} \tag{3.30}$$

It follows that $u_4 = u_3 + 3u_2 + 3u_1 + u_0$ and so on. His 'symbolic' method, will then have given $u_4 = (u+1)^3$. Equating the cells in tables 3.29 and 3.30 between the leftmost column and the diagonal then indicates the formula. For instance, he gets $u_4 - 2u_3 + u_2 = u_3 + u_2$. This leads to $u^2(u-1)^2 = u^2(u+1)^1$, and so to the formula above.

3.3.5 Bennett to Thompson, postcard, 29 Dec. 1938

Writing the previous letter got his mind working, and it kept working after he had sent it. The result is over 20 pages of miscellaneous notes and a postcard.[30] This postcard is simply a list of 'symbolic', or 'umbral' formulae for the Bell numbers, including the ones he has already found in eq. 3.25 and eq. 3.27.

n	0	1	2	3	4	5	6	7	8
u_n	1	1	2	5	15	52	203	877	4140

(3.31)

$$\begin{aligned}
u^{n+1} &= (u+1)^n && (3.32)\\
u(u-1)^m &= u^m && (3.33)\\
u(u-1)(u-2)\ldots(u-s+1) &= 1 && (3.34)\\
u^{n+1}(u-1)^m &= u^m(u+1)^n && (3.35)\\
u^{n+1}(u-1)(u-2)\ldots(u-s+1) &= (u+s)^n && (3.36)\\
u(u-1)(u-2)\ldots(u-s+1)(u-s)^m &= u^m && (3.37)
\end{aligned}$$

are all special cases of

$$u^{n+1}(u-1)(u-2)\ldots(u-s+1)(u-s)^m = u^m(u+s)^n \qquad (3.38)$$

3.3.6 Bennett to Thompson, 30 Dec. 1938

He is still not done, however, and the following day sees another letter. By now Bennett has fully understood how the number works relating to rhyme schemes, and he has worked out a few more of the properties.

[30] There is no reason to believe he had received a letter from Thompson between the previous letter and this.

> Yes.[31] For the rhyme-count for n letters take any partition of n, say a repeated α times, b repeated β times, c repeated γ times, etc., with the necessary condition $\alpha a + \beta b + \gamma c + \ldots = n$.
>
> The n letters are $A_1\ A_2\ A_3 \ldots A_\alpha$ each repeated a times, $B_1\ B_2\ B_3 \ldots B_\beta$ each repeated b times, etc., and as the equal A-groups are to be exchangeable and the equal B-groups are to be exchangeable, etc., the partition gives
>
> $$\frac{n!}{a!^\alpha \alpha! b!^\beta \beta! c!^\gamma \gamma! \ldots} \tag{3.39}$$
>
> permutations. The sum for all partitions gives the rhyme-total.

This is the exact same formula that Aitken gave in part 3.2.2 of his article, which is unknown to Bennett. Bennett, like Aitken, relates these numbers to partitions of numbers, giving as an example the partitions of the number 5.

$$\begin{array}{llcr} 5 & 5!/5! & = & 1 \\ 4.1 & 5!/(4!1!) & = & 5 \\ 3.2 & 5!/(3!2!) & = & 10 \\ 3.1^2 & 5!/(3!1!1!2!) & = & 10 \\ 2^2.1 & 5!/(2!2!1!2!) & = & 15 \\ 2.1^3 & 5!/(2!1!1!1!3!) & = & 10 \\ 1^5 & 5!/(1!1!1!1!1!5!) & = & 1 \\ \hline & u_5 & = & \underline{52} \end{array} \tag{3.40}$$

Summing the numbers in the right column in row 2 and 3, and also in row 4 and 5 gives the Stirling numbers $S(4,2)$ and $S(4,3)$, respectively. Bennett has also found the generating function and the link between this and Dobiński's result:

> The summation $\sum n!/a!^\alpha \alpha! b!^\beta \beta! \ldots$ can be effected by a generating function: for $u_n/n! =$ Coeff. x^n in $1 + (e^x - 1) + \frac{1}{2!}(e^x-1)^2 + \frac{1}{3!}(e^x-1)^3 \ldots$ wh. may be written symbolically as $e^{ux} = e^{e^x - 1}$.
>
> This leads readily to the expression of u_n in terms of the differenced series $1^n, 2^n, 3^n, 4^n \ldots$ I shd. not be surprised to find that all this was done by P.A. MacMahon.

[31] If this is a response to a letter that has gone missing, or what this 'yes' is intended to mean is not entirely clear.

3.3.7 Bennett to Thompson, 30 Dec. 1938

He probably sends this letter off right away, but his mind keeps working still and later on in the same day, he is inspired once again. This letter is marked 'Postultimate remark'.

> Any series of numbers $a\ \ b\ \ c\ \ d\ \ e\ \ f\dots$ may be converted into a fresh series by multiplying the terms by 1, 2, 3, 4, 5, 6... and adding the-term-next-before. The new series is
>
> $$a \quad 2b+a \quad 3c+b \quad 4d+c \quad 5e+d\dots$$
>
> Starting from a series of only 1 term, and that unity (in value), the series consecutively obtained by this mode of conversion are[[32]]
>
> | | | | | | | | Totals |
> |---|---|---|---|---|---|---|---|
> | 1 | | | | | | | 1 |
> | 1 | 1 | | | | | | 2 |
> | 1 | 3 | 1 | | | | | 5 |
> | 1 | 7 | 6 | 1 | | | | 15 |
> | 1 | 15 | 25 | 10 | 1 | | | 52 |
> | 1 | 31 | 90 | 65 | 15 | 1 | | 203 |
> | 1 | 63 | 301 | 350 | 140 | 21 | 1 | 877 |
>
> &c &c
>
> The numbers in this table are the <u>same</u> as those got by repeated differencing of $1^n, 2^n, 3^n, \dots$ [[and]] division by $1!, 2!, 3!\dots$; but they come in this way <u>much</u> more readily.

This table, giving the Stirling numbers of the second kind, was known at the time, but Bennett seems to be unaware of this.[33]

[32]Each entry in this table is found by taking a multiple of the entry directly above it, and adding the entry to the above left. The fifth row is therefore produced in the following way:

$$1 \quad 2\times 7+1 \quad 3\times 6+7 \quad 4\times 1+6 \quad 5\times 0+1.$$

[33]See for instance [34].

3.3.8 Bennett to Thompson, 1 Jan. 1939

Bennett's post-ultimate remark is followed by another letter, marking the end of his own work on the Bell numbers. He has now found an explanation for why the table he created in his previous letter works.

> The modes of self-generation of the Table
>
> m
>
> | n | 1 | 2 | 3 | 4 | 5 | ... | |
> |---|---|---|---|---|---|---|---|
> | 1 | 1 | | | | | ... | 1 |
> | 2 | 1 | 1 | | | | ... | 2 |
> | 3 | 1 | 3 | 1 | | | ... | 5 |
> | 4 | 1 | 7 | 6 | 1 | | ... | 15 |
> | | | | | | | ... | |
>
> when arrived at, seemed too simple not to have an 'explanation': and it turns out to be this :—
>
> Let the entry in row n col m be $(n \;\; m)$. It is the number of rhyme-systems of n lines when m different rhymes occur. A set of these from the Table are
>
> $$\begin{array}{c|c} (n \;\; m-1) & (n \;\; m) \\ \hline & (n+1 \;\; m) \end{array}$$
>
> The $(n+1 \;\; m)$ systems are got by taking jointly
>
> (i) each of the $(n \;\; m)$ and repeating any one of the rhymes already in it for the extra line
>
> (ii) each of the $(n \;\; m-1)$ and adding a new rhyme(solitary) for the extra line.
>
> Hence $(n+1 \;\; m) = m(n \;\; m) + (n \;\; m-1)$.
> This formula should come at the outset and lead to all the rest of the algebra!

What he has found here is the well-known recurrence relation for the Stirling numbers:

$$S(n+1,k) = k \times S(n,k) + S(n,k-1) \tag{3.41}$$

It was relatively well-known at the time as well, as explained by Bell [9].[34] Bennett continues:

> I looked in "Comb. Analysis" but did not come on
>
> $$1 \quad 2 \quad 5 \quad 15 \quad \&c.$$
>
> ...MacMahon would have seen this difference formula at a glance!

He is here referring to MacMahon's *Combinatory Analysis*, a two-volume treatise, first appearing in 1915 and 1916.

3.3.9 Aitken to Thompson, 4 Jan. 1939

Thompson is suitably impressed with Bennett's work, and forwards Bennett's letters to Aitken. Aitken, who never intended this correspondence to be much more than light entertainment, returns the letters with comments.

> G.T. Bennett has gone very quickly to the root of the sequence 1, 2, 5, 15, 52, 203, 877,..., and with his "post-ultimate" method (which I recognise in another guise as a known recurrence relation between the so-called "differences of zero", sc.[35] $\Delta^r O^s$) has made a point unobserved by me.

This "post-ultimate" method is the recurrence-formula from Bennett's last letter, even if the term was attached to the penultimate one. Aitken does not say exactly where he has seen these differences of zero, but the formula was certainly known. A version of it had for instance been published 100 years earlier by De Morgan [53, pg. 255].[36] Aitken continues:

> I find, as he suggests, that the sequence occurs in MacMahon (and in Muir also) as the solution of combinatory problems not quite the same, but in one-to-one correspondence with our rhyme scheme problem. The totals of rows in one of MacMahon's tables of enumerating functions gives precisely 1, 2, 5, 15

[34] See for instance d'Ocagne [21]. His K_m^p is the same as the Stirling number $S(m,p)$. Bell remarks that the properties discovered by d'Ocagne had been discovered by several English writers before this.

[35] Abbr. for *scilicet*, meaning 'that is to say'.

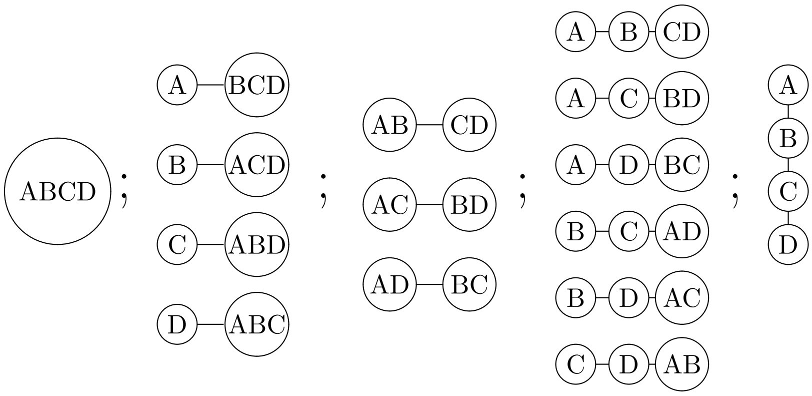
[36] De Morgan's formula is presented in connection with a table resembling the one Bennett gives in the previous letter. It gives the Stirling numbers of the second kind, in addition to other numbers. The formula is

$$(\Delta^n 0^m (/(2\times 3\times \ldots \times n) = (\Delta^{n-1}0^{m-1})/(2\times 3\times \ldots \times (n-1)) + n\times(\Delta^n 0^{m-1})/(2\times 3\times \ldots \times n).$$

Substituting $m-1$ for m, and then exchanging m and n gives Bennett's formula.

> and the rest, and he remarks, as all must who direct some consideration upon the matter, that they are coefficients of
>
> $$\frac{x^n}{n!}$$
>
> in the expansion in powers of
>
> $$e^{e^x-1} \quad .$$

It is not clear what he is referring to when he says he has found the sequence in 'MacMahon'. It would be natural to suspect MacMahon's *Combinatory Analysis*, but Bennett is correct when he says the sequence is not to be found there. Aitken's statement could indicate that the sequence is not there explicitly, but that it appears only implicitly as sums of rows of a 'table of enumerating functions'. Even if this is the case, it is still not to be found in MacMahon's book, that contains no tables satisfying this at all. It is not unlikely that what Aitken has found is a table of Stirling numbers of the second kind, in one of MacMahon's many other publications.

> The problem in which they arise most naturally appears to be, not the rhyme scheme problem or its paraphrase in abstract language, but what may be called the "Oxford Groups" problem. Given n persons, e.g. $n = 4$, A, B, C, D, in how many ways can they be grouped, allowing possible groups of one person? Thus:
>
> (1)
>
> 
>
>
> $G_4 = 1 + 4 + 3 + 6 + 1 = 15$ ways. In the same way $G_5 = 52$, $G_6 = 203$ etc.
>
> The identity of this with the rhyme scheme problem puzzled me for a moment, until I saw what the correspondence was. It is this: let persons in a group be

named with the same small letter, e.g. persons in ABCD will all be tagged with "a". Then scheme (1) is in exact correspondence with (Putting A, B, C, D in order and tagging them.)

(2)

$$\text{aaaa} \;;\; \begin{array}{l} \text{a bbb} \\ \text{b a bb} \\ \text{bb a b} \\ \text{bbb a} \end{array} \;;\; \begin{array}{l} \text{aa bb} \\ \text{a b a b} \\ \text{a bb a} \end{array} \;;\; \begin{array}{l} \text{a b cc} \\ \text{a c b c} \\ \text{a cc b} \\ \text{c a b c} \\ \text{c a c b} \\ \text{cc a b} \end{array} \;;\; \text{a b c d}$$

and these are the rhyme schemes.

Then I found in Muir's last volume of the "History of Determinants" (1900–1920)" yet another form for the numbers 1, 2, 5, 15, 52, ... which interested me, as it involved determinants [55, pg. 249]. The sequence of determinants

$$1\ ,\ \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}\ ,\ \begin{vmatrix} 1 & -1 & . \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}\ ,\ \begin{vmatrix} 1 & -1 & . & . \\ 1 & 1 & -1 & . \\ 1 & 2 & 1 & -1 \\ 1 & 3 & 3 & 1 \end{vmatrix}\ ,\ \cdots$$

made by writing the under-diagonal side of the determinant as a Pascal triangle of binomial coefficients and then putting elements -1 in the superprincipal diagonal, yields on evaluation our sequence 1, 2, 5, 15, ... This representation is due to a man called Anderegg, otherwise unknown to fame.[37]

Well, they are a most interesting set of numbers and they appear to arise in those problems in which we are concerned not merely with partitioning an integer, as 4 into $1 + 1 + 2$, but with putting the parts into an order as well, $1 + 1 + 2$, $1 + 2 + 1$, $2 + 1 + 1$.

MacMahon was incomparably the greatest of the combinatorialists, so great that his stature is not yet recognized. I greatly treasure two letters which I

[37] Muir attributes this result to Anderegg, and notes that he found it in [2].

> have from him, concerned with a theorem on dual symmetric functions upon which I stumbled in 1927.

This is Aitken's final letter on the rhyme scheme problem.

3.3.10 Bennett to Thompson, 18 Jan. 1939

Bennett sends Thompson a postcard, dated 13*th* January 1939, asking how the work on the number sequence is going. He emphasises his belief that the recurrence-formula should have come first, and everything else second. Thompson's response to this has not been found. Apparently, he forwarded an older letter, for Bennett then writes:

> An interesting relic, this 22-year old letter that you produce from your store! I have looked up PAM's paper (7th Memoir in the Partition of Numbers, Phil. Trans. A. Vol 217 1917 pp 81-113) and doubtless, if one travels assiduously up to the point, it is "all there". But the great man knew too much, and the giant machinery is used to attack even small problems (or so it seems). Hamiltonian Operators[38] and so on...: the apparatus of the specialist.
>
> As regards 1 2 5 15 ... and its anatomy, you jotted down originally only part of what Aitken has — the two tables of successive differences, for the numbers themselves and for the powers of the natural numbers. It interested me to find the set of formulae (one being comprehensive of the rest) in all of which u^n becomes u_n after expansion: and more particularly the generating formula $(m \;\; n+1) = m(m \;\; n) + (m-1 \;\; n)$ primordial but noticed last instead of first, as it happened.
>
> (Did I make this clear? ... You have not made any comment.). — It is quite likely that Aitken has all this: but I wd. suggest that you pass my notes to him, as contributions to the common store, to use as he will. [For, so far, I know only <u>of</u> him.] Could and would you?

The 22-year-old letter Bennett refers to is part of correspondence between MacMahon and Thompson.[39] Thompson wrote to MacMahon on the 2th of July 1917, on the paper by MacMahon that Bennett refers to here. The paper deals with the enumeration of partitions of 'multipartite numbers' into exactly k parts, giving the Stirling numbers of

[38] Bennett is slightly on the wrong side, as the term used in MacMahon's paper is 'Hammond Operators'.

[39] Thomson's letters to MacMahon were presumably, like the ones to Bennett, returned to Thompson on MacMahon's death.

the second kind. In his letter, Thompson considers how many ways three factors can combined, much like the Oxford Groups problem. He finds four of them, not including the case where all factors are taken apart, but there is a hand-written amendment, marked 5.1.39, adding the missing one.[40]

3.3.11 Thompson to Bennett, 19 Jan. 1939

Bennett is obviously not aware that Thompson has already forwarded the letters to Aitken. Since Aitken has now had his say on Bennett's contributions, Thompson decides to do Bennett the same courtesy, and forwards Aitken's letters to Bennett.

> I think I have shown Aitken all yr. recent batch of letters, dealing with our series: and here are his. I rather think you have beaten him on points, — but I am an unworthy judge.

He returns to this in a later letter, and explains why he believes Aitken to be outdone.

3.3.12 Bennett to Thompson, 27 Jan. 1939

Thompson has still not commented on Bennett's formula, which makes Bennett somewhat impatient.

> ii. Am I tiresomely persistent? The sweetly simple and obtrusively obvious difference equation $(n+1, m) = m(n, m) + (n, m-1)$ that I gave is still awaiting your recognition and assent, if may be. Did I fall short of making it clear? . . . If n lines of verse have m rhyme-sets then an extra line added will either repeat one of the m old rhymes or add a new one. That is really all about it. If the enumerative Table

$$\begin{array}{ccccc}
1 & & & & \\
1 & 1 & & & \\
1 & 3 & 1 & & \\
1 & 7 & 6 & 1 & \\
1 & 15 & 25 & 10 & 1 \\
 & \&c & \&c & \&c &
\end{array}$$

[40] MacMahon responds to this in a letter dated 7th of July 1917, but this does not mention his paper. It is unlikely that Thompson should have forwarded his own letter without also forwarding MacMahon's.

> has its actual material written down as it appears on the squared-paper page. The contents of each cell are got from the cell to the North and the cell to the NW, by adding an old or a new letter, respectively. The contribution above the dotted line in each cell comes from the N, and the part below from the NW.[[41]]
>
> — NB. At their first appearances the rhymes are here named in alphabetical order. For this method *baab* would be "wrong": it should be *abba*.[[42]]

He now starts commenting on Aitken's letters, in the opposite order, the newest letter first.

> iii. ACA4/1. If the alternative aspect were preferred (his "tagged Groupers") the lines of verse may be numbered (as commonly) and then the rhyme-system, say in the lines
>
> $$\begin{array}{ccccccccc} a & a & b & c & b & a & d & d & a \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array}$$
>
> may be recorded either by the alphabetic row above or by the numeral equivalent. (1269)(35)(4)(78)
>
> The former seems simpler, as a plain sequence with no brackets.
>
> iv. ACA4/1. Nevertheless I may continue, tentatively, to regard $u^{n+1}(u-1)(u-2)...(u-s)^m = u^m(u+s)^n$ as novel until it is made plagiary by an actual identifying reference. . . ? (It was the formula with half a dozen subcases got when m or n is zero and s unity)

This latter comment is a bit puzzling. Aitken says in the letter in question that he recognises Bennett's "post-ultimate" method as a well-known result, without specifying exactly which method he means. It would not be necessary for someone with Bennett's letters at hand, as it would be obvious which method he meant. It could be that Bennett understands which method Aitken is talking about, and that he is simply stating that

[41] This is not the first time Bennett has had to beg for Thompson's comments. In one of his letters during their correspondence on islands and hedges in 1933, he remarked

> Some day I am going to abduct you and keep you prisoned on bread and water until you make some remark on my arithmetical mode of presenting and enumerating the different modes of cellular subdivision (the islands with maritime counties). A voluntary surrender would still be in order.

(See his letter dated 24th of July 1933, ms 26075.)

[42] Aitken shows in his letter dated 4th of January 1939 that he does not have the same reluctance for writing *baab*. This is not a problem as long as one is aware that this is the same rhyme scheme as *abba*, which Aitken is.

he will, at the very least, consider this particular formula as new, until such a time comes that this is proven plagiary as well. His use of the words 'actual reference' could indicate another meaning. Since he does not have his own letters at hand, unless he made a draft, it is possible that he has forgotten which method of his that he referred to as "post-ultimate", and that he mistakenly thinks of this particular formula instead. If so, what he is saying is that he will keep regarding it as new, despite Aitken's words, until he is given a reference proving him wrong. He continues:

> v. ACA19/12 ... "clearly the most rapid way" ... Is it? — with binomial coefficients as multipliers? The (n, m) Table hopefully competes, — with the natural numbers as multipliers.

As touched on before, it is not unlikely that Aitken really thought this to be the most rapid way, but it is probable that most mathematicians would side with Bennett.

The next two points deal with two other letters of Aitken that were written between his second and third rhyme scheme letters. These deal with Fibonacci numbers and other matters not related to the Bell numbers, and were presumably forwarded by Thompson by mistake. At the end of the letter, Bennett returns to the topic at hand again.

> ix. Don't think me Nil Admirari.[43] I do however tend to believe that marvels melt into normality with understanding. "Omne (semi) ignotum pro mirifico"[44] I take to be a soundly minatory motto.

He finishes by drawing a table of the enumeration of rhyme schemes with $n = 1 \dots 5$. A rather meticulous drawing of a cubic curve is also attached to this letter. This appears to have no connection to the rhyme schemes at all, and has therefore not been included here.

[43] *Nil Admirari*: To be astonished at nothing.

[44] *Omne (semi) ignotum pro mirifico*: 'Everything that is (half) unknown is taken for marvellous'.

	1	2	3	4	5 $= n$	
1	a 1					1
2	aa ··· 1	··· ab 1				2
3	aaa 1	aba abb ··· aab 3	··· abc 1			5
4	aaaa ··· 1	abaa abab abba abbb aaba aabb ··· aaab 7	abca abcb abcc ··· abac abbc aabc 6	··· abcd 1		15
5 = n	aaaaa ··· 1	abaaa abaab ababa ababb &c. aaaba aaabb ··· aaaab 15	abcaa abcab abcac abcba abcbb &c. aabca aabcb aabcc ··· abaac ababc &c. aaabc 25	abcda abcdb abcdc abcdd ··· abcad abcbd &c aabcd 10	··· abcde 1	52

3.3.13 Thompson to Bennett, 1 February 1939

Bennett's pleas finally work, and Thompson fulfils his wishes by praising the equation.

i. Yes. I fear I omitted to acknowledge your difference equation $(n+1, m) = m(n, m) + (n, m-1)$. But I can assure you I understood, appreciated and admired it! It has (like the 3 per cents) a "sweet simplicity".[45]

[45] It is not clear what these '3 per cents' is referring to.

ii. It looks to me as though our series was not unconnected with our old problem of the cell-partitions in the dividing cells on the surface of an egg. The numbers don't agree; but I rather fancy that is merely because the conditions of the cell-problem exclude certain particular cases. However, I have not found a moment to look farther into this.

iii.

...

iv. I have doubtless lots more to learn with regard to ACA's formulae as well as your own. But it seems clear that your (n, m) Table beats ACA's binomial coefficients easily, — both for speed and simplicity.

...

He is here comparing Bennett's table of Stirling numbers of the second kind to Aitken's table with binomial coefficients. It is perhaps a bit unfair to choose these two for comparison, at least when looking at speed and simplicity, as this is not Aitken's fastest or simplest result. His table of extreme differences, Aitken's Array, is simpler than either of these two.[46] It is, however, no wonder that Thompson chooses the table with binomials for comparison, as Aitken himself believed it to be superior to his other methods.

3.4 Conclusion

The most curious aspect of Aitken's three letters on the rhyme schemes is not what is actually in them, but what is not. As we have seen, everything Aitken contributes to this discussion, except for identifying the Bell numbers as counts of rhyme schemes, is older material that he has already published, and yet he remains curiously silent on that point. Not even when Bennett appears to have outdone him does he give a reference to his five-year-old article. His only attempt at defending himself consists of implicitly saying that he has seen it all before. By admitting that Bennett has seen one thing that he himself has not, he is indirectly saying that the rest of it is known to him already.

[46] To produce the *nth* Bell number using Bennett's equation, one must perform $(n-2)$ multiplications and $(n-2)+(n-1)$ summations. A similar assessment of Aitken's Array gives $(n-1)$ summations.

Aitken's remarks that the generating function must be found by everyone 'who direct some consideration upon the matter' could be read as an attempt to trivialise Bennett's results, consciously done or not. The contents of the letter from Thompson that Aitken is responding to can only be guessed at, but considering how he wrote to Bennett that he believed Bennett had outdone Aitken, it is possible that this sentiment shone through when he was writing to Aitken as well.

When Aitken chooses not to drag his old paper into the light, this could very well be because he thinks it is too late. He has by now realised that others have written about these numbers before him, if in a slightly different guise. He might also think it unwise to 'show off'. Doing so would also raise the question of why he did not mention the paper first time around. He might not wish to let Thompson know that he, in a way, had hidden explanations from him when he first wrote on this mystical number sequence. Had Aitken agreed with Bennett's comment that 'marvels melt into normality with understanding', such a presentation would be rather dishonest. More likely, however, Aitken would not have agreed, and presumably did find these numbers exactly as intriguing as he let on, even with the knowledge he already had. Another matter is that he might have feared that confessing to having given a simplified version might cause offence, something he would be keen to avoid, considering his great respect for Thompson. Another possibility is that the paper has been mentioned, but in speech. This is not very likely, as there would then have been traces of this in the letters. If nothing else Thompson would be inclined to mention it to Bennett, and he does not.

As it is, the matter is probably not very important to him. The correspondence starts out as mere amusement to him, and he does not expect it to get so serious. The conclusion to the whole affair is that Thompson and Bennett think Aitken knows less than he actually does. This probably would not affect Thompson's opinion of Aitken much, as he knows Aitken already, but it is possible that Bennett ends up with a slightly less flattering impression of Aitken than Aitken strictly speaking deserved.

What this correspondence shows is that papers published in the *Notes* would have held interest to more than just teachers, if they had taken the time to read it. Aitken does not expect Thompson to have seen this article of his, which turns out to be correct. He cannot have expected Thompson to read the *Notes* at all, at least not in 1933. Interestingly enough, he does expect Thompson to have read the *Mathematical*

Gazette. In his letter dated 19th of December 1938 (3.3.2), he asks if Thompson has seen a certain proof appearing in the English journal.[47] He does not mention what journal this proof is found in, he only mentions the author, so he must expect Thompson to recognise it based on that. As it turns out, he is wrong, because he eventually has to furnish Thompson with the full reference, but this does say something about the status of the two journals, at least in Aitken's eyes. This is of course one man's opinion, but it bears remembering that Aitken knows the journal very well, after his long service as the Editor. When he seems to think it a lot more likely that an academic in Scotland has read the *Mathematical Gazette* than the *Notes*, he is probably correct. *The Mathematical Gazette* must have enjoyed a certain status that the *Notes* did not.

[47] This is C. Dudley Langford's 'A dissection proof', *The Mathematical Gazette* Vol. 22, No. 252 (Dec., 1938), pg. 492.

Chapter 4

The Road to Research

4.1 Introduction

4.1.1 The outset

This chapter deals largely with a prolonged discussion on the Society's publishing policy and its attitude towards its teachers. To set the scene, as it were, it is worth summarising the situation in 1926, when the debate was first mentioned. The Society had by then become dependent on grants and donations in order to cover the cost of their publications. The *Proceedings* had changed radically in the last five or six years, at least where authors were concerned, with most papers now written by academics. The habit of publishing all papers read to the society was beginning to slip, and the *Notes* was struggling. After the first long break in publishing, most likely caused by financial woes and lack of material after the war, the journal had been published for five years, but in 1926 it found itself yet again at a standstill. The teachers were becoming less involved in the Society than they had been before, with fewer teachers on the committee and the number of teacher members declining. Many teachers had left the Society between 1915 and 1920, and only one teacher joined, and that for one session only. The recruitment of teachers had improved somewhat in the 1920s, but the percentage of teacher members was definitely heading downhill. On the 2nd of June 1923, one of the current Editors of the *Proceedings* resigned. This was the teacher G.D.C. Stokes and on giving his reasons for doing so, he blamed pressure of work, but not only that [88]:

> [...] I also feel that the school aspect of mathematics (with which I am now necessarily more identified) is being steadily neglected by the Society simply

> on account of the expansion of the specialist research.

Such was the situation when the debate started.

4.1.2 The key players

Before delving into the debate itself, it is worthwhile introducing some of the key players in it. There were two main players, the first being a very important man of the day.

Edmund Taylor Whittaker

Edmund Taylor Whittaker (later Sir) was born in Lancashire in England and educated at Cambridge.[1] After a spell of teaching at Cambridge and working for some time as Royal Astronomer of Ireland in Dublin, he arrived in Edinburgh in 1912 to take up Chrystal's chair of mathematics at Edinburgh University. He was to remain in Edinburgh for the rest of his career.

Professor Whittaker was a very influential man. W. H. McCrea wrote in his obituary of Whittaker [52]:

> It may reasonably be claimed that no single individual in this century or the last had so far-reaching an influence upon its progress. If such a claim comes as a surprise to some present-day readers, it is probably because we are apt to forget the part that Whittaker played personally in bringing about so many of the developments that we now take for granted.

He certainly had a most profound influence on mathematics in Edinburgh. One of the first things he did when he arrived was to set up the Edinburgh Mathematical Laboratory, which received much attention, and inspired the first EMS Colloquium. He also gave inspiring research lectures to staff and post-graduates twice a week for the rest of his career. In these lectures, he not only presented his own research, but also gave his own take on recent developments in mathematics.

He was to have an equally profound effect on the EMS. Although he joined the Society on his arrival, in June 1912, it took two years for him to get further involved with the Society. When he did, he went straight to the top job. He was elected President in November 1914 for session 1915. He stayed on the committee for a total of 20 years,

[1] With the exception of the paragraph on his involvement with the EMS, the information in this section was found in [70].

mostly as ordinary committee member, but he was also Vice-President in session 1932. On the 5th of June 1937, he was elected as Honorary Member of the Society.

Professor Whittaker's prowess received many honours. He was elected a Fellow of the Royal Society in 1905 and of the Royal Society of Edinburgh in 1912. The LMS awarded him the Sylvester Medal in 1931 and the Copley Medal in 1954. The RSE also gave him two awards; the Gunning Victoria Jubilee Prize in 1929 and the Bruce-Preller prize in 1931. To top it all, he was knighted in 1945.

Professor Whittaker was described as a very kind and unselfish man, with a great interest in other people. He was a devout Christian, and converted to the Roman Catholic Church in 1930. He was later to give lectures on science and religion, one resulting in his book *Space and Spirit: Theories of The Universe And The Arguments For The Existence of God.*

With his great interest in current research, and his continuous work to improve it, it should come as no surprise that E. T. Whittaker was to drive the Society in that same direction, at the expense of the teachers.

Thomas Murray MacRobert

The second main player, who took a strong stance for the teachers, was the Professor of Mathematics at the University of Glasgow.[2] He was born in Dreghorn in Ayrshire and educated first at the University of Glasgow and then at Cambridge. On his return to Glasgow, he was employed as Professor G. A. Gibson's assistant in 1910. He was promoted to lecturer in mathematics in 1913, and was appointed to the Chair of Mathematics in 1927 when Gibson retired.

An interesting thing about MacRobert is that his first research paper was not published until 1916, when he had been working at Glasgow for six years. He followed the opposite pattern of what most mathematicians did, by publishing text-books in his early career and leaving his most important research for later. This did not prevent him from being elected a fellow of the Royal Society of Edinburgh on the 7th of March 1921.

MacRobert joined the EMS in February 1911, and was appointed ordinary committee member from session 1916. He stayed on the committee for two years only, but returned as Vice-President for session 1921, and President for the following year. After that, he

[2]With the exception of the paragraph on his involvement with the EMS, the information in this section was found in [75].

Edmund Taylor Whittaker (1873-1956)

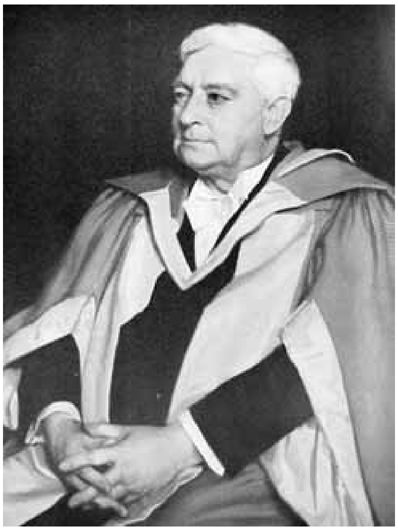


Figure 4.1: Thomas Murray MacRobert (1884-1962)

was Editor of the *Proceedings* for four years, before finishing as ordinary member of committee for three.

Professor MacRobert was a man of strong opinions. He was raised under strong political and religious influences. His father was a minister in the Evangelical Union, later Congregational Union, a protestant church. The senior MacRobert gave strong liberal support, which rubbed off on his son. The young MacRobert considered both following his father into the Church and becoming a politician, before settling for mathematics. Rankin describes him in an obituary as a man held in high respect and affection, who was very generous in helping others [61].

Others played a part too, such as Edward Thomas Copson. He was born in England and arrived in Edinburgh to take up a lectureship in mathematics under Whittaker. He joined the Society immediately, as one did in those days. He was the secretary from 1925 to 1930 and Editor of the *Proceedings* in two rounds, 1931–1933 and 1926–1938. While holding these other posts, he was Vice-President in session 1929 and President two years later, in session 1931. In 1934, he was ordinary member of committee. Copson was elected an Honorary Member of the Society in 1979.

Another President who was to involve himself greatly in this discussion was Herbert Westren Turnbull, the Professor of Mathematics at the University of St Andrews. He joined the Society in January 1922, and joined the Committee as ordinary member two years later. He held the post of Editor from session 1925 to session 1934. During this time, he was also Vice-President in session 1926 and President in session 1927. He stayed on as ordinary committee member from session 1935 to session 1941. Professor Turnbull was elected an Honorary Member of the Society in 1954.

Harold Stanley Ruse also deserves a mention. He would later become Professor of Mathematics at University College in Southhampton, and then Professor of Pure Mathematics at Leeds. He arrived in Edinburgh in 1928 to study for a research degree under Whittaker, and joined the Society right away. Ruse took on the office of Secretary after Copson, in November 1930, meaning that he was unfortunate enough to hold this post when the debate reached its climax in the spring of 1931.

4.2 A new policy

4.2.1 The first discussions, 1926–28

A new policy for the publications was discussed in the Committee on the 5th of June 1926 when they met in St Andrews.[3] The minutes from this meeting say very little on the contents of the discussion, but the minutes from the next one, in November that year, show that it concerned the culmination of the first series of the *Proceedings* and the start of a new one. The Committee approved of this scheme at the meeting on the 14th of January 1927.

This new series differed from the old one in quite a few ways. First of all, the appearance was different. The pages were larger, the cover thicker and the type changed. The volumes were also arranged differently, as explained in chapter 1. These cosmetic changes were only part of the story, however, as the Committee kept discussing the matter throughout the 45th and 46th sessions. Very little of this discussion was minuted, but other records shed more light on what transpired.

4.2.2 Correspondence

The Secretary, E. T. Copson, was corresponding with the President and Editor, Professor Turnbull, on the matter shortly after the Committee meeting in January 1927. These letters, and all other letters referred to in this chapter, are reproduced in full in Appendix C. Copson's first letter has sadly not been preserved, but the rest of this correspondence gives a rather good indication of the views he expresses in it. Copson is arguing for a research periodical, along the lines of the *Proceedings of the London Mathematical Society* (PLMS). He advocates a firmer line, where the level of the *Proceedings* is raised by rejecting more papers.

Turnbull's response shows he is not quite agreeing with this, and he gives two reasons for this [97].

> The whole conduct of the publications of the E.M.S. has been comparatively free & easy in the past, compared with that of the L.M.S. There are some advantages in both methods. First we must remember that we are guests in

[3] It is uncertain whether this was the start of the discussion, or a continuation of it, as no minutes are kept older than the 5th of February that year. It is likely that it is the beginning, as the publications had not been discussed earlier that spring, including one meeting where 'there was no business to bring before the Committee' [90, 5 March 1926].

> a country which is not England, & for that reason must not be too obviously anxious to mould the Societie's [sic.] *Proceedings* on London lines!

Turnbull's second letter even goes as far as saying Copson regards the EMS as a replica of the LMS, something MacRobert will not agree with [94].[4] Turnbull's other objection is that the membership of the EMS is rather more varied than that of the LMS. He feels this should be reflected in the *Proceedings* and has therefore, as Editor, accepted papers of a more general nature.[5] Copson has a counter-argument to this, as presented in a draft of his response to Turnbull. He believes such general papers would find a better home in the *Notes*, which is not in a very good state. He writes [18]:

> At present, the journal [*Notes*] is, for all practical purposes, dead. So far this session, only one paper has been received for publication in the *Notes*. I proposed a more rigorous rejection of papers from the *Proceedings* so as to have more matter for the *Notes*. For example, the paper of Dr McWhan's is very suitable for publication in the *Notes* but, to my mind at any rate, is not at all desirable in the *Proceedings*.

The paper he speaks of here is 'Proof of the conditions for a turning value of $f(x, y)$ without the use of Taylor's theorem'.[6] This paper had already been accepted for publication. It seems that lack of material is still the major issue for the *Notes*.

He continues:

> It seems to me that there are two alternatives open to us:-
>
> a keep the *Notes* going and reject more papers from the *Proceedings*, or
>
> b scrap the *Notes*.
>
> The Committee may feel that (b) is preferable, and decide to enlarge the scope of the *Proceedings* instead of restricting it. I should be very glad to hear your views on the subject; I have no personal opinions in the matter, but was supposing that (a) would be preferable.

[4]Copson does not confirm or deny this in any of the letters that have been preserved. It is in fact not mentioned at all.

[5]He gives Professor G. A. Gibson's paper 'Sketch on the history of mathematics in Scotland' as an example. This was published in two parts in PEMS, Vol. 1., no. 1, pp. 1–18, and no. 2, pp. 71–93. Professor Turnbull also argues that a good paper on the history of mathematics should have a natural home even in the most advanced of journals.

[6]It is to be found in the *Proceedings*, Series II, Vol. 1., No. 1, pp. 68–70.

When Copson here states that he has no personal opinions in the matter, it is not entirely obvious what he means. It is clear that he does have personal opinions where the *Proceedings* is concerned. His comments on Dr McWhan's paper leaves little doubt of that. Copson appears to have rather high ambitions on behalf of the periodical. It is possible that he is more indifferent to the *Notes*, and therefore more inclined to go along with the consensus of the Committee in that area. Even so, he would probably prefer not to enlarge the scope of the *Proceedings* the way he suggests. Perhaps he is trying to be a more neutral secretary, or perhaps all he means is it is not important enough for him to argue about it.

4.2.3 The teachers in 1926

The Honorary Treasurer Dr E. M. Horsburgh, a lecturer at Edinburgh University, also expresses his views in writing around this time. On realising that he will be unable to attend the committee meeting where 'the general management of the E.M.S.' is to be discussed, he sends a letter to Copson, explaining his own opinions on the matter [42]. He writes:

> I suppose we must recognise this new departure in publishing the Proc. E.M.S. as the outward and visible sign of the break with the Traditions of the early years of the Society. It stood then for scholastic mathematics.

As discussed in chapter 1, he is not entirely correct in this assessment of the Society's earliest days. It is, however, no wonder that it would look like this to him in 1926, especially considering how the real goals and aims of the foundation were not stated explicitly.

Dr Horsburgh draws the attention to how the Society is 'likely go get more papers than it can afford to publish', and stresses that the subscriptions fees cannot pay for the publications. He remarks with regret that it has often become necessary to ask authors to fund their own paper, and he continues:

> Also as the schoolmasters form the main strength of the Society it is desirable to cater for their requirements, as far as finances allow; since he who pays the piper should certainly have a say in the choice of tune.

It is not entirely clear what he means when he writes that the teachers 'form the main strength of the Society'. An analysis of the list of members from November 1926, organising the members by occupation, yields the following result:

Table 4.1: Members by occupation 1926

Teachers	68
Academics	104
Other Higher Education	22
Students	1
Other	34
Unknown	27

Only 26.6% are identified as teachers in secondary schools. It is possible that he considers those working in other forms of higher education as teachers, which would give a total of 90 and 35.2%. Only if the entire unknown-category turned out to be teachers, which is not very likely, does the teacher category surpass the academics enough to call it the main strength. Horsburgh may be speaking out of conviction; if considering the home members, the members actually living in Scotland, the teachers certainly form the main strength, with 60 of 155 Scotland-based members.[7] It is not unlikely that this would give the impression that the Society had more teacher members than it actually did.[8]

His point is, however, still valid, as the teachers still formed a sizeable chunk of the membership.

4.2.4 The discussions at the committee meetings

This correspondence shows that the discussion throughout that spring dealt with more than just the *Proceedings*. There appears to have been consensus that the *Proceedings* should be allowed to turn towards specialised research. It can be argued that this process had been going on for some time, as indicated by the rising percentage of unread papers from 1920 onwards. The custom of publishing the read papers as a rule must have been abandoned around this time, and the quality of the paper became the deciding factor in whether or not to publish.[9]

The real question was what ought do be done about the *Notes*. The next two meetings, in February and March, were devoted fully to this discussion, but with no result. Not until the May meeting was a decision reached, when it was decided to send

[7] The second largest group is the academics, with 36.

[8] Another possibility is that Horsburgh does this consciously, giving the home members more weight in the discussion. This would make very little sense, however, as the publication policies would matter even more to those members who could not attend the meetings, such as members abroad.

[9] This is presumably also when the journal became peer-reviewed, though this is not certain. The second constitution of 1931 certainly requires the papers in the *Proceedings* to be refereed.

out an appeal for more material for the *Notes*, which 'must not be allowed to cease'. This was repeated at the following meeting in June.[10]

The minutes mention a few of the main speakers at the February meeting, who were identified as Dr MacRobert, Professor Whittaker and Dr Copson. Professor Turnbull, who was not present at that particular meeting, was surely involved to a large extent at the following meetings. The two teachers on the Committee, G. S. Eastwood and J. B. Lockhart, who should perhaps more than anyone care for the future of the *Notes*, are not mentioned at all.

The disagreement that kept this discussion alive for so long seems not to have been on the existence of the *Notes*, but rather how the journal should be revived. On the 12th of March 1927, Turnbull writes in a letter that MacRobert has suggested sending a circular to secondary school maths teachers in Scotland as an attempt to recruit more teacher members; a circular which would hint about the improvement of the *Notes*.[11] He could hardly have done so if it was not yet clear whether the *Notes* should be improved or abandoned. All in all, there appears to have been some amount of optimism on behalf of the *Notes* at the time. Professor Turnbull remarks in another letter, dated 14th of May 1927, that he does not believe the lack of material will remain a problem for long. He writes [95]:

> I don't think the question of Notes & Research Papers need worry us. Once the *Notes* are established again as a regular thing, people will be glad enough to have their work printed in it. [...] I'll get Whittaker & a few others to send in some Notes (I have my eye on two such Notes already). Then the *Notes* will have the necessary prestige![12]

In the same letter he also reminds Copson to try to find local sub-Editors for the *Notes*. This, however, seems to have been a suggestion that was never acted on, as it is never mentioned again.

In addition to the suggested circular, another attempt was made to draw the attention of the teachers. For the meeting in Glasgow in May, Professor Whittaker was asked to lecture on Euclid, and he gave a talk entitled 'Definitions, axioms and existence-

[10]They also settled details regarding format for both journals at this meeting. It was agreed each volume of the *Proceedings* should consist of four parts, and that each part should cost 6/3. It was also determined that the *Notes* should be published 'in such a form that it could be bound with the *Proceedings*' and that the pagination in the *Notes* and the *Proceedings* should be separate.

[11]If such a circular was sent, which is likely enough, then it was done without any record of it happening.

[12]Whittaker never did this, though he had published in the *Notes* before. Turnbull, A. C. Aitken of Edinburgh University and N. McArthur of Glasgow University were all published in the next issue of the *Notes*.

theorems in Euclid'. At MacRobert's initiative, this meeting in Glasgow had a larger audience than usual, as the members of the Euclidean Society had been invited.[13] Dr MacRobert proposed this to the Secretary before the meeting, and although he did not say so directly, it is very likely that he was hoping to recruit more teacher members this way [49].

All in all, the discussion taking place in the spring of 1927 seems to have been on how to provide for the teacher members, and how to bring in even more of them, rather than whether teachers should be provided for at all. As Professor Turnbull pointed out in one of his many letters, 'The fact is that all our difficulties of adjustment are caused by the very success of the society, which is evidently expanding its scope' [99].

4.2.5 A new journal

In Turnbull's letter of the 14th of May, he also includes a rather cryptic remark regarding the Glasgow members of committee.

> On the whole I incline to the idea of separate *Notes*, same sized paper, etc, separate pagination. The main thing was to get the Glasgow Cee. to interest themselves in the *Notes* & that I think has been done.

Why he wanted this done is far from clear, but done it was. When the Committee met in Glasgow in December 1927, Professor MacRobert presented a scheme for the publications that he intended to explain more fully at the next meeting in Edinburgh.[14] He had prepared a document that was attached to the minutes from the meeting.

> 1. That in the future the society should produce two publications, '*The Proceedings of the Edinburgh Mathematical Society*' and '*The Journal of the Edin. Math. Soc.*'.
> 2. That in the *Proceedings* only research papers should be published.

[13] This was the predecessor of the Glasgow Mathematical Association. The EMS-member J. A. McBride, who was the President of the Euclidean Society in 1927, later described the foundation of it in a letter to the secretary of the EMS [51].

> [It had] the object of re-establishing Euclid's text as a book for School Geometry, but after some time it became quite clear to the organisers that the younger generation of mathematical teachers did not know Joseph's compatriot, and it gradually became a means of bringing together western mathematical teachers for purposes of discussion etc.
>
> Another idea was that it would be a recruiting ground for the Ed. Math. Soc.

[14] The Glasgow Committee meeting often had lower attendance, so important decisions were sometimes left for the Edinburgh meetings.

3. That the *Journal* should be devoted to articles of general interest, including historical papers, notes, reviews, and discussions on methods of teaching.
4. That the portion of the ordinary funds of the society available for these publications should be divided between the *Proceedings* and the *Journal* in the ratio 4 to 3.

The first three points summarised what the Committee had more or less agreed on already. The *Proceedings* was turning towards research only. A new teacher journal, or rather, the renaming and re-profiling of the *Notes* had been discussed before, though it had not entered the minutes before now.

The first time it was mentioned was in Horsburgh's letter of 4th of February 1927. He asked if the teachers might not want a journal with a more imposing title than the *Notes*. This journal, he explained, could contain notes and abbreviations of papers that teachers would find interesting, even if these papers did not 'profess to enlarge the boundaries of Mathematics' [42]. It was next mentioned by Professor Turnbull, who in a letter to Copson from November 1927, asked who was to take on 'the *Notes* or its successor' [98]. The name MacRobert suggests here, *The Journal of the Edinburgh Mathematical Society*, had not been recorded before. Judging by how staunchly he was to defend this name later, it is likely that it originated with him (See section 4.3).[15] The final point, regarding division of funds, appears to have been new as well, and this was the only point that the Committee could not agree on when they met to discuss the scheme on the 13th of January 1928. It was felt that this question was too important for them alone to decide on, and that it should be decided by the Society as a whole. There is no evidence that this matter was ever brought before the Society, and this division of funds is not mentioned in the minutes, or anywhere else, after this meeting.

On the 4th of May 1928, the Committee agreed to start publishing the *Journal* in its new form. Everything seemed settled, but something must have happened behind the scenes. When the next issue of the periodical appeared in January 1929, it was under the name of the *Notes*. The Committee never rescinded their decision to publish the *Journal*. The only document mentioning this is one of Turnbull's letters to Copson [96]:

Better call the *Notes*   *Math. Notes* for the time being. We must publish

[15] The name had possibly been mentioned earlier. They had certainly discussed names before, as Turnbull wrote in his letter to Copson 'Please don't let them call it 'Bulletin', for any other name would be sweeter' [98].

them in order to encourage the teachers.

Turnbull did not give any explanation as to why he felt it was better to keep calling it the *Notes*, but this decision gives the impression that matters were not quite as settled as the minutes would indicate. MacRobert's point 4 could be the key. The issue of funds had not been resolved and the Society's financial situation was far from stable. At the meeting in May 1928, when they agreed to start publishing the new *Journal*, they also decided to issue another part of the *Proceedings* as soon as possible. The Treasurer, the teacher J. B. Lockhart, warned the committee that next session's budget might not easily support two such publications that summer. The economy had been fairly strict for session 1928, due to an unexpected printer's bill in November 1927 of almost £100. In fact, things were bad enough for the Society to make yet another application for financial aid to the Royal Society that summer, which was again rejected. A bleak financial outlook could have made them reconsider their plans. Whatever the reason, the *Journal* lay abandoned. When the periodical was next mentioned in the minutes, in November 1929, it was only because the committee had agreed to publish records of meetings in it. This was only done for one issue.[16]

4.3 The Controversy

4.3.1 The Second Constitution

When the Committee met on the 7th of November 1930, they were in for a very long meeting. The newly appointed Secretary, H. S. Ruse, pointed out that the Society's rules needed revision. With the exception of certain rules, the constitution had not been revised at all since the foundation of the Society, and it was severely outdated. The Society's publications were, for instance, not mentioned at all.[17] Ruse also explained that the Society risked losing the support from the schoolteachers because of the rising standard of the *Proceedings*.[18] The discussion that followed was unusually long, stretching the meeting to a full two hours.[19] In the end, it was resolved to appoint a sub-committee to draw up a proposal for a new constitution. The Committee also decided that yet another effort should be made to expand the *Notes*. The name

[16]This was Volume 25, that appeared in January 1930.

[17]Copson had also mentioned this in his letter to Turnbull in 1927 [18].

[18]The minutes actually say 'publications', but rising standard was not an issue for the *Notes*. It had been published only three times since the discussions in 1927, once in 1929 and twice in 1930.

[19]This in contrast to how it works today, where 2 hours is the norm.

of the periodical was again a topic for discussion, and the sub-committee was asked to consider changing the name to *Journal of the Edinburgh Mathematical Society*, or similar.

This they did and they prepared a proposal for the next Committee meeting in Glasgow in December. This proposal incorporated the Society's publications, the updated setup of the Committee, and also the Society's library.[20] Their suggestions were for the most part accepted, but any official decision-making was postponed for the next meeting.[21] The discussion turned to the name of the *Notes*. Professor MacRobert made his case once again for the *Journal of the Edinburgh Mathematical Society*. This time, however, he was met with strong opposition from some of the other members. The minutes do not go into detail, but the main objection appears to have been a potential confusion with the *Journal of the London Mathematical Society*. This periodical was a rather different type of journal, closer in nature to the *Proceedings* than the *Notes*. It had not been an issue in 1927, possibly because the journal of the London society was so new then; it was only published for the first time in 1926. Other names were therefore suggested, such as *Bulletin of the Edinburgh Mathematical Society*, *Report of the Edinburgh Mathematical Society* and *Mathematical Notices*. As with the discussion on the rest of the rules, the matter was adjourned.

The next meeting took place on the 16th of January 1931 in Edinburgh. The new constitution was accepted, with the exception of the rules relating to the *Notes*. Professor MacRobert did not believe that the similarity in name and not in content of the *Journal of the LMS* and the potential *Journal of the EMS* would be a problem, and he expressed as much.

> Prof. MacRobert stated that the objections of certain of the other members to the name *Journal of the Edinburgh Mathematical Society* seemed to him absurd, since Scottish members would not feel the same objection to copying the London Mathematical Society as English members [90, 16 Jan. 1931].[22]

Professor Whittaker disagreed with this, and pointed out that the publications of the LMS were 'very widely read'. Not having reached a conclusion, the discussion on

[20] It also contained the requirement that papers written by non-members would only be considered when communicated by a member.

[21] This was presumably due to low attendance at the meeting.

[22] It is perhaps unexpected that MacRobert would say that copying the London Mathematical Society was a good idea, considering Professor Turnbull's cautious warnings to Copson in 1927. One might, however, argue that there was a distinction. Copson had claimed that the EMS was a replica of the LMS, whereas MacRobert here only wished to re-use a name. There is no evidence that MacRobert had any objection to copying the PLMS either, as long as the teachers were given another alternative.

the name was adjourned for the next meeting in February.

It is very unusual for a sentence such as 'The objections [...] seemed to him absurd' to enter the usually so dispassionate minutes, even if it was a direct quote. Any discussion would usually be toned down several notches for the minutes. When the phrase appears here, it could mean there was an unusually heated debate going on. The Secretary was, however, rather new, and perhaps not quite accustomed to the style of minute-taking. It will be seen shortly that he had not fully understood the severity of this debate, which he could hardly have avoided doing had there been a proper argument going on. That said, the discussion was surely lively enough, though perhaps not quite as lively as the minutes might indicate.

4.3.2 MacRobert's resignation

On the 6th of February the Committee met in Edinburgh. Professor MacRobert was not present, and the Committee agreed unanimously that the name *Mathematical Notes* should be changed to *Scottish Mathematical Journal*. After that, some minor details regarding the new constitution were discussed, and then he broke the news. Ruse reported that MacRobert had sent in his resignation from the Committee, with immediate effect.

In this letter, dated 28th of January 1931, Professor MacRobert announced that he had given it much consideration, but that he wished to resign from the Committee. He reminded Ruse that he had intended to resign at the beginning of the session, but that he had allowed himself to be talked into staying for one more year, in order to deal with the new constitution. This had now been resolved, with the exception of the *Notes*, and he had said everything he wished to say in that regard. He continued:

> During the last four years I have found myself so out of sympathy with the views of the majority of the committee that my resignation should add greatly to the harmony of the meetings, as well as to my own peace of mind.

He finished by stating he had had pleasant relations with all the committee members, and, perhaps as a sign of goodwill, offered to give a paper at the upcoming Glasgow meeting.

No one but the secretary seems to have known about this letter before this meeting, so naturally, this caused some stir. The Committee members, taken completely by surprise, must have felt that an important event such as this should have been mentioned much earlier, ideally before the meeting itself. The Secretary was instructed to issue agendas from thereon.[23] MacRobert's resignation sparked a long discussion on the future of the Society. The minutes say very little about the contents of it, except that it was decided to ask MacRobert to rethink his decision. More of the discussion is summarised in the agenda for the next meeting. Dr J. Hyslop, another Glasgow University man, had explained there were three different paths open to them [81]:

- A continuation on the present lines, but with a definite effort to cater for schoolteachers.
- The establishment of branch centres of the Society in Dundee, Perth, etc. thereby creating an organisation similar to that of the Math. Assoc. of Gt. Britain.
- A development as a purely research Society, with no effort to interest schoolteachers.

It would soon become clear that MacRobert's resignation represented a major change in the nature of the Society.

4.3.3 A new course

The next meeting took place on the 6th of March 1931, and it had by then been confirmed that Professor MacRobert would not be returning to the Committee. Professor Whittaker now took a much stronger stance on the future policy. He argued that the Society should in the future become purely a research society, leaving those whose main interest was pedagogic to the Glasgow Mathematical Association. He gave his reasons for saying so as such [90]:

> Experience has shown that it is impossible in the same society to cater both for researchers and those whose main interest is pedagogic'.

What 'experience' he is referring to can only be guessed at, but it is very likely that financial issues played a part. Professor Whittaker explained that the Society could not afford to publish both periodicals in the long term, seeing how they were dependent on grants in order to publish anything at all. He argued that it would be easier to receive

[23] This is what indicates that the Secretary had not fully understood the situation. If he had realised how important a discussion this was, and how it might affect the decision on the *Notes*, it would have been natural to mention the resignation at earliest opportunity.

grants from the Royal Society if the Society professed to be devoted purely to research.[24] As a consequence, Professor Whittaker felt that another organisation, ideally the GMA, should take over responsibility for the *Notes*, with some initial financial aid from the EMS.[25]

If there were any objections to this, it was not recorded. William Arthur, the Editor of the *Notes* and a Glasgow academic, agreed to approach the committee of the GMA to see if they were interested in such an offer. When the Committee met in May, it was to the news that Arthur had resigned from his positions as Editor of the *Notes* and committee member.

He had, as instructed, approached the GMA with an informal proposal. As he wrote to Professor Whittaker afterwards, the result had been in the negative [5]. Their committee had discussed it thoroughly, but the official response was that the GMA was too young to accept such financial responsibility, even with the offered aid from the EMS. Professor Whittaker had responded, expressing his disappointment, and repeating his view that the EMS could not support both periodicals. He urged Arthur to discuss the matter again with Professor MacRobert, who was a founder member of the GMA and very much involved in it. If this letter from Professor Whittaker was the trigger for Arthur, or if he had simply had the time to think matters through is unclear, but whatever the reason, he resigned on the 20th of April. Arthur wrote to Professor Whittaker on the same day, explaining that he had come to agree with him; that it had indeed become impossible for the EMS to cater for both teachers and researchers. Arthur could see no good explanation for why this was. He felt the Society owed their very existence to the teachers, and so felt obliged to provide something for them 'even on the expense of ambition' [4]. He believed that a change in the Society's 'character and purpose' was unavoidable and disagreed so strongly with this that he felt he had no choice but to resign.

Arthur did not blame Whittaker directly in this letter, but he did in the letter of resignation that he sent to the secretary. On explaining why he wished to resign, he wrote [6]:

> I do so because of strong disapproval of the change in character of the Society which is advocated by Professor Whittaker and appears certain to be carried

[24] The Society had as mentioned applied for grants from the RS twice before, both applications having been rejected.

[25] This argument might have lost some of its potency if a loss of the teachers meant a great loss of income in subscription fees. Considering how little of the printer's bills the subscriptions paid for, however, it is very likely that Whittaker thought this loss of fees would be more than made up for by higher grants. He might also have expected that more academics would wish to join when the *Proceedings* became even more research oriented.

> out in the near future. However the change may be effected, it involves the cessation of all attempts to cater for those whose interests are not solely centred in research, and to that I cannot be a party.

Either Whittaker saw this letter, or he realised himself that it was partly through his own actions that Arthur was now resigning. He sent another letter to Arthur, regretting his part in this, and also explaining more thoroughly why he believed the Edinburgh Mathematical Society should take this path. He argued that the EMS had already developed into a research society, it was no longer a society for teachers. Therefore, it was to him obvious that it should remain that. Professor Whittaker stressed that he believed the two societies, the one for research and the one for pedagogy, should be on the friendliest of terms, and he wished to support both.

At the Committee meeting in May, Professor Whittaker repeated his arguments from the previous meeting, as the personnel was rather different. He urged again that the time had come for two societies, not one. Considering the result from the enquiries with the GMA, he suggested that the EMS kept publishing the *Notes* until a suitable body willing to take over could be found. Dr T. P. Black suggested approaching the Educational Institute of Scotland.

Professor Whittaker did not get the approval he was hoping for in this. Copson, and others with him, was of a slightly different opinion. He agreed that the publications should be kept for research only, but he felt the papers read at the meetings should be of more general interest. This was agreed to in general, and various ways of achieving this was discussed. In the end, the committee appointed a subcommittee, consisting of Professor Whittaker, Dr Black and Dr McCrea, to consider this question and also to get in touch with the Educational Institute.

The new constitution, with exception of the rules regarding the *Notes*, was accepted at the ordinary meeting of the Society, directly following this committee meeting. At the wishes of Professor MacRobert, his resignation was not mentioned at the ordinary meetings. It was attempted to find a replacement, but without success. The decision to publish the *Scottish Mathematical Journal* was rescinded on the 1st of May 1931. The attempts at finding another body to take over responsibility for it failed, so the Committee eventually agreed to keep publishing it. The financial situation did not improve, and although this did not prevent the publication altogether, it did ensure it did not exceed 25 pages. When the Society was offered a rather large paper in 1938, intended for this journal, it was rejected on account of space. The *Notes* hobbled along until the last volume, the 44th, appeared in 1961.

4.3.4 The Glasgow Lapse and aftermath

Professor MacRobert's relationship with the EMS seemed to worsen. In October that year, it became clear that he would not be renewing his annual invitation for the Society to hold meetings in Glasgow. The Secretary sent out a circular, after an informal discussion by five of the committee members, with a proposed programme for the session. With the exception of the June meeting in St Andrews, all meetings were to be held in Edinburgh. It was suggested not to hold a meeting in January, and keep only one of the meetings for research papers. The rest would be devoted to talks of general interest, such as 'Careers for honours graduates in mathematics', and a lecture in connection with the Maxwell and Tait centenary. The responses from the Committee members were all positive, but a few interesting remarks were made, both on the programme and on what Professor Turnbull referred to as the Glasgow Lapse:

> Thinking over matters, I rather feel that we may be going too far in the other direction, by practically ignoring research papers. After all, some of us (and I speak personally) have learnt a great deal from hearing them read, even if we have not always understood every word. However, the programme is a brave gesture, and surely McR. must be gratified. [100]

It would seem the Committee was trying to please MacRobert, possibly hoping he would change his mind about the Glasgow meetings. Professor Turnbull's remarks on the programme presumably had an effect. In the end, a January meeting was held, and the extra time was allocated to research papers.

The actuary G. Lidstone, who seemed to have been absent from most of the relevant committee meetings in the previous year, placed parts of the blame on the 'old Glasgow-Edinburgh feeling' [46]. He expressed sympathy with the teachers, who he thought must consider the EMS to be 'the Publication Bureau of the Research-workers of Edinburgh'. The general talks, he wrote, would perhaps ease the situation somewhat, but he felt they were rather too general to be of much use to the teachers, and believed that a periodical would have suited their needs better.

He was not the only one making the Glasgow-Edinburgh connection, Dr Hyslop had apparently expressed similar sentiments.[26] Such a description of the problem may

[26] Dr Hyslop was in very poor health at the time, and a letter arrived from his father, James Hyslop, saying that his son had often discussed the 'unfortunate position as between Edinburgh and Glasgow' with him.

work on a superficial level, but it is unlikely to have been a major contributor to this Controversy. Professor MacRobert was a Glasgow man, true enough, but Professor Whittaker, Professor Turnbull and Dr Copson were all English. One could perhaps argue that Scottish-English relations were more to blame, but that will not explain it all either.

This programme must have had some success, and the following years saw a much higher percentage of general talks than before. As a means of gratifying MacRobert, however, it did not work; when the current Glasgow representative in 1932, Mr Robb, was asked if MacRobert would be able to invite the Society to Glasgow for the following session, the answer was a definite no.

4.3.5 The reasons for the controversy

Why did this seemingly rather simple discussion get so important? One side of this is why the debate lasted so long, but this is not too surprising, as the future of the Society was a very important topic that deserved due consideration. The more interesting question is why the discussion in 1931 grew to such proportions over something as trivial as a name of a periodical. The answer is of course that it was not about the name at all.

From MacRobert's point of view, it was simply the last straw. He had argued for such a journal for years. He got acceptance for it, only to see it become abandoned. When he relaunched the idea, he met strong opposition for reasons he found completely irrelevant, which must have felt very aggravating indeed. Whittaker on the other hand, probably believed quite strongly that his path was the best path for the Society. He was very ambitious on its behalf, as he was for mathematics in general. The reasons he gave for his objections to the name might have been genuine, but they were probably also influenced by his conviction that the journal was not the best option to begin with.

It should also be added that more personal issues may have affected the debate. MacRobert and Whittaker were very different as people and as academics, Whittaker being on the forefront of research and MacRobert not having published many research articles at the time.

If one is to speculate it is worth mentioning that they were very different in terms of religious beliefs. MacRobert was a Protestant, living in the Scottish city where religious sectarianism was at its worst, and Whittaker had converted to Catholicism in 1930. Such personal differences may have influenced their relationhip, after disagreeing with one another for so long, it is likely the relationship between them became somewhat strained. It is possible that the discussion would have been a much more pleasant affair, had it not been these two gentlemen who appeared as main opponents.

Fed up as MacRobert may have been of the internal skirmishes of the Committee, he still seemed inclined to remain on friendly terms with the Society. In his letter of resignation, he made sure to mention his offer to read a paper at the upcoming Glasgow meeting, as if to remove any doubt of his willingness to continue holding these. When he had changed his mind by October, this was most likely because of Whittaker's new strong stance on research. From MacRobert's point of view, the Committee began tearing down everything he had worked for during the past four years the very moment he walked out through the door. Hearing Arthur's reports from the following Committee meetings, and seeing how the Society tried to dispose of the *Notes* must be what made him sever the connection with the Society in the way he did.

4.3.6 Further developments

The high proportion of general talks that the Committee had such faith in was not to last, as shown by an incident in November 1937. The proposed programme for the session contained no meeting of interest to schoolteachers, which led the teacher Dr A. Inglis to object. He reminded the Society of an agreed policy that one meeting each year should be devoted to school matters, and asked why this it had not been done. He also requested talks relevant for teachers at the upcoming colloquium in 1938. His objections were followed by some correspondence between Dr Inglis and the secretary, Dr I. M. H. Etherington. The teachers had not been forgotten intentionally, and there were still several on the committee who sympathised with Dr Inglis and the other teachers. The Secretary was one of these, as was the President, the teacher G. Lawson.

Dr Etherington wrote to Dr Inglis on the 7th of December that year, informing him that the February meeting would now be filled with a discussion on the teaching of mathematics, rather than the research paper they had planned originally. Dr Etherington explained that he believed the blame for this negligence should be placed on the teachers of the Committee, who had not objected to the proposed syllabus in the first place. Dr Inglis made a few valuable observations in his response. On expressing the hope that as many teachers as possible would take advantage of this planned discussion, he wrote [43]:

> But there I am afraid is the snag! There is no doubt that Prof. Whittaker is right when he pointed out to me that very few schoolteachers attend meetings of the Society.[27] Whether that is due to the type of programme offered or to the laziness (or indifference) of teachers to attend, I am unable to say. Probably is it a combination of both.

Dr Inglis continued, talking about the necessity of a Scottish version of the Mathematical Association in England:

> There is no provision at all for the teachers of Mathematics in Scotland who wishes to know something of the progress made by others in his own craft. For this I blame primarily the teachers themselves, who are too indifferent to organise such an Association, and secondly I blame the Training Colleges. The latter seem to be staffed for the most part by those who do not have first hand knowledge of the teaching problems that confront us in the modern schools.

He explained his latter remark by referring to the type of summer courses provided for teachers by the Training Colleges, such as Country Dancing and Rural Gardening, and the complete lack of courses such as The Best Way of Teaching Logarithms.

4.3.7 A new generation of teachers

Dr Inglis touched on something important here. He called the teachers indifferent, and painted a vitally different picture to that of the Victorian schoolmaster of 1883. The teachers' dedication to their chosen field, to mathematics outside the teaching sphere seemed to be diminishing. The teaching colleges were presumably not so much to be blamed for this change as Inglis might think; Muir's talk in 1884 showed that the Scottish universities at the time were no better at encouraging individual study or research than the colleges seemed to be in the 1930s. The profession would appear to attract another kind of person than before, and there may be several reasons for this.

Firstly, the great expansion of the mathematical departments had created more academical jobs. Other areas had expanded too, so a student wanting to work with mathematics now had a lot more options than in 1883. Secondly, the attitudes of the

[27] According to Etherington, Professor Whittaker had approached Dr Inglis, explaining his views on the role of the teachers, most notably in connection with the Colloquium. This is not the only sign showing that Whittaker was very much involved with the Society's affairs, even after his election as an Honorary Member.

students in general might have changed.[28] Inter-war Europe was not a stable world, and it would only be natural for many in such times to focus more on job-security than personal advancement in a field.

In addition to this possible change of attitude, the contemporary mathematics was getting less accessible and less relevant to the teachers. The decline of Euclidean geometry, and the rising level of current research made it harder for the few who wish to stay in touch with the researching world to do so. Another Mackay, or another Muir would presumably have managed anyway, but as mentioned, they would now most likely be working elsewhere.

[28] Professor D'Arcy Thompson and Dr G. T. Bennett, who were introduced in chapter 3, talked about this in 1940. Sir D'Arcy wrote that the universities produced no one distinguished, because the students did not care about improving themselves, only about securing a steady job.

> It is just an extreme case of that lack of joy in one's work, that impatience to be done with it, that I am up against with my own students. I have, one way or another, about 100 students on my hands this year. Two of 'em are enjoying their work, doing their best, reading in odd hours, borrowing books, learning about beasts. Only two.

Sir D'Arcy was obviously talking about his own students, and not mathematics students, but there is no reason to believe such sentiments to be limited to students of zoology. Dr Bennett, who was a mathematician, found himself in agreement with this, and this could certainly hold for the students at large. These letters can be found in the D'Arcy Thompson Collection at the University of St Andrews, ms 26238-26240.

4.3.8 Conclusion

Although some teachers were still present and active in the 1930s, it is clear that the processes that eventually led to their departure altogether had already begun. The Society's increasing focus on specialised research, and the teachers' decreasing interest in research were two major contributors to this.

The time has come, then, to summarise what made the Society turn towards research. Professor Whittaker's influence can hardly be exaggerated here, though he cannot take the full credit, or the full blame. His arrival in Edinburgh had an enormous impact on the university and the research done there, which obviously had an effect on the Society, centred as it was around the University. He cannot, however, be given the credit for the expansion of the other Scottish Universities. The University of Glasgow, and the University of St Andrews, and the rest, had grown quite a lot, giving many more local academic members, members who took a greater interest in the Society than the teachers did. It is also worth mentioning that although Whittaker's arrival surely helped, the number of academics in mathematics at Edinburgh begun increasing before his time.

When Whittaker wrote to Arthur that the Society was already a research society, this was broadly speaking true. Perhaps this was a result of Whittaker's actions, but the timing is essential. The switch in the *Proceedings* towards a pure research periodical can be said to have begun in the early 1920s which was after the teachers' involvement with the Society had started to decline. It is of course possible that Whittaker had a hand also in this, by for instance fighting attempts to recruit more teachers. This would be taking it too far, however, as he was not hostile towards the teachers by any stretch of imagination. It is far more likely that the lack of teacher interest was a result of the war, along with the increasing levels of mathematics, and that Whittaker merely saw the opportunity that presented itself.

And was he so entirely wrong in doing so? Did he, and his fellow academics, force the Society in a direction it was not suited for? The answer must surely be no. As claimed in chapter 1, the Society was founded to continue where the University education left off, which is perfectly in line with what Whittaker wanted it to do. The large percentage of scholastic content in the early days was not the ultimate goal for the Society, it was largely a result of having many teacher members, teachers who worked to promote the mathematics in Scotland. When the mathematics in Scotland moved into new pastures, it was only fair that the Society followed suit. Sir Thomas Muir, who dreamt of the

Society becoming as successful as the London Mathematical Society, would most likely have been rather pleased.

This did not mean, however, that there could not be room for teacher members anymore, even if it was in the spirit of the thing to focus on research. The Committee was certainly willing to try keep them, so why did they fail? The more general programme of session 1932 did not work exactly as hoped, possibly because G. Lidstone was right when he said the talks were too general. Another explanation is simply that the teachers were for the most part based outside Edinburgh, and with a few, rare, exceptions, could not attend the meetings on a regular basis. The *Notes*, or an improved successor, would have had a much higher chance of success. This failed, however, when the new journal was abandoned for good. A lack of material may have exacerbated matters, but it was financial difficulties, and the Committee's choice to prioritise research, that truly prevented the periodical from succeeding. As the teachers retreated more and more from Society life, the will to keep the *Notes* going diminished.

In light of the interest in Bell numbers, as shown in chapter 3, perhaps the key lies in the Society's profiling of the *Notes*. Whenever the *Notes* are discussed in the Committee, it is treated as a journal meant for teachers, and academics are approached to provide the teachers with papers. As it was, the content did not really fit the shoes of a 'teacher journal', as it was broader than one would expect. Perhaps more academics would have been interested in submitting papers if the periodical had appeared more far-reaching than it did.

Chapter 5

The Higher Education of Women

5.1 Women in the early society

The women in the earlier days of the Society were few and far between. This was not due to any prejudice on the Society's behalf, but rather because very few women had the necessary mathematical background to take an interest in the Society. This will be discussed further in the following section, but before that, a short assessment of the Society's women might prove interesting.

Figure 5.1 shows the number of women members for the first 64 years of the Society's existence. The first four women to join were Flora Philip (1886), Jessie Chrystal Macmillan (1897), Charlotte Angas Scott (1897) and Margaret Forrest (1898).[1] They were the only ones to do so before 1900, but after that, the numbers began rising. The peaks of the graph in the early and the late 1920s could possibly be a result of the Society trying to recruit more women, because these peaks coincide nicely with the only appearances of women on the committee during these 64 years.

The women's participation in the organisation of the Society began modestly in session 1918 with Eleanor Pairman, who served on the rather short-lived Library Committee. With the exception of session 1919, the Library Committee was to have one or two women on it for the rest of its existence, which ended in session 1923. The first woman was elected onto the ordinary Committee in 1919, and the following decade saw a surprising number of female Committee members.

This ended abruptly in 1931 and no other woman was elected to the Committee dur-

[1]Chrystal Macmillan and Charlotte Angas Scott were relatively well-known in their time, Macmillan as a suffragette and Scott as a mathematician. More information on their interesting lives can be found at [58] and [35], respectively.

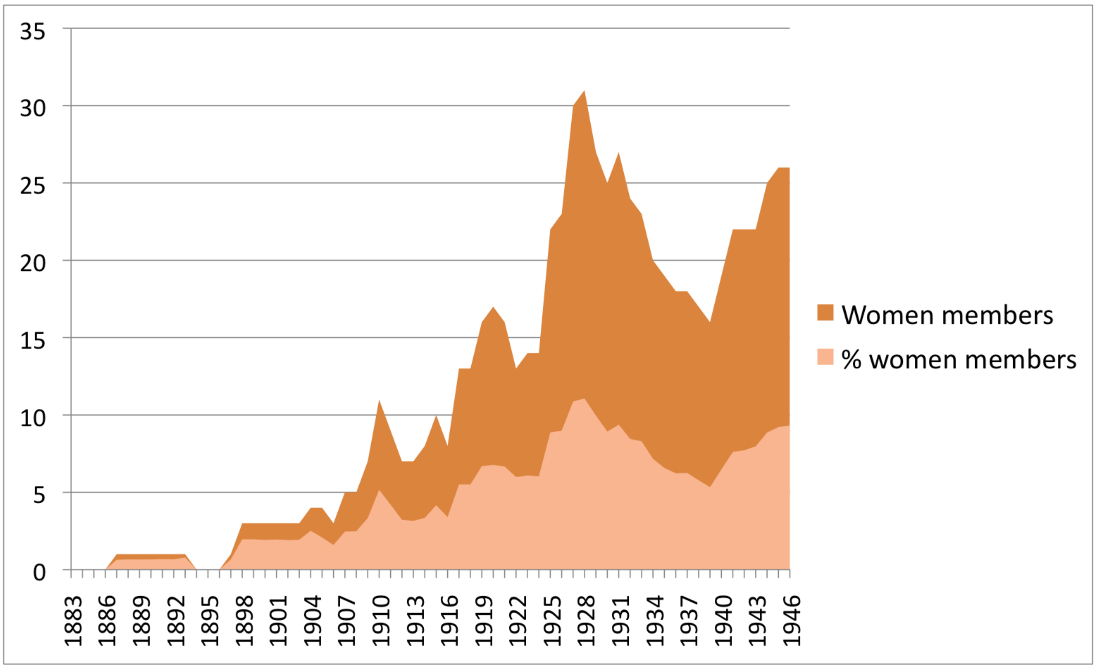


Figure 5.1: Women in the Society, 1883–1946

Table 5.1: Women on the Committee, 1883–1946

1920–21	Margaret P. White	Academic
1921–23	M. A. Dallas	Unknown
1922–23	Gertrude Pagan	Academic
1924	Marion Gray	Academic
1928–30	Lilias McGregor	Teacher

ing these 64 years. The year 1931 could indicate that this is related to the controversy of 1931, covered in chapter 4. That debate could certainly help explain the decline of female members in the 1930s, as many of them were teachers in the secondary schools. In that light, it is not unreasonable to assume that a female teacher would have been uninterested in joining the Committee, but there were female academics around as well. Whether these were uninterested in serving on the Committee, or simply not asked, is not known.

There were even fewer female authors. Only three, as far as has been established, were published in the *Proceedings* before 1946, and only two in the *Notes*. However, as many authors in both periodicals are identified by initials and surname only, it is possible that there were more female authors than this.[2]

[2]More information on the two authors Eleanor Pairman and Marion Gray can be found in [77] and [78], respectively.

Table 5.2: Female authors in *Proceedings* and *Notes*, 1883–1946

		The Proceedings
Eleanor Pairman	—	'On a difference equation due to Stirling', Vol. 36 (Series 1), 1918, pp. 40–62
Marion Gray	—	'The equation of telegraphy', Vol. 42 (Series 1), 1924, pp. 14–28
	—	'Particular solutions of the equation of conduction of heat in one dimension', Vol. 43, 1925, pp. 50–63.
Olga Taussky	—	'Matrices with finite period', joint paper with John Todd, Vol. 6 (Series 2), Issue 03,1940, pp. 128–134
		The Mathematical Notes
Nancy Walls	—	'An elementary proof of Morley's trisector theorem', Vol. 34, 1944, pp. 12–13
Agnes H. Waddell	—	'Curves formed by colonies of micro-organisms growing on a plane surface', Vol. 35, 1945, pp. 14–19

It is also worth mentioning that women took a great interest in the mathematical colloquia. The group photograph of all the participants from the first to take place, in 1913, is shown in figure 5.2. Of the 75 people in the picture, 7 are female. The group photograph of the colloquium in 1926 contains 79 people with 26 female (Fig. 5.3). Although some of these were the wives and daughters of the male participants, most of them were members of the colloquium.[3]

Most of the female members may be presumed to have had a university degree on their election. The first woman, Flora Philip, did not, but still had the necessary mathematical background. The following section, which appeared in the British Society for the History of Mathematics Bulletin [39], contains the results of the investigation into how and where she received this education.

[3]The version of this photograph found at [67] includes the names of the people in the picture.

Figure 5.2: The Edinburgh Mathematical Colloquium 1913

Figure 5.3: The Edinburgh Mathematical Colloquium 1926

5.2 How Flora got her Cap

This section is based on a lecture given at the History Splinter Group at the British Mathematical Colloquium in York, March 2008. The Edinburgh Association for the University Education of Women was an association providing women with higher education in a time when they were denied access to the Scottish universities. The section will give an outline of its history before assessing the performance of its mathematical class.

5.2.1 Flora Philip's love of mathematics

While studying the early days of the Edinburgh Mathematical Society, I came across its first female member. When the Society was founded in 1883, its members counted only men. This was hardly surprising, as it was intended as a society for university graduates, and women would not be allowed entrance to the Scottish universities for another six years. And yet a 'Miss Flora Philip' joined the Society as early as December 1886 [89, 10 Dec. 1886]. Miss Philip had no university degree at that time. This puzzled me; why would a woman, or indeed anyone, without proper mathematical training wish to join a society dealing in advanced mathematics? What could she possibly have to gain?

She must have had some proof of proficiency, or she would simply not have been allowed to join, but what? There were women's colleges in England, and Queen Margaret's College in Glasgow, but it turned out Miss Philip had not studied at any of these. My slightly unrealistic surmise that she had merely had an exceptionally skilled governess was proven wrong by a box labelled 'Information on the first women graduates of the University of Edinburgh', standing on top of a shelf at the Special Collections of Edinburgh University Library. Miss Philip was in fact awarded a degree by Edinburgh University, but not until 1893, when she and seven other women graduated M.A. [11]. They had not been allowed to matriculate until 1892, which left less than one year between matriculation and graduation. A governess could not possibly explain that much, so how did they do this? And where had Flora learnt enough mathematics to want to participate in the EMS?

The Association

What the eight women had in common was their attendance of the Edinburgh Association for the University Education of Women (EAUEW). This Association provided women with advanced teaching on various university topics, and the eight women of Edinburgh University had all studied under it.

The Association was established in 1867.[4] There was at that time no option but to leave for England for a woman who wished to pursue her studies beyond school level. This was what the Association's six founders wished to remedy when they met on the 15 October that year to found their Association.[5] One of the founding mothers stands out; Mrs Mary Crudelius (née Maclean), their first secretary, was the initiator and main force behind the movement. As explained in her biography [36], she was born in England, but her parents were Scottish, and she was sent to Edinburgh for schooling. This was where she met the German wool merchant Rudolph Crudelius and they eventually married and settled down in Leith. Mrs Crudelius had strong opinions on the subject of women's education and women's rights, but she was as far removed from militant feminism as it was possible for a feminist to get. She was described as a 'highly intelligent woman, possessed of great enterprise and much tact' [11, pg. 227]. These personal traits would greatly influence how the Association worked.

The six ladies began amassing supporters immediately. They drew up a prospectus to explain their goals and aims [27, 1867, pg. 7]:

> That this Association shall have for its object the advancement of the higher education of women and that its first work shall be the establishment of a high-class lecture scheme. That future operations shall not be decided upon, as time will show what educational wants are the most pressing.

The high-class lecture scheme was a very clear objective, and it was to become a great success, but the rest of the statement is rather vague. This was purposely done, and while perusing the rest of their prospectus you will not find what — to modern eyes — would have been natural to find there; a demand to enter the universities. They were in fact so good at avoiding this topic in general that it is not entirely clear if it was the Association's goal from the very beginning or not, though it was certainly the ultimate

[4]The history of the Association is described in full in [38], [37], and also in [10], the latter a pamphlet issued by the Association itself for their 100th anniversary. Most of the information on the general history of the Association is taken from these three sources.

[5]At least seven women were present, but the young and unmarried Sarah Mair was not considered to be a founder member.

goal for several of the key players, such as Professor Masson and Mrs Crudelius herself [36, 50]. Yet they kept quiet about it, and for good reasons. Mrs Crudelius and the others knew their contemporaries well. They feared that openly demanding entrance to universities would make them lose support. The other campaign in Edinburgh was to show how real this fear was.

Controversy

Sophia Jex-Blake, the well known feminist who would become one of the first female doctors in the UK, was soon to arrive in Edinburgh and begin her campaign for female medical students.[6] She and six other women, who Jex-Blake referred to as 'the Edinburgh Seven' [29], succeeded half-way, and were allowed to matriculate at Edinburgh University in 1869. Before they could graduate, however, it was decided that it had been illegal for the University to admit them in the first place, and so the seven found themselves out in the cold again. Jex-Blake and the rest had received much support, but also much resistance, including the infamous 'Surgeons' Hall Riot', where male students tried to physically prevent the women students from attending an examination. This sort of controversy did not suit the tactful Mrs Crudelius.

What gave the medical campaign such resistance was the idea of women entering the professions, an idea that did not go well with the Victorian feminine ideal. The Association was aware of this, and stated quite clearly that women in professions was not their aim at all. They presented other reasons for why women had a right to study, focussing on the benefits of mental training, the broadening of the mind, and — as one of their professors was to put it — the 'preparing [of] the mind for the after life' [82]. This careful approach had its effects. Within three months they had gathered no less than 160 members, 81 of these honorary ones, consisting of distinguished gentlemen who gave their support. The list is rather impressive, containing almost all of the professors of Edinburgh University — including the principal, Sir David Brewster — and also several ministers of the church, three lords, various heads of private schools and even an admiral.

By distancing themselves from the medical campaign, they managed to secure supporters even amongst Jex-Blake's enemies. Professor Robert Christison of Edinburgh University was one of Jex-Blake's fiercest opposers, being described as a 'rabid opponent of female doctors' [10, pg. 4], but he was also an honorary member of the ELEA.

[6] A more complete account of this campaign and the life of Sophia Jex-Blake is given in [66].

He even offered to give a course on dietics to them [15], though this never happened.

Even though they enforced this distance, it would be entirely wrong to say there was no connection between the two campaigns at all. Helen de Lacy Evans was one of the ELEA's founding mothers and also one of the Edinburgh Seven. Jex-Blake was a student of the Association, and Professor Masson supported both campaigns.

Professor David Masson, who held the chair of Rhetoric and English Literature, was one of the Association's earliest and strongest supporters. He took on a mentor role for the women in the Association, guiding them carefully through the political landscape of the university. Although openly confessing his belief that women should have the right to study at universities, he was very well aware of the controversial aspect of this, and did not encourage the Association to do the same. He became their first lecturer and opened their very first term with a course on English Literature in January 1868. The secretary reported this as an astonishing success, with over 400 ladies attending the first meeting, and 265 signing up for the course itself. The course was to become the most popular course of the Association, with a total of 2100 students up to 1892.[7]

The University structure

The performance of the ladies was not at all discouraging, and Professor Masson argued there was no time to lose in expanding their curriculum [50]:

> It seems clear that we must, next session, have an addition of classes & subjects. If after the mere pioneering of this session, we do not establish a more considerable nucleus of an Arts Faculty we shall have lost all & it shall deserve to be called a failure.

Their choice of subjects was in other words not random. The Association wished to provide an education resembling that of the M.A. of Edinburgh University as closely as possible and offered courses accordingly. Although other subjects did enter their syllabus, such as Biblical Criticism in 1872, this did not happen until they considered the core university subjects to be sufficiently established. The next two courses to see the light of day were Experimental Physics, given by Professor P. G. Tait and Logic and Mental Philosophy, by Professor Fraser, both given for the first time in 1868–69. Other courses followed, such as Moral Philosophy, Greek, Latin, Botany, and Chemistry.

[7] This is explained in [26, 24].

These were all given by university lecturers, usually by the professors themselves. At the wish of the Association, the professors shaped their respective ladies' courses by the university mould. This was done as closely as possible, albeit with an unavoidable reduction in number of lectures. The Association provided two lectures a week for 20 weeks. In mathematics, the university courses consisted of over 100 lectures each year, so the difference was quite substantial [102]. In addition to lectures, the professors gave homework in the form of reading or problems for solutions, and examinations were held several times a year. This work was not made compulsory, and many women were satisfied by just attending the lectures.

The lecturing hours put some restrictions on the student body. Since these were always given during the day, teachers, governesses and other women who had to support themselves through work were prevented from attending, to the Association's regret. Seeing as many housewives had very little time to pursue their studies, the larger part of the student body would have consisted of unmarried women of independent means, and younger women who still lived with their parents. Occasionally, housewives would sign up for classes they could not possibly attend, merely to support the Association with their fees. The secretary reported from the first session that the age of the students ranged from sixteen to sixty, if not older, but the main bulk lay somewhere between twenty-two and thirty-five.

The progress and performance of the women were assessed by the professors and their assistants in the Annual Report of the Association. These reports were in general all very favourable, in fact so favourable that one may suspect a certain partiality on the professors' behalf. Such a notion is strengthened by one report in particular, as explained in [12]. William Robertson Smith, later known as a theologian, was Professor Tait's assistant in 1868–69. His first version of his contribution to the Annual Report was so blunt regarding the shortcomings of the women that the Association was enraged. Although he had given them high praise as well, Smith was strongly encouraged to rewrite his report, which he did. An extract would explain why this new version did not succeed in soothing any hurt feelings [27, 1868–69, pg. 18]:

> There were one or two very good students at the top of the class, who were in every respect equal to the best University men; but below these there was a rather rapid descent to painstaking mediocrity.

However, he had retained the praise he had given as well, which was quite substantial, and he was told privately that his report was indeed the truest and the most favourable'

[86]. His was only one report, amongst many strictly positive ones, but it did show that the image was more diverse than the professors conveyed. One may of course argue that a certain amount of 'positive discrimination' on behalf of the professors may have been expected and indeed necessary, considering the women's great disadvantage of poorer schooling.

Certificates

The Association issued their own certificates for each course, but wanted to provide their students with something more substantial. They contacted Edinburgh University, in the hopes of establishing a proof of proficiency with more merit. After much discussion, the university agreed in 1872 to issue certificates to the women students. A pass in three certificate examinations in selected ELEA subjects would grant a woman the ordinary certificate, whereas passing an advanced exam in one of her chosen subjects would grant her the honours certificate. From the first in 1874 to the end of the scheme in 1893, 138 ordinary and 36 honours certificates were awarded. The Association itself also awarded an Association Diploma to fourteen women who had passed seven examinations or more. This was the closest alternative to an M.A.

The form of the certificate examination caused some discussion. According to a memo by Professor John Wilson [106], the ladies originally requested to be given exactly the same examination paper as the university graduates, but this was denied by the *Senatus Academicus* [105]. The *Senatus* felt it would be impossible to comply with this, on account of the vastly reduced number of lectures in the Association. They did agree, however, that the university standard should be applied to the women's examination papers, but they should only cover parts of the areas covered by corresponding university papers. The women were very disappointed, and felt they could not achieve their goal of a true alternative to the M.A. before they were tested on equal grounds. Their arguments did not persuade the *Senatus*, and although it did happened on a few occasions that the women were given the graduate examination paper, it was never the rule.

Even if the certificates were not ideal in the Association's eyes, they helped strengthen the bonds between the Association and the University. Details on the ELEA courses eventually appeared in the Edinburgh University Calendar. In 1879, the Association took even further steps towards the University, by renaming itself the Edinburgh Association for the University Education of Women (EAUEW) and so reflecting their ultimate goal in their name. By this time Mrs Crudelius had passed away. She died in 1877 after long periods of illness, which were not improved by her hard work for the Association.

The remaining women and their supporters lobbied hard for their cause and saw the fruits of their work in the passing of the Universities Act (Scotland) of 1889. This provided the change of law needed for the Scottish universities to admit women, and the University of Edinburgh was one of the first to do so in 1892. The Association's certificate examinations were then accepted as replacements for the university examinations, and women who had passed one of these were not required to sit for the corresponding exam at the university, hence allowing Flora Philip and the others to graduate with record speed.

The Association then turned its focus to other concerns, such as improving elementary education and providing halls of residences for women students, and continued to promote education for women until its end in the 1970s.

5.2.2 The mathematical classes

Mathematics was of great importance to them, and was offered for the first time in the third session. The subject was a mandatory part of the M.A. in those days, and a student at the Arts Faculty had to study mathematics for two years (or equivalent). An ELEA course on mathematics was therefore needed to complete their alternative M.A. Further, mathematics was considered to be 'lying altogether outside the domain of a lady's thought and studies' [27, 1869–70, pg. 11]. Women were said to be unfit for mathematics, and hence for university studies, so the Association had a point to prove. It would be difficult for their opposers to maintain that claim if the women demonstrably mastered the subject. Professor Masson even judged it to be so important that not providing it as soon as possible would be downright harmful to their cause. He wrote to Mrs Crudelius that 'until we add Mathematics we keep our case weak' [50].

A successful course on mathematics would therefore aid their cause greatly, but it was a double-edged sword. Failure would be disastrous and would fuel the argument of their opposers. This was a well-founded fear, for the ladies were very ill-prepared. According to the Professor of Mathematics at Edinburgh, the convictions mentioned above had 'banished mathematics from the ladies schools' [82], and they feared that few women would dare sign up for a mathematics course and that those who did would perform poorly.

They discussed various ways of countering this lack of mathematical schooling. Professor Masson and Mrs Crudelius hoped that they could encourage the ladies' schools into teaching more mathematics by announcing an intended course a few years ahead in time [50]. Others felt it would be better to provide the students with a preliminary course and help them pass the Local Examinations before arranging a proper ELEA course.[8] The latter was done in the end, through the St George's Hall Classes, but not until after mathematics had been introduced. The committee members of the Association did not agree on the best course of action, but they all agreed something had to be done before they could give a proper course, and the committee meetings for the winter of 1868 showed that they had no intention of expanding their curriculum for the following term [28].

Their change of heart was largely a result of a direct request from the ladies themselves, hence convincing the committee that a sufficient number of students would come forward. When the request was granted, it was presumably also a result of Professor Tait's success with his course on Experimental Physics, which was considerable, even when W. R. Smith's report was taken into account.

Professor Philip Kelland

And so they approached Philip Kelland, the Professor of Mathematics at Edinburgh.[9] He was more than willing to lecture to the ladies and was to become a strong supporter up until his death in 1879. Born in England in 1808, he was educated at Cambridge and tutored by William Hopkins, before his appointment to the chair of mathematics at Edinburgh in 1838, his competitors being Duncan Gregory and Edward Sang. There, he became known as a university reformer and a superb teacher, his one sin being a tendency to progress a little too rapidly for the average student, who had a lower 'standard of attainment' than Professor Kelland thought to be the case [83]. This was not to become a problem for the women, since he assumed very little on their behalf. Professor Kelland made less of a figure as a researcher, but as a teacher he was kind and inspiring and a well-suited coach for the ladies.

His introductory lecture was given on 11 November 1869, and The Scotsman summarised it on the following day. Professor Kelland expressed strong views on the subject of women's education. He went about it in a very organised manner, first discussing

[8] The Local Examinations were the entrance examinations to Edinburgh University. They had been open to girls from 1865, even if University was not.

[9] This biographical information was found in [19].

whether mathematics was suitable for women and then if it was necessary, before discussing the desirability, and finally the attainability of it. He argued that the intellectual division between men and women was 'like Goethe's giant, which had no strength except in its shadow' [82], a cultural division caused by differences in education alone. He felt that mathematics was particularly useful for the aforementioned broadening of the mind. On commenting upon the supposed masculinity of the subject, he argued that mathematics had about it 'a compactness, a symmetry of form, a beauty of feature that made him feel disposed rather to say that mathematics was essentially feminine' (ibid.).

Kelland's mathematical class

Thirty-five of the women present at his introductory lecture signed up for the course itself. Professor Kelland's contributions to the Annual Reports of the Association described the structure of his teaching. As the other ELEA professors, he strived to keep it on university lines, with weekly problems for solution and three to four exams a year. These exams were to continue even when the certificate examinations were established. The professor expressed in his first report some regret that the *vivâ voce* portion of the course had to be reduced, an unfortunate consequence of the compression of 200 lectures into 40, which also enforced a certain reduction in the syllabus.

What he taught is reflected in what he set as topics for the certificate examinations. These were printed in the Annual Report as soon as the certificate scheme was in place.[10] The ordinary examinations were to consist of questions on the first six books of Euclid and also algebra up to quadratic equations. The honours examinations were to test the students on advanced algebra, trigonometry, conic sections and Newton's *Principia*, Sec. 1. The reports also included synopses of the courses for the following year, and these synopses match the 'heads of examination' pretty well. However, there is reason to believe that what he announced was a simplified version of what he actually intended to do. The synopses always said that the development of algebra would depend on the students, and so was not fixed, and yet he always seemed teach up to and including quadratic equations, if not beyond. The synopsis for the year 1872–73 was not larger than usual, and yet Professor Kelland confessed in his report for that year that he had set them all questions on trigonometry [27, 1873–74, pg. 13]. Considering the low number of 'honours students' in later years, it seems very unlikely that all the students

[10] The Annual Reports and later the calendars were eventually bound together in three volumes, [27, 22, 23].

that year would have sat for the honours examination. It's therefore more likely that he chose to 'play it down' a little when announcing the course in order to soothe the fears of ladies who felt unprepared.

When comparing the synopses to the university ones, the reduction in syllabus becomes obvious. As described in the university calendars for these years [102], there were three university courses of mathematics, the first two being mandatory. The calendars also included the synopses of the courses, and the first two combined included not only analytical trigonometry, but also geometrical and analytical conic sections. These were areas not even the honours certificate of the ELEA would cover.

Considering how the synopses could be misleading, it may be wiser to turn to the examination papers themselves for more reliable data. The Annual Report sometimes included the examination questions and on comparing these to the graduate examination questions — published in the university calendar [102]— most of the impressions from the study of the synopses are strengthened. Joint and partly joint questions did appear, but they were rare, and most of the time the graduate examination questions were, as expected, more advanced than the certificate ones. The exception was the exam set in 1878, when the men and women were given exactly the same paper. This seemed to be slightly simpler than the average university exam, but it still extended the Association syllabus by far.

It is a bit puzzling that Kelland moved from giving one or two joint questions per paper to giving exactly the same paper, but it is not impossible. According to the Association's examination records [25] only two women — of seven students — passed the certificate examination that year, and no fails were recorded, so presumably only two presented themselves for examination. Creating a separate exam for these two students might have seemed more trouble than it was worth for the elderly Kelland. He may have found it simpler to set the same exam and perhaps ask the women to answer only a selection of the questions, if not all. Another possibility is that the low number of students allowed him the opportunity of greatly extending the syllabus for that particular year, and such a change would not appear in the announced synopsis.

Seven students were not many. The Annual Report also included attendance records, and the thirty-five from the first session was the highest attendance the course would ever have [23, 1888–89, pg. 26]. The number of students dropped rapidly, with only seventeen in the second term and then none; too few women had come forward and the Association felt it could not ask Professor Kelland to lecture. This pattern repeated itself twice over, seventeen students, then none, seventeen, then none, before the Association decided mathematics had to become a regular course, if it were to become a serious institution of education. From 1876 the course was there to stay, though occasionally with as few as six students.

One of the initial fears had therefore proven real, few women were tempted to do mathematics. Some did not dare trying, others were not allowed by their families, who sometimes believed it would be too stressful for their daughters [38]. The other fear, however, was removed; those who did sign up did very well. Though Kelland was always disappointed at the low number of students, he was always pleased with their performance. He wrote in the report from the second session of mathematics, on commenting on the averages received on the examinations that year [27, 1870–71, pg. 15]:

> These averages are high, — much higher than could have been hoped for,— much higher than found in ordinary Classes of Students who have devoted only five months to the study of the subject. The zeal and industry of the Class doubly compensates for the smallness of its numbers.

His reports gave a picture of students that were hard-working, motivated and persistent. The number of 'working students' were included in the Annual Reports for all subjects for the early years, and the percentage for mathematics varied from 70% to 94%. This was not that much higher than the average for the other classes, which lay between 60% and 70%, but getting classified as a 'working student' was easy. It sufficed to hand in one of all the essays or problems, without attending a single exam, and so the amount of work the 'working students' did, varied greatly. This was where the mathematical class truly stood out, having near perfect attendance at the examinations.

Their zeal also showed in the lecture records [26]. These, although incomplete, give the number of students with perfect attendance. The data covers only 1868–71, giving information for two of the three years for each of the courses given during that period. The norm was around 14% with perfect attendance, but the second year of mathematics had 47%, which was the highest recorded. The second place was held by the first session of Experimental Physics with 28%.

The course continued in a pretty much settled form, with a steadily decreasing number of students before reaching an all-time low of six in Kelland's final year. He passed away in the summer of 1879 after a period of failing health, but kept up his devotion to his students to the very last.

Professor George Chrystal

By October that year, he was succeeded by George Chrystal.[11] Professor Chrystal was born near Aberdeen, and attended the university there before going to Cambridge. He had held the chair of mathematics at St Andrews for two years prior to his arrival at Edinburgh. Chrystal was, like Kelland, a brilliant teacher and reformer. He was heavily involved in the setting up of the Scottish Leaving Certificate, but he was also a good researcher, and was to become well-known for his textbook on Algebra. Being twenty-eight when arriving at Edinburgh, he represented quite a change from the elderly Kelland.

There is little information on his first year of teaching the newly renamed EAUEW, but his second term introduced a senior course on mathematics. This was intended for those who wished to sit for the honours examinations, and who had already done the ordinary work. The senior course was given with semi-regular intervals, until it was cancelled in 1887–88. This system encouraged re-attendance of the class, something which was discouraged in other classes. Flora Philip, for instance, attended no less than three sessions in a row, first a senior class, then a junior with ensuing ordinary certificate examination, before finishing with another senior class and an honours certificate in 1886 [24].

Chrystal's mathematical class

The synopses for the junior and senior courses were to reflect the requirements for the ordinary and honours examinations respectively. If Professor Chrystal moved beyond what Professor Kelland had done, or if he was merely more honest in his descriptions is hard to say, but the matter of fact is that the synopses expanded gradually until the ordinary course equalled approximately half of the university requirements and the honours course took care of most of the rest.

The most common version of the topics for examination now read [23, 1885–86, pp. 34,36]:

> Ordinary Certificate Examination: Arithmetic, Euclid, Books I., II., III., IV., and VI., or their equivalent; Algebra to Quadratic Equations, including the Progressions.

[11] This biographical information is found in [71].

> Honours Certificate Examination: Euclid, I., II., III., IV., VI., and XI., or their equivalent; The Advanced portions of the Elements of Algebra, Trigonometry and Conic Sections, treated geometrically and analytically.

For comparison, the topics for the graduate examination were as follows [102, 1885–86, pg. 111]:

> Pass Examination: Arithmetic, Euclid, Books I., II., III., IV., VI., and XI or their equivalent, with their application to Mensuration. Algebra, Trigonometry, Conic Sections, Geometrical and Analytical.

The topics for examinations may not give a correct picture, since they do not indicate how much Trigonometry was taught in each case, but the examination papers were more accurate. For the ordinary certificate papers between 1885 and 1888, roughly half of the questions were found in the graduate pass examination as well. A similar study of the honours paper did not give the same amount of 're-used' questions, but the areas covered and the difficulty of questions seemed to match the rest of the pass examination pretty well.

The trend of devoted mathematical students was there to stay. Just before Kelland's death, the Annual Report of the Association was replaced by a calendar, resembling that of Edinburgh University, and the reports from the professors ceased to be printed. There are no reports by Chrystal, but there are other ways of assessing the students' zeal and diligence.

Records for the certificate examinations were kept in the 'Certificates in Arts' from 1876 until they ceased to be held in 1893. During these years, 149 students signed up for the mathematical class [23, 1888–89, pg. 26].[12] 45% of these sat and passed the ordinary certificate examination. The next subject on the list worth comparing with is Logic and Mental Philosophy with a pass percentage of 30.[13] The corresponding number for English Literature, with 1217 students in this particular period, was 10%.

It should be added that mathematics also had the highest recorded number of fails, with seven students failing their exam during these years. However, the complete lack of recorded fails from most of the other subjects may indicate a failure to record them.

The distinction between the mathematical class and the other classes becomes even more apparent when we look at the honours certificates. Of the thirty-six honours

[12] At least twelve of these were women who attended the class for a second or third time, but the original number is used in order to justly compare it with other subjects, where the such re-attendance has not been counted.

[13] Chemistry had 33%, but this course was given only twice between 1876 and 1893.

certificates that were awarded, ten of these had Mathematics as the special subject, and six of them achieved first class. This becomes even more impressive when it is added that these are the exact same numbers as for English Literature, who had a student body more than eight times as large. No other class managed to get close to such scores, except for Latin, with 163 students and four honours certificates, three of which achieved first class.

5.2.3 Conclusion

It is very tempting to agree with Professor Kelland and say that the lady students were indeed unusually good. Their reasons for being so were probably as diverse as the students themselves, though some general explanations may be given. It was not only Victorian men who believed women were unfit to do mathematics; many of their female contemporaries agreed, and passed such beliefs on to their daughters. With such ballast, it would take a little extra for a woman to take on the subject, for instance an unusual skill at it, or a strong desire to prove her contemporaries wrong, both of which were properties likely to enhance her performance. The low number of students was therefore very likely a contributor to the excellent results; it removed what would otherwise have been the bottom half of the class. A student who doubted her ability would simply avoid taking it.

Her freedom to do just that can also explain why she did so much better than expected compared with the university students. Not all men had an affinity for mathematics, but they all had to study the topic for two years if they wished to graduate. It is easily believed that a woman with a love for the subject could outdo a man who loathed it, even when he had many more years of experience than her.

More technical matters would also influence the result. The classes were smaller, allowing the professor to spend more time on each student, which, judging by the excellent reports on both Kelland and Chrystal, must have served as an encouragement. The later system with junior and senior classes would also encourage more students to aim for the honours certificate.

Was the University right in accepting the EAUEW ordinary certificate as proof of proficiency in the way they did? A detailed study of the university students would be required to answer this question fully, but a few aspects have become clear. The ordinary certificate examination was inferior to the university one, but that need not mean the women students would not have passed the university exam. No examiners expect their students to fully master every aspect of a syllabus in order to let them pass. Although the women were not taught as much as the men, when taking their hard work into account, saying they were taught enough does not seem like an outrageous claim.

As for Flora, Boog-Watson described her career [11] in his article on the first women graduates. Before her graduation, she had already worked for several years as a teacher, at the St George's School for Girls that the Association had set up. She married the solicitor George Stewart in 1893, and withdrew from academic life and the Edinburgh Mathematical Society. She was the only one of the first eight graduates to get married.

Appendix A: Tables

This section contains tables with data for the graphs in this thesis. The first three of these tables regard the classification of papers in the *Proceedings* by subject. The categories have been labelled, for ease of print:

- 1 — History and pedagogics
- 2 — Arithmetic and algebra
- 3 — Analysis
- 4.1 — Elementary geometry
- 4.2— Advanced geometry
- 5 — Applied mathematics
- 6 — Calculus of observations

The papers in *Proceedings* by subject, 1883–1926

	1	**2**	**3**	**4.1**	**4.2**	**5**	**6**
1883	1	0	0	1	1	0	0
1884	1	5	0	10	3	0	1
1885	4	0	7	12	3	3	0
1886	4	3	2	5	4	4	1
1887	3	3	3	7	1	2	0
1888	1	2	1	5	2	3	0
1889	2	2	1	6	5	4	1
1890	3	3	2	4	3	4	0
1891	1	3	1	10	2	2	0
1892	3	1	3	4	3	5	1
1893	4	5	2	6	2	5	1
1894	2	4	2	13	1	0	0
1895	5	1	4	9	7	0	0
1896	2	1	4	7	3	1	0
1897	1	2	0	6	5	2	0
1898	3	4	3	5	4	5	1
1899	1	3	1	2	3	0	0
1900	1	2	3	9	1	3	0
1901	0	4	2	7	2	1	0
1902	0	4	3	8	2	3	0
1903	2	7	3	3	5	1	0
1904	0	4	6	7	0	1	0
1905	1	5	0	4	5	1	1
1906	0	6	1	6	1	3	0
1907	1	3	3	5	3	1	0
1908	0	6	0	6	2	1	0
1909	0	3	0	6	4	1	0
1910	2	2	2	4	9	1	1
1911	0	1	0	0	10	0	1
1912	1	2	4	0	7	2	2
1913	1	3	2	2	2	3	1
1914	0	2	11	5	1	2	0
1915	0	1	14	0	5	2	0
1916	0	5	9	2	4	0	0
1917	0	5	2	1	2	0	1
1918	1	2	1	1	4	2	1
1919	0	1	4	1	4	0	0
1920	0	1	2	1	1	0	2
1921	1	0	4	0	2	1	0
1922	2	0	2	0	0	1	1
1923	1	1	5	0	2	4	0
1924	0	4	6	0	0	1	2
1925	2	5	9	0	2	1	1
1926	6	0	5	0	4	4	0
	63	**121**	**139**	**190**	**136**	**80**	**20**

The papers by teachers in *Proceedings* by subject, 1883–1926

	1	**2**	**3**	**4.1**	**4.2**	**5**	**6**
1883	1	0	0	1	0	0	0
1884	1	2	0	4	1	0	0
1885	4	0	2	8	0	0	0
1886	4	1	1	2	1	2	0
1887	3	0	0	4	0	0	0
1888	1	0	0	3	1	1	0
1889	1	1	0	3	1	0	0
1890	1	0	0	1	0	0	0
1891	1	0	0	5	1	1	0
1892	2	0	0	2	0	0	0
1893	1	0	0	5	1	0	0
1894	2	2	0	10	0	0	0
1895	4	0	0	3	1	0	0
1896	1	0	0	2	0	0	0
1897	1	2	0	3	1	0	0
1898	1	1	1	0	0	0	0
1899	0	0	0	2	2	0	0
1900	0	0	0	2	1	0	0
1901	0	0	0	1	0	0	0
1902	0	1	0	2	0	0	0
1903	2	4	0	1	1	0	0
1904	0	2	0	2	0	0	0
1905	1	1	0	1	0	0	0
1906	0	0	0	2	0	1	0
1907	1	0	0	2	0	0	0
1908	0	0	0	0	0	0	0
1909	0	1	0	1	2	0	0
1910	0	0	0	1	1	0	0
1911	0	0	0	0	4	0	0
1912	0	1	2	0	3	0	0
1913	0	0	1	2	1	0	0
1914	0	0	0	3	0	0	0
1915	0	1	1	0	1	0	0
1916	0	0	0	1	2	0	0
1917	0	0	0	0	2	0	0
1918	0	0	0	0	1	0	0
1919	0	0	0	1	2	0	0
1920	0	0	0	0	0	0	0
1921	0	0	0	0	1	0	0
1922	0	0	0	0	0	0	0
1923	0	0	0	0	0	1	0
1924	0	0	0	0	0	0	0
1925	0	0	1	0	0	0	0
1926	2	0	0	0	0	0	0
Total	**35**	**20**	**9**	**80**	**32**	**6**	**0**

The papers by academics in *Proceedings* by subject, 1883–1926

	1	**2**	**3**	**4.1**	**4.2**	**5**	**6**
1883	0	0	0	0	1	0	0
1884	0	3	0	3	2	0	1
1885	0	0	2	3	2	3	0
1886	0	1	1	1	2	2	0
1887	0	2	2	2	0	1	0
1888	0	2	1	2	1	2	0
1889	1	1	1	3	3	3	1
1890	1	3	2	3	1	4	0
1891	0	1	1	4	1	1	0
1892	1	1	3	1	3	5	1
1893	3	5	2	1	1	5	1
1894	0	1	1	2	1	0	0
1895	0	1	3	2	4	0	0
1896	1	0	2	2	2	1	0
1897	0	0	0	1	1	1	0
1898	2	0	1	4	0	1	1
1899	1	1	0	0	0	0	0
1900	1	1	2	3	0	2	0
1901	0	2	2	6	2	1	0
1902	0	2	3	2	1	2	0
1903	0	0	2	0	3	1	0
1904	0	2	3	3	0	0	0
1905	0	0	0	2	3	0	0
1906	0	3	1	2	0	0	0
1907	0	1	2	0	2	0	0
1908	0	2	0	2	2	1	0
1909	0	1	0	1	1	1	0
1910	2	2	2	1	7	1	1
1911	0	1	0	0	3	0	0
1912	0	1	2	0	3	2	0
1913	0	1	1	0	1	1	1
1914	0	1	8	2	1	1	0
1915	0	0	9	0	4	1	0
1916	0	3	6	0	1	0	0
1917	0	3	1	1	0	0	1
1918	1	2	1	0	1	1	1
1919	0	1	2	0	1	0	0
1920	0	1	2	1	0	0	1
1921	1	0	4	0	1	1	0
1922	1	0	2	0	0	1	0
1923	1	1	4	0	1	3	0
1924	0	4	6	0	0	1	2
1925	2	5	6	0	1	1	1
1926	3	0	3	0	4	1	0
Total	**22**	**62**	**96**	**60**	**68**	**52**	**13**

	Education	Research	Presentation	Historical
1883	0	1	0	0
1884	3	4	0	1
1885	2	5	1	3
1886	2	5	0	2
1887	3	0	0	2
1888	0	4	1	1
1889	1	1	1	0
1890	0	0	0	1
1891	1	4	0	1
1892	0	0	0	2
1893	0	4	0	2
1894	3	8	2	0
1895	1	4	0	0
1896	0	2	0	0
1897	3	3	0	0
1898	1	1	0	0
1899	0	4	0	0
1900	0	2	0	0
1901	0	1	0	0
1902	1	2	0	1
1903	1	4	0	1
1904	1	3	0	0
1905	0	2	0	1
1906	0	2	1	0
1907	0	2	0	1
1908	0	0	0	0
1909	0	3	0	0
1910	0	2	0	0
1911	0	4	0	0
1912	0	5	0	0
1913	0	3	0	1
1914	0	3	1	0
1915	0	3	0	0
1916	0	3	0	0
1917	0	2	0	0
1918	0	1	0	0
1919	0	3	0	0
1920	0	0	0	0
1921	0	1	0	0
1922	0	0	0	0
1923	0	1	0	0
1924	0	0	0	0
1925	0	1	0	0
1926	1	0	0	0

The papers by teachers in *Proceedings* by type, 1883–1926

	1883	**1884**	**1885**	**1886**	**1887**	**1888**	**1889**	**1890**	**1891**	**1892**	**1893**	**1894**	**1895**	**1896**	**1897**	**1898**
Teachers	1	8	11	9	5	6	4	1	6	2	6	12	6	2	6	2
Academics	1	10	9	7	8	8	10	14	8	12	13	5	8	8	3	9
OHE	0	0	2	0	0	0	0	1	0	0	0	1	3	3	3	5
Students	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Other	0	0	1	4	2	0	1	1	1	0	0	1	1	1	1	0
Unknown	0	2	2	1	2	0	0	0	2	1	0	1	3	2	1	4
Total	**2**	**21**	**25**	**21**	**17**	**14**	**16**	**17**	**17**	**15**	**19**	**20**	**21**	**16**	**14**	**20**

	1899	**1900**	**1901**	**1902**	**1903**	**1904**	**1905**	**1906**	**1907**	**1908**	**1909**	**1910**	**1911**	**1912**	**1913**	**1914**
Teachers	4	2	1	4	6	4	3	3	3	0	3	3	4	5	4	3
Academics	2	8	11	9	4	8	5	6	5	7	4	13	5	8	4	9
OHE	3	4	2	4	4	3	2	2	2	4	1	2	2	1	1	3
Students	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Other	1	2	1	1	2	3	4	4	6	3	4	1	1	0	0	1
Unknown	0	1	0	0	1	0	2	2	0	0	0	0	0	2	3	1
Total	**10**	**17**	**15**	**18**	**17**	**18**	**16**	**17**	**16**	**14**	**12**	**19**	**12**	**16**	**12**	**17**

	1915	**1916**	**1917**	**1918**	**1919**	**1920**	**1921**	**1922**	**1923**	**1924**	**1925**	**1926**
Teachers	3	3	2	1	3	0	1	0	0	0	1	2
Academics	11	9	7	6	4	5	7	4	10	12	14	13
OHE	1	1	0	1	0	2	1	0	0	0	1	0
Students	0	0	0	0	0	0	0	0	0	1	0	2
Other	1	1	0	1	1	0	0	1	1	0	0	2
Unknown	1	4	2	2	2	0	0	0	1	0	1	0
Total	**17**	**18**	**11**	**11**	**10**	**7**	**9**	**5**	**12**	**13**	**17**	**19**

Authors by occupation, *Proceedings*, 1883–1926

	1909	**1910**	**1911**	**1912**	**1913**	**1914**	**1915**	**1916**	**1920**	**1921**	**1922**	**1923**	**1924**
Teachers	11	9	2	4	7	1	7	2	4	1	2	1	1
Academics	2	3	1	2	2	3	9	9	4	3	1	3	5
OHE	2	3	4	0	2	1	1	2	1	0	1	0	0
Other	2	3	0	1	5	2	0	1	0	1	1	0	0
Students	1	0	0	0	0	0	0	0	0	0	0	0	0
Unknown	0	0	2	1	0	3	0	1	0	1	0	0	0
Total	**18**	**18**	**9**	**8**	**16**	**10**	**17**	**15**	**9**	**6**	**5**	**4**	**6**

	1925	**1929**	**1930**	**1932**	**1933**	**1935**	**1937**	**1939**	**1940**	**1943**	**1944**	**1945**
Teachers	1	3	1	0	0	0	0	1	1	1	1	1
Academics	4	3	10	9	5	5	9	5	4	1	5	3
OHE	0	2	0	0	0	0	0	0	0	0	2	1
Students	1	2	2	0	0	1	0	1	0	0	0	0
Other	0	0	0	0	0	0	0	0	0	0	0	0
Unknown	1	0	0	0	2	2	0	1	3	1	0	1
Total	**7**	**10**	**13**	**9**	**7**	**8**	**9**	**8**	**8**	**3**	**8**	**6**

Authors by occupation, *Notes*, 1909–1926

	1883	1884	1885	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	1898
Teachers	23	40	55	59	63	52	53	53	52	52	47	49	49	53	63	64
Academics	11	18	24	25	27	30	34	35	33	34	37	39	41	40	44	40
OHE	4	8	9	10	9	12	12	14	14	15	13	14	13	13	13	15
Students	8	7	9	9	8	5	5	0	0	1	1	2	1	1	1	1
Others	3	8	15	17	18	23	22	27	25	25	22	21	20	18	20	21
Unknown	9	11	35	33	32	29	26	22	22	22	8	8	8	9	10	12
Total	**58**	**92**	**147**	**153**	**157**	**151**	**152**	**151**	**146**	**149**	**128**	**133**	**132**	**134**	**151**	**153**

	1899	1900	1901	1902	1903	1904	1905	1906	1907	1908	1909	1910	1911	1912	1913	1914
Teachers	61	64	62	65	58	62	80	73	76	72	77	82	81	83	84	82
Academics	39	41	40	42	46	45	47	47	53	53	56	56	57	64	63	70
OHE	18	18	17	17	18	19	20	20	21	23	23	22	22	21	20	22
Students	3	1	1	1	0	1	1	0	0	2	2	2	2	2	2	3
Others	21	22	24	24	23	22	28	31	34	33	31	30	29	25	26	28
Unknown	11	10	10	8	10	10	16	17	19	18	21	21	23	22	27	34
Total	**153**	**156**	**154**	**157**	**155**	**159**	**192**	**188**	**203**	**201**	**210**	**213**	**214**	**217**	**222**	**239**

	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930
Teachers	83	73	66	65	59	62	65	58	61	64	66	68	76	73	63	64
Academics	75	79	81	80	85	93	94	95	101	100	103	104	108	109	113	119
OHE	23	22	22	21	22	22	19	20	20	21	22	22	24	24	23	24
Students	1	0	1	1	2	2	0	0	0	2	3	1	2	3	2	2
Others	28	27	29	31	34	32	31	22	24	26	31	34	35	36	33	35
Unknown	30	34	37	38	37	40	31	22	24	19	23	27	31	35	37	36
Total	**240**	**235**	**236**	**236**	**239**	**251**	**240**	**217**	**230**	**232**	**248**	**256**	**276**	**280**	**271**	**280**

	1931	1932	1933	1934	1935	1936	1937	1938	1939	1940	1941	1942	1943	1944	1945	1946
Teachers	70	68	60	59	59	57	54	57	57	53	48	45	44	44	46	47
Academics	125	125	128	128	135	133	141	147	153	151	151	152	146	148	144	145
OHE	23	23	23	23	25	27	24	24	27	28	28	26	25	25	26	22
Students	1	1	0	1	1	2	2	3	0	0	0	0	1	1	1	0
Others	36	36	37	37	35	31	31	29	29	28	29	32	32	32	30	32
Unknown	33	31	29	31	34	39	36	34	34	33	33	30	28	32	35	33
Total	**288**	**284**	**277**	**279**	**289**	**289**	**288**	**294**	**300**	**293**	**289**	**285**	**276**	**282**	**282**	**279**

Members by occupation 1883–1946

	1883	1884	1885	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	1898
Teachers	(2)5	(4)5	(4)8	(4)8	(4)7	(3)5	(3)4	(3)5	(2)5	(3)6	(3)6	(2)6	(2)6	(3)7	(4)7	(3)5
Acad.	(1)2	2	1	2	2	(1)4	(1)4	(1)4	(1)2	(1)3	(1)4	(2)4	(2)4	(1)3	2	3
OHE	0	0	0	0	0	1	1	1	2	1	0	0	0	0	1	(1)2
Others	0	0	0	0	1	0	0	0	(1)1	0	0	0	0	0	0	0
Student	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Unkn.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	**7**	**9**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**

	1899	1900	1901	1902	1903	1904	1905	1906	1907	1908	1909	1910	1911	1912	1913	1914
Teachers	(1)3	(3)6	(3)5	(3)5	(3)6	(3)7	(4)6	(4)7	(3)6	(2)6	(3)6	(3)6	(3)6	(3)6	(3)6	(2)5
Acad.	3	2	1	3	(1)2	(1)1	2	2	2	(1)2	(1)1	2	(1)2	(1)3	4	(1)4
OHE	(3)4	(1)2	2	1	1	1	2	2	(1)2	(1)2	2	(1)2	1	1	(1)1	(1)1
Other	0	0	(1)1	(1)1	1	1	0	0	0	0	0	0	0	0	0	0
Student	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Unkn.	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**11**	**10**	**10**	**9**	**10**	**9**	**10**	**11**	**10**

	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930
Teachers	(2)4	(2)5	(2)5	(2)5	(1)4	(1)3	(1)3	(1)3	(1)3	(1)4	4	3	2	(1)3	(1)4	(1)4
Acad.	(2)5	(2)5	(2)5	(2)5	(3)5	(3)7	(3)7	(3)7	(2)8	(2)8	(4)10	(3)9	(4)11	(3)9	(3)10	(3)11
OHE	1	1	1	1	1	1	1	1	(1)1	(1)1	1	1	1	1	0	0
Other	0	0	0	0	1	1	0	0	0	0	0	(1)2	1	1	1	1
Student	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Unkn.	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
Total	**10**	**11**	**11**	**11**	**11**	**12**	**12**	**12**	**12**	**14**	**15**	**15**	**15**	**14**	**15**	**16**

	1931	1932	1933	1934	1935	1936	1937	1938	1939	1940	1941	1942	1943	1944	1945	1946
Teachers	(1)3	(1)4	(1)4	(1)4	(1)4	(1)3	(2)5	(2)5	(1)4	(1)4	(1)2	(1)2	(1)2	(1)2	(2)2	(2)3
Acad.	(3)10	(3)9	(3)11	(3)11	(3)10	(3)12	(2)10	(2)10	(2)9	(3)10	(3)9	(3)9	(3)10	(3)10	(1)9	(2)10
OHE	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
Other	1	1	1	1	0	0	0	0	0	0	1	2	1	1	1	1
Student	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Unkn.	0	0	0	0	1	1	1	0	(1)1	0	0	0	0	0	(1)1	0
Total	**14**	**14**	**16**	**16**	**15**	**17**	**17**	**16**	**15**	**15**	**13**	**14**	**14**	**14**	**14**	**15**

Members of Committee by occupation, 1883–1946
Office-bearers in parentheses.

Editors of *Proceedings* and *Notes*

	The Proceedings			The Notes
1889	R. E. Allardice	W. Peddie		
1890	R. E. Allardice			
1891	R. E. Allardice			
1892	R. E. Allardice			
1893	C. G. Knott	A. J. Pressland		
1894	C. G. Knott	A. J. Pressland		
1895	C. G. Knott	W. J. Macdonald		
1896	C. G. Knott	W. J. Macdonald		
1897	C. G. Knott	W. J. Macdonald	C. Tweedie	
1898	C. G. Knott	W. J. Macdonald	C. Tweedie	
1899	C. G. Knott	J. Dougall / W. Anderson	C. Tweedie	
1900	C. G. Knott	W. A. Lindsay	C. Tweedie	
1901	W. A. Lindsay	C. Tweedie		
1902	W. A. Lindsay	C. Tweedie		
1903	W. A. Lindsay	W. L. Thomson		
1904	W. A. Lindsay	J. Turner		
1905	W. A. Lindsay	J. Turner		
1906	W. A. Lindsay	D. K. Picken		
1907	J. H. M. Wedderburn	D. K. Picken		
1908	J. H. M. Wedderburn	A. G. Burgess		
1909	J. Miller			
1910	J. Miller	N. McArthur		P.Pinkerton
1911	J. Miller	N. McArthur		
1912	R. J. T. Bell	N. McArthur		
1913	R. J. T. Bell	N. McArthur		P. Pinkerton
1914	R. J. T. Bell	D. M. Y. Sommerville		
1915	R. J. T. Bell	D. M. Y. Sommerville		
1916	R. J. T. Bell	A. Milne		G. D. C. Stokes
1917	R. J. T. Bell	A. Milne		G. D. C. Stokes
1918	R. J. T. Bell	A. Milne		G. D. C. Stokes
1919	R. J. T. Bell	A. Milne		G. D. C. Stokes
1920	A. Milne	G. D. C. Stokes		
1921	A. Milne	G. D. C. Stokes		
1922	A. Milne	G. D. C. Stokes		
1923	A. Milne	G. D. C. Stokes		

Editors of *Proceedings* and *Notes*

1924	A. Milne	T. M. MacRobert		P. Ramsay
1925	T. M. MacRobert	H. W. Turnbull		P. Ramsay
1926	T. M. MacRobert	H. W. Turnbull		P. Ramsay
1927	T. M. MacRobert	H. W. Turnbull		A. C. Aitken
1928	H. W. Turnbull			A. C. Aitken
1929	H. W. Turnbull	W. Saddler		W. Arthur
1930	H. W. Turnbull	W. Saddler		W. Arthur
1931	H. W. Turnbull	E. T. Copson		W. Arthur
1932	H. W. Turnbull	E. T. Copson		A. C. Aitken
1933	H. W. Turnbull	E. T. Copson		A. C. Aitken
1934	H. W. Turnbull	F. Bath		A. C. Aitken
1935	H. S. Ruse	F. Bath		A. C. Aitken
1936	H. S. Ruse	F. Bath	E. T. Copson	L. M. Brown
1937	H. S. Ruse	F. Bath	E. T. Copson	L. M. Brown
1938	F. Bath	E. T. Copson		L. M. Brown
1939	F. Bath	J. M. Hyslop		L. M. Brown
1940	F. Bath	J. M. Hyslop		L. M. Brown
1941	J. M . Hyslop			L. M. Brown
1942	G. Timms	A. Erdelyi		L. M. Brown
1943	G. Timms	A. Erdelyi		L. M. Brown
1944	G. Timms	A. Erdelyi		L. M. Brown
1945	L. M. Brown	A. Erdelyi		I. M. H. Etherington
1946	L. M. Brown	A. Erdelyi		I. M. H. Etherington

Appendix B: The rhyme scheme letters

Letters concerning the enumeration of rhyme schemes

Newspaper cutting attached to Aitken's letter of 6 December 1938.
From The Scotsman, Tuesday, December 20, 1938

Cosmocapeleion

St Andrews University,
December 18, 1938

SIR, - the fine word "Cosmocapeleion" stretched across the front of an old-fashioned draper's shop on the South Bridge all my life long, until a year or two ago when a shabbier architecture removed the landmark and obliterated the friendly word. When I was a boy I wondered vainly what it meant; that κοσμοκαχηλεῖον was Greek for a "World-shop," or "Universal Provider," I learnt later on; and now they tell me that the word suggested caps and capes to the South Bridge drapers and so signified for them a "General Outfitter." My own interest in the matter goes deeper than all this. It comes of the fact that Principal Sir James Donaldson once told me that the word was of his making, that he had coined it in his student days at the University hard by, and that the draper paid him no less than £5 for doing so. – I am &c. D'ARCY W. THOMPSON.

A. C. Aitken —StASC ms25629

54 Braid Road,
Edinburgh. 10
Dec. 6, 1938

Dear Sir D'Arcy,

"Cosmocapeleion" was not pronounced as accurately in South Bridge as you pronounced it last night. Early in my stay in Edinburgh I noted the name, rather faded and not very conspicuous, and I drew an opinion from a bystander near the Tron. Few people, I gathered, dared to pronounce the word, and those that did made of it something like “Cosmocapeleum”. As for its meaning, that had been the subject of conjecture; a common idea was that “capeleum” had something to with caps, or capes, or both. And so it appeared that Cosmocapeleion meant general outfitter, which was near enough. Like you, I was sorry to see it go, and to see Hunter’s unpretentious shop-front replaced by a garish modern one.

As for the numbers which enumerate rhyme-schemes. For one line, simply A, 1 member; two lines, AB and AA, 2 members; three lines, ABC, ABA, AAB, ABB, AAA, 5 members; four lines, one finds 15 types; 5 lines, 52, and so on. The sequence is

1, 2, 5, 15, 52, 203, 877, 4140...

These numbers, which I have met elsewhere in mathematics, have most curious properties. Construct from them a table of successive differences, and you find that they are their own extreme differences, as below; from which one can build them by entering each one, as it is found, on the far right and completing by summation a fresh line of the table, shown in red ink here:

$$
\begin{array}{cccccccc}
 & \Delta & \Delta^2 & \Delta^3 & \Delta^4 & \Delta^5 & \Delta^6 & \\
1 & & & & & & & \\
 & \underline{1} & & & & & & \\
2 & & \underline{2} & & & & & \\
 & 3 & & \underline{5} & & & & \\
5 & & 7 & & \underline{15} & & & \\
 & 10 & & 20 & & \underline{52} & & \\
15 & & 27 & & 67 & & 203 & \\
 & 37 & & 87 & & 255 & & 877 \\
52 & & 114 & & 322 & & & \\
 & 151 & & 409 & & & & \\
\underline{203} & & 523 & & & & & \\
 & 674 & & & & & & \\
\underline{\underline{877}} & & & & & & &
\end{array}
$$

This is a most rapid way of finding the first dozen or so. But they occur most strangely thus:

(e = 2.71828...)

$$
\begin{array}{lcr}
\frac{1^1}{1!} + \frac{2^1}{2!} + \frac{3^1}{3!} + \ldots \text{ad inf.} & = & e \\
\frac{1^2}{1!} + \frac{2^2}{2!} + \frac{3^2}{3!} + \ldots & = & 2e \\
\frac{1^3}{1!} + \frac{2^3}{2!} + \frac{3^3}{3!} + \ldots & = & 5e \\
\frac{1^4}{1!} + \frac{2^4}{2!} + \frac{3^4}{3!} + \ldots & = & 15e
\end{array}
$$

and so on.

And yet again, they occur in relation to tables of differences of powers in the natural numbers: for example, to obtain the fourth of them, form a table of 4th powers:

x	x^4	Δ	Δ^2	Δ^3	Δ^4
0	0				
		$\underline{1}$			
1	1		$\underline{14}$		
		15		$\underline{36}$	
2	16		50		$\underline{24}$
		65		60	
3	81		110		
		175			
4	256				

Divide the underlined differences by 1!, 2!, 3!, 4! and add. Thus $1 + 7 + 6 + 1 = 15$.

For 5th powers

	Δ	Δ^2	Δ^3	Δ^4	Δ^5
0					
	$\underline{1}$				
1		$\underline{30}$			
	31		$\underline{150}$		
32		180		$\underline{240}$	
	211		390		$\underline{120}$
243		570		360	
	781		750		
1024		1320			
	2101				
3125					

Divide by 1!, 2!, 3!, 4!, 5! and add. $1 + 15 + 25 + 10 + 1 = 52$

The separate numbers in such sums as these appear to enumerate the rhyme-types according to groups. For example 5, the number of 3-line types, appears from the difference-table of cubes as $1 + 3 + 1$. Here the first 1 refers to ABC, the 3 to AAB, ABA and ABB, and the last 1 to AAA.

In short, they are most fascinating and strange enumerants. Perhaps the most curious property is that if one takes the exponential series,

$$e^x = 1 + \tfrac{x}{1!} + \tfrac{x^2}{2!} + \tfrac{x^3}{3!} + \ldots \text{ad inf.},$$

truncates its first term and exponentiates once again, the result is

$$e^{\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots} = e^{e^x - 1} = 1 + \frac{x}{1!} + \frac{2x^2}{2!} + \frac{5x^3}{3!} + \frac{15x^4}{4!} \ \ldots$$

in which our friends appear as coefficients.

Yours sincerely,
A.C. Aitken

A. C. Aitken —StASC 25630

54 Braid Road
Edinburgh 10.
Dec. 19, 1938.

Dear Sir D'Arcy,

The enumeration 15 for 4-line rhyme-schemes is correct. We have:

a a a a	a a b b	b c a a
a a a b	a b a b	b a c a
a a b a	a b b a	a b c a
a b a a	a a b c	a b c d
b a a a	a b a c	
	b a a c	

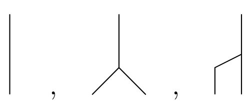
You probably discounted abcd as having <u>no</u> rhyme at all; for I should have explained that I included the null-rhyme-scheme, blank verse, as one possibility.

On consideration I observe that the simplest relation between the successive enumerants 1, 2, 5, 15, 52, 203, 877, ... is provided by the binomial numbers in the Pascal triangle. Thus, prefix a conventional enumerant 1 for the no-line scheme. (Like $a^0 = 1$.) Then

$$
\begin{array}{ccccc}
1 & \qquad & 1 & = & 1 \\
1 \quad 1 & & \overparen{1 \times 1} + \overparen{1 \times 1} & = & 2 \\
1 \quad 2 \quad 1 & & \overparen{1 \times 1} + \overparen{2 \times 1} + \overparen{1 \times 2} & = & 5 \\
1 \quad 3 \quad 3 \quad 1 & & \overparen{1 \times 1} + \overparen{3 \times 1} + \overparen{3 \times 2} + \overparen{1 \times 5} & = & 15 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 & & \overparen{1 \times 1} + \overparen{4 \times 1} + \overparen{6 \times 2} + \overparen{4 \times 5} + \overparen{1 \times 15} & = & 52 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 & & \text{etc.} & &
\end{array}
$$

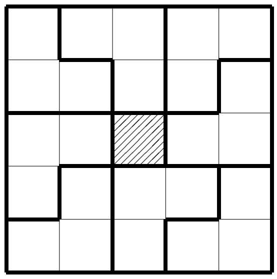
This is clearly the most rapid way of deriving them in succession.

Your island and hedges problem is very interesting. I am greatly reminded of Cayley's paper on enumerating the forms he called "trees".



Did you see Dudley Langford's proof by dissection that 8 times any of 0, 1, 3, 6, 10, 15, ... plus 1 gives a perfect square?

Ex.



$8 \times 3 + 1 = 25$. The shaded square is the added unit.
I think his dissection was more symmetrical, but the idea is clear.
All good wishes for Christmas and the New Year.

Yours sincerely,
A.C. Aitken

A. C. Aitken —StASC 25631

54 Braid Road,
Edinburgh, 10
Dec. 20, 1938

Dear Sir D'Arcy

Any mention of the Fibonacci numbers is always sure to draw me. All the romance of continued fractions, linear recurrence relations, surd approximations to integers and the rest lies in them; and they are a source of endless curiosity.

How interesting it is to see them striving to attain the unattainable, the golden ratio $\sqrt{5} - 1 : 2$ or $0.618034 : 1$.

$$\begin{aligned}
\frac{1}{2} &= 0.5 \\
\frac{2}{3} &= 0.6666\ldots \\
\frac{3}{5} &= 0.6000\ldots \\
\frac{5}{8} &= 0.625 \\
\frac{8}{13} &= 0.\dot{6}1538\dot{4} \\
\frac{13}{21} &= 0.\dot{6}1904\dot{7} \\
\frac{21}{34} &= 0.6176470\ldots \\
\frac{34}{55} &= 0.6181818\ldots
\end{aligned}$$

At $\dfrac{89}{144}$ we are getting near: $0.6180555\ldots$

Then again, to take $\frac{\sqrt{5}-1}{2} = 0.618034$ nearly, its reciprocal $\frac{\sqrt{5}+1}{2} = 1.618034$ and the square of that, 2.618034, all in A.P. . The square of 0.618034 is its complement 0.381966. So we build

$$\begin{array}{llclcr}
1 & + & 0 \times 0.381966 & = & 1 & \\
1.618034 & + & 1 \times 0.381966 & = & 2 & \\
2.618034 & + & 1 \times 0.381966 & = & 3 & \\
4.236068 & + & 2 \times 0.381966 & = & 5 & \\
6.854102 & + & 3 \times 0.381966 & = & 8 & \\
11.090170 & + & 5 \times 0.381966 & = & 13 & \\
17.944272 & + & 8 \times 0.381966 & = & 21 & \text{, etc} \\
\downarrow & & & & &
\end{array}$$

(powers of $\frac{\sqrt{5}+1}{2}$), and this is only one of hundreds of such relations.

Another set is

$$\begin{array}{llclcr}
1 & - & 0 \times 0.618034 & = & 1 & \\
1.618034 & - & 1 \times 0.618034 & = & 1 & \\
2.618034 & - & 1 \times 0.618034 & = & 2 & \\
4.236068 & - & 2 \times 0.618034 & = & 3 & \\
6.854102 & - & 3 \times 0.618034 & = & 5 & \\
11.090170 & - & 5 \times 0.618034 & = & 8 & \\
17.944272 & - & 8 \times 0.618034 & = & 13 & \text{, etc}
\end{array}$$

perhaps even more vivid.

Or again, square any Fibonacci number, add or subtract 1, and factorize the result. Thus

$$\begin{array}{rcccl}
21^2 + 1 & = & 442 & = & 13 \times 34, \\
21^2 - 1 & = & 440 & = & 8 \times 55, \\
34^2 + 1 & = & 1157 & = & 13 \times 89, \\
34^2 - 1 & = & 1155 & = & 21 \times 55;
\end{array}$$

the factors are again Fibonacci numbers. Or again, the sum of the squares of two consecutive Fibonacci numbers is a Fibonacci number.

$$5^2 + 8^2 = 89,$$

$$8^2 + 13^2 = 233, \quad \textit{etc.}$$

And so on, and so on. Indeed I have often thought that, excellent as was the emblem on Bernoulli's gravestone, the equiangular spiral and the inscription 'Eadem mutata resurgo', the Fibonacci numbers, with their phoenix-like quality of rising from their own ashes, or from the ashes of $(\frac{\sqrt{5}\pm1}{2})^n$), would have been an equally excellent symbol.

Yours sincerely,
A. C. Aitken.

Or again,

$$\begin{aligned} 1 &= 1, \\ 1+\frac{1}{1} &= \frac{2}{1} \quad , \\ 1+\frac{1}{1+\frac{1}{1}} &= \frac{3}{2} \quad , \\ 1+\frac{1}{1+\frac{1}{1+1}} &= \frac{5}{3} \quad , \textit{ etc} \end{aligned}$$

Sir D'Arcy Thompson — letter, StASC ms 26232

St Andrews
20th December, 1938

My dear Bennett,

Imprimis, A Happy Christmas to you! Secundo, I should like to know, at your leisure, what you think of that boy, Bertie Hynd, whom we send [sic] up to Emmanuel last term. He's a queer little beggar. Brains he has, but manners he has none; and he resembles a good many other Scotch men in these characteristics.

Thirdly, also at your leisure, I should like to know what you make of the enclosed paper, just received.

It is odd that, looking for a minimal-area configuration, he should hit on, not on Kelvin's 14-sider, but on one of the other two Archimedean ditto. But I don't see that he gets any forruder, at all, at all. Because it's pretty evident that these 'cubo-octahedra' of his won't close-pack; and again, the fact that they have four-face corners, to which another edge will have to be added in the 'pack', seems dead against any minimal-area quality, right away to begin with.

Fourthly, (en parenthese), Aitken has been writing me two or three extremely interesting letters on a certain series, which becomes more and more wonderful the more you look at it.

You begin with the idea of rhymes, or rhyme-types.

With	one line, you've	simple A, —	one type.
	two lines	AA or AB,	two ,,
	three ,,	ABC, ABA, AAB, ABB, AAA	five types
	four ,,		fifteen ..
	five		fifty-two,,

etc. The sequence goes on 52, 203, 877, 4140. . .

As for their properties, beginning gently, with a table of differences, they merely bob up again!

$$
\begin{array}{ccccccc}
 & \Delta & \Delta^2 & & & & \\
\underline{1} & & & & & & \\
 & \underline{1} & & & & & \\
\underline{2} & & \underline{2} & & & & \\
 & 3 & & \underline{5} & & & \\
\underline{5} & & 7 & & \underline{15} & & \\
 & 10 & & 20 & & \underline{52} & \\
\underline{15} & & 27 & & 67 & & 203 \\
 & 37 & & 87 & & 255 & \\
\underline{52} & & 114 & & 322 & & \\
 & 151 & & 409 & & & \\
\underline{203} & & 523 & & & & \\
 & 674 & & & & & \\
\underline{\underline{877}} & & & & & &
\end{array}
$$

Next, take any table of <u>powers</u> of the natural numbers (e.g. n^4) and make a difference-table of these:—

$$
\begin{array}{cccccc}
n^4 & & & & & \\
0 & & & & & \\
 & \underline{1} & & & & \\
1 & & \underline{14} & & & \\
 & 15 & & \underline{36} & & \\
16 & & 50 & & \underline{24} & \\
 & 65 & & 60 & & 0 \\
81 & & 110 & & & \\
 & 175 & & & & \\
256 & & & & &
\end{array}
$$

Divide the underlined differences by 1!, 2!, 3!, 4!,... and add. Thus $1 + 7 + 6 + 1 = 15$.

But even that's only the beginning of the story! Perhaps you know it all already. — (but perhaps not)

Yours sin.
DWT

A. C. Aitken —StASC 25632

P.S. Dudley Langford, in current number of *Math. Gazette.* A.C.A.

54 Braid Road,
Edinburgh 10.
Dec. 23, 1938

Dear Sir D'Arcy

Very many thanks for the reprint of "Excess and Defect..", which I have perused with equal profit and pleasure.

Let me say at once how warmly I agree with you (foot of p. 45) when you say that the continued fraction, though an elegant arithmetical device, is not simple to work with. As one deeply engaged in making mathematics of a practical kind amenable to the extraordinary people afforded by modern electrical machines, let me assure you that the continued fraction in practice is dead. I wouldn't look at it. What is living (and this is not surprising, for the machine is simply a supremely endowed robot using the simplest and most natural devices) is the <u>relation</u> of <u>recurrence</u>, which the Greeks used and upon which you insist. In lip service to current algebra I used a mention of continued fractions in my letter to you, but to me a continued fraction is simply a function, not only defined, <u>but also best evaluated</u>, by a recurrence such as

$$u_{n+2} + au_{n+1} + bu_n = 0,$$

of the second order. When one passes to recurrences of the 3rd and higher order,

$$u_{n+3} + a_1u_{n+2} + a_2u_{n+1} + a_3u_n = 0$$

and the like, the continued function is no longer of any sort of help. Its analogue is then a staircase of staircases, interlocked like the lattice constructions of carved ivory made by the ingenious Chinese, and such a generalized continued fraction is a mere horror to the computer. On the other hand a recurrence of the 3rd order

$$u_{n+3} = -a_1u_{n+2} - a_1u_{n+1} - a_3u_n[sic.]^{14}$$

is as easy, given by 3 initial terms u_0, u_1, u_2, as that of the second order, and the machine revels in it.

I believe you, utterly, when you deprecated the introduction of <u>even the</u> <u>mention</u> of continued fractions into the discussion of the views of Theon, Iamblicus and Proclus. Take my own case. As a youth, knowing nothing of theory of numbers but being good at mental arithmetic, I read in the

[14]Should be $-a_1u_{n+2} - a_2u_{n+1} - a_3u_n$

puzzle column of the "Strand" Magazine the statement that 17^2 is twice 12^2 plus 1; challenge, to find two pairs of higher numbers possessing this property. Well, with trial and error and some slight scribbling, I soon found the pair 99 and 70, noting with interest the intermediate pair 41 and 29 and the minus 1 in their case; and soon after that I noticed the recurrence $12+17=29$, $12+12+17=41$, $29+41=70$ and so on, and building on it got 577 and 408, noting 239 and 169 of the other type All this in some quarter hour in a public library in Dunedin, New Zealand. Going home, I proved the recurrence algebraically, worked out $\frac{7}{5}$, $\frac{17}{12}$, $\frac{41}{29}$, $\frac{99}{70}$, $\cdots$ up to $\frac{1393}{985}$ compared them with $\sqrt{2}$ and observed that the errors diminished successively in something which seemed to be tending to a G.P. of common ratio about $-1:6$. Now if I did that from natural interest and without preconceived ideas, I can easily believe the Greeks (or the Babylonians, whom, according to Dr Neugebauer, we have greatly underestimated) worked by a like instinct.

Not until long afterwards, when my own researches forced me in that direction, did I come to know the great and beautiful theorem that unifies all such matters in respect of recurrence relations of any order whatsoever, this namely: given a recurrence

$$u_{n+m} + a_1 u_{n+m-1} + a_2 u_{n+m-2} + \cdots + a_m u_n = 0,$$

with any m initial values u_0, $u_1, \cdots$, u_{m-1} to start it. Construct by recurrence u_m, u_{m+1}, u_{m+2}, $\cdots$,u_{m+n}, $\cdots$.

Then, except for certain sets of initial u's, the ratio

$$u_{n+m+1} : u_{n+m}$$

tends as n increases to the greatest root x_1 of the algebraic equation associated, viz.

$$x^m + a_1 x^{m-1} + a_2 x^{m-2} + \cdots + a_m = 0,$$

and further the rapidity of convergence is more and more in a G.P. of common ratio $\frac{x_1}{x_2}$, where x_2 is the next greatest root. (Exceptions when $x_1 = x_2$, but still $u_{n+m-1}/u_{n+m} \rightsquigarrow x_1$

This most beautiful result (on which I base my work in finding not only the greatest root, but all roots of an algebraic equation by the appropriate recurrences, so suitable to the machine) explains why the $\sqrt{2}$ series converges better than the Fibonacci series. For the $\sqrt{2}$ series is generated by the recurrence $u_{n+2} = 2u_{n+1} + u_n$ corresponding to the associated quadratic equation

$$x^2 - 2x - 1 = 0,$$

roots $1 \pm \sqrt{2}$, namely 2.4142.. and $-0.4142\ldots$ Now their ratio, my $\dfrac{x_2}{x_1}$, is $\frac{1-\sqrt{2}}{1+\sqrt{2}} = -1 : 3 + 2\sqrt{2} = -1 : 5.8284\ldots$, and this, had I only known it years ago in Dunedin, was the ratio I then recognized as about $-1:6$.

As for the Fibonacci series, the recurrence is

$$u_{n+2} = u_{n+1} + u_n,$$

and so the associated quadratic equation is

$$x^2 - x - 1 = 0,$$

roots $\frac{1\pm\sqrt{5}}{2}$, our old friends. (How glad I am you wrote about them! The Greeks could not have missed them.) Here the ratio is

$$1 - \sqrt{5} : 1 + \sqrt{5} = -1 : 2.618\ldots,$$

so that the G.P. measuring the rate of convergence shows still a side-to-side oscillation (negative ratio) but not nearly so strong a convergency.

Now as to $\sqrt[3]{2}$. Certainly $x^3 = 2$, or $x^3 - 2 = 0$ is a very specialized example of a cubic equation. The more general cubic $x^3 + a_1x^2 + a_2x + a_3 = 0$ is quite amenable, however, to methods of recurrence. Let us choose a rather favourable example. I have written down

$$x^3 - 8x^2 + 11x - 4 = 0.$$

I do not yet know its roots, but trial shows that the greatest lies between 6 and 7. Let us use recurrence, in this case

$$u_{n+3} = 8u_{n+2} - 11_{n+1} + 4u_n, [sic.]^{15}$$

trying, quite arbitrarily, $u_0 = 0$, $u_1 = 0$, $u_2 = 1$ for a beginning. In succession the sequence then develops as

$$0, 0, 1, 8, 53, 340, 3169, 13824, 88093, 561356, \cdots$$

well, that will be far enough. Take ratios:

[15] Should presumably read $8u_{n+2} - 11u_{n+1} + 4u_n$

$$\begin{array}{rcll} & & & \Delta \\ 8/1 & = & 8.000000 & \\ 53/8 & = & 6.625000 & \\ 340/53 & = & 6.415094 & \\ & & & -\ 35682 \\ 2169/340 & = & 6.379412 & \\ & & & -\ 5868 \\ 13824/2169 & = & 6.373444 & \\ & & & -\ 976 \\ 88093/13824 & = & 6.372468 & \\ & & & -\ 157 \\ 561556/88093 & = & 6.372311 & \end{array}$$ [16]

Here the convergence to the greatest root is palpable; and the successive difference of the convergents are clearly doing their best to establish a G.P. of ratio about 1 : 6. By ocular extrapolation I place the greatest root at $x_1 = 6.37228$, and I also observe, since $\frac{x_2}{x_1}$ is about 1/6, that x_2 is nearly 1. In fact I have methods by which from the above convergents alone x_1, x_2, x_3 could be quickly found to 15 or 20 decimals.

How strange and beautiful the theory of approximation is! What would Aristotle have said, though, to those bizarre modern sequences which, even in the limit, do <u>not quite</u> approximate? To Hermite's $e^{\pi\sqrt{43}}$, $e^{\pi\sqrt{163}}$etc. which are amazingly close to integers, and yet are not integers? . . . and, above all, to Ramanujan's series $1000\{\frac{1}{1001} + \frac{1}{1002^2} + \frac{3^1}{1003^3} + \frac{4^2}{1004^4} + \frac{5^3}{1005^5} +$ *etc ad inf.*$\}$ which does <u>not</u> converge to 1, as the geometric series $1000\{\frac{1}{1001} + \frac{1}{1002^2} + \frac{1}{1003^3} + \frac{2}{1004^4} + \cdots\}$ does, but to $0.99999999 \cdots \cdots \cdots \cdots 999875 \ldots$

$\downarrow$. . . a sequence of 437 nines $\downarrow$!!!

Here we are in a region which gave at least one French mathematician "la chair de poule' as he confessed; and it is fitting that these, and hundreds of other results equally exotic and strange (surviving in posthumous, notebooks, and many still unproved) should have come from the brain of one who was a high Brahmin and almost a yogi!

I had better stop: the subject is too congenial to me.

Yours sincerely,
A. C. Aitken.

G. T. Bennett— letter, StASC ms 26100

Royal Societies Club
St. James's Street,
S.W.1.
27.12.1938

D'A. Th.

(i) Your good wishes have worked. Temp. Fah. has gone up 20^o!

(ii) I did not encounter B.H. in his freshman's term. Will make enquiry when I go back.

(iii) This "Science" extract (Hancock) is very chaotic. Patient perusal and some generous guessing might make more of it than appears. But (forgive me!) it has not the light incisiveness of a holiday task. Shall I return it?

(iv) The number-series 1 2 5 15 52 203 877... is new to me, and I do not q. clearly make out its provenance as counts of rhyme-types. What is the precise definition? I see that the differencing of the powered natural numbers gives the numbers in the way you indicate — but not why. The one thing that I do see is that if the series is u_1, u_2,u_3 $\ldots u_n$,$u_{n+1}\ldots$& if $u_0 = 1$ is put as antecedent then symbolically $u_{n+1} = (u+1)^n$ where in the expanded binomial suffixes replace indices: L.E.

$$\begin{aligned} u_2 &= u_1 + u_0 \\ u_3 &= u_2 + 2u_1 + u_0 \\ u_4 &= u_3 + 3u_2 + 3u_1 + u_0 \\ &\&c. \end{aligned}$$

deriving each term from its predecessors. There is also an extension of this formula, in the shape

$$u^{n+1}(u-1)^m = u^m(u+1)^n.$$

ex. gr.

$$u_6 - 3u_5 + 3u_4 - u_3 = u_5 + 2u_4 + u_3$$

But you already have the whole "story", and these fragments are in it?

(v) Do Bestiaries include Birds? (Thank you.) Also the Ophidian of Eden?

G.T.B.

G. T. Bennett— postcard, StASC ms 26101

Royal Societies Club
St. James's Street
S.W.1 G.T.B
29.XII.38

n	0	1	2	3	4	5	6	7	8
u_n	1	1	2	5	15	52	203	877	4140

$$\begin{array}{lll}
(i) & u^{n+1} & = (u+1)^n \\
[ii] & u(u-1)^m & = u^m \\
(iii) & u(u-1)(u-2)\ldots(u-s+1) & = 1 \\
(iv) & u^{n+1}(u-1)^m & = u^m(u+1)^n \\
[v] & u^{n+1}(u-1)(u-2)\ldots(u-s+1) & = (u+s)^n \\
[vi] & u(u-1)(u-2)\ldots(u-s+1)(u-s)^m & = u^m \\
 & \text{are all special cases of} & \\
[vii] & u^{n+1}(u-1)(u-2)\ldots(u-s+1)(u-s)^m & = u^m(u+s)^n
\end{array}$$

Put u_n for u^n after expansion.[17]

G. T. Bennett— letter, StASC ms 26103

Royal Societies Club
St. James's Street
S.W.1
30.12.38

D'AT

Yes.[18] For the rhyme-count for n letters take any partition of n, say a repeated α times, b repeated β times, c repeated γ times, etc., with the necessary condition $\alpha a + \beta b + \gamma c + \ldots = n$.

The n letters are $A_1\ A_2\ A_3 \ldots A_\alpha$ each repeated a times, $B_1\ B_2\ B_3 \ldots B_\beta$ each repeated b times, etc., and as the equal A-groups are to be exchangeable and the equal B-groups are to be exchangeable, etc., the partition gives

$$\frac{n!}{a!^\alpha \alpha! b!^\beta \beta! c!^\gamma \gamma! \ldots} \tag{5.1}$$

permutations. The sum for all partitions gives the rhyme-total.

Ex.gr. $n = 5$

[17] This was originally added vertically to the right of the table.
[18] If this is a response to a letter that has gone missing, or what this 'yes' is intended to mean is not entirely clear.

Partitions[19]

5.	$\lfloor 5 / \lfloor 5$	=	1
4.1.	$\lfloor 5 / \lfloor 4 \; \lfloor 1$	=	5
3.2.	$\lfloor 5 / \lfloor 3 \; \lfloor 2$	=	10
3.1^2	$\lfloor 5 / \lfloor 3 . \lfloor 1 . \lfloor 1 . \lfloor 2$	=	10
$2^2.1$	$\lfloor 5 / \lfloor 2 \; \lfloor 2 \; \lfloor 1 \; \lfloor 2$	=	15
2.1^3	$\lfloor 5 / \lfloor 2 . \lfloor 1 . \lfloor 1 . \lfloor 1 . \lfloor 3$	=	10
1^5	$\lfloor 5 / \lfloor 1 \; \lfloor 1 \; \lfloor 1 \; \lfloor 1 \; \lfloor 1 \; \lfloor 5$	=	1
	u_5	=	52

The summation $\sum \frac{\lfloor n}{a!^\alpha \alpha! b!^\beta \beta! \ldots}$ can be effected by a generating function: for $\frac{u_n}{n!}$ = Coeff. x^n in $1 + [e^x - 1) + \frac{1}{2!}(e^x-1)^2 + \frac{1}{3!}(e^x-1)^3 \ldots$[20] wh. may be written symbolically as $e^{ux} = e^{e^x-1}$. This leads readily to the expression of u_n in terms of the differenced series $1^n, 2^n, 3^n, 4^n \ldots$

I shd. not be surprised to find that all this was done by P.A. MacMahon.

G.T.B.

G. T. Bennett— letter, StASC ms 26104

Royal Societies Club
St James's Street
S.W.1
30.XII.38

D'A.Th

Postultimate remark

Any series of numbers

$$a \quad b \quad c \quad d \quad e \quad f \ldots$$

may be converted into a fresh series by multiplying the terms by 1, 2, 3, 4, 5, 6… and adding the-term-next-before.
The new series is

$$a \quad 2b+a \quad 3c+d \quad 4d+c \quad 5e+d \ldots$$

Starting from a series of only 1 term, and that unity (in value), the series consecutively obtained by this mode of conversion are

[19]The notation $\lfloor x$ is an abandoned notation for $x!$.

[20]Bennett writes the factorials in this expression in the $\lfloor x$ -notation.

							Totals
1							1
1	1						2
1	3	1					5
1	7	6	1				15
1	15	25	10	1			52
1	31	90	65	15	1		203
1	63	301	350	140	21	1	877

&c &c

The numbers in this table are the same as those got by repeated differencing of $1^n, 2^n, 3^n, \ldots$ [illegible word[21]] division by $\lfloor\underline{1}$ $\lfloor\underline{2}$ $\lfloor\underline{3}$ $\ldots$; but they come in this way much more readily.

G.T.B.

G. T. Bennett— letter, StASC ms 26106

1.i.1939

D'A Th.

The modes of self-generation of the Table[22]

m

		1	2	3	4	5	...	
	1	1					...	1
n	2	1	1				...	2
	3	1	3	1			...	5
	4	1	7	6	1		...	15
							...	

when arrived at, seemed too simple not to have an 'explanation': and it turns out to be this :—

Let the entry in row n col m be $(n \quad m)$. It is the number of rhyme-systems of n lines when m different rhymes occur. A set of these from the Table are

$$\begin{array}{c|c} (n \quad m-1) & (n \quad m) \\ \hline & (n+1 \quad m) \end{array}$$

The $(n+1 \quad m)$ systems are got by taking jointly

(i) each of the $(n \quad m)$ and repeating any one of the rhymes already in it for the extra line

(ii) each of the $(n \quad m-1)$ and adding a new rhyme(solitary) for the extra line.

[21] This might be a rather careless representation of '&'. Replacing it by the word 'and' would not appear to alter the meaning of the sentence.

[22] This table has been changed slightly for ease of print.

Hence $(n+1 \quad m) = m(n \quad m) + (n \quad m-1)$.
This formula should come at the outset and lead to all the rest of the algebra!

I looked in "Comb. Analysis" but did not come on

$$1 \quad 2 \quad 5 \quad 15 \quad \&c.$$

. . . MacMahon would have seen this difference formula at a glance!

G.T.B.

A. C. Aitken — letter, StASC ms 25633

54 Braid Road,
Edin, 10
Jan. 4, 1939

Dear Sir D'Arcy,

Thank you for your New Year wishes, which I heartily reciprocate.

G.T. Bennett has gone very quickly to the root of the sequence 1, 2, 5, 15, 52, 203, 877,. . . , and with his "post-ultimate" method (which I recognise in another guise as a known recurrence relation between the so-called "differences of zero", sc. $\Delta^r O^s$)has made a point unobserved by me.

I find, as he suggests, that the sequence occurs in MacMahon (and in Muir also) as the solution of combinatory problems not quite the same, but in one-to-one correspondence with our rhyme-scheme problem. The totals of rows in one of MacMahon's tables of enumerating functions gives precisely 1, 2, 5, 15 and the rest, and he remarks, as all must who direct some consideration upon the matter, that they are coefficients of

$$\frac{x^n}{n!}$$

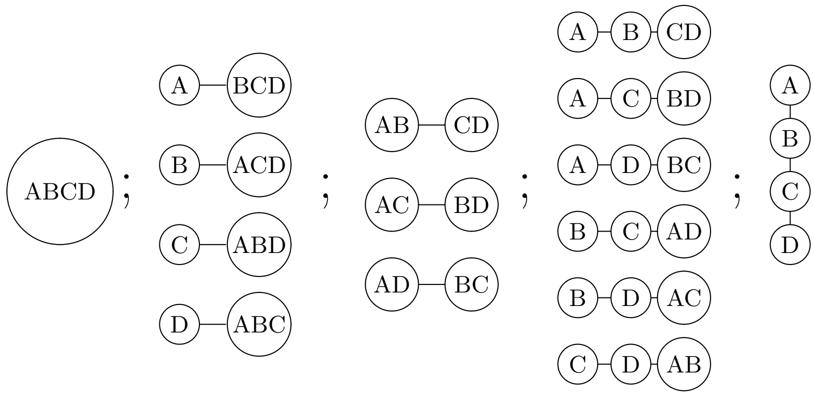
in the expansion in powers of

$$e^{e^x - 1} \quad .$$

The problem in which they arise most naturally appears to be, not the rhyme-scheme problem or its paraphrase in abstract language, but what may be called the "Oxford Groups" problem. Given n persons, e.g. $n = 4$, A, B, C, D, in how many ways can they be grouped, allowing possible groups of one person?

Thus:

(1)



$G_4 = 1 + 4 + 3 + 6 + 1 = 15$ ways. In the same way $G_5 = 52$, $G_6 = 203$ etc.

The identity of this with the rhyme-scheme problem puzzled me for a moment, until I saw what the correspondence was. It is this: let persons in a group be named with the same small letter, e.g. persons in ABCD will all be tagged with "a". Then scheme (1) is in exact correspondence with (Putting A, B, C, D in order and tagging them.)

(2)

aaaa	;	a bbb b a bb bb a b bbb a	;	aa bb a b a b a bb a	;	a b cc a c b c a cc b c a b c c a c b cc a b	;	a b c d

and these are the rhyme-schemes.

Then I found in Muir's last volume of the "History of Determinants" (1900–1920)" yet another form for the numbers 1, 2, 5, 15, 52, ... which interested me, as it involved determinants. The sequence of determinants

$$1\ ,\ \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}\ ,\ \begin{vmatrix} 1 & -1 & . \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}\ ,\ \begin{vmatrix} 1 & -1 & . & . \\ 1 & 1 & -1 & . \\ 1 & 2 & 1 & -1 \\ 1 & 3 & 3 & 1 \end{vmatrix}\ ,\ \ldots$$

made by writing the under-diagonal side of the determinant as a Pascal triangle of binomial coefficients and then putting elements -1 in the superprincipal diagonal, yields on evaluation our sequence 1, 2, 5, 15, ... This representation is due to a man called Anderegg, otherwise unknown to fame.

Well, they are a most interesting set of numbers and they appear to arise in those problems in which we are concerned not merely with partitioning an integer, as 4 into $1+1+2$, but with putting the parts into an order as well, $1+1+2$, $1+2+1$, $2+1+1$.

MacMahon was incomparably the greatest of the combinatorialists, so great that his stature is not yet recognized. I greatly treasure two letters which I have from him, concerned with a theorem on dual symmetric functions upon which I stumbled in 1927.

Thank you for the extract from "Science" by Harris Hancock. (The name is quite well known; — as author of a good text-book on function theory.) It is rather beyond me on the geometrical side, though the part on the minimizing of a determinant is very much in my line. I shall ask Edge and Bath what they think of it. One would almost imagine that a physical experiment could be made.

Hancock, if this extract be typical of him, does not excel in lucidity and grace. However, an American reviewer, rapping me over the knuckles a few years ago because I had praised Muir for a clear and leisurely style in exposition, held up L.E. Dickson's "History of the Theory of Numbers" as the ideal, referring to its "machine-gun fire of facts". Alas! I don't like a relentless machine-gun fire of facts; I like some gentle lawns and an occasional tree!

Yours sincerely,
A. C. Aitken.

G. T. Bennett— letter, StASC ms 26107

Royal Societies Club,
St. James's Street,
S.W.1
13.i.39

My dear Professor,

Did I not espy you in the Piccadilly Tube passage on Tuesday evening? I was busy wafting a friend away, or I should have accosted you.

I find that I shall be here for another week, and am wondering whether you would come here for lunch or dinner for a talk, if still at your Club.

How goes 1 1 2 5 15 52 203 &c.? The difference-equation seemed basic and crucial, and should have come initially and not finally.

I find much to admire at Burlington House.

Yours always sincerely
G.T.Bennett.

G. T. Bennett— letter, StASC ms 26108

Royal Societies Club,
St. James's Street,
S.W.1
18.1.39

D.W.T.

I did miss you then! Give me a moiety of our joint regrets.

An interesting relic, this 22-year old letter that you produce from your store! I have looked up PAM's paper (7th Memoir in the Partition of Numbers, Phil. Trans. A. Vol 217 1917 pp 81-113) and doubtless, if one travels assiduously up to the point, it is "all there". But the great man knew too much, and the giant machinery is used to attack even small problems (or so it seems). Hamiltonian Operators and so on...: the apparatus of the specialist.

As regards 1 2 5 15 ... and its anatomy, you jotted down originally only part of what Aitken has — the two tables of successive differences, for the numbers themselves and for the powers of the natural numbers. It interested me to find the set of formulae (one being comprehensive of the rest) in all of which u^n becomes u_n after expansion: and more particularly the generating formula $(m \quad n+1) = m(m \quad n) + (m-1 \quad n)$ primordial but noticed last instead of first, as it happened.

(Did I make this clear? ... You have not made any comment.). — It is quite likely that Aitken has all this: but I wd. suggest that you pass my notes to him, as contributions to the common store, to use as he will. [For, so far, I know only of him.] Could and would you?

G.T.B.

Sir D'Arcy Thompson— letter, StASC ms 26233

St. A. 19/1/-
19/1/39[added in pencil]

G.T.B.

I think I have shown Aitken all yr. recent batch of letters, dealing with our series: and here are his. I rather think you have beaten him on points, — but I am an unworthy judge.

Talking of PAM, I happen to have been looking over R.A. Fisher's book on 'The Design of Experiments'. It contains a long chapter on the Latin Square: and the idea of Combinatory Analysis runs through the book. But I cannot find a single mention either in text, notes or bibliography, of that praeclarum ac venerabile nomen PAM!

D.W.T.

G. T. Bennett— letter, StASC ms 26109

Emm. Coll. Cam. 27/1/39

D.W.T.
*****

i. Many thanks for the sight of ACA's 5 letters — here returned.

ii. Am I tiresomely persistent? The sweetly simple and obtrusively obvious difference equation $(n+1,m) = m(n,m) + (n,m-1)$ that I gave is still awaiting your recognition and assent, if may be. Did I fall short of making it clear? ... If n lines of verse have m rhyme-sets then an extra line added will either repeat one of the m old rhymes or add a new one. That is really all about it. If the enumerative Table

1				
1	1			
1	3	1		
1	7	6	1	
1	15	25	10	1
	&c	&c	&c	

has its actual material written down it appears as on the squared-paper page. The contents of each cell are got from the cell to the North and the cell to the NW, by adding an old or a new letter, respectively. The contribution above the dotted line in each cell comes from the N, and the part below from the NW.

— NB. At their first appearances the rhymes are here named in alphabetical order. For this method *baab* would be "wrong": it should be *abba*.

iii. ACA4/1. If the alternative aspect were preferred (his "tagged Groupers") the lines of verse may be numbered (as commonly) and then the rhyme-system, say in the lines

a	a	b	c	b	a	d	d	a
1	2	3	4	5	6	7	8	9

may be recorded either by the alphabetic row above or by the numeral equivalent.

(1269)(35)(4)(78)

The former seems simpler, as a plain sequence with no brackets.

iv. ACA4/1. Nevertheless I may continue, tentatively, to regard

$$u^{n+1}(u-1)(u-2)...(u-s)^m = u^m(u+s)^n$$

as novel until it is made plagiary by an actual identifying reference. . . ? (It was the formula with half a dozen subcases got when m or n is zero and s unity)

v. ACA19/12 . . . "clearly the most rapid way" . . . Is it? — with binomial coefficients as multipliers? The (n, m) Table hopefully competes, — with the natural numbers as multipliers.

vi. ACA20/12 The Fibonacci numbers often seem to steal thunder belonging elsewhere. His formulae, if made explicit, are special cases of

$$u_n^2 + (-1)^{n+m+1} u_m^2 = u_{n-m} u_{n+m}$$

and this latter holds for any series given by

$$u_{n+1} = \alpha u_n + u_{n-1}$$

where α is arbitrary. $\alpha = 1$ gives Fib: $\alpha = 2$ gives

$$0 \quad 1 \quad 2 \quad 5 \quad 12 \quad 29$$

This formula in its turn is only a very special case of Euler's Continuant Identity.

vii. ACA 23/12 'the ratio tends to the greatest root of the algebraic equation". This is near enough to the truth to excite legitimate curiosity as to its source. I have not seen it "about" yet. Is there any available reference?

viii. PAM was PAM. . . . . . . Fisher is Fisher

ix. Don't think me Nil Admirari. I do however tend to believe that marvels melt into normality with understanding. "Omne (semi) ignotum pro mirifico" I take to be a soundly minatory motto.
and

x. They send me your Picture. Most admirably formidable!

GTB.

***

	1	2	3	4	5 = n	
1	a 1					1
2	aa ⋯ 1	⋯ ab 1				2
3	aaa 1	aba abb ⋯ aab 3	⋯ abc 1			5
4	aaaa ⋯ 1	abaa abab abba abbb aaba aabb ⋯ aaab 7	abca abcb abcc ⋯ abac abbc aabc 6	⋯ abcd 1		15
5 = n	aaaaa ⋯ 1	abaaa abaab ababa ababb &c. aaaba aaabb ⋯ aaaab 15	abcaa abcab abcac abcba abcbb &c. aabca aabcb aabcc ⋯ abaac ababc &c. aaabc 25	abcda abcdb abcdc abcdd ⋯ abcad abcbd &c aabcd 10	⋯ abcde 1	52

Sir D'Arcy Thompson— letter, StASC ms 26235

St Andrews 1:2:39

G.T.B.

i. Yes. I fear I omitted to acknowledge your difference equation $(n+1, m) = m(n, m) + (n, m-1)$ But I can assure you I understood, appreciated and admired it! It has (like the 3 per cents) a "sweet simplicity".

ii. It looks to me as though our series was not unconnected with our old problem of the cell-partitions in the dividing cells on the surface of an egg. The numbers don't agree; but I rather fancy that is merely because the conditions of the cell-problem exclude certain particular cases. However, I have not found a moment to look farther into this.

iii. If you had to lecture in the forenoon, as I did today, on the osteology of the Kangaroo, look over people's shoulders in the Laboratory all afternoon, and deal with the fishery-statistics of Aberdeen in the evening, you would not have much time to amuse yourself either!

iv. I have doubtless lots more to learn with regard to ACA's formulae as well as your own. But it seems clear that your (n, m) Table beats ACA's binomial coefficients easily, — both for speed and simplicity.

v. In Dublin, last week, (whither I had gone to lecture on Lilliput and Broodingnag) I was given a handsome present of fircones of many species, large and small, with which I have since been exhibiting Fibonacci numbers to a none too interested Class.

vi. The Laboratory Boy was more interested therein than any student.

vii. During the three days I spent in Dublin, I heard no single soul allude to H., M. or even N.C.

viii. With regard to the essential difference between PAM and RAF, not only rem acu tetigisti: but you may be said to have 'had a glimpse of the obvious'.

ix. In an Edinburgh Auction-room last week, on my way to Dublin, I bought, for half-a-crown (2/6), Alexander Anderson's Problemata, Paris, 1612; with Poncelet's Propriété projectives and Marianini's Porisms, thrown in! Alex. Anderson being, as you will remember, brother to Janet, who married the Minister, and became the ancestress of the long line of "Academic Gregories".

x. That's all for the present. Q.E.F.

D.W.T-

No mention of ACA's largest root of an equation. — I asked for a reference[23]

G. T. Bennett— letter, StASC ms 26110

EMM COLL CAM
7.2.39

***

DWT

****

Glad to know that you were silently appreciating the difference-equation for the sub-classes of the rhyme-count. My approach to it was cancrizans: for I got the arithmetic before the substance!

[23] This was added in pencil by Bennett

Your Lab. Boy almost deserves the formula for the sum or difference of two Fib. squares. — A suitable floral frieze for a Botanic Lecture Room would put the fir-cone and the pine-apple and the sunflower &c &c all in their proper sequence, without being more explicit.

Trouble not at all about the largest root as got by ACA, for I have found his paper. He has, I see, no proper horror of mixing trigonometry with algebra. Very British!

If busy with the osteology of the kangaroo you must have in mind the way his jaw works when he feeds. Has the mandible 3 degrees of freedom (as for Man) or not more than 2, or only 1? My brother is excogitating these matters and finds the Zoologists not very informative. For the carnivora the movement is a simple hinge-movement, is it? I have been looking in the Museum, but the loss of all the cartilage leaves me doubtful of the working of the original mechanism.

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GTB

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Appendix C: The EMS letters

Stokes to Baker, 2 June 1923

53 Mosspark Drive
Mosspark
Glasgow 2nd June 1923

Dear Baker

I wish regretfully to give notice of resignation from the office of Co. Editor of the E.M.S. whenever it is found possible to appoint a successor. I do feel that i am not doing my fair share of the work partly owing to the multitude of new subjects I have to take up in connection with my new appointment. It is hopeless for me to keep up with any one specialist department of maths. now, and I also feel that the school aspect of mathematics (with which I am now necessarily more identified) is being steadily neglected by the Society simply on account of the expansion of the specialist research.

Should the committee find an enthusiast who would take up the Co-Editorship I am sure it would be to the Society's advantage to relieve me of office.

Yours faithfully
G.D.C. Stokes

Turnbull to Copson, 27 Jan. 1927

P.S. The Committee asked me to attend to the matter of type + and printing. I await a letter from you soon.

The University,
St Andrews
27 1 27

Dear Copson,

Thank you for your letter & suggestions. We had certainly better talk it over at the Committee on Friday week. Please would you be so good as to find out informally if Dr. Horsburgh would be kind enough to put me up that night?

The whole conduct of the publications of the E.M.S. has been comparatively free & easy in the past, compared with that of the L.M.S. There are some advantages in both methods. First we must remember that we are guests in a country which is not England, & for that reason must not be too obviously anxious to mould the Societie's [sic.] *Proceedings* on London lines! So I am glad you wrote to me in the first instance.

I am not sure that the *Proceedings* should be so undiluted severe as you propose, because the membership of the Society is relatively broader than that of the L.M.S. That is why I have welcomed papers of a more general interest, such as Gibson's History. In any case an expert paper on the History of a scientific subject - especially one like math^{cs}, which unlike some other sciences, is not a mushroom growth of the last few decades, is worthy of a place in the best journals.

So first we should be clear as to the main duty of the officers of the Society towards the members.

As to refereeing of papers, I agree with your[24] proposal in general [25] & in fact have acted on it as far as possible. But I am not prepared to abrogate the duties of Editor which I was asked to undertake along with MacRobert, unless the Society sees fit to alter the existing arrangement.

More when we meet. Meantime I shall correspond with MacRobert over the question. Thanks for writing.

Yours very sincerely,
H. W. Turnbull.

[24]The words'the main' are scored out and replaced by 'your'.
[25]The words 'in general' are added later.

Copson to Turnbull, 31 Jan. 1927 — Draft

16 Chambers Street,
Edinburgh
1926 January 31[26]

Dear Professor Turnbull,

Thank you [27] for your two letters. I am pleased that we seem to be [28] in agreement over the more important[29] points.[30]

First of all, I must disclaim any desire to encroach upon the duties [31] of the Editors. I have quite enough work to do for the Society as it is. But I feel that we ought to define what the duties of the various officers are. The rules contain no explicit mention of the duties of the Editors; the only rule which seems to have any bearing on Publications is Rule 11. [32], which reads 'Abstracts of all papers read before the Society shall be left for preservation in the hands of the Secretary.' This is truly an undesirable state of affairs; the rules should be brought up to date.

The procedure with regard to the publication of papers which I suggested was arranged so as to enable the Secretary and the Editors to work without much waste of time. As Secretary, I feel that I ought to see the papers before they are read so as to be able to provide interesting meetings for the members. Some papers may be read and provoke valuable discussion, others are best read by title and considered by members at their leisure. The procedure I proposed would allow me to discriminate between the two types of paper and enable me to preserve some sort of balance between the geometrical, the analytical and more applied interests of the members. I do not at all[33] desire to veto papers before the Editors have a chance to see them. In fact, in the past I always have submitted every paper I received to the Editors.

In the proposed procedure, I suggested that I should send the papers to referees; this was suggested merely to save time and cost of postage.

The next question that arises is that of the *Mathematical Notes*. At present, the journal is, for all practical purposes, dead. So for this session, only one paper has been received for publication in the *Notes*. I proposed a more rigorous rejection of papers from the *Proceedings* so as to have more matter for the *Notes*. For example, the paper of Dr McWhan's is very suitable for publication in the *Notes*, but, to my mind at any rate, is not at all desirable in the *Proceedings*. It seems to me that there are two alternatives open to us:- (a) keep the *Notes* going and reject more papers from the *Proceedings*, or (b) scrap the *Notes*. The Committee may feel that (b) is preferable, and decide to enlarge the scope of the *Proceedings* instead of restricting it. I should be very glad to hear your views on the subject; I have no personal opinions in the matter, but was supposing that (a) would be preferable.

[26] This date is wrong, the correct year is 1927.
[27] 'thanks' changed to 'thank you'
[28] The word 'are' was changed to 'seem to be'.
[29] The word 'some' was changed to 'the more important'.
[30] The term 'at least' is scored out.
[31] This originally read 'usurp the powers'.
[32] was 'no mention whatever of Editors and Publications, except possibly Rule 11
[33] The words 'at all' were added later.

I must, last of all, explain my action in regard to the type. I heard nothing at all from Lindsay's for a week after your visit to them, so I went to see what they were doing. They gave me a proof-page in pica[34]. This I showed to Professor Whittaker, Mr Gibb and Dr Aitken, and we all agreed that the type was far too large. By consulting the article on Typography in the Encyclopaedia Britannica, we found that the type in use by the L.M.S. and R.S.E. is small pica[35]. Accordingly I asked the printers to set up a specimen page in that type. This they have done. I do not feel that by doing so I have exceeded my powers as Secretary; but if I have, I am very sorry.

I have asked Dr Horsburgh to put you up, and I expect you will hear from him soon. If you intend to go to Prof. Eddington's lecture at 5 p.m., please come up to the Department; tea will be going then.

Yours very sincerely,
E T Copson

Turnbull to Copson, 1 Feb. 1927

2 Queens[sic] Terrace
The University,
St Andrews
1.2.1927

Dear Copson,

I have been suddenly called south on urgent family matters & I shall not be able to come to the Friday meeting. My wife has just had the sad news that her Mother has died.

Please will you explain my absence to all who are concerned?

I have asked MacRobert, if he agrees, to confirm the decision that the third specimen of type is best - small pica - so please will you carry on for me such details as may be necessary to get the printers to print the work? They are awfully slow over it.

I think if you explain to MacRobert the points you put in your second letter, they will clear things up a bit. But you must not expect him to agree with your idea that the E.M.S. is a replica of the L.M.S., for he won't!

Yours very sincerely
H.W. Turnbull

P.S. Saddler will come over.

[34] was underlined
[35] was underlined

MacRobert to Copson, 3 Feb. 1927

6 Lothian Gardens
Kelvinside N.
Glasgow
3rd Feb., 1927

Dear Copson

I received your letter today, and am quite willing to discuss the question of policy tomorrow.

As to the papers you mention, they have already been passed by the editors and sent on to the printers for publication, and consequently I consider that it would be very inadvisable to bring them into the discussion. As you know, Dr McWhan's paper was to be read at the Glasgow meeting, but as that meeting has now been postponed, you might have it read by title tomorrow night.

With Kind Regards,
Yours Sincerely
Thomas. M MacRobert

Horsburgh to Copson, 4. Feb. 1927

Friday 4th Feb 1927

Dear Mr Copson

I regret my absence from todays Committee meeting, but it was the only day on which my visit to Colville's (Motherwell) could be arranged.

I understand that the general management of the E.M.S. is to be the subject of a friendly talk in Committee; and that members are requested to state their views.

I suppose we must recognise this new departure in publishing the Proc. E.M.S. as the outward and visible sign of the break with the Traditions of the early years of the Society. It stood then for scholastic mathematics.

The Society now seems likely to get more papers than it can afford to publish. What is to be the procedure? It seems to me that this depends on the finances.

I do not see how more papers can well be published as already the annual subscriptions pay little more than half the working costs.

It seems to me that it will often be necessary to invite the contributor of a paper to become also a contributor to the cost of publication:- failing that paper would need to be cut down if too long, I suppose by the contributor.

This may not be a very pleasant duty for the Editors to oversee.

Also as the schoolmasters form the main strength of the Society it is desirable to cater for their requirements, as far as finances allow; since he who pays the piper should certainly have a say in the choice of tune.

It would be interesting to hear their views on this subject.

Would they (for example) think it desirable to have the '*Notes*' published under a more imposing title, so as to include '*Notes*' and abbreviations of papers, which, tho' they do not profess to enlarge the boundaries of Mathematics, may yet be of interest to teachers of mathematics, pure and applied?

But again all this turns on the available funds.

Yours sincerely
E M Horsburgh

Turnbull to Copson, 8 Feb. 1927

The University,
St Andrews
8 Feb 1927

My dear Copson,

Thank you for your letter & enclosure. Also for your kind message. It has been a rather bad time for Mrs Turnbull as she was ill with the 'flu also. I have left her in the south, & had to rush back alone.

Perhaps we could meet for a talk at the end of the month. I'll let you know. The fact is that all our difficulties of adjustment are caused by the very success of the society, which is evidently expanding its scope.

The book you send is evidently a result of the close connexion Indian Maths^{cs} has with Edinburgh. I think it would be well to review it for *Notes.* Shall I set Saddler to read it.[sic] Perhaps you will consult Aitken on the matter & tell me.

The type for *Proceedings* is excellent.

Sometime, either in March or at the St Andrews meeting I should like to communicate some remarks on determinants which might be generally interesting to scholastic members as well as university members. If March is already full up, please put it on later if you prefer. Even next autumn might be better then, because at St Andrews, we don't get so many general members.

I certainly understood the Colloq^{m} account should go into *Notes.* But more later when we meet.

Yours sincerely
H W Turnbull

Turnbull to Copson, 12 March 1927 — Postcard

Postcard addressed to E T Copson, 16 Chambers Street, Edinburgh

MacRobert thinks a circular sent to all Sec. school math [36] teachers in Scotland with information that

1. Prof^{r} Gibsons Colloq^{m} history lectures will appear in next *Proceedings.*

[36] The word 'math' is added later.

2. Hint about improvement of *Notes*.

will bring in new members, I think now would be a very good time for this.

If you agree will you do something short + pithy in this way, in the near future?

I am also writing to Aitken about drawing up a circular with fuller details, to go perhaps to our members with next billet of meeting.

H.W.T.
2 Queen's Tce, St. A.

12.3.27

MacRobert to Copson, 6 April 1927

6 Lothian Gardens
Kelvinside N.
Glasgow
6/4/27

Dear Copson

At a recent meeting of the Euclidean Society it was suggested that the E.M.S. might invite the members to their next Glasgow meeting.

Do you think there would be any objection to this? If the Edinburgh members of Committee are willing, I should get the Secretary of the Euclidean to send out invitations to his members. There would be no need for your to put anything about it in the Billet.

I hope you are having a good holiday.

With kind regards
I am
Your Sincerely
Thomas M. MacRobert

Turnbull to Copson, 21 April 1927

The University of St Andrews
21 4 27

Dear Copson,

To answer a few points of your letter.

Copies printed. McR & the Glasgow people agree about increasing this and we recommend 450 to 500 copies should be made. So please will you see Youngson about it if it commends itself to you at Edinburgh? It is a matter for the Sec^y & Treas^r I should say.

Paper. We all agree about the old being good enough.

Cover. MacRobert suggests blue, & I have asked for specimens. Thicker paper by all means.

Index. Yes, one per volume & current index on cover. But what about Bell's advertisement? The LMS advertise Camb. on the back. Perhaps you would let me have your views.

Next meet[in]g. Would Prof^r W. lecture on Euclid? It would be highly interesting. McR is asking him.

You would do the Society a good turn by getting things settled clearly with the publishers. I think we ought to be cautious about the 1^{st} series, as it may run short.

Will you ask Aitken about preparing an index & what help he wants?

The first part (70 pages) is well in hand, & but for the question of *Notes* could really appear by the Meeting at Gl. or soon after. Anyhow we must settle that. There should be a good turn out of Glasgow members & we can get a representative decision.

We are all fit, thank you. Derwent, alias DG, has just informed me that he wants to write an index of my book! I wish he could, but 'tis too precarious a venture.

Do you know anything of Haslam Jones (Queens. Oxon)? I am after a lecturer to succeed Guthrie, who goes to Cambridge.

Yours very sincerely
Herbert W. Turnbull

P.S. What are the power series $\sum a_n z^n$ when $a_n = \frac{1}{(n-1)!n!}$; $a_n = \frac{(n-3)!}{n-2!n-1!n!}$, $a_n = \frac{(n-4)!}{(n-3)!n-2!n-1!n!}$ etc? They turn up in the theory of determinants rather curiously. The 1st is related to Bessel Functions?

Turnbull to Copson, 14 May 1927

2 Queens Terrace
14 5 27

Dear Copson

Vaid—y's paper can be read by title (which I sent you)

I am suggesting the cover which you enclose, to McRobert [sic]. I am not enamoured of degrees etc; + in such a matter prefer to leave it to the practice of the country wherein the paper is printed. I'll forward your suggestion. Only let it be clear that the list of members should bear degrees etc.

Thank you for finding misprints. Are there enough to warrant my having another look through? The printers have an awkward habit of inserting new misprints after corrections have been made by the authors.

On the whole I incline to the idea of separate *Notes*, same sized paper etc, separate pagination. The main thing was to get the Glasgow Cee. to interest themselves in the *Notes*, + I think this has been done.

By all means send off free copies to other Universities.

I don't think the question of *Notes* + Research Papers need worry us. Once the *Notes* are established again as a regular thing, people will be glad enough to have their work printed in it. In any case, authors must leave it to some responsible officer of a Society to decide what happens to his paper. I'll get Whittaker + a few others to send in some *Notes* (I have my eye on two such Notes already). Then the *Notes* will have the necessary prestige! Don't forget to try + get a local subeditor of *Notes* in each area, who can keep in touch with Aitken or whoever is general editor of *Notes*.

We shall get it all fixed up in June.

I fear I shall miss the Edinburgh C.U. Camp. Our holiday is a week earlier + doesn't help me as I had hoped. I hope you will have a good time.

Yours sincerely
H W Turnbull

P.S. many thanks about the problem. Please solve it if possible

'Infinitesimal analysis of an arc in n-space' by Sen

Part is old work. But the rest which is heavy + rather dull, may be original. I'll find out more about it.

I suggest: Read by title.

MacRobert to Copson, 20 Oct. 1927

6 Lothian Gardens
Glasgow, N.W.
20/10/27

Dear Copson

Many thanks for your paper, which I will send on to Turnbull. I looked over it with much interest.

I must apologise for the delay in answering your very kind enquiry about Hyslop. I was waiting on word from his father.

His doctor thinks that a visit from any of his friends at present would upset him. If visitors are allowed later on I will let you know. The prospects of recovery seem to be brighter than they were.

Yours very sincerely
Thomas M MacRobert

P.S. I don't know if Turnbull mentioned to you that I intended to resign from the Editorship at the next meeting. This has been my intention all along, and the pressure of new work at present makes it impossible for me to carry on longer.

T.M.M

P.P.S. I have arranged for the meeting of the E.M.S. on 2nd Dec. and 9nd March at 6.30. p.m. I will let you know the arrangements for tea nearer the time.

Turnbull to Copson, 30 Nov. 1927

The University,
St Andrews
30 11 27

Dear Copson,

Thank you for returning the MS. I'll see if it can go in this next number without unduly delaying the publication.

Who would take on *Notes* or its successor? Please don't let them call it 'Bulletin', for any other name would be sweeter. By the time something is done, that account I wrote of the Colloquium will be a trifle ancient!

J. Williamson knows something about Division Algebras, and might tell us of it. Also, as you know, there are questions on the validity of q-numbers which might be discussed - especially if Darwin would come forth and uphold them.

Yours sincerely,
H.W. Turnbull

[Added in pencil in Copson's hand: Not Darwin. He has given up on q numbers entirely]

Turnbull to Copson, 17 Oct. 1928

2 Queens Terrace
The University,
St Andrews
17 Oct 1928

My dear Copson

Many thanks for your letter. One thing is evident: authors must get something in the way of offprints. Saddler also complains he has had no offprints. Can you get the printers to chop up 15 copies of proceed. and distribute the buts to the authors as far as possible? If not, can they reset 4 pages of Sen's paper and charge a nominal fee for it?

Those printers make me pretty wild, and I should like to propose to the Committee through yourself in my absence that we take steps to find out a better firm. How about Constables or Neill?

This sort of thing will simply get the Society a bad name through no fault of our own.

I expect Sen over here on Friday and will hand him his paper.

Better call the *Notes, Math. Notes* for the time being. We must publish them in order to encourage the teachers. Don't let Aitken delay too long in publishing them.

Yours v. sincerely,
H.W. Turnbull

MacRobert to Ruse, 28 Jan. 1931

10, The University, Glasgow, W.2
28th January, 1931

Dear Mr. Rouse [sic]

I have decided, after much consideration, to resign from the committee of the E.M.S. I ought to have done this years ago; and, as you know, I intended to withdraw at the beginning of this session, but agreed to remain on the committee this year in order to take part in the discussion of the new constitution. All the points of importance have now been dealt with except the title of the new journal, and regarding that I said all I wished to say at the last meeting.

During the last four years I have found myself so out of sympathy with the views of the majority of the committee that my resignation should add greatly to the harmony of the meetings, as well as to my own peace of mind.

My relations with all the members of the committee have been very pleasant, but for a great many reasons I wish to retire, and I have decided to take the step now.

Yours, with kind regards,
Thomas M. MacRobert

P.S. If there is any difficulty about the programme for the Glasgow meeting I could give a paper.

MacRobert to Ruse, 11 Feb. 1931

10, The University, Glasgow, W.2
11th February, 1931

Dear Mr. Ruse

It was very kind of the Committee to ask me to reconsider my resignation; but, while I fully appreciate their goodwill, I am quite decided that it is best for me to withdraw.

As I wrote to you before, my actions is entirely personal. In taking it, I do not represent any particular body of opinion. As regards Dr. Hyslop's suggestions, they are, I expect, entirely his own. His views and mine do not necessarily coincide. In such a society as the E.M.S. I think that each member should be on an equality with any other, and that members of a University Staff should, if they wish, act in independence of or even opposition to their chief.

I will give some notes on Fourier Integrals at the Glasgow meeting. I have heard that both Dr. Aitken and Mr. Ross are giving papers, so it will not be necessary for me to say much,

Many thanks for your letter.
Yours sincerely
Thomas M. MacRobert

Ruse to Committee Members, 26 Feb. 1931 — Agenda

Edinburgh Mathematical Society

Notice to Members of Committee

In accordance with a decision made at the last meeting, a copy of the agenda will be sent to each member about a week before future Committee Meetings.

The next Committee Meeting will be held at Glasgow at 5.30 p.m. on Friday, 6th March.

AGENDA

1) Resignation from Committee of Professor MacRobert, (and selection of nominee for vacancy if resignation is accepted).

2) Consideration of future policy of the Society, especially in regard to its relations with the Glasgow Mathematical Association.

(Discussion adjourned from the last meeting, at which Dr Hyslop suggested that there were three courses open to the Society, namely

(i) A continuation on the present lines, but with a definite effort to cater for school teachers.

(ii) The establishment of branch centres of the Society in Dundee, Perth, etc., thereby creating an organisation similar to that of the Math. Assoc. of Gt. Britain

(iii) A development as a purely research Society, with no effort to interest school teachers.)

3) As may arise.
26th Feb., 1931

H. S. Ruse, Hon. Sec.,
16 Chambers Street,
Edinburgh

McBride to Ruse, 12 March 1931

Oakbank,
Callander,
12th March 1931

Dear Mr Ruse,

Your letter was forwarded to me at Callander where I have been rusticating for five or six weeks. I have no objections to being put on the Committee of the Mathematical Society, although I do not think I was of much service when, some years ago, I served a term or two.

The Euclidean Society was formed in Glasgow about three years ago with the object of re/establishing Euclid's text as a book for School Geometry, but after some time it became quite clear to the organisers that the younger generation of math.l. teachers did not know Joseph's compatriot, and it gradually became a means of bringing together western math.l. teachers for purposes of discussion etc.

Another idea was that it would be a recruiting ground for the Ed. Math. Soc^{y}.

I shall be in Glasgow for the rest of this month, and, without saying anything about your letter shall gladly see Mr Arthur and the others, as indeed Mr Arthur has already asked me to do.

Yours faithfully
Ja. A. McBride

McBride to Ruse, 27 March 1931

The Literary Club,
198 Bath Street,
Glasgow.
27th March 1931

Dear Mr Ruse,

Since writing to you some weeks ago from Callander I have found that it will be necessary to go to Ireland for a considerable time and I intend to cross next week. It is unlikely that I could be in Edinburgh on the first of May, and I think it better to ask you to proceed no further with my nomination as a member of Committee of the Edinburgh Mathematical Society. Indeed I shall be so little in Scotland during next winter that I should only keep a more useful member out, and be of no service to the Society.

It is quite certain I shall be a good deal abroad —if not actually as far as America
Many thanks, however, for your kind proposals.

Yours faithfully
James A. McBride

Arthur to Whittaker, 26 March 1931

148, Carmunnock Road,
Cathcart,
Glasgow.
26th March, 1931

Dear Professor Whittaker,

I explained your proposals for co-operation to the Committee of the Glasgow Mathematical Association at a recent meeting. Considerable discussion followed, in which a large number of members took part. Those who spoke were almost unanimously against undertaking in the meantime, in any circumstance, responsibility for the production of a mathematical journal. The Association, though thriving, is still in its infancy, and the general feeling appears to be that, for the present at any rate, its policy should be a cautious one, and that financial commitments are to be avoided.

Since that meeting I have been in touch with several influential members of the Association and I find that their views are much the same as those expressed in committee.

As a result of my enquiries I am of opinion that, if definite official proposals on the lines of your suggestion should be submitted to the Association, there would be little likelihood of their being accepted.

Yours sincerely,
W. Arthur

Whittaker to Arthur, 4 April 1931

48 George Square,
Edinburgh
1931 April 4th

Dear Arthur

Many thanks for your letter. The decision of the Glasgow Mathematical Association is to me disappointing, as I had hoped that by their co-operation we might have had in future two mathematical periodicals, each excellent of its kind, published in Scotland. Now, I don't see clearly what should be done. It isn't possible for the Edin. Math. Soc. permanently to finance both publications, or even the *Proceedings* alone, without help: and the most likely possibility of help, namely a grant from the Royal Society's publication fund, we should probably not get unless we professed to be purely a research society, and dropped the *Notes* or *Journal* altogether.

Anyhow, will you please talk the situation over with Professor MacRobert, and then we can discuss it at the next Committee meeting: and if you have any suggestions, please let me have them, so that I can turn them over in my mind before then.

With kind regards
Yours sincerely
E. T. Whittaker

Arthur to Whittaker, 20 April 1931

20.4.31

Dear Professor Whittaker,

After giving a good deal of thought to the present position of the Edinburgh Mathematical Society, in the light of your recent proposals, I have come to the conclusion with you that it has become definitely impossible for the society to cater harmoniously for both teachers and those interested solely in advanced research. I can see no really good reason why this should be so, and why the Society should not, even on the expense of ambition, do something for the Mathematical teachers of Scotland to whom it owes its existence. However it seems now to be clear that an open change in the character and purpose of the Society is to be made, and as a strong dissentient from this policy I feel bound to resign my position as Editor of *Mathematical Notes* and a member of Committee. This I propose to do by the post which carries this letter. I have no doubt my resignation will facilitate the carrying out of the changes contemplated.

I need not add that I very much regret being constrained to this course, and that I shall keep pleasant recollections of association with yourself personally and with the other members of committee.

Yours sincerely,
W. Arthur

Arthur to Ruse, 20 April 1931

148, Carmunnock Road
Cathcart
Glasgow
20th April, 1931

My dear Ruse,

I beg to render my resignation of the office of Editor of *Mathematical Notes* and of membership of the Committee of the Edinburgh Mathematical Society. I do so because of strong disapproval of the change in character of the Society which is advocated by Professor Whittaker and appears certain to be carried out in the near future. However the change may be effected, it involves the cessation of all attempts to cater for those members whose interests are not solely centred in research; and to that I cannot be a party.

It is with great reluctance that I take this step, which means the breaking of close association in committee with men for whom I have a very high regard and with whom my relations have been uniformly happy.

The MSS which I have on hand I shall send to you in a day or two.

Yours sincerely,
W. Arthur

Whittaker to Arthur, 21 April 1931

1931 April 21

My dear Arthur

I am very sorry to hear of your decision, and particularly sorry it should be, in great measure, a consequence of my own activities in the Society.

As you know, I believe that the cause of mathematics in Scotland will be best served by having two societies instead of one. I hope to support them both so far as I am able, so I do not look on them in any way as rivals, but rather as friends who have agreed on a partition of duties. If the Edin. Math. Soc. had been at this moment what it was originally - namely, a school teacher's society - the natural thing would have been for me and my researchers to start the second society outside it: but there is no disguising the fact that for the last 20 years the Edin. Math. Soc. has been slowly changing its character, and that at the present time it is mainly a research society: so the common-sense solution is that it should continue as a research society. The foundation of the Glasgow Math. Association, which is evidently destined to be the other society, is an additional reason for solving the problem in this way.

I hope the Glasgow Math. Assoc. will extend its worth to the East of Scotland, where I shall always be glad to do anything I can to help it.

With kind regards,
Yours sincerely,
E. T. Whittaker

Ruse to Committee members, 14 Oct. 1931

Letter sent to the members of Committee:

16 Chambers St.
Edinburgh
14 Oct. 1931

Dear [name to be inserted]

Prof. MacRobert has not this year renewed his invitation to the Society to hold some of its meetings in Glasgow. The matter was discussed informally to-day by five members of the Committee, who beg to submit the following suggestions for your approval

1.That the January meeting be discontinued, an alteration in the Rules being made to that effect.

2.That all meetings be held in Edinburgh except that in June at St Andrews

3.That a course of addresses on general topics be given instead of the customary reading of research papers. The programme proposed is as follows:

November: Address by Retiring President. (Subject to be decided later)

December: Address by Dr McCrea in honour of the Maxwell and Tait centenary

February: 'Careers for honours graduates in mathematics' by Prof. Whittaker

March: Research papers

May: 'Mr. H. G. Forder's recent works on geometry in relation to school teaching' (It is proposed to ask Dr Black to deliver this address)

June: 'Mathematical Statistics', by Dr Aitken

I should be grateful for an early reply, if possible by return of post, since it is desired that a copy of the programme for the year should be sent to members of the Society before the next meeting, and also published in educational journals.

It is regretted that the unforeseen circumstances which have arisen necessitate the present informal procedure. A Committee meeting will be held as usual on 6 November, and the agenda will be sent to you before then.

Yours sincerely,
H. S. Ruse.
Hon. Sec.

Turnbull to Ruse, 15 Oct. 1931

The University of St Andrews
20 Oct 1931

My dear Ruse

Many thanks for your kind letter. We shall be delighted to welcome the Society in March and June. The Sat^y arrangement is excellent.

It is good of you to think of considering my suggestion. I should be quite content with the existing arrangements without adding a January meeting as an experiment. If only it is not stereotyped in the future. So just do as you think well. In any case I think that the definite programme should be a great inducement for a better attendance and for increased interest among teachers.

I am very sorry to hear of Hyslop's poor health.

With kind regards,
Yours sincerely
H. W. Turnbull

Turnbull to Ruse, 20 Oct. 1931

The University of St Andrews
15 Oct 1931

My dear Ruse

Thank you for your letter. I am very sorry that the Glasgow lapse has come; but that cannot be helped! I should have liked to have had the opportunity of inviting the Society to meet here besides in June. But I quite realise the need for a quick decision.

You have my blessing, and I hope that the *Proceedings* of the meetings will prosper.

Thinking over matters, I rather feel that we may be going too far in the other direction, by practically ignoring research papers. After all, some of us (and I speak personally) have learnt a great deal from hearing them read, even if we have not always understood every word. However, the programme is a brave gesture, and surely McR. must be gratified.

Yours sincerely
H. W. Turnbull

Copson to Ruse, 15 Oct 1931

Mayfield,
St Andrews,
Fife
15.10.31

Dear Ruse,

Thanks very much for your proof of Rutter's paper with its valuable corrections; also for the Committee's report. By the way, it looks as though the Committee intend to decide what my Presidential Address is to be about!

A. C. A. has the papers by Wilson and Mitra and can give you the titles; I don't remember them and want them, so let me know, please, when you find out. Carslaw has sent in two papers: -

1. A Trigonometrical Sum, by H.S. Carslaw and R. S. Lyons
2. On Cramer's Theorem concerning the Gibbs' Phenomenon, by H. S. Carslaw.

(Put the apostrophe in the right place, or people will wonder what David has been up to!)

I saw your O. J. paper yesterday — a pleasing affair. Don't they print well? I should imagine yours wasn't too easy! I sent one to Ferrar yesterday and am wondering what will happen to it.

Cheerio.
ETC

Eastwood to Ruse, 15 Oct. 1931

Craigie Lea.
Cowdenbeath. Fife.
15/10/31

Dear Mr. Ruse,

Many thanks for your note which I received this morning.

I am in cordial agreement with the members of the Committee in their suggestions contained in your note.

Yours sincerely,
Geo. S. Eastwood

Lidstone to Ruse, 16 Oct. 1931

Hermiston, 16.X.31

I concur to the recommendations, as above, made by the five members of the Committee who have had the opportunity of [illegible word] together and considering the matter with full knowledge of the circumstances. Without such knowledge, I do not quite see why the discontinuance of meetings in Glasgow should lead to a complete alteration in the nature of the meetings themselves, but I am sure there must be good and sufficient reasons for them.

G. Lidstone

Lidstone to Ruse, ??

Hermiston House
Hermiston, Currie
Midlothian

Dear Mr. Ruse,

Many thanks for writing me so fully. I confirm my concurrence in the decisions of the five members. It seems a great pity that the old Glasgow-Edinburgh feeling has turned up again. At the same time I must confess that I have great sympathy with teachers, and other moderate mathematicians, who feel that neither the *Proceedings* nor the meetings are of any use to them, since they consist almost entirely of matters much too high for them, and these not explained or taught but assumed and developed. No doubt such people look upon this Society as practically the Publication Bureau of the Research-workers of Edin. Univ. As such, I am personally interested in it though I have to 'trust where I cannot trace', but I can quite understand the school-teachers to whom even the small subscription is a consideration may feel that they are not getting much out of it. As a stop-gap the lectures or general talks which you are arranging may be all very well, but I doubt if they will be a permanent remedy as they are too general. I should have thought that what the teachers wanted (needed) was some development of the *Mathematical Notes*, encouragement to contribute them, and opportunities for discussing them at a proportion of the meetings, when they would have the advantage of learning from some of the more advances members. No doubt it may be said that this is practically turning the Society, partly at any rate, and overlapping with the *Math. Gaz.* But there may be much virtue in the 'partly', and I don't see how we can expect to retain the subscriptions of the un-advanced members unless we do something more for them.

Don't trouble to answer this. I am really not "in" this sort of thing, and therefore unable to express an opinion except on very general grounds.

Yours sincerely
G. Lidstone

Black to Ruse, 16 Oct. 1931

32 Eastfield.
Joppa.
16th Oct. 31

Dear Dr. Ruse,

I have received your communication regarding arrangements for the E.M. Society for the coming session.

I am extremely sorry about the Glasgow business but it would appear we have no option.

Please consider me as supporting the five members of Committee in all three suggestions they have made.

With regard to the May address on Forder's works on Geometry, I have carefully considered the matter and am willing to undertake the job. I appreciate very highly the kindness of those who make the suggestion that I should be asked to give this address and I will endeavour to justify their confidence.

Could you perhaps get from Professor Whittaker a complete list of Forder's works and let me have it at an early date?

Yours faithfully,
T. P. Black

Black to Ruse, 21 Oct. 1931

Trinity Academy
Leith
21 October 1931

Dear Mr Ruse,

Thanks for the note of Forder's proofs and also for the reference to the review.

With regard to the further advertisement of our lectures, I suggest that a letter be sent to the Chief Education Officer intimating to him the attempt we are making to interest teachers in our work and asking whether it might be possible to send a circular letter to the staff of schools concerned commending our lectures to the teachers.

I think it might be worth while extending the scope of such a scheme to include at least Midlothian. The Chief Education Officer of the latter county is, I think, a former Maths. Head.

Yours faithfully,
T. P. Black.

Hyslop to Ruse, 17 Oct. 1931

127 Irish Street,
Dumfries
17th Oct /31

Dear Sir,

Your letter of 14th inst addressed to my son Dr. Hyslop has been passed on here. I deeply regret that my son has been unwell for some time and is so unwell at present that he is unable to take up his duties at Glasgow. I may be able to see him this next week and if possible shall speak to him about your letter in case he has any suggestions to make.

I may say my son has discussed with me more than once the unfortunate position as between Edinburgh and Glasgow and I know how anxious he was to see the relationship improved, but I do not recall that he placed any blame upon an individual in Edinburgh. I am very sorry to add that I cannot hold out hope of assistance from Dr. Hyslop for some time so far as anything might depend upon him personally. If however I can get any useful suggestion meantime, I shall write to you again.

Yours faithfully
Jas. Hyslop

Etherington to Inglis, 7 Dec. 1937 — Draft

7 Dec 1937

Dear Dr Inglis,

As I think you were not at the meeting of this Society on Friday, I am writing to let you know of a change in the programme of meetings for the session, which was initiated by the President during the meeting.

The change is made in response to your criticism at the meeting on Nov. 5th. Mr Lawson had approached Dr Mackie of Leith Academy, and ascertained his willingness to open a discussion on the teaching of mathematics or some aspect of it; this will replace the "Research papers" on Feb. 4th. I hope you will regard this as satisfactory.

I had the intention of bringing the matter up for discussion at our Committee meeting on Friday, and was not in the least sorry to find that Mr Lawson had acted on his own initiative in approaching Dr Mackie. The Committee readily agreed.

I cannot say anything about the possibility of catering for teachers at the St Andrews Colloquium in the way you suggested. Our time in Committee meetings has been fully taken up in discussing the main lecture courses on pure and applied mathematics. As at previous colloquia, there will also be single lectures and discussions of an informal kind, but these have not yet been considered, as they are more easily arranged at short notice. (I understand that Professor Whittaker has explained to you his own views about this, and that he is not in favour of our arranging lectures specially for teachers.)

May I say that I am myself very sympathetic to your point of view, and would personally be very glad if the Edinburgh Mathematical Society could be a stimulating and progressive influence on mathematical teaching in Scotland. I have however to carry out the instructions of the Committee. As to why the programme of meetings for this session was originally drawn up in forgetfulness of teacher's interests, I think perhaps the responsibility really rests on the representatives of the teaching profession in the Committee. (The programme was considered at the June committee meeting at St Andrews, when they perhaps found it impossible to be present; but the agenda were circularised by post.)

Yours sincerely,
Hon. Sec.

Inglis to Etherington, 11 Dec. 1937

Morig,
Newport,
Fife
11th December 1937

Dear Dr Etherington,

Many thanks for your letter of the 7th December. I welcome very much the prospect of a discussion opened by Dr Mackie of Leith Academy on 4th of February, and I hope that many teacher-members will take the advantage of is. But there I am afraid is the snag! There is no doubt that Prof. Whittaker is right when he pointed out to me that very few school teachers attend meetings of the Society. Whether that is due to the type of programme offered or to the laziness (or indifference) of teachers to attend, I am unable to say. Probably is it a combination of both.

Concerning the Colloquium I suppose that the same thing applies also, but nevertheless if at least I have "planted the seed" which may possibly bear fruit sometimes in the future, then maybe I shall have done something. As you say discussions may be arranged on the spot, as it were.

Speaking generally of my own views is that it is time that there was some such organisation in Scotland as the Mathematical Association in England. There is no provision at all for the teachers of Mathematics in Scotland who wishes to know something of the progress made by others in his own craft. For this I blame primarily the teachers themselves, who are too indifferent to organise such an Association, and secondly I blame the Training Colleges. The latter seem to be staffed for the most part by those who do not have first hand knowledge of the teaching problems that confront us in the modern schools.This was made quite evident by the paper Mr Taylor gave last January. The Training Colleges provide Summer Courses for teachers in Infant and Junior Schools, and give courses in Rural Gardening, Country Dancing and -possibly- Elocution, but as for a course say on the best way of teaching logarithms to those who know no algebra, or the best way of teaching the convergence and divergence of series to those who have merely reached the standard of the leaving certificate, these problems are never attempted; perhaps because there is no one in the Training Colleges competent to deal with them, or perhaps because if someone did attempt to deal with them then he would have no audience. Through the Central Mathematics Committee of the Educational Institute I have put forward this point of view, but I doubt is if will come to anything.

As fas as I can see at present the only hope of such matters being systematically dealt with is the E.M.S., and hence my remarks at the meeting of 5th November.

I am very grateful to you for the trouble you have taken over this matter, trouble which may be of no avail as I have tried to explain, and I am sorry that you have had this extra work. However, perhaps in about a hundred year's time the name of Etherington will be regarded as that of the prime mover in the foundations of a Scottish Mathematical Association!

With many thanks and apologies,
Yours sincerely,
Alex. Inglis

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