EXPLAINING MEDIUM RUN SWINGS IN UNEMPLOYMENT – SHOCKS, MONETARY POLICY AND LABOUR MARKET FRICTIONS

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Explaining Medium Run Swings in Unemployment
- Shocks, Monetary Policy and Labour Market Frictions

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Submitted for the degree of Doctor of Philosophy (Economics) at the University of St. Andrews

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ABSTRACT

Explaining Medium Run Swings in Unemployment - Shocks, Monetary Policy and Labour Market Frictions

Ansgar Rannenberg

The literature trying to link the increase in unemployment in many western European countries since the mid of the 1970s to an increase in labour market rigidity has run into a number of problems. In particular, changes in labour market institutions do not seem to be able to explain the evolution of unemployment across time.

We conclude that a new theory of medium run unemployment swings should explain the increase in unemployment in many European countries and the lack thereof in the United States. Furthermore, it should also help to explain the high degree of endogenous unemployment persistence in the many European countries and findings suggesting a link between disinflationary monetary policy and subsequent increases in the NAIRU.

To address these issues, we first develop an endogenous growth sticky price model. We subject the model to an uncorrelated cost push shock, in order to mimic a scenario akin to the one faced by central banks at the end of the 1970s. Monetary policy implements a disinflation by following an interest feedback rule calibrated to an estimate of a Bundesbank reaction function. 40 quarters after the shock has vanished, unemployment is still about 1.8 percentage points above its steady state.

The model also partly explains cross country differences in the unemployment evolution by drawing on differences in the size of the disinflation, the monetary policy reaction function and wage setting.

We then draw some conclusions about optimal monetary policy in the presence of endogenous growth and find that optimal policy is substantially less hawkish than in an identical economy without endogenous growth.

The second model introduces duration dependent skill decay among the unemployed into a New-Keynesian model with hiring frictions developed by Blanchard/Gali (2008). If the central bank responds only to inflation and quarterly skill decay is above a threshold level, determinacy requires a coefficient on inflation smaller than one. The threshold level is plausible with little steady-state hiring and firing ("Continental European Calibration") but implausibly high in the opposite case ("American calibration"). Neither interest rate smoothing nor responding to the output gap helps to restore determinacy if skill decay exceeds the threshold level. However, a modest response to unemployment guarantees determinacy.

Moreover, under indeterminacy, both an adverse sunspot shock and an adverse technology shock increase unemployment extremely persistently.

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Introduction

Unemployment is one of the main social evils that afflict advanced economies. Despite this fact, there are significant gaps in the economic analysis of unemployment. Crucially, a detailed understanding of the medium-run evolution of unemployment is yet to be developed. Farther, it has remained a conundrum why unemployment has persistently increased in many Western European economies but not in the United States.

For a long time, the commonly accepted assessment of the European unemployment problem has put the blame on labour market institutions such as generous and elongated unemployment benefits, powerful unions and the tax wedge between the labour cost of the employer and take home pay. It has been argued that these “wage push" factors increase labour costs and therefore create unemployment. In the terminology of the simple wage setting / price setting framework proposed by Jackman et al. (1991) – which can be found in any intermediate macroeconomic textbook to this day – the aforementioned factors push up the wage setting curve. To reconcile the wage claims of workers and the wage employers are willing to pay, the later of which are described by the price setting curve, unemployment has to increase. In other words, the Non-Accelerating-Inflation-Rate-of-Unemployment (NAIRU) rises. In contrast, the low unemployment rate in the United States is attributed to the absence of protective labour market institutions and labour taxes.

In recent years, this received view has come under criticism. As the IMF (2003) notes, "Institutions explain a good deal of the cross-country differences in unemployment rates. However, they hardly account for the growing trend observed in most European countries and the dramatic fall in US unemployment in the 1990s."[1] Blanchard and Wolfers (2000) point out that "Many of these institutions were already present when unemployment was low (and similar across countries), and, while many became less employment-friendly in the 1970s, the movement since then has been mostly in the opposite direction. Thus, while labour market institutions can potentially explain cross country differences today, they do not appear able to explain the general evolution of unemployment over time."[2]

This suggests that a new approach to explain unemployment swings is needed. Such an approach should explain medium run swings in unemployment in advanced OECD economies without reference to changes in labour market institutions. Furthermore, such approach should explain a range of empirical findings associated with the unemployment nexus. Most prominently among these findings is the high endogenous persistence of unemployment in Europe but not in the United States.

To meet these requirements, we introduce an approach that focuses on shifts to the price setting curve to explain increases in unemployment. Put differently, we will concentrate on movements in the wage employers are willing to pay at a given level of employment relative to the wage negotiated by wage setters. This wage will among other things be affected by total factor productivity – or, in a dynamic context, total factor productivity growth. Roughly speaking, the common feature of the two models developed in this thesis is that they start off with a general equilibrium sticky price

[1] IMF (2003), p. 134.

[2] Blanchard and Wolfers (2000), p. C2.

model of the economy and then endogenise total factor productivity. This in turn implies that monetary policy and the monetary policy response to shocks, affect the NAIRU.

We should note that we do not aim to provide a country-by-country story. Rather, we aim to shed light on a set of empirical findings characterising unemployment and NAIRU dynamics in a number of European economies and on why US-American unemployment dynamics are different.

The thesis is structured as follows. Chapter one evaluates the two existing mainstream approaches aiming to explain the medium run evolution of unemployment and differences in the evolution of unemployment across advanced OECD countries. Having identified some major problems of the existing approaches, we then discuss additional evidence associated with the unemployment evolution nexus. We conclude with a list of five empirical findings which we want a new theory of medium run unemployment swings to explain.

In chapter two, we introduce endogenous growth as a capital stock externality into a New Keynesian general equilibrium model with unemployment. This feature implies that total factor productivity growth is driven by investment, which in turn is affected by monetary policy and aggregate demand. We explore how the introduction of endogenous growth alters the effects of a disinflation on unemployment, like those seen in many OECD countries at the beginning of the 1980s. A key finding is that in the presence of endogenous growth, the disinflation substantially increases unemployment for 10 to 20 years after inflation has been brought back to target. Furthermore, the way in which monetary policy induces the disinflation affects the size of the unemployment increase over the same horizon. We also investigate how cross country differences in wage setting and monetary policy shape the effect of a disinflation on unemployment in the presence of endogenous growth.

While the main focus of this thesis is a positive one, chapter three investigates the consequences of introducing endogenous growth for optimal monetary policy. More specifically, we are interested in whether the conventional wisdom holds that monetary policy should respond aggressively to inflation but little to the output gap. Our main finding is that in the presence of endogenous growth, monetary policy responds more aggressively to the output gap and less to inflation than in an otherwise identical economy without endogenous growth.

Chapter four analyses the consequences of a different mechanism to endogenise labour productivity. It adds duration dependent skill loss among the unemployed to a New Keynesian model with hiring frictions developed by Blanchard and Gali (2008). If – for any reason – unemployment increases, this will in turn increase the average unemployment duration and will thus lower the productivity of the average applicant. It is shown that depending on how fluid the labour market is, skill decay affects the determinacy requirements on the nominal interest feedback rule of the central bank. In particular, a coefficient on inflation larger than one is no longer sufficient to guarantee determinacy. We also look at the dynamics of the model under indeterminacy.

The conclusion discusses to which extent the approach introduced in this thesis can explain the five empirical findings about the medium run evolution of unemployment stated at the end of chapter one and suggests some directions for future research.

CHAPTER 1

The European Unemployment Conundrum

The goal of this chapter is to motivate the theoretical research conducted in the following chapter. We proceed in two steps. First, we evaluate the two existing mainstream approaches aiming to explain the medium run evolution of unemployment and differences in the evolution of unemployment across advanced OECD countries. Thus section 1.1 deals with multicountry - multiperiod panel data regressions trying to link the increase in continental European unemployment to changes in labour market rigidities (or labour market institutions), promoted by Nickell and others. This approach towards explaining unemployment has probably received the most attention in terms of empirical testing. Section 1.2 then deals with the efforts by Ljungqvist and Sargent to replicate the increase in European unemployment by varying the degree of "microeconomic turbulence" within a competitive search model. It also discusses the criticism of their theory by den Haan et al. (2005). Second, having identified some major problems of the existing approaches, we discuss additional evidence surrounding the unemployment evolution nexus which the models developed in this thesis aim to shed light on. Section 1.3 reviews the evidence on endogenous persistence and unit roots in OECD unemployment. Section 1.4 deals with Ball's attempts to link changes in the NAIRU in the 1980s and beyond to the occurrence and size of disinflations and to the behaviour of central banks during recessions. Section 1.5 concludes and gives a list of five features of the unemployment conundrum arising from the preceding discussion which we aim to shed light on in the following chapters.[1]

1.1. Pitfalls of the institutional Approach to explaining European Unemployment

This section aims to illustrate some of the shortcomings of the empirical literature trying to explain unemployment with the evolution of labour market institutions. The theory underlying this literature can be summarised as follows. Following Jackman et al. (1991), it is assumed that a given level of unemployment can be decomposed into the cyclical unemployment rate and the Non-Accelerating-Inflation-Rate-of-Unemployment (NAIRU). The former depends on aggregate demand, which can be proxied by the change in the inflation rate (assuming an expectation-augmented-Phillips-Curve type relationship between the cyclical unemployment rate and the change in inflation), the output gap or the change in the money supply. The NAIRU depends on the factors determining the real wage as targeted by wage setters and the real wage price setters, i.e. firms, are willing to pay. The real wage targeted by wage setters depends positively

[1] A vibrant and fruitful area of macro-labour economics is one which attempts to incorporate search and matching frictions into dynamic stochastic general equilibrium (DSGE) models. For instance, the seminal contribution of Mortensen and Pissarides (1994) shows that a real business cycle model with matching frictions can proxy the cyclical behaviour of job creation and destruction. Hall (2005) shows that replacing the assumptions of Nash bargained wages with a "wage norm" increases the volatility of job creation, thus enhancing the model's ability to match the data. Walsh (2003) is the first attempt to incorporate sticky prices into a model with matching frictions in the labour market. He shows that matching frictions help to create a hump shaped response of output to a money growth shock as suggested by VAR evidence. Ravenna and Walsh (2008) show how a Phillips Curve relating inflation to labour market variables, specifically unemployment, can be derived from a similar model. They estimate this Phillips curve and the canonical New Keynesian Phillips Curve on US data and reject the latter in favour of the former. Very recently, the success of matching frictions and nominal rigidities in explaining the business cycle dynamics of important macroeconomic variables has motivated policy analysis in models with such features. Recent papers include Sala et al. (2008), Thomas (2008) Faia (2008) and Faia (2009).
The goal of this literature is to jointly explain high frequency movements of unemployment, job creation and destruction, output and other real variables, and more recently also inflation and the money supply. By contrast, our goal is to shed light on low frequency movements in unemployment, therefore the following survey refrains from discussing these contributions in detail.

on variables which induce "wage push", for instance the generosity and duration of unemployment benefits, the power of unions or the tax wedge between the labour cost of the employer and take home pay. The real wage firms are willing to pay at a given level of employment depends negatively on the size of the mark-up and thus positively on the degree of product market competition.

Furthermore, unemployment is also affected by supply shocks which affect the position of the price setting relative to the wage setting schedule. For instance, a decline in productivity growth might decrease the real wage employees are willing to pay relative to the real wage demanded by wage setters. An increase in the price of imports might have the same effect by raising the cost of inputs or reducing the purchasing power of a given wage payment, which wage setters might respond to by demanding higher wages.[2] In addition, the real interest rate is also seen by some (see for instance Blanchard (2003)) as a variable which might affect the feasible real wage. An increase in the real interest rate would discourage capital accumulation, thus lowering the capital labour ratio and the marginal product of labour.

Thus a reduced form unemployment equation could be written as follows:

$$u = D'b_1 + z'b_2 + s'b_3 \tag{1.1}$$

where u, D, z and s denote unemployment, vectors of variables representing aggregate demand, labour market institutions and supply shocks, respectively. Where productivity growth is assumed to have a temporary effect on unemployment, researchers might include the deviation of the Solow residual from its trend value or the change in labour productivity growth. Likewise, if productivity growth is allowed to have a permanent

[2] There is disagreement about whether the wage setting schedule will ultimately adjust to such shocks, implying that the NAIRU is only temporarily affected.

effect, researchers might include the rate of labour productivity growth or total factor productivity (TFP) growth.

In the following section we discuss several studies conducted by experts in the field who implement the approach outlined above. Section 1.1.1 critically appraises several prominent and widely cited studies quantifying the share of unemployment movements over time explained by changes in institutions. Accordingly, the main focus is on the share of unemployment movements these studies do and do not explain. All of these studies use annual data. Following this discussion, several additional issues are considered, such as the problem of low robustness of coefficient estimates across different studies, to adding observations to the data set and to adding atheoretical variables like country or fixed effects, all of which we discuss in section 1.1.2. Section 1.1.3 deals with the problem of reverse causality.

1.1.1. Studies quantifying the Share of Unemployment Movements across Time explained by Changes in Institutions

An early study trying to quantify the share of unemployment movements explained by labour market institutions is conducted by Elmeskov et al. (1998) in a paper forming part of the OECD research following up the OECD's 1996 "Jobs Study". Their data stretches from 1983 to 1995 and covers 19 countries. They consider the following institutions:

- Active labour market programmes: expenditure per person unemployed relative to GDP per capita
- Unemployment benefits, measured as the average of unemployment benefit replacement rates for two earning levels, three family situations, and three duration categories

- Employment protection, measured via a simple ranking of countries
- Union density
- The tax wedge, measured as the total value of employers' and employees social security contributions and personal income tax paid divided by gross earnings plus the employers' social security contributions
- A minimum wage index, measured as the gross statutory minimum wage relative to the average wage.[3]

In addition, they include the output gap to control for demand induced fluctuations in unemployment, where potential output is measured as a Hodrick-Prescott Filter of actual output levels. Unlike the other studies in this area, they use a random effects model to account for unobserved heterogeneity between countries.[4] The study assesses the quantitative impact of institutions on unemployment by asking how much of the change in structural rather than actual unemployment is accounted for by institutional changes. They proxy the structural unemployment rate by subtracting the impact of the business cycle, as represented by the output gap times its coefficient, from the unemployment rate. It goes without saying that in doing so, all the issues arising in measuring potential output also affect this measure of structural unemployment. The authors then compare the change in this structural unemployment rate from 1983 to 1995 to the contributions of the individual institutions and the country specific effect. The country specific effect is calculated as a residual, which follows from the random effects assumption.[5] It turns out that this country specific effect explains most of the change in structural unemployment in almost every country, with the exception of

[3]See Elmeskov et. al. (1998), p. 244.
[4]See Elmeskov et. al (1998), p. 213-214. They test this specification against the alternative of correlation between the unobserved effects and the explanatory variables.
[5]See Elmeskov et. al (1998). The country specific effect is the difference between the structural unemployment rate and the institutional variables times their respective coefficients.

the Netherlands, the UK, Belgium and Ireland, in the latter of which it still explains about 50% of the change.[6] The authors conclude that "an important fraction of the estimated changes in structural unemployment cannot be accounted for by changes in the explanatory variables included in our analysis."[7]

Nickell (2005, 2002) et al investigate the role of institutions in explaining unemployment in 20 OECD countries from 1960 to 1995. All countries but the United States, Canada and Australia are Western European. The institutional variables are

- The benefit replacement rate, which is the before tax benefit entitlement as a percentage of previous earnings before tax
- A benefit duration index, which equals (0.6*replacement rate in 2nd/3rd year of an unemployment spell +0.4*replacement rate in 4th/5th year of an unemployment spell)/(replacement rate in the first year of an unemployment spell)
- Trade union density: ratio of total reported union members (less than retired and unemployed ones) over total employment, level and change.
- Wage bargaining coordination index on a (1-3) scale. It refers to mechanisms which make the wage bargainers internalize the effect of a wage deal on aggregate employment. This may be achieved by formally centralized bargaining or through institutions like employers federations.[8]
- An Employment protection index on a (0-2) scale
- Labour Taxes, which equal the of the payroll tax rate, the income tax rate and the consumption tax rate. Note that all of these percentages refer to different bases, but they are added up nevertheless. It is not exactly clear

[6]See Elmeskov et. al (1998), p. 220, Table 3a.
[7]Elmeskov et. al. (1998), p. 219.
[8]See Nickell et al (2002), p. 8.

why consumption taxes are used, as indirect taxes clearly affect unemployment benefits and thus the reservation wage in the same way they affect the real wage. Nickell et. al argue that unions might temporarily resist the reduction in wages resulting from an increase in indirect taxes and that this would temporarily raise unemployment.

- Owner occupation rate: The percentage of the housing stock classified as owner occupied. This is supposed to measure of labour mobility[9]

The authors also use various interactions of these institutions in their regression equation, where interaction refers to the product of two institutions. They also use time dummies and country specific time trends. The later has been criticised by Baker et al. (2007), who argue that there is "little theoretical justification for imposing a common time trend, and even less justification for including a separate time trend for each country. To the extent that unemployment in OECD economies is trended over time, the role of this kind of modelling ought to be to explain such a trend, not to control for it."[10]

According to the theoretical framework of Nickell et al. as summarised above, institutions affect the long run level of "equilibrium" unemployment (or the long run NAIRU) by affecting wage and price setting behaviour, thus determining the real wage employees demand for a given unemployment level and the real wage employers are willing to pay. But actual unemployment movements are driven in addition by fluctuations of the natural rate originating from supply shocks like productivity or oil price shocks, and by cyclical unemployment movements induced by demand changes.[11]

[9]See Nickel (2005), pp. 23-24.
[10]Baker et al. (2007), p. 62.
[11]See Layard, Nickell and Jackman (1991), pp. 12-18.

Therefore the following variables representing macroeconomic shocks are also included in the regression of Nickell et. al.:

- Productivity shocks, measured deviations of Total Factor Productivity (TFP) Growth from trend
- short run labour demand shocks measured by the residuals from a simple labour demand model.[12]
- real import price shocks, measured by proportional changes in real import prices weighted by the trade share
- the ex-post real interest rate[13]
- Change in the rate of growth of the nominal money stock to account for aggregate demand fluctuations. To us this seems an inadequate way to control for aggregate demand fluctuations. First, monetary policy is not the only force driving aggregate demand. Furthermore, theory would suggest that the effect of the money supply on aggregate demand would depend on the elasticity of money demand as well as on the effect of output on inflation, both of which might be unstable. Postulating an expectation-augmented-Phillips Curve type relationship between the deviation of unemployment from the NAIRU and the change in inflation would be both a more direct and a more comprehensive way to capture aggregate demand fluctuations. It is also the approach followed by Nickell (1997) and Jackman et al. (1991).[14]

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[12]Baker et al. (2002) criticize this way of constructing labour demand shocks because it might imply regressing unemployment on employment. They argue that if the labour demand model were misspecified, then the residual would simply measure employment movements (which are perhaps driven by an omitted variable). They see their critique justified by the fact that the coefficient estimate stands out as extremely high and extremely significant. See Baker et al, p.67 footnote 8.

[13]See Nickell (2005), p. 10.

[14]See Nickell (1997), p. 65, and Jackman et al. (1991).

Finally, to account for unexplained persistence in unemployment, the equation also includes the lagged unemployment rate.

The study finds that all institutions but employment protection and the level of union density are significant, although the change of union density does have a significant effect as well. Labour taxes are insignificant if the owner occupation rate is included. Bargaining coordination reduces unemployment, especially if it interacts with employment taxes and union density. The shocks are all significant except for the real interest rate and the change in the money supply, though this might have to do with the fact that the two variables would be expected to be highly correlated: an increase in the money growth rate would be expected to induce a reduction in the interest rate. Most notably, however, is that the coefficient on the lagged unemployment rate is as high as 0.86.[15] Hence each variable has a long run multiplier of more than 7 ($= \frac{1}{1-0.86}$), which is multiplied with the on impact coefficient. This means that to a large degree, unemployment is explaining itself, or as Nickell et al put it: "This reflects a high level of persistence and/or the inability of the included variables to explain what is going on."[16] The resulting empirical model explains the persistence in unemployment extremely well. However, it would certainly be desirable to explain the source of this high endogenous persistence.

The authors then conduct a dynamic simulation to illustrate the effect of institutions country by country. To do so, they fix the institutional variables at their 1965 level and compare the result to the fitted value of unemployment using the actual values of the institutional variables and past unemployment. According to their simulation, they can explain 55% of the rise in unemployment in OECD Europe from the

[15]See Nickell (2005), p. 14.
[16]See Nickell et al (2005), p. 15.

1960's to 1995. This result is not very impressive given the high internal persistence of unemployment they estimate. Turning to individual countries, institutions explain virtually nothing for Western Germany (where unemployment rose from about 1% to about 6%.), Finland and New Zealand. In the case of Germany, this probably has to do with the fact that institutions have changed very little. However, even among countries where the institutional variables make some contribution, significant bits of the evolution of unemployment are not explained by institutions. For instance, in Spain, unemployment rose from about 2% in 1960 to about 22% in 1995. According to the simulation, with institutions remaining fixed, there would still have been an increase in unemployment to about 17%. Though this is the most extreme example, the limits of the explanatory power of the institutional variables are obvious for other countries as well. For Ireland, the fixed-institution simulation gives a rise in unemployment from less than 5% to about 15%, which is striking. However, unlike most other countries, in the 1990s actual unemployment fell below the simulation with institutions being fixed. The simulation with the actual institution values yields an unemployment rate of 11%. Obviously there has been some favourable institutional reform, but this still leaves an increase in unemployment of about 6% not explained by institutions.[17] To a lesser degree, similar conclusions can be drawn from looking at the simulations for Australia, France, the UK and Italy. Baker et al. (2007) draw attention to the role of the interaction of the aforementioned country specific time trends and the high coefficient on the lagged dependent variable in picking up unemployment movements across time. In most countries, the coefficients of the country specific time trends imply an increase or decrease in unemployment of at least two percentage points over two decades.[18] The size

[17]See Nickell (2002), pp. 44-45.

[18]See Baker et al. (2007), pp. 26 and 27, footnote 13, and the regression results of Nickell et al. (2002), pp. 37-38. With a coefficient on lagged unemployment, of 0.87, to generate a change in

of unemployment movements driven by interactions between atheoretical variables and the coefficient on the lagged dependent variable suggests the need for a new theoretical model able to produce large medium run country specific swings in unemployment and endogenous unemployment persistence.

In addition, for a number of countries (Austria, Denmark, Japan, Norway, Sweden and Switzerland) the simulation with fixed institutions actually yields negative unemployment rates, sometimes substantially so. This sheds doubt on the validity of the whole simulation exercise: Maybe the "true" coefficients are sensitive to variations of the independent variables, implying that counterfactual simulations will yield only crude estimates of what would have actually happened had the exogenous variables evolved as assumed for the purpose of the fixed institution simulation.

The IMF (2003) conducted an empirical study very similar in spirit to the Nickell et al. approach. It features the same countries and uses Nickell et al.'s data, but it's scope extends until 1998. Due to a lack of data on this particular variable, benefit duration is absent from the model. However, it adds an index of central bank independence, the change in the inflation rate (which would seem to us a superior way to control for aggregate demand fluctuations for the reasons given above) and the output gap to the macroeconomic variables but does not include the change in money-supply growth and labour demand shocks. Various interaction terms are included as well. The authors carry out several regressions and also engage in fixed institutions simulations like Nickell et al. do, and the regression forming the basis for these simulations includes lagged unemployment in order to account for the persistence in unemployment.

unemployment of two percentage points, one needs a coefficient on the country specific time trend with an absolute value of 0.019. The only time trends with a coefficient with a lower absolute value are Finland, Germany, New Zealand and the United Kingdom.

The first regression includes only labour market institutions and hence no shocks or lagged dependent variables. All institutional variables are significant at the 5% level. This carries over to the other regressions. The residuals of this regression are interpreted as "Institution adjusted Unemployment Rates", that means the part of unemployment in each year not explained by institutions. They are displayed only for a selection of countries.[19] According to the authors, for each country, the "Institution adjusted Unemployment Rate" fluctuates around zero, suggesting it provides an unbiased explanation of the cross-country differences in unemployment. However, due to the way these residuals are trended over time, i.e. upwards in most of Europe and downwards in the U.S., institutions "hardly account for the growing trend observed in most European countries and the dramatic fall in U.S. unemployment in the 90's": The part of the unemployment rate not explained by institutions increases over time.[20] This confirms similar findings of Blanchard and Wolfers (2000) saying that while institutions might be able to explain differences in unemployment across countries, they cannot explain the evolution of unemployment over time.[21]

Moreover, as in Nickell et. al., the coefficient on unemployment in the regression that forms the basis for their simulations is pretty high, equalling 0.79, which corresponds to a long-run multiplier of about 4.76. Hence large parts of the evolution of unemployment remain again unexplained. Concerning the simulation results, these are only reported for six countries (Germany, France, Italy, Ireland, the Netherlands and the UK) to exemplify the impact of labour market institutions. In all of them, unemployment increases over time even when institutions are held fixed. As in Nickell et

[19]See IMF (2003), p. 134.
[20]IMF (2003), p. 134.
[21]See Blanchard (2000), p. C2.

al, institutions do not explain anything for Germany. For France and Italy, deteriorations in institutions contribute about 3.5% and 1.8% to the increase in the fitted value of unemployment in these countries from 1970 to 1998, whereas the increase in the fitted value amounts to about 9% and 5% respectively.[22] Thus especially for France, the institutional approach certainly leaves something to be wished for. This is also true for those countries which the IMF presents as examples of successful institutional reform, namely the United Kingdom, the Netherlands and Ireland. According to the simulation, in all these countries successful reform has reduced the unemployment rate by about 2.5%. Institutions, however, can not account for an increase of 4% in the UK and Ireland, and 3% in the Netherlands, respectively.

The most recent cross country/ cross time macro econometric estimation of the effects of policies and institutions on unemployment is conducted by Bassanini and Duval (2006). Bassanini and Duval consider a sample of 20 OECD countries over the period from 1982 to 2003. Their Baseline specification features the following variables:

- Tax wedge between labour cost and take home pay (for a single earner couple with two children, at average earnings levels)
- Unemployment benefit generosity, measured by the average of replacement rates over various earnings levels, family situations and durations of unemployment
- The degree of stringency of employment protection laws
- The average degree of stringency of product market regulation across seven non-manufacturing industries,
- Union membership rates to measure the bargaining power of unions

[22]See IMF (2003), pp. 138-141.

- The degree of centralisation/ coordination of wage bargaining, referred to as the degree of corporatism.[23]

To control for "the unemployment effects of aggregate demand fluctuations over the business cycle", Bassanini and Duval include the output gap measure of the OECD in addition to the institutional variables. They also include time dummies and country fixed effects.

Bassanini and Duval (2006) find that lowering the replacement rate and the tax wedge by ten percentage points, respectively, and reducing product market regulation by two standard deviations would lower unemployment by 1.2 and 2.8 percentage points, respectively. Increasing corporatism by one unit lowers unemployment by 1.4 percentage points.[24] Furthermore, the baseline equation is able to explain a substantial share of unemployment trends over the estimation period. More specifically, it explains 74% of the cross country variance of unemployment changes for 1982 - 2003. Labour market institutions explain 47% of the cross country variance of unemployment changes, while they explain 64% of non-cyclical unemployment changes. The latter are calculated as the difference between actual changes in unemployment from 1982 to 2003 and the change in unemployment that can be assigned to the change in the output gap- over that period.[25]

The authors also run an alternative specification in which they replace the output gap with the following shocks:

- A TFP shock, i.e. deviation of logarithm of TFP from its trend, which in turn was calculated by means of a Hodrick - Prescott filter

[23]See Bassanini and Duval (2006), p. 12.
[24]See Bassanini and Duval (2006), p. 16.
[25]See Bassanini and Duval (2006), p. 16.

- A Terms of Trade Shock, defined as the ratio of imports to output multiplied by the logarithm of their relative prices
- The ex-post long term real interest rate
- A labour demand shock, measured labour share purged from short run influences of factor prices[26]

They find that replacing the output gap with these variables has only minor effects on estimates of the effects of the institutional variables.

Hence the results of Bassanini and Duval seem to provide strong support for the hypothesis that changes in labour market institutions explain movements in unemployment across time. However, we believe that two qualifying remarks are in order. The first refers to the time period they consider. Of course the choice of a sample period is always arbitrary. However, it is hard to ignore that in the early 1980s, unemployment rose substantially across the OECD. From 1980 to 1982, in the countries in Bassanini and Duval's sample, the average unemployment rate increased by two percentage points. Only four countries saw their unemployment increase by less than one percentage point. Unemployment rates in Belgium, France, Italy, Spain and Germany, all countries with a history of high unemployment often blamed on rigid labour markets increased by 3.4, 1.5, 1.3, 3.7 and 2.8 percentage points respectively. The average unemployment increase from 1980 to its peak during the decade amounted to 3.8 percentage points. Hence ignoring the years 1980 and 1981 leaves out about 53% of the total average unemployment increase during that decade, while it leaves out 82%, 42%, 28%, 44% and 57% of the total unemployment increase for the countries just listed.

[26]See Bassanini and Duval (2006), p.15.

Explaining the highly persistent and non-cyclical increases in unemployment during the 1980s constitutes a major part of the European unemployment conundrum. The omission of 1980 - 1982 not only qualifies the conclusions of Bassanini and Duval concerning the power of institutions to explain changes in unemployment, but also raises the question of whether excluding these years of substantial unemployment variation affects both the coefficient estimates and the share of the cross country variance of unemployment changes over the sample period institutions can explain. For instance, Blanchard and Wolfers (2000) show that at the beginning of the 1980s in Italy, Germany or Great Britain, where unemployment rose substantially, unemployment benefit replacement rates either stayed flat or fell.[27]

Our second remark is that the equation is potentially miss-specified. As we have pointed out above, a reduced form equation to describe unemployment in line with the approach of Jackman et al. (1991) has to feature a variable controlling for deviations of unemployment from the NAIRU driven by aggregate demand fluctuations and the variables driving the NAIRU. The latter consists of the institutional variables and various shocks affecting the supply side. Bassanini and Duval (2006) suggest that it is possible to control for demand or "business cycle" effects either by the output gap *or* what are essentially supply side shocks. This is even more remarkable since they suggest that some of these variables might actually be non-stationary and such would be their effect on unemployment.[28]

The specification featuring the four supply shocks but no control for aggregate demand movements is also highly unusual. Almost every study looking for direct effects of labour market institutions on unemployment controls for deviations of unemployment

[27] See Blanchard and Wolfers (2000), pp. C14-C16.
[28] See Bassanini and Duval (2006), p. 16.

from the NAIRU by either the output gap, the change in inflation or (in case of Nickell (2002)) the change in the money supply, but not by supply shocks. Many studies – which as far as we know all work with annual data- then include supply shocks (like productivity growth, TFP shocks, terms of trade shocks...) in addition not as a replacement, see for instance Baccaro and Rei (2005), the IMF (2003), Nickell et al. (2002, 2005) and Nunziata (2002).

Hence in two of these four studies, the explanation of unemployment via labour market institutions has to rely on a high coefficient on the lagged dependent variable. As pointed out by Nickell et al. (2005), this could be interpreted as a failure of the exogenous variables to explain unemployment. Even with the help of the implied high long run multiplier, these studies leave huge bits of unemployment dynamics to be explained which are currently only captured by atheoretical variables. The latter is also true for the Elmeskov et al. study. A number of other criticisms were mentioned above. Finally, the fourth study estimates a specification which does not feature lagged unemployment. Here institutions explain almost two thirds of the cross-country variation of unemployment changes from 1982-2003. However, this result might be sensitive to the inclusion of the years 1980-1981, since unemployment rose substantially across the OECD from 1980 - 1982. More seriously, the equation might be misspecified.

1.1.2. Robustness

Coefficients of the institutional variables seem to vary strongly across different empirical studies, and seem to be very sensitive to adding new data, to the inclusion of additional variables and to how unobserved variables are controlled for. Baker et. al survey six recent papers (Nickell 1997, Elmeskov et al. 1998, Belot and van Ours (2002)[29], Nickell

[29]This was published as Belot and van Ours (2004), which can be found in our list of references.

et al. (2002), Blanchard and Wolfers (2000), Bertola et al. (2001)) and find that labour taxes and benefit duration are significant in all studies were they are included, and the replacement rate in all but one. However, the effect a 10 percentage point increase taxes and the replacement rate on unemployment ranges from 0.9 p.p. to 2.1 and from 0.1-1.3 p.p., respectively. The effect of an increase in benefit duration by one year ranges from 0.7% to 1.4%.[30] They conclude that "the range of estimated coefficients of the variables that were generally found to have a significant relationship with the unemployment rate is sufficiently large to both raise questions about the robustness of this result and also to obscure the potential trade-offs for policy makers."[31]

It might be argued that the variation of coefficients between studies is hard to avoid given the fact that the equations estimated often do not include the same institutional variables, or different measurements of them (though the Nickell-dataset is now widely used), cover different time periods, add different macroeconomic shocks, use different specifications of country and time effects etc. Therefore robustness experiments within a given study, which changes one aspect of the approach but leaving everything else the same, are of particular interest. Concerning the Nickell et al. study discussed above, Baker et. al. report that an earlier version (from 2001) of that paper whose data extended only to 1992 produced very different estimates of the coefficients of labour taxes, benefit duration, and bargaining coordination. In the 2002, their (long-run) impacts are reduced by more than 30%, 50% and 40%, respectively, as compared to the 2001 version.[32] It is striking that the addition of three years to a study which would

[30]See Baker et al (2002), pp.43-44.

[31]Baker et al. (2003), p. 44.

[32]See Baker et. al (2003), p. 35. These numbers can be easily checked by comparing the coefficients for the 2001 version of the Nickell et al study reported in Baker et al, p.47, fourth column of the table, with the coefficients reported in Nickell (2002), p. 37, column 2 of the table, or in Nickell (2005), column 3, after modifying them for the effects of the long-run multiplier. Sadly, I was not able to get hold of the earlier version of the Nickell paper.

otherwise stretch over 32 years using annual data leads to such major revisions in the coefficient estimates. Nickell et al.'s results are apparently not robust to the inclusion of additional data.

Baker et al. (2004) investigate the robustness of the results from the IMF (2003) study mentioned above. They re-estimate the specification underlying the simulation results discussed after modifying it as follows. They remove the country specific time trends for the reasons given above and introduce common time dummies instead. They introduce benefit duration as an additional variable (which was absent in the IMF (2003) study due to a lack of data) and replace the interaction terms (between employment protection and union density, union density and the tax wedge and central bank independence and bargaining coordination) with the same interactions used in Nickell et al. (2002, 2005) (i.e. Union density and bargaining coordination, replacement ratio and benefit duration, tax wedge and bargaining coordination).[33] They also use slightly different versions of the union density, replacement ratio and tax wedge variable.[34] When estimating this modified specification, only the tax wedge, the interaction between bargaining coordination and union density and between the tax wedge and bargaining coordination are significant, although the tax wedge only at the 10% level.

Belot and van Ours (2004) exemplify the robustness problem with respect to the inclusion of additional variables. They estimate the effect of labour taxes, the replacement rate, employment protection, centralization, interactions of benefits and taxes, coordination with employment protection and union density. The authors justify the

[33]See Baker et al. (2004), p. 15, and Nickell et al. (2002), p. 37, table 13.

[34]Baker et al. (2004) claim that the modifications of the variables are minor.

use of interactions by recurring to a rigorous model of price and wage setting. Furthermore, they include the change of inflation.[35] Their data ranges from 1960 to 1999 and consists of five year averages. In their first regression, they include all the variables but no interactions, and find that all variables are highly significant with the expected sign. They then add country fixed effects (recall that Nickel et al used country specific time trends instead) to account for unobserved heterogeneity between countries. This renders the replacement rate and union density insignificant, while employment protection is significant only at the 10% level. However, fit (adjusted R^2) improves to from 0.39 to 0.7 . Adding time period fixed effects instead renders again the replacement rate, employment protection, and also coordination insignificant. The tax rate is significant only at the 10% level, while fit improves to 0.46. Adding both time and country effects turns all institutional variables insignificant, while fit improves to 0.77.[36] Hence it seems quite possible that the first regression did not include the variables truly driving unemployment, or as the authors put it: "The results with respect to the relationship between labour market institutions and unemployment in the first column seem to be caused by fixed differences between countries and time periods and not by within country changes in labor market institutions."[37]

Adding the interaction terms in addition to the country and time effects leads to remarkable outcomes. Both union density and the replacement rate and union density are now significant, but the replacement rate with the wrong sign: an increase in the replacement rate by 10 percentage points reduces unemployment by 2.2 percentage points. The interaction effect of taxes and benefits is significant with a coefficient of 0.57, which is in line with theory. Assuming an about average tax rate of 0.4, this means

[35] See Belot and van Ours (2004), pp. 630-631 and pp. 634-635.
[36] See Belot/Van Ours (2004), p. 635.
[37] Belot and van Ours (2004), p. 636.

that the net effect of a 10 percentage point increase in the replacement rate amounts to a mere 0.08 (=-2.2+0.57*0.4*10) percentage point increase in unemployment. While this is result is obviously quantitatively small, the fact that it relies on the interaction of the tax rate with the replacement rate is at odds with conventional wisdom and the model Belot and van Ours develop in the paper: An increase in the replacement rate should increase unemployment at any tax rate.[38]

In conclusion, it seems that robustness of the estimated coefficients represents a serious problem of the institutional approach, and casts doubt on the policy conclusions drawn from regressions of unemployment on institutional variables.

1.1.3. Reverse Causality

One of the crucial underlying assumptions of panel data regressions of unemployment on labour market institutions is that the latter are exogenous and are not affected by those forces which are affecting unemployment or by unemployment itself. This assumption might be violated with respect to the tax wedge, but also with respect to the generosity and duration of unemployment benefits. For instance, in some countries the welfare state is organized as an insurance system, with the bodies providing the unemployment benefits, pensions health and care insurance having to break even at least in the long run. An economic downturn will result in a twofold worsening of the financial position of these institutions. Firstly, expenditure on unemployment benefits will increase with the number claimants, and secondly, as those becoming unemployed stop contributing and wages fall or rise at a slower pace the revenues of the system will decrease. This will then almost mechanically lead to an increase in the tax wedge in order to balance revenues and costs. If the government decides to funnel more

[38]See Belo and Van Ours (2004), pp. 636-637.

tax money into the system to prevent a rise in social security contributions, this will ultimately lead to tax rises elsewhere, given that the government also has to balance its budget to some extent. Hence an increase in unemployment might cause an increase in the tax wedge and so part of the estimated coefficient associated with it in empirical studies might be related to that. Similarly, an increase in unemployment might lead the government to extend the duration of unemployment benefits for certain groups which find it especially hard to find a job, like the elderly, though in this case the link would certainly be less mechanical. Blanchard (2007) views reverse causality as a major factor compromising the explanatory power of the macroeconometric evidence: "Asking these panel data regressions to tell us conclusively about causal effects of institutions, shocks, and interactions of shocks and institutions on unemployment is beyond what they can deliver. Causality is next to impossible to establish, as many institutional changes are triggered by labor market developments."[39]

Still, the problem is rarely addressed in empirical studies aimed at understanding the effects of labour market institutions and tends to be downplayed. The Nickell (2002) study recognizes that there might be an issue but then more or less dismisses it: "We have not faced up to the problem of the endogeneity of the institutional shifts. In certain cases this may be important but, overall, we do not feel this problem seriously distorts our results. In any event, the absence of suitable instruments ensures that we are unable to deal with the issue."[40] The IMF (2003) notes that "In line with the existing literature, all explanatory variables were assumed exogenous. Admittedly, some institutional variables like employment protection and benefit replacement ratio might react to higher unemployment as demands for greater insurance against unemployment

[39]Blanchard (2007), p. 415.
[40]Nickell et al (2002), p.2.

risk increase. However, the few existing empirical studies on the determinants of labour market institutions do not suggest any significant feedback effect of unemployment on the institutional framework, rather emphasizing strong complementarities among institutions and a significant role for "deeper" aggregate risk factors like the degree of trade openness."[41] However, two of the three studies the IMF cites (Rodrik 1999, Agell 1999) regress the size of government and general welfare expenditures (Rodrik) and labour market institutions (Agell) on measures of trade openness but not on unemployment.[42] The key argument of these studies is that higher trade openness increases aggregate risk which increases the desire for insurance which, due to the aggregate nature of the risk, can only be provided by the government. While these lines of research are interesting in themselves, they clearly address different issues. The third study listed by the IMF, which is conducted by Checci and Lucifora (2002), focuses solely on unionization, whereas the most interesting variables would be benefit generosity and duration and the tax wedge, both because they are the most likely to be affected by unemployment, but more importantly because these are the most consistently significant variables throughout a wide range of empirical studies, as discussed in the previous section.[43] Thus the research on the endogeneity of labour market institutions is clearly still in its early stages.

The Elmeskov et al. (1998) study discussed above does include a test of whether unemployment benefits as defined in this study and the tax wedge do Granger cause unemployment. A variable x is said to Granger cause a variable y if in a regression of y on lagged values of x (the number of lags being determined by suitable criteria) the lagged

[41]IMF (2003), pp. 146-147.
[42]See Rodrik (1998), p. 997-1032, and Agell (1999), p. F143-F164.
[43]See Checci and Lucifora (2002).

values of x are jointly significant.[44] They conduct the test separately for each country, with the data ranging from 1970 to 1995. They find that unemployment Granger causes unemployment benefits in Belgium, France, Italy, the UK, the United States and the Netherlands, though in the later the result is significant only at the 10% level. This is an interesting result because the list includes both high and low unemployment countries where benefit levels have been moving in opposite directions. Unemployment is found to Granger cause the tax wedge in Austria, Ireland and Norway.[45]

Baker et al. (2007) look for Granger causality running from unemployment to unemployment benefits and vice versa in a sample of 21 OECD countries and a longer time period ranging from 1962 to 2004. Unemployment benefits are measured as the Gross Replacement Rate, which is calculated as an average across family types, income levels and for different durations of unemployment. It is the same measure used in the Bassanini and Duval (2006) paper cited above.[46] The authors use four lags of unemployment in the regression. Even if only the first lag is considered, unemployment is found to Granger cause unemployment benefits at least the 5% level (i.e. the null that unemployment does not Granger cause unemployment benefits is rejected at this significance level) in Denmark, France, Ireland, Italy, Japan, the Netherlands, Portugal and the United Kingdom. If the significance of the remaining lags is considered as well, unemployment is found to Granger cause unemployment in Australia and the United States at the 5% level as well.[47] Hence in 10 out of 21 countries, unemployment is found to Granger cause unemployment benefits, and again the list includes countries with varying unemployment performance. By contrast, unemployment benefits are

[44]See Gujarati (2004), p. 698.

[45]See Elmeskov et. al. (1998), pp. 248-249.

[46]See Bassanini and Duval (2006), p. 106 for a definition. This represents the latest effort of the OECD to develop a benefit generosity variable available for a large set of countries.

[47]See Baker et al. (2006), p. 40, Table 6.

found to Granger cause unemployment only in Australia, Belgium, Finland, Ireland, the Netherlands, Norway and Switzerland at least the 5% level, while it causes unemployment benefits in Canada at the 10% level. These results suggest that the consistent finding of a significant relationship between unemployment benefits and unemployment is at least partly be due to causality running in the opposite direction.

1.2. Turbulence as a Source of Rising Unemployment in a Welfare State Economy

The problems of the research program in the spirit of Jackman, Layard and Nickel to explain the evolution of unemployment across time motivated Ljungqvist and Sargent to pursue an alternative approach. In a series of papers (1998, 2004, 2007) they aim to explain why the European welfare state was associated with (OECD-) average unemployment rates until the mid 1970s but with substantially higher unemployment rates thereafter even though there have been no major changes to labour market institutions during that period. Another motivating stylised fact is that the increase in unemployment has been accompanied by a decline in the probability of gaining employment rather than an increase in the separation rate, and thus an increase in unemployment duration.

In Ljungqvist and Sargent's approach, increased "Microeconomic turbulence" increases unemployment levels in an economy with generous unemployment benefits. Microeconomic turbulence is defined as the probability that a laid off worker suffers substantial skill loss. During "tranquil" times, on the other hand, the welfare state economy generates unemployment rates which are almost equal to the unemployment rates of the "laissez faire" economy, which offers no unemployment benefits to laid-off workers. Thus Ljungqvist and Sargent offer an explanation of why unemployment has increased in Europe without institutions becoming more unemployment unfriendly.

We will first summarise the original exposition (Ljungqvist and Sargent (1998)) of their argument, which was based on a simple competitive search model in sections 1.2.1 and 1.2.2. They refine their argument in later papers but the basic mechanisms determining the relationship between turbulence and unemployment remain the same. Ljungqvist and Sargent's argument has been attacked by Den Haan et al. (2001,2005). They restate the unemployment turbulence relationship in a matching model and show that it breaks down if there is even a small probability that workers whose matches have broken up endogenously suffer from skill loss. This critique is summarised in section 1.2.3.

1.2.1. Assumptions

Workers aim to maximise the present value of their income net of job search costs. Income, in case of having a job, is the product of the skill level and the wage, which is specified per skill unit. In case of unemployment, the welfare state economy offers unemployment benefits to its workers which replace 70% of previous earnings. They are funded by a payroll tax. Benefits are terminated if the worker rejects a job offer which would generate an income larger or equal to those benefits. With a fixed probability, skills can increase if the worker is in employment, thus generating earning rises of employed workers over time (there are 21 skill levels). They will deteriorate during periods of joblessness. What is more, the event of a layoff will cause an immediate skill loss. In the initial calibration, corresponding to "tranquil times" this skill loss is zero, while in the turbulent times calibration, the immediate skill loss a worker might suffer can be huge. Layoffs happen with a fixed probability. Workers also die with a fixed probability. Dead workers are replaced by an equal number of entrants which have the lowest skill level.

Unemployed workers determine the probability of receiving a wage offer by choosing their search intensity. Job search is costly and increases in the search intensity. The wage offer follows a continuous distribution.[48]

1.2.2. Benefit Traps and the Effect of Turbulence

This setup implies that in the welfare state economy, an unemployed worker with low skill levels but high past earnings will exert a low search effort. With a low skill level, the wage offer such a worker would have to receive in order to generate earnings exceeding their benefits is high and thus quite unlikely to occur. Accepting a job offer generating a lower income than the unemployment benefit will mostly not be worthwhile because building up the skills necessary for the workers income to exceed his unemployment benefit will take a long time, while there is still the danger of being laid off again in the meantime and thus ending up with a much lower unemployment benefit (as the wage on which to base the benefit is calculated has decreased as well). Apart from that, further skill deterioration is not much of an issue for these workers since they are close to the lowest skill level anyway. The low search intensity in conjunction with the high reservation wage means that few of those worker move into jobs.[49] Workers in the laissez faire economy, by contrast, always choose the highest search intensity, implying that they receive a job offer with probability one.[50]

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[48]See Ljungqvist and Sargent (1998), pp. 524-528.

[49]See Ljungqvist and (1998), pp. 529-531. Workers with very high skill levels and high benefits tend to exert low search efforts, too, but not as low as the low skilled/ high benefit workers, and search intensities increase as skills move towards the intermediate range as the unemployment spell lengthens.

[50]See Ljungqvist and Sargent (1998), p. 531.

In the "tranquil times" calibration, however, these differences do not lead to significantly different average unemployment rates. The reason is that with no immediate skill loss after layoff, the group described above is too small to matter.[51] In such a world, higher previous earnings, and thus higher benefits, are on average associated with higher current skills, which tend to reduce the reservation wage and tend to increase search intensity. To reach the lowest skill level while enjoying a high benefit therefore implies a very long unemployment spell which is unlikely to occur for the reasons just given. Even in the tranquil times calibration, however, a higher fraction of the unemployed in the welfare state economy are long-term unemployed, i.e. with spells exceeding six months (12.6% as compared to 9.8%) and twelve months (1.3% as compared to 0.7%).

An increase in the degree of economic turbulence, as defined above, causes a much less benign outcome for the welfare state economy. With Ljungqvist and Sargent's calibration of skill evolution during employment, workers who become unemployed will typically have accumulated the highest skill level.[52] Faced with large skill losses, many will thus end up in a situation as sketched above, i.e. low skills coupled with high benefits. Thus the share of the population "trapped" in the benefit system because they have little incentive to search for jobs and to accept job offers will be much higher than during tranquil times. Furthermore, moving to a higher skill level takes on average longer than with low turbulence and payroll taxes increase to sustain the benefits paid to the larger number of unemployed. This further reduces the attractiveness of job

[51] See Ljungqvist and Sargent (1998), pp. 447-448.

[52] See Ljungqvist and Sargent (1998), p. 540 and p. 527.

searches.[53] Hence average unemployment increases in the welfare state economy. By contrast, unemployment in the laissez-faire economy does not change.[54]

1.2.3. Stylised facts

Apart from generating an increase in unemployment in the welfare state economy as compared to the laissez-faire economy holding the welfare state constant, the welfare state economy also has a much larger fraction of long-term unemployment. For the highest degree of turbulence, 55.6% of the unemployed in the welfare state economy have been unemployed for longer than a year, as opposed to 0.6% in the Laissez-faire economy. This matches the fact that European OECD countries have larger fractions of unemployed with long spells than the U.S has. Average unemployment duration also strongly increases in the welfare state economy.[55] This is in line with the observed increase in the average duration of unemployment as unemployment increased in Europe.[56]

Concerning the laissez faire economy, the model can partly reproduce evidence on earnings of Gottschalk and Moffit (1994) for the U.S., who decompose earnings of individuals into two components, an individual specific mean and a transitory serially correlated component. They find that during the 1979 to 1987 period as opposed to the 1970 to 1978 period, both the dispersion of individual means and the standard deviation of transitory earnings have increased.[57] An increase of the degree of economic

[53] See Ljungqvist and Sargent (1998), p. 540.
[54] See Ljungqvist and Sargent (1998), p. 532 and p. 540.
[55] See Ljungqvist and Sargent (1998), p. 541.
[56] Ljungqvist and Sargent (2007) mention that feature of their approach more explicitly. See Ljungqvist and Sargent (2007), pp. 2140-2141 and pp. 2153-2155.
[57] See Ljungqvist and Sargent (1998), p. 518.

turbulence in the laissez-faire economy does the same but the changes are not as big as to match all of the increase observed by Gottschalk and Moffit.[58]

While the extent to which the model can match the evolution of stylised facts is remarkable, Ljungqvist and Sargent do not provide any direct evidence on the variable driving their model, namely the mean and variance of the immediate human capital losses suffered by laid-off workers. This is pointed out by Nickell et al. (2005), who also argue that there seems to be no evidence for increase in the rates of job-destruction, which could provide a rationale for the increase in the immediate loss of human capital assumed by Ljungqvist and Sargent.[59]

1.2.4. Turbulence in the Presence of Endogenous Separation

Den Haan et al. (2001,2005) and Ljungqvist and Sargent (2004) reconsider the the ability of turbulence to explain increases in unemployment within a simple matching model. Thus the model sketched above is modified in the following ways. The number of newly formed employment relationships is determined by a homogenous-of-degree-one matching function. Employment relationships consist of one firm and one worker. As the number of firms and workers is fixed, so is the matching probability. The productivity z of a newly formed relationship is drawn from a continuous distribution, where $v_i(z)$ denotes the distribution function and i denotes the skill level of the worker, which may be high or low. It is assumed that $v_h(z) < v_l(z)$, i.e. the distribution of z for high skilled workers stochastically dominates the distribution for low skilled workers. Each period an employment relationship persists, with a fixed probability, there will be a new draw from $v_i(z)$. Furthermore, with a fixed probability, employed

[58]See Ljungqvist and Sargent (1998), p. 542

[59]See Nickell et al. (2005), p.13.

low skilled workers will become high skilled workers. Employment relationships may be exogenously destroyed with a fixed probability.[60]

Unlike Ljungqvist and Sargent (1998), the wage payment in each relationship is determined via Nash bargaining between the worker and the firm after the new value of z has been observed by both parties. Thus employment relationships may also break up endogenously because the bargaining surplus turns negative due to an unfavourable draw of z.[61]

The main difference between Ljungqvist and Sargent (2004) and Den Haan et al. (2001,2005) lies in whether endogenous separation can trigger skill loss. Ljungqvist and Sargent (2004) assume that only exogenously laid off workers face skill loss with a fixed probability $\gamma^{d,x}$ (the degree of turbulence), while Den Haan et al. (2001,2005) allow skill loss to occur in the event of endogenous separation as well with a probability γ^d, where $\gamma^d = \varepsilon\gamma^{d,x}$ and ε is a small fraction. Accordingly, in Ljungqvist and Sargent (2004), turbulence continues to operate in a very similar fashion to Ljungqvist and Sargent (1998). An increase in turbulence increases the share of workers entering unemployment with high benefits (due to a high past skill level and high past earnings) but a low skill level and thus low earnings potential. If matched to a firm, their high benefit level means that they enter bargaining with a high outside option. It is then quite likely that the realisation of z is too low for the bargaining surplus to turn positive and thus the relationship is not formed. Accordingly, these workers have a low hazard rate of gaining employment. Thus as in Ljungqvist and Sargent (1998), an increase in

[60] See Den Haan et al. (2005), p. 1363.
[61] See Den Haan et al. (2005), p. 1363.

turbulence depresses the overall hazard rate, increases average unemployment duration and the average unemployment rate.[62]

Den Haan et al. (2005) show that these results change dramatically even for a very small probability of skill loss following an endogenous separation. For $\varepsilon = 0.03$, the unemployment-turbulence relationship turns weakly negative, while for $\varepsilon = 0.05$ more strongly so. This is due to declining endogenous separation among highly skilled workers. Since workers are aware that terminating the employment relationship can have strong adverse consequences for their future earnings due to the possibility of a skill downgrade, they are willing to accept lower wages in case z takes a low value. In other words, higher turbulence lowers the disagreement point of highly skilled workers and thus lowers the reservation value of z, thus lowering endogenous separation of highly skilled workers. This channel comes to dominate the channel emphasized by Ljungqvist and Sargent (1998, 2004) and thus the overall unemployment rate declines.[63]

Thus the unemployment-turbulence relationship turns out to be highly sensitive to whether skill loss is allowed to happen in the event of an endogenous separation. Ljungqvist and Sargent (2004) justify out ruling this possibility as follows: "We see quitters as people who are secure in their skills and inspired to change jobs to take advantage of evident opportunities to make better use of their current skills".[64] However, Den Haan et al. (2005) argue that "the quit/ layoff distinction is completely arbitrary in the context of Nash bargaining" since workers and firms mutually agree to terminate their relationship after observing the draw of z. Correspondingly, "exogenous separations can be viewed as responses to changes in z such that the surplus becomes

[62]See Ljungqvist and Sargent (2004), pp. 465-467. As in Ljungqvist and Sargent (1998), the average inflow rate into unemployment is not affected.
[63]See Den Haan et al. (2005), pp. 1372-1374, and p. 1361.
[64]Ljungqvist and Sargent (2004), p. 462.

permanently negative. Viewed in this way, there is no fundamental distinction, but only a quantitative difference, between exogenous and endogenous separation, as both are optimal responses to a deterioration in z. This view accords with our perturbation experiment, where the probability of skill loss after and endogenous separation is a (small) fraction of the probability of skill loss after an exogenous one." They add that labelling endogenous separation as employee quits is a departure from the established search and matching literature.[65]

1.3. Evidence on Endogenous Persistence and Unit Roots in Unemployment

A main motivation for the models developed in chapters two and four is the evidence for high endogenous persistence in European unemployment and lower endogenous persistence in the United States. In particular, there has been an ongoing discussion over the last two decades about whether unemployment has a unit root. Taken at face value, a unit root in unemployment does not appear plausible since it would imply that unemployment could in principle become negative or could exceed 100%. Thus if unit root behaviour is detected, it should be interpreted either as a local linear approximation to a global non-linear and stable process or as evidence for very high unemployment persistence indistinguishable from a unit root. Thus we view such evidence as a motivation to develop models which produce significant unemployment persistence.

Following Camarero et al. (2006), tests of the null of a unit root hypothesis against the alternative of stationarity or trend stationarity, can be divided into three groups. The first one consists of classical ADF type tests. The second group tests for the unit

[65]Den Haan et al. (2005), pp. 1374-1377.

root in the presence of structural breaks. The third group tests for a unit roots in a panel of countries.

An early example of the first group are Blanchard and Summers (1986), who reject the unit root for the United States but fail to do so for France, Germany and the United Kingdom. Indeed, today a broad consensus in the literature says that applying such tests to EU (or EEA) countries almost uniformly leads to a failure to reject the unit root, while the unit root tends to be rejected in the United States.[66].

However, unit root-like behaviour of a variable might in fact be generated by a stationary process with a structural break as shown by Perron (1989). Hence if one estimates an autoregressive equation without a break, the estimate of the autoregressive coefficient will be biased towards unity, rendering a failure to reject the unit root quite likely. The location of the break is somewhat arbitrary and could itself be the source of bias. Perron suggests to base the break location on graphical inspection of the series and historical events representing shocks which can be safely considered exogenous, which is followed by Mitchell (1993). Roed (1997) surveys this and other early tests along these lines and concludes that the results found using simple ADF tests continue to hold.[67] More recently, this consensus has been partly challenged by tests basing the break location on the unemployment data itself using statistical algorithms. Papell et al. (2000) test the unit root hypothesis for 16 OECD countries for unemployment data spanning from 1955 to 1997.[68] They allow for one structural break, the date of which is chosen in order to minimise the t-statistic on lagged unemployment after the break and a constant have been removed from the data.[69] This procedure of course maximises the

[66]See Camarero et al. (2006), pp. 168-169.
[67]See Roed (1997), p. 408.
[68]See Papell et al. (2000), p. 310. The countries are Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, Spain, Sweden, UK and USA.
[69]See Papell et. al. (2000), p. 311.

likelihood that the null is rejected. The unit root is rejected in Belgium, Canada, Denmark, Finland, Norway, Spain, Sweden, the United Kingdom and the United States. They fail to reject the unit root in Australia, France, Germany, Italy, Japan, Netherlands, while in Ireland, it is rejected only at the 10% level. Hence even when allowing for a structural break, the unit root can not be rejected in most of the bigger European economies with a history of high unemployment. However, the estimated autoregressive coefficients in the countries where the unit root has been rejected are quite low.

Papell et al. (2000) then search for multiple breaks in those countries where the unit root was rejected and detect 20 significant breaks, out of which 18 are positive. The United States have one positive and one negative break. Furthermore, the positive breaks are clustered in times of recession like for instance those at the beginning of the 1980s.[70] The authors conclude their results are "in accord with the view that, especially for Europe, increases in unemployment during recessions have lead to increase in long term unemployment."[71] Camarero et al. (2006) also conduct a unit root test with multiple structural breaks and offer the same interpretation of the break location.[72] Thus these results can be seen as motivation for a theory purporting that aggregate demand fluctuations can have long lasting effects on unemployment, even though a simple autoregressive process in unemployment does not always seem to be a good approximation to the underlying data generating process.

Arestis et al. (2000) conduct a very similar test on quarterly data from 23 OECD countries stretching for most countries from 1960 to between 1995 and 1997.[73] They

[70] Eight occur during 1973-1975, five from 1980 to 1982 and three from 1988 to 1991.
[71] See Papell et al. (2000), p. 314.
[72] See Camarero et al. (2006), pp. 178-179 and p. 180.
[73] The countries are Australia, Austria, Belgium, Canada, Denmark, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the United States and the United Kingdom.

find that the unit root is not rejected in 11 countries, namely Austria, France, Greece, Iceland, Ireland, Japan, Netherlands, Norway, Spain, Sweden and the USA. For Portugal, Italy and New Zealand, the result is sensitive to how the lag length is chosen. If the method leading to more lags is chosen, the unit root is not rejected in Italy and New Zealand.[74] The unit root is unambiguously rejected in Australia, Belgium, Canada, Denmark, Germany, Finland, Luxembourg, Switzerland and the UK. For Belgium and Denmark, the estimated persistence coefficients amount to 0.976, while for Germany, the estimate is still consistent with a value of 0.935.

Moreover, we would like to note that the two studies from the literature on the relationship between labour market institutions and unemployment which include a lagged dependent variable (i.e. Nickell et al. (2002, 2005) and IMF (2003)) can be interpreted as tests for endogenous persistence (if not a unit root) in unemployment in the presence of structural breaks, where the latter have a strong theoretical motivation rather than a purely statistical one. As was mentioned above, using annual data, the coefficient on lagged unemployment are 0.86 and 0.8, respectively, which is a substantial degree of persistence.

The third approach is motivated by the well known finding that standard unit root tests have low power against stationary alternatives in small samples. Testing the unit root hypotheses in a panel context promises to increase power by making use of cross sectional information. For instance, Song and Wu (1998) test for a unit root in a panel of 15 OECD countries on quarterly data stretching form 1972 to 1992.[75] They use a test developed by Levin and Lin (1992) which restricts the coefficient on lagged unemployment to be the same for all countries in the panel to improve power

[74] See Arestis et al. (2000), p. 402, table 1.

[75] The countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Sweden, U.K. and USA.

while allowing for country specific serial correlation, country specific and time specific effects.[76] They reject the unit root at the 10% level with only country specific effects and at the 5% level with both country and time specific effects. They estimate the coefficient on lagged unemployment as 0.976 and 0.957, respectively, which is clearly substantial.[77] Leon-Ledesma (2002) tests for a unit root in a panel of 51 US states and a panel of 12 EU countries using quarterly data from 1985 to 1999.[78] He uses an IPS test where the null is a unit root among all individuals in the panel, while the alternative says that at least some individuals have stationary unemployment rates. There is no cross country restriction on the speed of mean reversion under the alternative. As shown by Im et al. (2003), allowing for this amount of heterogeneity reduces size distortions while it also increases power relative to the Levin and Lin (1992) test.[79] Applying the IPS test to the European panel, he finds that the unit root is not rejected at the 10% level, while he is able to reject the unit root in the panel of US states.[80] Camarero and Tamarit (2004) test for a unit root in 19 OECD countries on annual data stretching from 1956 to 2001.[81] They use a SURADF test developed by Breuer (1999) which allows for allow for heterogeneous serial correlation, contemporaneous correlations among errors across individuals and different autoregressive parameters for each individual. Unlike the panel tests discussed so far, however, it allows to test the unit root for each individual country in the panel.[82] While the unit root is rejected in the United States, it is not rejected for Austria, Germany, Italy, Japan, Norway, New Zealand and Switzerland, while it is

[76] See Song and Wu (1998), p. 184.

[77] On an annual basis, this amounts to 0.91 and 0.84, respectively.

[78] The countries are Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden and UK.

[79] See Leon-Ledesma, p. 98.

[80] See Leon-Ledesma (2002), p. 101.

[81] The countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, New Zealand, Spain, Sweden, Switzerland, UK and USA.

[82] See Camarero and Tamarit (2004), p. 415.

rejected for France and Spain only at the 5% and 10% level, respectively. Chang et al. (2005) apply the same test to a to annual data stretching from 1961 to 1999 from ten European countries.[83] They reject the unit root only in Belgium and the Netherlands at the 5% and 10% level, respectively, while the unit root is not rejected in Denmark, Finland, France, Norway, Portugal, Ireland, Italy and the UK.

To sum it up, even though unit root tests using endogenously determined structural breaks and to some extend also panel unit root tests indicate that unit roots in European unemployment seem to be less widespread than previously thought there still seems to be quite a bit of evidence in favour of a unit root in unemployment or at least high endogenous persistence in Europe, especially in the bigger continental European economies. Among the post-1997 studies discussed above which are able to discriminate between Europe and the United States, the unit root in unemployment was not rejected for Germany, Italy, France, Spain, Ireland and the Netherlands in three out of four, five out of five, four out of five, two out of four, three out of five and three out of five studies testing for unit roots in these countries. Furthermore, for both Spain and Ireland one rejection is only significant at the 10% level. Interestingly, three out of three studies did not reject the unit root in Japan. Furthermore, in those studies featuring structural breaks, virtually all positive breaks are located during recessions, which is consistent with a model generating long lasting effects of aggregate demand fluctuations on unemployment.

[83] Belgium, Denmark, Finland, France, Ireland, Italy, Netherlands, Norway, Portugal and the UK. Note that apart from Norway, all these countries where also considered by Camarero and Tamarit (2005).

1.4. Monetary Policy and the NAIRU

This section summarizes three papers by Ball which try to establish a relationship between the monetary and macroeconomic policy stance and changes in the NAIRU in a set of 20 OECD countries during the 1980s and to some extend subsequently as well. Ball (1996) and Ball (2007) ask whether there exists a negative relationship between changes in inflation and changes in the NAIRU. Just as in the approach of Jackman et al. (1991), an increase in inflation is seen as evidence that unemployment is below the NAIRU due aggregate demand pressures. With some sort of hysteresis, this would increase the NAIRU itself. These papers are discussed in the following section. In section 1.4.2, we turn to Ball (1999), which focuses mainly the relationship between the change in the short term real interest rate during recessions and subsequent changes in the NAIRU. In Ball (1996, 1999), the analysis follows the OECD Jobs study in constructing the NAIRU data, while in Ball (2009) uses a method developed by Ball and Mankiw (2002).[84]

A general problem with Ball's analysis is that it lacks a well specified theoretical model which could more tightly motivate the way he looks at the relationship between disinflations and real interest rates on the one hand and NAIRU movements on the other. However, this thesis develops theoretical models which are able to replicate some of his empirical findings.

[84]See Ball (1996), p.3 and Ball (2009), p. 7.

1.4.1. The Relationship between the Change in Inflation and the Change in the NAIRU

Ball uses two measures of inflation dynamics: The size of the disinflation from 1980 to 1990 and the length of the longest disinflation during that period. Those matter because in standard NAIRU models, the former is related to the size of the unemployment increase, while the latter indicates for how long the actual unemployment rate exceeded the NAIRU. Ball also points towards empirical evidence saying that slower disinflations cause higher cyclical output losses.[85] He squares the length of disinflations but gives no further justification for that other than this resulted from "experimentation with functional forms".

Ball's preferred regressions features the change in the NAIRU as dependent variable and the interaction between benefit duration and the policy stance as independent variables. From the labour market hysteresis point of view, one would expect a fall of unemployment below the NAIRU to have a greater effect if unemployment benefits are paid for a long time. A longer benefit duration will cause less enthusiastic search activity among the newly unemployed, thus increasing the time required to find a job. This will increase skill loss and also reduce the competitive pressure on insiders bargaining with firms over wages. Furthermore, longer benefits facilitate the process of becoming accustomed to an unemployment lifestyle, thus enhancing the reduction in search activity.[86]

Hence Ball considers interactions of benefit duration with the inflation decrease and squared length. This yields an R^2 between 0.67 and 0.75, depending on whether both interactions are included or whether only the change in inflation is interacted with

[85] See Ball (1996), pp. 6-7.
[86] See Ball (1996), p. 13.

benefit duration. Ball concludes that while with the amount of data available it is not possible to decide which specification is superior, "a broad conclusion is robust: the explanatory power of macro policy variables increases greatly when we account for interaction with benefit duration."[87]

Ball then subjects this result to a series of robustness experiments, all of which basically confirm his original conclusions.[88] Most notably, he tries to show that the correlation between the change in the NAIRU and the change in inflation represents a causal relationship rather than increases in the NAIRU and inflation driven by an unobserved third variable which subsequently lead to large disinflations parallel to the rise in the NAIRU with no causal relationship between the two.[89] All in all, Ball's results seem quite supportive to the hypothesis that monetary policy, to the extent that it is responsible for the disinflations in the sample, indeed has an effect on the NAIRU.

In a more recent paper, Ball (2009) examines the relationship between major changes in inflation and major changes in the NAIRU over the period from 1980 to 2007 in a simple fashion.[90] He first defines episodes of major NAIRU changes as periods during which the NAIRU moves in the same direction and which also involve a change in the NAIRU of at least three percentage points within a period of ten years (or less).[91] Using this definition, he identifies eight episodes of NAIRU increases and nine episodes of NAIRU decreases.[92] For each of these episodes, he looks for large changes in inflation, defined as those exceeding three percentage points.

[87] Ball (1996), p. 12.
[88] These includes using and HP estimate of the NAIRU and a procedure to account for an imperfect purge of cyclical unemployment out of the NAIRU measure. See Ball (1996), pp. 13-15.
[89] See Ball (1996), p. 16-17.
[90] Trend inflation is measured as a centred nine quarter moving average, see Ball (2009), p. 16.
[91] See Ball (2007), p. 14.
[92] See Ball (2009), p. 15.

Ball then shows how the episodes of NAIRU increases and decreases are associated with changes in inflation. Six out of eight episodes of NAIRU increases involve major disinflations and no inflation run-up. Two countries (Sweden and New Zealand) each involve two disinflations and one run up in between. The size of the disinflations are in both countries larger than the intervening run-up, and the overall change in inflation over the three periods is highly negative in each case. Ball therefore views these countries as having disinflationary regimes overall.[93] He concludes that "if you find an episode with a NAIRU increase, it is always an episode with a major disinflation. To put the same result in a different way, a major disinflation is a necessary condition for a NAIRU increase."[94] He adds that "the reverse result does not hold: a disinflation is not sufficient for a NAIRU increase."[95]

The picture is more complex for episodes of NAIRU decreases. Out of the nine episodes of NAIRU decreases, only five include at least one inflation run-up, and four of these include one disinflation as well. However, Ball notes that the inflation run-ups and disinflations are of familiar sizes and thus the disinflations where not "overwhelmed" by larger disinflations.[96] Furthermore, four countries experience decreases in the NAIRU without major changes in inflation as defined by Ball. Ball notes all of these NAIRU decreases were preceded by large increases in the NAIRU and only partly reversed these increases.[97] Ball interprets this as suggesting that "hysteresis effects are long lived but not permanent. Tight monetary policy causes a rise in unemployment that lasts a long time, but eventually unemployment starts falling even if inflation is stable."[98] Thus

[93]See Ball (2009), p. 17.
[94]See Ball (2009), pp. 17-18.
[95]See Ball (2009), p. 18.
[96]See Ball (2009), p.19.
[97]See Ball (2009), p. 20.
[98]Ball (2009), p. 20.

an increase in inflation is a necessary condition for a decrease in the NAIRU if "mean reversion is not at work."

Finally, Ball looks at the inflation run-up/ NAIRU change relationship the opposite way. There are 13 inflation run-ups over the sample period. Ball excludes four of them either because the countries involved are not able to experience a major decline in the NAIRU due to its low level (Japan and Switzerland) or the inflation run-ups are interruptions of overall disinflationary regimes.[99] This leaves nine inflation run-ups, seven of which occurred during NAIRU decreases. The two that do not involve the smallest "major" disinflations in the sample. Ball concludes that "With some qualifications, an inflation run-up is sufficient for a NAIRU decrease."[100] Ball (1999) provides further evidence for a relationship between NAIRU reductions and inflation run-ups for 10 OECD countries between 1985 and 1997.[101]

1.4.2. Short-term Interest Rates and the NAIRU in the 1980s

Ball then turns towards the role of monetary easing as measured by interest rates during the deep recessions of the early 1980s. At first, he focuses on the policies of six countries (United States, Canada, United Kingdom, France, Italy, Germany). Those countries had very different outcomes both in the development of actual unemployment and the NAIRU. All those countries went into two recessions during the 1980-1984 period, with the exception of Germany, which had only one. Ball defines a recession as two quarters of falling GDP in row or a fall of GDP exceeding 2% in any quarter.[102] However, while in the United States and Canada, actual unemployment rose sharply

[99]The former two countries are Japan and Switzerland. The latter two country/ inflation-run-up pairs are the aforementioned Swedish and New Zealand episodes

[100]Ball (2009), p.22.

[101]See Ball (1999), pp. 212-220.

[102]See Ball (1999), p. 193.

but returned to its pre-recession value during the eight years following the pre-recession peak or even fell below this value (as in the USA), in the European countries (including the UK), unemployment remained between 3.8 percentage points and 5.8 percentage points higher than at the pre-recession peak.[103]

The turnarounds in unemployment in the North American countries were driven by strong recoveries pushing output growth above trend in the quarters especially after the second recession. These recoveries ensure that long run average growth rates across the two recessions, where Ball defines the 20 quarter average beginning at the pre-recession peak of the first recession as "long run", are close to estimates of potential output growth for those countries, whereas the long run growth rates for the European countries remain substantially below theirs.[104] At the same time, the reduction in inflation in the European countries were not substantially larger as after some time inflation just stopped falling although unemployment was still high. The average inflation reduction in the four European countries exceeds the average inflation reduction in the North American ones by just 2.3 percentage points.[105] Correspondingly, OECD estimates of the NAIRU show substantial increases for the four European Countries (between 2.4 and 3.3 percentage points) during the five years following the pre-recession peak while the NAIRU actually slightly declines during that period in Canada and more strongly in the United states.[106]

Ball then shows that the central banks of the two North-American countries reacted much more forcefully to the unfolding recessions than the four European countries. He measures the policy response by calculating the cumulative change in short-term real

[103]See Ball (1999), p. 201.
[104]See Ball (1999), pp. 199-200.
[105]See Ball (1999), p. 202.
[106]See Ball (1999), pp. 202-204.

and nominal interest rates between the pre-recession peak and the quarter after the trough for each recession. The relevant inflation rate is calculated using the average of two four quarter moving averages of present and past inflation rates.[107] Ball summarizes the results as follows: "In the four recessions in the NA2 [Canada and the United States, A.R.], the total change in the real interest rate from the peak to the quarter after the trough ranges from -1.4 to -5.4 percentage points, with an average of -3.4 percentage points. In the seven recessions in the E4[United Kingdom, France, Italy, Germany, A.R.], there is always less easing of policy, and often[UK, France 80-81] even a tightening: the change in the real interest rate ranges from -1.1 to +2.6 percentage points, with an average of +0.2 percentage points."[108] Ball shows that these differences are mostly due to different evolution of nominal interest rates and thus are caused by monetary policy.[109] Hence it seems that those countries experiencing no increase in the NAIRU also pursued a stronger countercyclical monetary policy.

Ball then moves to a larger sample of 19 OECD countries in order to check whether these conclusions carry over and to control for the effect of benefit duration, for the reasons discussed in the previous subsection.[110] For a lack of data, he now uses annual rather than quarterly output figures. This requires new identification procedures for recessions and the stance of monetary policy. The former is now defined as one or more consecutive years of growth below one percent a year, while the latter is measured by largest cumulative decrease in any part of the recession's first year, or the average

[107] See Ball (1999), pp. 193-196. Ball reports that he experimented with both t-3 through t and t-4 to t-1 averages. The former he considers to be more commonly used in the literature but criticizes for the fact that the period t inflation rate is unknown in period t. He reports that the "broad conclusions" of his analysis remain unaffected by the choice of the inflation rate but uses an average of the two in the paper.
[108] Ball (1999), p. 196.
[109] See Ball (1999), pp. 194-196.
[110] See Ball (1999), pp. 203-204.

of the largest cumulative decreases from each recession in case the country had two recessions.[111] Ball then uses this measure of policy and benefit duration to explain two variables: the change in the NAIRU from the peak before the first recession until five years after the peak, and this change divided by the change in actual unemployment over the same time period. The latter variable is called degree of hysteresis and accounts for the fact that the severity of recessions and thus the increase in actual unemployment vary over the sample and hence one would observe different increases in the NAIRU even if actual unemployment fed into the NAIRU to the same extent in all countries, i.e. if monetary policy and benefit duration had been the same.[112] In both equations, both the easing variable and the benefit duration variable are individually significant. Fit is substantially better when the degree of hysteresis is used as a dependent variable, with an adjusted R^2 of 0.62 as opposed to 0.43. This is in line with the fact mentioned above, namely that the countries in the sample experienced recessions of differing severity. Concerning the quantitative impact of the two variables on the degree of hysteresis, Ball points out that "The coefficient on maximum easing implies that raising that variable from 0 to 6 (Sweden's value, the highest in the sample) reduces the degree of hysteresis by 0.54. Reducing the duration of unemployment benefits from indefinite to half a year reduces the degree of hysteresis by 0.35. Thus policymakers can reduce hysteresis through both macroeconomic and labour market policy, and the former has somewhat larger effects."[113] Hence it seems that the effect of monetary policy on the NAIRU in the 80s is robust to different specifications of the policy stance.

[111] Ball notes that his dating criterion for recessions yields only two countries with two recessions and thus is stricter than the one used with quarterly data. See Ball (1999), p. 205.
[112] See Ball (1999), p. 205-206.
[113] Ball (1999), p. 207.

Stockhammer and Sturn (2008) extend this part of Ball's analysis in four ways. Firstly, they extend the sample to range from 1980 to 2003. Secondly, they use quarterly rather than annual data to measure the period of recession, the degree of hysteresis and the reaction of monetary policy. The degree of hysteresis is now given by the increase in the NAIRU during the five years following the business cycle peak divided by the greatest increase in actual unemployment from the quarter before the recession to at most 18 quarters later. The response of monetary policy is measured by the cumulated change of the ex post short-term real interest rate per quarter between the first quarter of the recession and the second quarter after the recession. Thirdly, they control for a larger set of labour market institutions and finally, they conduct a set of robustness checks.[114] They find that in the various specifications they estimate, monetary easing almost always has a highly significant and quantitatively substantial effect on the degree of hysteresis, with the coefficient on monetary easing ranging between 0.3 and 0.85.[115] This is substantially above Ball's (1999) estimate.

1.5. Conclusion

This chapter motivates the theoretical research conducted in the following chapters. For that purpose, we first critically examine what we consider the two leading mainstream approaches trying to explain the evolution of unemployment across time in advanced OECD countries and the fact that unemployment has persistently increased in many European countries but has not done so in the United States. The first of these is the macroeconometric work inspired by Jackman et al. (1991) trying to explain changes in unemployment via changes in labour market rigidities, also referred to as

[114] See Stockhammer and Sturn (2008), p. 2 and p. 5.
[115] See Stockhammer and Sturn (2008), pp. 8-15 and Table 1 and 2 at the end of the paper.

labour market institutions. We identify the following problems. First, the ability of changes in labour market institutions to explain the evolution of unemployment across time is limited, especially for several high unemployment European countries. This seems to be partly related to the fact that in many countries institutions have not changed very much. Furthermore, there appear to be robustness issues and problems of reverse causality: Some institutions like the generosity of unemployment benefits appear to have changed in response to increasing unemployment.

In response to the first of these issues, Ljungqvist and Sargent (1998, 2004) develop models able to generate an increase in unemployment for a given level of institutions. In their framework unemployment benefits interacts with "microeconomic turbulence", where the latter is defined as the probability that a worker suffers substantial skill loss following an exogenous separation. An increase in turbulence increases unemployment in an economy where unemployment compensation forms a large fraction of income in the previous job but will not affect unemployment in an economy where benefits are absent. The interaction of benefits and turbulence pushes a large share of workers into benefit traps from which they have no incentive to escape. However, den Haan et al. (2005) show that this result is not robust against allowing skill loss linked to endogenous separations, even if the probability of this to happen is only a tiny fraction of the probability of skill loss following an exogenous separation. We conclude from this that causes of large swings in OECD unemployment are by no means well understood and that the development of a new theory explaining such swings is called for. This is the overall goal of this thesis.

We then turn our attention to an additional set of empirical findings surrounding the unemployment nexus which we aim to explain. Section 1.3 examines the time series evidence on endogenous persistence and unit roots in unemployment. Although there is some controversy, there is quite a bit of evidence consistent with a unit root or at least high endogenous persistence in a set of large Western European countries with a history of high unemployment, including Germany, France, Italy and Spain. By contrast, for the United States, the unit root is rejected most of the time.

Studies which are more likely to reject the unit root even in the aforementioned countries are those allowing for structural breaks. While allowing for structural breaks makes the unit root more likely to be rejected even in many European countries, virtually all positive breaks are located during recessions. This is consistent with a model generating long lasting effects of aggregate demand fluctuations and thus of monetary policy on unemployment.

Section 1.4 examines Ball's OECD evidence surrounding the relationship between the NAIRU on the one hand and the movement of inflation on the other. Ball shows that a major disinflation seems to be a necessary condition for an increase in the NAIRU to occur. He also shows that there exists negative relationship between the change in inflation from 1980 to 1990 and the change in the NAIRU during that period. Finally, he finds a relationship between the amount of monetary easing during the recessions of the 1980s and the subsequent development of the NAIRU.

Hence we conclude that a new theory of medium run unemployment swings should be able to shed light on the following set of findings:

- The medium run swings in unemployment we observe in OECD countries, particularly the increase in unemployment in several Western continental European countries since the end of the 1970s. Our discussion in section 1.1

suggests that the theory should generate medium run swings without relying on changes in labour market institutions.

- The time series evidence we reviewed in section 1.3 saying that there is high *endogenous* unemployment persistence, or even unit root behaviour in a number of Western continental European countries, but much less persistence in the United States. That means that a temporary shock increasing unemployment today should have a lasting effect on unemployment long after it has passed.
- The time series evidence that positive structural breaks in unemployment seem to be predominantly located during recessions. This suggests that aggregate demand contractions may have long lasting effects on unemployment.
- The evidence produced by Ball that a major disinflation seems to have been a necessary condition for a major increase in the NAIRU to have happened. There is also evidence that the change in inflation over a ten year period is negatively related to the change in the NAIRU during that period. We discussed these findings in section 1.4.1
- The evidence produced by Ball that the amount of monetary easing during a recession is negatively related to the subsequent change in the NAIRU, which we discussed section 1.4.2.

CHAPTER 2

Shocks, Monetary Policy and Institutions: Explaining Unemployment Swings in Europe

This chapter examines the rise in European unemployment by introducing endogenous growth along the lines of Romer (1986) into a New Keynesian model featuring unemployment. We subject the economy to a one quarter non-serially correlated cost-push shock and let the central bank disinflate the economy. The purpose of the cost push shock is to create scenario akin to the second oil price shock and its aftermath. This temporary shock causes a persistent and substantial increase in unemployment, lasting over 10 to 20 years in an order of magnitude of one percentage points or more. The model also sheds light on some cross-country differences in the unemployment experience.

More precisely, we aim to shed light on the following set of stylised facts and empirical findings (this list is partly a recap of the list in of chapter one):

- Unemployment has increased substantially in many large European economies since the 1970s. Figure 2.1 displays quarterly unemployment rates from 1975 to 2000 for six selected European Economies and the United States. By contrast, there is no such persistent increase in the United States. Furthermore, note that unemployment increases relatively quickly, as for instance during the recessions at the beginning of the 80s, but reverts only relatively slowly, incompletely, or not at all. Moreover, there is evidence in favour of high *endogenous* unemployment persistence, or even unit root behaviour in a number of Western continental European countries. That means that any temporary shock increasing unemployment today will have a lasting effect on unemployment long after it has passed.

- There has been a decline in the growth rate of labour productivity (measured as output per hour worked) across OECD countries in the 1980s. This decline has been substantially larger in Western European economies than in the United States. Average annual productivity growth in Western European economies was 1.5% lower in the period from 1981 to 1990 than in the previous decade, while it declined by merely 0.2% in the United States.[1] Skoczylas and Tissot (2005) estimate changes in trend productivity growth for OECD economies from 1960 to 2004. They locate declines between one and 3.9% between 1976 and 1985 in nine Western European Economies but none in the United States.
- It is a consistent finding that a decline in productivity growth increases unemployment. Examples include Bassanini and Duval (2006), Pissarides and Vallanti (2005), Nickel (2002, 2005), Ball and Moffitt (2001), Blanchard and Wolfers (2000) and Fitoussi et al. (2000). Three of these studies (Bassanini and Duval, Blanchard and Wolfers, Fitoussi et al.) explicitly model interactions between decreases in productivity growth and labour market institutions. They find that macroeconomic shocks help to explain the evolution of unemployment across time while cross country-differences in institutions help to explain why

[1]The number is based on cross country averages for 1971-1980 and 1981-1990 of the productivity growth rates of Belgium, Denmark, Western Germany, Ireland, Spain, France, Italy, the Netherlands, Finland, Sweden, the United Kingdom and Norway. These rates are based on the series on GDP at constant prices and total hours worked from AMECO (2008).

in some countries unemployment responds more strongly to macroeconomic shocks than in others.

- As discussed in chapter one, based on evidence from advanced OECD economies, Ball (1999) argues that those central banks willing to aggressively lower real interest rates during the recessions of the early 1980s reduced the subsequent increase in the NAIRU in their countries.
- Based on OECD evidence, Ball (2009) shows that "if you find an episode with a NAIRU increase, it is always an episode with a major disinflation. To put the same result in a different way, a major disinflation is a necessary condition for a NAIRU increase."[2]
- There seems to be a negative medium run relationship between the change in inflation and the change in the NAIRU. This is illustrated in figure 2.2, which plots the change in the NAIRU against the change in CPI Inflation for 21 OECD countries from 1980 to 1990 and from 1990 to 2000. The negative correlation is not perfect but still obvious: Countries with a larger decrease in inflation suffered on average a larger increase in their NAIRU.[3] We mentioned that Ball (1996) is the first to draw attention to this link and also investigates it more formally.

Our motivation for addressing these issues by introducing endogenous growth into a sticky price model can be sketched as follows. The standard way to think about the effects of a monetary contraction is that it increases unemployment, lowers real wage growth, unit labour costs of firms and thus inflation. The decline in inflation induces the

[2]See Ball (2009), pp. 17-18.

[3]The data is taken from the OECD Economic Outlook. The countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, U.S.A.

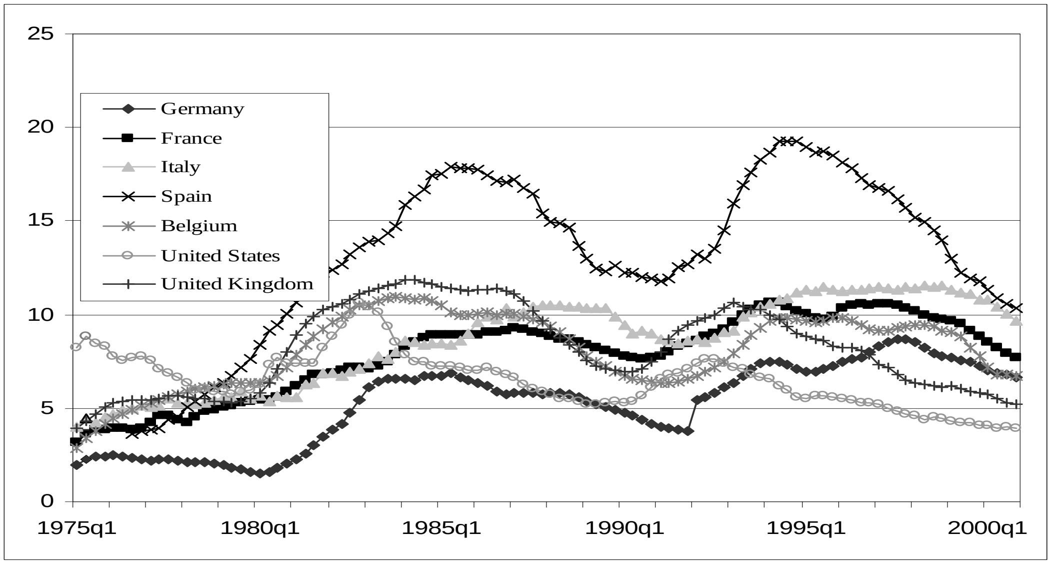

Figure 2.1. Unemployment for six European Economies and the United States

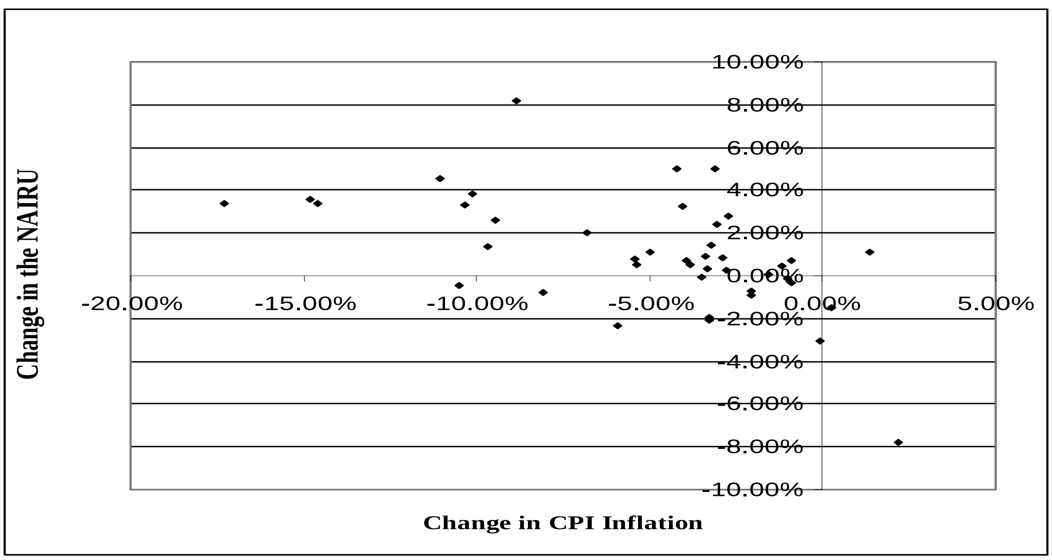

Figure 2.2. Change in CPI Inflation vs. Change in the NAIRU: 1980-1990, 1990-2000

central bank to lower the nominal and real interest rate, which would work to reverse the increase in unemployment.

Introducing endogenous growth inhibits the deflationary effects of a monetary contraction because it implies a strong link between investment and productivity growth. It thus creates a link between monetary policy and aggregate demand on the one hand and productivity growth on the other. A monetary contraction -since it lowers investment-thus also reduces productivity growth. The decline in productivity growth lowers the real wage growth rate associated with stable inflation. If real wage growth is rigid, stable inflation will then require an increase in the unemployment rate: the NAIRU increases. The central bank therefore engineers a very slow recovery of aggregate demand and unemployment to prevent an acceleration of inflation. The slow recovery of demand also slows down the recovery of investment and productivity growth.

Thus endogenous growth may be able to create endogenous unemployment persistence: A temporary shock increasing unemployment today -for instance an adverse cost push shock inducing the central bank to raise the interest rate- may also increase unemployment in the future. It may also be able to generate a persistent decline in productivity growth in response to a contraction of the economy. Furthermore, the response of the central bank to the shock will shape not just the short run, but also the medium run response of unemployment to the shock. The goal of this chapter is to explore the quantitative relevance of these mechanisms.

Our approach is in some respects similar to Comin and Gertler (2006) in that they also incorporate endogenous growth into a general equilibrium model in order to analyse the causal relationship between short and medium run movements of the economy. In particular, temporary mark-up shocks have lasting real effects in their model. However, in their flexible price real business cycle model, monetary policy, unemployment and inflation are absent, all of which are our focus of attention.

Our results resemble in some respects those of Sargent and Ljungqvist (1998, 2004, 2007) in that the model proposed here generates an increase in unemployment without relying on changes in labour market rigidity, while the "level" of labour market rigidity does matter. However, as we discussed earlier, their approach differs from ours in that in their model, unemployment increases via the interaction of unemployment benefits linked to past income and a permanent increase in "microeconomic turbulence".

This chapter is structured as follows: Section 2.1 develops a model which broadly reflects the mainstream consensus on the long and short run dynamics of unemployment as for instance developed by Jackman et al (1991). In this model, a temporary cost push shock only has a short lived effect on unemployment and the same is true for the monetary policy response to the shock. We coin this model "Jackman, Layard, Nickell", or JLN economy. We then add the New Growth extension. Section 2.2 calibrates the model to Western German data. Section 2.3 compares the second moments of important variables of the model to German data. Section 2.4 then discusses the response of the economy to a one quarter cost push shock calibrated to induce a disinflation of about 4 percentage points and focuses on the induced evolution of unemployment across time. It also looks into the tradeoffs policymakers face between stabilising inflation and stabilising unemployment. Section 2.5 adds a cross-country dimension to our analysis. First, we vary the size of the cost push shock and record the resulting changes in inflation and the NAIRU over a 10 year horizon. We then compare the differences in the unemployment response generated by a Bundesbank and a Federal Reserve Policy rule as estimated by Clarida et al. (1998), and finally we investigate the effects of differences in real wage rigidity between Europe and the United States. Section 2.6 concludes.

2.1. The Model

In this section we will develop a New Keynesian model with unemployment and endogenous growth which contributes to explaining the above findings. To stress the fact our results stem from the introduction of endogenous growth, we also present an otherwise identical model without endogenous growth which we take as the starting point of our analysis. This is a model to approximate the prevailing consensus on the relationship between unemployment and the NAIRU. We will refer to this model as the JLN economy. This consensus says that while unemployment both in the short and in the long run is determined by aggregate demand, only the NAIRU is consistent with stable inflation. Inflation targeting central banks will push unemployment towards this level. The NAIRU itself will be affected by any variable which directly increases wages in spite of excess supply in the labour market, increases the pricing power of firms or reduces the efficiency of the labour market to match jobs to workers[4]

Sections 2.1.1 to 2.1.4 develop the JLN economy, while section 2.1.5 introduces endogenous growth.

2.1.1. Households

The representative household is infinitely lived and chooses its consumption C_t, end-of-period holdings of the risk-less bond B_t, investment expenditure I_t and its next period's capital stock K_{t+1} in order to maximise the expected present value of its lifetime utility. C_t is a CES consumption, i.e. $C_t = \left[\int_0^1 (c_t(i))^{\frac{(\theta-1)}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$ where $c_t(i)$ denotes the different varieties in the basket while θ denotes the elasticity of substitution between those varieties. Following Smets and Wouters (2002), we assume external habit formation in consumption and adjustment costs in investment. The level of habit is denoted by

$$hab_{t-1} = jC_{t-1}$$

[4] See Nickell et al. (2002), pp. 2-3.

while investment adjustment costs imply the following capital accumulation equation:

$$
\begin{aligned}
K_{t+1} &= (1-\delta) K_t + I_t \left(1 - S\left(\frac{I_t}{I_{t-1}} - (1+g)\right)\right) \\
S(0) &= 0,\ S(0)' = 0,\ S(0)'' > 0
\end{aligned}
$$

Hence only a fraction $1 - S(.)$ of one unit of investment expenditure is actually turned into additional capital. This fraction decreases in the investment growth rate. The assumptions on the first derivative of the $S(.)$ function imply that adjustment costs vanish when the economy is growing at its steady state growth rate g.[5]

Following Danthine and Kurmann (2004), the representative household consists of a continuum of members who might be employed or unemployed but are all allocated the same level of consumption. Each household member supplies one unit of labour inelastically but derives disutility $G(e_t)$ from the effort e_t he or she supplies in their job. The share of unemployed members is the same for each household. These assumptions imply that although there are unemployed individuals in the economy, it is not necessary to track the distribution of wealth.

[5] There are two advantages of assuming investment adjustment costs and external habit formation. Firstly, it facilitates matching the second moments of investment and consumption, and secondly, it dampens the on-impact response of unemployment to the cost push shock in the simulation we are going to perform later, thus making it more reasonable. By contrast, the impact on the longer run response of unemployment, which is the main focus of this chapter, is rather small.

The cost of effort function of individual j $G(e_{t+i}(j))$ is of the form

$$G\left(e_t(j)\right) = \left(e_t(j) - \left(\begin{array}{c} \phi_0 + \phi_1 \log w_t(j) + \phi_2 f\left(n_t\right) \\ +\phi_3 \log w_t + \phi_4 \log w_{t-1} - \phi_5 \log b_t - \phi_8 \log\left(Y_{t-1}/\left(n_{t-1} - \overline{n} - n^s\right)\right) \end{array}\right)\right)^2,$$

$$\log b_t = \phi_6 \log\left(Y_{t-1}/\left(n_{t-1} - \overline{n} - n^s\right)\right) + (1 - \phi_6) \log w_{t-1} + \phi_7$$

$$f'\left(n_t\right) > 0, \phi_1, \phi_5, \phi_8 > 0, 1 \geqq \phi_6 \geqq 0, \phi_2, \phi_3, \phi_4 < 0, \phi_1 > -\phi_3,$$

where Y_t is private sector output. Note that the effort function enters the families' utility separately which implies that it is independent of the budget constraint.

The structure of the cost of effort function is motivated by the idea of "gift exchange" between the firm and the worker. The worker's gift to the employer is effort. The employer has to show his appreciation for the employees' contribution by paying an appropriate wage $w_t(j)$. A higher contemporary average wage w_t reduces effort because it represents a "reference level" to which the current employers' wage offer is compared. Put differently, it requires the firm to pay a higher wage if it wants to extract the same amount of effort. A higher average past real wage w_{t-1} boosts the workers' aspirations as well.[6] The aggregate employment level of non-overhead workers n_t summarizes labour market tightness. It is thus positively related to the workers' outside working opportunities, and thus also tends to reduce effort.

The view that wages have a big effect on workers morale and thus productivity because they signal to the worker how his contribution to the organizational goals

[6]See Danthine and Kurmann (2004), pp. 111-113. It would be desirable to have the individual workers past real wage $w_t(j)$ in the equation but that would considerably complicate the maximisation problem of the representative firm dealt with later, so we follow Danthine and Kurman in assuming a dependence of effort on the average wage. For the same reason we include average productivity rather than the respective firm's productivity.

is valued is supported by an extensive microeconomic survey conducted by Bewley (1998). Bewley found that wage changes (in particular wage cuts) seem to be especially important. Bewley interviewed over 300 business people, labour leaders and business consultants in search for an explanation why wages are rarely cut in recessions.[7] Bewley (2004) also finds that workers elicit higher effort if unemployment is high, layoffs are likely and new jobs are difficult to find. This motivates a role for n_t in the effort equation.[8]

The terms b_t and $(Y_{t-1}/(n_{t-1} - \overline{n} - n^s))$ represent a modification to the Danthine and Kurman (2004) cost of effort function. b_t denotes unemployment income. This will be chiefly unemployment benefits and black market income. It tends to lower the level of effort.[9] Workers want to be valued more than someone who receives benefits or does not have a legal job. b_t is linked both to past real wages and past productivity in the private sector, where Y_t denotes private sector output. This may reflect both the structure of benefits and the manner in which the black market is linked to the official economy. Productivity also has a direct effect on morale and effort as employees desire their due share of the companies' success. Unions might play a role in this to the extent that they instil a sense of entitlement among employees. Abowd et al. (2001) present evidence for a relationship between firm level wages and performance measures like value added and sales per employee. This suggests that there might be a relationship between effort and productivity.

[7] See Bewley (1998), pp. 459-490. A discussion of further evidence is Bewley (2004). Bewley also argues that his findings contradicts essentially all theoretical justifications of real wage rigidity not based on gift exchange considerations, like implicit contracts, insider outsider models or the efficiency wage models based on no-shirking conditions.

[8] See Bewley (2004), p. 10.

[9] Danthine and Kurman (2007) introduce the benefit level as a factor which, ceteris paribus, reduces effort.

Some of the household members supply overhead labour. They can be thought of as the owners of the monopolistically competitive firms. Overhead workers never become unemployed because no firm can produce without managerial staff. Furthermore, a share n^s of the workforce is employed by the government who is assumed to pay the same wage as the private sector. Government employees are funded by lump sum taxes.[10] We assume that these workers do not contribute to output but perform essential services without which the economy as a whole could not function, like policing, public transport and maintaining the infrastructure. All families have the same share of managers and government employees.[11]

Hence each period the household solves the following constraint maximisation problem by optimally choosing C_t, B_t, I_t, K_{t+1} and e_t :

$$U = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[u(C_{t+i} - hab_{t+i-1}) - (n_{t+i} - \overline{n})\, G(e_{t+i}) \right] \right\}, u\prime > 0, u\prime\prime < 0 \tag{2.1}$$

$$s.t.\ \ (n_t - \overline{n})\, w_t + r_t^k K_t + \frac{B_{t-1}}{P_t}(1 + i_{t-1}) + F_t \geq C_t + I_t \ + \frac{B_t}{P_t} + T_t \ and$$

$$K_{t+1} = (1-\delta)\, K_t + I_t \left(1 - S\left(\frac{I_t}{I_{t-1}} - (1+g) \right) \right),\ \ S\,(0) = 0,\ \ S\,(0)' = 0,\ \ S\,(0)'' > 0$$

---

[10]The level of the wage of state employees does not matter for the results because we assume lump sum taxes and a representative household. Checking (2.21) and (2.30) / (2.31) , which will be derived further below, reveals that real government expenditures $n^s w_t$ cancel out since they are part of aggregate demand, but also part of output. The government is assumed to employ workers directly. This implies that their wages are counted as output of the government at cost. This is what is conventionally done in the national accounts. Thus the demand generated by the government automatically generates its own supply, without any direct effect on private sector output.
The representative household will not increase its consumption if the wage of its members employed by the state increases because the increase in wage income is exactly offset by the higher tax burden and marginal benefits and costs of alternative actions are not affected by lump sum taxes.

[11]The reason for introducing both state employees and overhead workers $\overline{n}$ is to achieve a reasonable calibration of steady state values. In the Romer (1986) endogenous growth model, the level of employment affects the growth rate. This is due to the fact that the marginal product of capital is an increasing function of employment. The marginal product of capital governs the growth rate by determining the willingness of households to save. To achieve a reasonable steady state growth rate, we remove a fraction of the labour force from the "productive" sector by assuming that they perform necessary tasks without which the productive sector could not operate (managerial work in case of overhead workers, policing etc. in case of the state employees).

The second line denotes the budget constraint in real terms, where P_t denotes the price index of the consumption basket. The household's period t real income consists of real wage income $(n_t - \overline{n})\, w_t$ earned by the non-overhead workers, where w_t denotes the real wage, interest income on the real value of the risk-less bonds they bought in the previous period $i_{t-1}\frac{B_{t-1}}{P_t}$, where i_t denotes the nominal interest rate earned by holding bonds from period t to period t+1, the operating profits of the monopolistically competitive firms $\digamma_t$ which are accrued by the overhead workers, and rental income r_t^k K_t earned from renting the household's capital stock to the firms. Households have to pay lump sum taxes T_t to the government.

Setting up the lagrangian and denoting the lagrange multipliers of the budget constraint and the capital accumulation constraint as λ_t and $\lambda_t q_t$, respectively, yields the following first order conditions with respect to consumption, capital and investment:

$$u'(C_t - hab_{t-1}) = \beta E_t \left[u'(C_{t+1} - hab_t) \frac{1}{1 + \pi_{t+1}} \right] [1 + i_t] \tag{2.2}$$

$$\lambda_t = u'(C_t - hab_{t-1})$$

$$\beta E_t \left(\lambda_{t+1} r_{t+1}^k + \lambda_{t+1} q_{t+1} (1 - \delta) \right) = \lambda_t q_t$$

$$\begin{aligned} \lambda_t q_t \left[\left(1 - S\left(\frac{I_t}{I_{t-1}} - (1+g) \right) \right) - \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}} - (1+g) \right) \right] & \\ + \beta E_t \left[\lambda_{t+1} q_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 S'\left(\frac{I_{t+1}}{I_t} - (1+g) \right) \right] &= \lambda_t \end{aligned} \tag{2.3}$$

Note that with this notation, q_t denotes the present discounted value of the future profits associated with buying an additional unit of capital today, also known as Tobin's q.

The first order condition with respect to effort is

$$e_t(j) = \phi_0 + \phi_1 \log w_t(j) + \phi_2 f(n_t) + \phi_3 \log w_t + \phi_4 \log w_{t-1} - \phi_5 \log b_t - \phi_8 \log \left(Y_{t-1} / \left(n_{t-1} - \overline{n} - n^s\right)\right) \tag{2.4}$$

The employer takes this relationship into account and sets the wage as part of his cost minimisation problem. In section 2.1.3, we show that cost minimisation implies an equation relating real wage growth positively to employment and negatively to the lagged private sector labour share.

2.1.2. Price Setting and Nominal Rigidities

Each firm produces one of the variants of the output good in the CES basket. Households spread their expenditures across the different varieties in the basket in a cost minimising fashion. Assuming that investment expenditure stretches over these variants in precisely the same way as consumption demand, we can write the demand for variant j as $y_t(i) = Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta}$, where $p_t(i)$ denotes the price of variety i. Following Rotemberg (1983) we assume that the representative firm faces quadratic costs if it alters its individual price inflation from a reference level $\Pi - 1$. This is the steady state level of inflation in the economy. These costs arise because frequent price changes are bad for the reputation of the company. Convincing customers to remain with the company in spite of price volatility is costly. Additional costs arise because deviating from the "standard" level of inflation requires the firm to engage in a costly re-optimisation process. This has to be carried out by highly paid marketing professionals, while price changes close to average inflation can be decided by lower paid "frontline" staff. Both kinds of costs are likely to increase in the firms' output as well. We assume the following functional form:

$$AC_{t+i}(i) = \frac{\varphi}{2}(\frac{p_{t+i}(i)}{p_{t+i-1}(i)} - \Pi)^2 y_{t+i}(i)$$

The firm j chooses its price $p_{t+i}(j)$ in order to maximise

$$\sum_{i=0}^{\infty} E_t \left[\rho_{t,t+i} \left[\frac{p_{t+i}(i)}{P_{t+i}} y_{t+i}(i) - mc_{t+i} y_{t+i}(i) - AC_{t+i}(i) \right] \right]$$

where $\rho_{t,t+i}$ denotes the discount factor used to discount real profits earned in period t+i back to period t. Note that because households own the firms, we have $\rho_{t,t+i} = \beta^i \frac{u'(C_{t+i})}{u'(C_t)}$. Furthermore, note that marginal cost mc_{t+i} is the same across all firms, which we will prove in section 2.1.3. Differentiating with respect to $p_t(i)$ and noting that, as all firms are identical, $p_t(i) = P_t$ holds ex post, yields

$$\begin{aligned} (1-\theta) + \theta mc_t - \varphi \left(\frac{P_t}{P_{t-1}} - \Pi \right) \frac{P_t}{P_{t-1}} + \theta \frac{\varphi}{2} (\frac{P_t}{P_{t-1}} - \Pi)^2 \\ + E_t \left[\rho_{t,t+1} \varphi \frac{Y_{t+1}}{Y_t} \left(\frac{P_{t+1}}{P_t} - \Pi \right) \frac{P_{t+1}}{P_t} \right] = 0 \end{aligned}$$

which is a nonlinear version of the standard New Keynesian Phillips curve. It is, however, a consistent feature of empirical estimations of Phillips curves that specifications which include lagged inflation as well ("hybrid" Phillips curves") perform better than purely forward looking Phillips Curves. This is because inflation has inertia.[12] Backward looking elements are easily introduced into the price setting considerations of the firm by assuming that the reference level of inflation does not remain constant over time. Instead, we assume that it equals last period's inflation, i.e. $\Pi_t = \frac{P_{t-1}}{P_{t-2}}$. If the inflation rate becomes higher for several periods, firms will mandate frontline staff to handle price increases of that size in order to keep costs low. Customers will get used to

[12] See for instance Gali and Gertler (2000).

the different pace of price changes as well, reducing the adverse effect of a given change in prices on the reputation of the firm. Hence we have

$$(2.5)\qquad \begin{aligned} (1-\theta)+\theta mc_t-\varphi\left(\frac{P_t}{P_{t-1}}-\frac{P_{t-1}}{P_{t-2}}\right)\frac{P_t}{P_{t-1}}+\theta\frac{\varphi}{2}(\frac{P_t}{P_{t-1}}-\frac{P_{t-1}}{P_{t-2}})^2 & \\ +E_t\left[\rho_{t,t+1}\varphi\frac{Y_{t+1}}{Y_t}\left(\frac{P_{t+1}}{P_t}-\frac{P_t}{P_{t-1}}\right)\frac{P_{t+1}}{P_t}\right] &= 0 \end{aligned}$$

The experiment we want to conduct later is a disinflation following an inflationary shock. Inflation is brought into the economy by a so-called "cost-push shock" u_t widely used in the New Keynesian literature.[13] This shock increases current inflation, holding the values of past inflation and marginal costs constant, and is added directly to the Phillip's curve equation:

$$(2.6)\qquad (1-\theta)+\theta mc_t-\varphi\left(\left(\frac{P_t}{P_{t-1}}-u_t\right)-\frac{P_{t-1}}{P_{t-2}}\right)\left(\frac{P_t}{P_{t-1}}-u_t\right)$$
$$+\theta\frac{\varphi}{2}(\left(\frac{P_t}{P_{t-1}}-u_t\right)-\frac{P_{t-1}}{P_{t-2}})+E_t\left[\rho_{t,t+1}\varphi\frac{Y_{t+1}}{Y_t}\left(\frac{P_{t+1}}{P_t}-\left(\frac{P_t}{P_{t-1}}-u_t\right)\right)\frac{P_{t+1}}{P_t}\right]=0$$

It is easily shown that up to first order, this Phillips Curve resembles very closely specifications which are obtained by Woodford (2003) under the assumption of Calvo contracts and full indexation of the prices of those firms which can not re-optimise prices to past inflation.[14] It is a forward looking accelerationist Phillips Curve. If present and future marginal costs are at their steady state level and present and future values of the cost push shock are zero, inflation will remain constant. It will accelerate or decelerate otherwise. Hence the model has a well defined NAIRU.

[13]See for instance Clarida et al. (1999), p.1665 and p. 1667.

[14]See Woodford (2003), p. 215. In fact, the coefficients on expected future inflation and the coefficient on lagged inflation exactly match Woodfords' results.

2.1.3. Cost Minimisation and Efficiency Wages

The production technology is a Cobb Douglas production function,

$$Y_t(i) = AK_t(i)^{\alpha}(TFP_t e_t(i)\,(n_t(i) - \overline{n}))^{1-\alpha}$$

where the output of firm i $Y_t(i)$ depends on the capital stock of firm i $K_t(i)$, the efficiency of its workers $e_t(i)$ and the number of non-overhead workers $n_t(i) - \overline{n}$. In the Danthine and Kurman model (2004), in a first stage the firm minimises its cost of producing a given amount of output. Capital and labour are hired in economy-wide factor markets. However, the firm does not take the real wage as given but sets it taking into account the relationship between effort and wages given by (2.4).[15] Hence the firm's problem is:

$$\begin{aligned}\min_{K_t(i),n_t(i),w_t(i),e_t(i)} r_t^k K_t(i) + w_t(i)(n_t(i) - \overline{n}) s.t. Y_t(i) &= AK_t(i)^{\alpha}(TFP_t e_t(i)\,(n_t(i) - \overline{n}))^{1-\alpha} \\ \textit{and } e_t(i) = \phi_0 + \phi_1 \log w_t(i) + \phi_2 f\,(n_t) + \phi_3 \log w_t & \\ +\phi_4 \log w_{t-1} - \phi_5 \log b_t - \phi_8\,(Y_{t-1}/\,(n_{t-1} - \overline{n} - n^s)) &\end{aligned}$$

This gives rise to the lagrangian

$$\begin{aligned}\pounds\,(K_t(i), n_t(i), w_t(i), e_t\,(i)\,, mc_t\,(i)\,, \zeta_t) &= r_t^k K_t(i) + w_t(i)(n_t(i) - \overline{n}) \\ &+ mc_t\,(i)\left(Y_t(i) - AK_t(i)^{\alpha}(TFP_t e_t(i)\,(n_t(i) - \overline{n}))^{1-\alpha}\right) \\ &+\zeta_t \left(\begin{array}{c} e_t(i) - (\phi_0 + \phi_1 \log w_t(i) + \phi_2 f\,(n_t) + \phi_3 \log w_t) \\ +\phi_4 \log w_{t-1} - \phi_5 \log b_t - \phi_8\,(Y_{t-1}/\,(n_{t-1} - \overline{n} - n^s)) \end{array}\right)\end{aligned}$$

[15] See Danthine and Kurman (2004), pp. 114-115.

The first order conditions for capital and labour are

$$
\begin{aligned}
r_t^k &= \alpha mc_t\,(i)\,\frac{Y_t(i)}{K_t(i)} \\
w_t(i) &= (1-\alpha)mc_t\,(i)\,\frac{Y_t(i)}{n_t(i)-\overline{n}}
\end{aligned}
$$

where $mc_t(i)$ and r_t^k refer to real marginal costs of firm i and the capital rental rate. It will be shown below that even though all firms set the wage individually, firms will find it optimal to set the same wage and the same efficiency level. Dividing the two first order conditions gives $\frac{K_t(i)}{n_t(i)-\overline{n}} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}$. Thus the capital labour ratio is the same across firms. It is then easily shown using the production function that the same holds for the output-capital ratio and the output-to-productive labour ratio, implying that marginal costs are the same across all firms as well. Hence we can write

$$
r_t^k = \alpha mc_t \frac{Y_t}{K_t} \tag{2.7}
$$

$$
w_t = (1-\alpha)mc_t \frac{Y_t}{n_t - n^s - \overline{n}} \tag{2.8}
$$

$$
Y_t = A{K_t}^{\alpha}(TFP_t\phi_1\,(n_t - n^s - \overline{n}))^{1-\alpha} \tag{2.9}
$$

Substituting $\frac{K_t(i)}{n_t(i)-\overline{n}} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}$ into equation (2.7) yields

$$
mc_t = \frac{\left(r_t^k\right)^{\alpha} w_t^{1-\alpha}}{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}(\phi_1 TFP_t)^{1-\alpha}} \tag{2.10}
$$

Note that we have used the fact that firms find it optimal to set $e_t(i) = \phi_1$, which we will prove now. The firm's first order conditions with respect to the real wage and effort are

$$
\begin{aligned}
n_t(i) - \overline{n} &= \frac{\zeta_t \phi_1}{w_t(i)} \\
\zeta_t &= (1-\alpha) mc_t \frac{Y_t(i)}{e_t(i)}
\end{aligned}
$$

Combining those with the first order condition with respect to labour yields $e_t(i) = \phi_1$. Substituting this back into the effort function (2.4), we note that, as the firm's wage depends only on aggregate variables which are the same for all firms, it must indeed hold that $w_t(i) = w_t$. We then substitute for $\log b_t$, solve for $\log w_t$ and impose the balanced growth restriction $\frac{\phi_5+\phi_8-\phi_4}{\phi_1+\phi_3} = 1$.[16] Thus we arrive at a real wage Phillips Curve with a labour share term:

$$
\log w_t - \log w_{t-1} = a + b * f\left(n_t\right) + c \log\left(\frac{w_{t-1}\left(n_{t-1} - \overline{n} - n^s\right)}{Y_{t-1}}\right), \tag{2.11}
$$

$$
\text{with } a = \frac{\phi_0 - \phi_1 + \phi_5\phi_7}{\phi_1+\phi_3},\ b = -\frac{\phi_2}{\phi_1+\phi_3} > 0 \text{ and } c = -\frac{(\phi_5\phi_6+\phi_8)}{\phi_1+\phi_3} < 0
$$

The details of the derivation are given in appendix A.1. Equation (2.11) is very close to a specification derived by Blanchard and Katz (1999) from intuitively plausible relationships between average wages, the reservation wage and productivity.[17] The growth rate of the real wage w_t is positively related to employment and negatively to the labour share. The effect of the labour share stems from the direct impact of

[16] This restriction ensures that steady state employment is constant even though the economy is growing in the steady state. This does not seem too restrictive; it simply says that an increase in the log of the time t real wage in the economy (including firm i) has in absolute value the same net effect on effort (remember we have $\phi_1 + \phi_3 > 0$) as an increase in the exogenous reference as represented by $\log w_{t-1}$, $\log b_t$ and $\log\left(Y_{t-1}/\left(n_{t-1} - \overline{n} - n^s\right)\right)$.

[17] Blanchard and Katz (1999) specify the wage as a function of productivity and the reservation wage, the latter of which is in turn a convex combination of average wages and productivity, just as b_t in our model.

productivity on effort ϕ_8 and the indirect impact through benefits ϕ_6. If these are absent, i.e. $\phi_8 = 0$ and $\phi_6 = 0$, we have c=0.

Empirical estimates of (2.29) (usually replacing n_t with the unemployment rate) or variants thereof repeatedly find c=0 (or even c>0) for the United states but, c<0 for European countries.[18] The difference could be due to the direct effect of productivity on effort being close to zero in the U.S. but positive in Europe because of a larger influence of unions who establish the idea that the reference wage should be linked to productivity, as is also argued by Blanchard and Katz (1999). Using individual data on compensation matched with firm level data on performance and inputs, Abowd et al. (2001) find that the relationship between firm level wages and performance measures like value added and sales per employee is stronger in France than in the United States. One could also imagine that benefits are linked more closely to productivity in Europe because policymakers are more likely to believe in concepts of relative poverty rather than absolute poverty and therefore would aim to link benefits to a country's overall income.[19]

For now we allow $f(n_t)$ to take two different forms: n_t and -log$(1 - n_t)$. The advantage of the log specification is twofold. First, it has the appeal that wage growth will become very high if unemployment moves close to zero. In the context of the wider model, this rules out negative unemployment rates.

The second advantage of the log specification is that it introduces downward rigidity in the real wage or real wage growth in a crude but simple fashion. There is some evidence for downward real wage and real wage growth rigidity. For instance, Bewley's (1998) survey finds that employers are extremely reluctant to cut pay due to the adverse

[18]See Blanchard and Katz (1999), p.73, Blanchard and Katz (1997), p.62, OECD(1997), p. 21 and Cahuc and Zylberberg (2004), pp.484-486.
[19]See Abowd et al. (2001), pp. 429-433.

effects on morale. His results do not allow a clear cut distinction of whether it is a reduction in nominal or in real wages employers deem harmful. However, the fact that employers mention the adverse effect of a wage decline on the standards of living as a reason why wage cuts harm morale point suggests a reluctance to implement real wage cuts.[20] Econometric studies trying to detect downward real wage rigidity focus on the skewness of the distribution of wage changes. They try to gauge the "notional" distribution which would hold if wages were flexible using various approaches. They then compare the notional to the actual distribution to find the extent of downward real wage rigidity. Bauer et al. (2003) investigate nominal wage changes of western German workers between 1976 and 1997. The lower bound of the nominal wage change for those individuals subject to real rigidity is allowed to vary over time and to be above the rate of inflation. They find that the percentage of wage changes constrained by downward real rigidity varies between 37% and 16% over the period they consider.[21] Furthermore, a large fraction of those constrained workers experience real wage increases.[22] The estimated "sweep up" effect on average wage growth is substantial and varies between 3% in the 70s to less than 1% in the 90s.[23] Holden and Wolfsberg (2007) consider industry wide annual real wage changes for manual workers from seven sectors and 19 OECD countries between 1973 and 1999.[24] They measure the degree of downward real wage rigidity by the fraction of the real wage cuts which would have taken place under the notional distribution but did not take place because the wage follows the actual distribution. They find that in the "core" region (Austria, Belgium, France, Germany, Luxembourg, and the Netherlands) 6.3% of all real wage cuts are prevented but only

[20]See Bewley (1998), p. 477.
[21]See Bauer et al. (2004), p. 17 and p.37.
[22]See Bauer et al. (2004), p.15.
[23]See Bauer et al. (2004), p. 17.
[24]See Holden and Wolfsberg (2007), p.13.

2.7% in the "Anglo" (Canada, Ireland, New Zealand, the United Kingdom, and the United States) region. The fraction of cuts larger or equal to 2% prevented amounts to 18.8% in the "Core" and to 11.7% in the Anglo region.[25]

The size of the overhead labour force remains to be determined. Following Rotemberg and Woodford (1999), we assume that in the steady state, all economic profit generated by the monopolistically competitive firm goes to the overhead staff. This is justified because setting up production is impossible without overhead labour and the firm's profit is thus essentially equal to the collective marginal product of its overhead staff. We assume that the overhead staff splits this profit equally. Hence the firm ends up with zero profits, which eliminates any incentive for market entry. Christiano et al. (2005) also assume a fixed cost of production to eliminate profits among monopolistically competitive firms, although they do not specify the origin of the cost.[26] The details of the derivation can be found in appendix A.2.

2.1.4. Monetary Policy

Monetary Policy is assumed to follow a simple Taylor type nominal interest rate rule. The exact specification will vary across simulations. In the baseline, the nominal interest rate reacts to current inflation, the lagged output gap and the lagged nominal interest rate:

$$i_t = (1-\rho)\left(\bar{i} + \psi_\pi \pi_t + \frac{\psi_Y}{4} gp_{t-1}\right) + \rho i_{t-1} \tag{2.12}$$

$\bar{i}$, ρ and gp_t denote the long-run real interest rate, the degree of interest rate smoothing and the output gap, respectively. ψ_π and ψ_Y denote the long run coefficients on

[25] See Holden and Wolfsberg (2007), pp. 20-21.
[26] See Rotemberg and Woodford (2004), p. 17, and Christiano et al. (2005), p. 15.

inflation and the output gap. The central bank responds to the lagged value of the output gap but the current deviation of inflation from its target, which without loss of generality is assumed to be zero. Note that this implies a zero steady state inflation rate.

There are a couple of advantages associated with characterising monetary policy via an interest feedback rule like (2.12) rather than a rule determining the money supply. Firstly, almost all central banks target the interest rate rather than the money supply.[27] Clarida and Gertler (1996) argue that this is true even for the Bundesbank, in spite of the public focus on monetary targeting. Indeed, it is sometimes argued that central banks do not control the money supply since the money multiplier varies as the desired reserve holdings of commercial banks change. Secondly, assuming an interest rate feedback rule means that we avoid having to make assumptions about money demand. Thirdly, and most importantly, interest rate feedback rules with interest rate smoothing have been estimated and have been found to be successful at explaining interest rate movements in various countries, including Germany.[28] Specifying monetary policy as an interest rate feedback rule thus allows us to calibrate the behaviour of monetary policy in line with the data by drawing on such estimates. Finally, an interest rate feedback rule allows us to vary the emphasis the central bank places on output and inflation stabilisation in a simple fashion.

The output gap is the percentage deviation of total output, i.e. private sector plus the output of government employees, from its natural level. We calculate the output of government employees by simply adding up their wages, following the convention of national accounts. We assume that government employees earn the same wage as in the

[27] See Woodford (2003), pp. 24-31, and Walsh (2003), p. 29.
[28] See for example Clarida and Gertler (1996), Clarida et al. (1999) and Clausen and Meier (2003).

private sector. For total output, we then have $Output_t = Y_t + w_t n^s$, while total natural output is given by $Output_t^n = Y_t^n + w_t^n n^s$. w_t^n and Y_t^n denote the wage rate and the private sector output level consistent with natural employment, or the NAIRU. Thus we have

$$gp_t = \frac{Output_t - Output_t^n}{Output_t^n} \tag{2.13}$$

$Output_t^n$ denotes the output level which would set marginal costs equal to its long run level μ^{-1}, given the capital stock and the previous period's real wage. As can be obtained from equation (2.6), this would ensure that in the absence of cost push shocks, inflation is neither rising nor falling. The employment level corresponding to this output level will be referred to as "natural employment" n_t^n. The natural levels of output and employment are derived by first substituting the equation for the rental on capital (2.7) into (2.10) and setting $mc_t = \mu^{-1}$. The natural levels of output, employment and the real wage are then given by the values of Y_t^n n_t^n and w_t^n solving

$$\begin{aligned}
\mu^{-1} &= \frac{(n_t^n - n^s - \overline{n})^\alpha w_t^n}{A(1-\alpha)(\phi_1 TFP_t)^{1-\alpha} K_t^\alpha} \\
\log w_t^n - \log w_{t-1} &= a + b * (n_t^n - \overline{n}) + c \log\left(\frac{w_{t-1}(n_{t-1} - \overline{n} - n^s)}{Y_{t-1}}\right) \\
Output_t^n &= AK_t{}^\alpha (TFP_t \phi_1 (n_t^n - n^s - \overline{n}))^{1-\alpha} + w_t^n n^s
\end{aligned} \tag{2.14}$$

2.1.5. Introducing Endogenous Growth

We introduce endogenous growth following Romer (1986). We assume that investing firms discover ways to produce more efficiently and that knowledge is a public good. Therefore total factor productivity TFP_t is assumed to be proportional to the aggregate capital stock rather than the individual firm's capital stock. This implies that there are now constant returns to capital at the economy wide level, allowing per capita output to grow. However, there are still decreasing returns to capital at the firm level.

Hence we replace TFP_t with K_t in the above equations. The equations for private sector output and marginal costs are given by

$$mc_t = \frac{\left(r_t^k\right)^\alpha w_t^{1-\alpha}}{A\alpha^\alpha(1-\alpha)^{1-\alpha}(\phi_1 K_t)^{1-\alpha}} \tag{2.15}$$

$$Y_t = AK_t(\phi_1\left(n_t - n^s - \overline{n}\right))^{1-\alpha} \tag{2.16}$$

while total output is given by

$$Output_t = AK_t(\phi_1\left(n_t - n^s - \overline{n}\right))^{1-\alpha} + w_t n^s \tag{2.17}$$

The capital stock now has a stronger effect on both marginal costs and output than in the JLN economy. This can be seen by first eliminating Y_t in equation(2.7) using (2.9) and then substituting the resulting expression into (2.10), which yields

$$mc_t = \frac{w_t}{A(1-\alpha)\left(n_t - n^s - \overline{n}\right)^{-\alpha}\left(\phi_1 TFP_t\right)^{1-\alpha} K_t^\alpha} \tag{2.18}$$

for the JLN economy and, after again setting $TFP_t = K_t$

$$mc_t = \frac{w_t}{A(1-\alpha)\left(n_t - n^s - \overline{n}\right)-^\alpha \phi_1^{1-\alpha} K_t} \tag{2.19}$$

for the New Growth economy. Hence an increase in the capital stock by 1% for a given employment level reduces marginal costs by 1%. In the absence of endogenous growth the effect is only α%. The unitary elasticity of marginal costs with respect to the capital stock in the New Growth economy implies that the real wage-to-capital ratio drives marginal cost for a given level of employment. We will return to this relationship in section 2.4.

(2.18) and (2.19) also reveal the intuition for this result. In each equation, the denominator is the marginal product of labour. Hence (2.18) and (2.19) express marginal cost as marginal unit labour costs. An increase of the capital stock increases the marginal product of labour by $\alpha\%$ in the JLN economy but by 1% in the New Growth economy. This is of course due to the fact that there are decreasing returns to capital in the JLN economy but constant returns in the New Growth economy.

Furthermore, Y_t^n, n_t^n and w_t^n are now determined by

$$\mu^{-1} = \frac{\left(n_t^n - n^s - \overline{n}\right)^{\alpha} w_t^n}{A\left(1-\alpha\right)\left(\phi_1\right)^{1-\alpha} K_t} \tag{2.20}$$

$$\log w_t^n - \log w_{t-1} = a + b * \left(n_t^n - \overline{n}\right) + c \log\left(\frac{w_{t-1}\left(n_{t-1} - \overline{n} - n^s\right)}{Y_{t-1}}\right)$$

$$Y_t^n = AK_t(\phi_1\left(n_t^n - n^s - \overline{n}\right))^{1-\alpha} \qquad + w_t^n n^s$$

The assumption that technological progress is simply a by-product of capital accumulation is clearly a strong simplification. However, the capital stock externality assumption can thus be seen as a convenient short cut to a model with more realistic microfoundations but similar implications at the aggregate level. Acemoglu (2009) notes that also a more explicit modelling of technological change frequently leads to linearity of output in the produced input, for instance in expanding variety type models.[29] A famous example are expanding-variety-type models, like the one developed by Romer (1990).[30] Furthermore, Comin and Gertler (2006) show in that their real business cycle expanding-varieties-type endogenous growth model, a temporary adverse mark-up

[29]See Acemoglu (2009), p. 402 and p. 440.
[30]See Acemoglu (2009), p. 440.

shock reduces not only output but TFP growth. This is because the drop in output lowers R&D investment and expenditure on the adoption of new technologies.[31] The capital stock externality we assume produces a similar relationship: A reduction in output will lower employment, the marginal product of capital, capital stock growth and thus total factor productivity growth.

2.1.6. The Aggregate Equations

This section summarises the models aggregate equations developed above for convenience of the reader and introduces explicit functional forms where that has not yet been done above. As many of the economy's variables are growing in the steady state $(Y_{t,}C_t, I_t, w_t, K_t)$, simulation of the model requires normalising those variables with a cointegrated variable. It is very convenient from a technical point of view to normalise with respect to the capital stock. How that is done is shown in appendices I and III to this chapter.

Aggregate demand is the sum of consumption, investment, the amount of price adjustment costs and government expenditure:

$$AD_t = C_t + I_t + \frac{\varphi}{2}(\pi_t - \pi_{t-1})^2 Y_t + w_t n^s \tag{2.21}$$

We will assume logarithmic utility so that the consumption Euler equation becomes

$$1/\left(C_t - hab_{t-1}\right) = \beta\left(1 + i_t\right) E_t\left[\frac{1}{\left(C_{t+1} - hab_t\right)\left(1 + \pi_{t+1}\right)}\right] \tag{2.22}$$

The level of habit is given by

$$hab_{t-1} = jC_{t-1}$$

[31] See Comin and Gertler (2006), pp. 542-543.

Following Schmitt-Grohe and Uribe (2005), we assume

$$S\left(\frac{I_t}{I_{t-1}} - (1+g)\right) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - (1+g)\right)^2.$$ [32]

Hence Investment expenditures is governed by the following equations:

$$\lambda_t = \frac{1}{C_t - hab_{t-1}}$$

$$\beta E_t\left(\lambda_{t+1} r_{t+1}^k + \lambda_{t+1} q_{t+1}(1-\delta)\right) = \lambda_t q_t$$

$$\lambda_t q_t\left[\left(1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - (1+g)\right)^2\right) - \frac{I_t}{I_{t-1}}\kappa\left(\frac{I_t}{I_{t-1}} - (1+g)\right)\right] + \beta E_t\left[\lambda_{t+1} q_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 \kappa\left(\frac{I_{t+1}}{I_t} - (1+g)\right)\right] = \lambda_t \tag{2.23}$$

while capital accumulation is given by

$$K_{t+1} = (1-\delta) K_t + I_t\left(1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - (1+g)\right)^2\right) \tag{2.24}$$

The capital rental is given in both models by

$$r_t^k = \alpha mc_t \frac{Y_t}{K_t} \tag{2.25}$$

[32] The functional form we use has the following advantages:

- It fulfils the requirements stated in equation (2.1), for instance convexity of S(.) in the investment growth rate (which is necessary for a maximum to exist).
- Using a second order polynomial in the investment growth rate makes for algebraic convenience.
- The recursive solution of the model is approximated (only) up to second order anyway. This I implies that the quantitative results would not be affected by using a higher order polynomial for S(.). For instance, even with the chosen second order polynomial, in equations (2.23) and (2.24), investment (or the investment growth rate) occurs up to third order. But the fact that we only take a second order Taylor approximation means that the presence of a third order term does not matter. Thus using a higher order polynomial would not make a difference (perhaps kappa would have to be chosen slightly differently).
- The fact that the recursive solution of the model is only approximated up to second order also renders using a nonlinear functional form more complicated than a polynomial futile.

However, with endogenous growth, we can write r_t^k as a function of employment and marginal costs alone, namely as

$$r_t^k = \alpha mc_t A(\phi_1 (n_t - n^s - \overline{n}))^{1-\alpha} \tag{2.26}$$

Marginal cost in the JLN economy becomes

$$mc_t = \frac{\left(r_t^k\right)^\alpha w_t^{1-\alpha}}{A\alpha^\alpha(1-\alpha)^{1-\alpha}(\phi_1 TFP_t)^{1-\alpha}} \tag{2.27}$$

while in the presence of endogenous growth, we have

$$mc_t = \frac{\left(r_t^k\right)^\alpha w_t^{1-\alpha}}{A\alpha^\alpha(1-\alpha)^{1-\alpha}(\phi_1 K_t)^{1-\alpha}} \tag{2.28}$$

Wages are set according to equation (2.11):

$$\log w_t - \log w_{t-1} = a + b * f(n_t) + c \log\left(\frac{w_{t-1}(n_{t-1} - \overline{n} - n^s)}{Y_{t-1}}\right) \tag{2.29}$$

Total output in the absence of endogenous growth is given by private sector output Y_t plus the output of state employees:

$$Output_t = AK_t{}^\alpha(TFP_t\phi_1 (n_t - \overline{n} - n^s))^{1-\alpha} + w_t n^s \tag{2.30}$$

while in the presence of endogenous growth, we have

$$Output_t = AK_t((n_t - \overline{n} - n^s)\phi_1)^{1-\alpha} + w_t n^s \tag{2.31}$$

Markets clear:

$$AD_t = Output_t$$

The evolution of prices is determined by the Phillips Curve, where we replace the stochastic discount factor by its definition $\rho_{t,t+1} = \beta\frac{u'(C_{t+1}-hab_t)}{u'(C_t-hab_{t-1})} = \beta\frac{C_t-hab_{t-1}}{C_{t+1}-hab_t}$

$$(1-\theta)+\theta mc_t-\varphi\left(\left(\frac{P_t}{P_{t-1}}-u_t\right)-\frac{P_{t-1}}{P_{t-2}}\right)\left(\frac{P_t}{P_{t-1}}-u_t\right)+\theta\frac{\varphi}{2}(\left(\frac{P_t}{P_{t-1}}-u_t\right)-\frac{P_{t-1}}{P_{t-2}})^2$$

$$+\beta E_t\left[\frac{C_t-hab_{t-1}}{C_{t+1}-hab_t}\varphi\frac{Y_{t+1}}{Y_t}\left(\frac{P_{t+1}}{P_t}-\left(\frac{P_t}{P_{t-1}}-u_t\right)\right)\frac{P_{t+1}}{P_t}\right]=0 \tag{2.32}$$

Finally, monetary policy is specified by equation (2.12)

$$i_t=(1-\rho)\left(\bar{i}+\psi_\pi\pi_t+\frac{\psi_Y}{4}gp_{t-1}\right)+\rho i_{t-1} \tag{2.33}$$

with gp_t as defined in (2.13) with natural output as determined in (2.14) for the JLN economy and as determined in (2.20) for the New Growth economy.

2.2. Calibration

Wherever we can draw on empirical evidence to pin down the model parameters, we calibrate the model to Western German data. Germany is the largest economy in Western Europe, it has a history of high unemployment, and the monetary policy of the Bundesbank was widely regarded to exert a strong influence on monetary policy in other Western European countries.[33]

The calibration of the non-monetary policy model parameters for the experiment described above is presented table 2.1. We distinguish between three different types of parameters. The first set is calibrated according to standard values in the literature.

[33]For instance, Clarida et al. (1998) estimate Taylor rules for Britain, Italy and France and show that the German short rate has an effect on the short rate in these countries between 0.6 and 1.14. Furthermore, they find that none of these central bank pursued an active monetary policy, i.e. the estimate of the inflation coefficient is significantly below one. See Clarida et al. (1998), pp. 1054-158. Thus using estimates for these countries would have raised determinacy issues which we would like to avoid in this chapter.

This set contains the discount factor β, the output elasticity of employment $1-\alpha$, the elasticity of substitution between varieties of goods θ, the depreciation rate δ, and the price adjustment cost parameter φ. φ is calibrated as to generate a marginal cost coefficient in the linearised version of equation (2.6) which would also be generated in a Calvo Phillips Curve with full backward indexing of unchanged prices and a probability of no re-optimisation of 2/3.[34]

The second set, consisting of n^s a, b and c, is based on evidence from German data. The share of government employees n^s has been set to 0.18, which corresponds to a share of government expenditure in GDP of 0.14.[35] This is somewhat below the average share of government consumption expenditure in German GDP from 1970-1990, which is 0.19. However, our main results are robust to increasing n^s.[36]

We estimate the wage setting equation (2.29) on German data on labour costs per employee, unemployment (instead of employment, as is done in the empirical literature) and productivity per employee ranging from 1970q1 to 2000q4, using both the log and the level specification.[37] The results are discussed in appendix A.7 . For the simulation, we decided to let employment enter (2.29) linearly for two reasons. Since the New Growth economy is quite a non-standard framework, we would like to facilitate the interpretation of our simulation results by using the simpler specification. Furthermore, simulating the model with the wage setting function featuring -log$(1-n_t)$ and b and c calibrated according to the point estimates yielded explosive paths for the model's variables. Therefore we use the linear specification in the simulation but use

[34]See for instance Danthine and Kurmann (2004), p. 119.

[35]This share is given by $\frac{w*n^s}{Output} = (1-\alpha)mc\frac{Y_t}{n-n^s-\overline{n}} / \left(Y + (1-\alpha)mc\frac{Y}{n-n^s-\overline{n}} * n^s\right)$. This can be simplified to yield $\frac{w*n^s}{Output} = \frac{(1-\alpha)/\mu*n^s}{n-n^s-\overline{n}+(1-\alpha)/\mu*n^s}$

[36]See Statistisches Bundesamt Wiesbaden (2006a).

[37]We would have preferred to estimate on pre-reunification data alone but needed to extend the dataset to 2000q4 to get significant coefficients.

a smaller coefficient for b than the point estimate, namely 0.08. It can be checked in the appendix A.7 that this is less than one standard deviation away from the point estimate. Furthermore, it is still higher than the effect of a change in unemployment in the equation featuring -log$(1 - n_t)$ if the unemployment rate is at the sample average. The calibration of c is consistent with both estimates of that coefficient. The intercept a is calibrated to achieve a steady state unemployment rate of 4%.

The third set consists of the three "free" parameters A, κ and j the production function shifter, the parameter indexing adjustment costs and the degree of habit formation. Given the calibration of α, β, θ, n^s, and δ, the value of A was calibrated to achieve a reasonable steady state growth rate. The other two parameters were calibrated to match second moments of the investment-capital ratio and the consumption capital ratio. The results of the moment comparison are discussed in the following section.

α	β	j	A	θ	δ	ϕ_1	φ	$\overline{n}$
0.33	0.99	0.4	0.38	6	0.025	0.452	30	0.1793
n^s	$\bar{i}$	g_{TFP}	σ_u	a	b	c	κ	
0.18	0.0181	0.0079	0.003	-0.1123	0.08	-0.1	0.65	

Table 2.1. Baseline Calibration of non-policy Parameters

Turning to the monetary policy parameters, the baseline calibration of the monetary policy reaction function is taken from Clausen and Meier (2003), who estimate the interest feedback rule (2.12) for the Bundesbank over the period from 1973 to 1998 for quarterly data.[38] They use a real time measure of the output gap in order to account for the fact that the central bank's information set does not include future levels of GDP. Therefore they argue that the estimate of potential output underlying the output gap measure should be based only on GDP levels known up to the quarter when the decision

[38] See Clausen and Meier (2003), p. 22.

on the interest rate is made.[39] An important additional benefit of this procedure is that the potential output estimate will evolve in a manner depending more strongly on past values of actual output than in a procedure which uses the full sample of output values. If the economy is characterised by endogenous growth, we would expect that this method is superior at detecting the path of potential output. High past output will trigger high investment and thus will also increase potential output.

Clausen and Meier's preferred estimates are reported in table 2.2 which in fact correspond to the original coefficients proposed by Taylor (1993) to characterise the policy of the Federal Reserve. Their estimate of the output gap coefficient is of particular interest because the Bundesbank was often perceived as paying less attention to output than the Fed. This is also borne out by other Taylor-rule estimates, one of which we discuss in turn.

ψ_π	ψ_Y	ρ
1.5	0.52	0.75

Table 2.2. Baseline Calibration of the Policy Rule: Clausen and Meier (2003)

In section 2.5, we compare the effects of monetary policies estimated for the Bundesbank and the Federal Reserve and thus need internationally comparable estimates. Therefore we would like to draw on a study conducted by Clarida et al. (1998) which uses the same methodology to estimate policy rules for different countries. Their rule is estimated on monthly data stretching from 1979 to 1993. A quarterly data version of their specification would be

[39]See Clausen and Meier (2003), p. 2. Note that because Taylor rules are usually estimated using annualised inflation and interest rate data, the coefficient on the output gap has to be divided by 4 to adapt it to quarterly frequency.

$$(2.34) \qquad i_t = (1-\rho)\left(\bar{i} + \psi_\pi E_t\left(\frac{\pi_{t+1}+\pi_{t+2}+\pi_{t+3}+\pi_{t+4}}{4}\right) + \frac{\psi_Y}{4} gp_t\right) + \rho i_{t-1}$$

Hence the central bank responds to a one year forecast of inflation, the current output gap and the lagged interest rate.[40] The point estimates for the Bundesbank and the Federal Reserve are replicated in table 2.3.[41] Clearly, the small coefficient on the output gap corresponds more to the conventional wisdom on how the Bundesbank was conducting policy.

	ψ_π	ψ_Y	ρ
Bundesbank	1.31	0.25	0.91
Federal Reserve	1.83	0.56	0.97

Table 2.3. Forward looking Interest Rate Rules: Clarida et al. (1998)

2.3. Some Moment Comparisons

We now compare the second moments generated by stochastic simulations of the two models to the corresponding empirical moments for German data. The moment comparison serves two purposes: First, it informs the calibration of the free parameters κ and j. These were calibrated with an eye to matching the standard deviation of the investment to capital ratio relative to the output to capital ratio, and also the persistence of the consumption to capital ratio, as measured by the first to fifth order autocorrelation. Furthermore, we want to instil trust in our simulation results in section 2.4 by showing that the moments generated by the New Growth economy are broadly in line with the data, while the JLN economy fails to match the persistence in the

[40]See Clarida et al. (1998), p. 1039 and p.1042.

[41]See Clarida et al. (1998), p.1042 for the estimate for the Bundesbank and p. 1045 for the estimate for the Federal Reserve.

real variables. In order to see whether our results are robust, we carry out the same comparison for the JLN economy. For both economies, we consider the moments of the model's variables for two different cases. In the first case, the monetary policy reaction function is as estimated by Clausen and Meier (2003), i.e. our baseline case. In the second case, the policy reaction function is as estimated by Clarida et al. (1998). We generate the models' second moments by conducting a stochastic simulation by randomly drawing a value for u_t 200000 times, where u_t is assumed to be normally distributed and to have a standard deviation of 0.003. The standard deviation of the shock was chosen to set the standard deviation of employment close to its value in the data. We solve the model by employing a second order approximation to the policy function using the approach of Schmitt-Grohe and Uribe (2004a). We use the software Dynare to implement the solution and conduct the simulation.

We consider the following variables: The ratios of (total) output, consumption, investment and real wages to capital, denoted as F_t, D_t, R_t and H_t respectively (recall that we have to normalise all the trended variables with the capital stock to render them stationary), employment n_t (measured as linearly detrended log hours), the unemployment rate, the nominal interest rate i_t, inflation π_t (measured as the change in the consumer price index (CPI)), productivity growth p_t (measured as change in real GDP per hour worked), capital stock growth g_t, and the investment rate I/Y. From those, we compute the following moments: The standard deviations of n_t (which in our model, since n_t is the employment rate, is same as the standard deviation of the unemployment rate) and g_t, the coefficient of variation of F_t, and the ratio between the coefficients of variation of D_t and R_t, the standard deviations of employment, capital stock growth inflation, the nominal interest rate and the coefficient of variation of F_t.[42] We will refer to theses ratios below as the "relative coefficients of variation" or the "relative standard deviations" without explicitly mentioning the coefficient of variation of F_t each time. Furthermore, we look at the cross-correlation of all variables with F_t and the autocorrelation of each variable up to the fifth order.

The construction of the data for F_t, D_t, R_t and H_t is discussed in appendix A.6. The raw data was obtained from the Federal Statistical Office of Germany (Statistisches Bundesamt), except for the nominal interest rate and the inflation data which was obtained from the "International Financial Statistics" CD-ROM. The data ranges from 1970:Q1 to only 1990:Q4 because reunification is associated with a big drop in F_t, D_t and R_t, which would distort the moments. Furthermore, there are strong theoretical reasons to believe that all variables other than hours, inflation and the nominal interest rate are stationary. This is why we do not detrend or filter them. However we adjust the sample to induce stationarity if stationarity is not confirmed for the full sample by either an ADF test (by rejecting the null of a unit root) or by the KPSS test (by not rejecting the null of stationarity). Where we have to detrend, we use a linear time trend. The details are given in appendix A.6. The one exception is the unemployment rate: We include this variable in spite of stationarity being strongly rejected even after removing a substantial amount of observations from the sample. This is not surprising since, as we discussed in chapter one, there is indeed quite a bit of evidence that the German unemployment rate is not stationary. Nevertheless, for the purpose of this moment comparison, we interpret the data as saying that unemployment is highly persistent

[42] Note that when solving the models, we express the variables in absolute rather than log (or percentage) deviations from their steady state, as is more common in the literature. Thus to render our statistics comparable, we have to compute the coefficients of variation of those variables which are not naturally expressed as percentages. For example, note that for the variance of the log deviation of F_t from its mean, we have $Var\left(\frac{dF_t}{E_t(F_t)}\right) = \frac{Var(dF_t)}{(E_t(F_t))^2} = \frac{Var(F_t)}{(E_t(F_t))^2}$. Hence $sd.\left(\frac{dF_t}{E_t(F_t)}\right) = \frac{sd(F_t)}{E_t(F_t)}$

but assume that it is stationary, as it is in our model. Furthermore, note that while in the model, the second moments of employment and unemployment are identical, this will not be the case in the data.

Table 2.4 reports the various standard deviations, relative standard deviations and cross-correlations with the output capital ratio F_t listed above. Column 1 contains the data, while column 2 and 3 refer to the baseline policy reaction function. The standard deviation of employment for the New Growth economy is on the mark because we have calibrated the standard deviation of the cost push shock accordingly. Note that the empirical standard deviation of employment happens to be extremely close to the standard deviation of the unemployment rate, implying that we can match both at the same time. The coefficient of variation of F_t for the New Growth economy (NGE) is considerably smaller than in the data. It is almost equal to the standard deviation of employment, which is also true for the JLN economy. The relative coefficient of variation of D_t in the New Growth model is somewhat lower than in the data, while in the JLN economy, it is far too low. The relative coefficient of variation of R_t is somewhat lower in the New Growth economy than in the data, while it is way too high in the JLN economy.

Of particular interest is the relative volatility of capital stock growth. As we show in section 2.4, movements in capital stock growth drive our results in the New Growth economy. We would thus prefer the relative volatility of capital stock not to be too high. The relative standard deviation of capital stock growth g_t in the New Growth economy is somewhat higher than in the data, although it is higher still in the JLN economy. On the other hand, the standard deviation of g_t relative to the standard deviation of employment n_t is a little smaller than in the data for the New Growth economy (0.0766

Moments	**Data**	JLN	NGE	CGG: JLN	CGG: NGE
$(sd.D_t/meanD_t)\,/\,(sd.F_t/meanF_t)$	1.0165	0.6573	0.8773	0.685	0.8704
$(sd.R_t/meanR_t)\,/\,(sd.F_t/meanF_t)$	**2.7836**	3.2060	2.5184	3.364	2.5736
$sd.n_t/(sd.F_t/meanF_t)$	**0.6907**	0.9763	1.0875	0.9688	1.0932
$sd.g_t/(sd.F_t/meanF_t)$	**0.0601**	0.1072	0.0840	0.1125	0.0857
$sd.F_t/meanF_t$	**0.0311**	0.0115	0.0192	0.0077	0.0215
$sd.(1-n_t)/(sd.F_t/meanF_t)$	**0.6578**	0.9763	1.0875	0.9688	0.023
$sd.n_t$, $sd.(1-n_t)$	**0.0215, 0.0205**	0.0112	0.0209	0.0074	0.0235
$sd.\,(I_t/Y_t)$	**0.0122**	0.0048	0.0053	0.0035	0.0061
$sd.\pi_t/\,(sd.F_t/meanF_t)$	**0.1835**	0.3645	0.2001	0.8801	0.1982
$sd.i_t/(sd.F_t/meanF_t)$	**0.1952**	0.2254	0.1418	0.0868	0.1081
$sd.g_t$	**0.0019**	0.0012	0.0016	0.0009	0.0018
$corr(D_t, F_t)$	**0.8704**	0.95	0.9923	0.8863	0.9906
$corr(R_t, F_t)$	**0.9284**	0.9317	0.9953	0.8898	0.9948
$corr(n_t, F_t)$	**0.6902**	0.7970	0.999	0.8001	0.9991
$corr(i_t, F_t)$	**0.3068**	-0.6772	0.0830	0.0188	0.8804
$corr(\pi_t, F_t)$	**0.2505**	-0.5071	-0.0901	0.1471	0.2263
$corr(p_t, F_t)$	**0.2390**	-0.1966	0.7587	-0.2452	0.8262
$corr(H_t, F_t)$	**0.4123**	0.4476	-0.6729	0.4468	-0.7258

Table 2.4. Relative Standard Deviations and Cross-Correlations

as compared to 0.0870 in the data). This discrepancy stems from the fact that the New Growth economy understates the volatility of F_t and matches the volatility of n_t while the standard deviation of g_t in the data actually exceeds its value in the model. Since the focus of this chapter is on explaining movements in unemployment, we place a greater importance on the relative volatility of g_t with respect to n_t than the volatility of g_t with respect to F_t and thus conclude that g_t is not too volatile.

Turning to the nominal variables, the relative standard deviation of π_t is very close to the data for the New Growth economy, but too high in the JLN economy. Both models come reasonably close to matching the relative standard deviation of the nominal interest rate i_t as well, although the JLN economy does slightly better.

Turning to the cross-correlations, what is most striking is that for the JLN economy, $corr(\pi_t, F_t)$, $corr(i_t, F_t)$ and $corr(p_t, F_t)$ are wrongly signed. They are negative whereas those calculated from the data are positive. A negative correlation of inflation and the interest rate with output is what we would expect in a standard sticky price New Keynesian model. A positive cost push shock raises inflation and thus the nominal as well as the real interest rate via the interest feedback rule and lowers output. Correctly matching the correlation of output with inflation and the nominal interest rate is generally perceived as a difficulty in New Keynesian models if demand shocks are absent.[43] Furthermore, the decline in output following a positive cost push shock lowers employment and thus increases labour productivity.

The New Growth economy produces wrong signs for $corr(\pi_t, F_t)$, though the absolute value is much smaller than for the JLN Economy, and $corr(H_t, F_t)$. The magnitudes of $corr(D_t, F_t)$ and $corr(R_t, F_t)$ are not too far away from the data for both models, while for $corr(n_t, F_t)$, both models produce considerably too high values. It is particularly interesting that the New Growth model produces a positive correlation between the output capital ratio and the nominal interest rate.

Tables 2.5 and 2.6 report the autocorrelation up to the fifth order for the data and the baseline case. For those variables for which we do not reject the null of stationarity over the full sample we use the dataset starting in 1970 rather than the reduced dataset in order not to unnecessarily sacrifice information. When the i-th order autocorrelation of a variable is within ± 0.1 of the corresponding autocorrelation in the sample, it is printed in bold. A number in italics means that the value is closer to the data than the i-th order autocorrelation of the same variable in the competing model. Concerning the variables F_t, D_t,and n_t, we observe that the New Growth economy is matching the persistence in the data quite closely. By contrast, R_t, $1 - n_t$, g_t, i_t and I_t/Y_t are considerably less persistent in the New Growth model than in the data, although they

[43] See for instance Nolan and Thoenissen (2005), pp. 25-26.

Order of Autocorrelation	Data	JLN	NGE	Data	JLN	NGE
	F_t	F_t	F_t	i_t	i_t	i_t
1	**0.88**	**0.89**	**0.93**	**0.9**	**0.8**	**0.84**
2	**0.78**	0.65	**0.82**	**0.75**	0.49	*0.58*
3	**0.65**	0.4	**0.71**	**0.58**	0.22	*0.38*
4	**0.58**	0.22	**0.63**	**0.39**	0.06	*0.25*
5	**0.47**	0.08	*0.58*	**0.23**	-0.00	**0.2**
	D_t	D_t	D_t	π_t	π_t	π_t
1	**0.91**	**0.88**	**0.94**	**0.35**	**0.45**	**0.42**
2	**0.85**	0.65	**0.85**	**-0.16**	0.1	0.07
3	**0.78**	0.4	**0.76**	**0.21**	-0.07	-0.09
4	**0.71**	0.22	**0.71**	**0.6**	-0.11	-0.11
5	**0.65**	0.11	**0.68**	**0.17**	-0.08	-0.07
	R_t	R_t	R_t	p_t	p_t	p_t
1	**0.96**	**0.9**	**0.94**	**-0.03**	0.53	0.84
2	**0.92**	0.68	**0.82**	**-0.18**	0.07	0.67
3	**0.86**	0.45	*0.7*	**-0.02**	*-0.21*	0.53
4	**0.81**	0.24	*0.6*	**0.37**	-0.31	**0.47**
5	**0.74**	0.1	*0.54*	**0.04**	-0.27	*0.42*
	n_t	n_t	n_t	H_t	H_t	H_t
1	**0.93**	**0.88**	**0.94**	**0.92**	**0.99**	**0.99**
2	**0.84**	0.65	**0.84**	**0.89**	**0.95**	**0.97**
3	**0.73**	0.34	**0.74**	**0.85**	**0.9**	**0.94**
4	**0.62**	0.20	**0.66**	**0.82**	**0.84**	**0.92**
5	**0.51**	0.08	*0.62*	**0.78**	**0.78**	0.89

Table 2.5. Autocorrelations, Baseline

Order of Autocorrelation	Data	JLN	NGE	Data	JLN	NGE
	$1-n_t$	$1-n_t$	$1-n_t$			
1	**0.98**	**0.88**	**0.94**			
2	**0.96**	0.65	*0.84*			
3	**0.93**	0.34	*0.74*			
4	**0.9**	0.20	*0.66*			
5	**0.86**	0.08	*0.62*			
	g	g	g	I/Y	I/Y	I/Y
1	**0.97**	**0.9**	**0.94**	**0.95**	**0.91**	**0.94**
2	**0.94**	0.68	*0.82*	**0.91**	0.71	**0.82**
3	**0.89**	0.45	*0.7*	**0.87**	0.49	*0.69*
4	**0.85**	0.24	*0.6*	**0.82**	0.3	*0.58*
5	**0.79**	0.1	*0.54*	**0.75**	0.15	*0.51*

Table 2.6. Autocorrelations, Baseline (continued)

are still considerably closer to the data than in the JLN economy Conversely, all these

Order of Autocorrelation	Data	JLN	NGE	Data	JLN	NGE
	F_t	F_t	F_t	i_t	i_t	i_t
1	**0.88**	**0.84**	**0.96**	**0.9**	**0.91**	**0.99**
2	**0.78**	0.57	**0.87**	**0.75**	**0.73**	0.98
3	**0.65**	0.30	*0.77*	**0.58**	**0.55**	0.96
4	**0.58**	0.10	*0.69*	**0.39**	**0.42**	0.93
5	**0.47**	-0.01	*0.64*	**0.23**	**0.34**	0.91
	D_t	D_t	D_t	π_t	π_t	π_t
1	**0.91**	0.77	**0.96**	**0.35**	0.66	*0.49*
2	**0.85**	0.46	**0.90**	**-0.16**	0.36	0.14
3	**0.78**	0.23	**0.83**	**0.21**	0.12	-0.06
4	**0.71**	0.13	**0.78**	**0.6**	-0.03	-0.13
5	**0.65**	0.12	**0.74**	**0.17**	-0.1	-0.11
	R_t	R_t	R_t	p_t	p_t	p_t
1	**0.96**	**0.92**	**0.96**	**-0.03**	0.34	0.9
2	**0.92**	0.72	**0.86**	**-0.18**	*-0.00*	0.76
3	**0.86**	0.48	*0.75*	**-0.02**	*-0.20*	0.64
4	**0.81**	0.26	*0.65*	**0.37**	-0.26	*0.55*
5	**0.74**	0.08	*0.58*	**0.04**	-0.23	*0.49*
	n_t	n_t	n_t	H_t	H_t	H_t
1	**0.93**	**0.82**	**0.96**	**0.92**	**0.99**	**0.99**
2	**0.84**	0.53	**0.88**	**0.89**	**0.95**	**0.98**
3	**0.73**	0.24	**0.8**	**0.85**	**0.91**	**0.95**
4	**0.62**	0.03	*0.73*	**0.82**	**0.85**	0.93
5	**0.51**	-0.1	*0.68*	**0.78**	**0.8**	0.90

Table 2.7. Autocorrelations, Clarida et al.

Order of Autocorrelation	Data	JLN	NGE	Data	JLN	NGE
	$1-n_t$	$1-n_t$	$1-n_t$			
1	**0.98**	0.82	**0.96**			
2	**0.96**	0.53	**0.88**			
3	**0.93**	0.24	*0.8*			
4	**0.9**	0.03	*0.73*			
5	**0.86**	-0.1	*0.68*			
	g	g	g	I/Y	I/Y	I/Y
1	**0.97**	**0.92**	**0.96**	**0.95**	**0.94**	**0.95**
2	**0.92**	0.72	**0.86**	**0.91**	0.78	**0.85**
3	**0.88**	0.48	*0.75*	**0.87**	0.58	*0.73*
4	**0.83**	0.26	*0.65*	**0.82**	0.37	*0.62*
5	**0.78**	0.08	*0.58*	**0.75**	0.19	*0.54*

Table 2.8. Autocorrelations, Clarida et al. Reaction Function

variables show far too little persistence in the JLN economy (and for all variables less than in the New Growth economy): The autocorrelations are dying off too quickly.

For π_t, both models produce very similar autocorrelations. They match the first order empirical autocorrelation but all the remaining ones are incorrectly signed. For p_t, both models produce incorrectly signed first and second order autocorrelations. The JLN economy then does match the sign of the third order autocorrelation but produces wrong signs for the remainder. The New Growth economy produces a wrong sign for the third order autocorrelation but almost matches the fourth and matches the sign of the fifth. For the real wage to capital ratio H_t, both models match the first to fourth order autocorrelation, though the JLN economy comes closer to the data. The New Growth economy fails to match the fifth order autocorrelation, while the JLN economy does.

Thus the New Growth economy's second moments are indeed broadly in line with the data. It does mostly better than the JLN economy, with a few exceptions. In particular, the JLN economy fails to match the persistence of the real variables.

We now turn to the moments for the case where the models feature the reaction function estimated by Clarida et al. (1998), i.e. equation (2.34), instead of equation (2.33). The relative standard deviations and cross correlations can be obtained from columns 4 and 5 of table 2.4 (Clarida et al.=CGG). The performance of the two models seems quite robust to the change in the reaction function, with a couple of exceptions. Concerning the standard deviations and cross correlations, both models perform worse at matching the relative standard deviation of i_t. The relative standard deviation of inflation becomes far too high in the JLN economy. For the New Growth economy $corr(D_t, F_t)$, $corr(R_t, F_t)$ and $corr(n_t, F_t)$ are almost unchanged while $corr(D_t, F_t)$ is somewhat reduced (and thus brought closer to the data) for the JLN economy. $corr(\pi_t, F_t)$ becomes positive in both models, with the New Growth model coming very close to the data.

Concerning the autocorrelations, which are reported in tables 2.7 and 2.8, note that they generally increase slightly in the New Growth model, much so in case of i_t, but decrease in the JLN economy, with the exception of i_t and π_t.

Thus we conclude that the New Growth model is still better at matching the second moments discussed here, in particular the persistence in the data, than the JLN economy.

2.4. Explaining the Evolution of Unemployment over Time

We now discuss the response of the New Growth and the JLN economies to a cost push shock. We aim to create a scenario akin to the one faced by central banks in Western Europe at the end of the 1970s and the beginning of the 1980s, the time of the second oil price shock. That means we would like to create a situation where an exogenous force increases annual inflation several percentage points above its target level, forcing the central bank to increase the interest rate to bring inflation back to target. Therefore u_t is set equal to 0.03 for the first quarter. To put it differently, for given values of marginal cost, past and expected inflation, inflation in that quarter is increased by three percentage points. In the baseline simulation, this will give rise to a disinflation of a bit more than 4.6 percentage points over 5 years, if we compare annual rates in the first and the sixth year. This is at the lower end of disinflations actually experienced during that period. For instance, in Germany, annual inflation was at 6.3% in 1981, which was then reduced to -0.1% in 1986, which is a rather small disinflation compared to the UK, France or Italy where inflation declined by 8.6, 10.8 and 13.7 percentage points over the same period, respectively.

An alternative way to generate a disinflation would have been a reduction of the inflation target of the central bank. However, this approach would have rendered a disinflation of a given size much less costly in terms of the increase in unemployment necessary to bring inflation back to target. The reason for this is rooted in the effect of future expected inflation on current inflation. The cost push shock generates a less favourable trade-off between inflation and unemployment than a reduction in the inflation target. Moreover, as mentioned above, we aim to simulate the effect of an exogenous supply shock on the model economies rather than a reduction in the inflation target.

Note that there is no endogenous persistence in the shock itself beyond the first quarter, implying that any persistence in the path of the variables and in particular unemployment beyond that point is endogenous. This section focuses on understanding the induced evolution of unemployment and inflation over time. We first examine the results under the baseline calibration. Section 2.4.1 restricts itself to comparing the evolution of unemployment and inflation in the New Growth and the JLN economy, as well as describing the paths of the NAIRU and productivity growth. It turns out that in the New Growth economy, the cost push shock causes a persistent increase in unemployment and the NAIRU as well as a persistent decline in productivity growth. Section 2.4.2 develops the intuition for the persistent increase in unemployment. The increase is due to the interaction of a decline in capital stock growth (which with endogenous growth implies a decline in total factor productivity growth) with rigid real wage growth. Section 2.4.3 shows how varying the output gap coefficient ψ_Y in the interest feedback rule affects the unemployment increase over the short and medium run. Section 2.4.4 examines the robustness of our results against varying the parameter indexing the degree of investment adjustment costs κ and the slope of the real wage growth function b.

We again solve the model by employing a second order approximation to the policy function using the approach of Schmitt-Grohe and Uribe (2004a) and using the software Dynare to implement the solution and conduct the simulation. The simulation is conducted under perfect foresight.

In all figures, the period zero value is the steady state value of the respective variable. Furthermore, when we refer to "Baseline" in figures or in the text, we always mean the New Growth economy in its baseline calibration. The abbreviation "NGE" used in the figures refers to "New Growth Economy".

2.4.1. Unemployment and the NAIRU in the New Growth and JLN Economies

Figure 2.3 plots the response of actual unemployment for the JLN and the New Growth economies to the one quarter cost push shock. In the JLN economy, unemployment increases by about 3 percentage points on impact but starts declining after reaching a maximum of 10.4%. It then quickly falls and in quarter 8 practically returns to its steady state value and then slightly undershoots for some time. Unemployment would be expected to increase because the cost push shock will increase inflation which will ultimately lead to an increase in ex ante real interest rates via equation (2.33). As consumers and investors are forward looking, this causes a contraction of aggregate demand on impact. Figure 2.4 plots the inflation rate, which peaks in quarter one at a value of about 3.8% and then quickly declines back to zero. In both economies, inflation then turns negative and approaches it's steady state from below.[44]

[44]The fact that there is persistent deflation, especially in the New Growth economy, is caused the fact that we calibrated the steady state inflation rate to be zero. If the inflation target were about 2%, this would imply a quarterly steady state inflation rate of about 0.5%. The inflation trajectory in figure 2.4 would be shifted upwards accordingly and thus deflation would be limited to three quarters.

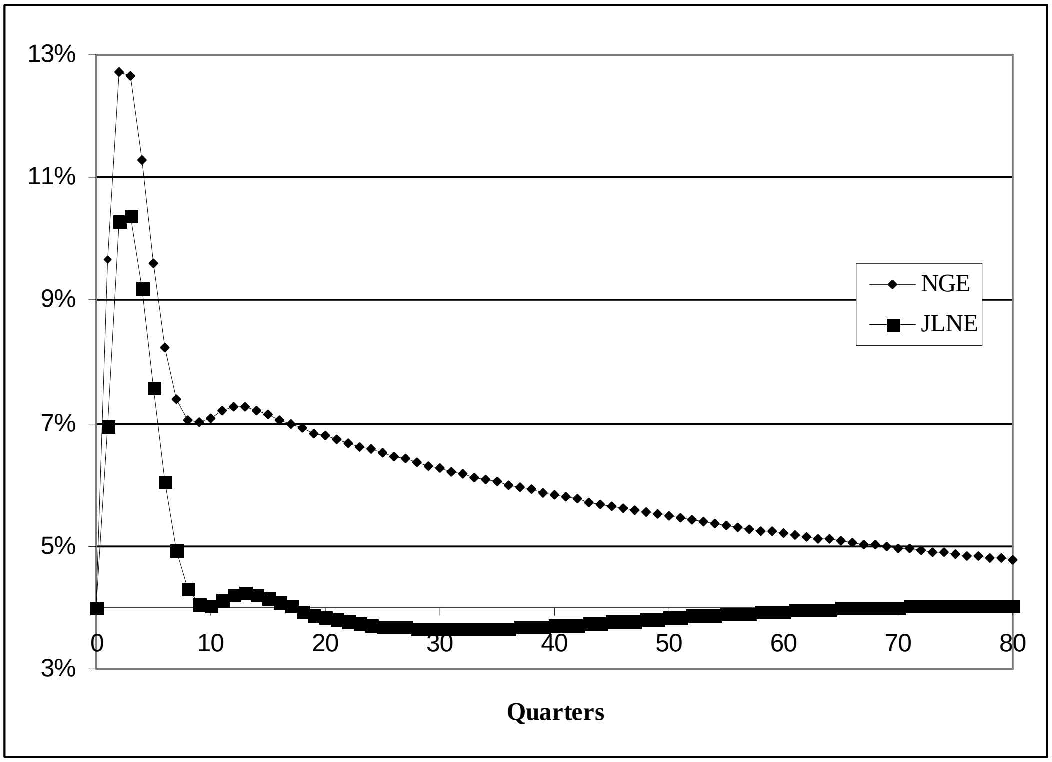

Figure 2.3. Unemployment in the New Growth and the JLN Economy

By contrast, in the New Growth economy, unemployment increases by more on impact than in the JLN economy. Even more important, the increase is far more persistent. After about 11 quarters (10 quarters after the end of the shock), when unemployment is already undershooting in the JLN economy, only a bit more than half of the on-impact loss in employment has vanished and employment is still about 3.2 percentage points below its steady state value. What is more, employment growth in the New Growth economy then becomes very slow: quarterly increases are only around 0.06 percentage points per quarter or less. As can be seen in table 2.9, in the New Growth economy, after 10 years unemployment is still about 1.8 percentage points above its steady state value and after 15 years the difference is still about 1.2 percentage points.

As observed in many European countries, unemployment increases quickly but reverts only very slowly. What is more, this slow mean reversion is endogenous: The

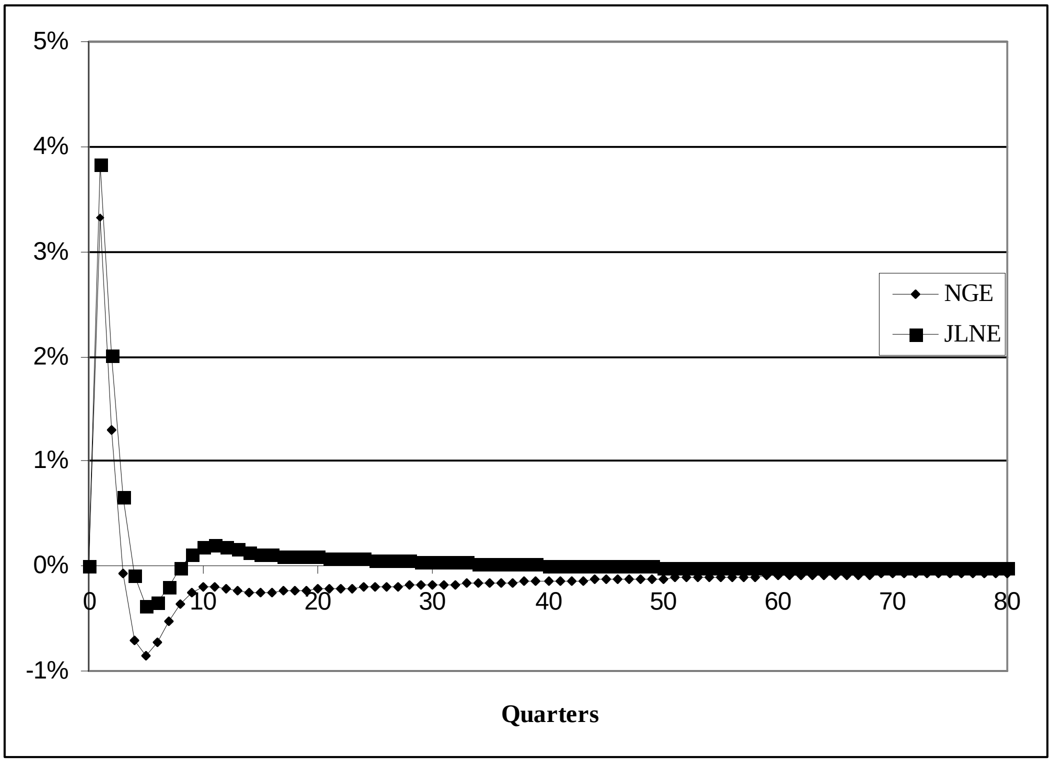

Figure 2.4. Inflation in the New Growth and the JLN Economy

exogenous shock which increases unemployment in the first place vanishes after one quarter but unemployment remains high. This is in line with the time series evidence we discussed in chapter one saying that unemployment rates in Germany and several other big European economies display high endogenous persistence.

Furthermore, Figure 2.5 reveals that the persistent increase in actual unemployment is matched by an increase in the NAIRU, as after six quarters, actual unemployment falls below the NAIRU, which gradually increases during and after the recession. A glance at Figure 2.4 shows that inflation (after peaking in quarter 1 at a quarterly rate of about 3.3%) indeed stops declining at about the same time that actual unemployment falls below the NAIRU, as we would expect from the definition of the NAIRU. Thus the disinflation engineered by the central bank, while clearly successful, has come at a

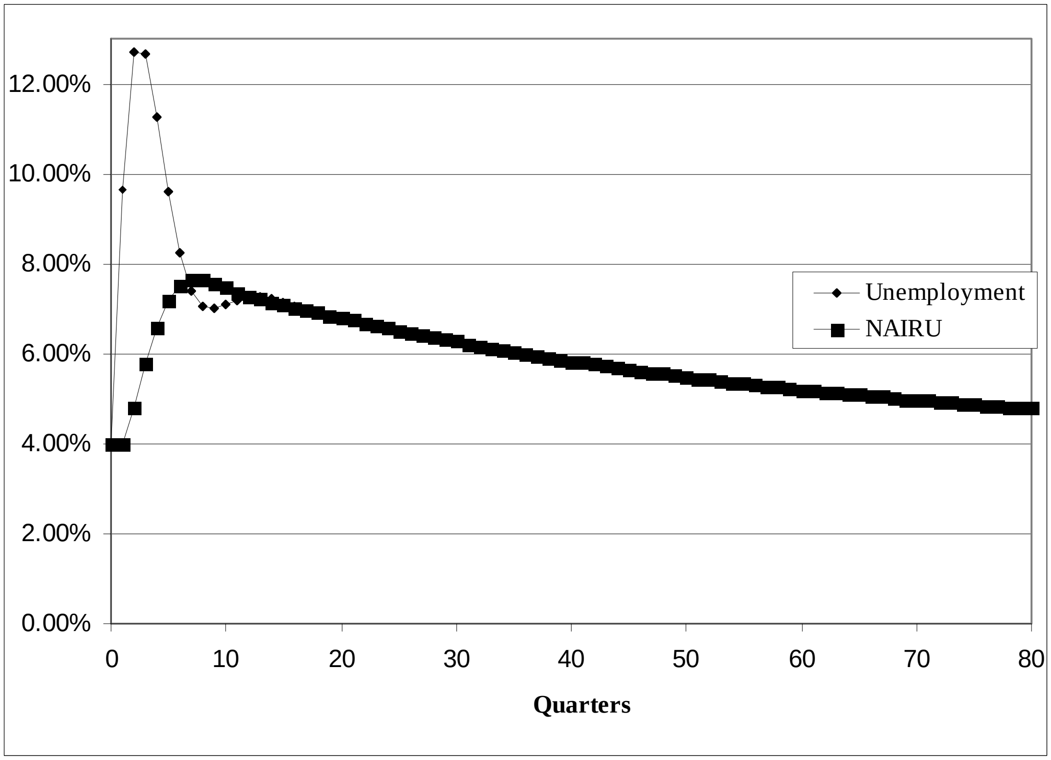

Figure 2.5. New Growth Economy, Baseline - Unemployment and NAIRU

cost beyond a temporary reduction in employment: The unemployment level consistent with constant inflation has increased.

Quarters	10	20	30	40	50	60	70	80
Baseline	3.1	2.8	2.3	1.8	1.5	1.2	1	0.8
$\psi_Y = 5$	1.9	1.5	1.2	1.0	0.8	0.6	0.5	0.4
$\kappa = 0.5$	3.5	3.1	2.5	2	1.6	1.3	1.1	0.9
$\kappa = 0.8$	2.8	2.6	2.1	1.7	1.4	1.1	0.9	0.7
$b = 0.14$	0.8	0.6	0.3	0.1	0.1	0	0	0
$b = 0.06$	4.3	4.3	4.0	3.7	3.5	3.2	3	2.8

Table 2.9. Unemployment Deviation from the Steady State in the New Growth Economy

Associated with the increase in unemployment in the New Growth economy is a persistent slowdown in labour productivity growth. This is in line with the evidence cited above. After 10 quarters it falls short of its steady state value by about 0.21% per quarter or 0.88% at an annualised rate, while 40 quarters after the shock it is still about 0.13% lower than in the steady state, or 0.54% at an annualised rate. Average annualised productivity growth over the first 10 years after the shock equals 2.46%. Assuming that average productivity growth before the shock hit equalled its steady state rate of 3.42%, this implies a decline of average productivity growth from one decade to the next of 0.96%. Interestingly, average German productivity growth did decline by 1.44% from the 1970s to the 1980s.[45]

2.4.2. Understanding the Evolution of Unemployment in the New Growth Economy

We know from equation (2.29) that an increase in unemployment will reduce real wage growth which will tend to lower marginal costs. Hence there must be a strong countervailing force pushing marginal costs up in order to explain why inflation stops falling. Figure 2.6 shows that while real wage growth drops sharply, in quarter two, the growth rate of the capital stock falls by even more and remains considerably below real wage growth for about nine quarters. After that they are about equal. In the New Growth economy, slower capital stock growth entails slower technological progress and thus slower growth of labour productivity. We showed more formally above (see equation (2.19)) that therefore in the New Growth economy, the real wage-to-capital ratio drives marginal cost. Figure 2.7, which plots the deviations of marginal cost and the real wage-to-capital ratio from their steady state values confirms that it is the movement of the real wage-to-capital ratio which drives marginal cost back up, as both move broadly in parallel.

[45]Productivity is measured as real GDP per hour worked. The data was taken from the Federal Statistical Office of Germany (Statistisches Bundesamt). See Statistisches Bundesamt Wiesbaden (2007b). A sophisticated analysis of changes in trend productivity growth by Skoczylas and Tissot (2005) finds a negative break for Germany in 1979 of -2.75%

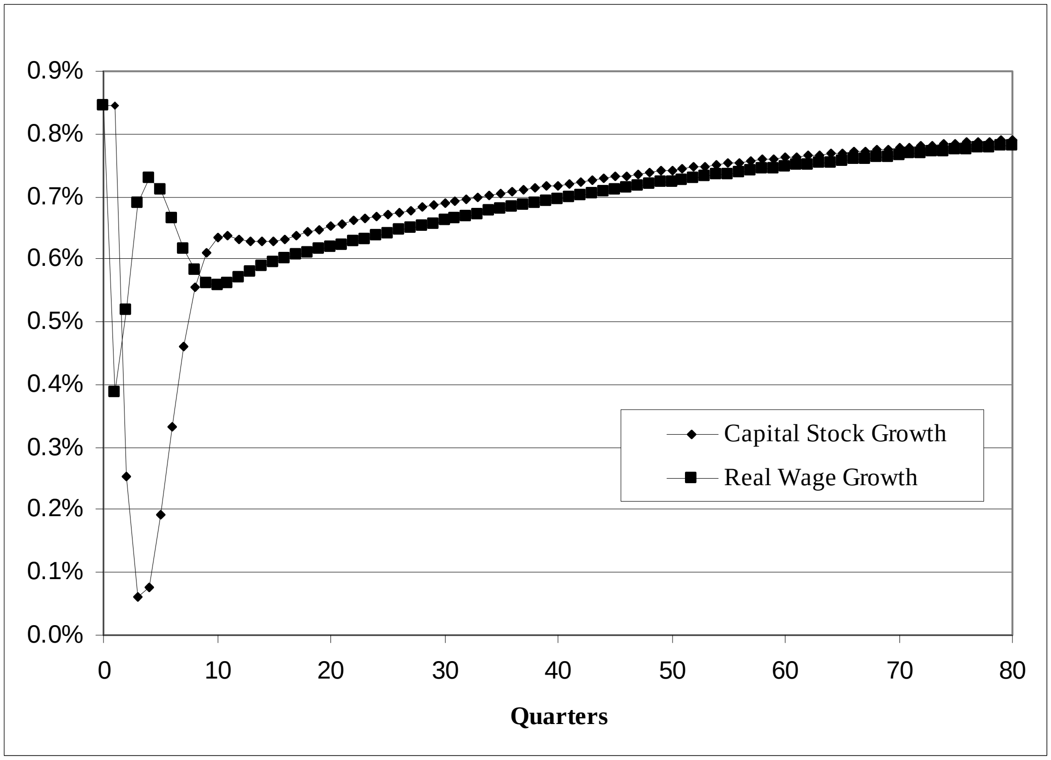

Figure 2.6. New Growth Economy, Baseline - Capital Stock Growth and Real Wage Growth

By contrast, in the JLN economy, the effect of the capital stock on marginal costs is much weaker, as shown by equation (2.18). The major determinant of marginal costs apart from the real wage is total factor productivity TFP_t. This grows exogenously no matter whether output and investment are contracting or growing. Thus marginal costs or, to put it differently, the permissible, non-inflationary rate of real wage growth are much less affected by changes to the capital stock.

Turning back to the New Growth economy, the recovery of actual employment has to slow down after about six quarters because unemployment arrives at a level beyond which any reduction would cause inflation to accelerate as it pushes real wage growth above the growth rate of the capital stock and thus pushes up marginal cost. A quick reduction in unemployment would also turn the output gap positive. Both of these developments would trigger interest rate increases via the policy rule. In fact this is

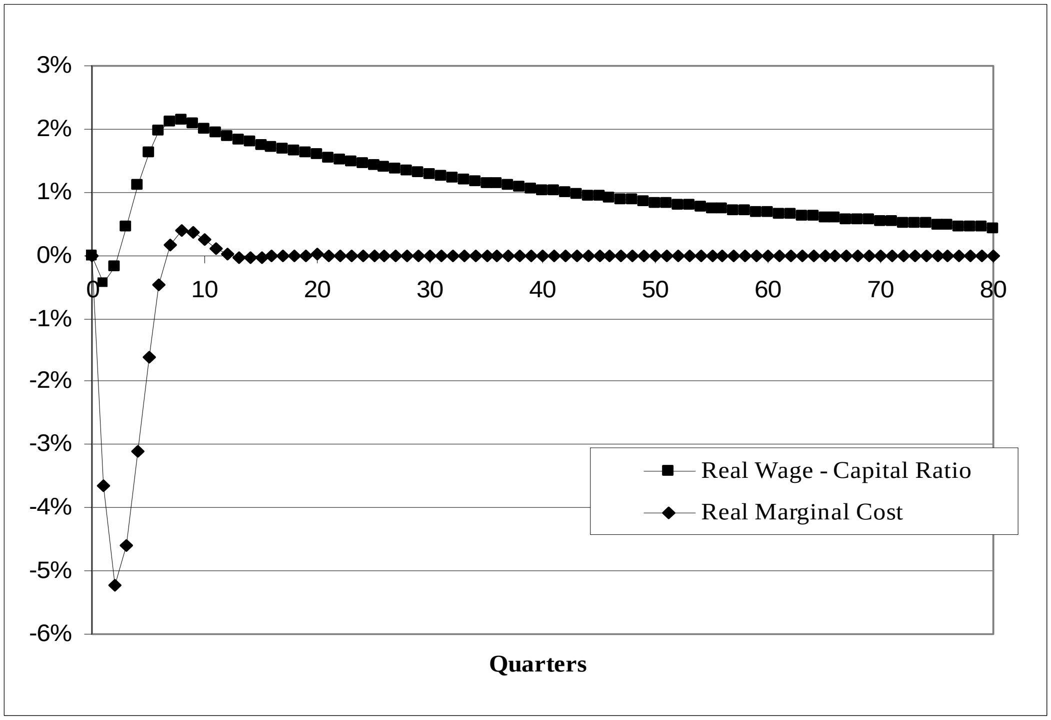

Figure 2.7. New Growth Economy, Baseline - Real Wage-to-Capital Ratio and Marginal Costs

already happening: Inflation is picking up and actual unemployment is falling below the NAIRU in quarters six and seven, respectively, as can be obtained from Figures (2.4) and (2.5). Correspondingly, figure 2.8 shows that the central bank stops lowering the real interest rate $i_t - E_t\pi_t$ after 8 quarters, when it is 0.45 percentage points (about 1.81 percentage points at an annualised rate) below the steady state value, and begins to tighten again.

Note that this level of the real interest rate, while below its steady state value, is not sufficiently low to promote a fast recovery of capital stock growth and thus a fast decline of the NAIRU and unemployment. Figure 2.9 summarises the benefits from investing by plotting the present discounted value of an additional unit of capital, q_t. q_t recovers quickly after the shock has passed and reaches its steady state value of one after five quarters. It then slightly exceeds it's steady state level for six quarters. The first order conditions (2.23) determine the investment growth rate, which due to the fast recovery of q_t, moves much closer to its steady state value as well. However, the capital stock growth rate depends on the investment-to-capital ratio, as can be seen from the capital accumulation equation (2.24). The investment-to-capital ratio has declined during the recession and the subsequent period of slow growth. To move the investment-to-capital ratio and thus capital stock growth back to its steady state would require an investment growth rate exceeding the steady state. An above-steady-state investment growth rate would have to be induced by an above-steady-state value of q_t. This in turn would require a lower real interest rate.

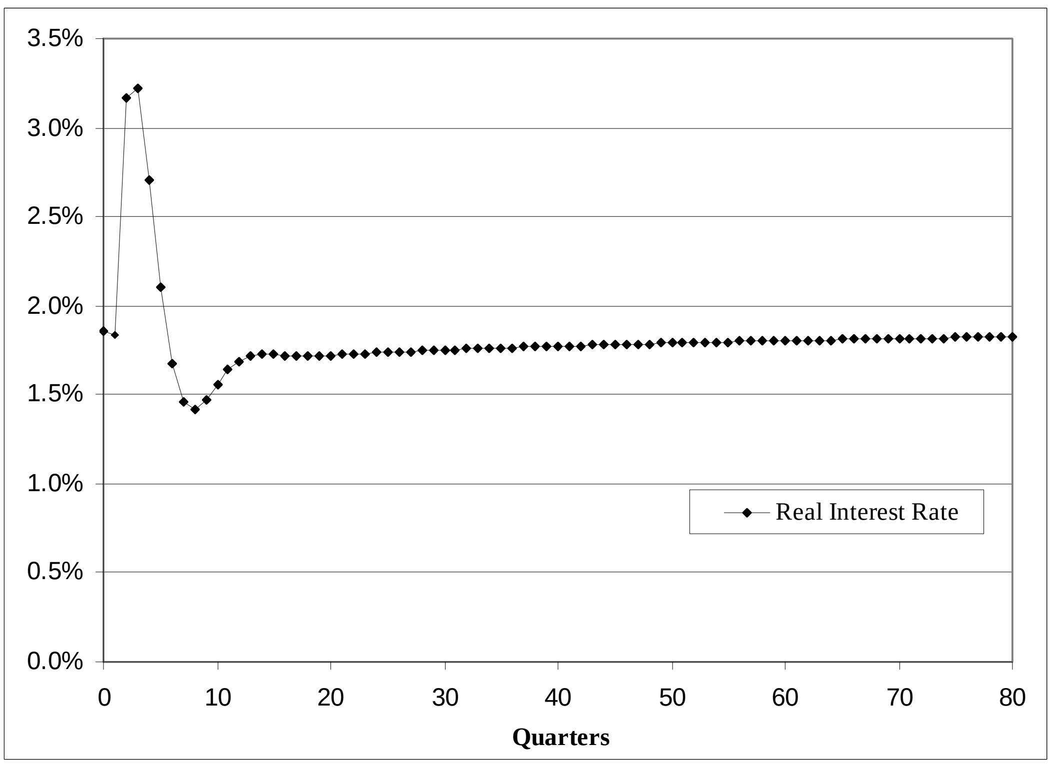

Figure 2.8. New Growth Economy, Baseline -Real Interest Rate

The speed of recovery is then governed by the relative growth rates of real wages and the capital stock. From quarter 9 onwards, the capital stock grows slightly faster than real wages. This slowly lowers the real wage-capital ratio (see figure 2.7), and a slow reduction in unemployment as higher productivity growth implies that firms can

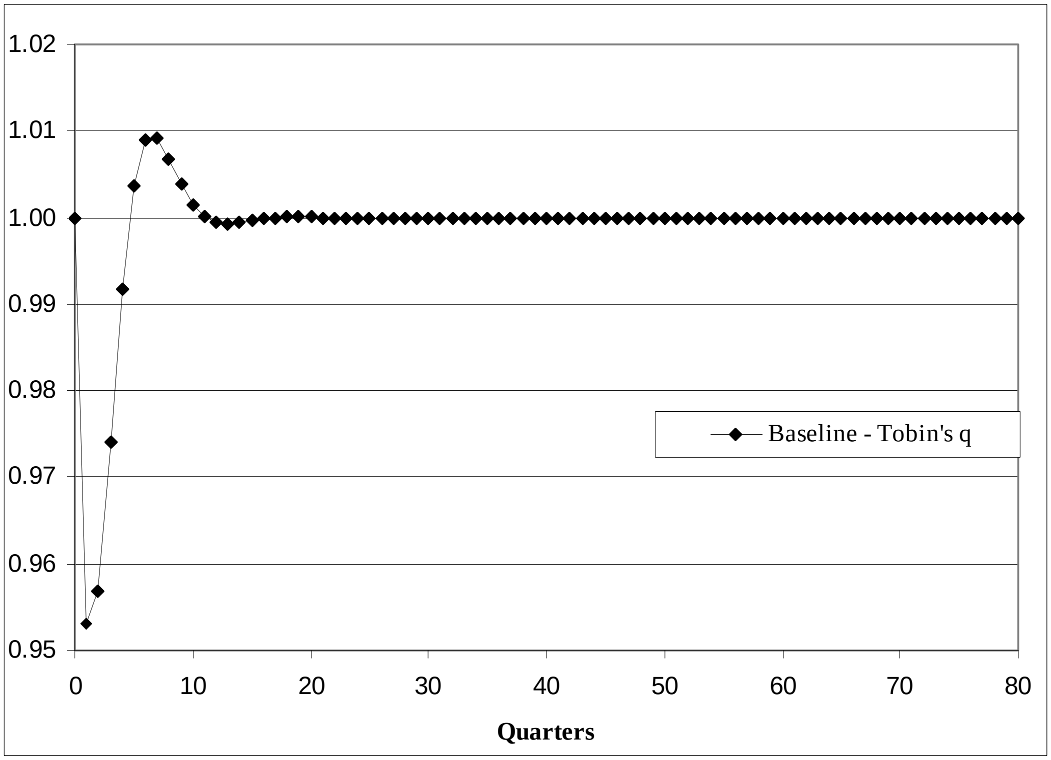

Figure 2.9. New Growth Economy, Baseline - Tobin's q

accommodate the increased real wage growth associated with a tighter labour market without facing an increase in marginal costs. This, in turn, again increases capital stock growth by increasing the marginal product of capital.

Before we move on to discuss the inflation-unemployment trade-off in the New Growth economy, note that the above discussion implies that in the New Growth economy, the causal link between monetary policy and productivity growth runs both ways. The real interest rate affects productivity growth via its direct and indirect (via AD_t) effects on investment, while productivity growth affects the real interest rate via its effect on inflation and the output gap which are arguments in the monetary policy rule.

2.4.3. The Inflation-Unemployment Trade-Off

These results provoke the question as to how changes to the central bank's reaction function would affect the long-run paths of employment and inflation. Intuition suggests that a stronger weight on the output gap in the reaction function would lead to a smaller decrease in employment not just in the short but also in the long run. As investment would be squeezed less, there would be a smaller decline in capital stock growth. This would accommodate higher non-inflationary output and employment after the recovery from the recession. This would further induce the central bank to set a lower interest rate than it would otherwise have done in order to move output closer to the higher potential output level. To show this we increase the coefficient on the output gap, ψ_y, to 5, leaving all other parameters the same. The corresponding evolution of unemployment can be obtained from figure 2.10. Indeed unemployment increases considerably less in the short run (in fact it decreases on impact), and after 40 quarters it is still about 0.8 percentage points lower than in the Baseline case, as can be obtained from the second line of table 2.9. Hence a less hawkish monetary policy has indeed very long-lasting benign effects on unemployment.

The lower increase in unemployment comes at the cost of a considerably stronger short-run inflation surge. While in the baseline simulation, inflation peaks at a (quarterly) rate of 3.3%, it now increases as high as 4.9% in the first quarter (figure 2.11), while the annual inflation rate over the first year amounts to 15%. Note however that the increase in inflation is only temporary. After 10 quarters, it has already decreased to 0.42%. Thus the stronger acceleration in inflation is a short-run phenomenon. The gain in employment is of more persistent nature.

As mentioned above, Ball (1999) finds that during the recessions of the early 1980s, countries whose central banks aggressively lowered interest rates experienced smaller increases in the NAIRU than those which did not. Ball calculates the difference between the NAIRU in the year before the recession and five years after. He defines a recession

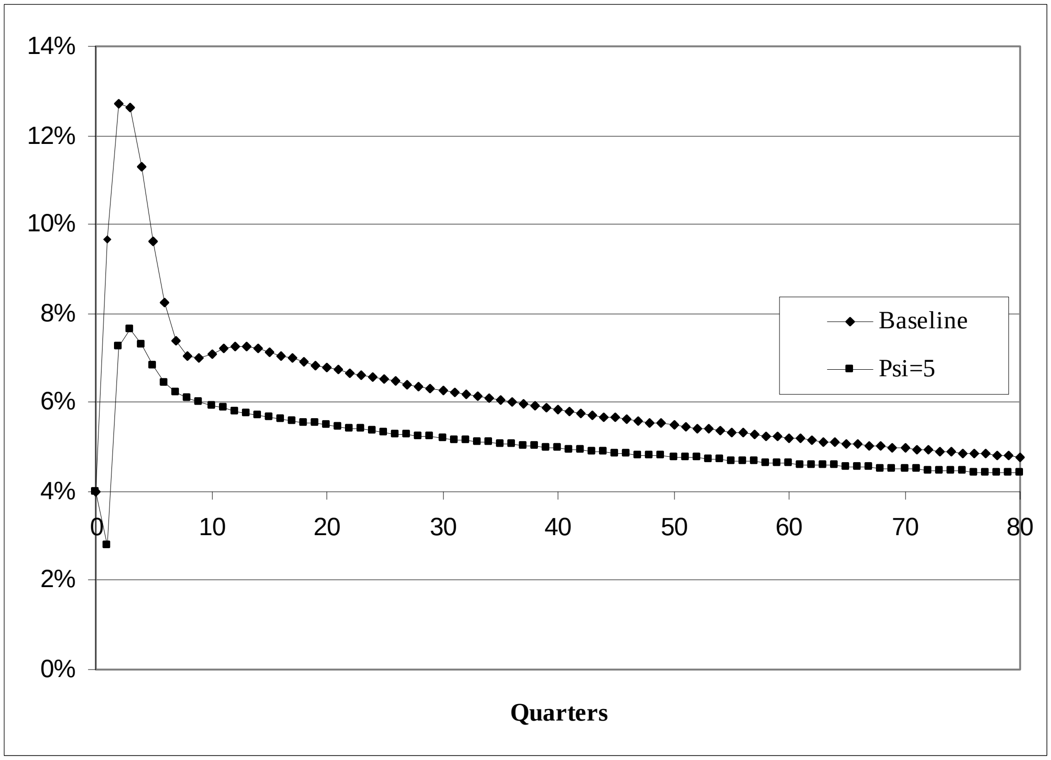

Figure 2.10. New Growth Economy, Baseline and $\psi_y = 5$ – Unemployment

as one year with GDP growth below 1%. He regresses the change in the NAIRU on the maximum reduction of the ex-post real interest rate during any time of the recession's first year, which he refers to as maximum easing.[46] The coefficient on maximum easing is -0.42 and is significant at the 10% level.[47] We try to replicate this relationship with our New Growth model by varying the output gap coefficient between 0 and 4, leaving everything else the same, thus obtaining data on maximum easing and the five year change in the NAIRU. Our resulting coefficient on maximum easing is negative as well and varies between -0.24 and -1.16. This is for the most part consistent with Ball's estimate.

[46]Ball controls for the duration of unemployment benefits.
[47]See Ball (1999), p. 207.

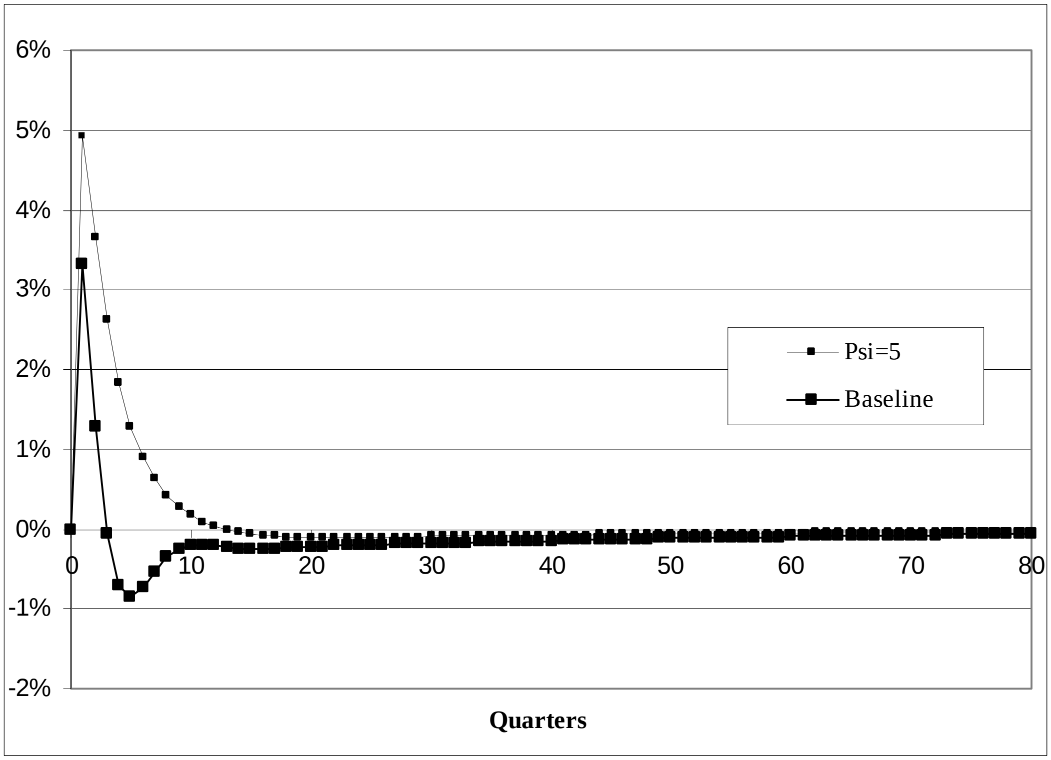

Figure 2.11. New Growth Economy, Baseline and $\psi_y = 5$ – Inflation

2.4.4. Robustness

We now check the robustness of the above results against changes to the parameter indexing investment adjustment costs κ and against changes in b, the effect of unemployment on real wage growth. We restrict ourselves to the response of unemployment. Figure 2.12 reports the response of unemployment for $\kappa = 0.5$, 0.8 and the baseline value. Clearly the response of unemployment is somewhat sensitive to varying κ : The path of unemployment is between 0.1 and 0.4 percentage points above and below the baseline path for $\kappa = 0.5$ and 0.8, respectively, as can be obtained from table 4. With lower values of κ, unemployment is increased as compared to the baseline in the short and in the long run since investment declines by more in response to the cost push shock. This reduces both aggregate demand and capital stock growth, thus increasing actual unemployment and the unemployment rate consistent with constant inflation. Correspondingly, a higher value of κ limits the drop in investment and thus unemployment is reduced as compared to the baseline.

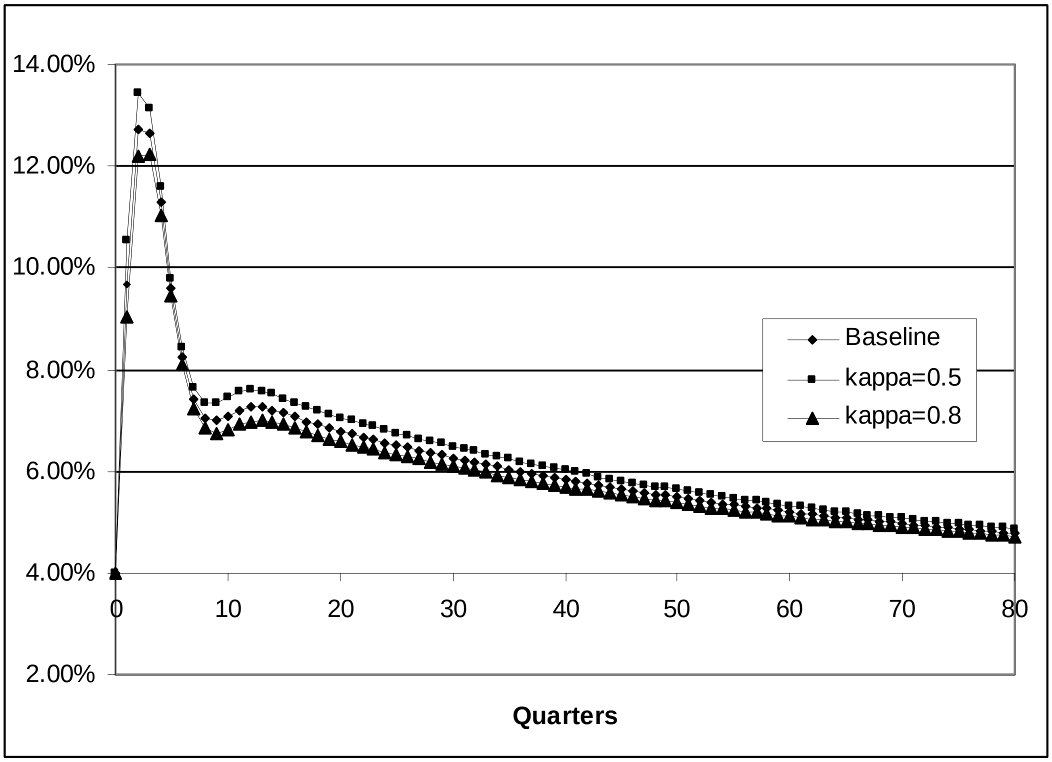

Figure 2.12. New Growth Economy, various Values of κ - Unemployment

Figure 2.13 displays the response of unemployment under values for b two standard deviations above (i.e. a value of 0.14) and two standard deviations below (i.e. 0.06) the point estimate reported in appendix A.7 and the baseline calibration. Clearly the response of unemployment is very sensitive to the value of b both in the short and in medium run. Unemployment peaks at 11% if $b = 0.14$ but at 13.5% for $b = 0.06$. The unemployment increase is also a lot less persistent if wages are more flexible: After 10 quarters, unemployment is merely 0.8 percentage points higher than in the steady state if $b = 0.14$, while after 40 quarters unemployment has almost returned to its steady state. By contrast, if $b = 0.06$, the response of unemployment becomes very

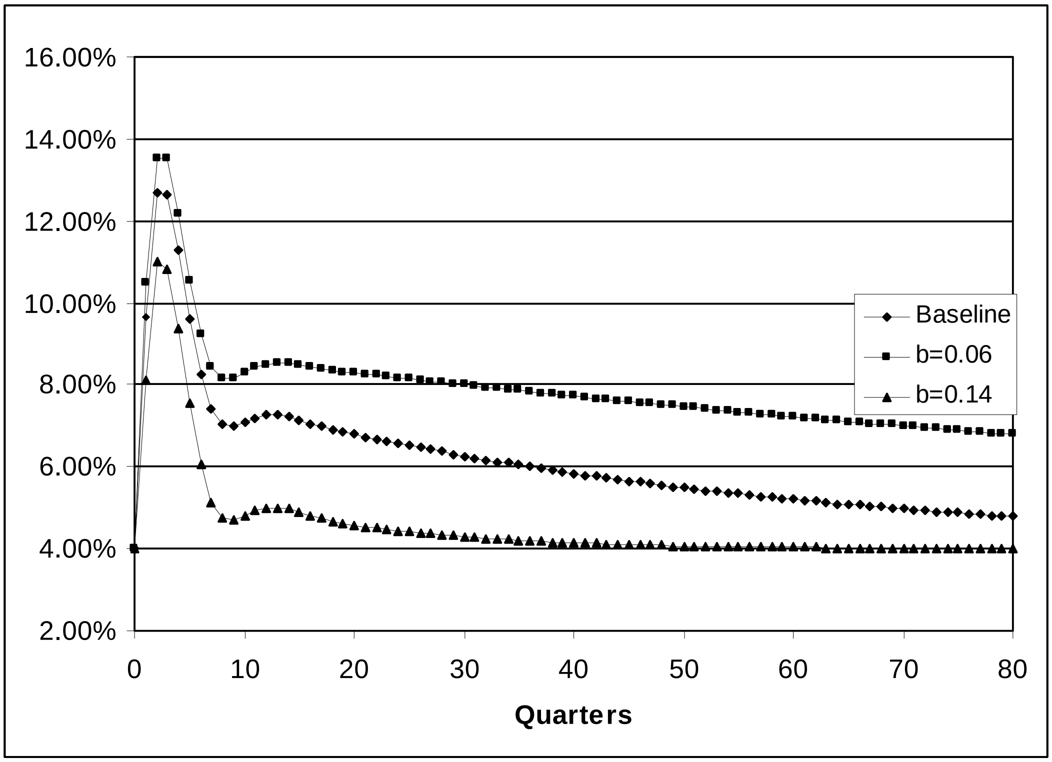

Figure 2.13. New Growth Economy, various Values of b - Unemployment

persistent. After 10 quarters, unemployment exceeds its steady state by 4.3 percentage points, while after 40 quarters unemployment still exceeds its steady state by about 3.7 percentage points. This implies that 39% of the peak deviation of unemployment from its steady state is still present after 10 years.

The great sensitivity of the unemployment response to changes in b is not surprising. The New Growth economy generates a persistent increase in unemployment through the reduction in the productivity growth rate implied by the drop in investment during the recession induced by the cost push shock. The fall in productivity growth implies that the real wage growth rate consistent with constant inflation declines and thus the unemployment rate increases. However, the increase in unemployment necessary to enforce that decline in the real wage growth rate will depend on how strongly wage growth responds to changes in unemployment. Hence with $b = 0.14$, the central bank has to increase unemployment by a lot less for a given reduction in productivity growth than under $b = 0.06$. This in turn increases the marginal product of capital (relative to a scenario with a lower b), implying a higher investment growth rate and thus a faster recovery of capital stock growth and thus productivity growth.

2.5. Cross Country Aspects

The previous section shows that our New Keynesian model with endogenous growth is able to produce a persistent increase in unemployment as a consequence of a disinflation. This is an important result because economists have been struggling to explain the evolution of unemployment in continental Europe over time. This begs the question as to whether we can also use the model to replicate differences in the evolution of unemployment across countries. We address this issue in three different ways in this section. For that purpose, we will draw on the differences in the size of the disinflation across the OECD, in (estimates of) the policy reaction function coefficients between the Bundesbank and the Federal Reserve and in real wage rigidity.

We noted earlier that there is a negative correlation between the change in inflation and the change in the NAIRU. Ball (1996) investigated this for the 1980s and we plotted it over two decades and across 21 OECD countries in figure 2.2. There are various possible reasons why countries might have differently sized disinflations. Economies might differ in the way they respond to global supply shocks, perhaps due to differences in energy intensity of production. Their past record of monetary policy might differ as well, (in the sense that some central banks have let inflation spiral more out of bounds than others, leading to larger deviations of inflation from target), as might choices of how much to disinflate (a central bank might just be willing to accept a higher inflation rate following a supply shock). Finally, exchange rate volatility might differ as well.

Incorporating these various sources of inflation volatility into our model would be far beyond the scope of this chapter. However, we do try to mimic their inflationary impact by varying the size of the cost push shock.

We vary the size of the cost push shock from 0.01 to 0.05, leaving all other parameters unchanged. Then we calculate the change of the inflation rate from year 1 to year 10 and the change of the NAIRU from the first quarter of year one to the first quarter of year 10, and plot the latter against the former in figure 2.14.[48] There is a clear negative correlation. The slope of the line varies between -0.41 and -0.56, which is not too far away from the simple regression coefficient of -0.33 (or -0.36 if, like Ball (1996) we exclude Greece) resulting from a regression of the change in the NAIRU on the change in inflation using the OECD data presented earlier.

We now examine whether observable cross-differences in the monetary policy rule imply different unemployment paths in response to the cost push shock. To get a proper idea of the effects of these it is obviously important to have comparable estimates. Therefore we make use of the estimates by Clarida et al. (1998), who estimate equation (2.34) using the same methodology for Germany and the United States. The coefficient estimates are reproduced in table 2.3.

We now repeat the same experiment we conducted in the previous section for both the estimate of equation (2.34) for the Bundesbank and the estimate of equation (2.34) for the Federal Reserve. The first two lines of Table 2.10 show the deviation of unemployment from steady state for both set of coefficients. Note first that the persistent

[48]We take the difference of the first quarter of both years since the NAIRU moves up very fast during the first four quarters. Differencing the annual averages of the two years would create a misleading impression of the correlation between the medium run change in the NAIRU (by unduly reducing this change) and the change in inflation. The quarterly movements of the NAIRU in the OECD data are very slow and re-doing figure one with the difference in the NAIRU between 1980 quarter1 1990 quarter one rather than with the differences in the annual averages as is the case now would not change the result.

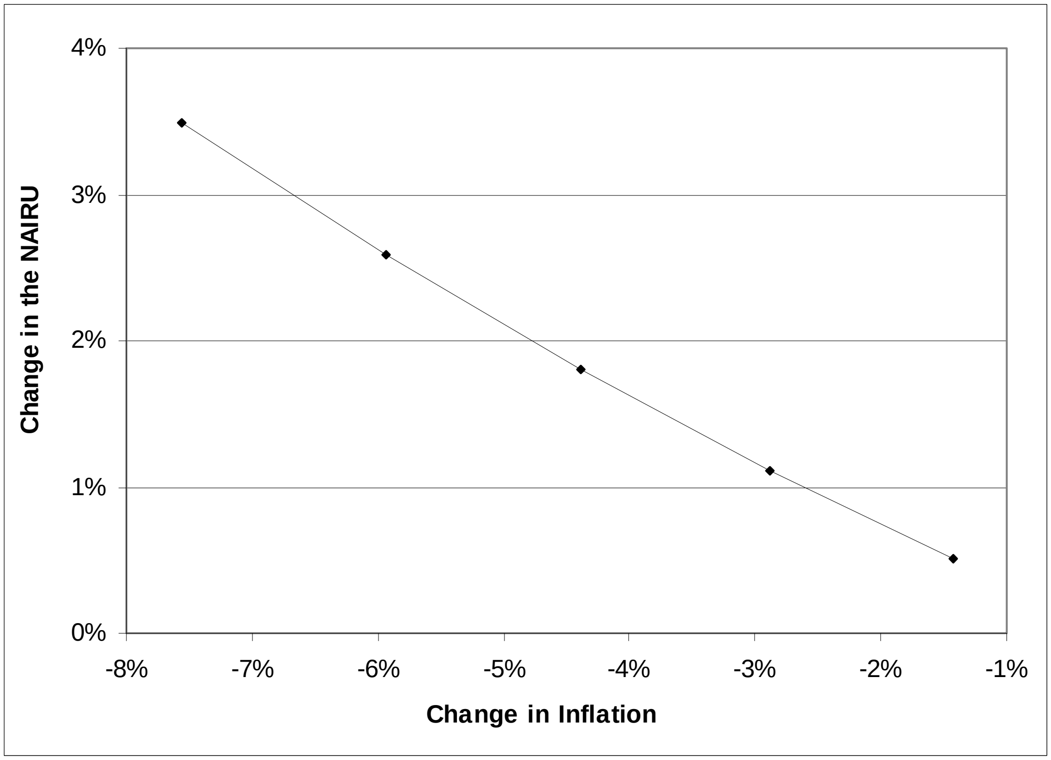

Figure 2.14. Change in Inflation vs. Change in the NAIRU over 10 Years

increase in unemployment with the policy rule as specified and estimated by Clarida et al. for the Bundesbank is substantially higher than the increase we saw with the policy rule used in the Baseline. This illustrates that, in terms of the unemployment effects which are the subject of this chapter, we were quite conservative in specifying and calibrating our Baseline policy rule. Apart from that, unemployment is persistently higher under the Bundesbank rule than under the Federal Reserve one, though the difference is for the most part less than one percentage point. For instance after 10 years, unemployment and the NAIRU are about 0.5 percentage points higher under the Bundesbank Rule than under the Federal Reserve rule.

It is, however, informative to take a look at the standard errors associated with Clarida et al.'s estimate. For instance, the coefficient on the lagged interest rate ρ has a standard error of 0.03. Thus a value for ρ of 0.06 is still consistent (at a 5% level of confidence) with Clarida et al.'s estimate. The third row of table 2.10, shows the implied evolution of unemployment if we set $\rho = 0.91$. The resulting unemployment trajectory is substantially lower than before. After 40 quarters, the unemployment and the NAIRU are now 1.1 percentage points lower than under the Bundesbank rule, while after 50 quarters, the difference is still 1 percentage point. In the same manner, we can also make use of the standard error of the estimate of ψ_Y, which equals 0.16. Increasing ψ_Y to 0.88 yields the employment trajectory shown in the final row of table 2.10, which is again lower than with the point estimate. After 40 quarters, unemployment and the NAIRU are about 0.9 percentage points lower than under the Bundesbank policy rule. Thus in the New Growth model, differences in policy function parameters consistent with Clarida et al.'s estimate can contribute to explaining the different evolution of the unemployment rate in Germany as compared to the United States.

Accordingly, differences in monetary policy also explain differences in the change in the productivity growth rate between Germany and United States from the 1970s to the 1980s. As noted above, average US productivity growth declined by only about 0.18% from the 1970s to the 1980s, whereas the decline in Germany was about 1.4%. Table 2.11 displays the difference between average annualised productivity growth during the first decade after the shock and the decade before the shock.[49] Thus within the New Growth model, differences in monetary policy would account for between 0.24 and 0.6 percentage points of the difference between Germany and the United States in the decline in productivity growth.

Finally, we explore the effects of the observed cross continental differences in the nature of real wage rigidity. As was mentioned above, empirical estimates of wage setting

[49]As above we assume that during the decade before the shock hits, the average productivity growth rate equals its steady state.

Unemployment Deviation from the Steady State, Percentage Points								
Quarter	10	20	30	40	50	60	70	80
Bundesbank	3.8	3.8	3.1	2.5	2.1	1.7	1.4	1.1
Federal Reserve	2.9	3.1	2.5	2	1.7	1.3	1.1	0.9
Federal Reserve, $\rho = 0.91$	2.1	2.1	1.7	1.4	1.1	0.9	0.7	0.6
Federal Reserve, $\psi_Y = 0.88$	2.4	2.5	2	1.6	1.3	1.1	0.9	0.7

Table 2.10. Results for Clarida et al.'s Policy Rules

Change in ten Year Average productivity Growth, Percentage Points	
Bundesbank	-1.28
Federal Reserve	-1.04
Federal Reserve, $\rho = 0.91$	-0.68
Federal Reserve, $\psi_Y = 0.88$	-0.88

Table 2.11. Results for Clarida et al.'s Policy Rules

functions repeatedly find that real wage growth is negatively related to the labour share in Europe but not in the United States. Therefore, in our final experiment aimed at highlighting cross country dimensions, we set $c = 0$ in the Baseline calibration, leaving everything else as in the Baseline. The resulting deviation of unemployment from its steady state can be obtained from table 2.12. Clearly, the increase in unemployment is persistently lower. After 40 quarters, unemployment is only 0.6 percentage points higher than in the steady state, compared to 1.7 percentage points in the Baseline. Average annualised productivity growth is only 0.36% lower than in the previous decade as opposed to 0.96% in the baseline calibration.

10	20	30	40	50	60	70	80
0.9	1.1	0.9	0.7	0.6	0.5	0.4	0.3

Table 2.12. c=0 - Percentage Point Deviation of Unemployment from its Steady State for selected Quarters

Within our model, c=0 would arise if there is no direct effect of productivity on effort and if benefits are not linked to productivity. We suggested above that these results might be rooted in stronger unions and perhaps a stronger link between unemployment benefits and productivity in Europe. Blanchard and Wolfers (2000) find that the impact of macroeconomic shocks on unemployment is affected by the labour market structure. They find that both unobservable macroeconomic shocks (captured by a time effect) as well as a one percentage point reduction in total factor productivity growth increase unemployment by more the higher is union density.[50] This result is confirmed by Fitoussi et al. (2000).[51] In that sense, our model provides some theoretical support to the notion that both "shocks and institutions" (Blanchard and Wolfers) are crucial to explaining the cross country evidence on the evolution of unemployment.

2.6. Conclusion

This chapter develops a New Keynesian model with unemployment and endogenous growth to explain the persistent increase in continental European unemployment and the lack thereof in the United States. We calibrate key parameters like the coefficients in the wage setting equation and the interest feedback rule of the central bank to Western German data. The model economy is hit with a one quarter cost-push shock calibrated to induce a disinflation of an order of magnitude seen at the beginning of the 1980s in many industrialised OECD economies. We perform the same experiment on a model without endogenous growth which we coin the JLN economy.

Under the baseline calibration, after ten years, unemployment will still be about 1.8 percentage points above its pre-shock value. As observed in many European countries, unemployment increases quickly but reverts only very slowly. What is more, this slow mean reversion is endogenous: The exogenous shock which increases unemployment in the first place vanishes after one quarter but unemployment remains high. This is in line with the time series evidence we discussed in chapter one saying that unemployment

[50]See Blanchard and Wolfers (2000), pp. C20-C28.
[51]See Fitoussi et al. (2000), p. 250.

rates in Germany and several other big European economies display high endogenous persistence.

At the same time, inflation stops declining soon after the cost push shock has vanished, implying that the successful disinflation has increased the NAIRU. Unsurprisingly, no such effect is seen in the JLN economy, where unemployment is back to its steady state after about two years.

The high degree of endogenous unemployment persistence in the New Growth economy is due to the interaction of the investment - productivity growth relationship with rigid real wage growth. In the New Growth economy, for a given employment level, capital stock growth determines labour productivity growth. Hence the real wage-to-capital ratio is the main driver of marginal costs. Thus, although real wage growth declines as employment contracts, marginal cost returns back to its steady state level soon after the shock has vanished because capital stock growth declines by even more. This stops the disinflation and lowers the real wage growth rate associated with stable inflation. Since real wage growth is rigid, stable inflation then requires an increase in the unemployment rate: the NAIRU increases. The central bank therefore engineers only a slow recovery of aggregate demand and unemployment since a faster decline of unemployment would push inflation above its target and render the output gap positive. The slow recovery of demand also slows down the recovery of investment and productivity growth.

The model thus also contributes to explaining the productivity slowdown observed across advanced OECD economies, and why negative shocks to productivity growth are frequently significant variables in regressions of unemployment on this variable and others. Furthermore, the amount of monetary easing during the recession associated with the disinflation is negatively related to the subsequent increase in the NAIRU as found by Ball (1999). If the central bank aggressively lowers the real interest rate as soon as the economy is in recession, this lowers the decline in investment and productivity growth and thus the increase in the NAIRU.

Finally, apart from generating a persistent increase in unemployment, the model also contributes to explaining cross country differences in the unemployment evolution. Varying the size of the cost push shock generates a relationship between the change in the inflation rate and the change in the NAIRU over a ten year horizon similar to a relationship in the data first observed by Ball (1996). Using comparable policy rule estimates of Clarida et al. (1998) for the Bundesbank and the Federal Reserve, while holding the cost-push shock constant, creates a higher persistent unemployment increase with the latter than with the former. Finally, taking account of a well established cross-continental difference in the structure of the wage setting function, namely the absence of a labour share term, also proves informative. In the absence of the labour share term, we see a lower medium run increase in unemployment and the NAIRU since real wage growth adjusts more flexibly to the decline in productivity growth caused by the cost push shock. The size of the labour share term in wage setting can be linked, if coarsely so, to features of the labour market like union density or the benefit system. Thus the chapter lends support to the view that, as suggested by Blanchard and Wolfers (2000), it is both "shocks and institutions" which are at the heart of explaining the evolution of unemployment across time and the differences across countries.

CHAPTER 3

Optimal Simple Monetary Policy Rules in a New Keynesian – Endogenous Growth Model

Conventional wisdom among monetary economists says that central banks should mainly focus on fighting inflation. This priority is found to be optimal in a wide range of small and medium scale macroeconomic models. Examples include but are not limited to Schmitt-Grohe and Uribe (2004b), (2004c) and (2005), or for an open economy context Senay (2008). Costs of inflation arise in the form of price dispersion in the presence of Calvo contracts. As not all firms can re-optimise their price every period, their prices will diverge as will their output quantities. This is inefficient as each good generates declining marginal utility for the consumer, implying that more purchases have to be made in order to reach a given utility level. Other costs of inflation arise from monetary frictions in the form of cash-in-advance constraints or transaction-cost technologies. These factors all work to focus an optimising central bank on stabilising inflation rather than output. In addition in these models, temporary shocks to the exogenous driving processes like total factor productivity do not have very long lasting effects on real variables. To the extent they do, this persistence is generated by autocorrelation in the exogenous variables.

In the New Keynesian New Growth model developed in the previous chapter, a non-auto correlated one period cost-push shock affects employment, capital stock growth and the other real variables over more than two decades in a non-trivial fashion. Therefore it is interesting to reassess the conventional wisdom on the central banks stabilisation priorities. This chapter aims to do so by optimising the coefficients of an interest feedback rule for the economy with and without endogenous growth.

As in chapter two, the only source of disturbance we consider is a cost push shock since a cost push shock generates a trade-off between stabilising output and stabilising inflation.[1] A demand shock, by contrast, does not create this sort of trade-off and is therefore less interesting in the present context.

We proceed as follows. First, in section 3.1, we replace our assumption of quadratic costs of price adjustment by the more commonly used assumption of Calvo contracts. The reason for this modification is that that Calvo contracts is the most commonly used way of modelling nominal rigidity in the literature on optimal monetary policy and that Calvo contracts imply larger welfare costs of inflation than quadratic costs of price adjustment. Let us assume for the moment that we searched for an optimal simple rule in the economies developed in chapter two and found that in the presence of endogenous growth, the central bank responds less to inflation and more to the output gap. Then one could argue that this shift in priorities might be due to the low welfare costs associated with quadratic costs of price adjustment rather than to endogenous growth. To avoid this charge, we use Calvo contracts. Note that this modification has only minor consequences for the positive properties of the models. Up to first order, both assumptions lead to the same Phillips curve as long as under Calvo contracts, non-reoptimised prices are fully indexed to past inflation, which is what we will assume most of the time.

[1] An alternative way to generate a trade-off between stabilising inflation and stabilising output would have been a productivity shock. However, to analyse the effects of a productivity shock would have been a lot more difficult as it would directly affect more than one of the two model's equations, while the cost push shock affects only the Phillips curve. We leave this extension for future research.

Thus as far as the non-policy components of the model are concerned, the *baseline* model used for policy evaluation in this chapter deviates from the model developed in chapter two only in the source of nominal rigidity. We also maintain the calibration introduced in the previous chapter and thus will discuss calibration issues only in so far as required by new features we ad to the model. Section 3.2 introduces a transaction cost for consumption purchases to motivate a money demand by households, following Schmitt-Grohe and Uribe (2005). This modification also works to increase the cost of inflation and thus provides another check of the robustness of our main result. Section 3.3 shows how we are measuring welfare and the welfare costs of suboptimal policies. Section 3.4 introduces the policy rule and discusses the grid. Section 3.5 summarises the equations of the modified model. Section 3.6 illustrates how the trade-off between stabilising real variables and stabilising inflation differs between the endogenous growth and the JLN economy. Section 3.7 presents the result from the grid search and discusses some properties of the rules found to be optimal in the respective scenarios. Section 3.8 computes the welfare costs associated with the policy rule estimates for the Bundesbank by Clausen and Meier (2003) and Clarida et al. (1998). Section 3.9 concludes.

3.1. Calvo Contracts

We replace the assumption of quadratic costs of price adjustment with Calvo (1983) contracts. The main reason is that Calvo price setting is generally held to involve larger costs of inflation. As before, the aggregate consumption and the investment good bought by the household is a CES basket of varieties. Households spread their consumption and investment expenditures across the various varieties in the basket in a cost minimising way, subject to achieving a certain number of baskets. This number is the sum of C_t, which determined by the consumption Euler equation, and I_t, which is determined by the investment first order condition. Under Calvo pricing, the prices of the varieties in the basket can differ because only a fraction of firms can re-optimise their prices each quarter. Thus the quantities bought of each variety differ since households buy more of the cheaper and less of the more expensive varieties. Because the marginal contribution of each variety to the basket is diminishing, the decline in the basket from consuming less of the more expensive varieties more than offsets the increase in the basket from consuming more of the cheaper varieties. To put it differently, to reach a certain value of $C_t + I_t$, consumers have to pay more if there is price dispersion.[2]

Under the assumptions about the degree of indexing of non-optimised prices to past inflation we are going to make, the resulting Phillips curve will be identical to the one we used so far up to a first order approximation.

With Calvo pricing, a random fraction of firms ω is not allowed to re-optimise their prices every period. Those firms instead index their prices to past inflation, where the degree of indexing is given by χ, so that a price of a firm j allowed to reset its price in period t equals $P_t(j)\left(\frac{P_{t+i-1}}{P_{t-1}}\right)^{\chi}$ in period t+i. Hence the firm maximises

$$E_t\left[\sum_{i=0}^{\infty}\omega^i\rho_{t,t+i}Y_{t+i}\left[\begin{array}{c}\left(\frac{P_t(j)}{P_{t+i}}\right)^{1-\theta}\left(\frac{P_{t+i-1}}{P_{t-1}}\right)^{\chi(1-\theta)}\\ -mc_{t+i}\left(\frac{P_t(j)}{P_{t+i}}\right)^{-\theta}\left(\frac{P_{t+i-1}}{P_{t-1}}\right)^{-\theta\chi}\end{array}\right]\right]$$

where $\rho_{t,t+i} = \beta^i\frac{u'(C_{t+i}-hab_{t+i-1})}{u'(C_t-hab_{t-1})}$ denotes the stochastic discount factor employed by the household to discount real profits. The first order condition can be rearranged to get

$$\widetilde{p}_t = \frac{\theta E_t\left[\sum_{i=0}^{\infty}(\omega\beta)^i u_c\left(C_{t+i}-hab_{t+i-1}\right)mc_{t+i}Y_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\theta}\left(\frac{P_{t+i-1}}{P_{t-1}}\right)^{-\theta\chi}\right]}{(\theta-1)E_t\left[\sum_{i=0}^{\infty}(\omega\beta)^i u_c\left(C_{t+i}-hab_{t+i-1}\right)Y_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}\left(\frac{P_{t+i-1}}{P_{t-1}}\right)^{\chi(1-\theta)}\right]}$$

[2]See for instance Walsh (2003), p. 521.

where $\widetilde{p}_t = \frac{\widetilde{P}_t}{P_t}$, and $\widetilde{P}_t$ denotes the price set by the fraction of firms which has been allowed to reoptimse. To get rid of the infinite sums, we define two artificial variables G_t and M_t, one for the numerator and one for the denominator. Thus we have

$$
\begin{aligned}
G_t &= \theta E_t \left[\sum_{i=0}^{\infty} (\omega\beta)^i u_c \left(C_{t+i} - hab_{t+i-1}\right) mc_{t+i} Y_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\theta} \left(\frac{P_{t+i-1}}{P_{t-1}} \right)^{-\theta\chi} \right] \\
M_t &= (\theta - 1) E_t \left[\sum_{i=0}^{\infty} (\omega\beta)^i u_c \left(C_{t+i} - hab_{t+i-1}\right) Y_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\theta-1} \left(\frac{P_{t+i-1}}{P_{t-1}} \right)^{\chi(1-\theta)} \right]
\end{aligned}
$$

which can be rewritten, assuming log utility, as

$$
\begin{aligned}
(3.1) \qquad G_t &= \theta (mc_t) \frac{Y_t}{C_t - hab_{t-1}} + \omega\beta (1+\pi_t)^{-\chi\theta} E_t \left((1+\pi_{t+1})^{\theta} G_{t+1} \right) \\
(3.2) \qquad M_t &= (\theta - 1) \frac{Y_t}{C_t - hab_{t-1}} + \omega\beta (1+\pi_t)^{\chi(1-\theta)} E_t \left((1+\pi_{t+1})^{\theta-1} M_{t+1} \right)
\end{aligned}
$$

The price index evolves according to

$$
\begin{aligned}
P_t^{1-\theta} &= (1-\omega) \widetilde{P}_t^{1-\theta} + \omega \left(P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\chi} \right)^{1-\theta} \quad \text{or} \\
1 &= (1-\omega) \widetilde{p}_t^{1-\theta} + \frac{\omega}{(1+\pi_t)^{1-\theta}} (1+\pi_{t-1})^{\chi(1-\theta)}
\end{aligned}
$$

Thus we have

$$
(3.3) \qquad \widetilde{p}_t^{1-\theta} = \frac{1 - \omega (1+\pi_t)^{\theta-1} (1+\pi_{t-1})^{\chi(1-\theta)}}{(1-\omega)}
$$

We now want to again introduce a cost push shock, i.e. a shock increasing inflation given marginal costs. Due to the more complicated structure of the Phillips Curve, instead of subtracting u_t from π_t, we instead add $u_t * \mu^{-1} \frac{\omega}{1-\omega} * \frac{1+\chi\beta}{1-\omega\beta}$ to mc_t in (3.1) and (3.2). This implies that up to first order approximation, we have a Phillips curve with a cost push shock added on the right hand side, just as in chapter two.

The resource cost induced by the inefficient price dispersion present in the Calvo model is captured by the variable S_t, which enters the recourse constraint in the following way

$$AD_t = S_t (C_t + I_t) \tag{3.4}$$

$$S_t = (1-\omega) \cdot \tilde{p}_t^{-\theta} + \omega \left(\frac{1+\pi_t}{(1+\pi_{t-1})^{\chi}} \right)^{\theta} S_{t-1} \tag{3.5}$$

as shown by Schmitt-Grohe and Uribe (2005).[3] S_t is always greater than or equal to one. It increases the recourse costs of a given amount of consumption and investment goods.

For future reference, it will be useful to substitute (3.3) into (3.5), then take a second order approximation of the resulting expression and the unconditional expectation thereof. This (as is shown in the appendix B.1) yields

$$ES_t = 1 + \frac{\omega\theta}{(1-\omega)^2} \left[\frac{1}{2} \left[1+\chi^2 \right] - \chi AC (d\pi_t) \right] E (d\pi_t)^2 \tag{3.6}$$

where $AC(d\pi_t)$ and $E(d\pi_t)^2$ denote the autocorrelation and variance of inflation, respectively. Thus price dispersion depends positively on the variance of inflation, as the term in the outer brackets is always positive $AC(d\pi_t) < 1$ and $\chi \leq 1$.[4] If there is some indexing ($\chi > 0$), it also depends negatively on the autocorrelation of inflation. Note also that the effect of the variance of inflation increases in χ. Note that an increase in the degree of nominal rigidity ω increases the effect of the inflation variance on mean price dispersion. If prices remain fixed for longer on average, they will revert

[3] For a derivation of the law of motion for S_t see Schmitt-Grohe and Uribe (2005), pp. 18-19.

[4] The expression in brackets clearly decreases in $AC(d\pi_t)$. If we set $AC(d\pi_t) = 1$, the term in brackets becomes $\frac{1}{2}\left[1+\chi^2\right] - \chi$, which will be zero if $\chi^2 - 2\chi + 1 = 0$.The single solution to this equation is $\chi = 1$. For $0 \leq \chi \leq 1$, $\frac{1}{2}\left[1+\chi^2\right] - \chi$ will be larger than or equal to zero.

less quickly to whatever the average price level turns out to be in the future. Hence an increase in inflation in a given period will have a stronger effect on price dispersion than it otherwise would.

We set $\omega = 0.67$, implying that firms re-optimise their prices about every three quarters, and $\chi = 1$, implying that non-optimising firms index their prices to past inflation.

3.2. Transaction Cost

As a deviation from the baseline, we will introduce a transaction cost for consumption. Following Schmitt Grohe and Uribe (2005), consumers have to pay a cost $l(v_t)$, where $v_t = \frac{C_t}{m_t^h}$ and m_t^h denotes real money holdings by the household, and $l'(v_t)$, $l''(v_t) > 0$. Thus the household faces higher transaction costs if it increases its purchases for a given amount of money holdings in period t, and the costs increase at an increasing rate. We also follow Schmitt Grohe and Uribe in adopting the following functional form:

$$l(v_t) = \phi_1 v_t + \phi_2 / v_t - 2\sqrt{\phi_1 \phi_2}, \phi_1, \phi_2 > 0 \tag{3.7}$$

One of the great advantages of this functional form is that the resulting money demand function can be quite accurately approximated up to second order over a relevant interest rate interval.

With the transaction technology summarised by (3.7), and all other assumptions about the household remaining the same, the representative household now faces the problem to maximise

$$U = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[u(C_{t+i} - hab_{t+i-1}) - (n_{t+i} - \overline{n})\, G(e_{t+i}) \right] \right\}, u\prime > 0,\ u\prime\prime < 0$$

$$G\left(e_t(j)\right) = \left(e_t(j) - \left(\begin{array}{c} \phi_0 + \phi_1 \log w_t(j) + \phi_2 (n_t - \overline{n}) + \phi_3 \log w_t \\ +\phi_4 \log w_{t-1} - \phi_5 \log b_t - \phi_8 \log\left(Y_{t-1} / \left(n_{t-1} - \overline{n} - n^s\right)\right) \end{array} \right) \right)^2$$

$$s.t.\ \ (n_t - \overline{n})\, w_t + r_t^k K_t + \frac{B_{t-1}}{P_t}(1 + i_{t-1}) + F_t + m_{t-1}^h \frac{P_{t-1}}{P_t}$$

$$\geq C_t \left(1 + l\left(\frac{C_t}{m_t^h} \right) \right) + I_t \ + \frac{B_t}{P_t} + T_t \ + m_t^h \ and$$

$$l(\frac{C_t}{m_t^h}) = \phi_1 + \phi_2 / v_t - 2\sqrt{\phi_1 \phi_2}, \phi_1, \phi_2 > 0$$

$$K_{t+1} = (1 - \delta)\, K_t + I_t \left(1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - (1 + g) \right)^2 \right)$$

Note how the budget constraint is being modified by the introduction of the transaction cost. Households have to pay the transaction cost for each unit of consumption. If they want to reduce the transaction cost, they have to hold money m_t^h, implying that they are going to have less wealth to allocate towards investment or bonds.

Maximising utility yields the following first order conditions for consumption, bonds and money (the first order conditions for capital, investment and effort are unaffected), assuming that $u(C_{t+i} - hab_{t+i-1}) = \ln\left(C_{t+i} - hab_{t+i-1}\right):$

$$\frac{1}{C_t - hab_{t-1}} = \lambda_t \left(1 + l(v_t) + \frac{C_t}{m_t^h} l'\left(v_t\right) \right) \tag{3.8}$$

$$1 = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t \left(1 + \pi_{t+1}\right)} \right) (1 + i_t) \tag{3.9}$$

$$v_t^2 l'\left(v_t\right) = 1 - \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t \left(1 + \pi_{t+1}\right)} \right) \tag{3.10}$$

Combining (3.9) and (3.10) yields the demand for money:

$$v_t^2 l'(v_t) = 1 - \frac{1}{1+i_t} \tag{3.11}$$

or, applying our assumed functional form for $l(v_t)$

$$v_t^2 = \frac{\phi_2}{\phi_1} + \frac{1}{\phi_1}\frac{i_t}{1+i_t} \tag{3.12}$$

Note that v_t is increasing in the interest rate: As the yields on bonds increases, holding money becomes more costly relative to the increase in transaction costs associated with reducing money holdings. Thus the household holds less money. Note also that a higher inflation rate, to the extent that it implies a higher nominal interest rate i_t, will also tend to lower money holdings. As this increases $l(v_t)$, the introduction of transaction costs thus creates another reason why inflation is costly. This will provide a useful check of our baseline results.

Given that consumption now comes with a transaction cost, we also have to modify the aggregate demand equation, which now reads

$$AD_t = S_t\left(C_t\left(1 + l\left(\frac{C_t}{m_t^h}\right)\right) + I_t\right)$$

Although the transaction cost thus affects aggregate demand, it turns out that it has in fact a negligible effect on the second moments and impulse responses.

We will calibrate the coefficients ϕ_1 and ϕ_2 as follows: (3.12) implies the following demand for log money $\ln\left(m_t^h\right) = \ln\left(C_t\right) - \frac{1}{2}\ln\left(\frac{\phi_2}{\phi_1} + \frac{1}{\phi_1}\frac{i_t}{1+i_t}\right)$. Hence around the steady state, the annualised semi elasticity of money demand, $\varepsilon_{mi} = \frac{1}{4}\frac{d\ln\left(m_t^h\right)}{di_t}$, is given by

$$\varepsilon_{mi} = -\frac{1}{8}\frac{1}{\phi_2\left(1+\bar{i}\right)^2 + \left(1+\bar{i}\right)\bar{i}}$$

We calibrate ε_{mi} to an empirical estimate of a M1 log money demand function on German data by Clausen (1998) using short term interest rates and output as the arguments. We would of course prefer an estimate based on a specification including consumption but as far as we are aware these estimates do not exist in the empirical literature. Clausens estimate of ε_{mi} is -2.93, but we also check our results against an estimate of Luetekepohl et al (1999), who estimate ε_{mi} to be -5.11.[5] Using our estimate for ε_{mi}, we can back out ϕ_2. We then calibrate ϕ_1 from (3.12) using the average household money to consumption ratio for Germany from 1970q1 to 1990q4. The average of v_t is obtained by calculating the average of $C_tP_t/M1_t$ over this time period, and further assuming that households hold a fixed share of M1. We calibrate this share to equal 0.54 as this was the average annual share in Germany from 1970 to 1990.[6]

We show in appendix B.2 to this chapter that a more negative ε_{mi} implies a larger increase in transaction costs as a response to higher interest rates, i.e. $\frac{\partial^2 l_t}{\partial i_t \partial \varepsilon_{mi}} < 0$. This is because if ε_{mi} is more negative, people will reduce their money balances by more in response to an increase in the nominal interest rate, and thus transaction costs will increase by more.[7]

[5] See Clausen (1998), p. 737, and Luetkephol et al., (1999), p. 516. A problem with these estimates is that the underlying equations are misspecified for our purpose because, like all empirically estimated money demand equations, use output rather than consumption in their equation.

[6] We are grateful to DEUTSCHE BUNDESBANK- Kommunikation for emailing us the relevant data.

[7] That $\frac{\partial^2 l_t}{\partial i_t \partial \varepsilon_{mi}} < 0$ is not as self evident as it may seem. There are two effects at work here: On the one hand, a more negative value of ε_{mi} means that as the nominal interest rate increases, the consumption money ratio v_t increase by more thus implying a higher rise in transaction cost. The countervailing effect stems from the fact that the calibration method backs out the transaction cost parameters (ϕ_1

3.3. Measuring Welfare

Our welfare measure is going to be a quadratic approximation to the representative households utility. However, the endogenous growth sticky price model has multiple distortions rendering the steady state inefficient. As shown by Sutherland (2002) and Woodford (2003), an inefficient steady state implies that the second order approximation to the households welfare would feature a term linear in one or more of the model's variables. The recursive solution describing the path of these variables needs to be second order accurate as well to render the approximation to the household's welfare second order accurate. Under a first order approximation to the recursive solution of the model, we would ignore certain second order terms belonging to the second order accurate approximation to the household's welfare.[8]

To address this issue we use a second order approximation to the model's solution following Schmitt-Grohe and Uribe (2004a). Our welfare measure is the households expected utility, conditional on the state of the economy being the non-stochastic steady state in period 0. This implies that the stochastic shocks are equal to zero as well. Schmitt Grohe and Uribe (2004c) detail how to compute welfare for a given policy under these assumptions.[9] As in this paper, we are using the solution method of Schmitt Grohe and Uribe (2004a) as implemented in the software Dynare, we will use a somewhat different notation to illustrate their approach.

and ϕ_2) such that the money demand function implies a given ε_{mi}. Both ϕ_1 and ϕ_2 will be lower if ε_{mi} is more negative. As can be seen from (3.7) , the effect of this on $l^{'}(v_t)$, is ambiguous. Hence a higher ε_{mi} might decrease the cost of increasing v_t. The net effect on $\frac{\partial l_t}{\partial i_t}$ would be ambiguous.

[8]See Sutherland (2002), p. 5, and Woodford (2003), pp. 383-388.

[9]See Schmitt-Grohe and Uribe (2004c), pp.19 and pp. 28-30.

The equilibrium conditions of most rational expectation models can be written in the following fashion:

$$\begin{aligned} E_t\left[f\left(y_{t+1}, y_t, y_{t-1}, u_t\right)\right] &= 0 \\ u_t = \sigma \varepsilon_t & \\ E_t\left(\varepsilon_t \varepsilon_t^{'}\right) = \Sigma_\varepsilon & \\ E_t\left(\varepsilon_t \varepsilon_{t+i}^{'}\right) &= 0, i \neq 0 \end{aligned}$$

In this notation, y denotes a vector of endogenous variables, ε_t a vector of mean zero and variance one random variables which might be intratemporary correlated. All predetermined, or state variables are denoted with a t-1 subscript.

The non explosive recursive solution to this set of equations, if it exists, can be written as

$$y_t = g\left(y_{t-1}, u_t, \sigma\right) \tag{3.13}$$

In our model, the endogenous variables are $F_t = \frac{Y_t}{K_t}$, $D_t = \frac{C_t}{K_t}$, $R_t = \frac{I_t}{K_t}$, n_t, $g_t^K = \frac{K_{t+1}}{K_t}$, π_t, S_t, i_t, $H_t = \frac{w_t}{K_t}$, r_t^k, mc_t, q_t, G_t, M_t, $\Lambda_t = \frac{\lambda_t}{K_t}$ and $\widetilde{V}_t$, our stationarised welfare measure, to be defined below in more detail. Among those, the variables which enter the equilibrium conditions in a predetermined fashion and thus would form part of y_{t-1} are $D_{t-1}, R_{t-1,}g_t^K, \pi_{t-1}, s_{t-1}, i_{t-1}, H_{t-1}$. We have a single stochastic shock in our model denoted by u_t, which is not auto correlated.

We measure welfare using the households utility function. Utility maximisation implies that the disutility of effort is always zero[10], so that the relevant welfare measure is

$$
\begin{aligned}
(3.15) \qquad V_0 &= E_0\left[\sum_{i=0}^{\infty}\beta^t \ln\left(\widetilde{C}_t\right)\right] \\
\widetilde{C}_t &= C_t - jC_{t-1}
\end{aligned}
$$

Since we are dealing with a growth model, $C_t, \widetilde{C}_t$ and V_t will all be trended. Thus we will have to rewrite (3.15) to contain only stationary variables. Furthermore, we would like to express welfare in a difference equation which we can add to the other difference equations to jointly solve them. We proceed as follows:

$$
\begin{aligned}
V_0 &= E_0\left[\sum_{t=0}^{\infty}\beta^t \ln\left(\widetilde{C}_{-1}\prod_{s=0}^{t}\left(1+g_s^{\widetilde{c}}\right)\right)\right] = \frac{\ln\left(\widetilde{C}_{0-1}\right)}{1-\beta} + E_0\left[\sum_{t=0}^{\infty}\beta^i \ln\left(\prod_{s=0}^{t}\left(1+g_s^{\widetilde{c}}\right)\right)\right] \\
\frac{\widetilde{C}_t}{\widetilde{C}_{t-1}} &= 1+g_t^{\widetilde{c}}
\end{aligned}
$$

Note that $g_t^{\widetilde{c}}$ is a stationary variable and that $\widetilde{C}_{0-1}$ is independent of a policy which is implemented in period t=0. Note also that if we move $\frac{\ln\left(\widetilde{C}_{0-1}\right)}{1-\beta}$ to the other side, the right hand side of the equation will be stationary. We define $\widetilde{V}_0 = V_0 - \frac{\ln\left(\widetilde{C}_{-1}\right)}{1-\beta}$ and write $\widetilde{V}_0 =$ $V_0 - \frac{\ln\left(\widetilde{C}_{-1}\right)}{1-\beta} = E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\sum_{s=0}^{t}\ln\left(1+g_s^{\widetilde{c}}\right)\right)\right] = \frac{\ln\left(1+g_0^{\widetilde{c}}\right)}{1-\beta} + E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\sum_{s=1}^{t}\ln\left(1+g_s^{\widetilde{c}}\right)\right)\right]$
$= \frac{\ln\left(1+g_0^{\widetilde{c}}\right)}{1-\beta} + E_0\left[\sum_{i=0}^{\infty}\beta^t\left(\sum_{s=0}^{t}\ln\left(1+g_{s+1}^{\widetilde{c}}\right)\right)\right] = \frac{\ln\left(1+g_0^{\widetilde{c}}\right)}{1-\beta} + \beta E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\sum_{s=0}^{t}\ln\left(1+g_{s+1}^{\widetilde{c}}\right)\right)\right]$
$= \frac{\ln\left(1+g_0^{\widetilde{c}}\right)}{1-\beta} + \beta E_0\widetilde{V}_1$

[10]This is because the optimal effort level is given by

$$
\begin{aligned}
& e_t(j) = \phi_0 + \phi_1 \log w_t(j) + \phi_2\left(n_t - \overline{n}\right) + \phi_3 \log w_t \\
(3.14) \qquad & +\phi_4 \log w_{t-1} - \phi_5 \log b_t - \phi_8\left(Y_{t-1}/\left(n_{t-1} - \overline{n} - n^s\right)\right)
\end{aligned}
$$

Thus we have expressed welfare in period 0 by means of a forward looking difference equation featuring only stationary variables. $\widetilde{V}_t$ is now a variable in our model, solving the model will yield a recursive equation expressing it as function of the state vector and the stochastic shock:

$$\widetilde{V}_t = g\left(y_{t-1}, u_t, \sigma\right)$$

A second order approximation to g yields

$$\widetilde{V}_t \approx \widetilde{V} + g_y dy_{t-1} + g_u u_t + \frac{g_{\sigma\sigma}}{2}\sigma^2 + \frac{1}{2} g_{yy} dy'_{t-1} dy_{t-1} + \frac{1}{2} g_{yu} u'_{t-1} dy_{t-1} + \frac{1}{2} g_{uu} u_t^2$$

[11],where dy_t denotes the deviation of a variable from its steady state y, while $\widetilde{V}$ denotes the non-stochastic steady state level of welfare, and all partial derivatives are evaluated at the non-stochastic steady state. We are interested in expected welfare at t=0, i.e. $\widetilde{V}_0$, which is a function of y_{-1} and u_0. Moreover, as mentioned above, we are interested in $\widetilde{V}_0$ conditional on the initial state being the non stochastic steady state, i.e. $dy_{-1} = 0$ and $u_0 = 0$. Our welfare measure is accordingly written as

$$\widetilde{V}_0\left(y_{-1} = y, u_0 = 0, \sigma\right) \approx \widetilde{V} + \frac{g_{\sigma\sigma}}{2}\sigma^2 \tag{3.16}$$

where the first term is independent of policy in our model. We perform a grid search over various interest feedback rule coefficients in order to find the coefficient vector maximising $\widetilde{V}_0\left(y_{-1} = y, u_0 = 0\right)$.

Following Schmitt-Grohe and Uribe (2004c), we can also compute the welfare costs of different policies, and express them as a percentage of consumption under the optimal

[11]Schmitt Grohe and Uribe (2004a) show that $g_\sigma$and $g_{y\sigma}$ are equal to zero. See Grohe and Uribe (2004), p. 763.The second order approximation to a stochastic model differs from its non-stochastic counterpart only in the constant $\frac{1}{2} g_{\sigma\sigma}\sigma\sigma$.

policy. Let V_0^r denote welfare under the optimal, or reference policy regime, while V_0^a denotes welfare under an alternative policy regime. From (3.15), we have

$$V_0^r = E_0\left[\sum_{i=0}^{\infty}\beta^t \ln\left(C_t^r - jC_{t-1}^r\right)\right] \tag{3.17}$$

$$V_0^a = E_0\left[\sum_{i=0}^{\infty}\beta^t \ln\left(C_t^a - jC_{t-1}^a\right)\right] \tag{3.18}$$

It is useful to rewrite (3.17) as

$$V_0^r = \ln\left(C_0^r - jC_{-1}\right) + E_0\left[\sum_{t=1}^{\infty}\beta^t \ln\left(C_t^r - jC_{t-1}^r\right)\right]$$

noting that C_{-1} is independent of policy. Let λ denote the percentage of consumption we will have to take away from consumers from 0 to infinity under the optimal policy regime to make them as bad off as under the alternative one. It must then hold that

$$\begin{aligned} V_0^a &= \ln\left(\left(1-\lambda\right)C_0^r - jC_{-1}\right) + E_0\left[\sum_{t=1}^{\infty}\beta^t \ln\left(\left(1-\lambda\right)\left(C_t^r - jC_{t-1}^r\right)\right)\right] \\ &= \ln\left(\left(1-\lambda\right)C_0^r - jC_{-1}\right) + \frac{\beta\ln\left(1-\lambda\right)}{1-\beta} + E_0\left[\sum_{t=1}^{\infty}\beta^t \ln\left(C_t^r - jC_{t-1}^r\right)\right] \end{aligned}$$

We can thus write

$$\begin{aligned} V_0^r - V_0^a &= \ln\left(C_0^r - jC_{-1}\right) - \ln\left(\left(1-\lambda\right)C_0^r - jC_{-1}\right) - \frac{\beta\ln\left(1-\lambda\right)}{1-\beta} \\ &= \ln\left(\frac{C_0^r - jC_{-1}}{\left(1-\lambda\right)C_0^r - jC_{-1}}\right) - \frac{\beta\ln\left(1-\lambda\right)}{1-\beta} \end{aligned}$$

As before, we would like to replace all trended variables by stationary variables. Under our assumption that C_{-1} is the same under all policy regimes, we have $V_0^r - V_0^a =$

$\widetilde{V}_0^r - \widetilde{V}_0^a$. We thus have

$$\text{(3.19)}\quad \widetilde{V}_0^r - \widetilde{V}_0^a = \ln\left(\frac{\left(1+g_0^{c^r}\right)-j}{\left(1+g_0^{c^r}\right)(1-\lambda)-j}\right) - \frac{\beta\ln(1-\lambda)}{1-\beta}$$

$$\text{(3.20)}\quad = \ln\left(\left(1+g_0^{c^r}\right)-j\right) - \ln\left(\left(1+g_0^{c^r}\right)(1-\lambda)-j\right) - \frac{\beta\ln(1-\lambda)}{1-\beta}$$

where $g_0^{c^r} = \frac{C_0^r}{C_{-1}} - 1$. We now take a second order approximation to λ. As $\widetilde{V}_0^r$, $\widetilde{V}_0^a$ and $g_0^{c^r}$ are all functions of the state variables, which we denote as $g^r(y_{t-1}, u_t, \sigma)$, $g^a(y_{t-1}, u_t, \sigma)$ and $g^{c^r}(y_{t-1}, u_t, \sigma)$, so will $\lambda : \lambda = g^\lambda(y_{t-1}, u_t, \sigma)$. Furthermore, we assume that the initial state is the non-stochastic steady state, so that we can write

$$\lambda_0(y_{-1} = y, u_0 = 0, \sigma) \approx \frac{g_{\sigma\sigma}^\lambda}{2}\sigma^2$$

as there are no welfare costs in the non-stochastic steady state. To find $g_{\sigma\sigma}^\lambda$ from (3.20), we first take the first derivative with respect to σ, which yields

$$g_\sigma^r - g_\sigma^a = \frac{g_\sigma^{g^{c^r}}}{1+g^{c^r}-j} - \left(\frac{g_\sigma^{g^{c^r}}(1-\lambda) - \left(1+g^{c^r}\right)g_\sigma^\lambda}{\left(1+g^{c^r}\right)(1-\lambda)-j}\right) + \frac{\beta g_\sigma^\lambda}{(1-\beta)(1-\lambda)}$$

Schmitt-Grohe and Uribe (2004a) show that the first derivative of the g function with respect to σ is zero. Thus $g_\sigma^r = g_\sigma^a = g_\sigma^{g^{c^r}} = 0$, and hence $g_\sigma^\lambda = 0$ as well. Taking the second derivative, and using this information, we have

$$g_{\sigma\sigma}^\lambda = \frac{g_{\sigma\sigma}^r - g_{\sigma\sigma}^a}{\frac{\beta}{1-\beta} + \frac{1+g}{1+g-j}}$$

and thus

$$\lambda_0(y_{-1} = y, u_0 = 0, \sigma) = \frac{g_{\sigma\sigma}^r - g_{\sigma\sigma}^a}{\frac{\beta}{1-\beta} + \frac{1+g}{1+g-j}}\frac{1}{2}\sigma^2$$

where g denotes the steady state growth rate of the economy. Substituting (3.16) and multiplying both sides by 100 yields the welfare cost of a the alternative policy as a percentage of the consumption stream under the optimal policy:

$$100\lambda_0 \left(y_{-1} = y, u_0 = 0, \sigma\right) = \frac{\widetilde{V}_0^r - \widetilde{V}_0^a}{\frac{\beta}{1-\beta} + \frac{1+g}{1+g-j}} 100$$

3.4. Monetary Policy

We maximise welfare by appropriately choosing the coefficients of the following simple interest feedback rule:

$$i_t = (1-\rho)\,\bar{i} + (1-\rho)\,\phi_\pi \pi_t + (1-\rho)\,\phi_{gap} gap_t + \rho i_{t-1} \tag{3.21}$$

We are searching over the following intervals: $\rho = [0, 0.8]$, step size 0.1, $\phi_\pi = [1.2, 8.2]$, step size 0.5 and $\phi_{gap} = [0, 8]$, step size 0.5. Note that we are restricting ourselves to policy rules which would be commonly expected to yield determinate results, which indeed turns out to be the case. Furthermore, we do not consider "pathological" rules featuring negative coefficients on either inflation or the output gap.

Other than the baseline non-policy calibration, we will also consider the case of no indexation to past inflation among non optimising price setters $(\varkappa = 0)$, a zero coefficient on the labour share in wage setting $(c = 0)$ and an increase of the probability ω that a firm can not change its price to 0.75 to see whether our results are robust. Furthermore, we will introduce a transaction cost for consumption purchases which creates a demand for money. This will increase the costs of inflation and will therefore provide a useful check for our results.

Finally, note that we are not considering the case of $\rho = 0.9$. Allowing for this, though, would not change the basic thrust of our results regarding the priorities of the central bank with respect to the stabilisation of the output gap versus the stabilisation of inflation. However, we have two reasons to distrust the accuracy of our welfare measure for the policies found optimal in this case. Firstly, we find that for the policy found optimal for the baseline, the second order accurate standard deviation of inflation differs significantly from the standard deviation calculated from a simulation of a second order accurate solution to the model (which amounts to a fourth order accurate approximation to the standard deviation). As our second order accurate approximation to welfare incorporates the effects of second order accurate second moments of the model, this sheds doubt on the accuracy of our welfare measure for this case. More specifically, as shown above, the mean degree of price dispersion, representing the costs of inflation, depends on the inflation variance. If our measurement of the inflation variance is biased downwards, so will the welfare costs of inflation.

Secondly, comparing welfare across the various deviations from the baseline, we find some counterintuitive results. For instance, we find that welfare levels under the respective optimal policy are lower for $\chi = 0$ and $c = 0$ than in the Baseline. But both a lower degree of indexation and lower real wage rigidity should facilitate the task of the central bank rather than make it more difficult. These issues disappear when we restrict ρ to equal 0.8 or less.

3.5. Aggregate Equations

For future reference, we will list the aggregate equations of the modifies model. Aggregate demand is given by

$$AD_t = S_t (C_t + I_t) \text{ [Baseline]} \tag{3.22}$$

$$AD_t = S_t \left(C_t \left(1 + l \left(\frac{C_t}{m_t^h} \right) \right) + I_t \right) \text{ [Transaction cost]} \tag{3.23}$$

where S_t evolves according to

$$S_t = (1-\omega) \cdot \tilde{p}_t^{-\theta} + \omega \left(\frac{1+\pi_t}{(1+\pi_{t-1})^{\chi}} \right)^{\theta} S_{t-1} \tag{3.24}$$

The marginal utility of consumption evolves according to

$$\begin{aligned} \frac{1}{C_t - hab_{t-1}} &= \lambda_t \text{ [Baseline]} \\ \frac{1}{C_t - hab_{t-1}} &= \lambda_t \left(1 + l(v_t) + \frac{C_t}{m_t^h} l'(v_t) \right) \text{ [Transaction cost]} \end{aligned} \tag{3.25}$$

and

$$1 = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t (1+\pi_{t+1})} \right) (1+i_t) \tag{3.26}$$

The level of habit is given by

$$hab_{t-1} = jC_{t-1} \tag{3.27}$$

Money demand, in the presence of transaction costs, is given by

$$v_t^2 = \frac{\phi_2}{\phi_1} + \frac{1}{\phi_1} \frac{i_t}{1+i_t} \text{ [Transaction cost]} \tag{3.28}$$

Investment expenditures is governed by the following equations:

$$\beta E_t \left(\lambda_{t+1} r_{t+1}^k + \lambda_{t+1} q_{t+1} (1-\delta) \right) = \lambda_t q_t \tag{3.29}$$

$$\lambda_t q_t \left[\left(1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - (1+g) \right)^2 \right) - \frac{I_t}{I_{t-1}} \kappa \left(\frac{I_t}{I_{t-1}} - (1+g) \right) \right] \tag{3.30}$$

$$+\beta E_t \left[\lambda_{t+1} q_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 \kappa \left(\frac{I_{t+1}}{I_t} - (1+g) \right) \right] = \lambda_t \tag{3.31}$$

while capital accumulation is given by

$$K_{t+1} = (1-\delta) K_t + I_t \left(1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - (1+g) \right)^2 \right) \tag{3.32}$$

The capital rental is given by

$$r_t^k = \alpha m c_t \frac{Y_t}{K_t} \tag{3.33}$$

In the equations that follow, total factor productivity will evolve according to

$$\begin{aligned} TFP_t &= (1+g)\, TFP_{t-1} \text{ [JLNE]} \\ TFP_t &= K_t \text{ [New Growth]} \end{aligned} \tag{3.34}$$

Marginal cost are given by

$$mc_t = \frac{\left(r_t^k \right)^\alpha w_t^{1-\alpha}}{A \alpha^\alpha (1-\alpha)^{1-\alpha} (\phi_1 TFP_t)^{1-\alpha}} \tag{3.35}$$

Wages are set according to

$$\log w_t - \log w_{t-1} = a + b * (n_t - \overline{n}) + c \log \left(\frac{w_{t-1} (n_{t-1} - \overline{n} - n^s)}{Y_{t-1}} \right) \tag{3.36}$$

$$b > 0, c < 0.$$

Total output is given by private sector output Y_t plus the output of state employees:

$$Output_t = AK_t{}^{\alpha}(TFP_t \phi_1 (n_t - \overline{n} - n^s))^{1-\alpha} + w_t n^s \text{ [JLNE]} \tag{3.37}$$

$$Output_t = A_t K_t ((n_t - \overline{n} - n^s) \phi_1)^{1-\alpha} + w_t n^s \text{ [New Growth]} \tag{3.38}$$

Markets clear:

$$AD_t = Output_t$$

The evolution of prices is determined by the equations for the two artificial variables G_t and M_t and the law of motion of the price index:

$$G_t = \theta \left(mc_t + u_t * \mu^{-1} \frac{\omega}{1-\omega} * \frac{1+\chi\beta}{1-\omega\beta} \right) \frac{Y_t}{C_t} \tag{3.39}$$

$$+\omega\beta (1+\pi_t)^{-\chi\theta} E_t \left((1+\pi_{t+1})^{\theta} G_{t+1} \right) \tag{3.40}$$

$$M_t = (\theta - 1) \frac{Y_t}{C_t}$$

$$+\omega\beta (1+\pi_t)^{\chi(1-\theta)} E_t \left((1+\pi_{t+1})^{\theta-1} M_{t+1} \right) \tag{3.41}$$

$$\left(\frac{G_t}{M_t} \right)^{1-\theta} = \frac{1 - \omega (1+\pi_t)^{\theta-1} (1+\pi_{t-1})^{\chi(1-\theta)}}{(1-\omega)}$$

The interest feedback rule is given by

$$i_t = (1-\rho) \bar{i} + (1-\rho) \phi_\pi \pi_t + (1-\rho) \phi_{gap} gap_t + \rho i_{t-1} \tag{3.42}$$

with

$$gp_t = \frac{Output_t - Output_t^n}{Output_t^n} \tag{3.43}$$

and natural output, natural employment and the natural wage being determined by

$$\begin{aligned}
\mu^{-1} &= \frac{\left(n_t^n - n^s - \overline{n}\right)^{\alpha} w_t^n}{A\left(1-\alpha\right)\left(\phi_1 TFP_t\right)^{1-\alpha} K_t^{\alpha}} \\
\log w_t^n - \log w_{t-1} &= a + b * \left(n_t^n - \overline{n}\right) + c \log\left(\frac{w_{t-1}\left(n_{t-1} - \overline{n} - n^s\right)}{Y_{t-1}}\right) \\
Output_t^n &= AK_t{}^{\alpha}(TFP_t\phi_1\left(n_t^n - n^s - \overline{n}\right))^{1-\alpha} + w_t^n n^s \qquad (3.44)
\end{aligned}$$

3.6. Illustration of the Tradeoffs between stabilising Inflation and stabilising real Variables

We will now illustrate how the tradeoffs between stabilising inflation and stabilising real variables differ between the JLN and the New Growth economy, referred to as "JLNE" and "NGE" in the figures and tables printed below. Let's turn first to the JLN economy. Figure 3.1 displays the impulse response functions of inflation for $\phi_\pi = 1.2$

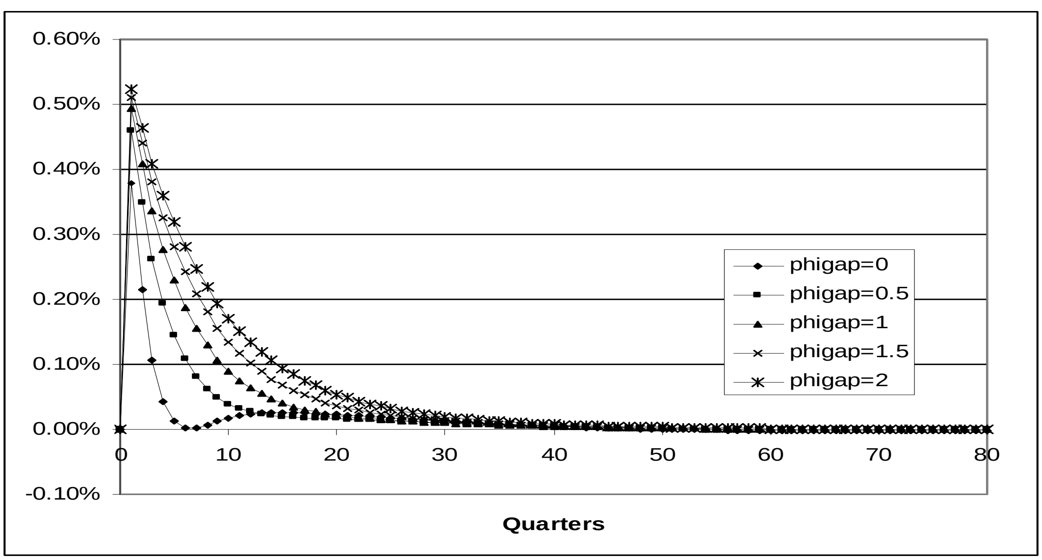

Figure 3.1. JLNE - Inflation for $\phi_\pi = 1.2$, $\rho = 0$, and various Values of ϕ_{gap}

and values of ϕ_{gap} between 0 and 2. Clearly, the smaller the output gap coefficient, the less inflation rises on impact and the faster it returns to zero. On-impact responses vary between 0.38% and 0.52%. While for $\phi_{gap} = 0$, inflation is back to target after 5 quarters, it takes about 40 quarters with $\phi_{gap} = 2$. Correspondingly, the less emphasis the central bank places on output gap stabilisation, the stronger employment falls on impact, as can be obtained from Figure 3.2. However, employment bounces back very quickly. After about 10 quarters, the distance to the steady state has shrunken to about 0.0189%. Indeed, it returns faster to the steady state than under the alternative policy rules which put a larger weight on the output gap. The sharp on-impact drop in

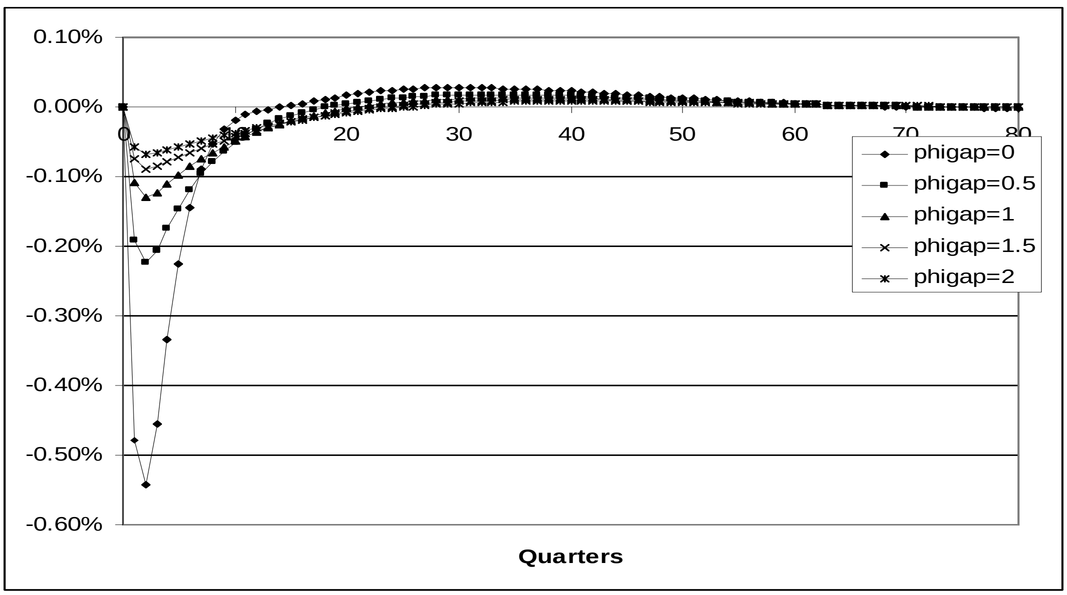

Figure 3.2. JLNE - Employment for $\phi_\pi = 1.2$, $\rho = 0$, and various Values of ϕ_{gap}

employment with $\phi_{gap} = 0$ means that real wage growth collapses and inflation is quickly forced out of the system, thus creating scope for lowering the real interest rate and increasing employment. By contrast, avoiding a large on impact drop in employment means that inflation remains high for longer, partly as a consequence indexing of non-re-optimised prices to past inflation. Therefore employment has to stay below its steady state somewhat longer in order to keep real wage growth muted. Correspondingly, figure 3.3 shows that habit adjusted consumption growth (the variable driving the household's welfare) drops more sharply the less the central bank reacts to output. However for all five rules, it returns to its steady state within about 4 years and then remains slightly above it for some time.

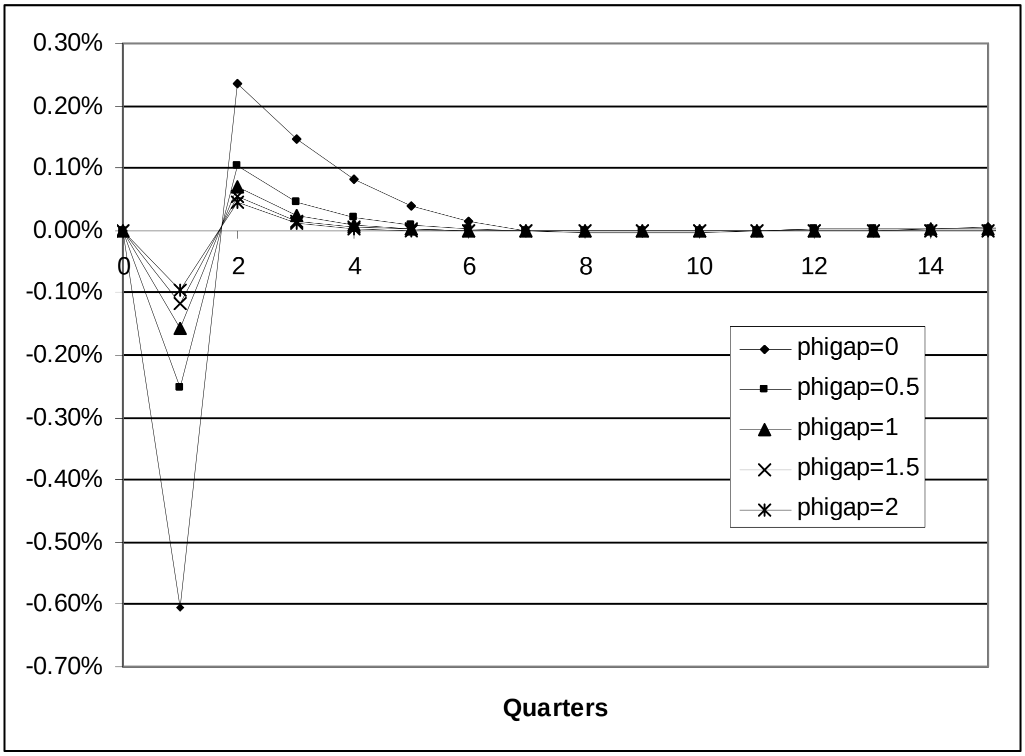

Figure 3.3. JLNE - $g_t^{\tilde{c}}$ for $\phi_\pi = 1.2$, $\rho = 0$, and various Values of ϕ_{gap}

This above trend growth rate brings consumption back to its pre-shock trajectory. Hence a more hawkish monetary policy response to a one time cost push shock cannot permanently lower consumption. This is not surprising since total factor productivity grows exogenously and thus the capital-to-effective labour ratio has declined after the recession, which increases the marginal product of capital. Hence capital accumulation stays above trend until the capital stock, output and thus consumption have returned to their pre-shock trajectories.

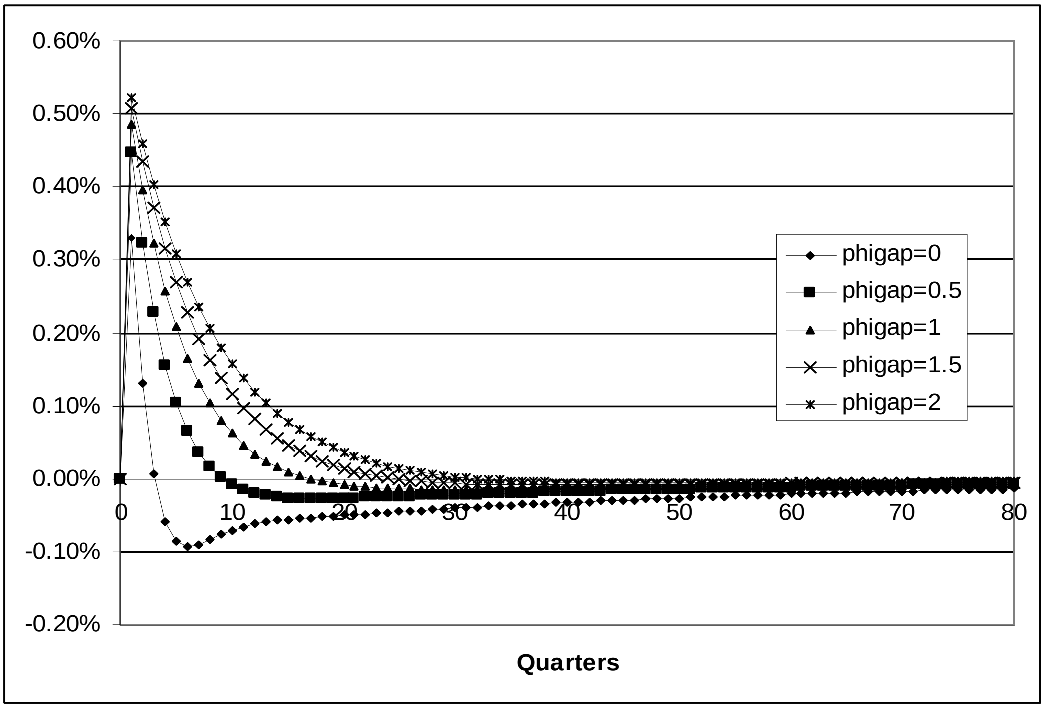

Figure 3.4. New Growth Economy - Inflation for $\phi_\pi = 1.2$, $\rho = 0$, and various Values of ϕ_{gap}

The results are very different for the New Growth economy. While Figure 3.4 shows that the initial increase and the persistence of inflation still increase in ϕ_{gap}, not responding to the output gap has substantially stronger real effects than in the JLN economy. From figure 3.5, it can be obtained that employment remains persistently below its steady state. Not only does employment decline more on impact than for instance with $\phi_{gap} = 1$ (the triangle), it also remains persistently lower for more than 15 years. Similarly, habit adjusted consumption falls a lot more if $\phi_{gap} = 0$ than for larger values of ϕ_{gap}, as can be obtained from figure 3.6. While it returns above its steady state for a couple of quarters, it then falls back below the steady state and remains there. It also remains below the path it would take if ϕ_{gap} would exceed zero.

This implies that after a one-off adverse shock, consumption is permanently lower under $\phi_{gap} = 0$ than if policy responds more strongly to output.

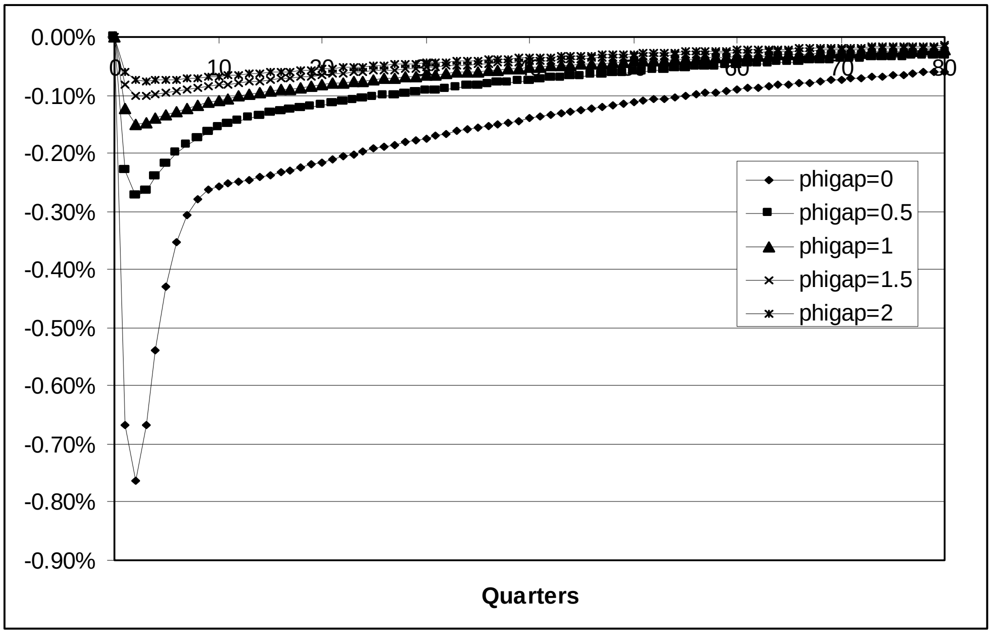

Figure 3.5. New Growth Economy - Employment for $\phi_\pi = 1.2$, $\rho = 0$, and various Values of ϕ_{gap}

The underlying reason for the permanent effect of monetary policy on the post-shock consumption path is that in the presence of endogenous growth, the capital stock and by implication output and consumption are fully path dependent. This is a well known property of growth models with an AK type production technology, i.e. a technology where output is linear in the producible input. If an adverse shock temporarily lowers capital stock growth by lowering the marginal product of capital and increasing the real interest rate, then the capital stock will never return to its pre-shock growth path - unless a sufficiently big favourable shock occurs. This is illustrated by figure 2.6 of chapter two: Note that capital stock growth stays persistently below its steady state value but never exceeds it, which would be necessary to move the capital stock back to its pre-shock trajectory. However, the central bank can influence how much and how persistently capital stock growth declines in response to the cost push shock. If it raises the interest rate very aggressively in spite of a negative output gap, the decline in capital stock growth will be larger and more persistent. Thus the distance of the

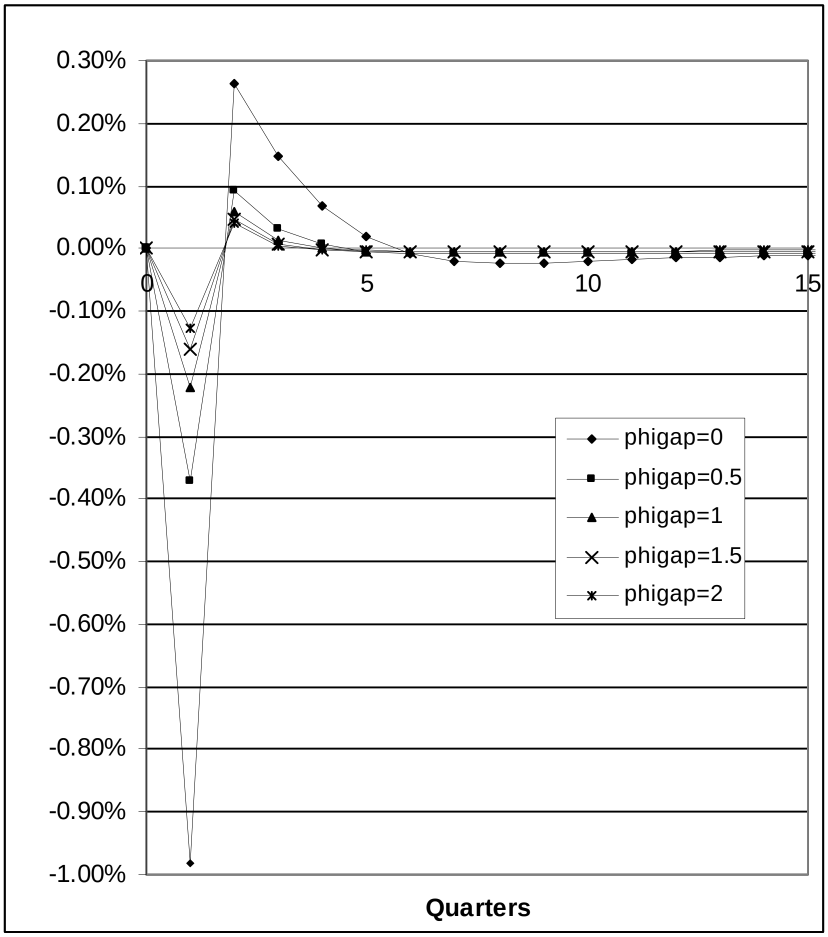

Figure 3.6. New Growth Economy - $g_t^{\tilde{c}}$ for $\phi_\pi = 1.2$, $\rho = 0$, and various Values of ϕ_{gap}

post-shock trajectory of the capital stock from the path the capital stock would have taken in the absence of the shock will be larger. The same will be true for the distance of the post-shock trajectory of consumption from the path consumption would have taken in the absence of the shock.

Obviously the trade-off between stabilising inflation and stabilising real variables is different in the New Growth economy: stabilising inflation implies much more volatility in employment and consumption growth than in the JLN economy. What is more, a failure to respond to the output gap and an aggressive response to inflation increase the uncertainty of the long run consumption path far more in the New Growth than in the JLN economy. This is because a cost push shock in the New Growth economy has a permanent effect on the future output and consumption trajectory, which will be amplified by a hawkish monetary policy. By contrast, in the JLN economy a hawkish monetary policy only increases the fluctuations of consumption around an exogenously determined trajectory.

3.7. Stabilisation Priorities in the Presence and Absence of Endogenous Growth

Based on the discussion in the previous section we would expect that in the New Growth economy, an optimising central bank will respond more strongly to the output gap and less to inflation than in the JLN economy in order to reduce the uncertainty of the future consumption path. Table 3.1 shows that this is indeed the case. It contains the optimal rules under the various scenarios. The first column shows the respective deviation from the baseline case. The first three columns display the optimal coefficients in the JLN economy, while the final three columns display the optimal coefficients under the endogenous growth economy. Obviously, the central bank is always, with the exception of $\varkappa = 0$, more hawkish in the JLN than in the New Growth economy. With the exception of the scenario where $\varkappa = 0$, the coefficient on inflation in the New Growth Economy is always smaller than the coefficient in the JLN economy, while the coefficient on the output gap is larger. Clearly, the central bank is aware that in the New Growth economy, a hawkish monetary policy amplifies the permanent effect of a cost push shock on consumption and therefore chooses to stabilise output.

Tables 3.2 and 3.3 further illustrate the policymakers motivation by displaying a couple of unconditional first second moments for the nominal variables and S_t (table 3.2) and a couple of real variables (table 3.3). In both tables, the first row shows the moments generated by the JLN economy with the policy rule optimal in this economy,

	JLNE			NGE		
Scenario	ϕ_π	ϕ_{gap}	ρ	ϕ_π	ϕ_{gap}	ρ
Baseline	6.7	1.5	0.8	1.2	3	0.8
$\varkappa = 0$	1.2	8	0.8	1.2	8	0.8
$c = 0$	8.2	2	0.8	1.2	4.5	0.8
$\omega = 0.75$	2.7	0.5	0.8	1.2	2	0.8
Money: $\varepsilon_{mi} = -5.11$	8.2	0	0	1.2	0.5	0.8
Money: $\varepsilon_{mi} = -2.93$	1.2	0	0.6	1.2	1	0.8

Table 3.1. Optimal Rules for the JLNE and the New Growth Economy

while the second row displays the moments generated by applying this rule to the New Growth economy. Finally, the third row shows the moments generated by applying the rule optimal in the New Growth economy to the New Growth economy.[12] Note that the moments of the nominal variables S_t and shown in table 3.2 are remarkably similar across the first two rows: The policy rule optimal in the JLN economy induces very similar behaviour of inflation, price dispersion, and the nominal interest rate in both the New Growth economy and the JLN economy. By contrast, this policy causes substantially higher volatility of habit adjusted consumption growth $g_t^{\widetilde{c}}$. This can be seen from column two of table 3.3. Under the policy optimal in the JLN economy, the standard deviation of $g_t^{\widetilde{c}}$ is about 0.17% (or 0.68% at an annualised rate) higher in the New Growth economy than in the JLN economy. The standard deviation of employment is higher by 0.69%.

By contrast, the policy optimal in the New Growth economy reduces the standard deviation of $g_t^{\widetilde{c}}$ far below its value in the JLN economy. The likely reason for this was discussed in the previous section. In the New Growth economy, a given deviation of $g_t^{\widetilde{c}}$ from its steady state will be associated with a permanently different consumption trajectory (unless a future shock moves the economy in the opposite direction). Thus a

[12] All the second moments are based on a first order approximation to the solution of the model as this will yield a second order accurate approximation to the second moments of the model. As we are taking a second order accurate solution to welfare, this is the welfare relevant way to measure the second moments.

given standard deviation of habit adjusted consumption growth generates more uncertainty regarding the future path of consumption in the New Growth than in the JLN economy. The policy maker chooses to reduce this uncertainty and thus stabilises $g_t^{\tilde{c}}$ far more than in the JLN economy. He also strongly reduces its autocorrelation in order to make sure that a given deviation of habit adjusted consumption growth from its trend does not persist.

Correspondingly, the optimal policy in the New Growth economy puts a much smaller priority on stabilising inflation. The mean inflation rate is 1.27% per quarter, or about 5.1% at an annualised rate. More important for welfare considerations, the standard deviation of inflation is about 2.5 times as large as in the JLN economy, which tends to increase mean price dispersion $\mathbf{ES}_t$, as can be obtained from equation (3.6) . The autocorrelation of inflation also increases when moving to the policy optimal in the New Growth economy, which will tend to lower $\mathbf{ES}_t$, but this effect is dominated by the increase in $sd.\ \pi_t$.[13] Thus price dispersion is also higher, namely 1.0006 as opposed to 1.0003 under the rule optimal for the JLN economy.

The welfare cost of applying the policy optimal in the JLN economy to the New Growth economy, shown in the final column, amounts to 0.36% of consumption under the optimal policy. Clearly, a policy optimal in an economy in which the monetary policy response to a cost push shock has only short lived effects on consumption can be quite costly in an endogenous growth economy where the monetary policy response to a shock has permanent effects on the consumption trajectory.

Correspondingly, the response of inflation to a one standard deviation shock in the New Growth economy is much stronger and more persistent under the optimal rule

[13] Note that equation (3.6) features the variance of inflation, implying that. $sd\ \pi_t$ enters $E_t S_t$ with a square.

Scenario	$\mathbf{E}\boldsymbol{\pi}_t$	*sd.* π_t	$\mathbf{AC}\boldsymbol{\pi}_t$	$\mathbf{ES}_t$	*sd.* i_t	$\mathbf{ACi}_t$
JLNE, $[6.7, 1.5, 0.8]$	0.0001	0.0041	0.47	1.0003	0.0048	0.71
NGE, $[6.7, 1.5, 0.8]$	0.0001	0.004	0.46	1.0003	0.0042	0.68
NGE, $[1.2, 3, 0.8]$	0.0127	0.0101	0.84	1.0006	0.0071	0.93

Table 3.2. Selected Moments of nominal Variables

Scenario	*sd.* $g_t^{\widetilde{c}}$	$\mathbf{ACg}_t^{\widetilde{c}}$	*sd.* n_t	$\mathbf{ACn}_t$	**Welfare cost**
JLNE, $[6.7, 1.5, 0.8]$	0.0074	-0.0191	0.0115	0.84	0%
NGE, $[6.7, 1.5, 0.8]$	0.0091	-0.058	0.0184	0.93	0.36%
NGE, $[1.2, 3, 0.8]$	0.0037	-0.32	0.004	0.84	0%

Table 3.3. Selected Moments of real Variables

than under the rule optimal for the JLN economy, as can be obtained from figure 3.7. Inflation increases by 0.56% on impact and then decreases only gradually. After 10 quarters, it is still 0.07% above its steady state. It becomes negative after 22 quarters. Under the rule optimal for the JLN economy, inflation increases on impact by mere 0.35%, and returns to zero after 4 quarters. Turning to the real variables, under the optimal rule, employment actually increases on impact and then starts declining, as can be obtained from figure 3.8. Its maximum distance to the steady state is about 0.1% , which is reached after about 5 quarters. By contrast, under the rule optimal for the JLN economy, employment falls by 0.61% on impact, declines further and then recovers until after about nine quarters, the recovery slows down. Employment remains substantially below its value under the optimal policy. Habit adjusted consumption growth, which is displayed in figure 3.9, falls by about 0.55% on impact for the policy optimal in the JLN economy, while it increases by 0.15% under the optimal policy. In quarter 3, consumption growth under the policy optimal for the JLN economy actually increases above it's value for the optimal policy, but only for four quarters. After that, consumption growth under the policy optimal for the JLN economy remains persistently below consumption growth under the optimal policy, although that is not easily obtainable from the figure.

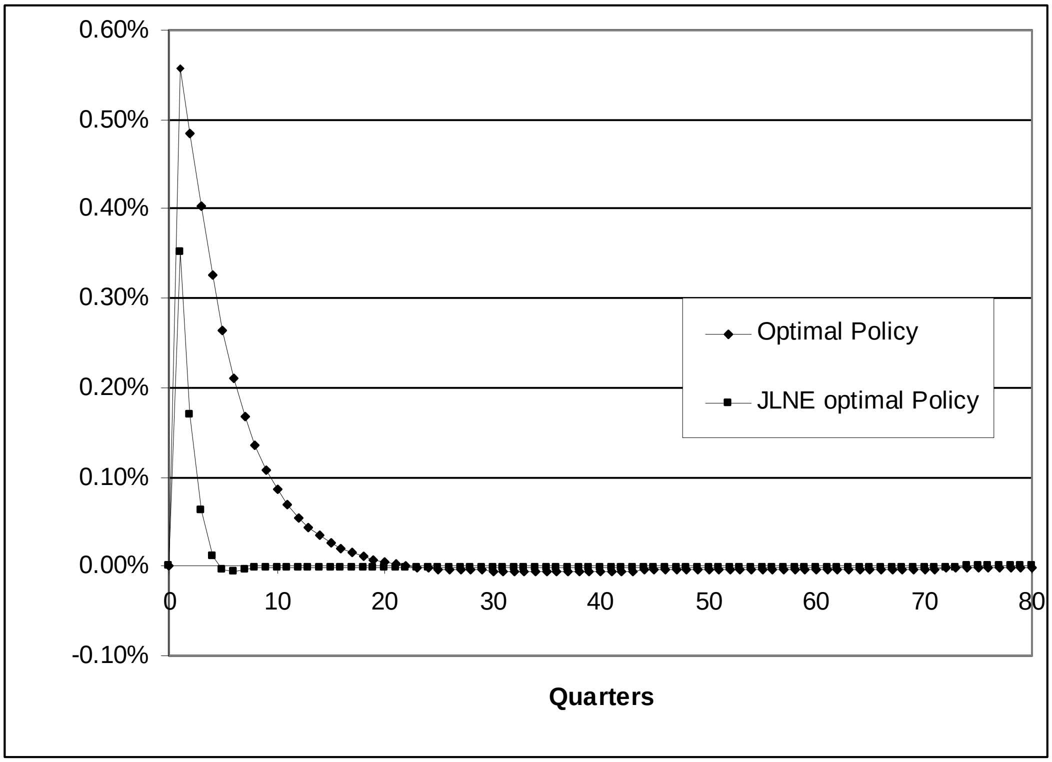

Figure 3.7. NGE - Inflation for the optimal Policy and the JLNE optimal Policy

We now want to gain some intuition for how the deviations from the baseline scenario affect optimal policy within the New Growth economy. If indexation is removed $(\varkappa = \mathbf{0})$, inflation today will have no effect in itself on inflation tomorrow and thus is easier to stabilise. Hence the central bank focuses more on stabilising output.[14] As can be obtained from the second row of table 3.4, even with this stronger emphasis on stabilising output, the autocorrelation coefficient of inflation collapses as compared to the baseline case, as do its mean and standard deviation, and the mean of S_t.

Concerning the case of $c = 0$, this will make wages a lot more flexible, implying that a given reduction in output and employment will do more to reduce inflation. This reduces the increase in inflation volatility associated with an increase in the output gap coefficient and thus induces a somewhat stronger focus on stabilising output.

[14]This is indeed a result which can be shown within a simple New Keynesian three equation model featuring a hybrid Phillips curve plus the appropriate welfare measure.

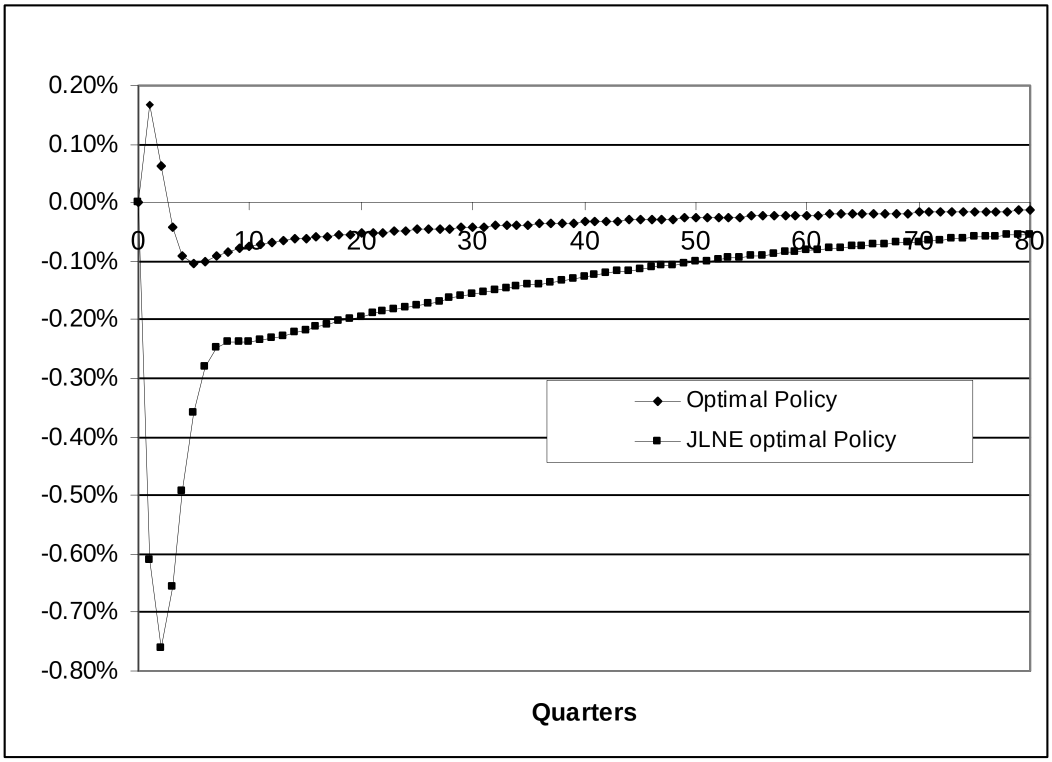

Figure 3.8. Employment for the optimal Policy and the JLNE optimal Policy

By contrast, increasing the probability that a firm cannot change its price ω to 0.75 instead of 0.67 means that a larger reduction in marginal cost and thus in output is required to reduce inflation by a given amount. Furthermore, as obvious from (3.6), a given amount of inflation variance also creates more mean price dispersion if ω is higher. Hence the optimal output gap coefficient is lower than in the baseline. Nevertheless, the standard deviation of inflation and mean price dispersion are still substantially higher with $\omega = 0.75$ than in the baseline, as can be obtained from table 3.4, as is the standard deviation of $g_t^{\tilde{c}}$ (see table 3.5).

Turning to the scenario with the transaction cost technology, we observe that the optimal rule features a much smaller coefficient on output than in the baseline, 0.5 in the case of $\varepsilon_{mi} = -5.11$ and 1 in the case of $\varepsilon_{mi} = -2.93$. A stronger emphasis on output gap stabilisation would imply higher mean inflation and thus a higher nominal

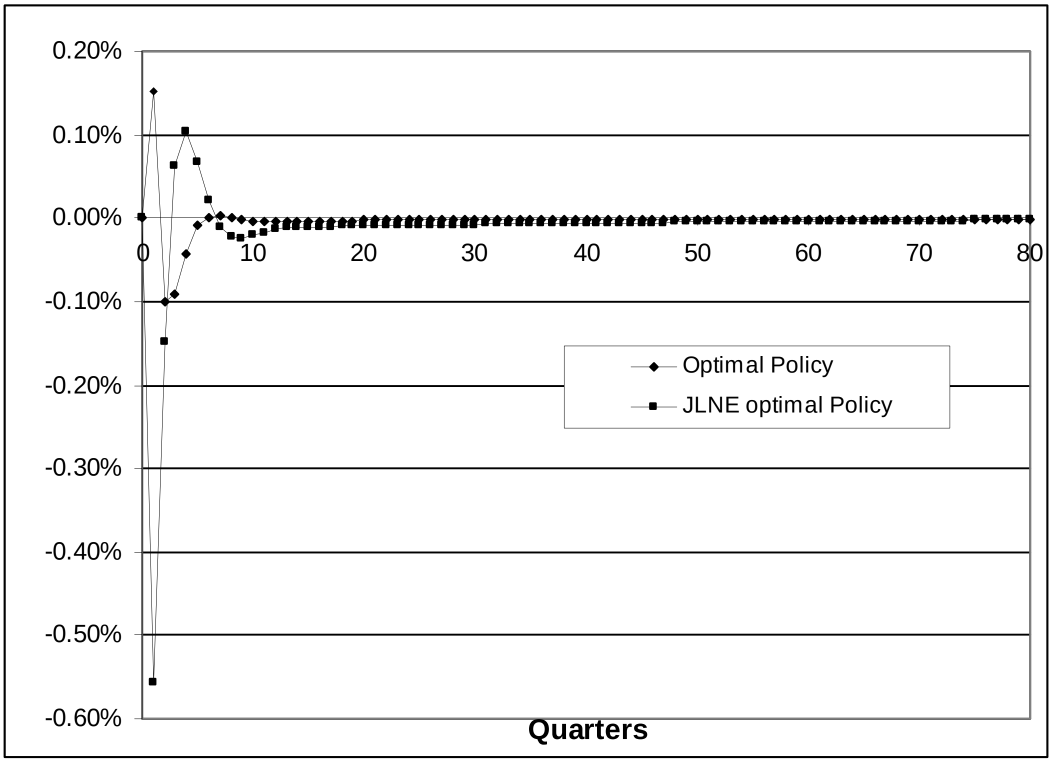

Figure 3.9. Habit adjusted Consumption Growth for the optimal Policy and the JLNE optimal Policy

Scenario	$\mathbf{E}\boldsymbol{\pi}_t$	$sd.\ \pi_t$	$\mathbf{AC}\boldsymbol{\pi}_t$	$\mathbf{ES}_t$	$\mathbf{Ei}_t$	$sd.\ i_t$	$\mathbf{ACi}_t$
Baseline	0.0127	0.0101	0.84	1.0006	0.0313	0.0071	0.93
$\varkappa = 0$	0.002	0.003	-0.01	1.0002	0.0207	0.0003	0.14
$c = 0$	0.0206	0.0118	0.88	1.0006	0.0395	0.0094	0.94
$\omega = 0.75$	0.0158	0.011	0.87	1.0012	0.0344	0.008	0.94
Money: $\varepsilon_{mi} = -5.11$	0.0011	0.0052	0.58	1.0004	0.0196	0.0025	0.92
Money: $\varepsilon_{mi} = -2.93$	0.0031	0.0066	0.69	1.0005	0.0217	0.0029	0.88

Table 3.4. New Growth Economy: Selected Moments of nominal Variables induced by the optimal Policy Rules

Scenario	$sd.\ g_t^{\widetilde{c}}$	$\mathbf{ACg}_t^{\widetilde{c}}$	$sd.\ n_t$	$\mathbf{ACn}_t$	$\mathbf{Ev}_t$	$sd.\ v_t$
Baseline	0.0037	-0.32	0.004	0.84		
$\varkappa = 0$	0.0005	-0.42	0.0003	0.65		
$c = 0$	0.0035	-0.35	0.0026	0.69		
$\omega = 0.75$	0.0044	-0.29	0.0059	0.88		
Money: $\varepsilon_{mi} = -5.11$	0.0036	0.46	0.0143	0.97	0.8764	0.0447
Money: $\varepsilon_{mi} = -2.93$	0.0026	0.26	0.0093	0.97	0.8899	0.0296

Table 3.5. New Growth Economy: Selected Moments of real Variables induced by the optimal Policy Rules

interest rate. This in turn would increase v_t, as can be obtained from equation (3.28), thus increasing transaction costs $l(v_t)$. Table 3.6 confirms that for both values of ε_{mi}, the means of v_t and i_t under the rule optimal in the baseline case considerably exceeds their values under the respective optimal rule. For instance, for $\varepsilon_{mi} = -5.11$, the mean interest and inflation rates exceed their values under the optimal rule by about 1.2%. This increases $\mathbf{Ev}_t$ to 1.08 as opposed to 0.88 under the optimal rule. Hence the policymaker reduces the output coefficient to reduce the mean interest rate and thus the transaction cost.

The optimal rule features a higher output gap coefficient if $\varepsilon_{mi} = -2.93$ than if $\varepsilon_{mi} = -5.11$. As mentioned in section 3.2, $\frac{\partial^2 l_t}{\partial i_t \partial \varepsilon_{mi}} < 0$. Hence a given increase in the mean nominal interest rate will increase transaction costs by less if ε_{mi} is less negative. Hence the policymaker is willing to accept a larger mean interest rate if $\varepsilon_{mi} = -2.93$ and thus chooses a higher output gap coefficient.

Scenario	$\mathbf{E}\boldsymbol{\pi}_t$	$\mathbf{Ei}_t$	$\mathbf{Ev}_t$
$\varepsilon_{mi} = -5.11$, [1.2 0.5 0.8]	0.0011	0.0196	0.8764
$\varepsilon_{mi} = -5.11$, [1.2 3 0.8]	0.013	0.0316	1.0801
$\varepsilon_{mi} = -2.93$, [1.2 0.5 0.8]	0.0031	0.0217	0.8899
$\varepsilon_{mi} = -2.93$, [1.2 3 0.8]	0.0129	0.0315	0.9868

Table 3.6. Selected First Moments from the New Growth Economy with Transaction Costs

Given that stabilising output seems to be a more important priority in the New Growth than in the JLN economy, it is interesting to compute the costs of deviating from the optimal value of ϕ_{gap}. Table 3.7 displays the welfare costs of for the optimal values of ϕ_π and ρ, which in the New Growth economy are the same for all deviations from the baseline, and for all the values of ϕ_{gap} from our grid. Clearly the costs of not responding to output at all are substantial in all settings but vary from 0.17% for c=0 to about 3% for $\omega = 0.75$. Increasing ϕ_{gap} from 0 to 0.5 increases welfare substantially for

	Welfare costs					
phigap	Baseline	chi=0	c=0	omega=0.75	epsilonmi=-5.11	epsilonmi=-2.93
0	0.6536%	0.3753%	0.1742%	3.0318%	0.3893%	0.4526%
0.5	0.1501%	0.1398%	0.0932%	0.1993%	0.0000%	0.0120%
1	0.0770%	0.1052%	0.0637%	0.0443%	0.0318%	0.0000%
1.5	0.0365%	0.0821%	0.0393%	0.0025%	0.1055%	0.0275%
2	0.0142%	0.0655%	0.0223%	0.0000%	0.1997%	0.0753%
2.5	0.0036%	0.0529%	0.0116%	0.0111%	0.3048%	0.1346%
3	0.0000%	0.0430%	0.0053%	0.0260%	0.4154%	0.2000%
3.5	0.0004%	0.0350%	0.0019%	0.0410%	0.5282%	0.2684%
4	0.0030%	0.0284%	0.0004%	0.0547%	0.6415%	0.3378%
4.5	0.0068%	0.0229%	0.0000%	0.0667%	0.7542%	0.4074%
5	0.0112%	0.0182%	0.0004%	0.0770%	0.8660%	0.4765%
5.5	0.0157%	0.0141%	0.0012%	0.0857%	0.9766%	0.5449%
6	0.0202%	0.0106%	0.0023%	0.0930%	1.0858%	0.6123%
6.5	0.0246%	0.0075%	0.0036%	0.0990%	1.1936%	0.6788%
7	0.0287%	0.0047%	0.0049%	0.1040%	1.3001%	0.7442%
7.5	0.0325%	0.0022%	0.0062%	0.1079%	1.4052%	0.8087%
8	0.0361%	0.0000%	0.0074%	0.1111%	1.5090%	0.8722%

Table 3.7. Welfare Costs of Deviating from the respective optimal Rule in the New Growth Economy: Varying the Output Gap Coefficient

all scenarios. At the same time, increasing ϕ_{gap} above the respective optimal coefficient reduces welfare relatively slowly in the absence of consumption transaction costs. For the baseline, c=0, and $\omega = 0.75$, setting $\phi_{gap} = 8$ imply welfare costs of 0.0361%, 0.0074% and 0.1111% respectively. Not responding to output at all is costlier in each case. By contrast, in the presence of a transaction cost of holding money, we find that reacting too much to the output gap quickly becomes very costly.

Responding more to inflation than under the optimal policy can also be quite costly, as shown by table 3.8.

Virtually all central banks nowadays believe in some sort of model along the lines of our JLN economy, namely a model where temporary shocks only have very short lasting effects on real variables. It is therefore interesting to look at the welfare costs of applying the optimal rule based on the JLN economy if "in truth" the economy features endogenous growth. This is done in Table 3.9. Other than in the case of $\varkappa = 0$, the

phipi	Baseline	chi=0	c=0	omega=0.75	epsilonmi=-5.11	epsilonmi=-2.93
1.2	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%
1.7	0.4396%	0.1079%	0.2290%	0.8676%	0.1484%	0.2329%
2.2	0.4701%	0.1583%	0.2483%	0.9610%	0.1507%	0.2311%
2.7	0.4665%	0.1870%	0.2480%	0.9814%	0.1473%	0.2196%
3.2	0.4562%	0.2053%	0.2431%	0.9838%	0.1452%	0.2098%
3.7	0.4449%	0.2178%	0.2371%	0.9809%	0.1448%	0.2024%
4.2	0.4343%	0.2267%	0.2310%	0.9767%	0.1457%	0.1983%
4.7	0.4247%	0.2332%	0.2253%	0.9727%	0.1478%	0.1956%
5.2	0.4163%	0.2382%	0.2200%	0.9695%	0.1507%	0.1945%
5.7	0.4090%	0.2420%	0.2152%	0.9673%	0.1543%	0.1946%
6.2	0.4027%	0.2450%	0.2108%	0.9661%	0.1583%	0.1957%
6.7	0.3974%	0.2474%	0.2069%	0.9659%	0.1627%	0.1975%
7.2	0.3921%	0.2492%	0.2034%	0.9667%	0.1674%	0.2001%
7.7	0.3889%	0.2507%	0.2003%	0.9684%	0.1723%	0.2031%
8.2	0.3857%	0.2519%	0.1976%	0.9708%	0.1773%	0.2066%

Table 3.8. Welfare Costs of Deviating from the respective optimal Rule
New Growth Economy: Varying the Inflation Coefficient

welfare costs are quite substantial and range from 0.18% to 0.88% of consumption under the respective optimal rule.

By contrast, pursuing a policy optimal in the endogenous growth economy if the actual production function is neoclassical causes much lower welfare costs relative to per period consumption under the respective optimal rule, as can be obtained from Table 3.10. Thus if there is uncertainty about whether the economy features endogenous growth or not and policy aims to minimise the maximum welfare costs relative to consumption under the respective optimal rule, it should in each scenario choose the policy optimal in the presence of endogenous growth.

For the New Growth economy in the baseline calibration, we also consider two alternative policy rule specifications replacing the output gap in equation (3.21) with two alternative real variables, the current growth rate of the capital stock g^k_{t+1} and the growth rate of habit adjusted consumption $g^{\tilde{c}}_t$.The policy vectors yielding the highest welfare for these specifications are [3.2 1.5 0.8] and [8.2 3 0] , thus placing a higher emphasis on the stabilisation of inflation than the rule featuring the output gap. However, welfare under these rules falls way short of welfare generated by the maximum welfare achievable if the central bank does respond to the output gap instead. Welfare costs amount to 0.35% and 0.28%, respectively.

Calibration	Welfare Cost
Baseline [6.7, 1.5, 0.8]	0.36%
$\varkappa = 0$ [1.2 8 0.8]	0%
$c = 0$ [8.2, 2, 0.8]	0.24%
$\omega = 0.75$	0.89%
$\varepsilon_{mi} = -5.11$, [8.2, 0, 0]	0.54%
$\varepsilon_{mi} = -2.93$, [1.2 0 0.6]	0.27%

Table 3.9. Welfare Cost of applying Policy optimal under JLNE to New Growth Economy

Calibration	Welfare Cost
Baseline [1.2 3 0.8]	0.03%
$\varkappa = 0$ [1.2 8 0.8]	0%
$c = 0$ [1.2 4.5 0.8]	0.03%
$\omega = 0.75$	0.02%
$\varepsilon_{mi} = -5.11$, [1.2, 0.5, 0.8]	0.12%
$\varepsilon_{mi} = -2.93$, [1.2 1 0.8]	0.1%

Table 3.10. Welfare Cost of applying Policy optimal under the New Growth Economy to the JLNE

3.8. Implied costs by two Estimates of Policy Rules for the Bundesbank

How do the rules estimated for the Bundesbank by Clausen and Meier (2003) and Clarida et al. (1998) perform in the New Growth and the JLN economy across the various scenarios? Table 3.11 displays the welfare costs of applying these two rules to the New Growth and the JLN economy. The coefficient estimates are printed in brackets. Clearly both policy calibrations imply significant welfare costs compared to the respective optimal rule in the New Growth economy. They range from 0.14% to 0.91% for the Clausen and Meier (2003) estimate, and from 0.28% to 3.02% for the Clarida et al. (1998) estimate. Note that we do not provide results for the scenario of $\chi = 0$ if policy is as estimated by Clarida et al. Two reasons made us suspicious of our welfare value in this case. First, it proved extremely sensitive to small changes in the parameters. Second, most of the second moments do not exist, and when we tried to simulate the second order accurate solution, the simulation exploded.

By contrast, in the JLN economy, both estimated policy rules do much less harm, as can be obtained from table 3.12. For every scenario considered, the welfare loss expressed as a percentage of consumption under the optimal rule is always greater in the New Growth economy than in the JLN economy. Furthermore, within the JLN economy, the welfare loss is always greater under the rule estimated by CGG than under the rule estimated by Clausen and Meier.

Calibration	**Clausen and Meier** [1.5 0.125 0.75]	**Clarida et al.** [1.31 0.25 0.91]
Baseline	0.35%	0.66%
$\varkappa = 0$	0.2%	[no reliable measurement]
$c = 0$	0.15%	0.28%
$\omega = 0.75$	0.91%	3.03%
$\varepsilon_{mi} = -5.11$	0.14%	0.42%
$\varepsilon_{mi} = -2.93$	0.17%	0.47%

Table 3.11. Welfare Cost of Bundesbank Policies in the New Growth Economy

Calibration	**Clausen and Meier** [1.5 0.125 0.75]	**Clarida et al.** [1.31 0.25 0.91]
Baseline	0.0021%	0.0555%
$\varkappa = 0$	0.0135%	0.0303%
$c = 0$	0.0029%	0.03166%
$\omega = 0.75$	0.0578%	0.0666%
$\varepsilon_{mi} = -5.11$	0.079%	0.11%
$\varepsilon_{mi} = -2.93$	0.0031%	0.0373%

Table 3.12. Welfare Cost of Bundesbank Policies in the JLNE

3.9. Conclusion

The goal of this chapter is to assess whether the conventional wisdom concerning the priorities of monetary policy still holds in the presence of endogenous growth. For that purpose we first modify the New Growth economy and the JLN economy developed in chapter two by replacing the assumption of quadratic costs of price adjustment by the more commonly used assumption of Calvo contracts. This generates higher costs of inflation. We then search for a simple optimal rule which maximises the households welfare in the two economies. As in chapter two, the only source of volatility is a cost push shock.

Our main result is that in the JLN economy, the optimal simple rule features a large inflation coefficient and a much lower output coefficient, just as conventional wisdom would suggest. By contrast, in the presence of endogenous growth, this hawkish monetary policy is no longer optimal. The optimal inflation coefficient is the lowest we allow in the grid, while the coefficient on the output gap is higher than in the JLN economy. The reason for this result seems to be that a failure to respond to the output gap and an aggressive response to inflation increase the uncertainty of the future consumption path far more in the New Growth than in the JLN economy. This is because a cost push shock in the New Growth economy has a permanent effect on the future consumption trajectory absent in the JLN economy. This permanent effect will be amplified by a hawkish monetary policy. Therefore the central bank opts to respond more strongly to output than in the JLN economy in spite of the implied higher mean price dispersion and the associated efficiency loss.

This result is qualitatively robust against a variety of changes to the baseline scenario, i.e. variations to the degree of real wage rigidity and nominal price stickiness and the introduction of a transaction cost for consumption, the latter of which implies a transaction demand for money. An exception is the case of no indexation of non-reoptimised prices to past inflation, for which the optimal policy is the same in both the endogenous growth and the JLN economy.

Looking across the different deviations from the baseline within the New Growth economy, we find that the optimal coefficient on output is higher than in the baseline if the real wage is more flexible and lower if nominal rigidity is higher. Furthermore, the presence of a transaction cost of consumption also reduces the optimal output gap coefficient relative to the baseline. This is because the mean inflation rate and thus the mean nominal interest rate and by implication the mean consumption to money ratio increase in the output gap coefficient, implying that consumers have to pay higher transaction costs. The output gap coefficient is lower the more negative the semi elasticity of money demand.

We also find that if there is uncertainty about whether the economy features endogenous growth or not and policy aims to minimise maximum welfare loss measured as percentage of per period consumption under the respective optimal rule, it should in each scenario choose the policy optimal in the presence of endogenous growth. This is because welfare costs measured in this way are always higher if we apply the policy rules optimal in the JLN economy to the New Growth economy than vice versa. For instance, in the baseline scenario, the welfare cost in New Growth economy of applying the JLN optimal policy amounts to 0.36% of consumption under the true optimal policy, while vice versa the welfare cost equals 0.03% of consumption under the JLN optimal rule.

Finally, we examine the welfare costs of the monetary policies of the German Bundesbank, as estimated by Clarida et al. (1998) and by Clausen and Meier (2003). We find that in the New Growth economy, both rules are highly suboptimal, while in the JLN economy, welfare costs measured as a percentage of consumption under the respective optimal rule are much lower.

CHAPTER 4

The Taylor Principle and (In-) Determinacy in a New Keynesian Model with hiring Frictions and Skill Loss

The idea that skill loss among the unemployed might generate a relationship between the actual and the natural unemployment rate is an old one. In fact, Phelps (1972) himself emphasized this mechanism when developing the concept of the natural rate of unemployment. In this chapter, we introduce skill loss among the unemployed along the lines of Pissarides (1992) into a New Keynesian model with hiring frictions developed by Blanchard and Gali (2008). We assume that workers who remain unemployed for one quarter or longer lose a fraction of their skills per quarter of their unemployment spell. The share of those unemployed for more than one quarter affects the willingness of firms to create jobs as firms are matched with different types of workers according to their share in the job seeking population. Our goal is to investigate the effects of introducing skill loss on macroeconomic stability and unemployment persistence under varying monetary policy rules and degrees of skill loss. As far as we are aware, this question has not been addressed so far within a state-of-the-art general equilibrium framework.

Our key results are as follows. Firstly, for sufficiently high levels of skill loss, a nominal interest rate feedback rule with a coefficient on inflation exceeding one does not guarantee determinacy if the quarterly skill loss percentage is large enough. If the central bank responds only to inflation, the coefficient on inflation has to be less than one. This does not depend on whether the central bank responds to current, expected future, or lagged inflation.

Secondly, let us denote the level of skill decay which switches the determinacy requirement on the coefficient on inflation in the interest feedback rule to less than one as the "critical level." We find that the critical level of skill decay will be implausibly high if we adopt what Blanchard and Gali deem an "American" calibration of labour market flows, i.e. a high job finding probability and a high job destruction rate. By contrast, if we adopt Blanchard and Gali's "continental European" calibration, with little hiring and firing, the critical skill loss percentage will be a lot lower, about 2.5% per quarter.

Thirdly, if skill loss is above the critical level, responding to the output gap (as defined in the New Keynesian literature) in addition to inflation decreases the determinacy region further. Considered jointly, these results suggest that indeterminacy is a realistic scenario in Europe, but not in the United States. Estimates of interest feedback rules suggest that the Federal Reserve, as well as the Bundesbank and the ECB respond more than one-for-one to inflation and pay some attention to the output gap as well. At the same time, an active monetary policy will be associated with indeterminacy at low and plausible levels of skill decay -larger than or equal to 2.5%- but implausibly high levels of skill decay in the United States.

Finally, if monetary policy induces indeterminacy, sunspot shocks can affect the endogenous variables. Under the continental European calibration, with skill loss above its critical level and the inflation coefficient in the interest feedback rule larger than one, a one quarter adverse sunspot shock has very persistent effects on unemployment.

Thus in the model developed in this chapter, endogenous, highly persistent unemployment fluctuations will arise under an active monetary policy if the labour market has "European" flow characteristics, but not if it has American characteristics. Given that indeterminacy is a realistic scenario in Europe, the model may be able to shed light on the fact that we observe highly persistent unemployment fluctuations in many European countries but not in the United States.[1]

Our results can be compared to an evolving literature showing that under certain circumstances, an active monetary policy does not guarantee determinacy. For instance, Batini et al. (2006), using several estimated variants of a standard DSGE model, consider the determinacy properties of an inflation forecast based rule. They find that this rule becomes increasingly prone to indeterminacy as the forecast horizon increases from two to four quarters. Levin et al. (2003) investigate which inflation and output gap forecast based rules robustly induce determinacy across four macroeconometric models and the canonical New Keynesian model. They find that only rules with an inflation forecast horizon not exceeding one year, an explicit response to the current output gap and a substantial degree of policy inertia robustly guarantee determinacy across all five models.

A particular branch of this literature focuses on models in which monetary policy has some indirect or direct effect on the supply side. These papers regularly find that some sort of restriction on the inflation coefficient in the interest feedback rule and/or

[1]It might seem puzzling that this chapter in contrast to chapters two and three focuses so much on determinacy issues. The chief reason for this is simply that in the models used in the previous chapters, the problem of indeterminacy never arose as long as the coefficient on inflation in the interest feedback rule of the central bank exceeded one. By contrast, in the model developed in this chapter, plausible calibrations of the parameters affect the requirements that the interest feedback rule has to meet. Similarly, sunspot shocks only become relevant when there is indeterminacy, which is why they where absent from the previous chapters. Furthermore, as mentioned above, given an active monetary policy, small values of skill decay do induce indeterminacy and thus endogenous persistent unemployment fluctuations under some calibrations of labour market flows but not under others. This allows drawing conclusions on why persistent unemployment fluctuations occur in Europe but not in the United States.

some response to output are necessary to ensure determinacy. For instance, Kurozumi and van Zandweghe (2008) and Carlstrom and Fuerst (2005) find, in a New Keynesian model with capital, that an interest feedback rule where the interest rate only responds to expected inflation limits the permissible inflation responses to an extremely small range above but very close to one. With such a rule, higher future inflation increases the ex-ante real interest rate and thus the expected future capital rental via the no arbitrage condition. This in turn increases expected inflation. Kurozumi and van Zandweghe (2008) also show that even a modest response to current output (as opposed to the output gap as used in this chapter) substantially widens the permissible range. A response to the lagged interest rate above a certain threshold has a similar effect. Duffy and Xiao (2008) qualify their results by showing that in the presence of capital stock adjustment costs, even a modest response to expected future output is enough to guarantee determinacy.

Surico (2008) considers a New Keynesian model with a cost-channel along the lines of Ravenna and Walsh (2006), where the nominal interest rate has a direct positive effect on inflation since firms have to borrow working capital to pay wages. He shows that, if the interest rate responds to current inflation, determinacy requires an upper bound on the inflation coefficient, which, however, is too high to form a relevant constraint for monetary policy. Tuesta and Llosa (2009) investigate the same model with a purely forward-looking rule and show that determinacy is unattainable if the central bank responds only to expected inflation.

All of the cited results have in common that the determinacy problem caused by a monetary policy rule responding to inflation alone is never caused by the active response to inflation per se, but to the timing subscript of inflation in the interest feedback rule.

By contrast, in the model proposed here, it is the Taylor principle itself -the idea that an increase in inflation should sooner or later cause an increase in the real interest rate- which creates scope for self-fulfilling prophecies if quarterly skill loss among the unemployed is above the critical level.

The change in the determinacy requirement appears to be caused by a change in the long-run relationship between marginal cost and unemployment from negative to positive if skill decay crosses the threshold. Thus if skill decay is above the critical level, a persistent increase in unemployment will ultimately increase marginal cost and thus inflation. If the central bank responds more than one-for-one to inflation, this would increase the real interest rate, which lowers demand and thus validates the increase in unemployment: Hence we have a self-fulfilling prophecy.

This chapter is structured as follows. Section 4.1 discusses some empirical evidence for skill loss among the unemployed while section 4.2 derives the model. Section 4.3 analyses determinacy in the absence of skill loss, i.e. in the original Blanchard and Gali (2008) model. Section 4.4 derives the marginal cost equation in the presence of skill loss and shows how the effect of unemployment on marginal cost is affected by the introduction of skill loss. Section 4.5 analyses determinacy in the model with skill loss. Section 4.6 discusses the response of the model under the European calibration to an adverse sunspot shock and an averse technology shock. Section 4.7 concludes.

4.1. Evidence for Skill Loss

Direct, quantifiable evidence for skill loss during unemployment is difficult to obtain. An idea of the size of skill decay over time can be gained from the literature on wage loss upon worker displacement. This literature has produced evidence based on panel regressions showing that the wage upon reemployment depends negatively on the duration of the unemployment spell. Skill decay during unemployment is usually seen as one of the factors causing this relationship, although the evolution of reservation wage due to other factors (for instance depletion of an unemployed person's wealth) would be expected to have an impact as well. Evidence along these lines include Addison and Portugal (1989) for American male workers displaced and reemployed between 1979 and 1984, Pichelmann and Riedel (1993) for Austrian workers between 1972 and 1988, Gregory and Jukes (2001) for British male workers between 1984 and 1994, Gregg and Tominey (2005) for male youths and Gangji and Plasman (2007) for Belgian workers. Their findings on the effect of a one-year unemployment spell on the real wage are -39%, -24%, -11%, -10% and -8% respectively.[2] Pichelmann and Riedel (1993) explicitly ask whether the earnings penalty arising from duration diminishes during the two years following the unemployment spell and find that it does not. Gregg and Tominey (2005) find that the wage penalty associated with a year of youth unemployment is still present at age 42.[3]

Furthermore, Nickell et al. (2002) look at three four year periods from 1982 to 1997. They ask how the earnings loss is changed if the unemployment spell exceeded 6 months and find an additional permanent earnings loss between 6.8% and 10.6%.[4] To which extent these numbers reflect skill depreciation depends on the movement of

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[2]For Addison and Portugal (1989), we have calculated the annual earnings penalty using the lower coefficient on log(duration) in their two preferred specifications (Table 3, columns 5 and 6), p. 294. Duration is measured in weeks. For Pichelmann and Riedel (1993), we had to resort to the same procedure, see p. 8 in that paper for the results. Their coefficient estimates for the effect on the real wage is reported in table 2, p. 8. The results of Gregory and Jukes (2001) are reported on page F619, while the results of Gangji and Plasman (2007) are reported on page 18, table 2.

[3]See Gregg and Tominey (2005), p. 502 and pp. 505-506.

[4]See Nickel et al. (2001), p. 17.

the reservation wage in general and in particular its responsiveness to the respective workers' human capital evolution.

There is also evidence suggesting that unemployed workers become less attractive employees as their unemployment spell lengthens. Jackman et al. (1991) cite various studies showing that morale and motivation decline the longer a person remains unemployed.[5] The stylised fact that the probability of an unemployed person of leaving unemployment increases with the unemployment duration (see for instance Machin and Manning (1999)[6]) is also seen by some as evidence for skill loss among the unemployed. It is, however, a priori unclear whether this represents "true" duration dependence, i.e. the worsening of an individual's employment probability over time, or merely individual heterogeneity, possibly unobserved. In the latter case different individuals have different hazard rates of leaving unemployment as a result of differing individual characteristics, such as their education. The individuals with higher hazard rates will leave the unemployment pool quickly, implying that the average hazard rate of a cohort of unemployed falls as the unemployment spell lengthens. However, Jackman et al (1991) argue that in the presence of pure individual heterogeneity, and under certain assumptions about its nature, the ratio of the average hazard rate and the hazard rate of new entrants into unemployment would have to be constant as the average hazard rate moves up or down. They find that for British data, the average hazard rate declines in fact much more than the hazard rate of new entrants. Van den Berg and van Ours confirm this result using other "eyeball" tests [7] and a more formal non-parametric estimation method[8]. Using the same method, they also find negative duration dependence for the United States.[9]

[5]See Jackman, Layard and Nickell (1991), p. 259.
[6]See Machin and Manning (1999), p. 3100.
[7]van der Berg and van Ours (1994a), p. 23.
[8]See Van der Berg and van Ours (1994b), p. 442.
[9]Van der Berg and van Ours (1996), p. 123.

The model discussed below does not actually model duration dependence (although it could be extended to do so). However, overall, we view the evidence above as indicating that workers are less efficient at work the longer they have been unemployed.

4.2. The Model

In this section we add skill loss along the lines of Pissarides (1992) to the Blanchard and Gali (2008) model. We first go through the optimisation problems of households and firms and then show what the expressions for marginal cost and the Phillips Curve look like in the absence and in the presence of skill loss.

4.2.1. Households

The economy is populated by a continuum of representative and infinitely lived households. A household consists of a continuum of members who supply labour to firms. They might be employed or unemployed. The household derives income from wage payments, bond holdings, and firms' profits. It allocates its income to buying a CES basket of consumption goods and a risk-less bond to maximise

$$E_t \sum_{t=0}^{\infty} \log C_t$$

where C_t denotes consumption, subject to the budget constraint

$$N_t W_t + \frac{B_{t-1}}{P_t}(1 + i_{t-1}) + F_t \geq C_t + \frac{B_t}{P_t}$$

where P_t, N_t, W_t, B_t, i_t and F_t denote the price level, hours worked by the members of the household, the real wage, bonds, the nominal interest rate and the profits of firms. Consumption is governed by the usual first order condition

$$\frac{1}{C_t} = [1 + i_t] \beta E_t \left[\frac{1}{1 + \pi_{t+1}} \frac{1}{C_{t+1}} \right]$$

where π_t denotes the inflation rate.

4.2.2. Firms

There are two types of firms. Final goods firms indexed by i produce a differentiated product using the intermediate good $X_t(i)$ in the linear technology

$$Y_t(i) = X_t(i)$$

They produce the varieties in the CES basket of goods consumed by households. The demand curve for variety i resulting from the household spreading its expenditures across varieties in a cost minimising way is given by $c_t(i) = C_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta}$, where $c_t(i)$, $p_t(i)$ and P_t denote consumption and price of variety i and the price level of the consumption basket, respectively. We will assume that final goods firms face nominal rigidities in the form of Calvo (1983) contracts, i.e. only a randomly chosen fraction $1 - \omega$ of firms can re-optimise its price in a given period. They accordingly maximise

$$E_t \left[\sum_{i=0}^{\infty} (\omega\beta)^i \frac{C_t}{C_{t+i}} C_{t+i} \left[\left(\frac{p_t(j)}{P_{t+i}} \right)^{1-\theta} - mc_{t+i} \left(\frac{p_t(j)}{P_{t+i}} \right)^{-\theta} \right] \right]$$

where mc_t denotes real marginal costs. The price index evolves according to

$$P_t^{1-\theta} = (1 - \omega) (p_t^*(j))^{1-\theta} + \omega (P_{t-1})^{1-\theta}$$

where $p_t^*(j)$ denotes the price set by those firms allowed to reset their price in period t. Taking first order approximations of both the final goods first order condition and the law of motion of the price index and combining the resulting equations yields the familiar New Keynesian Phillips curve relating inflation in period t to expected $t+1$ inflation and period t marginal costs. The marginal cost of the final goods firms equals the real price of the intermediate good, $\frac{P_t^I}{P_t}$.

Intermediate goods firms operate under perfect competition and are owned by households. As is common in the matching literature, we assume that a fixed fraction δ of jobs is destroyed each period. This can be thought of as an idiosyncratic productivity shock and implies that even with constant employment, there are constantly flows in and out of employment. Thus employment of firm j evolves according to

$$N_t(j) = (1-\delta) N_{t-1}(j) + H_t(j)$$

Where $H_t(j)$ denotes the amount of hiring in firm j. Aggregate hiring is accordingly given by

$$H_t = N_t - (1-\delta) N_{t-1} \tag{4.1}$$

Note that the lower is δ, the more H_t will depend on the change as opposed to the level of employment.

The Intermediate good firms employ labour to produce intermediate goods $X_t(j)$. Following Pissarides (1992), we assume that the productivity of a newly hired worker is the product of exogenous technology A_t^P and the skill level of worker of type i A^i. Thus the productivity $prod_t^i$ of a worker of type i is given by

$$prod_t^i = A_t^P A^i$$

We follow Pissarides (1992) by making the following assumptions. A^i equals one if he is short term unemployed, i.e. if he lost his job in period t. Unemployed workers loose a fraction δ_s of their skills per quarter if they remain unemployed for one quarter or longer. Skill decay continues for the duration of the unemployment spell. We assume further, following Pissarides (1992), that the unemployed regain all their skills after one quarter of employment, that intermediate goods firms meet workers according to their share among job seekers and that they hire any worker they meet.[10] Finally, when firms decide whether to hire or not they know the state of exogenous technology A_t^P but not which type of worker they are going to meet.

We denote the average skill level of the newly hired as A_t^L.The productivity of a newly hired worker expected by the firm when deciding whether to hire is denoted by A_t and is accordingly given by

$$A_t = A_t^P A_t^L \tag{4.2}$$

A_t^L is given by

$$A_t^L = \sum_{i=0}^{\infty} \beta_s^i s_t^i \tag{4.3}$$

where $\beta_s = 1 - \delta_s$ and s_t^i denotes the share of those unemployed i periods among job seekers. Note that $A_t^L < 1$ if $\delta_s > 0$ and equal to one if $\delta_s = 0$. We will refer to A_t^L as the average skill level in period t rather than the expected skill level to avoid confusion when we refer to $E_t A_{t+1}^L$.

[10]See Pissarides (1992), pp. 1371-1391. In contrast to Pissarides, skill loss does not stop after one quarter in our model.

The shares of the various groups among the number of job seekers, denoted as U_t, are given by

$$s_t^i = \frac{\delta N_{t-1-i} \prod_{j=1}^{i} (1 - x_{t-i})}{U_t} \tag{4.4}$$

where x_t denotes labour market tightness, defined as the ratio between aggregate hiring H_t and U_t, i.e.

$$x_t = \frac{H_t}{U_t} \tag{4.5}$$

We interpret labour market tightness x_t as the probability of an unemployed person to move into employment in period t. For instance, s_t^2 is calculated as follows: δN_{t-3} workers loose their jobs in period $t-2$. A fraction x_{t-2} moves right back into employment while a fraction $(1 - x_{t-2})$ remains unemployed and keeps looking for jobs in period $t-1$. Of those $\delta N_{t-3} (1 - x_{t-2})$ workers, a fraction $(1 - x_{t-1})$ does not find a job during $t-1$ and is still unemployed at the end of that period. Dividing those $\delta N_{t-3} (1 - x_{t-2}) (1 - x_{t-1})$ unemployed by U_t then gives the share of those unemployed for two periods among job seekers in period t.

U_t consists of those who did not find a job at the end of period t-1 and those whose jobs were destroyed at the beginning of t:

$$U_t = 1 - N_{t-1} + \delta N_{t-1} = 1 - (1 - \delta) N_{t-1} \tag{4.6}$$

As in the Blanchard Gali model, we assume that the real wage is rigid. We assume that the wage of a worker depends on his individual productivity in exactly the same way as in Blanchard Gali (2008): The wage W_t^i of a worker who has been unemployed for i periods is given by $W_t^i = \Theta' \left(A_t^P A_t^i\right)^{1-\gamma}$, with $0 \leq \gamma \leq 1$. This means that there are five different wage levels. Accordingly, the real wage the firm expects to pay when it decides to hire is given by

$$W_t = \Theta' \left(\sum_{i=0}^{\infty} \beta_s^{i(1-\gamma)} s_t^i \right) \left(A_t^P\right)^{1-\gamma} \tag{4.7}$$

Note that for $\delta^S = 0$, this collapses to $W_t = \Theta' \left(A_t^P\right)^{1-\gamma}$ as in Blanchard Gali. This is the wage a firm expects to pay when it decides whether to hire. Θ' is backed out to support a desired steady state combination of x, δ and n. This is shown in appendix C.2. For future reference, we denote the skill dependent part of the real wage as

$$W_t^L = \left(\sum_{i=0}^{\infty} \beta_s^{i(1-\gamma)} s_t^i \right) \tag{4.8}$$

As in Blanchard and Gali (2008), we assume that every hire generates a cost G_t which is proportional to the productivity of a newly hired worker

$$G_t = A_t B' x_t^{\alpha} \tag{4.9}$$

where B' denotes a constant. The intuition behind (4.9) is that if hiring is high relative to the number of job seekers, it takes on average longer to fill a vacancy. Since posting a vacancy is costly, hiring costs increase in x_t.[11]

The intermediate goods firms will hire additional workers until the hiring costs of an additional worker equal the present discounted value of the profits generated by this worker. However, unlike in the Blanchard and Gali model, we have to take account of

[11]Hence equation (4.9) can be viewed as a short cut to a model which would specify a matching function and thus allow to derive the expected time necessary to fill a vacancy and hence the expected cost of filling a vacancy. See Blanchard and Gali (2008), p. 8.

the skill level of the workforce hired in period t as well as their wage schedule change in period $t+1$, as all hired workers who remain employed upgrade to the full skill level after one quarter. Thus we have

$$G_t = \frac{P_t^I}{P_t} A_t^P A_t^L - W_t + E_t \left[\sum_{i=1}^{\infty} (1-\delta)^i \beta^i \frac{u_C(C_{t+i})}{u_C(C_t)} \left(\frac{P_{t+i}^I}{P_{t+i}} A_{t+i}^P - W_{t+i}^0 \right) \right] \tag{4.10}$$

where $\frac{P_t^I}{P_t}$ denotes the real price of intermediate goods while $\beta^i \frac{u_C(C_{t+i})}{u_C(C_t)}$ denotes the stochastic discount factor of the representative household. The terms $\frac{P_t^I}{P_t} A_t^P A_t^L - W_t$ and $E_t \left[\sum_{i=1}^{\infty} (1-\delta)^i \beta^i \frac{u_C(C_{t+i})}{u_C(C_t)} \left(\frac{P_{t+i}^I}{P_{t+i}} A_{t+i}^P - W_{t+i}^0 \right) \right]$ represent the flow profit generated in period t (when the worker has just been hired) and the present discounted value of profits generated in period $t+1$ and after, respectively. Note that due to our assumption that the worker regains all his skills after one period, the expression for the flow profit in period t is different from the expression for the flow profit in period $t+1$ and after. Rewriting this equation as a difference equation, noting that the real price of intermediate goods firms equals the marginal cost of final goods firms (hence $\frac{P_t^I}{P_t} = mc_t$) and that with log utility, $\frac{u_C(C_{t+i})}{u_C(C_t)} = \frac{C_t}{C_{t+i}}$, we have

$$\begin{aligned} \text{(4.11)}\ mc_t A_t^P A_t^L &= W_t + G_t \\ &\quad -\beta(1-\delta) E_t \left[\frac{C_t}{C_{t+1}} \left(G_{t+1} + mc_{t+1} A_{t+1}^P - W_{t+1}^0 - \left(mc_{t+1} A_{t+1}^P A_{t+1}^L - W_{t+1} \right) \right) \right] \end{aligned}$$

The left hand side represents the real marginal revenue product of labour, which depends on the period t average skill level among applicants. Clearly, an increase in the quality of the average period to job seeker A_t^L will reduce period t marginal cost. The right hand side features the period t real wage W_t and the period t hiring costs G_t, and, with a negative sign, the present expected value of hiring costs saved (G_{t+1}) by hiring the worker in t rather than $t+1$. While an increase in hiring cost today means increasing production is more costly, an increase in future expected hiring costs will induce intermediate goods firms to shift hiring into the present, thus lowering the price of intermediate goods and thus marginal cost.

In addition, the right hand side also includes the present expected value of the $t+1$ difference between the real profit generated by a fully skilled worker (with productivity A_{t+1}^{P} and real wage W_{t+1}^{0}) and a $t+1$ newly hired worker (with productivity $A_{t+1}^{P}A_{t+1}^{L}$ and real wage W_{t+1}). This represents an additional benefit of hiring today rather than tomorrow not present in the Blanchard Gali model. For further reference note that this benefit decreases in A_{t+1}^{L} and increases in W_{t+1} and mc_{t+1}. Thus an expected higher $t+1$ skill level will increase marginal cost in period t (since it reduces the benefit from hiring today), while a higher expected average real wage for the t+1 newly hired and a higher expected t+1 price of intermediate goods (i.e. higher t+1 marginal cost) will decrease it.

While A_t is the relevant level of productivity at the margin, the average productivity of the whole workforce after adding the newly hired will be different because those employees who remained in employment from t-1 to t are all fully skilled. The average productivity level A_t^A is then given by

$$A_t^A = A_t^P \left[s_t^N A_t^L + \left(1 - s_t^N\right)\right] \tag{4.12}$$

where s_t^N denotes the share of the newly hired in period t employment, which is given by

$$s_t^N = \frac{H_t}{N_t} = \frac{N_t - (1-\delta)\, N_{t-1}}{N_t} \tag{4.13}$$

To set up the production function, we have to use $A_t^A N_t$ for gross output. Hence the production function becomes

$$C_t = A_t^A N_t - B' x_t^\alpha A_t^P A_t^L H_t = A_t^A N_t - B' x_t^\alpha A_t^P A_t^L \left(N_t - (1-\delta) N_{t-1}\right) \tag{4.14}$$

4.2.3. Marginal Cost and Phillips Curve the Absence of Skill Loss

The assumption of hiring costs made by Blanchard and Gali has interesting consequences for the Phillips Curve, which we would like to highlight next. It is well known that monopolistic competition and Calvo pricing as found in the final goods firms lead to, up to first order, the familiar New Keynesian Phillips curve relating inflation to expected future inflation and marginal costs (a lower case variable with a hat denotes the percentage-deviation of this variable from its steady state, unless otherwise stated):

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t, \ \lambda = \frac{(1-\beta\omega)(1-\omega)}{\omega} \tag{4.15}$$

Concerning marginal cost, in appendix C.5 we show that combining log-linear approximations of equations (4.11) to (4.14) combined with log linear approximations to (4.5), (4.6) and (4.1) allows one to express the percentage deviation of marginal cost from its steady state as

$$(4.16) \quad \widehat{mc}_t = -a_1^L \widehat{a}_t^L + w_1^L \widehat{w}_t^L + a_2^L E_t \widehat{a}_{t+1}^L - w_2^L E_t \widehat{w}_{t+1}^L - p_0 \widehat{a}_t^P - p_1 E_t \widehat{a}_{t+1}^P$$

$$+h_0^{'} \widehat{n}_t + h_L^{'} \widehat{n}_{t-1} + h_F^{'} E_t \widehat{n}_{t+1} - h_c E_t \widehat{mc}_{t+1}$$

$$where$$

$$\begin{aligned}
h_c &= \beta(1-\delta)\frac{(1-A^L)}{A^L}, \; g = B' x^{\alpha} \\
h_F^{'} &= -\beta(1-\delta)\left(\frac{\alpha g M}{\delta} - \xi_0^{'} X\right) \\
h_0^{'} &= \left(\frac{\alpha g M}{\delta}\right)\left(1+\beta(1-\delta)^2(1-x)\right) + \beta(1-\delta)\left(\xi_1^{'} - \xi_0^{'}\right) X \\
h_L^{'} &= -\left(\frac{\alpha g M}{\delta}\right)(1-\delta)(1-x) - \beta(1-\delta)\xi_1^{'} X \\
a_1^L &= 1 - gM + \beta(1-\delta)\frac{A^L \delta(1-g)}{A^A - Ag\delta} X \\
a_2^L &= \beta(1-\delta)\left[1 - gM + \frac{A^L \delta(1-g)}{A^A - Ag\delta} X\right] \\
w_1^L &= \frac{M}{A^L} W, \; w_2^L = \beta(1-\delta)\frac{M}{A^L} W \\
p_0 &= \Phi' + \beta(1-\delta) X, \; p_1 = \beta(1-\delta)\frac{\gamma M(\Theta' - W)}{A^L} \\
X &= gM + \frac{1 - A^L - M(\Theta' - W)}{A^L} \\
\xi_0^{'} &= \frac{A^L(1-g(1+\alpha))}{A^A - Ag\delta} \\
\xi_1^{'} &= \frac{(1-\delta)\left((1+\alpha(1-x))A^L g + (1-A^L)\right)}{A^A - A^L g\delta} \\
\Phi' &= 1 - gM - (1-\gamma)\frac{M}{A^L} W
\end{aligned}$$

Smaller case variables with hats denote the percentage deviation of a variable from its steady state and M denotes the steady state mark-up of final goods firms.

We consider first the case of no skill loss, i.e. $\delta_s = 0$. In this case we have $A^L = A^A = 1$, $\Theta' = W$ and $\widehat{a}_t^L = \widehat{w}_t^L$. This yields

$$
\begin{aligned}
(4.17) \qquad \widehat{mc}_t &= h_0 \widehat{n}_t + h_L \widehat{n}_{t-1} + h_F E_t \widehat{n}_{t+1} - p_0 \widehat{a}_t \\
h_0 &= \left(\frac{\alpha g M}{\delta} \right) \left(1 + \beta (1-\delta)^2 (1-x) \right) - \beta (1-\delta) g M (\xi_1 - \xi_0) \\
h_L &= -\left(\frac{\alpha g M}{\delta} \right) (1-\delta)(1-x) - \beta (1-\delta) g M \xi_1 \\
h_F &= -\beta (1-\delta) g M \left(\left(\frac{\alpha}{\delta} \right) - \xi_0 \right) \\
\xi_0 &= \frac{1 - g(1+\alpha)}{(1-\delta g)} \\
\xi_1 &= \frac{g(1-\delta)(1+\alpha(1-x))}{(1-\delta g)}
\end{aligned}
$$

Hence marginal cost depends positively on current employment but negatively on lagged employment. An increase in $\widehat{n}_t$ increases labour market tightness and thus marginal cost, while an increase in $\widehat{n}_{t-1}$ reduces the amount of hiring necessary to achieve a given amount of employment in period t and thus reduces marginal cost. Marginal cost also depends negatively on $E_t \widehat{n}_{t+1}$, as higher $t+1$ employment in implies higher hiring costs in that period, thus increasing the benefit of creating jobs today and correspondingly lowering the price of intermediate goods.

Note that the effect of lagged and lead employment, relative to the effect of current employment, increases the less "fluid" the labour market is, i.e. the lower the separation rate δ and the steady state job finding rate x for a given level of employment.[12] Assume for instance, for the sake of example, that we have $N = 0.9$ and, unrealistically $x = 0.9$, implying a separation rate of $\delta = 1$. In this case, we have $h_L = h_F = 0$. In this scenario,

[12]Note the following steady state cross coefficient restriction between δ, x, and $N : \delta = \frac{x(1-N)}{N(1-x)}$, for values of $x \leq N$ and $N < 1$.

all workers are fired at the beginning of the period. This implies that hiring and hence the cost of hiring depend only on N_t and that the cost of hiring in the future is irrelevant for job creation today because no job lasts longer than one period anyway. As we lower the job finding rate and by implication δ, the values of h_L and h_F increase.

Using the relationship $\widehat{n}_t = \frac{\widehat{u}_t}{-(1-u)}$ (where $\widehat{u}_t$ denotes the percentage point, not the percentage deviation of unemployment from its steady state) and (4.15), we arrive at the Phillips Curve:

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} - \kappa_0 \widehat{u}_t + \kappa_L \widehat{u}_{t-1} + \kappa_F E_t \widehat{u}_{t+1} - \lambda p_0 \widehat{a}_t \\ \kappa_0 &= \frac{\lambda h_0}{1-u}, \kappa_L = \frac{-\lambda h_L}{1-u}, \kappa_F = \frac{-\lambda h_F}{1-u} \end{aligned} \tag{4.18}$$

For future reference, we note that in the Blanchard Gali model we always have $\kappa_0 - \kappa_L - \kappa_F > 0$.[13] This means that a "permanent" increase in unemployment (i.e. an equal sized increase in $\widehat{u}_t, \widehat{u}_{t-1}$ and $\widehat{u}_{t+1}$) reduces inflation because the effect of current unemployment dominates the effect of lagged and lead unemployment.

The fact that lead as well as lagged unemployment have positive effects on price setting, and thus inflation through their effect on marginal costs, clearly distinguishes the Phillips Curve in the Blanchard and Gali model from its counterpart in the canonical New Keynesian model. The presence of a lagged unemployment term in the Phillips Curve is commonly associated with (partial) labour market hysteresis. In the Blanchard and Gali model the effect of lagged unemployment works through the effect on

[13]This is easily shown: $\kappa_0 - \kappa_L - \kappa_F = \frac{\lambda}{1-u}(h_0 + h_L + h_F)$
$= \frac{\lambda}{1-u}\frac{\alpha g M}{\delta}\left(1 + \beta(1-\delta)^2(1-x) - (1-\delta)(1-x) - \beta(1-\delta)\right) > 0$. using the fact that that $1 - \delta = \frac{N-x}{N(1-x)}$, this can be simplified to $(1-N)x^2 + (N-x)N(1-\beta) > 0$. This holds for all permissible values of x, β and N since the maximum value x can take without violating $\delta \leq 1$ is N.

price setting. Jackman et al. (1991) jointly estimate a wage and a price setting equation featuring both the level and the change in the unemployment rate for 19 OECD countries, and find that the change in the unemployment rate has a positive effect on the real wage employers are willing to pay (given the change in the inflation rate and the level of unemployment) in all countries except for the United States.[14] This implies that lagged unemployment has a negative effect on the real wage employers are willing to pay and thus boosts inflation.

The difference between the United States and other, mostly European OECD economies concerning the role of lagged unemployment found by Jackman et al., is at least qualitatively reflected by (4.18) if the "American" and "European" calibrations of Blanchard and Gali are adopted, respectively. The two parameterisations are displayed in table 4.1. The two calibrations differ in that in the United States, steady state unemployment is lower and the labour market is more fluid, with a high steady state job finding probability x of 0.7, which (given u) backs out a high separation rate of 0.12. In continental Europe, unemployment is higher, with $u = 0.1$, and there are less flows in and out of unemployment, with $x = 0.25$ which backs out a separation rate δ of only 0.04. The calibration of x is based on American and European evidence on the average monthly job finding rate. Furthermore, Blanchard and Gali (2008) set $\alpha = 1$ since this is consistent with estimates of matching functions. They set $B' = 0.12$, which implies a fraction of hiring costs in GDP of about one percent under the American calibration , and correspondingly a lower fraction under the continental European calibration since

[14]See Jackman et al (1991), pp. 401-408.

x is lower.[15] Plugging these parameters into (4.18), we get

$$\pi_t = 0.99E_t\pi_{t+1} - 0.083\widehat{u}_t + 0.02\widehat{u}_{t-1} + 0.056E_t\widehat{u}_{t+1} - \lambda\Phi\gamma\widehat{a}_t \text{ [''American'']}$$

$$\pi_t = 0.99E_t\pi_{t+1} - 0.143\widehat{u}_t + 0.063\widehat{u}_{t-1} + 0.079E_t\widehat{u}_{t+1} - \lambda\Phi\gamma\widehat{a}_t \text{ [''Continental European'']}$$

The weight of lagged unemployment relative to the coefficients on current and lead unemployment is clearly higher under the continental European calibration than under the American one, as found by Jackman et al. The reduction in δ and x as we move from the American to the continental European calibration increases all three coefficients but the proportional increase is clearly the biggest for lagged unemployment.[16]

Parameter	"American"	"Continental European"
β	0.99	0.99
λ	0.08	0.08
θ	6	6
M	1.2	1.2
α	1	1
x	**0.7**	**0.25**
u	**0.05**	**0.1**
$\boldsymbol{\delta}$	**0.12**	**0.04**
B'	0.12	0.12
g	**0.084**	**0.03**

Table 4.1. Blanchard and Gali's Calibration

4.3. Determinacy in the Blanchard and Gali Model

We now explore under what conditions the Taylor principle ensures determinacy in the Blanchard Gali model. For that purpose, we first write our model as a system

[15] See Blanchard and Gali (2008), p.27.

[16] This is due to the fact that the absolute value of the coefficient on $\widehat{n}_{t-1}$ in the equation relating $\widehat{x}_t$ to $\widehat{n}_{t-1}$ and $\widehat{n}_t$ equals $\frac{(1-\delta)(1-x)}{\delta}$. The coefficient on $\widehat{n}_t$, which equals the coefficient on $\widehat{n}_{t+1}$ in $\widehat{x}_{t+1}$, depends only on $1/\delta$. Once we substitute out $\widehat{x}_t$ and $\widehat{x}_{t+1}$ in the marginal cost equation, this is multiplied with $(1-\delta)$ as the effect of future expected hiring costs depends on the likelihood that a job survives. Thus as δ and x both decrease, we will see a bigger increase of the coefficient on lagged employment (lagged unemployment) than on lead employment (lead unemployment).

in $\pi_t, \widehat{u}_t, \widehat{c}_t, \widehat{i}_t$ and $\widehat{a}_t$ and close it by adding an interest feedback rule. The full model consists of

$$\begin{aligned}
\pi_t &= \beta E_t \pi_{t+1} - \kappa_0 \widehat{u}_t + \kappa_L \widehat{u}_{t-1} + \kappa_F E_t \widehat{u}_{t+1} - \lambda p_0 \widehat{a}_t \\
\widehat{c}_t &= \widehat{a}_t - c_0 \widehat{u}_t - c_1 \widehat{u}_{t-1},\ c_0 = \frac{\xi_0}{1-u},\ c_1 = \frac{\xi_1}{1-u} \\
\widehat{c}_t &= E_t \widehat{c}_{t+1} - \left(\widehat{i}_t - E_t \pi_{t+1}\right) \\
\widehat{a}_t &= \rho_a \widehat{a}_{t-1} + e_t,\ e_t\ i.i.d. \sim \left(0, \sigma^2\right) \\
\widehat{i}_t &= \phi_\pi \pi_t + \phi_u \widehat{u}_t,\ \phi_\pi \geq 0,\ \phi_u \leq 0
\end{aligned} \tag{4.19}$$

The second equation is a log-linear approximation to equation (4.14) in the absence of skill loss. These equations can be reduced to system of three first order difference equations with variables π_t, $\widehat{u}_t$ and an auxiliary variable $\widehat{u}_t^L = \widehat{u}_{t-1}$ and the forcing process $\widehat{a}_t$:

$$\begin{pmatrix} E_t \pi_{t+1} \\ E_t \widehat{u}_{t+1} \\ \widehat{u}_{t+1}^L \end{pmatrix} = A \begin{pmatrix} \pi_t \\ \widehat{u}_t \\ \widehat{u}_t^L \end{pmatrix} + b \widehat{a}_t \tag{4.20}$$

where A is a 3x3 coefficient matrix and b is a 3x1 coefficient vector. This system has one predetermined endogenous variable, $\widehat{u}_t^L$, and two endogenous jump variables, π_t and $\widehat{u}_t$. To check for determinacy, we can thus apply proposition C.2 from Woodford (2003) to matrix A.[17] This is done in appendix C.1. The result is summarised in the following proposition:

[17] See Woodford (2003), p.672-673.

Proposition 1. *Consider the system described by* (4.19) *equilibrium is determinate if and only if* $\phi_\pi - \phi_u \frac{(1-\beta)}{\kappa_0 - \kappa_L - \kappa_F} > 1$ *and a set of other conditions discussed in the appendix are met, which however hold under reasonable restrictions on the parameters. Proof: appendix C.1*

The interpretation of this condition is analogous to the one derived in Woodford (2003) for the canonical New Keynesian model, since it also says that in the long run, a one-percentage-point increase in inflation should trigger an increase in the nominal interest rate of more than one.[18] In this chapter, if inflation increases permanently by one percentage point, this will increase the nominal interest rate directly by ϕ_π and indirectly through the reduction in unemployment, which amounts to $\frac{(1-\beta)}{\kappa_0 - \kappa_L - \kappa_F}$, times the coefficient on unemployment in the interest feedback rule, ϕ_u (which is restricted to be negative). Hence it suffices for determinacy to set $\phi_\pi > 1$.

4.4. Marginal Cost and Phillips Curve in the Presence of Skill Loss

The main difference between (4.16) and (4.17) is the presence of the $-a_1^L \widehat{a}_t^L + a_2^L E_t \widehat{a}_{t+1}^L + w_1^L \widehat{w}_t^L - w_2^L E_t \widehat{w}_{t+1}^L$ term, the $-p_1 E_t \widehat{a}_{t+1}^P$ term and the $-h_c E_t \widehat{mc}_{t+1}$ term in (4.16) The intuition for the impact of these on marginal costs was already provided in section 3.2. In this section we will express both the period t skill level of the average job seeker and the skill dependent real wage as a function of past employment alone. We also characterise the implied relationship between marginal cost and unemployment, and how the long run relationship between marginal cost and unemployment is shaped by the skill loss percentage δ_s and the job finding probability x.

To fully determine marginal cost, we will express both the skill level and the skill dependent component of the real wage as a function of past employment. In appendix

[18] Woodford (2003), p. 254.

C.3 we show after linearising (4.3), (4.4) and (4.6), (4.5), and (4.1), we can express the percentage deviation of the average skill level from its steady state $\widehat{a}_t^L$ as weighted infinite sum of past employment rates

$$\widehat{a}_t^L = \sum_{i=1}^{\infty} a_i^n \widehat{n}_{t-i}, \ a_i^n = \frac{1}{u}(1-x)^i \left(\beta_s^{i-1} - \beta_s^i\right) \tag{4.21}$$

and analogously for $\widehat{w}_t^L$ (using (4.8) instead of (4.3))

$$\widehat{w}_t^L = \sum_{i=1}^{\infty} w_i^n \widehat{n}_{t-i}, \ w_i^n = \frac{1}{u}(1-x)^i \left(\beta_s^{(1-\gamma)(i-1)} - \beta_s^{(1-\gamma)i}\right) \tag{4.22}$$

For both equations, the coefficients on past employment a_i^n and w_i^n are zero for $\delta_s = 0$ and larger than zero for $\delta_s > 0$. Higher past employment means that the unemployment spell of the average job seeker will be shorter. This increases the average skill level and by implication also increases his real wage.

Furthermore, both a_i^n and w_i^n decrease in the steady state job finding probability x. If people move quickly out of unemployment, the effect of $t-i$ employment on the average skill level in period t is lower since the additional worker employed in period $t-i$ had a high probability to find a job in period $t-i+1$ or after that anyway. Analogously, the effect of employment on the skill-dependent part of the real wage declines as well.

For the marginal cost of firms, what matters is not merely the direction of the effects of past employment on labour productivity and the real wage of the newly hired, but also their relative magnitude. We would also like to know how the latter depends on δ_s and x. Furthermore, what matters for our reasoning below will be the derivatives of the joint effects of past employment on the skill level and the real wage rather than the derivatives of the individual a_i^n coefficients. They are summarised by the following proposition. [19]

Proposition 2. *Let* $a^n = \sum_{i=1}^{\infty} a_i^n$ *and* $w^n = \sum_{i=1}^{\infty} w_i^n$*. Then* $a^n = \frac{1-x}{u}\frac{1-\beta_s}{1-(1-x)\beta_s}$, $w^n = \frac{1-x}{u}\frac{1-\beta_s^{1-\gamma}}{1-(1-x)\beta_s^{1-\gamma}}$ *and* $a^n > w^n$ *if and only if* $\gamma > 0$ *and* $\beta_s < 1$*. Furthermore,* $\frac{\partial a^n}{\partial \delta_s} = \frac{1-x}{u}\frac{x}{(1-(1-x)\beta_s)^2} > 0$ *and* $\frac{\partial w^n}{\partial \delta_s} = \frac{1-x}{u}(1-\gamma)\frac{x\beta_s^{-\gamma}}{\left(1-(1-x)\beta_s^{1-\gamma}\right)^2} > 0$. $\frac{\partial a^n}{\partial \delta^s} > \frac{\partial w^n}{\partial \delta^s}$ *if* β_s *is close to 1 and* $\gamma > 0$*. Furthermore,* $\frac{\partial a^n}{\partial x} = -\frac{1}{u}\frac{1-\beta_s}{(1-(1-x)\beta_s)^2} < 0$ *and* $\frac{\partial w^n}{\partial x} = -\frac{1}{u}\frac{1-\beta_s^{1-\gamma}}{\left(1-(1-x)\beta_s^{1-\gamma}\right)^2} < 0$*. Finally,* $\frac{\partial a^n}{\partial x} > \frac{\partial w^n}{\partial x}$ *if and only if* β_s *is close to one and* γ *is sufficiently large. Proof: appendix C.4.*

This proposition says that for positive skill loss and real wage rigidity, the joint effect of past employment levels on the quality of the average job seeker will always dominate the joint effect of past employment levels on the real wage. Furthermore, an increase in the quarterly skill-loss percentage will increase both the joint effect of past employment on the quality of the average job seeker and the real wage. However, if skill loss is small and there is real-wage rigidity, an increase in the quarterly skill-loss percentage will have a larger impact on the joint effect of past employment on the average skill level than on the effect of employment on the real wage.

Under qualitatively the same conditions, an increase in the job finding probability will reduce the joint effect of past employment on the quality of the average job seeker by more than the effect of past employment on the average real wage. These conditions are easily fulfilled for reasonable calibrations and in any case for the calibrations we will employ below.

The above implies that in the presence of real wage rigidity ($\gamma > 0$)

[19] As we show in appendix C.3, for $\delta_s > 0$, the relative magnitude of the a_i^n and w_i^n coefficients and in the case of $\frac{\partial a_i^n}{\partial \delta_s}$ and $\frac{\partial w_i^n}{\partial \delta_s}$ also the sign will depend on i.

- with positive skill loss ($\delta_s > 0$) a "permanent" increase in unemployment (decrease in employment) increases the ratio between the (average) wage of the newly hired and their average productivity, while a decline in unemployment (an increase in employment) decreases this ratio. More formally, for a given increase in unemployment $\Delta\widehat{a}^L < \Delta\widehat{w}^L$
- the size of the increase of the ratio between productivity and the real wage increases in δ_s. Hence if δ_s is higher, $\Delta\widehat{w}^L - \Delta\widehat{a}^L$ will be higher as well.
- the size of the increase in the gap between productivity and the real wage decreases in x. Hence if x is higher, $\Delta\widehat{w}^L - \Delta\widehat{a}^L$ will be smaller.

We now turn to the meaning of all this for the relationship between unemployment and marginal cost. Note that $a_1^L > a_2^L$ and $w_1^L > w_2^L$ if $\delta, \beta > 0$, as will be the case for reasonable calibrations. Hence we can obtain from (4.16) that a permanent increase in the average skill level will lower marginal cost and an increase in the (skill dependent component of) the real wage will increase it. This is because the gain from hiring today rather than tomorrow originating from the skill appreciation is uncertain and is being discounted. The same is true for the effect of the factors affecting this gain on marginal cost.

Furthermore, as can be obtained from their definitions, $a_1^L - a_2^L$ and $w_1^L - w_2^L$ will be quite close for sensible calibrations. We have seen that if unemployment increases permanently, both $\widehat{a}_t^L$ and $\widehat{a}_{t+1}^L$ decline by a larger amount than $\widehat{w}_t^L$ and $\widehat{w}_{t+1}^L$ if $\gamma > 0$. This means that unemployment increases marginal cost via this channel, the more so higher the degree of skill loss δ_s. Thus we would expect an increase in δ_s to make the link between unemployment and marginal cost less negative.

We now turn to characterize the effect of unemployment on marginal costs and how this effect depends on δ_s more rigorously. First, we quasi-difference (4.21) and (4.22), which yields

$$\begin{aligned}
\widehat{a}_t^L &= (1-x)\left(\frac{1}{u}(1-\beta_s)\widehat{n}_{t-1} + \beta_s \widehat{a}_{t-1}^L\right) \\
\widehat{w}_t^L &= (1-x)\left(\frac{1}{u}\left(1-\beta_s^{1-\gamma}\right)\widehat{n}_{t-1} + \beta_s^{1-\gamma}\widehat{w}_{t-1}^L\right)
\end{aligned}$$

Substituting these equations into (4.16) and using $\widehat{n}_t = \frac{-\widehat{u}_t}{1-u}$ yields

$$\begin{aligned}
\lambda\widehat{mc}_t &= -a^*\widehat{a}_t^L + w^*\widehat{w}_t^L - \kappa_0^*\widehat{u}_t + \kappa_L^*\widehat{u}_{t-1} + \kappa_F^* E_t\widehat{u}_{t+1} \\
&\quad -h_c E_t \lambda\widehat{mc}_{t+1} - \lambda(p_0 + \rho_a p_1)\widehat{a}_t^P \\
\widehat{a}_t^L &= (1-x)\left(-(1-\beta_s)\frac{\widehat{u}_{t-1}}{u(1-u)} + \beta_s\widehat{a}_{t-1}^L\right) \\
\widehat{w}_t^L &= (1-x)\left(-\left(1-\beta_s^{1-\gamma}\right)\frac{\widehat{u}_{t-1}}{u(1-u)} + \beta_s^{1-\gamma}\widehat{w}_{t-1}^L\right) \\
a^* &= \lambda\left(a_1^L - a_2^L(1-x)\beta_s\right) \\
w^* &= \lambda\left(w_1^L - w_2^L(1-x)\beta_s^{1-\gamma}\right) \\
\kappa_0^* &= \lambda\frac{\left(h_0' + (1-x)\left(a_2^L\frac{(1-\beta_s)}{u} - w_2^L\frac{\left(1-\beta_s^{1-\gamma}\right)}{u}\right)\right)}{1-u} \\
\kappa_L^* &= \frac{-\lambda h_L'}{1-u},\ \kappa_F^* = \frac{-\lambda h_F'}{1-u}
\end{aligned} \tag{4.23}$$

Setting $\widehat{mc}_{t+1} = \widehat{mc}_t = \widehat{mc}$, $\widehat{u}_{t+1} = \widehat{u}_t = \widehat{u}_{t-1} = \widehat{u}$, $\widehat{a}_t^L = \widehat{a}_{t-1}^L = \widehat{a}^L$ and $\widehat{w}_t^L = \widehat{w}_{t-1}^L = \widehat{w}^L$ and ignoring exogenous technology, we can write

$$\lambda\widehat{mc} = -\frac{\left[\kappa_0^* - \kappa_F^* - \kappa_{L1}^* - a^*\frac{(1-\beta_s)(1-x)}{u(1-u)(1-(1-x)\beta_s)} + w^*\frac{\left(1-\beta_s^{1-\gamma}\right)(1-x)}{u(1-u)\left(1-(1-x)\beta_s^{1-\gamma}\right)}\right]}{1+h_c}\widehat{u}$$

$$(4.24) \qquad = -\kappa\widehat{u}$$

$$\kappa = \frac{h_0^{'} + h_L^{'} + h_F^{'} - \left[\frac{(1-x)}{u}\frac{(1-\beta_s)\left(a_1^L - a_2^L\right)}{(1-(1-x)\beta_s)} - \frac{(1-x)}{u}\frac{\left(1-\beta_s^{1-\gamma}\right)\left(w_1^L - w_2^L\right)}{\left(1-(1-x)\beta_s^{1-\gamma}\right)}\right]}{(1+h_c)(1-u)}\lambda$$

$-\kappa$ gives the effect of a "permanent" increase in unemployment on marginal cost. Most conveniently, substituting the definitions of $h_0^{'}$, $h_L^{'}$ and $h_F^{'}$ yields

$$h_0^{'} + h_L^{'} + h_F^{'} = \frac{\alpha g M}{\delta}\left[1 + \beta(1-\delta)^2(1-x) - (1-\delta)(1-x) - \beta(1-\delta)\right]$$

which happens to be exactly the same as $h_0 + h_L + h_F$, is thus always positive and independent of δ^s.Hence in κ only the term in the squared brackets and h_c actually depend on skill loss. The term in the squared bracket will be zero if there is no skill loss ($\beta_s = 1$), implying that $\kappa > 0$ and thus a negative effect of a "permanent" increase in unemployment on marginal cost.

With positive skill loss, the squared bracket represents the "skill loss channel" from unemployment to marginal cost. The first term gives the decline of the skill level of the average applicant caused by the decline in $\widehat{n}$ associated with the increase in $\widehat{u}$ (note that $\frac{(1-x)}{u}\frac{(1-\beta_s)}{(1-(1-x)\beta_s)} = a^n$) times the net effect of a permanent skill level decline on marginal cost ($\left(a_1^L - a_2^L\right)$). The second term gives the decline of the skill-dependent real wage caused by the decline in $\widehat{n}$ associated with the increase in $\widehat{u}$ (Note that $\frac{(1-x)}{u}\frac{\left(1-\beta_s^{1-\gamma}\right)}{\left(1-(1-x)\beta_s^{1-\gamma}\right)} = w^n$) times the net effect of a permanent skill decline in the skill dependent real wage on marginal cost (-$\left(w_1^L - w_2^L\right)$).

As δ_s grows, we would expect the squared bracket to grow as well if the real wage is rigid. As was pointed out above, an increase in δ_s means that the gap between productivity and the real wage shrinks at a faster rate as unemployment increases. This would lower κ. To check our intuition, we take the derivative of κ with respect to δ_s and arrive at the following proposition:

Proposition 3. *Let κ be as in* (4.24) *and let δ_s close to zero. Then $\frac{\partial\kappa}{\partial\delta_s} < 0$ if $\gamma > \frac{B'x^{\alpha}M\beta(1-\delta)}{1-B'x^{\alpha}M(1-\beta(1-\delta))}$. Proof: appendix C.6.*[20]

Accepting the restriction on δ_s, the condition for $\frac{\partial\kappa}{\partial\delta_s} < 0$ is easily fulfilled for the calibrations adopted in this chapter since $B'x^{\alpha}M\beta(1-\delta)$ is a small number, while $1 - B'x^{\alpha}M(1-\beta(1-\delta))$ is very close to one.[21]

Thus an increase in δ_s indeed makes the effect of unemployment on marginal costs less negative. This raises the possibility of κ turning negative as δ_s increases. To put it differently, an increase in unemployment would then cause an increase rather than a decrease in marginal cost, and, by implication, inflation. This has consequences for the determinacy properties of the interest feedback rule of the central bank which we will come back to in the following section.

We are also interested in how a change in x for a given unemployment rate will affect κ and $\frac{\partial\kappa}{\partial\delta_s}$. It is easy to show that in the absence of skill loss, $\frac{\partial\kappa}{\partial x} > 0$. Hence in the

[20] *A more general proof without restrictions on δ_s would have been desirable but struck us as impossible due to the complexity of the expression resulting from $\frac{\partial\kappa}{\partial\delta_s}$.*

[21] One might wonder why the condition in the proposition does not simply say $\gamma > 0$. For better understanding, not first that this is merely a sufficient not a necessary and sufficient condition. As can be obtained from appendix C.6, the necessary and sufficient value of γ would be lower. Furthermore, it can obtained from (4.11) that even if there is no real wage rigidity and thus W_t would move by the same percentage as A_t^L the effects of a decline or increase in the average skill level would not be neutral. This is because the t+1 flow profit associated with hiring in t $mc_{t+1}A_{t+1}^P - W_{t+1}^0$ does not depend on the skill level of the average applicant. Thus a permanent decline in A_t^L affect mc_t in some way even if there is no real wage rigidity. The resulting effect can be obtained from (4.24) by setting $\gamma = 0$ in the squared bracket: $\left(\left(a_1^L - a_2^L\right) - \left(w_1^L - w_2^L\right)\right)\frac{(1-x)}{u}\frac{(1-\beta_s)}{(1-(1-x)\beta_s)}$.

absence of skill loss, the effect of a permanent increase in unemployment on marginal cost will be more negative. This is due to the reasons discussed earlier. Introducing skill loss adds two opposing forces of a change in x on both κ and $\frac{\partial \kappa}{\partial \delta_s}$. On the one hand, as was shown above, an increase in x will lower in absolute value the negative effect of past unemployment on the skill level a^n and, to a lesser extent, the effect of past unemployment on the real wage w^n. On the other hand, an increase in x given u will increase δ, implying that the gain associated with the skill appreciation of a worker hired today becomes more uncertain. This is reflected in the fact that both a_2^L and w_2^L decrease as δ increases, thus reducing the effect of the $\widehat{a}_{t+1}^L$ and $\widehat{w}_{t+1}^L$terms in (4.16). Concerning the effect on of a change of x on $\frac{\partial \kappa}{\partial \delta_s}$, we are able to prove the following proposition:

Proposition 4. *Let κ be as in* (4.24)*, δ_s close to zero and α close to 1. Then $\frac{\partial^2 \kappa}{\partial \delta_s \partial x} > 0$ if $x < \frac{4-u-\sqrt{u^2+8u}}{4}$ Proof: appendix C.6*[22]

This condition holds a for a wide range of reasonable calibrations of x and u, including those considered in this chapter. Hence an increase in the job finding probability x reduces the effect of δ_s on κ. To put it differently, if the job finding probability is higher, increasing δ_s will still weaken the (negative) link between marginal cost and employment, but to a lesser extent than in a less fluid labour market.

The model developed above features multiple links between unemployment and marginal costs. To sum up what we have learned, a permanent increase in unemployment has the following four effects on marginal cost in period t:

[22] *Again this is a sufficient condition not a necessary and sufficient one, which can be obtained from appendix C.6. For instance, the condition reported has γ set equal to zero. In fact the maximum value of x increases in γ.*

(1) The increase in period t unemployment lowers period t hiring costs (-$h_0^{'}/(1-u)$), which tends to lower marginal cost. The strength of this channel increases in the job finding probability x.

(2) The increase in period $t+1$ unemployment lowers period t+1 hiring cost, which tends to increase marginal cost $(-h_F^{'}/(1-u))$. The strength of this channel decreases in x.

(3) An increase in period $t-1$ unemployment increases period t hiring costs by increasing the amount of hiring necessary to reach a given level of employment $(h_L^{'}/(1-u))$. The strength of this channel decreases in x.

(4) An increase in period t to $t-\infty$ unemployment increases $\widehat{a}_t^L - \widehat{w}_t^L$ and $\widehat{a}_{t+1}^L - \widehat{w}_{t+1}^L$. The net effect of this is to increase marginal cost. $\left(\frac{(1-x)}{u(1-u)}\frac{(1-\beta_s)\left(a_1^L-a_2^L\right)}{(1-(1-x)\beta_s)} - \frac{(1-x)}{u(1-u)}\frac{\left(1-\beta_s^{1-\gamma}\right)\left(w_1^L-w_2^L\right)}{\left(1-(1-x)\beta_s^{1-\gamma}\right)(1-u)} > 0 \text{ if } \delta_s > 0\right)$. The strength of this channel increases in the skill loss percentage δ_s and decreases in x.

Note that effects 1-3 are already present in the model without skill loss, while the fourth effect arises from the introduction of skill loss among the unemployed.

Note that like in chapter two, we endogenise total factor productivity of the marginal worker. Introducing skill decay among the unemployed renders it a positive function of past employment and thus of past levels of aggregate demand. Thus just as with endogenous growth, lower aggregate demand today implies productivity will be lower than it otherwise would have been, implying that ceteris paribus marginal costs will be higher as well.

4.5. Determinacy in the Model with Skill Loss

We now investigate which policy rules guarantee determinacy in the presence of skill loss. The first question we are interested in is whether $\phi_\pi > 1$ is still a sufficient condition to establish determinacy for varying levels of skill loss. Thus we consider current, forward and backward looking rules where the interest rate responds only to inflation. We are dealing with the following system:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t \tag{M1}$$

$$\lambda \widehat{mc}_t = -a^* \widehat{a}_t^L + w^* \widehat{w}_t^L - \kappa_0^* \widehat{u}_t + \kappa_L^* \widehat{u}_{t-1} + \kappa_F^* E_t \widehat{u}_{t+1} \tag{M2}$$

$$-h_c E_t \lambda \widehat{mc}_{t+1} - \lambda \left(p_0 + \rho_a p_1\right) \widehat{a}_t^P$$

$$\widehat{a}_t^L = (1-x)\left(-\left(1-\beta_s\right)\frac{\widehat{u}_{t-1}}{u\,(1-u)} + \beta_s \widehat{a}_{t-1}^L\right) \tag{M3}$$

$$\widehat{w}_t^L = (1-x)\left(-\left(1-\beta_s^{1-\gamma}\right)\frac{\widehat{u}_{t-1}}{u\,(1-u)} + \beta_s^{1-\gamma} \widehat{w}_{t-1}^L\right) \tag{M4}$$

$$\widehat{c}_t = \widehat{a}_t^P + c_L \widehat{a}_t^L - c_0^* \widehat{u}_t - c_1^* \widehat{u}_{t-1} \tag{M5}$$

$$c^L = \frac{A^L \delta\,(1-g)}{A^A - A^L g \delta},\ c_0^* = \frac{\xi_0^{'}}{1-u},\ c_1^* = \frac{\xi_1^{'}}{1-u}$$

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \left(\widehat{i}_t - E_t \pi_{t+1}\right) \tag{M6}$$

$$\widehat{a}_t^P = \rho_a \widehat{a}_{t-1}^P + e_t,\ e_t\ i.i.d. \sim \left(0, \sigma^2\right) \tag{M7}$$

$$\widehat{i}_t = \phi_\pi E_t \pi_{t+j},\ \phi_\pi \geq 0,\ -1 \leq j \leq 1 \tag{M8}$$

$(M5)$ is derived in appendix A.7. Unfortunately, unlike in the original Blanchard/ Gali model, we cannot establish the conditions for determinacy analytically.[23] Therefore we solve the model numerically using the software Dynare and perform a grid search for values of δ_s between 0 and 0.07 (step size: 0.005) and values of ϕ_π between 0 and 3

[23]The model with skill decay has three forward looking variables and three state variables. This implies that we can not apply the conditions derived by Woodford (2003) which we used to derive the determinacy conditions for the Blanchard and Gali model. As far as we are aware, there is no straightforward way to analytically determine the eigenvalues of a 5x5 system.

(step size: 0.1). All other parameters are set to meet Blanchard and Gali's "Continental European" calibration as reproduced in table 4.1. We then repeat the grid search for the "American" calibration.[24] The determinacy regions for the current looking rule are graphed in figures 4.1 and 4.2. The area between the two lines denotes the determinacy region in both graphs (including the points situated on these lines). For the European calibration, for values of $\delta_s \geq 0.025$, the standard requirement on ϕ_π to guarantee determinacy is reversed: A unique equilibrium now requires $\phi_\pi \leq 0.9$. The determinacy regions for the backward and forward looking rules (not shown) are almost identical. In particular, under the Continental European calibration, the drop of the maximum value of ϕ_π to 0.9 for $\delta_s \geq 0.025$ carries over. This suggests that it is not the timing of the active response to inflation but the active response to inflation per se which induces indeterminacy.

By contrast, for the American calibration, $\phi_\pi > 1$ does guarantee determinacy for the whole range considered here. The determinacy regions for the current, forward and backward looking policy rule are completely identical. Experimentation suggest that for the current looking rule, the $\phi_\pi \leq 0.9$ requirement only becomes relevant at $\delta_s \geq 0.225$.

The intuition for this result can be gained by showing how the effect of a "permanent" increase in unemployment on marginal costs depends on δ_s. As can be seen from (4.24), in the absence of skill loss this effect is negative since $h_0^{'} + h_L^{'} + h_F^{'} > 0$. However, as we have shown in the previous section, $\frac{\partial \kappa}{\partial \delta_s} < 0$. Thus as we increase δ_s, κ will ultimately turn negative. Figure 4.3 plots κ against δ_s for both the European (broken line) and the American (solid line) calibration. Note that under the continental

[24] Note that the following results discussed below also hold if we use the same lower unemployment rate under the American calibration as under the European calibration. For a given job finding probability, this implies a lower job destruction rate.

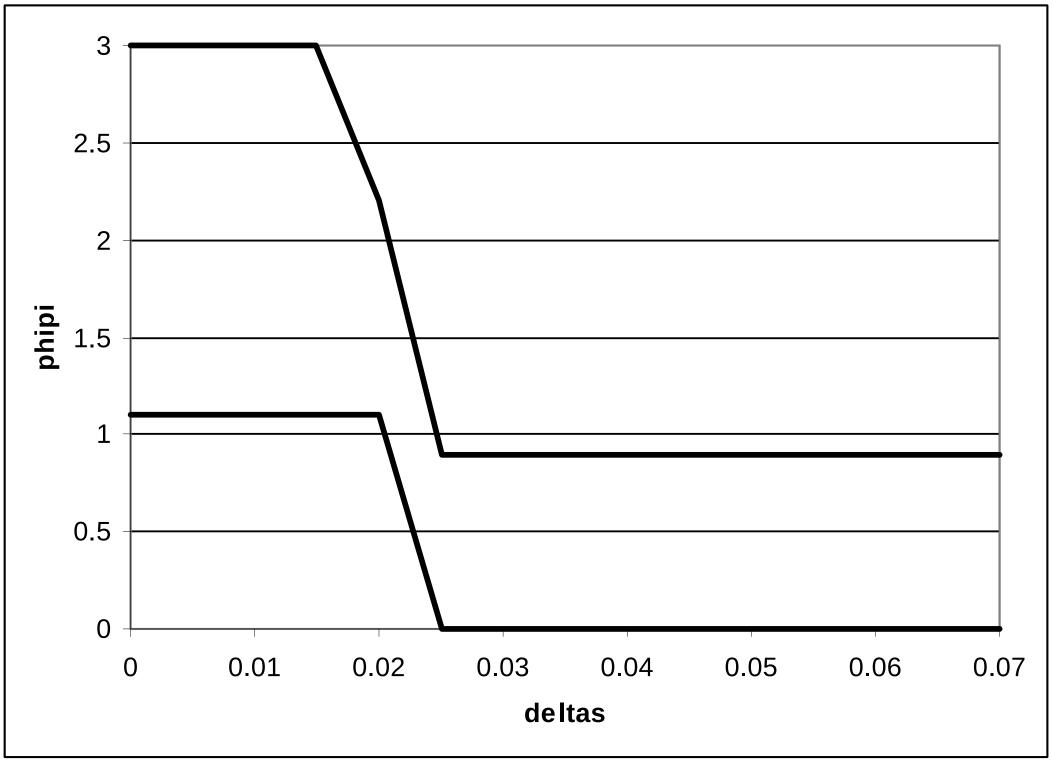

Figure 4.1. Continental European Calibration: Determinacy Region

European calibration, the level of skill loss for which this expression turns negative is the same for which the determinacy requirement switches to $\phi_\pi \leq 0.9$, i.e. 0.025. Thus if marginal costs, and thus inflation, increases in response to a persistent increase in the unemployment rate, the central bank should lower the real interest rate. This policy lowers the real interest rate, hence increases demand therefore does not validate the increase in unemployment. By contrast, with $\phi_\pi \geq 1$, there is scope for sunspot equilibria if δ_s exceeds its respective critical value: A persistent increase in unemployment will ultimately lead to an increase in inflation and (as $\phi_\pi \geq 1$) the real interest rate, irrespective of whether the central bank responds to lagged, current or expected future inflation. This lowers demand and thus validates the increase in unemployment. In the next section, when we display the impulse response function to a sunspot shock, we show that this is in fact exactly what happens.

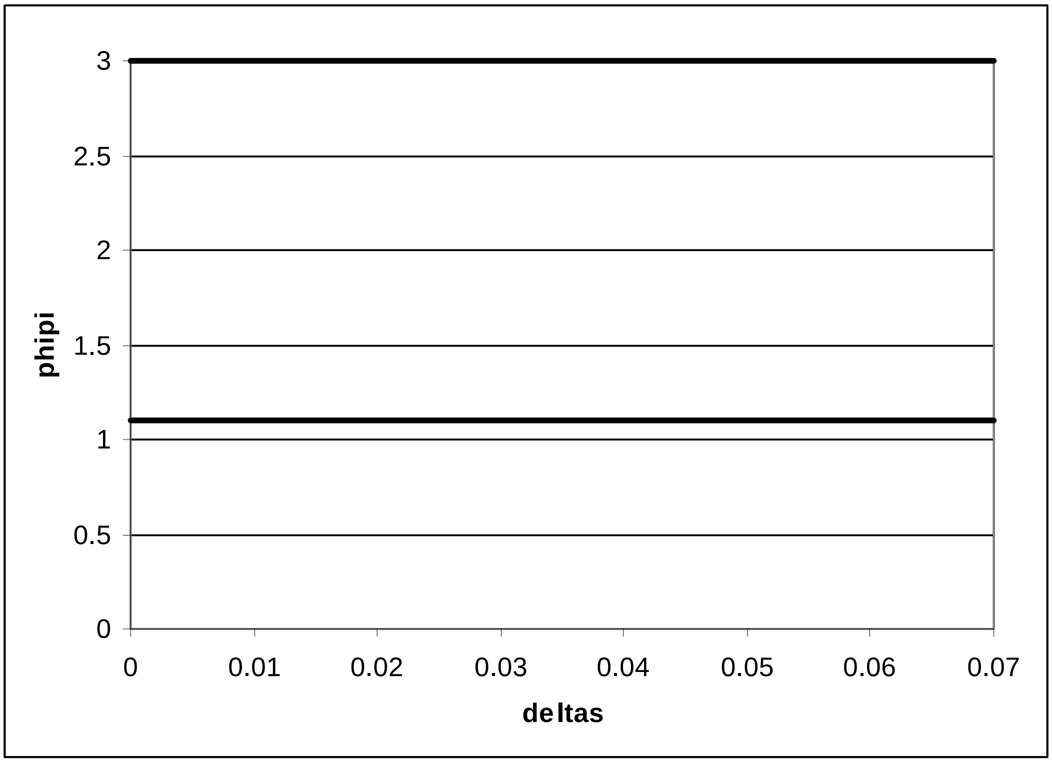

Figure 4.2. US Calibration: Determinacy Region

This leaves the question why this critical value is so much higher for the American than for the continental European calibration. The chief reason for this is that due to the more fluid labour market associated with the American calibration, for $\delta_s = 0$, κ is a lot higher than under the continental European calibration. The intuition for that was discussed in section 4.2.3: The higher the job destruction probability δ, the lower is the effect of lagged and lead unemployment on period t marginal cost. The reason is that with higher δ, period t hiring and thus period t hiring cost depend less on period $t-1$ employment since more jobs are destroyed as we move from period $t-1$ to period t. Similarly, the possibility to save hiring costs by moving job creation from $t+1$ to t is also reduced since fewer jobs survive from period t to $t+1$. Hence the effect of $t+1$ hiring costs and thus period t employment on marginal cost is reduced as well. Furthermore, we have shown in the previous section that if x is higher, the effect of δ_s

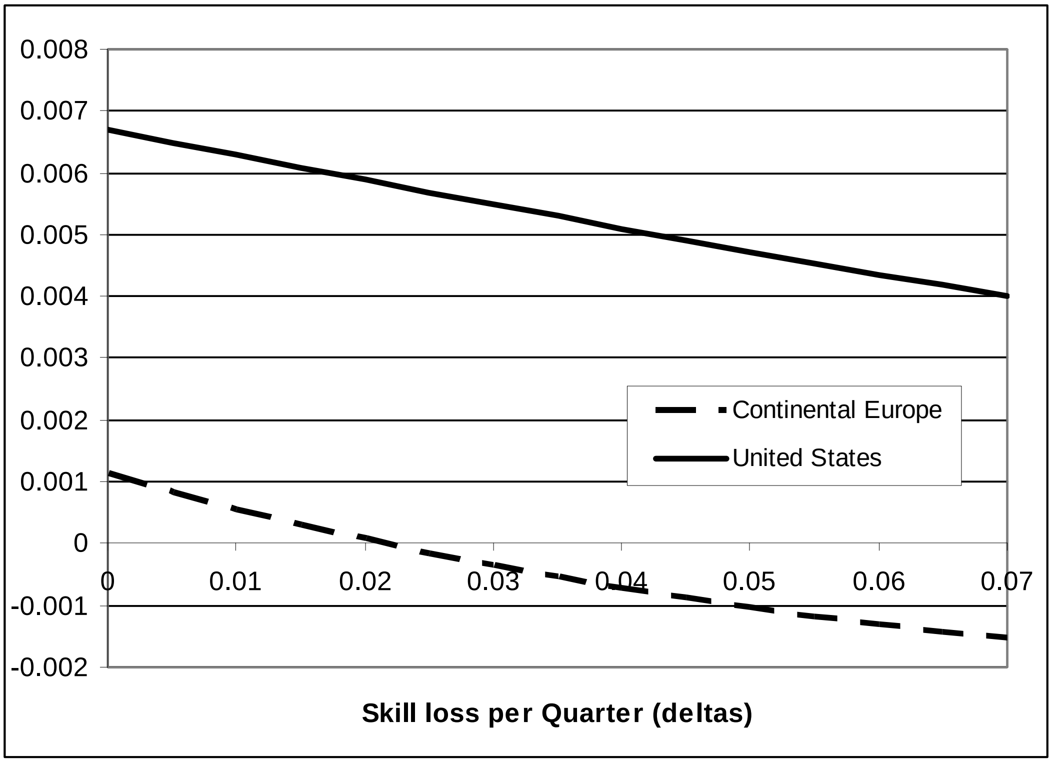

Figure 4.3. kappa

on κ will be less ($\frac{\partial^2\kappa}{\partial\delta_s\partial x} > 0$). Therefore under the American calibration, κ decreases a little less as δ_s increases than under the European calibration.

We now check whether interest rate smoothing would help to restore determinacy. Therefore we replace M8 by $\widehat{i}_t = (1-\phi_i)\,\phi_\pi\pi_t + \phi_i\widehat{i}_{t-1}$ and perform a grid search over ϕ_π, ϕ_i and δ_s, with $\phi_\pi = [0,\ 3]$, $\phi_i = [0,\ 1]$ and $\delta_s = [0,\ 0.07]$. The determinacy requirement on ϕ_π remains almost unaffected.[25] In particular, determinacy requires $\phi_\pi \leq 0.9$ if $\delta_s \geq 0.025$ independently of the degree of interest rate smoothing. This result is in line with the intuition given above as even with interest rate smoothing, if $\phi_\pi > 1$, an increase in inflation ultimately increases the real interest rate.

[25]Only for $\rho = 0.8$ and $\rho = 0.9$ does smoothing make a difference in that for $\delta^s = 0.02$, the maximum value for ϕ_π increases to 2.3 and 2.5, respectively. For $\delta^s \geq 0.025$, the maximum value of ϕ_π th drops to 0.9, as for all other degrees of smoothing.

We investigate next whether responding to the output gap in addition to inflation helps to restore determinacy under the European calibration. As is standard in the New Keynesian literature, we define potential output Y_t^n as the output level including hiring costs at which final goods firms charge their desired mark-up, implying that marginal cost is at its steady state. The associated unemployment rate is denoted as u_t^n. As marginal cost is affected by both lead unemployment and lead marginal cost, when deriving u_t^n, we will further assume that if unemployment is at its natural level in period t, it will be expected to be at its natural level in period t+1 as well.[26] Thus we are dealing with the following system:

[26]The following results are broadly robust against relaxing this assumption.

$$
\begin{aligned}
\pi_t &= \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t \\
\lambda \widehat{mc}_t &= -a^* \widehat{a}_t^L + w^* \widehat{w}_t^L - \kappa_0^* \widehat{u}_t + \kappa_L^* \widehat{u}_{t-1} + \kappa_F^* E_t \widehat{u}_{t+1} - h_c E_t \lambda \widehat{mc}_{t+1} \\
&\quad -\lambda \left(p_0 + \rho_a p_1\right) \widehat{a}_t^P \\
\widehat{a}_t^L &= (1-x) \left(-\left(1-\beta_s\right) \frac{\widehat{u}_{t-1}}{u(1-u)} + \beta_s \widehat{a}_{t-1}^L \right) \\
\widehat{w}_t^L &= (1-x) \left(-\left(1-\beta_s^{1-\gamma}\right) \frac{\widehat{u}_{t-1}}{u(1-u)} + \beta_s^{1-\gamma} \widehat{w}_{t-1}^L \right) \\
\widehat{c}_t &= \widehat{a}_t^P + c_L \widehat{a}_t^P - c_0^* \widehat{u}_t - c_1^* \widehat{u}_{t-1}, \; c^L = \frac{A^L \delta (1-g)}{A^A - A^L g \delta}, \\
c_0^* &= \frac{\xi_0^{'}}{1-u}, \; c_1^* = \frac{\xi_1^{'}}{1-u} \\
\widehat{c}_t &= E_t \widehat{c}_{t+1} - \left(\widehat{i}_t - E_t \pi_{t+1} \right) \\
\widehat{a}_t^P &= \rho_a \widehat{a}_{t-1}^P + e_t, \; e_t \; i.i.d. \sim \left(0, \sigma^2\right) \\
\widehat{u}_t^n &= \frac{\kappa_F^* E_t \widehat{u}_{t+1}^n + \kappa_L^* \widehat{u}_{t-1} + -a^* \widehat{a}_t^L + w^* \widehat{w}_t^L - \lambda p_0 \widehat{a}_t^P - \lambda p_1 E_t \widehat{a}_{t+1}^P}{\kappa_0^*} \\
\widehat{y}_t &= \widehat{a}_t^P + y^L \widehat{a}_t^L - y_0 \widehat{u}_t - y_1 \widehat{u}_{t-1}, \; y_L = \frac{\delta A^L}{A^A}, \; y_0 = \frac{A^L}{A^A (1-u)}, \\
y_1 &= \frac{\left(1-A^L\right)(1-\delta)}{A^A (1-u)} \\
\widehat{y}_t^n &= \widehat{a}_t^P + y^L \widehat{a}_t^L - y_0 \widehat{u}_t^n - y_1 \widehat{u}_{t-1} \\
\widehat{i}_t &= \phi_\pi \pi_t + \phi_y \left(\widehat{y}_t - \widehat{y}_t^n \right), \; \phi_\pi, \phi_y \geq 0
\end{aligned}
$$

The equation for $\widehat{u}_t^n$ was derived by setting $\widehat{mc}_t = \widehat{mc}_{t+1} = 0$ and $\widehat{u}_{t+1} = \widehat{u}_{t+1}^n$ in the marginal cost equation, while the equation describing the deviation of output including hiring costs from its steady state is derived in appendix C.5. Clearly $\widehat{u}_t^n$ depends on past values of actual unemployment as well as its own future value.

We perform a grid search over ϕ_π, ϕ_y and δ_s, with $\phi_\pi = [0,\ 3]$, $\phi_y = [0,\ 3]$ (step size 0.1) and $\delta_s = [0,\ 0.07]$. We find that responding to the output gap extends the determinacy region if $\delta_s < 0.025$ but reduces it if $\delta_s \geq 0.025$. For example, figure 4.4 plots the lowest value of ϕ_π compatible with determinacy against ϕ_y for $\delta_s = 0$. Clearly the lower bound of ϕ_π declines as ϕ_y increases. By contrast, figure 4.5 plots the highest value of ϕ_π compatible with determinacy for the case of $\delta_s = 0.025$. The upper bound of ϕ_π is declining, thus reducing the determinacy region.

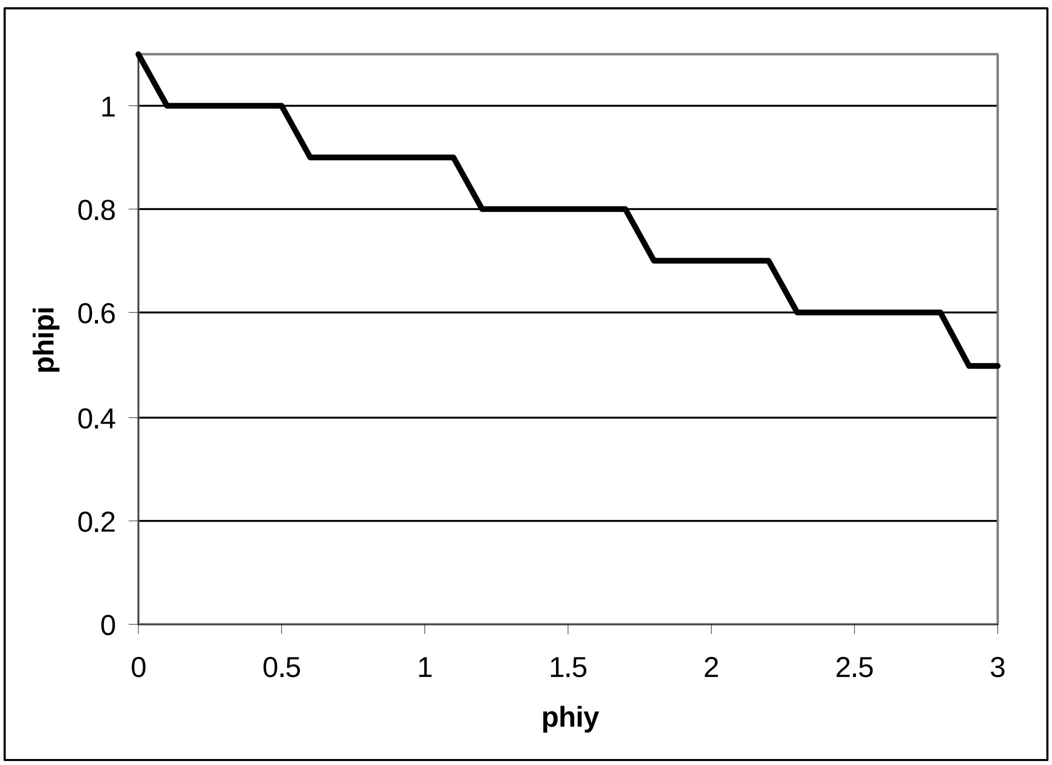

Figure 4.4. Continental Europe, $\delta_s = 0$, lower Bound of the Determinacy Region

Intuition for this result can be gained from the effect of actual unemployment on natural unemployment. It is easy to see that $\widehat{y}_t - \widehat{y}_t^n = -y_0 \left(\widehat{u}_t - \widehat{u}_t^n\right)$. Hence the output gap depends positively on $\widehat{u}_t^n$. Solving $\widehat{u}_t^n$ forward (ignoring the exogenous productivity process) yields $\widehat{u}_t^n = \sum_{i=0}^{\infty} \left(\frac{\kappa_F^*}{\kappa_0^*}\right)^i \left[\kappa_L^* \widehat{u}_{t-1} + a^* \widehat{a}_t^L - w^* \widehat{w}_t^L\right]$. Let us again assume for

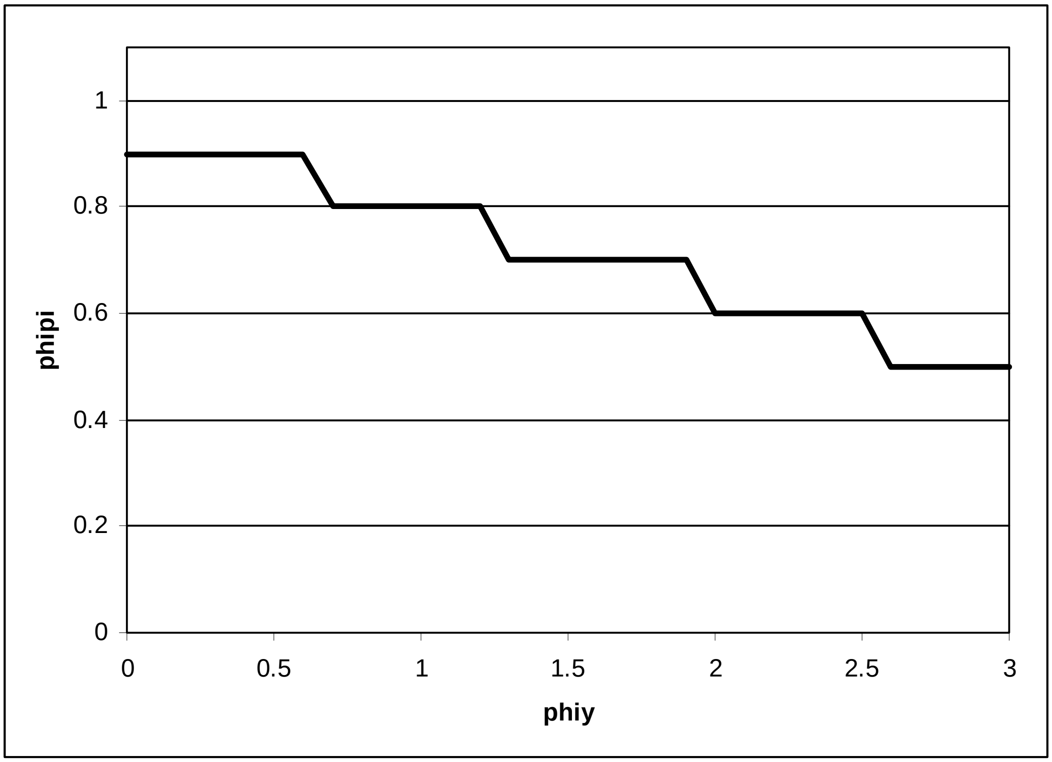

Figure 4.5. Continental Europe, $\delta_s = 0.025$, upper Bound of the Determinacy Region

simplicity that $\widehat{u}_{t+i} = \widehat{u}$. We then have

$$\widehat{u}^n = \frac{\kappa_L^* + a^* \frac{(1-\beta_s)(1-x)}{u(1-u)(1-(1-x)\beta_s)} - w^* \frac{\left(1-\beta_s^{1-\gamma}\right)(1-x)}{u(1-u)\left(1-(1-x)\beta_s^{1-\gamma}\right)}}{\kappa_0^* - \kappa_F^*} \widehat{u}$$

If $\frac{\partial \widehat{u}^n}{\partial \widehat{u}} < 1$, then an increase in unemployment increases natural unemployment less than one for one. It thus lowers the output gap and tends to lower real interest rate. This should stabilise unemployment. By contrast, if $\frac{\partial \widehat{u}^n}{\partial \widehat{u}} > 1$, an increase in unemployment will increase $\widehat{u}^n$ more than one for one and thus tend to increase the real interest rate. In this case responding to the output gap is actually destabilising. Moreover, note that $\frac{\partial \widehat{u}^n}{\partial \widehat{u}} > 1 \Leftrightarrow \kappa_0^* - \kappa_F^* - \kappa_L^* - a^* \frac{(1-\beta_s)(1-x)}{u(1-u)(1-(1-x)\beta_s)} + w^* \frac{\left(1-\beta_s^{1-\gamma}\right)(1-x)}{u(1-u)\left(1-(1-x)\beta_s^{1-\gamma}\right)} < 0$, implying that $\kappa < 0$. As was shown above, this will be true if $\delta^s \geq 0.025$. Hence responding to the output gap will tend to destabilise the economy precisely when responding more than one for one to inflation tends to destabilise the economy as well.

If natural output tracks actual output too closely for the output gap to be a stabilising argument in the policy rule, then perhaps the deviation of unemployment from its steady state (rather than its natural) value will help to achieve determinacy. Thus we consider the policy rule $\widehat{i}_t = \phi_\pi \pi_t + \phi_u \widehat{u}_t$ and conduct a grid search over δ_s, ϕ_π and ϕ_u, with $\phi_\pi = [0,\ 3]$, $\phi_u = [0,\ -3]$ (step size 0.1) and $\delta_s = [0,\ 0.07]$. It turns out that responding to unemployment has a strong stabilising effect. Setting $\phi_u = -0.1$ guarantees determinacy for $\phi_\pi \geq 0.2$ if $\delta_s \leq 0.02$ and for the full interval of ϕ_π for $0 < \delta_s \leq 0.035$. For higher values of δ^s the upper bound of ϕ_π again begins to decline. For $\phi_u = -0.2$, determinacy is guaranteed for the full interval of ϕ_π as long as $\delta_s \leq 0.055$. Finally, for $\phi_u \leq -0.3$, the equilibrium is determinate for any combination of ϕ_π and δ_s. Thus a modest response to unemployment restores determinacy and in doing so is robust against variations in δ_s.

Let us assume that our model in its respective calibrations of u, x, and δ indeed captures major differences between the continental European and the US economy. Furthermore, note that the value of δ_s for which the value of ϕ_π begins to be bounded above under the American Calibration seems implausibly high. With $\delta_s = 0.225$, a worker would have lost about 64% after one year of unemployment. By contrast, the critical value of δ_s for the continental European calibration seems a lot more plausible. It would imply a skill loss of about 9.6% after one year of unemployment. Note also that estimates of interest feedback rules suggest that the Federal Reserve as well as the Bundesbank and the ECB respond more than one for one to inflation and pay some attention to the output gap as well.[27] Hence we conclude that indeterminacy is a far more realistic scenario in Europe than in the United States.

Moreover, in the 1970s many central banks moved away from a "Keynesian" monetary policy which focuses on stabilising unemployment to a policy which aggressively targets inflation but pays little attention to unemployment. Within the model proposed here, with $\delta^s \geq 0.025$, this move would cause no determinacy problem with a fluid American labour market but would induce indeterminacy if labour market flows were low as in continental Europe. In the next section we will investigate the dynamics of unemployment and inflation for the European calibration with $\delta_s \geq 0.025$ and $\phi_\pi > 1$.

4.6. Dynamics under Indeterminacy

The previous section showed that a policy rule which increases the nominal interest rate more than one for one with inflation might quite likely imply indeterminacy if the flow - characteristics of the labour market are "continental European" in the sense that there is little hiring and firing and there is some skill loss among the unemployed. This renders indeterminacy a realistic scenario for the continental European calibration. Since there is an infinite number of stable equilibria, self-fulfilling prophecies can generate endogenous fluctuations of unemployment and other endogenous variables. In this section we investigate the response of the model under the continental European calibration and with skill loss being at the critical level of 0.025 to a sunspot shock and a non-correlated technology shock. The main focus of the discussion is to illustrate that the responses of unemployment, marginal cost, inflation and the real interest rate to

[27] See for instance Clarida et al. (1998), Orphanides (2001) and Clausen and Meier (2003) for reaction function estimates for the Fed and the Bundesbank and Gorter et al. (2008) and Sauer and Sturm (2003) for estimates for the ECB.

the sunspot shock are perfectly in line with the intuition given in the previous section for why $\delta_s \geq 0.025$ (and thus $\kappa < 0$) combined with $\phi_\pi > 1$ induces indeterminacy

To solve the indeterminate model, we follow a solution method proposed by Lubik and Schorfheide (2003). Their method builds on an approach by Sims (2002). Sims proposed to solve linear rational expectation (RE) models by solving for the vector of expectational errors $\eta_t = q_t - E_{t-1}q_t$, where q_t is a vector of variables over which agents form expectations. Thus the linear RE model is cast in the following form

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi \varepsilon_t + \Pi \eta_t \tag{4.25}$$

where ε_t denotes an i.i.d vector of structural shocks and all variables with a t and t-1 subscript are observable at time t, and all variables with a t-1 subscript are predetermined. Any system of first order difference equations can be brought into this form by replacing $E_t q_{t+1}$ with $y_t = E_t q_{t+1}$ and the adding an equation reading $q_t = y_{t-1} + \eta_t$. Thus there will be an expectation error for each forward-looking variable. Note that all variables on the right hand side except for η_t are either predetermined or exogenous.

The system has a stable solution if there exists a vector η_t as a function of the exogenous shocks ε_t to eliminate the explosive components of y_t. The solution is a unique solution if the vector of structural shocks ε_t uniquely determines the vector of expectational errors η_t. The solution will not be unique if the number of expectation errors exceeds the number of explosive components in y_t.[28] This opens the door for sunspot shocks to affect the endogenous variables. Lubik and Schorfheide suggest to interpret these shocks as belief shocks that trigger reversion of forecasts of the endogenous variable. Suppose that due to a sunspot the expectation of q_t between t and t-1

[28]See Lubik and Schorfheide (2003), pp. 276-277.

is revised by v_t. Hence

$$q_t = (E_{t-1}q_t + v_t) + \widetilde{\eta}_t$$

where the term in brackets denotes the revised forecast and $\widetilde{\eta}_t$ is the error associated with this revised forecast.[29] Thus (4.25) can be written as

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \begin{bmatrix} \Psi & \Pi \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ v_t \end{bmatrix} + \Pi\widetilde{\eta}_t \tag{4.26}$$

If the solution is unique, v_t will not appear in the solution.

We then assume that the effects of the sunspot shock v_t and the structural shock ε_t to the forecast error are orthogonal to each other. This is a standard assumption in the literature on indeterminate linear rational expectations models. It means we are restricting our attention to a subset of the set of solutions of the indeterminate model. This solution can be picked up easily by casting $M1$ to $M8$ in the form of (4.26).

We thus have $y_t = \begin{bmatrix} x_t^\pi & x_t^u & x_t^{mc} & x_t^n & x_t^c & \widehat{a}_t^P & \pi_t & \widehat{u}_t & \widehat{mc}_t^{'} & \widehat{u}_t^n & \widehat{c}_t & \widehat{a}_t^L & \widehat{w}_t^L & \widehat{i}_t \end{bmatrix}'$, $\varepsilon_t = e_t$ and $v_t = \begin{bmatrix} e_t & v_t^\pi & v_t^u & v_t^{mc} & v_t^n & v_t^c \end{bmatrix}'$ with $x_t^q = E_t q_{t+1}$, the v_t^q denoting the belief shock associated with the forecast of the t+1 value of variable q and $\widehat{mc}_t^{'} = \lambda mc_t$. The matrices Γ_0, Γ_1, Ψ and Π are to be found in appendix C.7.

Note that the way the model is written, we have five belief shocks - one for each forward looking variable. However, the effects of those shocks on the forecast errors, and thus on the endogenous variables, will not generally be independent from each other. For instance, if there is one stable root too many, as is the case under the calibration we are dealing with, there is one degree of freedom. That means we can choose the value of one endogenous variable and then the stable solution for the remaining ones

[29]See Lubik and Schorfheide (2003), p. 279.

will be pinned down as well. For instance, it will be possible to reproduce the dynamics produced by v_t^{π} with a suitable value of v_t^u, v_t^{mc}, v_t^n or v_t^c.

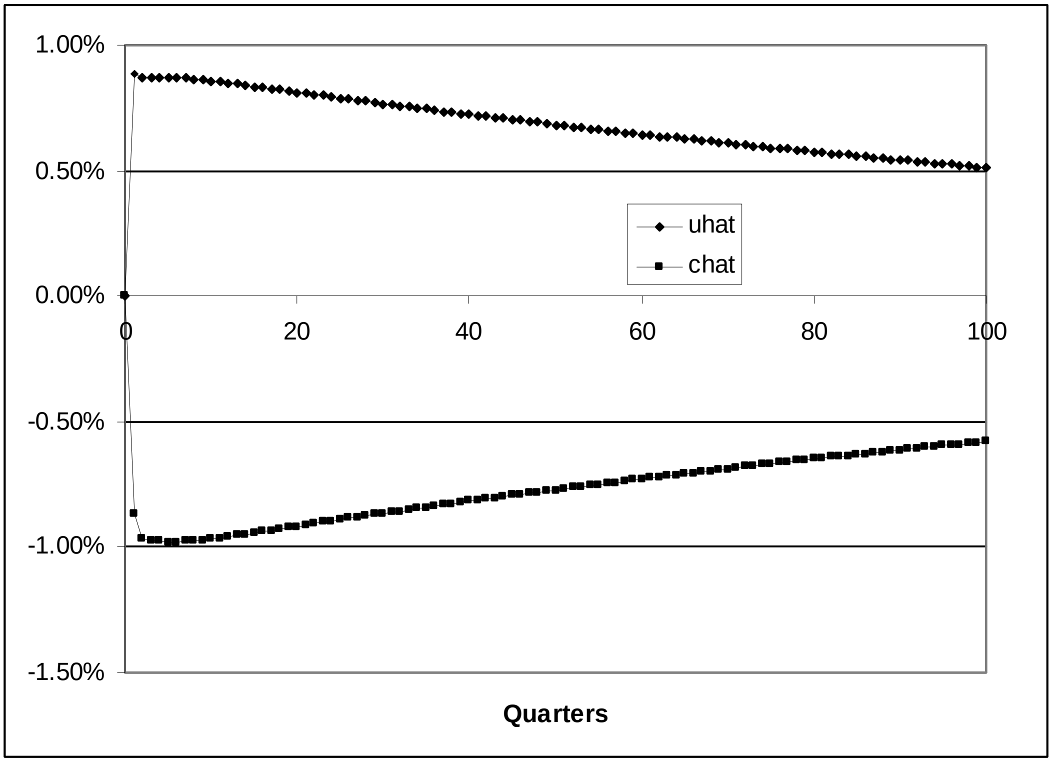

Figure 4.6. European Calibration, -2% Consumption Belief Shock - $\widehat{u}_t$ and $\widehat{c}_t$

We assume that the central bank responds only to inflation and set $\phi_\pi = 1.5$. When looking at the impulse response of the technology shock, we set $\rho_a = 0$.

We first consider the effects of a -2% belief shock to consumption, i.e. $v_0^c = -0.02$. Figure 4.6 displays the deviation of unemployment from its steady state (in percentage points) and output net of hiring costs (in percent), i.e. consumption. Unemployment increases by about 0.9 percentage point, while consumption declines by a bit less than 0.9% and then declines somewhat further. The increase in unemployment is very persistent: after 10 years, unemployment is still about 0.72 percentage points above its steady state while after 25 years (100 quarters) it still exceeds its steady state by 0.51%.

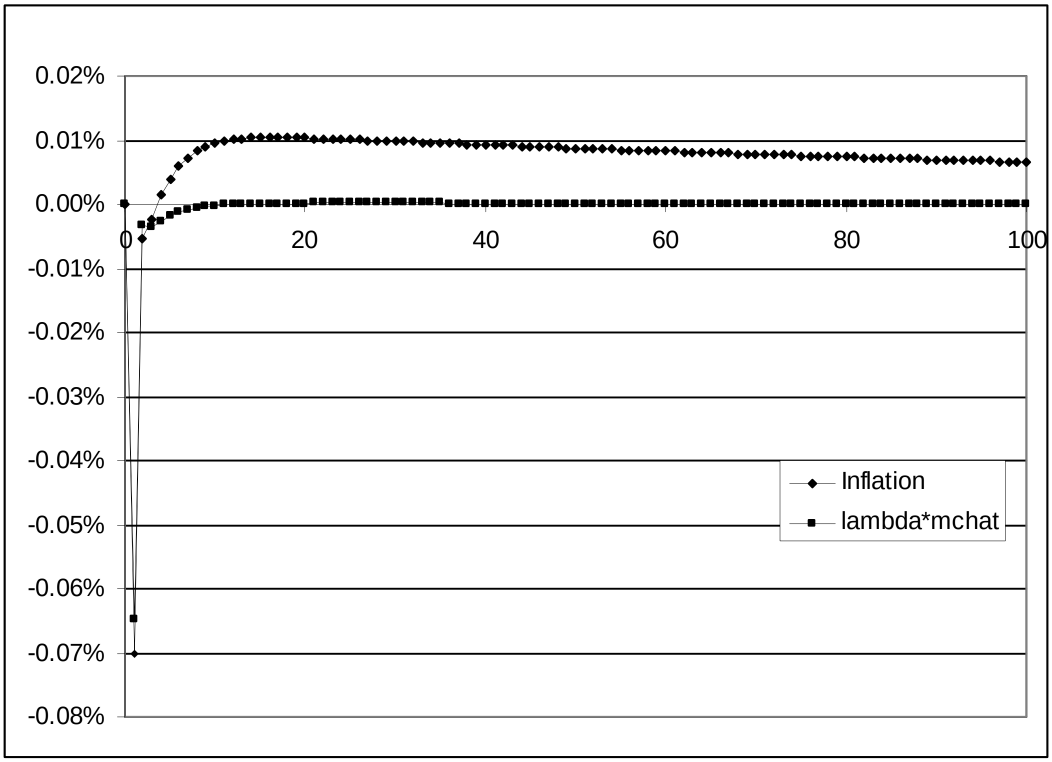

Figure 4.7. European Calibration, -2% Consumption Belief Shock - Inflation and $\lambda \widehat{mc}_t$

Figure 4.7 shows that $\lambda \widehat{mc}_t$ falls by 0.06% on impact and then starts increasing and turns positive in quarter 13. Since we have chosen a value of δ_s such that κ is smaller than zero (see Figure 4.3, the "Continental Europe" line), we would expect the persistent increase in unemployment to ultimately turn marginal cost positive. However, as long as the history of high unemployment is short, the skill loss among job seekers has not yet sufficiently built up to turn marginal cost positive. In terms of the four effects of an increase in unemployment on marginal cost listed at the end of section five, effect number four has not yet gained enough momentum such that the joint positive impact of effects 2, 3 and 4 dominate the negative impact of effect 1. To illustrate how the dynamic of the skill decline matches with sign change and dynamic of $\lambda \widehat{mc}_t$, consider how the skill level evolves in response to a "permanent" change in the unemployment rate:

$$\widehat{a}_t^L = a^n \left((1-x)^t \beta_s^t - 1 \right) \frac{\widehat{u}}{(1-u)}$$

where a^n is the effect of a permanent increase in employment on the skill level of the average applicant and can be obtained from proposition 2. Note that for $t \to \infty$, as $(1-x)^t \beta_s^t \to 0$, this expression gives the effect of a permanent increases in unemployment on the skill level. In Figure 4.8, we plot $\widehat{a}_t^L$ (as defined in this equation) as a percentage of the change of $\widehat{a}_\infty^L$ after an infinite number of periods, i.e. $\left(1-(1-x)^t \beta_s^t\right) \times 100$. The curve is rather steep at the beginning but then flattens out. With an unemployment history of twelve quarters, which happens to be the case in quarter 13, the decline in $\widehat{a}_t^L$ has reached 97.7% of its total and the rate of change has decreased to about 0.5 percentage points. Thus $\lambda \widehat{mc}_t$ turns positive after the decline in the skill level resulting from the increase in unemployment has almost reached its maximum. Note also that the dynamics of $\lambda \widehat{mc}_t$ and $\widehat{a}_t^L$ are similar in that the rate of increase of $\lambda \widehat{mc}_t$ is at its highest during those first 13 quarters but then gradually declines.

Inflation declines to -0.08% on impact but turns positive in quarter four. It then keeps rising until it reaches a maximum of 0.01% in quarter 17. Inflation is pushed faster above zero because it responds not just to current but also to expected future values of marginal costs. Correspondingly, we would expect the ex ante real interest rate to ultimately increase as well. Figure 4.9 shows that $(i_t - E_t \pi_{t+1})$ declines on impact but begins to increase in quarter two and begins to exceed its steady state value in quarter 5 and then remains persistently above it. The persistent increase in the real interest rate validates the initial decline in consumption and the associated increase in

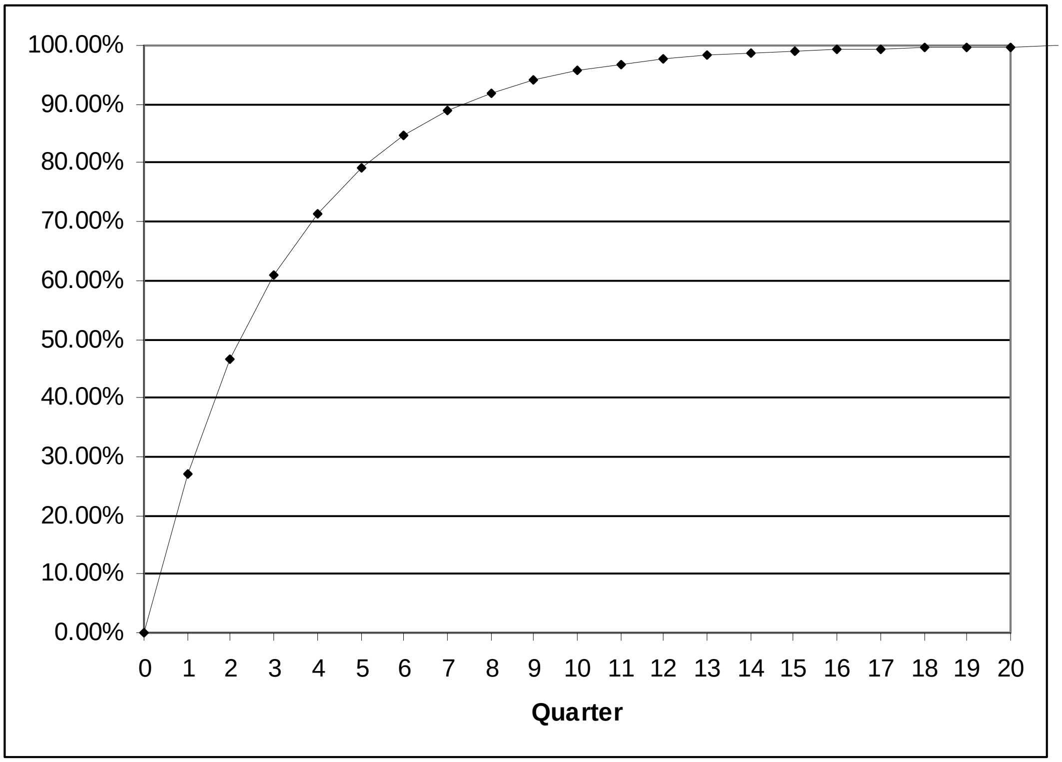

Figure 4.8. Skill Decline following a permanent Unemployment Increase - Percentage of total Drop

unemployment. Hence the response of unemployment, marginal cost inflation and the real interest rate is just as we would expect from our discussion of why indeterminacy occurs for $\delta_s \geq 0.025$ (and thus $\kappa < 0$) combined with $\phi_\pi > 1$.

We now turn to why the responses of unemployment and the other endogenous variables to the sunspot shock are so persistent. A highly persistent response of the endogenous variables to a sunspot shock in a linear indeterminate model is not uncommon.[30] Formally speaking, this might be due to the eigenvalues being continuous in the model's parameters. In that case, increasing or decreasing a parameter critical for determinacy until indeterminacy arises would result in a new stable eigenvalue just below one. For instance, increasing δ_s from 0 to 0.025 while holding ϕ_π constant at 1.5 monotonously decreases the modulus of the smallest (in modulus) unstable eigenvalue

[30]See Clarida et al. (2000), pp. 172-174 and Weder (2000), pp. 292-293.

from 1.015 to 0.9929. This new stable eigenvalue has a larger modulus than all other stable eigenvalues and will thus govern the persistence of the system, implying a highly persistent response of the economy to shocks.

We suspect that the economic intuition for the very slow return of unemployment to its steady state is related to the value of κ (recall that the effect of a permanent change in unemployment on marginal cost is given by $-\kappa$) implied by $\delta_s = 0.025$, which equals -0.00016. This is not only negative but also close to zero. Hence a given persistent decline in unemployment causes only a small decline in marginal cost and inflation. Hence the resulting decline in the nominal and the real interest rate will be small as well, implying only a small increase in aggregate demand. Hence only a small decline in unemployment is validated.

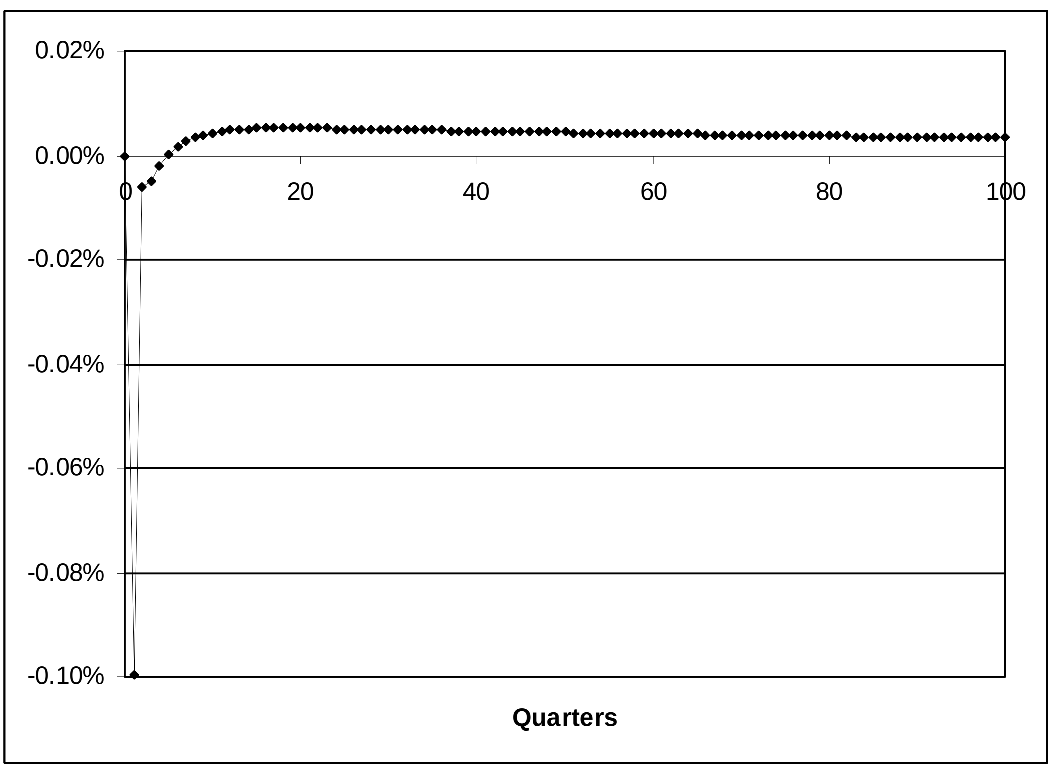

Figure 4.9. European Calibration, Consumption Belief Shock: Ex-ante real Interest Rate

We now turn to the effects of a non-correlated technology shock of -2%, i.e. $e_0 = -0.02$. Figure 4.10 shows that unemployment and consumption both decline by about 1%, but in quarter 2 unemployment increases to about 0.65 percentage points above its steady state value. Unemployment and consumption then display a similar degree of persistence as in response to a consumption belief shock. Figure 4.11 shows that inflation and $\lambda\widehat{mc}_t$ both increase on impact. Both turn negative in the next period due to the increase in unemployment. $\lambda\widehat{mc}_t$ turns positive in quarter 15 due to the fact that unemployment persistently increases, while inflation again turns positive faster. This then ultimately implies an above-steady-state real-interest rate.

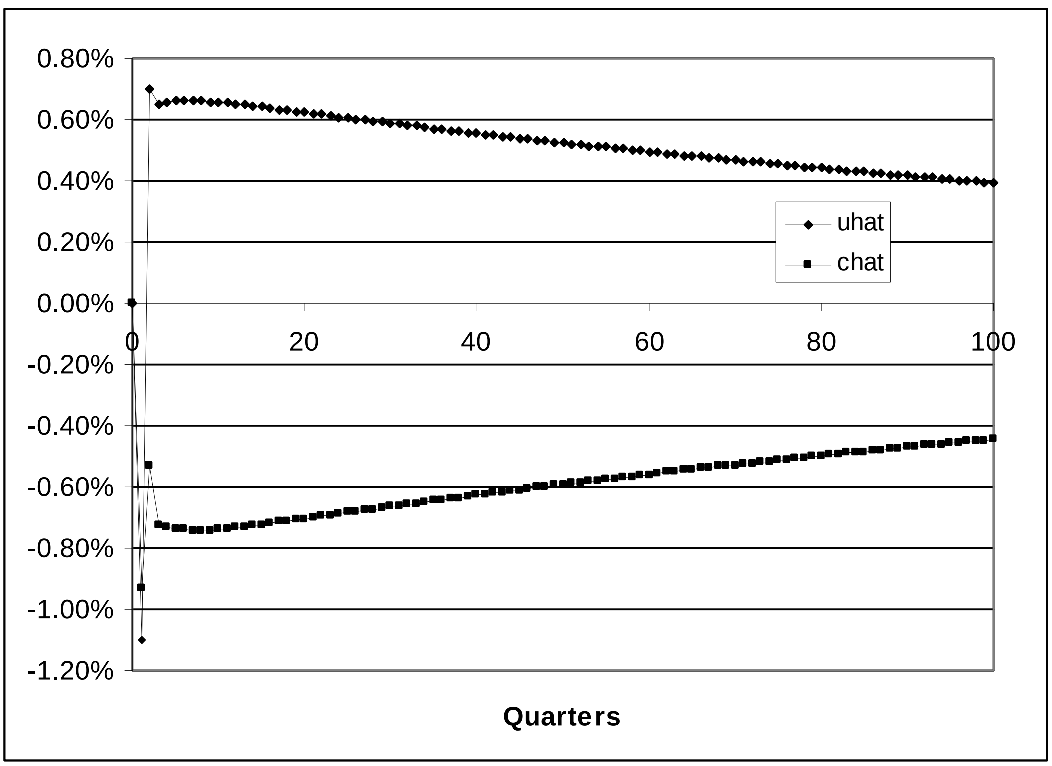

Figure 4.10. European Calibration, -2% Technology Shock

Thus both shocks can potentially trigger extremely persistent increases in unemployment under the continental European calibration if skill loss exceeds its critical level and the central bank reacts more than one-for-one to inflation. This is clearly a

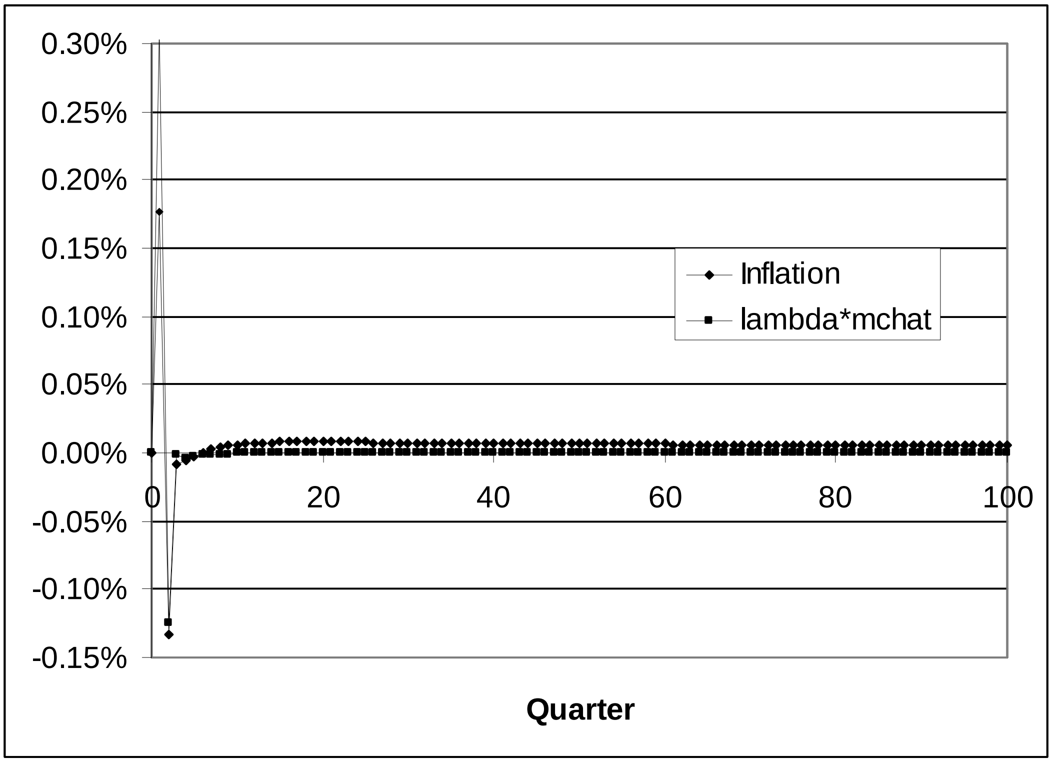

Figure 4.11. European Calibration, -2% Technology Shock - Inflation and $\lambda\widehat{mc}_t$

very interesting result given the persistent increase in unemployment in many Western European countries since the end of the 1970s.

4.7. Conclusion

This chapter adds skill loss among the unemployed as an additional labour-market friction to the model of Blanchard and Gali (2008) and shows the implications of this modification for determinacy. We assume that an unemployed person loses a set fraction of her skills during every quarter of her unemployment spell but regains all her skills after one quarter of employment. Firms that decide to hire meet workers according to their shares in the market. We first show that in the Blanchard and Gali (2008) model, a coefficient on inflation larger than one in an interest feedback for the nominal interest rate guarantees determinacy.

We then show that the introduction of skill loss increases the (positive) effect of past unemployment on marginal costs. An increase in past unemployment rates increases the share of the longer-term unemployed and thus worsens the quality of the pool of job seekers. If the quarterly skill-loss percentage is increased to or above a critical level, the combined positive effects of lagged and lead unemployment exceed the negative effect of current unemployment. In such a scenario, if the central bank responds only to inflation, determinacy requires a coefficient on inflation in the feedback rule smaller than one. This holds regardless of whether the central bank responds to current, lagged or expected future inflation.

We also show that the critical skill loss percentage is much lower, and a lot more plausible, if the flow characteristics of the labour market are "Continental European" (Blanchard and Gali (2008)) in the sense that there is little hiring and firing going on. By contrast, under and an "American" calibration of inflow and outflow rates, the implied critical skill loss percentage is implausibly high. This is largely due to the fact that even in the original Blanchard and Gali model lagged and lead unemployment matter a lot more for marginal costs under the continental European than under the American calibration.

Furthermore, neither interest rate smoothing nor responding to the output gap (as commonly modelled in the New Keynesian literature) help to restore determinacy under the continental European calibration if skill loss is above its critical level. As empirical estimates of interest-feedback rules frequently find that the Federal Reserve and Bundesbank as well as the ECB respond more than one-for-one to inflation, this might mean that indeterminacy and thus sunspot-driven dynamics are a much more likely phenomena in continental Europe than in the United States. By contrast, a modest response to unemployment guarantees determinacy for the full range of skill-decay percentages and inflation coefficients in the interest feedback rule we consider.

Finally, we compute the response of the model under the European calibration with skill loss above its critical level and the coefficient on inflation larger than one in the interest-feedback rule to an adverse sunspot shock and an adverse non-correlated technology shock. It turns out that the response of unemployment is extremely persistent. Thus this admittedly quite stylised model potentially contributes to explaining the persistent increase in unemployment observed in continental Europe since the late 1970s. It also suggests the following story: The shift of monetary policy away from a "Keynesian" approach towards aggressive inflation targeting might have been unproblematic in the fluid labour market of the United States but might have been a source of instability and persistent unemployment fluctuations in Western continental Europe with its much less fluid labour market. Hence the model developed here is able to match the first three of the five empirical findings listed in the conclusion to chapter one.

Finally, note that like in chapter two, we endogenise total factor productivity of the marginal worker. Just as with endogenous growth, lower aggregate demand today implies productivity will be lower than it otherwise would have been, implying that ceteris paribus marginal costs will be higher as well. However, in contrast to the sticky price endogenous growth model developed in chapter two, the model developed in this chapter, once calibrated to match continental European features, generates unemployment fluctuations endogenously as a result of multiple stable equilibria. Furthermore, these fluctuations are also more persistent than those created by the cost push shock in chapter two.

CHAPTER 5

Conclusion

In Chapter one we conduct a critical survey of the mainstream approaches aiming to explain the medium run evolution of unemployment in advanced OECD economies, and why unemployment has persistently increased in many European countries but not in the United States. We conclude that the existing approaches are wanting and that a new approach towards explaining medium run swings of unemployment is called for. We also conclude that such a theory should shed light on the following set of empirical findings:

- the medium run swings in unemployment we observe in OECD countries, particularly the increase in unemployment in several Western continental European countries since the mid of the 1970s. The theory should generate medium run swings without relying on changes in labour market institutions.
- the time series evidence showing that there is high *endogenous* unemployment persistence, or even unit root behaviour, in a number of Western continental European countries, but much less persistence in the United States. This means that a temporary shock in Europe increasing unemployment today will have a lasting effect on unemployment long after it occurred.
- the time series evidence showing that structural breaks in unemployment seem to be predominantly located during recessions. This suggests that aggregate demand contractions may have long lasting effects on unemployment.

- the evidence saying that a major disinflation seems to have been a necessary condition for a major increase in the NAIRU. There is also evidence that the change in inflation over a ten year period is negatively related to the change in the NAIRU during that period.
- the evidence saying that the amount of monetary easing during a recession is negatively related to the subsequent change in the NAIRU.

The models developed in this thesis go some way towards achieving these goals and in addition yield a set of other insights. In chapter two, we introduce endogenous growth via a capital stock externality into a New Keynesian model with unemployment. Unemployment arises due to efficiency wages. We refer to this model as the "New Growth economy" and to an otherwise identical model without endogenous growth as the "JLN economy". We calibrate the models to German data and hit the economy with a cost push shock which generates disinflation whose size is at the lower end of what was observed in many OECD countries at the beginning of the 1980s. In the New Growth economy, this one quarter shock causes a persistent and substantial increase in unemployment, lasting for 10 to 20 years in an order of magnitude of one percentage point or more. The drop in investment during the disinflation implies a slowdown in labour productivity growth, which, in conjunction with rigid real wage growth, increases the NAIRU. The model thus also contributes to explaining the productivity slowdown observed across advanced OECD economies from the 1970s to the 1980s. Furthermore, the amount of monetary easing during the recession associated with the disinflation turns out to be negatively related to the subsequent increase in the NAIRU.

The model also contributes to explaining the cross-country differences in unemployment evolution. Varying the size of the cost push shock generates a relationship between the change in the inflation rate and the change in the NAIRU over a ten year horizon similar to the relationship observed in the data. We also shed light on why unemployment has increased by less in the United States, by drawing on empirically observable differences in monetary policy and wage setting. The differences in wage setting can be related, if coarsely so, to features of the labour market such as union density or the benefit system. Thus the chapter also supports the view that, as suggested by Blanchard and Wolfers (2000), it is both "shocks and institutions" which are at the heart of explaining the evolution of unemployment across time and the differences across countries.

While the main focus of this thesis is a positive one, chapter three investigates the consequences of introducing endogenous growth for optimal monetary policy. We search for a simple optimal rule which maximises the households welfare in the two JLN and the New Growth economy. The main result is that in the JLN economy, the optimal simple rule features a large inflation coefficient and a much lower output coefficient, just as conventional wisdom would suggest. By contrast, in the presence of endogenous growth, this hawkish monetary policy is no longer optimal. The optimal inflation coefficient is the lowest in the grid, while the coefficient on the output gap is higher than in the JLN economy. This result is qualitatively robust against a variety of changes to the baseline scenario, i.e. variations to the degree of real wage rigidity and nominal price stickiness as well as the introduction of a transaction cost for consumption, the later of which implies a transaction demand for money.

Chapter four sheds light on some of the findings above in a different way. It adds skill loss among the unemployed to a New Keynesian model with hiring frictions developed by Blanchard and Gali (2008). An unemployed person looses a fixed fraction of her skills during every quarter of her unemployment spell but regains all her skills after one quarter of employment. We find that if skill decay exceeds a threshold level (about 2.5% per quarter), a nominal interest rate feedback rule with a coefficient on (current, expected future or lagged) inflation exceeding one induces indeterminacy. The change in the determinacy requirement appears to be related to a change in the long run relationship between marginal cost and unemployment from negative to positive if skill decay exceeds the threshold level.

Furthermore, the threshold skill decay percentage is implausibly high if we adopt Blanchard and Gali's (2008) "American" calibration of labour market flows, i.e. a high job finding probability and a high job destruction rate. By contrast, if we adopt their "continental European" calibration, the critical skill loss percentage is low and plausible. This result is robust against adding the lagged interest rate or the output gap to the policy rule. By contrast, a modest response to unemployment guarantees determinacy for the full range of skill decay percentages and inflation coefficients. Thus under the European calibration with skill decay above the threshold and an active monetary policy with no response to unemployment, there is indeterminacy and thus scope for sunspot driven fluctuations. Indeed, a contractive one quarter consumption sunspot shock which lowers output and increases unemployment on impact increases unemployment extremely persistently.

Empirical estimates of interest feedback rules frequently find that both the Federal Reserve and ECB respond more than one for one to inflation, as did the Bundesbank. The model of chapter four suggests that in continental Europe with its not very fluid labour market, such a policy will induce endogenous persistent swings in unemployment. This suggests that shift of monetary policy away from a "Keynesian" approach towards aggressive inflation targeting was unproblematic in the fluid labour market of the United States but was a source of instability and persistent unemployment fluctuations in continental Europe.

There are numerous possible extensions to the research conducted in this thesis. We restrict ourselves a selected few. At the moment, the analysis in chapter two focuses on replicating the persistent increase in unemployment in response to the disinflations of the 1980s by hitting the economy with a cost push shock. It would be interesting to extend the focus to the 1990s when unemployment remained high in some European countries like France, but started declining in others like Spain. This would quite likely require considering other shocks as well, for instance fiscal policy shocks to model the effects of a policy aimed at reducing the deficit and the debt to GDP ratio to meet the Maastricht criteria or demand shocks like the breakdown of the European exchange rate mechanism at the beginning of the 1990s. Given the ability of endogenous growth to generate persistence in real variables, it would be interesting to estimate the model to rigorously check the extent to which endogenous growth reduces the persistence in the exogenous driving processes needed to generate a given degree of persistence in the endogenous variables. Estimation would also allow one to pin down which shocks are most important in explaining movements in unemployment, assuming that the model is reasonably correctly specified.

The model in chapter four makes very strong predictions about the determinacy requirements on the policy rule, but, as models making strong predictions sometimes go, it is at the same time highly stylised. An obvious extension to make it less so would be to assume a production function featuring capital. On the one hand, since the marginal product of labour would then be decreasing as employment increases, this would make the relationship between unemployment and marginal cost more negative. On the other hand, a persistent decrease in employment and the associated reduction in the productivity of the average applicant would increase the capital to effective labour ratio, inducing firms to lower their capital stock. Our prior would be that the two effects would offset each other but this needs to be rigorously checked. Furthermore, it would be interesting to see whether the indeterminacy results carry over to a more sophisticated modelling of wage determination, like efficiency wages or Nash bargaining. This might also increase the scope for institutional features of the labour market, like the generosity of unemployment benefits, to affect the determinacy results, and thus might enhance the ability of the model to relate cross-country differences in unemployment dynamics to the "level" of institutions interacting with an unsuitable monetary policy. Finally, it would be interesting to relax assumption that recovery of skills after reemployment takes only one quarter since, since this might be overly restrictive. For instance, Gregg and Tominey (2005) find that the wage penalty associated with a year of youth unemployment is still present at age 42.[1]

We leave all these extension to future research.

[1] See Gregg and Tominey (2005), p. 502 and pp. 505-506.

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APPENDIX A

Appendix to Chapter 2

A.1. Derivation of the Real Wage Phillips Curve

The firm's first order conditions with respect to the real wage and effort are

$$
\begin{aligned}
n_t(i) - \overline{n} &= \frac{\zeta_t \phi_1}{w_t(i)} \\
\zeta_t &= (1-\alpha) mc_t \frac{Y_t(i)}{e_t(i)}
\end{aligned}
$$

Combining those with the first order condition with respect to labour yields $e_t(i) = \phi_1$. Substituting this back into the effort function (2.4), we note that, as the firm's wage depends only on aggregate variables which are the same for all firms, it must indeed hold that $w_t(i) = w_t$. Substituting for $\log b_t$ and rearranging then yields

$$
\begin{aligned}
(\phi_1 + \phi_3) \log w_t = (\phi_5 (1-\phi_6) - \phi_4) \log w_{t-1} + \phi_1 - \phi_0 + \phi_5 \phi_7 - \phi_2 f(n_t) \\
- (\phi_5 \phi_6 + \phi_8) \log \left(1 / \left(\frac{Y_{t-1}}{n_{t-1} - \overline{n} - n^s} \right) \right)
\end{aligned}
$$

Subtracting $(\phi_5 \phi_6 + \phi_8) \log w_{t-1}$ on both sides and dividing by $(\phi_1 + \phi_3)$ then yields

$$
\begin{aligned}
\text{(A.1)} \qquad \log w_t &= \frac{\phi_1 - \phi_0 + \phi_5 \phi_7}{\phi_1 + \phi_3} - \frac{\phi_2}{\phi_1 + \phi_3} f(n_t) + \frac{\phi_5 + \phi_8 - \phi_4}{\phi_1 + \phi_3} \log w_{t-1} \\
&\quad - \frac{(\phi_5 \phi_6 + \phi_8)}{\phi_1 + \phi_3} \log \left(\frac{w_{t-1} (n_{t-1} - \overline{n} - n^s)}{Y_{t-1}} \right)
\end{aligned}
$$

Hence with the coefficient restrictions imposed above, the wage depends positively on the past real wage and non-managerial employment. It will be above its market clearing level and thus there is unemployment in the economy.

Note that the last term in brackets is in fact the private sector labour share. If this were constant in the steady state, as it would be at a constant employment level, equation $(A.1)$ could be solved for a long run real wage if $\frac{\phi_5+\phi_8-\phi_4}{\phi_1+\phi_3} < 1$. As mentioned above however, the economy is growing in the steady state. Therefore the real wage must grow in the steady state as well. Thus a wage setting function simply relating the wage level to employment would not be consistent with a stable employment level. The easiest way to deal with the issue therefore is to set $\frac{\phi_5+\phi_8-\phi_4}{\phi_1+\phi_3} = 1$. This does not seem too restrictive; it simply says that an increase in the log of the time t real wage in the economy (including firm i) has in absolute value the same net effect on effort (remember we have $\phi_1 + \phi_3 > 0$) as an increase in the exogenous reference as represented by $\log w_{t-1}$, $\log b_t$ and $\log \left(Y_{t-1}/\left(n_{t-1} - \overline{n} - n^s\right)\right)$. Thus we arrive at a real wage Phillips Curve with a labour share term:

$$\text{(A.2)} \qquad \log w_t - \log w_{t-1} = a + b * f\left(n_t\right) + c \log\left(\frac{w_{t-1}\left(n_{t-1} - \overline{n} - n^s\right)}{Y_{t-1}}\right),$$

$$\text{with } a = \frac{\phi_0 - \phi_1 + \phi_5\phi_7}{\phi_1 + \phi_3},\ b = -\frac{\phi_2}{\phi_1 + \phi_3} > 0 \text{ and } c = -\frac{(\phi_5\phi_6 + \phi_8)}{\phi_1 + \phi_3} < 0$$

A.2. Determination of the Overhead Labour Force

Following Rotemberg and Woodford (1999), we assume that in the steady state, all economic profit generated by the monopolistically competitive firm goes to the overhead staff. This is justified because setting up production is impossible without overhead labour and the firm's profit is thus essentially equal to the collective marginal product of its overhead staff. We assume that the overhead staff splits this profit equally. Hence the firm ends up with zero profits, which eliminates any incentive for market entry. Christiano et al. (2005) also assume a fixed cost of production to eliminate profits among monopolistically competitive firms, although they do not specify the origin of the cost.[1]

For simplicity, we assume the amount of overhead workers required to enable production is such that the real wage for overhead and non-overhead workers will be exactly the same in the steady state. These assumptions allow for a straightforward way to determine the amount of overhead and non-overhead workers as a function of total employment. Zero profit requires

$$\frac{\mu - 1}{\mu} Y - w\overline{n} = 0$$

where $\frac{\mu-1}{\mu}$ is the share of firms' profits in output. Substituting $w_t = (1-\alpha)\frac{1}{\mu}\frac{Y_t}{n-n^s-\overline{n}}$ gives, after some manipulation

$$\frac{\mu - 1}{1 - \alpha} = \frac{\overline{n}}{n - n^s - \overline{n}} \equiv \overline{s}$$

This is the ratio of overhead labour to productive labour, which we call $\overline{s}$. Solving for $\overline{n}$ then gives

$$\overline{n} = \frac{\overline{s}}{1 + \overline{s}} (n - n^s)$$

A.3. Normalised Version of the New Growth Model

As we are dealing with two growth models, we have to stationarise all variables which would otherwise be trended in order to be able to solve the model. This appendix applies this normalisation to the New Growth model. The resulting equations are those which

[1] See Rotemberg and Woodford (2004), p. 17, and Christiano et al. (2005), p. 15.

have been solved and simulated. We define $\frac{C_t}{K_t}$, $\frac{hab_{t-1}}{K_t}$ $\frac{Y_t+w_t n^s}{K_t}$, $\frac{I_t}{K_t}$ and $\frac{w_t}{K_t}$ as D_t, Hab_{t-1}, F_t, R_t and H_t, while the gross capital stock growth rate $\frac{K_{t+1}}{K_t} - 1$ is defined as g_{t+1}^k.

We directly apply the normalisation to the equations of the aggregate demand block:

$$F_t = D_t + R_t + \frac{\varphi}{2}(\pi_t - \pi_{t-1})^2 (F_t - H_t n^s) + H_t n^s \tag{A.3}$$

Consumption (remember $hab_{t-1} = jC_{t-1}$, thus $Hab_{t-1} = j\frac{D_{t-1}}{1+g_t^k}$

$$\begin{aligned} & 1/\left(D_t - Hab_{t-1}\right) && \text{(A.4)} \\ = \; & \beta E_t \left[(1+i_t) / \left((1+\pi_{t+1})(D_{t+1} - Hab_t)\left(1+g_{t+1}^k\right)\right)\right] && \text{(A.5)} \\ Hab_t = \; & j\frac{D_t}{1+g_{t+1}^k} \end{aligned}$$

Investment:

$$\beta E_t \left(\frac{1}{(D_{t+1} - Hab_t)\left(1+g_{t+1}^K\right)} \left(r_{t+1}^k + q_{t+1}(1-\delta)\right) \right) = \frac{1}{D_t - Hab_{t-1}} q_t \tag{A.6}$$

$$\begin{aligned} & \frac{1}{D_t - Hab_{t-1}} q_t \left[\begin{array}{c} \left(1 - \frac{\kappa}{2}\left(\frac{R_t}{R_{t-1}}\left(1+g_t^k\right) - (1+g)\right)^2\right) \\ -\frac{R_t}{R_{t-1}}\kappa\left(\frac{R_t}{R_{t-1}}\left(1+g_t^k\right) - (1+g)\right) \end{array} \right] && \text{(A.7)} \\ & + \beta E_t \left[\begin{array}{c} \frac{1}{D_{t+1} - Hab_t} q_{t+1} \\ \left(\frac{R_{t+1}}{R_t}\left(1+g_{t+1}^k\right)\right)^2 \kappa \left(\frac{R_{t+1}}{R_t}\left(1+g_{t+1}^k\right) - (1+g)\right) \end{array} \right] \\ & = \frac{1}{D_t - Hab_{t-1}} \end{aligned}$$

From (2.24) we have

$$g_{t+1}^{k}=-\delta+R_{t}\left(1-\frac{\kappa}{2}\left(\frac{R_{t}}{R_{t-1}}\left(1+g_{t}^{k}\right)-(1+g)\right)^{2}\right) \tag{A.8}$$

The rental on capital becomes:

$$r_{t}^{k}=\alpha mc_{t}\left(F_{t}-H_{t}n^{s}\right) \tag{A.9}$$

Substituting $(A.9)$ into (2.28) and multiplying by $\frac{K_{t}^{\frac{\alpha}{1-\alpha}}}{K_{t}^{\frac{\alpha}{1-\alpha}}}$ yields

$$mc_{t}=\frac{\left(F_{t}-H_{t}n^{s}\right)^{\frac{\alpha}{1-\alpha}}H_{t}}{X} \tag{A.10}$$

where $X=A^{\frac{1}{1-\alpha}}(1-\alpha)\phi_{1}$.

The wage setting function $\ln w_{t}=\ln w_{t-1}+a+b\left(n_{t}-\overline{n}\right)+c\log\left(\frac{w_{t-1}\left(n_{t-1}-\overline{n}-n^{s}\right)}{Y_{t-1}}\right)$ can be rewritten as (using equation (2.8)) $\ln H_{t}=a+b\left(n_{t}-\overline{n}\right)+\ln\left(\frac{w_{t-1}}{K_{t-1}\left(1+g_{t}^{k}\right)}\right)+c\log\left(\left(1-\alpha\right)mc_{t-1}\right)=a+b\left(n_{t}-\overline{n}\right)+\ln\left(\frac{H_{t-1}}{\left(1+g_{t}^{k}\right)}\right)+c\log\left(\left(1-\alpha\right)mc_{t-1}\right)$

$$H_{t}=\exp(a+b\left(n_{t}-\overline{n}\right))\frac{H_{t-1}}{\left(1+g_{t}^{k}\right)}\left(\left(1-\alpha\right)mc_{t-1}\right)^{c} \tag{A.11}$$

Employment: from $Output_t = AK_t((n_t - \overline{n} - n^s)\,\phi_1)^{1-\alpha} + w_t n^s$, we have

$$F_t = A((n_t - \overline{n} - n^s)\,\phi_1)^{1-\alpha} + H_t n^s \tag{A.12}$$

The Phillips Curve and the Policy rule do not contain any trended variables and therefore does not need to be normalised. However, we will substitute the real profits stochastic discount factor by its definition, i.e. $\rho_{t,t+1} = \beta\frac{u'(C_{t+1}-Hab_t)}{u'(Hab_{t-1})} = \beta\frac{C_t - Hab_{t-1}}{C_{t+1}-Hab_t}$, which gives

(A.13)

$$(1-\theta) + \theta mc_t - \varphi\left(\left(\frac{P_t}{P_{t-1}} - u_t\right) - \frac{P_{t-1}}{P_{t-2}}\right)\left(\frac{P_t}{P_{t-1}} - u_t\right) + \theta\frac{\varphi}{2}(\left(\frac{P_t}{P_{t-1}} - u_t\right) - \frac{P_{t-1}}{P_{t-2}})^2$$

$$+\beta\frac{D_t - Hab_{t-1}}{F_t}\varphi E_t\left[\frac{F_{t+1}}{D_{t+1}-Hab_t}\left(\frac{P_{t+1}}{P_t} - \left(\frac{P_t}{P_{t-1}} - u_t\right)\right)\frac{P_{t+1}}{P_t}\right] = 0 \tag{A.14}$$

Replacing $\frac{P_t+i}{P_{t-1}+i} = 1 + \pi_{t+i}$ gives

$$(1-\theta) + \theta mc_t - \varphi\left((\pi_t - u_t) - \pi_{t-1}\right)(1 + \pi_t - u_t) + \theta\frac{\varphi}{2}((\pi_t - u_t) - \pi_{t-1})^2 \tag{A.15}$$

$$+\beta\frac{D_t - Hab_{t-1}}{F_t}\varphi E_t\left[\frac{F_{t+1}}{D_{t+1}-Hab_t}\left(\pi_{t+1} - (\pi_t - u_t)\right)(1+\pi_{t+1})\right] = 0 \tag{A.16}$$

For natural output, natural employment and the natural real wage we have

$$\mu^{-1} = \frac{(F_t^n)^{\frac{\alpha}{1-\alpha}}\,H_t^n}{X} \tag{A.17}$$

$$F_t^n = A_t((n_t^n - \overline{n} - n^s)\,\phi_1)^{1-\alpha} + n^s H_t^n$$

$$H_t^n = \exp(a + b\,(n_t^n - \overline{n}))\frac{H_{t-1}}{(1+g_t^k)}\left((1-\alpha)\,mc_{t-1}\right)^c \tag{A.18}$$

given last periods wage/ capital ratio H_{t-1} and this periods capital stock growth rate g_t^k (which was also determined in the t-1 by the then investment decision). The output gap gp_t is then calculated as

$$gp_t = \frac{Output_t - Output_t^n}{Output_t^n}\left(\frac{K_t}{K_t}\right) = \frac{F_t - F_t^n}{F_t^n} \tag{A.19}$$

A.4. Steady State Relations

This Appendix shows how to calculate the steady state values for the system developed in Appendix I. We will first derive a steady state relation between the level of employment and the steady state growth rate for the New Growth Economy.

First apply the fact that in the steady state, $g_t^K = g$ to $(A.7)$ which yields $q = 1$. We then apply this to $(A.6)$ which yields

$$\beta\left(r_{t+1}^k + (1-\delta)\right) = (1+g) \tag{A.20}$$

In the New Growth economy, we now replace the capital rental with equation $(A.9)$ and, after using $(A.12)$ and noting that in the steady state we have $mc = \mu^{-1}$, arrive at

$$g = \left[\beta\left[(1-\delta) + \alpha\mu^{-1}A((n - \overline{n} - n^s)\,\phi_1)^{1-\alpha}\right]\right] - 1 \tag{A.21}$$

This is the steady state growth rate which is borne out by the marginal product of capital in the endogenous growth economy. It is easily verified that it is increasing and concave in employment. It is straightforward to show that in the steady state, the real wage implied by the desired mark-up grows at the same rate as output and the capital stock by using $mc_t = \mu^{-1}$ and $r_t^k = r^k$ on (2.28). This yields

$$w_t = K_t \phi_1 \left(\frac{\mu^{-1} A \alpha^{\alpha} (1-\alpha)^{1-\alpha}}{(r^k)^{\alpha}} \right)^{1/(1-\alpha)} \tag{A.22}$$

Hence in the steady state, the real wage has to grow at the same rate as the capital stock. This means that equation $(A.22)$ is actually the dynamic, endogenous growth version of the familiar macroeconomic textbook price setting function: It gives the real wage growth rate compatible with marginal costs remaining constant and at it's long run level. Unlike the textbook price setting function, this real wage growth rate is not constant but increases in employment: A higher steady state employment level implies a higher marginal product of capital, which triggers higher investment and thus faster capital stock- and thus productivity growth. Accordingly, the steady state levels of employment an the growth rate are determined by the intersection of $(A.21)$ with the wage setting function (2.11), (making again use of the fact that $mc = \mu^{-1}$ in the steady state).

In practice, we choose a desired steady state employment rate (here 0.96) and then compute the wage setting function intercept a to support this value, given g, b and α and $\overline{n}$.

Having determined g and n, the determination of the steady state values of $F_t, D_t, R_t, H_t, r_t^k$ and i_t is now straightforward. For F we have

$$F = A((n - \overline{n} - n^s)\,\phi_1)^{1-\alpha} \tag{A.23}$$

from the production function. For R_t, we have from the capital accumulation equation in $(A.8)$

(A.24) $$R = g + \delta$$

D can then be determined as a residual via

(A.25) $$D = F - R$$

H is computed using the cost-minimisation first order condition for labour (2.8)

(A.26) $$H = (1-\alpha)\mu^{-1}\frac{F}{n - \overline{n} - n^s}$$

r^k is computed via

(A.27) $$r^k = \alpha\mu^{-1}A((n - \overline{n} - n^s)\,\phi_1)^{1-\alpha}$$

The steady state value of i_t is computed from $(A.4)$

(A.28) $$i = \frac{1+g}{\beta} - 1$$

Note that this is also the intercept of the interest rate rule $\bar{i}$ of the central bank.

A.5. Normalised Version of the JLN Economy

Most of the equations from Appendix II just carry over to the JLN economy. However, there are a few changes related to the production function and the marginal cost equation. The aggregate production function is now $Output_t = AK_t^{\alpha}\left(TFP_t\phi_1\left(n_t - \overline{n} - n^s\right)\right)^{1-\alpha} +$

$w_t n^s$. Dividing both sides by K_t gives

$$F_t = \left(l_t \phi_1 \left(n_t - \overline{n} - n^s\right)\right)^{1-\alpha} + H_t n^s \tag{A.29}$$

where l_t is defined as $\frac{TFP_t}{K_t}$. This variable evolves according to

$$l_t = \frac{1 + g_{TFP}}{1 + g_t^K} l_{t-1} \tag{A.30}$$

In the JLN model, it convenient to normalise the real wage with respect to TFP_t rather than with respect to K_t, while all the remaining normalisations carry over to the JLN model. Denoting $\frac{w_t}{TFP_t}$ as H_t^{nc}, we have from (2.27), after making use of (2.25)

$$mc_t = \frac{F_t^{(\alpha/(1-\alpha))} H_t^{nc}}{A^{1/(1-\alpha)} \left(1 - \alpha\right) \phi_1} \tag{A.31}$$

Concerning the capital rental, we employ the JLN expression for F_t to have

$$r_t^k = \alpha mc_t A l_t^{1-\alpha} \left(\left(n_t - \overline{n} - n^s\right) \phi_1\right)^{1-\alpha} \tag{A.32}$$

The normalised wage setting equation becomes

$$H_t^{nc} = \exp(a + b\left(n_t - \overline{n}\right)) \frac{H_{t-1}^{nc}}{\left(1 + g_{TFP}\right)} \left(\left(1 - \alpha\right) mc_{t-1}\right)^c \tag{A.33}$$

All the remaining equations are just the same as in the New Growth version. The computation of the steady state values in the neoclassical model is slightly different. The steady state growth rate (of output, consumption, the capital stock, the real wage) is now given by the parameter g_{TFP} rather than being endogenously determined, which means we have $g = g_{TFP}$. Hence we can compute the steady state real interest rate from $(A.28)$, while we compute r^k from $(A.20)$. From $(A.33)$, we have the steady state employment rate. Setting $mc_t = \mu^{-1}$in $(A.32)$ then gives the steady state value for l_t as

$$l = \frac{1}{(n_t - \overline{n} - n^s)\,\phi_1}\left(\frac{r^k \mu}{\alpha A}\right)^{1/(1-\alpha)} \tag{A.34}$$

which allows us to compute F from $(A.29)$. Rearranging $(A.31)$ then yields H^{nc}.

A.6. Construction of the Dataset

This appendix explains the construction of the dataset for F_t, D_t, R_t and H_t. The German federal statistical office ("Statistisches Bundesamt") supplies annual data for the capital stock in constant prices of the year 2000.[2] Thus we had to construct quarterly observations for the capital stock. We decided on the following method. We first calculated the annual change. Then we allocated the total changed to the four quarters according to the share these quarters had in real gross fixed investment. This yields a beginning of the quarter value for the capital stock.

Our data on real output, consumption and investment expenditure was preferably also to be in prices of 2000. However, the Statistisches Bundesamt only supplies chained indices for these variables.[3] We therefore used nominal GDP, consumption and investment 2000 to recursively calculate our series in absolute numbers. As the indices for post and pre reunification years have different bases, we used the ratio of unified Germany to Western Germany from 1991 to downscale the index for each variable.

Furthermore, as the total labour force in our model is normalised to one, Output, consumption and investment are essentially expressed in per capita terms in our model,

[2]See statistisches Bundesamt (2006b), table 3.2.19.1.

[3]See Statistisches Bundesamt (2006a), table 2.3.2, Statistisches Bundesamt (2007a), table 2.3.2, Statistisches Bundesamt (2006b), table 3.3.1 and Statistisches Bundesamt (2007b), table 2.1.1.

as is the capital stock. Hence case of F_t, D_t and R_t, the number of inhabitants cancels out and we can divide real GDP by our capital stock measure, and accordingly for D_t and R_t. By contrast, H_t is computed by multiplying the real wage as measured in the previous section times the average number of hours worked across the sample and then dividing by the capital stock. We tried a linear trend for hours worked instead but this would have turned our measure of H_t non-stationary.

The null of stationarity is rejected at the 5% level for D_t and F_t using the KPSS test.[4] After removing the years 70 to 72, we are not rejecting the null of stationarity anymore at the 10% level for these variables. For H_t, the null of stationarity is not rejected at the 5% level for the full sample. For R_t, the unit root can be rejected over the entire sample at the 5% level using an ADF test, as is the case for g_t and the savings rate. The same holds for the nominal interest rate, and so we do not detrend this variable either. We do detrend the inflation rate, because the null of stationarity is rejected for this variable using a KPSS test.

A.7. Estimation of the Wage Setting Function

We estimate the real wage growth function using German data ranging from 1970q1 to 2000q4. Our dataset includes Western German data up to 1991q4 and following that data for the unified country. All data is taken from a publication of the German "Statistisches Bundesamt", all of which has been seasonally adjusted.[5] When estimating the function, we replace the employment rate with one minus the unemployment rate. As a measure for labour costs, we use the "Arbeitnehmerentgeld" per employee, which is employee compensation including the full tax wedge. This is

[4]Following Hobijn et al. (2004), we use the Newey-West bandwidth selection procedure in combination a Quadratic Spectral (QS) window. See Hobjn et al., p. 486 and p.500.

[5]See Statistisches Bundesamt (2006a) and Statistisches Bundesamt (2007a).

deflated using the GDP price index since we are interested in a measure of labour costs. We use the real wage divided by productivity as a measure of the labour share. This is a standard procedure in estimations of this type of wage equation.[6] Denoting the unemployment rate as U, we aim to estimate the following equation: $\Delta \log w_t = a + bf(U_t) + c \log(w_{t-1}/(GDP_{t-1}/N_{t-1}) + d92Q1$, where N_t denotes the total number of employees $d92Q1$ denotes an intercept dummy equalling one in 1992q1 and zero everywhere else. The latter is to account for reunification. We allow $f(U_t)$ to take two different forms: U_t and $\log(U_t)$.

Note for both specifications we use Newey-West serial correlation consistent standard errors because the Breusch-Godfrey LM test for serial correlation rejects the hypotheses of no serial correlation at the 5% for a postulated maximum degree of autocorrelations of 1 and 4. With respect to the dynamics of the residuals, both specifications are almost identical. The estimation result is reported in table A.1, where WG denotes the change in log real wages, REALWH denotes the real wage and PRODH denotes real GDP per employee. Note that according to standard criteria of model selection (adjusted R^2, AIC and SIC), the specification featuring the log of the unemployment rate dominates the specification with the level. The standard error of regression is smaller as well and the coefficient on $\log(U_t)$ is more efficiently estimated, even after accounting for the different dimensions of the variables.

[6]Cahuc and Zylberberg (2004), p. 486 and OECD (1997), p. 21 and OECD(1997), p. 21.

Dependent Variable: WGH
Method: Least Squares
Date: 06/24/09 Time: 16:15
Sample (adjusted): 1970Q2 2000Q4
Included observations: 123 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.059858	0.021068	-2.841159	0.0053
LOG(U)	-0.003268	0.000738	-4.425339	0
D92Q1	-0.054809	0.000942	-58.18199	0
LOG(REALWH(-1)/PRODH(-1))	-0.071423	0.027284	-2.6178	0.01

R-squared	0.356794	Mean dependent var	0.003606
Adjusted R-squared	0.340578	S.D. dependent var	0.010246
S.E. of regression	0.00832	Akaike info criterion	-6.70834
Sum squared resid	0.008237	Schwarz criterion	-6.616887
Log likelihood	416.5629	F-statistic	22.00353
Durbin-Watson stat	2.460618	Prob(F-statistic)	0

Table A.1. Wage Setting Function, Estimate I

Dependent Variable: WGH
Method: Least Squares
Date: 06/24/09 Time: 16:22
Sample (adjusted): 1970Q2 2000Q4
Included observations: 123 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.06101	0.022333	-2.731817	0.0073
U	-0.103018	0.021799	-4.725811	0
D92Q1	-0.055994	0.001092	-51.27665	0
LOG(REALWH(-1)/PRODH(-1))	-0.09415	0.030879	-3.048977	0.0028

R-squared	0.35073	Mean dependent var	0.003606
Adjusted R-squared	0.334362	S.D. dependent var	0.010246
S.E. of regression	0.008359	Akaike info criterion	-6.698957
Sum squared resid	0.008315	Schwarz criterion	-6.607504
Log likelihood	415.9859	F-statistic	21.42762
Durbin-Watson stat	2.415394	Prob(F-statistic)	0

Table A.2. Wage Setting Function, Estimate II

APPENDIX B

Appendix to Chapter 3

B.1. Second Order Approximation to S_t as a Function of π_t

We want to take a second order approximation to S_t to see which forces drive mean price dispersion. From the law of motion of the price index we have

$$\tilde{p}_t^{1-\theta} = \frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}$$

Substituting this into the law of motion of S_t, given by

$$S_t = (1-\omega)\cdot \tilde{p}_t^{-\theta} + \omega\left(\frac{1+\pi_t}{\left(1+\pi_{t-1}\right)^{\chi}}\right)^{\theta} S_{t-1}$$

we have

$$S_t = (1-\omega)\cdot\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{\theta}{\theta-1}} + \omega\left(\frac{1+\pi_t}{\left(1+\pi_{t-1}\right)^{\chi}}\right)^{\theta} S_{t-1}$$

Thus we have expressed price dispersion as a function of inflation alone. We now take a second order approximation to this expression. We thus have to calculate $\frac{\partial S_t}{\partial \pi_t}$,

$\frac{\partial^2 S_t}{(\partial \pi_t)^2}, \frac{\partial S_t}{\partial \pi_{t-1}}, \frac{\partial S_t}{\partial \pi_{t-1} \vartheta \pi_t}, \frac{\partial S_t}{\partial S_{t-1}}$ and evaluate them at $\pi = 0$ and $S = 1$. The derivatives are

$$
\begin{aligned}
\frac{\partial S_t}{\partial \pi_t} &= (1-\omega)\frac{\theta}{\theta-1}\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{1}{\theta-1}} \\
&\quad \frac{(-\omega)}{1-\omega}(\theta-1)\,\pi_t^{\theta-2}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)} \\
&\quad +\omega\theta\left(\frac{(1+\pi_t)}{(1+\pi_{t-1})^{\chi}}\right)^{\theta-1} S_{t-1}\frac{1}{(1+\pi_{t-1})^{\chi}} \\
&= -\omega\theta\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{1}{\theta-1}}\left(1+\pi_t\right)^{\theta-2}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)} \\
&\quad +\omega\theta\frac{\left(1+\pi_t\right)^{\theta-1}}{\left(1+\pi_{t-1}\right)^{\chi\theta}}S_{t-1} \\
&= -\omega\theta+\omega\theta = 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 S_t}{\left(\partial\pi_t\right)^2} &= \frac{-\omega\theta}{1-\theta}\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{2-\theta}{\theta-1}} \\
&\quad \frac{(-\omega)}{1-\omega}\left(1+\pi_t\right)^{\theta-2}(\theta-1)\left(1+\pi_t\right)^{\theta-2}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)} \\
&\quad -\omega\theta\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{1}{\theta-1}} \\
&\quad (\theta-2)\left(1+\pi_t\right)^{\theta-3}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)} \\
&\quad +\omega\theta\frac{\left(1+\pi_t\right)^{\theta-2}}{\left(1+\pi_{t-1}\right)^{\chi\theta}}S_{t-1}(\theta-1) \\
&= \frac{\omega^2}{1-\omega}\left(1+\pi_t\right)^{2\theta-4}\theta\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{2-\theta}{\theta-1}} \\
&\quad \left(1+\pi_{t-1}\right)^{\chi(1-\theta)} \\
&\quad -\omega\theta(\theta-2)\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{1}{\theta-1}} \\
&\quad \left(1+\pi_t\right)^{\theta-3}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)} \\
&\quad +\omega\theta\frac{\left(1+\pi_t\right)^{\theta-2}}{\left(1+\pi_{t-1}\right)^{\chi\theta}}S_{t-1}(\theta-1) \\
&= \frac{\omega^2}{1-\omega}\theta-\omega\theta^2+2\omega\theta+\omega\theta^2-\omega\theta \\
&= \frac{\omega^2}{1-\omega}\theta+\omega\theta=\frac{\omega\theta}{1-\omega}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\vartheta^2 S_t}{\vartheta \pi_t \vartheta \pi_{t-1}} &= -\omega\theta \left(1+\pi_t\right)^{\theta-2} \left(1+\pi_{t-1}\right)^{\chi(1-\eta)} \\
&\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{2-\theta}{\theta-1}} \\
&\frac{(-\omega)}{1-\omega}\left(1+\pi_t\right)^{\theta-1}\chi\left(1-\theta\right)\frac{\left(1+\pi_{t-1}\right)^{\chi(1-\theta)-1}}{\theta-1} \\
&-\omega\theta\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{1}{\theta-1}} \\
&\left(1+\pi_t\right)^{\theta-2}\chi\left(1-\theta\right)\left(1+\pi_{t-1}\right)^{\chi(1-\theta)-1} \\
&-\omega\theta\chi\theta\frac{\left(1+\pi_t\right)^{\theta-1}}{\left(1+\pi_{t-1}\right)^{\chi\theta+1}}S_{t-1} \\
&= -\frac{\omega^2\theta\chi}{1-\omega}\left(1+\pi_t\right)^{2\theta-3}\left(1+\pi_{t-1}\right)^{2\chi(1-\theta)-1} \\
&\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{2-\theta}{\theta-1}} \\
&-\omega\theta\left(1+\pi_t\right)^{\theta-2}\chi\left(1-\theta\right)\left(1+\pi_{t-1}\right)^{\chi(1-\theta)-1} \\
&\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{1}{\theta-1}} \\
&-\omega\chi\theta^2\frac{\left(1+\pi_t\right)^{\theta-1}}{\left(1+\pi_{t-1}\right)^{\chi\theta+1}}S_{t-1} \\
&= -\frac{\omega^2\theta\chi}{1-\omega} - \omega\theta\chi\left(1-\theta\right) - \omega\chi\theta^2 \\
&= -\frac{\omega^2\theta\varkappa}{1-\omega} - \omega\theta\chi = \frac{-\omega\theta\chi}{1-\omega}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial S_t}{\partial \pi_{t-1}} &= (1-\omega)\frac{\theta}{\theta-1}\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{1}{\theta-1}} \\
&\quad \frac{(-\omega)}{1-\omega}\chi\left(1-\theta\right)\left(1+\pi_{t-1}\right)^{\chi(1-\theta)-1} \\
&\quad -\omega\theta\chi\left(1+\pi_t\right)^{\theta}\left(1+\pi_{t-1}\right)^{-\chi\theta-1}S_{t-1} \\
&= \left(1+\pi_{t-1}\right)^{\chi(1-\theta)-1}\chi\theta\omega\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{1}{\theta-1}} \\
&\quad -\omega\chi\theta\left(1+\pi_t\right)^{\theta}\left(1+\pi_{t-1}\right)^{-\chi\theta-1}S_{t-1} = \chi\theta\omega-\omega\chi\theta = 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 S_t}{\left(\partial \pi_{t-1}\right)^2} &= \frac{\chi\theta\omega}{\theta-1}\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{2-\theta}{\theta-1}} \\
& \frac{(-\omega)}{1-\omega}\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)-1}\chi\left(1-\theta\right) \\
& +\left[\chi\left(1-\theta\right)-1\right]\left(1+\pi_{t-1}\right)^{\chi(1-\theta)-2}\chi\theta\omega \\
& \left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{1}{\theta-1}} \\
& -\omega\theta\chi\left(1+\pi_t\right)^{\theta}\left(1+\pi_{t-1}\right)^{-\chi\theta-2}S_{t-1}\left(-\chi\theta-1\right) \\
&= \frac{\chi^2\theta\omega^2}{1-\omega}\left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{2-\theta}{\theta-1}} \\
& \left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)-1} \\
& +\chi\theta\omega\left[\chi\left(1-\theta\right)-1\right]\left(1+\pi_{t-1}\right)^{\chi(1-\theta)-2} \\
& \left(\frac{1-\omega\left(1+\pi_t\right)^{\theta-1}\left(1+\pi_{t-1}\right)^{\chi(1-\theta)}}{(1-\omega)}\right)^{\frac{1}{\theta-1}} \\
& +\left(\chi\theta+1\right)\omega\theta\chi\left(1+\pi_t\right)^{\theta}\left(1+\pi_{t-1}\right)^{-\chi\theta-2}S_{t-1} \\
&= \frac{\chi^2\theta\omega^2}{1-\omega}+\chi\theta\omega\left[\chi\left(1-\theta\right)-1\right]+\left(\chi\theta+1\right)\omega\theta\chi \\
&= \frac{\chi^2\theta\omega^2}{1-\omega}+\chi^2\theta\omega-\chi^2\theta^2\omega-\chi\theta\omega+\chi\theta\omega+\chi^2\theta^2\omega \\
&= \frac{\chi^2\theta\omega^2}{1-\omega}+\chi^2\theta\omega=\frac{\chi^2\theta\omega}{1-\omega}
\end{aligned}
$$

$$
\frac{\vartheta S_t}{\vartheta S_{t-1}}=\omega\left(\frac{1+\pi_t}{\left(1+\pi_{t-1}\right)^{\chi}}\right)^{\theta}=\omega
$$

$$
\frac{\partial^2 S_t}{\left(\partial S_{t-1}\right)^2}=0
$$

$$
\frac{\partial S_t}{\partial S_{t-1}\partial \pi_t}=\omega\theta\frac{\left(1+\pi_t\right)^{\theta-1}}{\left(1+\pi_{t-1}\right)^{\chi\theta}}=\omega\theta
$$

$$\frac{\partial S_t}{\partial S_{t-1}\partial \pi_{t-1}} = -\chi\theta\omega\left(1+\pi_t\right)^{\theta}\left(1+\pi_{t-1}\right)^{-\chi\theta-1} = -\chi\theta\omega$$

A second order Taylor expansion to S_t will be of the following form (noting that $\frac{\vartheta S_t}{\vartheta \pi_t} = \frac{\vartheta S_t}{\vartheta \pi_{t-1}} = 0$ if evaluated at the zero inflation steady state)

$$\begin{aligned}
S_t &\approx \overline{S} + \frac{\partial S_t}{\partial S_{t-1}} dS_{t-1} + \frac{\partial S_t}{\partial \pi_t} d\pi_t + \frac{\partial S_t}{\partial \pi_{t-1}} d\pi_{t-1} + \frac{1}{2}\frac{\partial^2 S_t}{\left(\partial \pi_t\right)^2}\left(d\pi_t\right)^2 + \frac{1}{2}\frac{\partial^2 S_t}{\left(\partial \pi_{t-1}\right)^2}\left(d\pi_{t-1}\right)^2 \\
&\quad + \frac{\partial^2 S_t}{\partial \pi_t \partial \pi_{t-1}}\left(d\pi_t d\pi_{t-1}\right) + \frac{\partial S_t}{\partial S_{t-1}\partial \pi_t}\left(d\pi_t dS_{t-1}\right) + \frac{\partial S_t}{\partial S_{t-1}\partial \pi_{t-1}}\left(d\pi_{t-1} dS_{t-1}\right) \\
&= \overline{S} + \frac{\partial S_t}{\partial S_{t-1}} dS_{t-1} + \frac{\partial^2 S_t}{\partial \pi_t \partial \pi_{t-1}}\left(d\pi_t d\pi_{t-1}\right) + \frac{1}{2}\frac{\partial^2 S_t}{\left(\partial \pi_t\right)^2}\left(d\pi_t\right)^2 + \frac{1}{2}\frac{\partial^2 S_t}{\left(\partial \pi_{t-1}\right)^2}\left(d\pi_{t-1}\right)^2 \\
&\quad + \frac{\partial^2 S_t}{\partial \pi_t \partial \pi_{t-1}}\left(d\pi_t d\pi_{t-1}\right) + \frac{\partial S_t}{\partial S_{t-1}\partial \pi_t}\left(d\pi_t dS_{t-1}\right) + \frac{\partial S_t}{\partial S_{t-1}\partial \pi_{t-1}}\left(d\pi_{t-1} dS_{t-1}\right)
\end{aligned}$$

We now take the unconditional expectation of S_t. Note that $E\left(d\pi_t dS_{t-1}\right) = E\left(d\pi_{t-1} dS_{t-1}\right) = 0$ because as can be seen from above, up to first order, S_t follows a deterministic autoregressive process of the form $dS_t = \frac{\partial S_t}{\partial S_{t-1}} dS_{t-1}$. This implies that up to first order, the variance of S_t is zero as and will be its covariance with π_t and π_{t-1}. Hence we write

$$\begin{aligned}
ES_t &= \overline{S} + \omega E dS_{t-1} + \frac{1}{2}\frac{\omega\theta}{1-\omega} E\left(d\pi_t\right)^2 + \frac{1}{2}\frac{\chi^2\theta\omega}{1-\omega} E\left(d\pi_{t-1}\right)^2 - \frac{\omega\theta\chi}{1-\omega} E\left(d\pi_t d\pi_{t-1}\right) \\
ES_t &= \overline{S} + \frac{1}{2}\frac{\omega\theta}{\left(1-\omega\right)^2}\left[1+\chi^2\right] E\left(d\pi_t\right)^2 - \frac{\omega\theta\chi}{\left(1-\omega\right)^2} E\left(d\pi_t d\pi_{t-1}\right) \\
ES_t &= 1 + \frac{1}{2}\frac{\omega\theta}{\left(1-\omega\right)^2}\left[1+\chi^2\right] E\left(d\pi_t\right)^2 - \frac{\omega\theta\chi}{\left(1-\omega\right)^2} AC\left(d\pi_t\right) E\left(d\pi_t\right)^2
\end{aligned}$$

B.2. How the Elasticity of Money Demand affects the Output Cost of an Interest Rate Increase

$$
\begin{aligned}
v_t &= \sqrt{\frac{\phi_2}{\phi_1} + \frac{1}{\phi_1}\frac{i_t}{1+i_t}} \\
\varepsilon_{mi} &= -\frac{1}{8}\frac{1}{\phi_2\left(1+\bar{i}\right)^2 + \left(1+\bar{i}\right)\bar{i}} \\
\phi_2 &= \frac{-\bar{i}}{1+\bar{i}} - \frac{1}{8\varepsilon_{mi}\left(1+\bar{i}\right)^2} = \frac{-8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i} - 1}{8\varepsilon_{mi}\left(1+\bar{i}\right)^2} \\
\phi_1 &= \left(\phi_2 + \frac{\bar{i}}{1+\bar{i}}\right)\frac{1}{\overline{v}^2} = \left(\frac{-1}{8\varepsilon_{mi}\left(1+\bar{i}\right)^2}\right)\frac{1}{\overline{v}^2} \\
\frac{\phi_2}{\phi_1} &= \overline{v}^2\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}+1\right) \\
l(v_t) &= \phi_1 v_t + \phi_2/v_t - 2\sqrt{\phi_1\phi_2} \\
&= \frac{1}{2}\left(\phi_1\phi_2 + \phi_1\frac{i_t}{1+i_t}\right)^{1/2} + \frac{\phi_2}{\left(\frac{\phi_2}{\phi_1} + \frac{1}{\phi_1}\frac{i_t}{1+i_t}\right)^{1/2}} - 2\sqrt{\phi_1\phi_2} \\
\frac{dl}{di_t} &= \frac{\frac{-1}{16\varepsilon_{mi}\left(1+\bar{i}\right)^4\overline{v}^2}}{\left(\frac{8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}+1}{64\left(1+\bar{i}\right)^2\varepsilon_{mi}^2\overline{v}^2} - \frac{\bar{i}}{8\varepsilon_{mi}\left(1+\bar{i}\right)^2\overline{v}^2}\right)^{1/2}} \\
&\quad - \frac{\overline{v}^2\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}+1\right)}{2\left(1+\bar{i}\right)^2\left(\overline{v}^2\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}+1\right) - 8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}\right)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
= \quad & \frac{-1}{(-1)\left(256\left|\varepsilon_{mi}\right|\left(1+\bar{i}\right)^{8}\overline{v}^{4}\right)^{1/2}\left(\frac{8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}+1}{64\left(1+\bar{i}\right)^{2}\left|\varepsilon_{mi}\right|^{2}\overline{v}^{2}}+\frac{\bar{i}}{8\left|\varepsilon_{mi}\right|\left(1+\bar{i}\right)^{2}\overline{v}^{2}}\right)^{1/2}} \\
& -\frac{\overline{v}^{2}\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}+1\right)}{2\left(1+\bar{i}\right)^{2}\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}\left(\overline{v}^{2}-1\right)+\overline{v}^{2}\right)^{3/2}} \\
= \quad & \frac{1}{\left(4\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}+1\right)\overline{v}^{2}\left(1+\bar{i}\right)^{2}-\bar{i}32\varepsilon_{mi}\left(1+\bar{i}\right)^{4}\overline{v}^{2}\right)^{1/2}} \\
& -\frac{\overline{v}^{2}\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}+1\right)}{2\left(1+\bar{i}\right)^{2}\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}\left(\overline{v}^{2}-1\right)+\overline{v}^{2}\right)^{3/2}} \\
= \quad & \frac{1}{2\overline{v}\left(1+\bar{i}\right)\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}+1-\bar{i}8\left(1+\bar{i}\right)^{2}\varepsilon_{mi}\right)^{1/2}} \\
& -\frac{\overline{v}^{2}\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}+1\right)}{2\left(1+\bar{i}\right)^{2}\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}\left(\overline{v}^{2}-1\right)+\overline{v}^{2}\right)^{3/2}} \\
= \quad & \frac{1}{2\overline{v}\left(1+\bar{i}\right)\left(8\varepsilon_{mi}\bar{i}\left(1+\bar{i}-\left(\bar{i}^{2}+2\bar{i}+1\right)\right)+1\right)^{1/2}} \\
& -\frac{\overline{v}^{2}\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}+1\right)}{2\left(1+\bar{i}\right)^{2}\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}\left(\overline{v}^{2}-1\right)+\overline{v}^{2}\right)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
= \quad & \frac{1}{2\overline{v}\left(1+\bar{i}\right)\left(-8\varepsilon_{mi}\bar{i}\left(\bar{i}^{2}+\bar{i}\right)+1\right)^{1/2}} \\
& -\frac{\overline{v}^{2}\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}+1\right)}{2\left(1+\bar{i}\right)^{2}\left(8\varepsilon_{mi}\left(1+\bar{i}\right)\bar{i}\left(\overline{v}^{2}-1\right)+\overline{v}^{2}\right)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial l}{\partial i_t \partial \varepsilon_{mi}} &= -\frac{(-8)\,\bar{i}\left(\bar{i}^2+\bar{i}\right)\left(-\varepsilon_{mi}8\bar{i}\left(\bar{i}^2+\bar{i}\right)+1\right)^{-3/2}}{4\overline{v}\left(1+\bar{i}\right)} - \frac{\overline{v}^2}{2\left(1+\bar{i}\right)^2} \\
&\quad \frac{\begin{array}{c} 8\bar{i}\left(1+\bar{i}\right)\left(8\varepsilon_{mi}\bar{i}\left(1+\bar{i}\right)\left(\overline{v}^2-1\right)+\overline{v}^2\right)^{3/2} \\ -\left(8\varepsilon_{mi}\bar{i}\left(1+\bar{i}\right)+1\right)\frac{3}{2}\left(\varepsilon_{mi}\bar{i}\left(1+\bar{i}\right)\left(\overline{v}^2-1\right)+\overline{v}^2\right)^{1/2}8\bar{i}\left(1+\bar{i}\right)\left(\overline{v}^2-1\right) \end{array}}{\left(8\varepsilon_{mi}\bar{i}\left(1+\bar{i}\right)\left(\overline{v}^2-1\right)+\overline{v}^2\right)^3} \\
&= \frac{2\bar{i}^2}{\overline{v}\left(-\varepsilon_{mi}8\bar{i}\left(\bar{i}^2+\bar{i}\right)+1\right)^{3/2}} - \frac{\overline{v}^2}{2\left(1+\bar{i}\right)^2} \\
&\quad \frac{\begin{array}{c} 8\bar{i}\left(1+\bar{i}\right)\left(8\varepsilon_{mi}\bar{i}\left(1+\bar{i}\right)\left(\overline{v}^2-1\right)+\overline{v}^2\right) \\ -\left(8\varepsilon_{mi}\bar{i}\left(1+\bar{i}\right)+1\right)12\bar{i}\left(1+\bar{i}\right)\left(\overline{v}^2-1\right) \end{array}}{\left(8\varepsilon_{mi}\bar{i}\left(1+\bar{i}\right)\left(\overline{v}^2-1\right)+\overline{v}^2\right)^{5/2}} \\
&= \frac{2\bar{i}^2}{\overline{v}\left(-\varepsilon_{mi}8\bar{i}\left(\bar{i}^2+\bar{i}\right)+1\right)^{3/2}} - \frac{\overline{v}^2}{2\left(1+\bar{i}\right)^2} \\
&\quad \frac{(64)\,\bar{i}^2\left(1+\bar{i}\right)^2\left(\overline{v}^2-1\right)+8\bar{i}\left(1+\bar{i}\right)\overline{v}^2-\left(96\bar{i}^2\left(1+\bar{i}\right)^2+12\bar{i}\left(1+\bar{i}\right)\left(\overline{v}^2-1\right)\right)}{\left(8\varepsilon_{mi}\bar{i}\left(1+\bar{i}\right)\left(\overline{v}^2-1\right)+\overline{v}^2\right)^{5/2}} \\
&= \frac{2\bar{i}^2}{\overline{v}\left(-\varepsilon_{mi}8\bar{i}\left(\bar{i}^2+\bar{i}\right)+1\right)^{3/2}} - \frac{\overline{v}^2}{2\left(1+\bar{i}\right)^2} \\
&\quad \frac{-32\varepsilon_{mi}\bar{i}^2\left(1+\bar{i}\right)^2\left(\overline{v}^2-1\right)-4\bar{i}\left(1+\bar{i}\right)\overline{v}^2+12\bar{i}\left(1+\bar{i}\right)}{\left(8\varepsilon_{mi}\bar{i}\left(1+\bar{i}\right)\left(\overline{v}^2-1\right)+\overline{v}^2\right)^{5/2}} \\
&= \frac{2\bar{i}^2}{\overline{v}\left(-\varepsilon_{mi}8\bar{i}\left(\bar{i}^2+\bar{i}\right)+1\right)^{3/2}} \\
&\quad + \frac{\overline{v}^2}{2\left(1+\bar{i}\right)^2}\frac{32\varepsilon_{mi}\bar{i}^2\left(1+\bar{i}\right)^2\left(\overline{v}^2-1\right)+4\bar{i}\left(1+\bar{i}\right)\overline{v}^2-12\bar{i}\left(1+\bar{i}\right)}{\left(8\varepsilon_{mi}\bar{i}\left(1+\bar{i}\right)\left(\overline{v}^2-1\right)+\overline{v}^2\right)^{5/2}}
\end{aligned}
$$

The first term is clearly positive but will be very small as it involves $\bar{i}^2$, and the interest rate will be quite a small number. The second fraction will be negative as the first term in the numerator $(32\varepsilon_{mi}\bar{i}^2\left(1+\bar{i}\right)^2\left(\overline{v}^2-1\right))$ is negative and the second term

$4\bar{i}\left(1+\bar{i}\right)\overline{v}^2 < 12\bar{i}\left(1+\bar{i}\right)$ for values of $\overline{v}^2$ smaller than 3 (or $\overline{v} < 1.73$), which seems reasonable. The second fraction would be expected to be one order of magnitude higher than the first due to the $\bar{i}^2$ multiplying the first fraction.

In our calibration, we have $\bar{i} = 0.018626929$ and $\overline{v} = 1.06$, while for ε_{mi} we have -5.11 and -2.93. This yields $\frac{\partial l}{\partial i_t \partial \varepsilon_{mi}} = -0.075010667 < 0$ and $\frac{\partial l}{\partial i_t \partial \varepsilon_{mi}} = -0.066545052 < 0$. Hence a more negative elasticity of money demand increases the output costs of an interest rate rise, and thus the costs of an increase in inflation.

APPENDIX C

Appendix to Chapter 4

C.1. Determinacy in the Blanchard and Gali Model

We show in this section that, for reasonable calibrations, the condition stated in proposition one ensures determinacy in the Blanchard Gali model. Woodford (2003) derives conditions for determinacy for a linear rational expectations model of the form

$$\begin{pmatrix} E_t z_{t+1} \\ x_{t+1} \end{pmatrix} = A \begin{pmatrix} z_t \\ x_t \end{pmatrix} + b e_t$$

$$where\ A = \begin{pmatrix} \frac{1+\frac{\kappa_F \phi_\pi}{c_0}}{\beta+\kappa_F/c_0} & \frac{\kappa_0+\kappa_F\frac{c_1+\phi_u-c_0}{c_0}}{\beta+\kappa_F/c_0} & -\frac{\kappa_L+\frac{c_1}{c_0}}{\beta+\kappa_F/c_0} \\ \frac{-\beta\phi_\pi+1}{\kappa_F+c_0\beta} & \frac{-\beta(c_1-c_0+\phi_u)+\kappa_0}{\kappa_F+c_0\beta} & \frac{\beta c_1-\kappa_L}{\kappa_F+c_0\beta} \\ 0 & 1 & 0 \end{pmatrix},\ b = \begin{pmatrix} \frac{-\frac{\kappa_F\rho_a}{c_0}+\lambda\Phi\gamma}{\beta+\kappa_F/c_0} \\ \frac{-\beta(1-\rho_a)-\kappa_F/c_0+\lambda\Phi\gamma}{\kappa_F+c_0\beta} \\ 0 \end{pmatrix}$$

where z_t is a 2x1 vector of endogenous jump variables, x_t is single endogenous predetermined variable and e_t is a vector of disturbances. This is exactly the kind of system we are dealing with. The rational expectations equilibrium will be determinate if and only if the matrix A has exactly one eigenvalue inside the unit circle, i.e. with modulus smaller than 1 and the two other eigenvalues outside the unit circle. If the characteristic equation is written in the form

$$\mu^3 + A_2\mu^2 + A_1\mu + A_0 = 0$$

Woodford shows that it will have two roots outside and one root inside the unit circle if and only if either (Case I)

$$\begin{aligned} 1 + A_2 + A_1 + A_0 &< 0 \text{ and} \\ -1 + A_2 - A_1 + A_0 &> 0 \end{aligned}$$

or (Case II)

$$\begin{aligned} 1 + A_2 + A_1 + A_0 &> 0 \\ -1 + A_2 - A_1 + A_0 &< 0 \\ A_0^2 - A_0 A_2 + A_1 - 1 &> 0 \end{aligned}$$

or (Case III) the first two conditions of Case II and

$$\begin{aligned} A_0^2 - A_0 A_2 + A_1 - 1 &< 0 \\ |A_2| &> 3 \end{aligned}$$

As would be expected, some of the resulting expressions will be quite complicated functions of the deep parameters. We therefore do not aspire to give a completely general proof. Rather, we will make the assumption throughout that g is a very small number. $g = Bx^{\alpha}$, and B will be calibrated to such that the fraction of total hiring costs in GDP δBx^{α} does not exceed a small fraction of GDP (Blanchard and Gali set them equal to 1% of GDP for the "American" and even less for the continental European calibration). In Blanchard and Gali, it comes out as 0.03. This also implies that $\xi_1 < \xi_0$, and both c_1 and ξ_1 will be small. Furthermore, we will assume that $\kappa_F - \kappa_L > 0$, which will be the case if $\kappa_F - \kappa_L = Mg\lambda \frac{(1-\delta)}{1-u} \left[\frac{\alpha}{\delta} [1 - x - \beta] + \beta [\xi_1 + \xi_0] \right] > 0$. This condition holds for values of x and associated values of δ which are not too small. For the calibration considered in this chapter, $\kappa_F - \kappa_L > 0$ for $x \geq 0.015$ and $\delta = 0.0017$, both of which is far below empirically reasonable values for these parameters.

Our first task is to derive the characteristic equation. To make the algebra easier, we first write our matrix A in a more general form:

$$A = \begin{pmatrix} \frac{1+\frac{\kappa_F \phi_\pi}{c_0}}{\beta+\kappa_F/c_0} & \frac{\kappa_0+\kappa_F\frac{c_1+\phi_u-c_0}{c_0}}{\beta+\kappa_F/c_0} & -\frac{\kappa_L+\frac{c_1}{c_0}}{\beta+\kappa_F/c_0} \\ \frac{-\beta\phi_\pi+1}{\kappa_F+c_0\beta} & \frac{-\beta(c_1-c_0+\phi_u)+\kappa_0}{\kappa_F+c_0\beta} & \frac{\beta c_1-\kappa_L}{\kappa_F+c_0\beta} \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 1 & 0 \end{pmatrix}$$

The characteristic equation is then given by

$$\mu^3 + (-a_{11} - a_{22})\,\mu^2 + (-a_{23} + a_{11}a_{22} - a_{12}a_{21})\,\mu + a_{11}a_{23} - a_{21}a_{13} = 0$$

Hence we can determine A_2, A_1 and A_0 as

$$\begin{aligned} A_2 &= -a_{11} - a_{22} = \frac{-c_0\,(1+\beta) - \kappa_F\phi_\pi - \kappa_0 + \beta\,(c_1+\phi_u)}{\kappa_F + c_0\beta} \\ A_1 &= \frac{-\,(1+\beta)\,c_1 + \kappa_L + c_0 - \phi_u + \phi_\pi\kappa_0}{\kappa_F + c_0\beta} \\ A_0 &= \frac{c_1 - \phi_\pi\kappa_L}{\kappa_F + c_0\beta} \end{aligned}$$

We first look at the second condition of Case I. We have

$$-1 + A_2 - A_1 + A_0 > 0$$

implying

$$\frac{2}{1-u}\frac{(\xi_1 - \xi_0)\,(1+\beta)}{\kappa_0 + \kappa_L + \kappa_F} - 1 > \phi_\pi$$

or

$$\frac{2}{1-u}\frac{\left(g\left(1-\delta\right)\left(1+\alpha\left(1-x\right)\right)+g\left(1+\alpha\right)-1\right)\left(1+\beta\right)}{\kappa_0+\kappa_L+\kappa_F}-1>\phi_\pi$$

This condition will never be fulfilled by positive values of ϕ_π under the assumptions made.

Thus we conclude that Case 1 is not relevant and turn to Case 2. The first condition implies

$$\phi_\pi-\phi_u\frac{\left(1-\beta\right)}{\kappa_0-\kappa_L-\kappa_F}>1 \tag{C.1}$$

The second condition is implied by the fact that the second condition of Case 1 is violated, while the third condition implies

$$\begin{aligned}\text{(C.2)}\qquad &\phi_\pi\left[-\phi_\pi\kappa_L\left[\kappa_F-\kappa_L\right]-c_1\kappa_L+c_0\left[\beta\kappa_0-\kappa_L\right]+\kappa_0\left[\kappa_F-\kappa_L\right]+\kappa_L\phi_u\beta\right]\\ &+c_1\left[1-\beta+c_0\left(1-\beta^2\right)-\kappa_F\right]+\left[\kappa_L+c_0\left(1-\beta\right)-\kappa_F-\phi_u\right]\left[\kappa_F+c_0\beta\right]>0\end{aligned}$$

Not that if $\kappa_F<\kappa_L$, this expression will be monotonously increasing in ϕ_π. Hence in that case, if $(C.2)$) holds for $\phi_\pi=1$, it will hold for $\phi_\pi>1$ as well. Hence we set $\phi_\pi=1$ and $\phi_u=1$(since permissible, i.e. negative values of ϕ_u make $(C.2)$) more likely to be met), which allows us to write the condition as

$$\begin{aligned}&\kappa_0\left(\kappa_F-\kappa_L\right)+c_0\left[\beta\kappa_0-\kappa_L-\beta\kappa_F\right]+\kappa_L\left(\kappa_L-c_1\right)+c_1\left(1-\beta\right)\\ &+c_0c_1\left(1-\beta^2\right)+c_0^2\left(1-\beta\right)\beta+c_0\kappa_F\left(1-\beta\right)+c_0\beta\kappa_L-\kappa_F^2-c_1\kappa_F>0\end{aligned}$$

This will usually be fulfilled. Given that c_0 slightly larger than 1 and that κ_L and κ_F are in the same order of magnitude but smaller than 1, and c_1 is quite small, $c_0\beta\kappa_L > \kappa_F^2 + c_1\kappa_F$.

If we assume $\kappa_F - \kappa_L > 0$, there is still an issue of $(C.2)$ being violated for sufficiently large values of ϕ_π since ϕ_π^2 has a negative coefficient. We will now show that under the assumptions already made, if $A_0^2 - A_0A_2 + A_1 - 1 > 0$ becomes violated, we will already be in a situation where $|A_2| > 3$ and thus Case III kicks in. Let us first consider the terms in $(C.2)$ not involving ϕ_π. Those can be written as

$$c_1\left[1-\beta+c_0\left(1-\beta^2\right)\right]+\kappa_F\left[\kappa_L-c_1\right]+\kappa_F c_0\left(1-\beta\right)+c_0\beta\kappa_L-\kappa_F^2+c_0^2\left(1-\beta\right)-c_0\beta\kappa_F > 0.$$

The term $-\kappa_F^2$ is dominated by $c_0\beta\kappa_L$ under the assumptions already made and all the other terms but $-c_0\beta\kappa_F$ are positive. It is not clear that $-c_0\beta\kappa_F$ is being dominated by any of the other terms. Therefore, in the next step, we disregard all the other terms not involving ϕ_π except for $-c_0\beta\kappa_F$. If the modified condition is fulfilled, so will be $(C.2)$ .Hence we look for which ϕ_π we have (still for $\phi_u = 0$)

$$-\phi_\pi^2\kappa_L\left[\kappa_F-\kappa_L\right]+\phi_\pi\left[-c_1\kappa_L+c_0\left[\beta\kappa_0-\kappa_L\right]+\kappa_0\left[\kappa_F-\kappa_L\right]\right]-c_0\beta\kappa_F > 0$$

or

$$\phi_\pi^2-\frac{\left[-c_1\kappa_L+c_0\left[\beta\kappa_0-\kappa_L\right]+\kappa_0\left[\kappa_F-\kappa_L\right]\right]}{\kappa_L\left[\kappa_F-\kappa_L\right]}\phi_\pi+\frac{c_0\beta\kappa_F}{\kappa_L\left[\kappa_F-\kappa_L\right]} < 0$$

The polynomial on the left hand side has two solutions $\phi_{\pi 1}$ and $\phi_{\pi 2}$ and the inequality will be fulfilled if ϕ_π lies between. Hence we have

$$\phi_{\pi 1,2} = \frac{-c_1\kappa_L + c_0\left[\beta\kappa_0 - \kappa_L\right] + \kappa_0\left[\kappa_F - \kappa_L\right]}{2\kappa_L\left[\kappa_F - \kappa_L\right]} \pm\sqrt{\frac{\left(-c_1\kappa_L + c_0\left[\beta\kappa_0 - \kappa_L\right] + \kappa_0\left[\kappa_F - \kappa_L\right]\right)^2}{4\kappa_L^2\left[\kappa_F - \kappa_L\right]^2} - \frac{c_0\beta\kappa_F}{\kappa_L\left[\kappa_F - \kappa_L\right]}}$$

Since we now assume $\kappa_F > \kappa_L$, the expression under the root will always be positive, as will the expression outside of the root. This also implies that we can focus on the larger of the two solution since $\sqrt{\frac{(-c_1\kappa_L + c_0[\beta\kappa_0 - \kappa_L] + \kappa_0[\kappa_F - \kappa_L])^2}{4\kappa_L^2[\kappa_F - \kappa_L]^2} - \frac{c_0\beta\kappa_F}{\kappa_L[\kappa_F - \kappa_L]}} > \frac{-c_1\kappa_L + c_0[\beta\kappa_0 - \kappa_L] + \kappa_0[\kappa_F - \kappa_L]}{2\kappa_L[\kappa_F - \kappa_L]}$ and thus the smaller solution will be will be negative. Hence the relevant lower bound is $(C.1)$ The larger of the two roots will be at least as big as the term outside the brackets. Hence condition $(C.2)$ will still be met under the assumptions made if

$$\phi_\pi < \frac{-c_1\kappa_L + c_0\left[\beta\kappa_0 - \kappa_L\right] + \kappa_0\left[\kappa_F - \kappa_L\right]}{2\kappa_L\left[\kappa_F - \kappa_L\right]} \tag{C.3}$$

We now turn to condition $|A_2| > 3$ from Case III to see what it implies for ϕ_π. For the "large" values of ϕ_π which are of interest here, A_2 will most likely be negative, so we consider the inequality $-A_2 > 3$, which can be written as

$$\phi_\pi > \frac{-c_0 + 2c_0\beta - \kappa_0 + c_1\beta}{\kappa_F} + 3 \tag{C.4}$$

We would like to check whether at the point $(C.3)$ becomes violated $(C.4)$ is already met. Hence we are asking whether

$$\frac{-c_1\kappa_L + c_0\left[\beta\kappa_0 - \kappa_L\right] + \kappa_0\left[\kappa_F - \kappa_L\right]}{2\kappa_L\left[\kappa_F - \kappa_L\right]} > \frac{-c_0 + 2c_0\beta - \kappa_0 + c_1\beta}{\kappa_F} + 3$$

holds. This can be written as

$$
\begin{aligned}
-c_1\kappa_L\kappa_F + c_0\left[\beta\kappa_0 - \kappa_L\right]\kappa_F + \kappa_0\left[\kappa_F - \kappa_L\right]\kappa_F \\
-2c_0\kappa_L\left[\kappa_F - \kappa_L\right] + 4c_0\beta\kappa_L\left[\kappa_F - \kappa_L\right] \\
-2\kappa_0\kappa_L\left[\kappa_F - \kappa_L\right] + c_1\beta 2\kappa_L\left[\kappa_F - \kappa_L\right] + 6\kappa_L\kappa_F\left[\kappa_F - \kappa_L\right] &> 0
\end{aligned}
$$

or

$$
\begin{aligned}
-c_1\kappa_L\kappa_F + c_0\left[\beta\kappa_0 - \kappa_L\right]\kappa_F + 2c_0\kappa_L\kappa_F - 2c_0\kappa_L^2 - 4\beta c_0\kappa_L\kappa_F \\
+4\beta c_0\kappa_L^2 + \left[\kappa_F - \kappa_L\right]\left(\kappa_0\kappa_F + 2\kappa_L\kappa_0 - 6\kappa_L\kappa_F\right) \\
-c_1\beta 2\kappa_L\left[\kappa_F - \kappa_L\right] > 0
\end{aligned}
$$

or

$$
\begin{aligned}
-c_1\kappa_L\kappa_F + c_0\kappa_F\left[\beta\kappa_0 - \kappa_L - \beta\kappa_L\right] + \kappa_L\left(2\beta c_0\kappa_L - \beta c_0\kappa_F\right) \\
+2c_0\kappa_F\kappa_L - 2c_0\kappa_L^2 - 2c_0\beta\kappa_L\kappa_F + 2\beta c_0\kappa_L^2 \\
+\left[\kappa_F - \kappa_L\right]\left(\kappa_0\kappa_F + 2\kappa_L\kappa_0 - 6\kappa_L\kappa_F\right) - c_1\beta 2\kappa_L\left[\kappa_F - \kappa_L\right] &> 0
\end{aligned}
$$

or

$$
\begin{aligned}
-c_1\kappa_L\kappa_F + c_0\left[\beta\kappa_0 - \kappa_L\right]\kappa_F + 2c_0\kappa_L\kappa_F - 2c_0\kappa_L^2 - 4\beta c_0\kappa_L\kappa_F + 4\beta c_0\kappa_L^2 \\
+\left[\kappa_F - \kappa_L\right]\left(\kappa_0\kappa_F + 2\kappa_L\kappa_0 - 6\kappa_L\kappa_F\right) - c_1\beta 2\kappa_L\left[\kappa_F - \kappa_L\right] &> 0
\end{aligned}
$$

Note also that

$$
\begin{aligned}
& 2c_0\kappa_F\kappa_L - 2c_0\kappa_L^2 - 2c_0\beta\kappa_L\kappa_F + 2\beta c_0\kappa_L^2 - c_1\kappa_L\kappa_F - c_1\beta 2\kappa_L\left[\kappa_F - \kappa_L\right] \\
= \; & 2\kappa_L c_0\left(1-\beta\right)\left[\kappa_F - \kappa_L\right] - c_1\kappa_L\kappa_F - c_1\beta 2\kappa_L\left[\kappa_F - \kappa_L\right] \\
= \; & -2\kappa_L\left[\kappa_F - \kappa_L\right]\left[c_1\beta - c_0\left(1-\beta\right)\right] - c_1\kappa_L\kappa_F
\end{aligned}
$$

Thus we can write

$$
\begin{aligned}
& c_0\kappa_F\left[\beta\kappa_0 - \kappa_L - \beta\kappa_L\right] + \beta c_0\kappa_L\left(2\kappa_L - \kappa_F\right) \\
& + \left[\kappa_F - \kappa_L\right]\left(\kappa_0\kappa_F + 2\kappa_L\kappa_0 - 6\kappa_L\kappa_F\right) - 2\kappa_L\left[\kappa_F - \kappa_L\right]\left[c_1\beta - c_0\left(1-\beta\right)\right] - c_1\kappa_L\kappa_F > 0
\end{aligned}
$$

Since (using $\kappa_0 > \kappa_F + \kappa_L$) $\kappa_0\kappa_F + 2\kappa_L\kappa_0 - 6\kappa_L\kappa_F > \left(\kappa_F - \kappa_L\right)^2 - \kappa_L\left(\kappa_F - \kappa_L\right) = \left(\kappa_F - \kappa_L\right)\left(\kappa_F - 2\kappa_L\right)$we can write

$$
\begin{aligned}
& \kappa_F c_0\left(\beta\kappa_0 - \kappa_L - \beta\kappa_L\right) + \beta c_0\kappa_L\left(2\kappa_L - \kappa_F\right) + \\
& \left[\kappa_F - \kappa_L\right]^2\left(\kappa_F - 2\kappa_L\right) - 2\kappa_L\left[\kappa_F - \kappa_L\right]\left[c_1\beta - c_0\left(1-\beta\right)\right] - c_1\kappa_L\kappa_F > 0
\end{aligned}
$$

The first term is clearly positive. The second term will be positive as long as $\kappa_F < 2\kappa_L$. If $\kappa_F - \kappa_L$ increases, in that case the first term would increase and at a larger rate as both $\left(\beta\kappa_0 - \kappa_L - \beta\kappa_L\right)$ and κ_F would increase. In this case we would also see the third term switch from negative to positive, which would otherwise also be negative. The final two terms are negative. We believe it is safe to assume that this condition holds. For values of κ_L and κ_F which are close, $\kappa_F c_0\left(\beta\kappa_0 - \kappa_L - \beta\kappa_L\right) + \beta c_0\kappa_L\left(2\kappa_L - \kappa_F\right)$ will be in a higher order of magnitude than $\left[\kappa_F - \kappa_L\right]^2\left(\kappa_F - 2\kappa_L\right) - 2\kappa_L\left[\kappa_F - \kappa_L\right]\left[c_1\beta - c_0\left(1-\beta\right)\right] - c_1\kappa_L\kappa_F$. For values of κ_F substantially higher than κ_L, the order of magnitude of $\kappa_F c_0 (\beta\kappa_0 - \kappa_L - \kappa_L)$ will increase and $[\kappa_F - \kappa_L]^2 (\kappa_F - 2\kappa_L)$ would turn positive. Thus the second condition of case III will be satisfied for values of ϕ_π violating $(C.2)$.

Thus we have shown, under the assumptions made, that $\phi_\pi - \phi_u \frac{(1-\beta)}{\kappa_0 - \kappa_L - \kappa_F} > 1$ guarantees the existence of a unique rational expectations equilibrium in the Blanchard/ Gali model.

C.2. Relevant steady State Values in the Model with Skill Loss

As was mentioned in the text, we start by assuming values for u and x. This allows to write the steady state values of δ, s^i, A^L and A^A as

$$\begin{aligned}
\delta &= \frac{ux}{(1-u)(1-x)} \\
s^i &= x(1-x)^i \\
A^L &= \sum_{i=0}^{\infty} s^i \beta_s^i = \frac{x}{1-(1-x)\beta_s}
\end{aligned}$$

and

$$A^A = s^N A^L + \left(1 - s_t^N\right) = \delta A^L + 1 - \delta$$

This allows to back out Θ by first noting that in the steady state, we can write (4.11) as

$$A^L \left[\frac{1}{M} - g\left(1 - \beta\left(1-\delta\right)\right)\right] + \beta\left(1-\delta\right)\left[\frac{1-A^L}{M}\right] = \Theta'\left[\beta\left(1-\delta\right) + \frac{W}{\Theta'}\left(1 - \beta\left(1-\delta\right)\right)\right]$$

From (4.7), we have

$$W = \Theta' \sum_{i=0}^{\infty} s^i \beta_s^{i(1-\gamma)} = \Theta' \frac{x}{1-(1-x)\beta_s^{1-\gamma}} \tag{C.5}$$

and, for W^L

$$W^L = \sum_{i=0}^{\infty} s^i \beta_s^{i(1-\gamma)} = \frac{x}{1-(1-x)\,\beta_s^{1-\gamma}}$$

which we use to solve for Θ' as

$$\Theta' = \frac{1/M - g\,(1-(1-\delta)\,\beta) + \frac{(1-\delta)\beta}{M}\left(1-A^L\right)}{(1-\delta)\,\beta + \frac{x}{1-(1-x)\beta_s^{1-\gamma}}\,(1-(1-\delta)\,\beta)}$$

C.3. Deriving the Laws of Motion for $\widehat{a}_t^L$ and $\widehat{w}_t^L$

A log linear approximation to the skill level A_t^L is given by

(C.6)
$$\widehat{a}_t^L = \frac{\sum\limits_{i=0}^{\infty} ds_t^i \beta_s^i}{A^L}$$

The shares of the various groups of the unemployed are given by

$$s_t^i = \frac{\delta N_{t-1-i} \prod\limits_{j=1}^{i} (1-x_{t-i})}{U_t}$$

This can be log-linearised as

$$ds_t^i = s^i \left[\widehat{n}_{t-1-i} - \widehat{U}_t + \sum_{j=1}^{i} \frac{-x}{1-x}\widehat{x}_{t-j}\right]$$

Log linear approximations to x_t and U_t are given by $\widehat{x}_{t-j} = \frac{\widehat{n}_{t-j}-(1-\delta)(1-x)\widehat{n}_{t-1-j}}{\delta}$ and

$\widehat{U}_t = -\frac{(1-\delta)x}{\delta}\widehat{n}_t$ yields

$$\begin{aligned}
ds_t^i &= s^i \left[\widehat{n}_{t-1-i} + \frac{1-\delta}{\delta} x \widehat{n}_{t-1} - \sum_{j=1}^{i} \frac{-x}{1-x} \frac{\widehat{n}_{t-j} - (1-\delta)(1-x)\widehat{n}_{t-1-j}}{\delta}\right] \\
&= s^i \left[\widehat{n}_{t-1-i} + \frac{1-\delta}{\delta} x \widehat{n}_{t-1} - \frac{x}{1-x}\left[\sum_{j=1}^{i} \frac{\widehat{n}_{t-j}}{\delta} - \frac{(1-\delta)(1-x)}{\delta}\sum_{j=1}^{i}\frac{\widehat{n}_{t-j-1}}{\delta}\right]\right] \\
&= s^i \left[\widehat{n}_{t-1-i} + \frac{1-\delta}{\delta} x \widehat{n}_{t-1} - \frac{x}{1-x}\left[\begin{array}{c} \sum_{j=2}^{i} \frac{\widehat{n}_{t-j}}{\delta} + \frac{\widehat{n}_{t-1}}{\delta} \\ -\frac{(1-\delta)(1-x)}{\delta}\sum_{j=2}^{i}\frac{\widehat{n}_{t-j}}{\delta} - \frac{(1-\delta)(1-x)}{\delta}\widehat{n}_{t-j-1} \end{array}\right]\right] \\
&= s^i \left[\begin{array}{c} \widehat{n}_{t-1-i}\left[1 + \frac{x(1-\delta)}{\delta}\right] + \frac{1-\delta}{\delta} x \widehat{n}_{t-1} - \frac{x}{1-x}\frac{\widehat{n}_{t-1}}{\delta} \\ -\frac{x}{1-x}\left(\frac{1}{\delta} - \frac{(1-\delta)(1-x)}{\delta}\right)\sum_{j=2}^{i}\frac{\widehat{n}_{t-j}}{\delta} \end{array}\right] \\
&= s^i \left[\begin{array}{c} \widehat{n}_{t-1-i}\left[1 + \frac{x(1-\delta)}{\delta}\right] - \frac{x}{1-x}\left(\frac{1}{\delta} - \frac{(1-\delta)(1-x)}{\delta}\right)\widehat{n}_{t-1} \\ -\frac{x}{1-x}\left(\frac{1}{\delta} - \frac{(1-\delta)(1-x)}{\delta}\right)\sum_{j=2}^{i}\frac{\widehat{n}_{t-j}}{\delta} \end{array}\right] \\
&= s^i \left[\widehat{n}_{t-1-i}\left[1 + \frac{x(1-\delta)}{\delta}\right] - \left(\frac{x^2}{\delta(1-x)} + x\right)\sum_{j=1}^{i}\frac{\widehat{n}_{t-j}}{\delta}\right]
\end{aligned}$$

We now use $\delta = \frac{ux}{(1-x)(1-u)}$ and $(1-\delta) = \frac{1-u-x}{(1-x)(1-u)}$ to eliminate δ in the $1 + \frac{(1-\delta)x}{\delta}$

and $\frac{x^2}{\delta(1-x)} + x$. This yields

$$\begin{aligned}
1 + x\frac{(1-\delta)}{\delta} &= 1 + x\frac{(1-u-x)}{(1-x)(1-u)}\frac{(1-x)(1-u)}{ux} = 1 + \frac{1-u-x}{u} = \frac{1-x}{u} \\
\frac{x^2}{\delta(1-x)} + x &= \frac{x^2}{\frac{ux}{(1-x)(1-u)}(1-x)} + x = \frac{x}{u}(1-u) + x = \frac{x}{u}
\end{aligned}$$

Hence we can write

$$ds_t^i = s^i\left[-\frac{x}{u}\sum_{j=1}^{i}\widehat{n}_{t-j} + \frac{1-x}{u}\widehat{n}_{t-1-i}\right] \tag{C.7}$$

Substituting this into $(C.6)$ yields

$$
\begin{aligned}
\widehat{a}_t^L &= \frac{\sum\limits_{i=0}^{\infty}\beta_s^i s^i\left[-\frac{x}{u}\sum\limits_{j=1}^{i}\widehat{n}_{t-j}+\frac{1-x}{u}\widehat{n}_{t-1-i}\right]}{A^L} \\
&= \frac{1}{A^L}\left[\frac{1-x}{u}\sum_{i=0}^{\infty}\beta_s^i s^i\widehat{n}_{t-1-i}-\frac{x}{u}\sum_{i=0}^{\infty}\sum_{j=1}^{i}\beta_s^i s^i\widehat{n}_{t-j}\right] \\
&= \frac{1}{A^L}\left[\frac{1-x}{u}\sum_{i=0}^{\infty}\beta_s^i s^i\widehat{n}_{t-1-i}-\frac{x}{u}\sum_{q=1}^{\infty}\beta_s^q s^q\sum_{j=1}^{q}\widehat{n}_{t-j}\right] \\
&= \frac{1}{A^L}\left[\frac{1-x}{u}\sum_{i=0}^{\infty}\beta_s^i s^i\widehat{n}_{t-1-i}-\frac{x}{u}\sum_{q=1}^{\infty}\beta_s^q s^q\left(\widehat{n}_{t-1}+\widehat{n}_{t-2}+\widehat{n}_{t-3}...+\widehat{n}_{t-q}\right)\right] \\
&= \frac{1}{A^L}\left[\begin{array}{c}\frac{1-x}{u}\sum\limits_{i=0}^{\infty}\beta_s^i s^i\widehat{n}_{t-1-i} \\ -\frac{x}{u}\left[\beta_s^1 s^1\widehat{n}_{t-1}+\beta_s^2 s^2\left(\widehat{n}_{t-1}+\widehat{n}_{t-2}\right)+\beta_s^3 s^3\left(\widehat{n}_{t-1}+\widehat{n}_{t-2}+\widehat{n}_{t-3}\right)......\right]\end{array}\right] \\
&= \frac{1}{A^L}\left[\begin{array}{c}\left[\frac{1-x}{u}\sum\limits_{i=0}^{\infty}\beta_s^i s^i\widehat{n}_{t-1-i}\right] \\ -\frac{x}{u}\left[\begin{array}{c}\left(\sum\limits_{q=1}^{\infty}\beta_s^q s^q\right)\widehat{n}_{t-1}+\left(\sum\limits_{q=2}^{\infty}\beta_s^q s^q\right)\widehat{n}_{t-2} \\ +\left(\sum\limits_{q=3}^{\infty}\beta_s^q s^q\right)\widehat{n}_{t-3}+\left(\sum\limits_{q=4}^{\infty}\beta_s^q s^q\right)\widehat{n}_{t-4}...\end{array}\right]\end{array}\right] \\
&= \frac{1}{A^L}\left[\begin{array}{c}\frac{1-x}{u}\left[s^0\widehat{n}_{t-1}+\beta_s^1 s^1\widehat{n}_{t-2}+\beta_s^2 s^2\widehat{n}_{t-3}+\beta_s^3 s^3\widehat{n}_{t-4}......\right] \\ -\frac{x}{u}\left[\begin{array}{c}\left(\sum\limits_{q=1}^{\infty}\beta_s^q s^q\right)\widehat{n}_{t-1}+\left(\sum\limits_{q=2}^{\infty}\beta_s^q s^q\right)\widehat{n}_{t-2} \\ +\left(\sum\limits_{q=3}^{\infty}\beta_s^q s^q\right)\widehat{n}_{t-3}+\left(\sum\limits_{q=4}^{\infty}\beta_s^q s^q\right)\widehat{n}_{t-4}...\end{array}\right]\end{array}\right] \\
&= \frac{1}{A^L}\left[\begin{array}{c}\left[s^0\frac{1-x}{u}-\frac{x}{u}\left(\sum\limits_{q=1}^{\infty}\beta_s^q s^q\right)\right]\widehat{n}_{t-1}+\left[s^1\frac{1-x}{u}-\frac{x}{u}\left(\sum\limits_{q=2}^{\infty}\beta_s^q s^q\right)\right]\widehat{n}_{t-2} \\ +\left[s^2\frac{1-x}{u}-\frac{x}{u}\left(\sum\limits_{q=3}^{\infty}\beta_s^q s^q\right)\right]\widehat{n}_{t-3}+\left[s^3\frac{1-x}{u}-\frac{x}{u}\left(\sum\limits_{q=4}^{\infty}\beta_s^q s^q\right)\right]\widehat{n}_{t-4}\end{array}\right] \\
&= \frac{1}{A^L}\sum_{i=1}^{\infty}\left[\left[\beta_s^{i-1}s^{i-1}\frac{1-x}{u}-\frac{x}{u}\left(\sum_{q=i}^{\infty}\beta_s^q s^q\right)\right]\widehat{n}_{t-i}\right]
\end{aligned}
$$

Using $A^L = \sum_{i=0}^{\infty} s^i \beta_s^i = \frac{x}{1-(1-x)\beta_s}$ and $s^i = x(1-x)^i$ we can write $\left(\sum_{q=i}^{\infty} \beta_s^q s^q\right) = x \sum_{q=i}^{\infty} \beta_s^q (1-x)^q = \beta_s^i (1-x)^i x \sum_{q=0}^{\infty} \beta_s^q (1-x)^q = \beta_s^i (1-x)^i A^L$ and thus arrive at

$$\widehat{a}_t^L = \frac{x}{u} \sum_{i=1}^{\infty} \left[\left[\frac{\beta_s^{i-1} (1-x)^i}{A^L} - \beta_s^i (1-x) \right] \widehat{n}_{t-i} \right]$$

This can be rewritten as

$$\begin{aligned} \widehat{a}_t^L &= \sum_{i=1}^{\infty} \frac{1}{u} (1-x)^i \left(\beta_s^{i-1} - \beta_s^i\right) \widehat{n}_{t-i} \\ &= \sum_{i=1}^{\infty} a_i^n \widehat{n}_{t-i}, \; a_i^n = \frac{1}{u} (1-x)^i \left(\beta_s^{i-1} - \beta_s^i\right) \end{aligned}$$

Thus in the presence of skill loss ($\beta_s < 1$), the deviation of the average skill level from its steady state depends positively on a weighted sum of past employment rates. The a_i^n coefficient depend on β_s and thus δ^s :

$$\frac{\vartheta a_i^n}{\vartheta \delta^s} = -\frac{1}{u} (1-x)^i \left((i-1)\beta_s^{i-2} - i\beta_s^{i-1}\right)$$

For $\beta_s = 1$ ($\delta^s = 0$), this is clearly positive. Thus the larger the quarterly skill decay among the unemployed, the larger is the effect of past employment on the skill level. For $\beta_s < 1$, we have

$$\frac{\vartheta a_i^n}{\vartheta \delta^s} \gtreqless 0 \Leftrightarrow i \lesseqgtr \frac{1}{1-\beta_s}$$

Hence the effect of δ^s on a_i^n will become negative if i is sufficiently large. Furthermore, $\frac{\vartheta a_i^n}{\vartheta x} < 0$ if $\beta_s < 1$.

To express the component of the real wage depending on the skill of the worker as a function of past employment rates, we follow an analogous process. A log linear approximation to W_t^L is given by

$$\widehat{w}_t^L = \frac{\sum\limits_{i=0}^{\infty} ds_t^i \beta_s^{i(1-\gamma)}}{W^L}$$

Note that the only difference to $(C.6)$ is that β_s and A^L are replaced by $\beta_s^{(1-\gamma)}$ and W^L, respectively. Substituting $(C.7)$ and going through exactly the same process as before thus gives us

$$\widehat{w}_t^L = \sum_{i=1}^{\infty} w_i^n \widehat{n}_{t-i},\ w_i^n = \frac{1}{u}\left(1-x\right)^i \left(\beta_s^{(1-\gamma)(i-1)} - \beta_s^{(1-\gamma)i}\right)$$

and, as with the a_i^n coefficients,

$$\begin{aligned}
\frac{\partial w_i^n}{\partial \delta^s} &= -\frac{1}{u}\left(1-x\right)^i \left(1-\gamma\right)\left(\left(i-1\right)\beta_s^{(1-\gamma)(i-1)-1} - i\beta_s^{(1-\gamma)i-1}\right) \\
\frac{\partial w_i^n}{\partial \delta^s} &> 0 \ \Leftrightarrow \delta^s = 0 \\
\frac{\partial w_i^n}{\partial \delta^s} &\gtreqless 0 \Leftrightarrow i \lesseqgtr \frac{1}{1-\beta_s} \\
\frac{\partial w_i^n}{\partial x} &< 0 \text{ iff } \beta_s < 1.
\end{aligned}$$

Furthermore, we have $\frac{\partial a_i^n}{\partial \delta^s} > \frac{\partial w_i^n}{\partial \delta^s}$ if $\delta^s = 0$ and $\gamma > 0$.

We now turn to express $\widehat{a}_t^L$ and $\widehat{w}_t^L$ as a function of their t-1 values and past employment. For $\widehat{a}_t^L$ we have

$$\begin{aligned}
\widehat{a}_t^L &= \frac{1}{u}(1-x)(1-\beta_s)\widehat{n}_{t-1}+\frac{1}{u}\sum_{i=2}^{\infty}(1-x)^i\left(\beta_s^{i-1}-\beta_s^i\right)\widehat{n}_{t-i} \\
&= \frac{1}{u}(1-x)(1-\beta_s)\widehat{n}_{t-1}+\frac{1}{u}\sum_{i=1}^{\infty}(1-x)^{i+1}\left(\beta_s^{i}-\beta_s^{i+1}\right)\widehat{n}_{t-i} \\
&= \frac{1}{u}(1-x)(1-\beta_s)\widehat{n}_{t-1}+\beta_s(1-x)\frac{1}{u}\sum_{i=1}^{\infty}(1-x)^i\left(\beta_s^{i-1}-\beta_s^i\right)\widehat{n}_{t-i}
\end{aligned}$$

and thus

$$\widehat{a}_t^L=(1-x)\left(\frac{1}{u}(1-\beta_s)\widehat{n}_{t-1}+\beta_s\widehat{a}_{t-1}^L\right) \tag{C.8}$$

Correspondingly for $\widehat{w}_t^L$ we have

$$\widehat{w}_t^L=(1-x)\left(\frac{1}{u}\left(1-\beta_s^{1-\gamma}\right)\widehat{n}_{t-1}+\beta_s^{1-\gamma}\widehat{w}_{t-1}^L\right) \tag{C.9}$$

C.4. On the relative Size of a_i^n and w_i^n, $\frac{\partial a_i^n}{\partial \delta^s}$ and $\frac{\partial w_i^n}{\partial \delta^s}$ if $\delta_s>0$

In the following we assume $\delta_s>0 \Leftrightarrow \beta_s<1$.

We have $a_i^n>w_i^n$ if

$$\left(\beta_s^{i-1}-\beta_s^i\right)>\left(\beta_s^{(1-\gamma)(i-1)}-\beta_s^{(1-\gamma)i}\right)$$

or

$$i<\frac{\ln(1-\beta_s)-\ln\left(\beta_s^{\gamma}-\beta_s\right)}{-\gamma\ln\beta_s}$$

Note that for this expression is always positive (as it should be).Thus a_i^n will turn smaller than w_i^n for large enough i.

The relative size $\frac{\partial a_i^n}{\partial \delta^s}$ and $\frac{\partial w_i^n}{\partial \delta^s}$ also depends on i. We have

$$\frac{\partial a_i^n}{\vartheta \delta^s} \gtreqless \frac{\partial w_i^n}{\partial \delta^s} \Leftrightarrow (1-\gamma)\,\beta_s^{(1-\gamma)i-1}\left((i-1)\,\beta_s^{-1}-i\right) \gtreqless \beta_s^{i-1}\left((i-1)\,\beta_s^{-1}-i\right)$$

Two cases have to be considered: $(i-1)\,\beta_s^{-1}-i\ (i-1)\,\beta_s^{-1}-i \lesseqgtr 0 \Leftrightarrow i \lesseqgtr \frac{1}{1-\beta_s}$. If $(i-1)\,\beta_s^{-1}-i\ (i-1)\,\beta_s^{-1}-i<0$, this implies $\frac{\partial a_i^n}{\partial \delta^s} \gtreqless \frac{\partial w_i^n}{\partial \delta^s}$ if

$$\frac{1}{\gamma}\left(\frac{\ln(1-\gamma)}{\ln \beta_s}-1\right) \gtreqless i$$

Now given that we look at the case $i<\frac{1}{1-\beta_s}$, we ask whether $\frac{1}{\gamma}\left(\frac{\ln(1-\gamma)}{\ln \beta_s}-1\right)>i$ is implied by that assumption. Thus we ask whether

$$\frac{1}{\gamma}\left(\frac{\ln(1-\gamma)}{\ln \beta_s}-1\right)>\frac{1}{1-\beta_s}$$

This is not necessarily fulfilled but will be met for the range of β_s used in this chapter if $\gamma>0.38$. Thus if this hold, for $i<\frac{1}{1-\beta_s}$, we have $\frac{\partial a_i^n}{\partial \delta^s}>\frac{\partial w_i^n}{\partial \delta^s}$.

If $(i-1)\,\beta_s^{-1}-i\ (i-1)\,\beta_s^{-1}-i>0$, we have $\frac{\partial a_i^n}{\partial \delta^s} \gtreqless \frac{\partial w_i^n}{\partial \delta^s}$ if

$$\frac{1}{\gamma}\left(\frac{\ln(1-\gamma)}{\ln \beta_s}-1\right) \lesseqgtr i$$

As for the calibration considered here, with $i=\frac{1}{1-\beta_s}$ we have $\frac{1}{\gamma}\left(\frac{\ln(1-\gamma)}{\ln \beta_s}-1\right)>i$, this means that as i becomes equal to $\frac{1}{1-\beta_s}$, we have $\frac{\partial a_i^n}{\partial \delta^s}<\frac{\partial w_i^n}{\partial \delta^s}$.However, it is also clear that as i increase, we will have $i>\frac{1}{\gamma}\left(\frac{\ln(1-\gamma)}{\ln \beta_s}-1\right)$ and hence $\frac{\partial a_i^n}{\partial \delta^s}>\frac{\partial w_i^n}{\partial \delta^s}$. Thus, for the minimum value of β_s considered here $\gamma>0.38$, we have three different cases depending on the value of i. For sufficiently low values of i, we have $\frac{\partial a_i^n}{\partial \delta^s}>\frac{\partial w_i^n}{\partial \delta^s}$. There is then an intermediate range where $\frac{1}{1-\beta_s}<i<\frac{1}{\gamma}\left(\frac{\ln(1-\gamma)}{\ln \beta_s}-1\right)$ where we have $\frac{\partial a_i^n}{\partial \delta^s}<\frac{\partial w_i^n}{\partial \delta^s}$. Finally, for $i>\frac{1}{\gamma}\left(\frac{\ln(1-\gamma)}{\ln \beta_s}-1\right)$, we have again $\frac{\partial a_i^n}{\partial \delta^s}>\frac{\partial w_i^n}{\partial \delta^s}$.

Since the relative size of a_i^n and w_i^n as well as the effect of an increase in δ^s on them depends on i, it is interesting to look how the combined effect of past employment on the skill level instead to look at the "net" impact of an increase in past employment on the real wage and the skill level and how this impact is affected by δ^s. The sum of the a_i^n and w_i^n is given by

$$\begin{aligned} a^n &= \sum_{i=1}^{\infty} a_i^n = \frac{1-x}{u} \frac{1-\beta_s}{1-(1-x)\beta_s} \\ w^n &= \sum_{i=1}^{\infty} w_i^n = \frac{1-x}{u} \frac{1-\beta_s^{1-\gamma}}{1-(1-x)\beta_s^{1-\gamma}} \end{aligned}$$

Thus $a^n > w^n$ if $\frac{1-\beta_s}{1-(1-x)\beta_s} > \frac{1-\beta_s^{1-\gamma}}{1-(1-x)\beta_s^{1-\gamma}}$ or

$$\gamma > 0$$

Hence the combined effect of an increase in past employment on the average skill level is always higher than the impact on the real wage if there is some real wage rigidity. Turning towards $\frac{\partial a^n}{\partial \delta^s}$ and $\frac{\partial w^n}{\partial \delta^s}$, we have

This then gives

$$\begin{aligned} \frac{\partial a^n}{\partial \delta^s} &= \frac{1-x}{u} \frac{x}{\left(1-(1-x)\beta_s\right)^2} > 0 \\ \frac{\partial w^n}{\partial \delta^s} &= \frac{1-x}{u} (1-\gamma) \frac{x\beta_s^{-\gamma}}{\left(1-(1-x)\beta_s^{1-\gamma}\right)^2} > 0 \end{aligned}$$

Thus the combined effect of past employment on the skill level of the average job seeker and the average real wage increases in δ^s Furthermore, for some real wage rigidity ($\gamma > 0$) and values of β_s not too much smaller than one, $\frac{\vartheta a^n}{\vartheta \delta^s} > \frac{\vartheta w^n}{\vartheta \delta^s}$.

Concerning the effect of a change in x, we have

$$\begin{aligned}\frac{\partial a^n}{\partial x} &= -\frac{1}{u}\frac{1-\beta_s}{\left(1-(1-x)\,\beta_s\right)^2} < 0\\ \frac{\partial w^n}{\partial x} &= -\frac{1}{u}\frac{1-\beta_s^{1-\gamma}}{\left(1-(1-x)\,\beta_s^{1-\gamma}\right)^2} < 0\end{aligned}$$

We have $\frac{\partial a^n}{\partial x} < \frac{\partial w^n}{\partial x}$ if

$$2x\beta_s^{-\gamma} + \beta_s^{1-2\gamma} + 1 + \beta_s^{2-\gamma} > 2x + \beta_s^{2(1-\gamma)} + \beta_s + \beta_s^{-\gamma}$$

Comparing each of the terms on the left and right hand side, we see that for $\gamma > 0$, all terms on the left hand side are greater than corresponding term on the right hand side except for $\beta_s^{2-\gamma}$, which is smaller than $\beta_s^{-\gamma}$.The difference between the two will grow as β_s declines and thus at some point the inequality would be violated. However, it can be checked numerically that for the calibrations employed in this chapter the condition is easily fulfilled.

C.5. Derivation of the marginal Cost Equation and the Output Equation in the Model with Skill Loss

This appendix derives the Phillips Curve and the remaining linearised model equations. Linearising (4.11) yields

$$\text{(C.10)} \qquad \widehat{mc}_t = -\left(1 - Mg\right)\widehat{a}_t^P$$

$$-\left(1 - Mg\right)\widehat{a}_t^L + \frac{M}{A^L}W\widehat{w}_t + M\alpha g\widehat{x}_t$$

$$-\left(1-\delta\right)\beta E_t\left[\begin{array}{c} X\left(\widehat{c}_t - \widehat{c}_{t+1}\right) + \left(\frac{1-A^L}{A^L}\right)\widehat{mc}_{t+1} + \left[\left(\frac{1-A^L}{A^L}\right) - \frac{(1-\gamma)\Theta M}{A^L} + Mg\right]\widehat{a}_{t+1}^P \\ -\left(1-Mg\right)\widehat{a}_{t+1}^L + \frac{M}{A^L}W\widehat{w}_{t+1} + M\alpha g\widehat{x}_{t+1} \end{array}\right]$$

with $X = gM + \frac{1-A^L-M(\Theta'-W)}{A^L}$ and $g = Bx^{\alpha}$. From (4.7) and (4.8), we see that the average wage can be written up to first order as

$$\text{(C.11)} \qquad \widehat{w}_t = \left(1-\gamma\right)\widehat{a}_t^P + \widehat{w}_t^L$$

where

$$\text{(C.12)} \qquad \widehat{w}_t^L = \left(1-x\right)\left(\frac{1}{u}\left(1-\beta_s^{1-\gamma}\right)\widehat{n}_{t-1} + \beta_s^{1-\gamma}\widehat{w}_{t-1}^L\right)$$

Using $(C.11)$ on $(C.10)$ gives

$$\text{(C.13)} \qquad \widehat{mc}_t = -\left(1-Mg\right)\left(\widehat{a}_t^L - \beta\left(1-\delta\right)E_t\widehat{a}_{t+1}^L\right)$$

$$\text{(C.14)} \qquad +\frac{M}{A^L}W\left[\widehat{w}_t^L - \left(1-\delta\right)\beta E_t\widehat{w}_{t+1}^L\right]$$

$$-\Phi'\widehat{a}_t^P - \beta\left(1-\delta\right)\left[\frac{1-\left(1-\gamma\right)\Theta' M}{A^L} - \Phi'\right]E_t\widehat{a}_{t+1}^P + M\alpha g\widehat{x}_t$$

$$-\beta\left(1-\delta\right)E_t\left[X\left(\widehat{c}_t - \widehat{c}_{t+1}\right) + \left(\frac{1-A^L}{A^L}\right)\widehat{mc}_{t+1} + M\alpha g\widehat{x}_{t+1}\right]$$

$$\Phi' = 1 - gM - \left(1-\gamma\right)\frac{M}{A^L}W$$

Linearising (4.14) yields

$$\widehat{c}_t = \frac{A^A}{A^A - A^L g\delta}\widehat{a}_t^A - \frac{Ag\delta}{A^A - A^L g\delta}\left(\widehat{a}_t^L + \widehat{a}_t^P\right) + \xi_0^{'}\widehat{n}_t + \xi_1^{'}\widehat{n}_{t-1} \tag{C.15}$$

with $\xi_0^{'} = \frac{A^L(1-g(1+\alpha))}{A^A - Ag\delta}$ and $\xi_1^{'} = \frac{(1-\delta)\left((1+\alpha(1-x))A^L g + \left(1-A^L\right)\right)}{A^A - A^L g\delta}$ Linearising (4.12) yields

$$\widehat{a}_t^A = \frac{A^L\delta}{A^A}\widehat{a}_t^L + \widehat{a}_t^P - \frac{\left(1-A^L\right)(1-\delta)}{A^A}\left(\widehat{n}_t - \widehat{n}_{t-1}\right) \tag{C.16}$$

Substituting this into $(C.15)$ yields

$$\begin{aligned} \widehat{c}_t &= \widehat{a}_t^P + c^L\widehat{a}_t^L + \xi_0^{'}\widehat{n}_t + \xi_1^{'}\widehat{n}_{t-1} \\ c^L &= \frac{A^L\delta(1-g)}{A^A - A^L g\delta} \end{aligned} \tag{C.17}$$

Substituting $(C.17)$ into $(C.13)$ yields

$$\begin{aligned} \widehat{mc}_t &= a_1^L\widehat{a}_t^L + a_2^L E_t\widehat{a}_{t+1}^L + \frac{M}{A^L}W\left[\widehat{w}_t^L - \beta(1-\delta)E_t\widehat{w}_{t+1}^L\right] \\ &\quad -p_0\widehat{a}_t^P - p_1 E_t\widehat{a}_{t+1}^P + M\alpha g\widehat{x}_t \\ &\quad +\beta(1-\delta)\left[\begin{array}{c} X\left(\xi_0^{'} - \xi_1^{'}\right)\widehat{n}_t + X\xi_0^{'}E_t\widehat{n}_{t+1} \\ -\beta(1-\delta)\xi_1^{'}\widehat{n}_{t-1} - \left(\frac{1-A^L}{A^L}\right)\widehat{mc}_{t+1} - M\alpha g\widehat{x}_{t+1} \end{array}\right] \\ a_1^L &= 1 - gM + \beta(1-\delta)\frac{A^L\delta(1-g)}{A^A - Ag\delta}X \\ a_2^L &= \beta(1-\delta)\left[1 - gM + \frac{A^L\delta(1-g)}{A^A - Ag\delta}X\right] \\ p_0 &= \Phi' + \beta(1-\delta)X \\ p_1 &= \beta(1-\delta)\frac{\gamma M(\Theta' - W)}{A^L} \end{aligned}$$

Using $\widehat{x}_t = \frac{\widehat{n}_t - (1-\delta)(1-x)\widehat{n}_{t-1}}{\delta}$ then yields

$$
\begin{aligned}
\widehat{mc}_t &= -a_1^L \widehat{a}_t^L + a_2^L E_t \widehat{a}_{t+1}^L + w_1^L \widehat{w}_t^L - w_2^L E_t \widehat{w}_{t+1}^L - p_0 \widehat{a}_t^P - p_1 E_t \widehat{a}_{t+1}^P \\
&\quad + h_0^{'} \widehat{n}_t + h_L^{'} \widehat{n}_{t-1} + h_F^{'} E_t \widehat{n}_{t+1} - h_c E_t \widehat{mc}_{t+1} \\
h_c &= \beta (1-\delta) \frac{\left(1 - A^L\right)}{A^L} \\
h_F^{'} &= -\beta (1-\delta) \left(\frac{\alpha g M}{\delta} - \xi_0^{'} X \right) \\
h_0^{'} &= \left(\frac{\alpha g M}{\delta} \right) \left(1 + \beta (1-\delta)^2 (1-x)\right) + \beta (1-\delta) \left(\xi_1^{'} - \xi_0^{'} \right) X \\
h_L^{'} &= -\left(\frac{\alpha g M}{\delta} \right) (1-\delta)(1-x) - \beta (1-\delta) \xi_1^{'} X
\end{aligned}
$$

We now substitute $(C.8)$ and $(C.9)$, which, after rearranging, yields

$$
\begin{aligned}
\widehat{mc}_t &= -\left(a_1^L - a_2^L (1-x) \beta_s\right) \widehat{a}_t^L + \left(w_1^L - w_2^L (1-x) \beta_s^{1-\gamma}\right) \widehat{w}_t^L \\
&\quad - \left[h_0^{'} + (1-x) \left(a_2^L \frac{(1-\beta_s)}{u} - w_2^L \frac{\left(1-\beta_s^{1-\gamma}\right)}{u} \right) \right] \widehat{n}_t \\
&\quad + h_L^{'} \widehat{n}_{t-1} + h_F^{'} E_t \widehat{n}_{t+1} - h_c E_t \widehat{mc}_{t+1} - p_0 \widehat{a}_t^P - p_1 E_t \widehat{a}_{t+1}^P
\end{aligned}
$$

Using $\widehat{n}_t = \frac{-\widehat{u}_t}{1-u}$ then yields

$$
\begin{aligned}
\lambda \widehat{mc}_t &= -a^* \widehat{a}_t^L + w^* \widehat{w}_t^L - \kappa_0^* \widehat{u}_t + \kappa_L^* \widehat{u}_{t-1} + \kappa_F^* E_t \widehat{u}_{t+1} - \lambda h_c E_t \widehat{mc}_{t+1} - \lambda (p_0 + \rho_a p_1) \widehat{a}_t^P \\
a^* &= \lambda \left(a_1^L - a_2^L (1-x) \beta_s\right) \\
w^* &= \lambda \left(w_1^L - w_2^L (1-x) \beta_s^{1-\gamma}\right) \\
\kappa_0^* &= \lambda \left[h_0^{'} + (1-x) \left(a_2^L \frac{(1-\beta_s)}{u} - w_2^L \frac{\left(1-\beta_s^{1-\gamma}\right)}{u} \right) \right] \\
\kappa_L^* &= \frac{-\lambda h_L^{'}}{1-u}, \;\; \kappa_F^* = \frac{-\lambda h_F^{'}}{1-u}
\end{aligned}
$$

The equation for output including hiring costs is derived as follows. We have $Y_t = A_t^A N_t$. Linearising gives $\widehat{y}_t = \widehat{a}_t^A + \widehat{n}_t$ which, using $(C.16)$ can be written as

$$\widehat{y}_t = \widehat{a}_t^P + \frac{1}{A^A}\left[A^L\delta\widehat{a}_t^L + A^L\widehat{n}_t + \left(1-A^L\right)(1-\delta)\,\widehat{n}_{t-1}\right]$$

Using $\widehat{n}_t = \frac{-\widehat{u}_t}{1-u}$ gives the equation used in the text.

C.6. Proof of Propositions on signs of $\frac{\partial\kappa}{\partial\delta_s}$ and $\frac{\partial^2\kappa}{\partial\delta_s\partial x}$

Marginal cost is given by

$$\lambda\widehat{mc}_t = -h_c E_t\lambda\widehat{mc}_{t+1} + \kappa_F^* E_t\widehat{u}_{t+1} - \kappa_0^*\widehat{u}_t + \kappa_{L1}^*\widehat{u}_{t-1} - a^*\widehat{a}_t^L + w^*\widehat{w}_t^L - \lambda p_0\widehat{a}_t^P - \lambda p_1 E_t\widehat{a}_{t+1}^P$$

If all variables stay constant over time and ignoring technology, we have

$$\begin{aligned}
\lambda\widehat{mc} &= -\frac{\left[\kappa_0^* - \kappa_F^* - \kappa_{L1}^* - a^*\frac{(1-\beta_s)(1-x)}{u(1-u)(1-(1-x)\beta_s)} + w^*\frac{\left(1-\beta_s^{1-\gamma}\right)(1-x)}{u(1-u)\left(1-(1-x)\beta_s^{1-\gamma}\right)}\right]}{1+h_c}\widehat{u} \\
&= -\frac{\begin{array}{c} h_0^{'} + h_L^{'} + h_F^{'} + \frac{(1-x)}{u} \\ \left[\begin{array}{c} a_2^L\left(1-\beta_s\right) - w_2^L\left(1-\beta_s^{1-\gamma}\right) - \left(a_1^L - a_2^L\left(1-x\right)\beta_s\right)\frac{(1-\beta_s)}{(1-(1-x)\beta_s)} \\ + \left(w_1^L - w_2^L\left(1-x\right)\beta_s^{1-\gamma}\right)\frac{\left(1-\beta_s^{1-\gamma}\right)}{\left(1-(1-x)\beta_s^{1-\gamma}\right)} \end{array}\right] \end{array}}{\left(1+h_c\right)\left(1-u\right)}\lambda\widehat{u} \\
&= -\frac{\begin{array}{c} h_0^{'} + h_L^{'} + h_F^{'} + \frac{(1-x)}{u} \\ \left[\begin{array}{c} \left(1-\beta_s\right)\frac{a_2^L(1-(1-x)\beta_s) - a_1^L + a_2^L(1-x)\beta_s}{(1-(1-x)\beta_s)} \\ + \left(1-\beta_s^{1-\gamma}\right)\frac{-w_2^L\left(1-(1-x)\beta_s^{1-\gamma}\right) + w_1^L - w_2^L(1-x)\beta_s^{1-\gamma}}{\left(1-(1-x)\beta_s^{1-\gamma}\right)} \end{array}\right] \end{array}}{\left(1+h_c\right)\left(1-u\right)} \\
&\quad \lambda\widehat{u} \\
&= -\frac{h_0^{'} + h_L^{'} + h_F^{'} + \frac{(1-x)}{u}\left[\frac{-(1-\beta_s)\left(a_1^L - a_2^L\right)}{(1-(1-x)\beta_s)} + \frac{\left(1-\beta_s^{1-\gamma}\right)\left(w_1^L - w_2^L\right)}{\left(1-(1-x)\beta_s^{1-\gamma}\right)}\right]}{\left(1+h_c\right)\left(1-u\right)}\lambda\widehat{u}
\end{aligned}$$

We can express $h_0^{'} + h_L^{'} + h_F^{'}$

$$h_0^{'} + h_L^{'} + h_F^{'} = \frac{\alpha g M}{\delta}\left[1 + \beta\left(1-\delta\right)^2\left(1-x\right) - \left(1-\delta\right)\left(1-x\right) - \beta\left(1-\delta\right)\right]$$

Using $\delta = \frac{ux}{(1-x)(1-u)}$ and $(1-\delta) = \frac{1-u-x}{(1-x)(1-u)}$, this can be rewritten as

$$\begin{aligned}
h_0^{'} + h_L^{'} + h_F^{'} &= \frac{\alpha g M\left(1-x\right)\left(1-u\right)}{ux} \\
&\quad \left[1 + \beta\frac{\left(1-u-x\right)^2}{\left(1-x\right)\left(1-u\right)^2} - \frac{1-u-x}{\left(1-u\right)} - \beta\frac{1-u-x}{\left(1-x\right)\left(1-u\right)}\right] \\
&= \frac{\alpha g M}{ux}\left[\begin{array}{c} \left(1-x\right)\left(1-u\right) + \beta\frac{(1-u-x)^2}{(1-u)} \\ -\left(1-x\right)\left(1-u-x\right) - \beta\left(1-u-x\right) \end{array}\right] \\
&= \frac{\alpha g M}{ux}\left[x\left(1-x\right) + \beta\left(1-u-x\right)\frac{\left(1-u-x\right)-\left(1-u\right)}{1-u}\right] \\
&= \frac{\alpha g M}{ux}\left[x\left(1-x\right) - \beta\left(1-u-x\right)\frac{x}{1-u}\right] \\
&= \frac{\alpha M B x^{\alpha}}{u\left(1-u\right)}\left[\left(1-u-x\right)\left(1-\beta\right) + ux\right] > 0
\end{aligned}$$

due to the restrictions on the parameters. Furthermore, we have $a_1^L - a_2^L = (1 - Bx^\alpha M)[1 - \beta(1-\delta)]$ and $w_1^L - w_2^L = \frac{M}{A^L} W[1 - \beta(1-\delta)]$. Hence we can now write:

$$\begin{aligned}
\lambda\widehat{mc} &= -\frac{\begin{array}{c}\frac{\alpha M B' x^\alpha}{(1-u)}\left[(1-u-x)(1-\beta)+ux\right] \\ +(1-x)\left[1-\beta(1-\delta)\right]\left[\frac{-(1-\beta_s)(1-Bx^\alpha M)}{(1-(1-x)\beta_s)} + \frac{\left(1-\beta_s^{1-\gamma}\right)\frac{WM}{A^L}}{\left(1-(1-x)\beta_s^{1-\gamma}\right)}\right]\end{array}}{u(1+h_c)(1-u)}\lambda\widehat{u} \\
&= -\kappa\widehat{u} \\
\kappa &= \frac{\left[\begin{array}{c}\frac{\alpha M B x^\alpha}{(1-u)}\left[(1-u-x)(1-\beta)+ux\right]+ \\ (1-x)\left[1-\beta(1-\delta)\right]\left[\frac{-(1-\beta_s)(1-B'x^\alpha M)}{(1-(1-x)\beta_s)} + \frac{\left(1-\beta_s^{1-\gamma}\right)WM}{A^L\left(1-(1-x)\beta_s^{1-\gamma}\right)}\right]\end{array}\right]}{u(1+h_c)(1-u)}\lambda
\end{aligned}$$

We will now show that $\frac{\partial\kappa}{\partial\beta_s} > 0$ and thus $\frac{\vartheta\kappa}{\vartheta\delta^s} < 0$ if β_s is not too far away from 1. A more general proof seems impossible. We have

$$\frac{\partial\kappa}{\partial\beta_s} = \frac{-\frac{\partial h_c}{\partial\beta_s}\kappa(1+h_c)}{(1+h_c)^2}$$

$$+\frac{\lambda(1-x)}{u(1-u)}\left[\frac{[1-\beta(1-\delta)]}{(1+h_c)}\left[\begin{array}{c}\frac{(1-B'x^\alpha M)[(1-(1-x)\beta_s)-(1-\beta_s)(1-x)]}{(1-(1-x)\beta_s)^2} + M \\ \dfrac{\begin{array}{c}\left[-\beta_s^{-\gamma}(1-\gamma)W + \left(1-\beta_s^{1-\gamma}\right)\frac{\vartheta W}{\vartheta\beta_s}\right] \\ A^L\left(1-(1-x)\beta_s^{1-\gamma}\right) \\ -\left(1-\beta_s^{1-\gamma}\right)W\left[\begin{array}{c}\frac{\vartheta A^L}{\vartheta\beta_s}\left(1-(1-x)\beta_s^{1-\gamma}\right) \\ -A^L(1-x)(1-\gamma)\beta_s^{-\gamma}\end{array}\right]\end{array}}{\left(A^L\left(1-(1-x)\beta_s^{1-\gamma}\right)\right)^2}\end{array}\right]\right]$$

It is easily shown that $\frac{\partial h_c}{\partial\beta_s} = -\beta(1-\delta)\frac{\partial A^L}{\partial\beta_s}\frac{1}{(A^L)^2} < 0$. For $\kappa > 0$, this implies that $\frac{-\frac{\partial h_c}{\partial\beta_s}\kappa(1+h_c)}{(1+h_c)^2} > 0$. Furthermore, since the range of values of β_s are those for which κ is positive, or "just" negative, we can safely write $\frac{\vartheta\kappa}{\vartheta\beta_s} > 0$ if

$$\frac{(1 - B'x^{\alpha}M)\,x}{(1-(1-x)\,\beta_s)^2}$$
$$+MA^LW\frac{\left[-\beta_s^{-\gamma}(1-\gamma) + \left(1-\beta_s^{1-\gamma}\right)\frac{\vartheta W}{\vartheta\beta_s}\frac{1}{W}\right]\left(1-(1-x)\,\beta_s^{1-\gamma}\right) - \left(1-\beta_s^{1-\gamma}\right)W\left[\frac{\vartheta A^L}{\vartheta\beta_s}\frac{1}{A_L}\left(1-(1-x)\,\beta_s^{1-\gamma}\right) - (1-x)(1-\gamma)\,\beta_s^{-\gamma}\right]}{\left(A^L\left(1-(1-x)\,\beta_s^{1-\gamma}\right)\right)^2} > 0$$

Further simplifying this yields

$$\frac{(1 - Bx^{\alpha}M)\,x}{(1-(1-x)\,\beta_s)^2}$$
$$+\frac{MW\left[-x(1-\gamma)\,\beta_s^{-\gamma} + \left(1-(1-x)\,\beta_s^{1-\gamma}\right)\left(1-\beta_s^{1-\gamma}\right)\left(\frac{\vartheta W}{\vartheta\beta_s}\frac{1}{W} - \frac{\vartheta A^L}{\vartheta\beta_s}\frac{1}{A_L}\right)\right]}{A^L\left(1-(1-x)\,\beta_s^{1-\gamma}\right)^2} > 0$$

Using $W = \Theta' W^L$,

We now set $\beta_s = 1$. This gives $W = \Theta' = \frac{1}{M} - g\left[1-\beta(1-\delta)\right]$ and, $\left(1-\beta_s^{1-\gamma}\right) = 0$ and $\left(1-(1-x)\,\beta_s^{1-\gamma}\right) = x$, means that our inequality becomes

$$\frac{(1 - B'x^{\alpha}M) - \left[\frac{1}{M} - B'x^{\alpha}\left[1-\beta(1-\delta)\right]\right]M(1-\gamma)}{x} > 0$$

Or

$$\gamma > \frac{B'x^{\alpha}M\beta(1-\delta)}{1 - B'x^{\alpha}M(1-\beta(1-\delta))}$$

This is easily fulfilled under the calibrations considered in this chapter.

For the case of $\beta_s = 1$, we now show that $\frac{\partial^2\kappa}{\partial\beta_s\partial x} < 0$ if α is close to one (as we assume in the chapter) and the other parameters have a calibration of "reasonable" magnitude. This means that the effect of the skill level on κ is weakened if the labour market becomes more fluid. For $\beta_s = 1$ and $\alpha = 1$ we have (noting that $\frac{\partial h_c}{\partial \beta_s} = -\frac{\beta(1-u-x)}{(1-u)x}$

$$
\begin{aligned}
\frac{\partial \kappa}{\partial \beta_s} &= \kappa \frac{\beta(1-u-x)}{(1-u)x} \\
&+ \frac{\lambda}{u(1-u)^2}\left[\begin{array}{c} [(1-x-u)(1-\beta)+ux] \\ \left[\frac{(1-B'x^{\alpha}M)}{x} - (1-\gamma)\left[\frac{1}{x} - MB\left[\frac{(1-\beta)(1-u)+x(u+\beta-1)}{(1-x)(1-u)}\right]\right]\right] \end{array}\right] \\
&= A_1 + A_2 \quad where \\
A_1 &= \kappa \frac{\beta(1-u-x)}{(1-u)x} \\
&= \frac{\lambda M B' \beta}{(1-u)^3 u}\left[(1-\beta)\left(1-2u-2x+u^2+2ux+x^2\right)+ux-ux^2-u^2x\right] \\
A_2 &= \frac{\lambda}{u(1-u)^2}\left[\begin{array}{c} [(1-x-u)(1-\beta)+ux] \\ \left[\frac{(1-B'xM)}{x} - (1-\gamma)\left[\frac{1}{x} - MB'\left[\frac{(1-\beta)(1-u)+x(u+\beta-1)}{(1-x)(1-u)}\right]\right]\right] \end{array}\right]
\end{aligned}
$$

Differentiating this with respect to x gives

$$
\frac{\partial^2 \kappa}{\partial \beta_s \partial x} = \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial x}
$$

$$
\frac{\partial A_1}{\partial x} = \frac{\lambda M B' \beta}{(1-u)^3 u}\left[-2(1-\beta)(1-u-x)+u(1-2x-u)\right]
$$

$$\frac{\partial A_2}{\partial x} = \frac{\lambda}{u\left(1-u\right)^2}$$

$$\left[\begin{array}{c} \left(\beta+u-1\right)\left[\frac{(1-B'xM)}{x}-\left(1-\gamma\right)\left[\frac{1}{x}-MB'\left[\frac{(1-\beta)(1-u)+x(u+\beta-1)}{(1-x)(1-u)}\right]\right]\right] \\ +\left[\left(1-x-u\right)\left(1-\beta\right)+ux\right] \\ \left[\begin{array}{c} \frac{-B'Mx-(1-B'Mx)}{x^2}-\left(1-\gamma\right) \\ \left[\frac{-1}{x^2}-\frac{MB'}{(1-u)}\left[\frac{(u+\beta-1)(1-x)+(1-\beta)(1-u)+x(u+\beta-1)}{(1-x)^2}\right]\right] \end{array}\right] \end{array}\right]$$

$$= \frac{\lambda}{u\left(1-u\right)^2}$$

$$\left[\begin{array}{c} \left(\beta+u-1\right)\left[\frac{(1-B'xM)}{x}-\left(1-\gamma\right)\left[\frac{1}{x}-MB'\left[\frac{(1-\beta)(1-u)+x(u+\beta-1)}{(1-x)(1-u)}\right]\right]\right] \\ +\left[\left(1-x-u\right)\left(1-\beta\right)+ux\right]\left[\frac{-1}{x^2}-\left(1-\gamma\right)\left[\frac{-1}{x^2}-\frac{MB'u\beta}{(1-u)(1-x)^2}\right]\right] \end{array}\right]$$

Note that

$$\begin{aligned} & \frac{\left(1-B'xM\right)}{x}-\left(1-\gamma\right)\left[\frac{1}{x}-MB'\left[\frac{\left(1-\beta\right)\left(1-u\right)+x\left(u+\beta-1\right)}{\left(1-x\right)\left(1-u\right)}\right]\right] \\ = & \frac{\left(\gamma-B'xM\right)}{x}+\left(1-\gamma\right)MB'\left[\frac{\left(1-\beta\right)\left(1-u\right)+x\left(u+\beta-1\right)}{\left(1-x\right)\left(1-u\right)}\right] \\ = & \frac{\left(\gamma-B'xM\right)\left(1-x\right)\left(1-u\right)+x\left(1-\gamma\right)MB'\left[\left(1-\beta\right)\left(1-u\right)+x\left(u+\beta-1\right)\right]}{x\left(1-x\right)\left(1-u\right)} \end{aligned}$$

and that

$$\begin{aligned} & \frac{-1}{x^2}-\left(1-\gamma\right)\left[\frac{-1}{x^2}-\frac{MB'u\beta}{\left(1-u\right)\left(1-x\right)^2}\right] \\ = & -\frac{\gamma}{x^2}+\frac{\left(1-\gamma\right)MBu\beta}{\left(1-u\right)\left(1-x\right)^2} \\ = & \frac{-\gamma\left(1-u\right)\left(1-x\right)^2+x^2\left(1-\gamma\right)MBu\beta}{x^2\left(1-u\right)\left(1-x\right)^2} \end{aligned}$$

Thus

$$\frac{\partial A_2}{\partial x}=\frac{\lambda}{u\left(1-u\right)^2}\left[\begin{array}{c}\frac{(\beta+u-1)}{x(1-x)(1-u)}\left[\begin{array}{c}\left(\gamma-BxM\right)\left(1-x\right)\left(1-u\right)\\+x\left(1-\gamma\right)MB'\left[\left(1-\beta\right)\left(1-u\right)+x\left(u+\beta-1\right)\right]\end{array}\right]\\+\left[\frac{(1-x-u)(1-\beta)+ux}{x^2(1-u)(1-x)^2}\left(-\gamma\left(1-u\right)\left(1-x\right)^2+x^2\left(1-\gamma\right)MB'u\beta\right)\right]\end{array}\right]$$

We can then write

$$\begin{aligned}\frac{\partial^2\kappa}{\partial\beta_s\partial x}&=\frac{\partial A_1}{\partial x}+\frac{\partial A_2}{\partial x}\\&=\frac{\lambda}{u\left(1-u\right)^3}\\&\left[\begin{array}{c}MB'\beta\left[-2\left(1-\beta\right)\left(1-u-x\right)+u\left(1-2x-u\right)\right]\\+\frac{(\beta+u-1)}{x(1-x)}\\\left[\begin{array}{c}\left(\gamma-B'xM\right)\left(1-x\right)\left(1-u\right)\\+x\left(1-\gamma\right)MB'\left[\left(1-\beta\right)\left(1-u\right)+x\left(u+\beta-1\right)\right]\end{array}\right]\\+\left[\frac{(1-x-u)(1-\beta)+ux}{x^2(1-x)^2}\left(-\gamma\left(1-u\right)\left(1-x\right)^2+x^2\left(1-\gamma\right)MB'u\beta\right)\right]\end{array}\right]\end{aligned}$$

As can be easily checked, setting $\beta = 1$ makes $\frac{\partial^2 \kappa}{\partial \beta_s \partial x}$ more positive. Thus if $\frac{\partial^2 \kappa}{\partial \beta_s \partial x} < 0$ for $\beta = 1$, then $\frac{\partial^2 \kappa}{\partial \beta_s \partial x} < 0$ for $\beta < 1$ as well. Hence $\frac{\partial^2 \kappa}{\partial \beta_s \partial x} < 0$ if

$$
\begin{aligned}
MBu\left(1-2x-u\right) \\
+\frac{u\left[\left(\gamma - B'xM\right)\left(1-x\right)\left(1-u\right) + x^2\left(1-\gamma\right)MB'u\right]}{x\left(1-x\right)} \\
+\frac{u\left(-\gamma\left(1-u\right)\left(1-x\right)^2 + x^2\left(1-\gamma\right)MB'u\right)}{x\left(1-x\right)^2} < 0
\end{aligned}
$$

$$
\begin{aligned}
MB'\left(1-2x-u\right)x\left(1-x\right)^2 & \\
+\left[\left(\gamma - B'xM\right)\left(1-x\right)\left(1-u\right) + x^2\left(1-\gamma\right)MB'u\right]\left(1-x\right) & \\
-\gamma\left(1-u\right)\left(1-x\right)^2 + x^2\left(1-\gamma\right)MB'u & < 0 \\
MB'\left(1-2x-u\right)x\left(1-x\right)^2 & \\
+\left(\gamma - B'xM\right)\left(1-x\right)^2\left(1-u\right) + x^2\left(1-\gamma\right)MB'u\left(1-x\right) & \\
-\gamma\left(1-u\right)\left(1-x\right)^2 + x^2\left(1-\gamma\right)MB'u & < 0 \\
MB'\left(1-2x-u\right)x\left(1-x\right)^2 & \\
-MBx\left(1-x\right)^2\left(1-u\right) & \\
+x^2\left(1-\gamma\right)MB'u\left(1-x\right) + x^2\left(1-\gamma\right)MB'u & < 0 \\
\left(1-2x-u\right)\left(1-x\right)^2 - \left(1-x\right)^2\left(1-u\right) & \\
+x\left(1-\gamma\right)u\left(1-x\right) + x\left(1-\gamma\right)u & < 0 \\
-2x\left(1-x\right)^2 + x\left(1-\gamma\right)u\left(1-x\right) + x\left(1-\gamma\right)u & < 0 \\
-2\left(1-x\right)^2 + \left(1-\gamma\right)u\left(1-x\right) + \left(1-\gamma\right)u & < 0 \\
-2\left(x^2 - 2x + 1\right) + \left(1-\gamma\right)u\left(1-x\right) + u\left(1-\gamma\right) & < 0 \\
-2x^2 + 4x - 2 + \left(1-\gamma\right)u - x\left(1-\gamma\right)u + u\left(1-\gamma\right) & < 0 \\
-2x^2 + x\left(4 - \left(1-\gamma\right)u\right) + 2u\left(1-\gamma\right) - 2 & < 0
\end{aligned}
$$

The polynomial on the left hands side has two solutions and the inequality will hold for values of x to the left of the smaller solution or to the right of the larger one. We have

$$x_{1,2} = \frac{4 - (1-\gamma)\, u \pm \sqrt{(1-\gamma)^2 u^2 + 8u\,(1-\gamma)}}{4}$$

Since the root will be larger than $(1-\gamma)\, u$, we have $x_1 > 1$, which is outside the permissible range for x. For x_2, we have

$$x_2 = \frac{4 - (1-\gamma)\, u - \sqrt{(1-\gamma)^2 u^2 + 8u\,(1-\gamma)}}{4}$$

Clearly x_2 increases in γ. Thus the larger γ, the larger is the maximum value of x consistent with $\frac{\partial^2 \kappa}{\partial \beta_s \partial x} < 0$. Setting $\gamma = 0$, we have

$$x_2 = \frac{4 - u - \sqrt{u^2 + 8u}}{4}$$

Thus for $x < x_2 = \frac{4-u-\sqrt{u^2+8u}}{4}$, which is easily fulfilled for the range of parameters we consider in this chapter, we have $\frac{\partial^2 \kappa}{\partial \beta_s \partial x} < 0$ and thus $\frac{\partial^2 \kappa}{\partial \delta_s \partial x} > 0$.

C.7. Model Equations in the Form required by Sims' (2000) Code

We first use the interest feedback rule to substitute $\widehat{i}_t$ out of the Euler equation (not the policy rule employed here is $\widehat{i}_t = \phi_\pi \pi_t + \phi_u \widehat{u}_t$). We can then write the system in the form

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \begin{bmatrix} \Psi & \Pi \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ v_t \end{bmatrix} + \Pi \widetilde{\eta}_t$$

$$\text{with } y_t = \begin{bmatrix} x_t^{\pi} \\ x_t^{u} \\ x_t^{mc} \\ x_t^{n} \\ x_t^{c} \\ \widehat{a}_t^P \\ \pi_t \\ \widehat{u}_t \\ \widehat{mc}_t' \\ \widehat{u}_t^n \\ \widehat{c}_t \\ \widehat{a}_t^L \\ \widehat{w}_t^L \\ \widehat{i}_t \end{bmatrix}, \Gamma_0 = [\Gamma_0^1 \; \Gamma_0^2]$$

$$
\Gamma_0^1 = \begin{pmatrix}
\beta & 0 & 0 & 0 & 0 & 0 \\
0 & -\kappa_F^* & h_c & 0 & 0 & \lambda\left(p_0 + \rho_a p_1\right) \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
-1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & \kappa_F^* & 0 & -\lambda\left(p_0 + \rho_a p_1\right) \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
\Gamma_0^2 = \begin{pmatrix}
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & \kappa_0^* & 1 & 0 & 0 & a^* & -w^* & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & c_0^* & 0 & 0 & 1 & -c_L & 0 & 0 \\
(1-\rho)\,\phi_\pi & (1-\rho)\left(\phi_u - \phi_y y_0\right) & 0 & (1-\rho)\,\phi_y y_0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & -\kappa_0^* & 0 & -a^* & w^* & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}
$$

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{L1}^{*} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-(1-x)(1-\beta_s)}{u(1-u)} & 0 & 0 & 0 & (1-x)\,\beta_s & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-(1-x)\left(1-\beta_s^{1-\gamma}\right)}{u(1-u)} & 0 & 0 & 0 & 0 & (1-x)\,\beta_s^{1-\gamma} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_1^{*} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho \\ 0 & 0 & 0 & 0 & 0 & \rho_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\kappa_{L1}^{*} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Psi = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \Pi = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$