DETERMINACY AND LEARNING STABILITY OF ECONOMIC POLICY IN ASYMMETRIC MONETARY UNION MODELS

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Abstract

This thesis examines determinacy and E-stability of economic policy in monetary union models. Monetary policy takes the form of either a contemporaneous or a forecast based interest rate rule, while fiscal policy follows a contemporaneous government spending rule. In the absence of asymmetries, the results from the closed economy literature on learning are retained. However, when introducing asymmetries into monetary union frameworks, the determinacy and E-stability conditions for economic policy differ from both the closed and open economy cases.

We find that a monetary union with heterogeneous price rigidities is more likely to be determinate and E-stable. Specifically, the Taylor principle, a key stability condition for the closed economy, is now relaxed. Furthermore, an interest rate rule that stabilizes the terms of trade in addition to output and inflation, is more likely to induce determinacy and local stability under RLS learning. If monetary policy is sufficiently aggressive in stabilizing the terms of trade, then determinacy and E-stability of the union economy can be achieved without direct stabilization of output and inflation.

A fiscal policy rule that supports demand for domestic goods following a shock to competitiveness, can destabilize the union economy regardless of the interest rate rule employed by the union central bank. In this case, determinacy and E-stability conditions have to be simultaneously and independently met by both fiscal and monetary policy for the union economy to be stable. When fiscal policy instead stabilizes domestic output gaps while monetary policy stabilizes union output and inflation, fiscal policy directly affects the stability of monetary policy. A contemporaneous monetary policy rule has to be more aggressive to satisfy the Taylor principle, the more aggressive fiscal policy is. On the other hand, when monetary policy is forward looking, an aggressive fiscal policy rule can help induce determinacy.

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Contents

Introduction

Background and Motivation

The relationship between the expectations of economic agents and the economy in which they operate makes identifying how expectations are formed key for successfully modelling an economy. Some of the earlier economic frameworks assumed perfect foresight and thus equated expectations with actual outcomes. However, this assumption was soon deemed to be too strong, since it is both unrealistic and fails to explicitly emphasize the role of expectations. The notion of *rational expectations* (RE) was characterized by Muth (1961) and is today a benchmark assumption in macroeconomics.[1] Under RE, it is assumed that agents know the structure of the economy as well as the values of the structural parameters. This differs from perfect foresight in that under RE, agents do not know the future values of random shocks to the economy. That is, agents know the probability distribution of future endogenous variables conditional on exogenous variables, but they do not know the specific values of future exogenous variables.

Rational expectations has been a key assumption in New Keynesian dynamic stochastic general equilibrium (DSGE) models used to derive policy feedback rules with welfare optimizing properties.[2] These models are often reduced by log-linearization around

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[1] While it has been argued that Hurwicz (1946) was the first in using the terminology "rational expectations", he does not offer an explicit definition of the term, but instead touches on what we refer to as *learning* in this thesis.

[2] See Clarida et al. (1999) and Woodford (2003) for examples.

some defined steady state, with policy recommendations based on the welfare properties of an associated *rational expectations equilibrium* (REE). However, economic policy recommendations based on the assumption of RE could lead to undesired levels of volatility and instability for two main reasons.

First, as discussed by e.g. Clarida et al. (2000), Rotemberg & Woodford (1998,1999) and Woodford (1999), if a policy leads to more than one stationary REE, then it could be the case that either an inferior equilibrium is attained or that agents fail to coordinate towards any one equilibrium. It may also be the case that no stationary REE exists for a given policy rule and under such a policy the economy would follow an explosive path. Given this, a policy rule should only be considered if it leads to a unique stationary REE or *determinacy*.

Second, although RE is a useful benchmark since it simplifies analysis significantly, it is a strong and restrictive assumption. In practice, economic theory puts forward frameworks attempting to describe the structure of the economy, while econometricians estimate actual parameter values. The notion of agents behaving like econometricians by using some least squares method (most commonly *recursive least squares* (RLS)) to estimate parameter values, rather than having RE, is known in the literature as *learning*. Given that small forecast errors are likely to occur, so that the economy could temporarily deviate slightly from the REE path, the question is whether the state of the economy approaches the REE over time. That is, if agents' estimates of the economic parameter values approach and eventually reach their true values as more data becomes available. The E-stability principle states that an REE is locally asymptotically stable under recursive least squares learning if and only if it is *expectationally stable* (*E-stable*).[3] If a policy rule leads to an REE that is not E-stable, then under recursive least squares learning, economic agents can collaborate towards this REE with a probability of zero. Such a policy rule should hence be avoided by policy makers, even if it results in high welfare gains under RE.

While determinacy and E-stability conditions for economic policy rules, in particular monetary policy rules, have been derived using both closed and open economy models as discussed below, there is a gap in the literature as determinacy and E-stability of policy has not yet been considered in the framework of a monetary union. In this thesis we aim to close this gap by examining determinacy and E-stability in two-country monetary union models. We show that in the absence of certain asymmetries in a monetary union, the results from the closed economy literature on learning are retained. However, when different asymmetries are introduced, the requirements for determinacy and E-stability of economic policy differ from those found in both the closed and open economy literature.

The following section describes some key economic issues of a monetary union and gives an overview of how these can be modelled. This is followed by a brief discussion of the monetary policy rules of the type considered in this thesis. We then summarize related work on determinacy and E-stability from the closed and open economy literature. An outline of the thesis is then given before presenting a summary of the main findings and contribution of this thesis.

[3] See Evans & Honkapohja (*p.30*, 2001).

A Monetary Union

Key Characteristics

A monetary union consists of two or more countries that share a common currency and a common central bank. Although the member countries do not have sovereignty of monetary policy, they do have sovereign fiscal authorities. There are both potential costs and benefits of being in a monetary union and these should be considered carefully before joining.

The main benefits of a monetary union are the elimination of exchange rate uncertainty and transaction costs when trading with other member countries, improved price transparency and thereby price convergence, and economic and financial integration. If the common central bank is hard-nosed, i.e. very inflation averse, then a monetary union could also imply improved price stability for its member countries. While exchange rate transaction costs can be estimated, other benefits of a monetary union, such as economic integration, are less tangible. It is hence difficult to quantify the benefits of a monetary union in practice. Nonetheless, it is clear that the higher the level of intra-union trade, the greater are the benefits of joining a monetary union. This is because more trade implies greater gains from not having exchange rate transaction costs and from not needing to hedge against exchange rate volatility.

The main cost of joining a monetary union is losing sovereignty of monetary policy. Countries often have fundamentally different economic structures and levels of efficiency, which implies that they are affected differently by economic shocks. In a monetary union, a country cannot use monetary policy to tackle these shocks according to the specific needs of the domestic economy. There is also a cost in not having an exchange rate to restore competitiveness within the monetary union, following asymmetric economic shocks. As pointed out by Mundell (1961) in his pioneering work, the cost of giving up control of national monetary policy and hence of joining a monetary union is lower, the more synchronized member countries' business cycles are. This is because a higher correlation between business cycles implies a lower relative magnitude of country-specific to union-wide shocks and while the union central bank has no means of tackling country-specific shocks it can mitigate the effects of union-wide shocks. There are contrasting views about the effect of trade on the correlation between business cycles and hence about its effect on the cost of joining a monetary union. Krugman (1993) argues that a higher level of integration results in more specialization and hence in a lower correlation of shocks. The views of the European Commission (1990) in a publication promoting the launch of a European single currency contrast this however, as trade is argued to lead to a higher level of synchronization of shocks. Empirical evidence presented by Babetskii (2005) support the viewpoint of the European Commission with trade found to have a positive impact on the correlation between the demand shocks of the 10 EU accession countries and those of the EU-15. The evidence on the effect of trade on supply shocks is less conclusive however. In *chapter 1* of this thesis, we put forward a monetary union framework with trade in inputs and show analytically that a higher level of trade increases the correlation between union members' supply shocks.

Given that countries of a monetary union do not have sovereignty of monetary policy, fiscal policy takes a greater role as a stabilization instrument of domestic shocks. Sargent & Wallace (1981) point out the importance of fiscal and monetary policy coordination and show that if monetary policy targets price stability, then excessive levels of unfinanced government expenditure can lead to speculative hyperinflation. The *convergence criteria* of the European Economic and Monetary Union (EMU) therefore stipulates a limit on budget deficits of 3% of GDP and on government debt of 60% of GDP. However, although additional restrictions may be imposed on fiscal policy in a monetary union, it plays an important role as an instrument for economic stabilization.

Modelling a Monetary Union

Monetary union models emphasize some of the key features that distinguishes a monetary union economy from that of an individual country. Although the member countries of a monetary union share the same currency and central bank, they are subject to different exogenous shocks and face other economic asymmetries. In this thesis we use different two-country monetary union models to examine the impact that such asymmetries has on determinacy and E-stability of economic policy.[4] We define asymmetries as economic characteristics that make the two member countries distinguishable in some fundamental way. In the absence of such asymmetries, the union economy can be regarded as one

[4] While multiple country models have been put forward, see for example Clausen & Wohltmann (2005), two-country models have been found to be both useful and convenient for examining key features of monetary unions.

country for the purposes of economic policy. In that case, there is no cost of having a monetary union.

The seminal paper by Benigno (2004) presents a micro-founded two-country monetary union model with sticky prices and shows that heterogeneous price rigidities has important implications for optimal monetary policy. In particular, he finds that it is welfare improving for policy to attach a higher weight to a country with a higher degree of price stickiness, than the weight suggested by its economic size. This is because a shock to the economy of the country with the higher degree of price rigidity causes larger distortions to the union economy. Benigno (2004) also finds that it is welfare improving for monetary policy to stabilize the terms of trade in addition to output and inflation in this case. The assumption of heterogeneous price rigidities is supported by empirical evidence put forward by Benigno & Lopez-Salido (2006) suggesting that price rigidities do in practice vary across countries, with Italy and the Netherlands having more flexible prices than Germany, France and Spain. In *chapter 3* of this thesis, we examine how heterogeneous price rigidities affect determinacy and E-stability of monetary policy.

Using a static new Keynesian model of a monetary union, Bofinger & Mayer (2007) show that fiscal policy has an important role as a stabilizer of idiosyncratic shocks in a monetary union. Beetsma & Jensen (2005) extend the model of Benigno (2004) by allowing for fiscal policy to be used to stabilize shocks. They conclude that when monetary policy stabilizes union-wide output and inflation, government spending rules that support demand for domestic goods following shocks to competitiveness, can improve welfare in a monetary union. In *chapter 4*, we consider the effects of such policy coordination on the determinacy and E-stability of the monetary union economy.

The economic frameworks are combined with government policy rules to give the economic environment. We focus on interest rate feedback rules in particular.

Monetary Policy Rules

Taylor (1993) identified that a simple interest rate rule, in which the nominal interest rate is adjusted to deviations in output and inflation from their target levels, closely tracked the federal funds rate between 1987 and 1992. Following this, interest rate feedback rules that stabilize macroeconomic variables are referred to as Taylor rules in the literature. More recent evidence by Molodtsova et al. (2009) suggests that the policy rates of both the Federal Reserve and the European Central Bank closely followed simple Taylor rules reacting to either contemporaneous or expected future values of output and inflation, between 1999:Q4 and 2007:Q4. They find that the interest rate rule reacting to expected future values gives a closer fit than that reacting to contemporaneous data. Furthermore, a feedback Taylor rule was found to better describe movements in the ECB's policy rate than the Federal Reserve's. Taylor (1999) finds that since the 1980s, the Federal Reserve has followed a Taylor rule in which the policy interest rate is adjusted by more than one-for-one to changes in inflation. This is known as the Taylor principle and implies that interest rates are increased sufficiently to increase real interest rates and thus to contract the economy, following a rise in inflation.

While there previously was an explicit role for money supply targeting in the literature of monetary economics, the consensus today is to supplement a model economy with a Taylor rule, as pointed out by Friedman (2003). In this thesis, as in the majority of related research, we will derive determinacy and E-stability conditions for different Taylor rules. We consider both contemporaneous and forecast based interest rate rules. While contemporaneous Taylor rules serve as an interesting benchmark case and a natural starting point, evidence suggests that central banks do consider expectations in their policy analysis.[5] We now summarize the existing literature on determinacy and E-stability.

Determinacy and E-stability

This thesis examines determinacy and E-stability conditions for economic policy rules in New Keynesian DSGE models of a monetary union. In each case, we log-linearize the model economy around some defined steady state under the assumption of sticky prices and consider the rational expectations equilibria that result when different policy rules are implemented. For determinacy, we then require that there exists a unique stationary REE. If a policy rule results in several stationary REE (indeterminacy) or none at all, then this will lead to an undesirable economic outcome. Such policy rules should hence be avoided by policy makers. Furthermore, even if an equilibrium is stationary under rational expectations, it is not necessarily stable under recursive least squares learning. Results by Evans & Honkapohja (1999, 2001) show that E-stability is the necessary and sufficient condition for

[5] As pointed out by Honkapohja & Mitra (2005), the inflation reports of the Bank of England discuss private sector forecasts, while the monthly bulletin of the European Central Bank considers both internal and external forecasts (see Bank of England (2003) and European Central Bank (2003), respectively).

an REE to be locally asymptotically stable under recursive least squares learning. In this thesis, we consider E-stability of stationary minimal state variable (MSV) solutions derived under the RE assumption. Although explosive solutions could be E-stable, such equilibria should still be avoided by policy makers and are hence not considered here.[6] Consistent with related research, our results on the stability under adaptive learning are local.

We propose to only use policy feedback rules that are both determinate and E-stable. A useful result by McCallum (2007) suggests that determinacy is sufficient but not necessary for E-stability, when current period data is in the information set. He proves this for a large class of models permitting any number of lags and leads. The models we consider here fall into this category, but since there could exist multiple stationary REE that are all E-stable, we examine both determinacy and E-stability. Our results for a monetary union are compared with those from the closed and open economy literature summarized below.

Closed Economy Models

The seminal paper by Bullard & Mitra (2002) examines determinacy and E-stability conditions for monetary policy rules in the closed economy forward-looking sticky price model of Woodford (1999). Employing the methodology of Evans & Honkapohja (1999, 2001), they consider four different specifications of an interest rate rule that stabilizes the output gap and inflation. It is found that for a contemporaneous or forecast based policy rule, the Taylor principle is both necessary and sufficient for E-stability. While this is true for determinacy of the contemporaneous Taylor rule, a forward looking rule must satisfy an

6 See Evans & Honkapohja (2001, p. 219) for a discussion about learning stability of explosive solutions.

additional constraint in order to induce a unique stationary REE. Specifically, the interest rate rule must not be too aggressive, in particular in its stabilization of the output gap. Consequently, for a forecast based monetary policy rule, there exists a region for which policy is indeterminate and E-stable.

Bullard & Mitra (2007) find that including lagged values of the interest rate in the Taylor rule, so as to smoothen the path of interest rates, increases the determinate and E-stable region for monetary policy. In contrast to inertia in the interest rate rule, Wang & Wong (2005) examine the effect of inertia in the Phillips curve on determinacy and E-stability of monetary policy. They find that the higher the degree of inertia is, the more aggressive policy needs to be to ensure a unique stationary and E-stable REE.

Rather than looking at general specifications of Taylor rules, Evans & Honkaphoja (2003) use the closed economy model of Clarida et al. (1999) to examine determinacy and E-stability of different specifications of a monetary policy rule derived as optimal under RE. They find that a fundamentals based rule, in which the central bank adjusts interest rates to exogenous shocks, is neither determinate nor E-stable. However, an interest rate rule that also reacts to expected future changes in inflation and output is found to govern both a unique stationary REE and local stability under adaptive learning. Evans & Honkaphoja (2003) argue that this result is a case for the monetary authorities to explicitly consider the expectations of the private sector when setting policy.[7] Honkapohja & Mitra (2005) show that if the central bank instead uses internal forecasts in the policy rule, this may be

[7] While Evans & Honkaphoja (2003) consider the case of discretionary monetary policy, Evans & Honkaphoja (2006) extend the result to optimal monetary policy under commitment.

an additional source of instability. Preston (2008) considers the case in which economic agents form their decisions based on forecasts into the indefinite future rather than one period ahead and concludes that this could make the optimal policy rule, considered E-stable by Evans & Honkaphoja (2003), unstable under RLS learning. He proposes price targeting policy as more likely to induce stability under adaptive learning than an inflation targeting rule.

Open Economy Models

In recent years, the analysis of adaptive learning stability has been extended to the open economy literature. The open economy introduces some additional features that have implications for the determinacy and E-stability of monetary policy rules.

First, the issue of what inflation the central bank should target, that in producer prices or that in consumer prices, now becomes of relevance. Bullard & Schaling (2006) find that, for the two-country model of Clarida, Galí & Gertler (2002), determinacy and E-stability conditions must be met by each country in isolation when policy targets inflation in producer prices. On the other hand, when the central banks stabilize inflation in consumer prices, determinacy and E-stability of monetary policy has international spill-over effects. Llosa & Tuesta (2008) employ the small open economy of Galí & Monacelli (2005) and demonstrate that an aggressive forecast based interest rate rule reacting to consumer prices rather than producer prices, is more likely to induce unstable equilibria under RLS learning.

Second, monetary policy in the open economy also chooses an exchange rate regime. Wang (2006) examines learning stability in the two-country model of Benigno & Benigno

(2006). He finds that if a country pegs its interest rate to that of the other country, then the determinacy and E-stability of both countries depends on the policy rule of the second country. This coincides with the results of Bullard & Schaling (2006). Furthermore, Llosa & Tuesta (2008) show that when the exchange rate is stabilized in addition to the output gap and inflation, then the Taylor principle known from the closed economy does not need to hold for there to exist a unique stationary REE that is stable under RLS learning. Wang (2006) also finds that a managed exchange rate can relax the conditions required for determinacy and E-stability.

Finally, the degree of openness in trade has implications for stability of policy. Bullard & Schaling (2006) interpret the size of home country as being its degree of openness, so that when it approaches one it becomes a closed economy, while as it approaches zero it becomes a small open economy. They find that the smaller a country is in size, i.e. the more open it is in terms of trade, the more aggressive does monetary policy have to be to guarantee determinacy and E-stability. Llossa & Tuesta (2008) find similar results.

Fiscal Policy and Learning

Introducing fiscal policy as an instrument for tackling exogenous economic shocks has an effect on both determinacy and learning stability of monetary policy.

Leeper (1991) considers combinations of fiscal and monetary policy rules in which policy is either active or passive. He defines passive fiscal policy as one where the fiscal authorities adjust taxes to finance spending, while active fiscal policy does not adhere to a balanced budget.[8] Similarly, passive monetary policy adjusts the money supply to support fiscal spending, while active monetary policy pursues aggressive inflation stabilization independently of fiscal policy, so as to satisfy the Taylor principle. It is concluded that determinacy results when either policy is active while the other is passive. So, the Taylor principle does not have to be satisfied by the monetary authorities if fiscal policy is sufficiently aggressive in stabilizing prices. If both monetary and fiscal policy are passive however, the system becomes indeterminate with multiple stationary rational expectations equilibria. On the other hand, if the interest rate rule satisfies the Taylor principle, while fiscal policy ignores the budget constraint, then there exists no stationary solutions to the system. Although Leeper (1991) examines determinacy, he does not consider learning stability.

Evans & Honkapohja (2005) introduce learning into a flexible price model to analyse liquidity traps. They find that while the superior of two equilibria is stable under adaptive learning, the liquidity trap is not.[9] Large negative economic shocks can therefore lead to deflationary spirals with falling prices and output. Switching to a more aggressive monetary policy rule at low inflation levels was found to be an effective measure for preventing this. However, when prices are sticky rather than flexible, Evans, Guse & Honkapohja (2008) show that relying solely on aggressive monetary policy is not sufficient to avoid a downward spiral of output and inflation, following a large negative shock. They thus introduce a fiscal policy rule in the form of the tax feedback rule considered by Leeper

[8] Under active fiscal policy, taxes are increased to at least cover interest payments on newly issued debt.

[9] Eusepi (2007) finds that liquidity traps are indeed learnable for some forward looking policy rules in a non-linearized global framework.

(1991). A joint stabilization effort of monetary and fiscal policy is found to be sufficient to prevent a destabilization of the system. Indeed, as pointed out by Evans (2008), an aggressive fiscal policy is on its own sufficient to prevent the economy from reaching the liquidity trap.

Thesis Outline

Before examining determinacy and E-stability of economic policy, we put forward a monetary union model derived from micro-principles under the assumption of RE, in *chapter 1*. The reduced form of this model takes the same form as those models considered in later chapters and it gives some useful insights about the key features of monetary union models and thereby serves as a useful benchmark. The model extends the two-country open economy model of Clarida et al. (2002), by allowing for intermediate goods or inputs to be traded and by giving it monetary union characteristics. The latter implies eliminating the bilateral exchange rate and introducing a common central bank. While allowing for trade in inputs we also assume home bias in production of final goods, so that firms in general use inputs produced domestically more efficiently than they use those produced abroad. This has both empirical merit and also allows us to isolate the specific effects that trade in inputs has on our results, as explained in *chapter 1*.[10]

We derive the loss function of the union central bank by aggregating the expected utility of households. We then look at optimal discretionary monetary policy as the union

[10] See for example McCallum (1995), Wolf (2000), Hillbury & Hummels (2002) and Lopez, Pagoulatos & Gonzalez (2006) for evidence from North American data and Head & Mayers (2000) for evidence of home bias within the EU.

central bank minimizes its loss function subject to the constraint of only having one interest rate to tackle shocks from both union countries. Different specifications of the optimal interest rate rule are then derived under the assumption of RE.

The assumption of rational expectations is relaxed in *chapter 2* as we examine determinacy and E-stability for general specifications of Taylor rules and the optimal interest rate rules derived under discretion in *chapter 1*. The Taylor rules considered adjust interest rates to either contemporaneous or expected future values of union inflation and the union output gap. We are specifically interested in how the level of trade in inputs affects our results and how these differ from those from the closed economy literature.

In *chapter 3*, we will examine the effect of heterogeneous price rigidities on the determinacy and E-stability of monetary policy.[11] To do this, we make use of the two-country model put forward by Benigno (2004), in which one country is assumed to have a higher degree of price rigidity than the other. When price rigidities differ across countries, monetary policy has an impact on relative variables such as the terms of trade, since prices in each country adjust by different amounts, following a change in the interest rate. We hence consider contemporaneous and forward looking Taylor rules reacting to the terms of trade in addition to inflation and the output gap. Using the framework of Benigno (2004) simplifies our analysis significantly since the model already incorporates heterogeneous price rigidities. However, we conjecture that the effects that heterogeneous price rigidities

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[11] While we consider heterogeneous price rigidities, we assume homogeneous learning in that all agents use the same learning algorithm. Honkapohja & Mitra (2006) consider the case of learning heterogeneity and concludes that when such heterogeneities are persistent rather then transient, then RLS learning stability may not result even if the characteristics of all individual agents and the aggregate economy are consistent with E-stability.

has on determinacy and RLS learning stability in *chapter 3* would also prevail for the model considered in *chapters 1* and *2*, as well as for a general class of monetary union models.

While the analysis of preceding chapters focuses on monetary policy, the role of fiscal policy is of particular interest in a monetary union. Hence, in *chapter 4* we will employ the model of Beetsma & Jensen (2005) and explicitly investigate the determinacy and E-stability of a monetary union economy when monetary and fiscal policy are both actively responding to exogenous shocks. While monetary policy stabilizes shocks to union output and inflation, fiscal policy uses government expenditure to support domestic demand following asymmetric shocks. We will consider three different government spending rules; one where government spending is adjusted to changes in the terms of trade, one where it reacts to relative inflation and one that stabilizes the domestic output gap. Each of these is combined with a contemporaneous and a forward looking Taylor rule in turn, giving six different policy combinations to consider.

Contribution and Main Results

In *chapter 1*, we will show that the optimal discretionary monetary policy rule in a monetary union is isomorphic to that familiar from the closed economy literature.[12] While interest rates are adjusted to changes in the union output gap and inflation, it has no means of tackling country-specific shocks. A higher level of trade in inputs is found to increase the

[12] See Clarida, Galí & Gertler (1999) for an example of an optimal discretionary monetary policy rule for a closed economy model.

correlation between such shocks and thereby reduce the cost of not having two independent central banks.

In *chapter 2*, we will show that the results from the closed economy literature on learning are retained for the monetary union model derived in *chapter 1*. Looking at different specifications of the optimal interest rate rule, our results replicate those of Evans & Honkapohja (2003), suggesting that a fundamentals based policy rule is both indeterminate and unstable under RLS learning, while an optimal policy rule that explicitly takes into account the expectations of the private sector, induces a unique stationary and E-stable REE. When looking at general Taylor rules, the results by Bullard & Mitra (2002) are replicated and the Taylor principle is found to be a required condition for both determinacy and E-stability. We find that these results are unaffected by the relative sizes of the union countries and the level of trade in inputs. Hence, even with home bias in production, a monetary union can be regarded as a closed economy concerning the learning stability of monetary policy. In following chapters we introduce other asymmetries for which these results do not hold.

When introducing heterogeneous price rigidities in *chapter 3*, we find that the Taylor principle, is no longer a required condition for determinacy and E-stability. In fact, the greater the asymmetry in price rigidities, the larger is the stability region for monetary policy. An interest rate rule that stabilizes the terms of trade in addition to output and inflation further increases the stability region for monetary policy. If interest rates are sufficiently aggressive in its stabilization of the terms of trade, then no stabilization of output or inflation is needed to induce a determinate and E-stable equilibrium. While Llosa & Tuesta (2008) obtain similar results for a contemporaneous Taylor rule that stabilizes the exchange rate in addition to output and inflation, they find that if the interest rate rule is forecast based and aggressive, then it is actually less likely to induce determinacy. In our case, the determinacy and E-stability region increases for both a contemporaneous and forward looking policy rule. Our results support the argument for a central bank to take the terms of trade into consideration when setting policy.

Our most interesting finding highlights the importance of prudent fiscal spending in a monetary union. In *chapter 4*, we find that when fiscal policy rules react to either the terms of trade or relative inflation, then monetary and fiscal policy have to satisfy determinacy and E-stability conditions independently of one another, for the union economy to induce a unique stationary REE that is stable under RLS learning. While the stability conditions for monetary policy in this case are those known from the closed economy literature, an overly aggressive fiscal policy rule can eliminate all stationary equilibria, regardless of the monetary policy rule employed. This contrasts the findings by Leeper (1991) who suggests that both monetary and fiscal policy must be aggressive simultaneously, for the system to be explosive. Furthermore, when a relative inflation fiscal policy rule is combined with a forward looking Taylor rule, then fiscal policy also directly affects the learning stability of monetary policy. Specifically, if monetary policy is indeterminate but E-stable, then an aggressive fiscal policy rule can make monetary policy unstable under RLS learning.

When fiscal policy instead stabilizes the domestic output gaps while monetary policy reacts to shocks in union output and inflation, then fiscal policy does not have to satisfy stability conditions independently of monetary policy. Instead, fiscal policy increases the indeterminate and E-unstable region for which the Taylor principle is violated. Hence, the determinacy and E-stability region for a contemporaneous interest rate rule is smaller, the more aggressive fiscal policy is in its stabilization efforts. Thus instead of eliminating unstable liquidity traps, as in the case investigated by Evans et al. (2008), aggressive fiscal policy can destabilize the economy of a monetary union. However, if monetary policy is forward looking and sufficiently aggressive to satisfy the Taylor principle, then an aggressive fiscal policy rule can actually help induce determinacy.

Chapter 1
A Monetary Union Model with Input Trade

Chapter Overview

This chapter presents a two-country general equilibrium model of a monetary union. In addition to trade in final goods, we also allow for trade in intermediate goods. This causes labour markets to have spillover effects on the neighboring economy. As a result of this, the correlation between the union members' supply shocks increases with the level of trade, which in turn decreases the cost of having relinquished sovereignty of monetary policy.

We look at optimal discretionary monetary policy under the assumption of sticky prices and find that the central bank stabilizes union-wide shocks since it has no means of tackling country-specific shocks.

The assumption of rational expectations is maintained throughout this chapter. However, this assumption will be relaxed in chapter 2, when we use the model presented here to look at determinacy and E-stability of different monetary policy rules. The results found in the following chapter will then be used as a benchmark case in later chapters, when considering learning in monetary union models with heterogeneities.

1.1 Introduction

The potential costs and benefits of forming a monetary union have long been debated among economists and with the introduction of the euro in January 1999, this became more than merely an academic exercise.

While the benefits of introducing a single currency include the elimination of exchange rate uncertainty and transaction costs, as well as improved price transparency and financial integration, there is a cost in that countries relinquish the sovereignty of monetary policy. This also eliminates the use of a flexible exchange rate to stabilize shocks to competitiveness and leaves fiscal policy as the sole instrument at a country's disposal to offset country-specific shocks.

However, because budgets are determined by a large set of variables being mainly partisan, fiscal policy is a less flexible instrument for economic stabilization than monetary policy is. Furthermore, sustainable budget deficits can undermine the credibility of the union single currency, since a monetary expansion may be expected to pay off government debt.[1] The convergence criteria of the EMU therefore stipulates a limit on budget deficits of 3% of GDP and on government debt of 60% of GDP.

Given that fiscal policy is less than a perfect substitute for monetary policy, a country should join a monetary union only if the expected long term benefits of doing so exceed the long term costs of not having independent monetary policy.

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1 Sargent & Wallace (1981) present a model in which a sustainable fiscal deficit forces the central bank to provide seigniorage, which in turn leads to self-fulfilling levels of hyper inflation.

As pointed out by Mundell (1961) in his pioneering work, the cost of giving up control of national monetary policy is lower, the more synchronized member countries' business cycles are. This is because a higher correlation of business cycles implies a lower relative magnitude of country-specific to union-wide shocks, ceteris paribus. While union-wide shocks can be tackled by a union central bank, this is not true of country-specific or asymmetric shocks.[2]

It has been suggested that mobility of labour and price flexibility could reduce the costs of forming a monetary union for countries with asymmetric shocks.[3] In our model, trade in inputs causes a direct link between the domestic labour market and foreign output and prices. Furthermore, we find that a higher level of input trade, induced by a lower level of home bias in final goods firms' production technology, increases the correlation between member countries' supply shocks and hence increases business cycle correlations. This lowers the magnitude of country-specific to union-wide shocks and thereby reduces the cost of having relinquished sovereignty of monetary policy.

We extend the two country model put forward by Clarida, Gali & Gertler (2002) (CGG (2002), hereafter) to include trade in intermediate goods or inputs and we also mod-

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[2] This is partly because a change in the interest rates affects prices and output similarly in all countries when price rigidities are the same for all union members. In *chapter 3* we will look at the case of heterogeneous price rigidities. In that case, monetary policy does affect relative variables and can therefore stabilize asymmetric shocks, to an extent. However, the current target inflation of the ECB, the Harmonized Index of Consumer Prices (HICP), suggests that overall price stability is the main concern of the ECB.

[3] Mundell (1961) assumed stationary expectations in addition to labour immobility and rigidities in wages and prices. This seminal paper became a key reference point for the opponents of a single European currency (see for example Murray et al. (2003)). However, in later work, Mundell (1973) allowed for private sector expectations to be conditional on the exchange rate regime. He then suggests that a common currency, which eliminates exchange rate uncertainty, would lead to a higher level of international portfolio diversification through financial markets. Hence, Mundell was ultimately a proponent of a European single currency.

ify it by giving it monetary union characteristics.[4] The latter implies eliminating the bilateral exchange rate and introducing a common central bank. We further introduce home bias in input trade, which can both be justified empirically and also allows us to isolate the specific effect that trade in inputs has on our results. We use two extreme cases as benchmarks; one in which there is no trade in inputs and complete home bias, and the other in which there is no home bias and complete openness in trade. While the former case is isomorphic to the cooperative case in CGG (2002), the latter is isomorphic to a closed economy. In this chapter, we assume that all agents have rational expectations. This assumption will be relaxed in the following chapter.

In the following section we set up the micro foundations of our model and in turn introduce three different types of agents; households, final goods firms and intermediate goods firms. *Section 1.3* then identifies the equilibrium steady state of the model and look at deviations from this under the assumptions of flexible and sticky prices. We introduce the central bank's loss function, derived from micro principles, in *section 1.4* and use this to derive optimal discretionary monetary policy under the assumption of sticky prices à la Calvo (1983). *Section 1.5* concludes the chapter.

1.2 The Model

We present a two-country monetary union model consisting of a continuum of agents spread over the unit interval, of which $[0, n)$ reside in home country (H) and $(n, 1]$ re-

[4] CGG (2002) show that cooperation between the two countries' independent central banks is welfare improving of the Nash equilibrium case in which each central bank takes the other's actions as exogenous.

side in foreign country (F). Following CGG (2002) we introduce three types of agents in each country: *(i)* households, *(ii)* intermediate firms and *(iii)* final goods firms.

Households supply labour domestically, while indiscriminately consuming final goods produced in both countries.

Intermediate firms use domestically supplied labour to produce inputs, or intermediate goods, which are sold to final goods firms. We assume that intermediate firms produce inputs under monopolistic competition. In addition, intermediate firms face a fixed probability ρ of changing their price in each period as proposed by Calvo (1983).

Final goods firms are perfectly competitive and have home bias in technology so that in general they use domestically produced inputs more efficiently than they use inputs produced abroad. Consequently, they have a higher demand for domestic intermediate goods.

The higher the level of home bias in technology of final goods firms is, the lower is the level of trade in inputs. At the two extremes we have; no trade in inputs and complete home bias in production, or no home bias and complete openness in trade. The former of these two benchmarks is the case considered in CGG (2002), while the latter case mimics a closed economy, as will be shown later in this chapter.

We assume that the number of households, intermediate firms and final goods firms is the same in each country. That is, n for home country and $(1 - n)$ for foreign country.

1.2.1 Households

The representative household of home country faces the lifetime utility function:

$$U_t(h) = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s^H(h)) + L\left(\frac{M_s(h)}{P_s^H}\right) - V(N_{H,s}(h)) \right] \right\} \tag{1.1}$$

Where E_t denotes household expectations given the period t information set, $\beta \epsilon (0,1)$ is the household discount factor, $\frac{M_s(h)}{P_s^H}$ is the holdings of the union currency deflated by the domestic consumer price index and $N_{H,s}(h)$ is the amount of labour hours supplied. While $U(\cdot)$ and $L(\cdot)$ are assumed concave, $V(\cdot)$ is taken to be convex.

Consumer preferences are given by a Cobb-Douglas function as in Obstfeld & Rogoff (1998);

$C_t^H \equiv \frac{(C_{H,t}^H)^n (C_{F,t}^H)^{1-n}}{n^n(1-n)^{1-n}}$,

implying the cost minimizing price index;

$P_t^H \equiv (P_{H,t}^H)^n (P_{F,t}^H)^{1-n}$.

Households in foreign country face an analogous utility function with their preferences specifying their consumption index as;

$C_t^F \equiv \frac{(C_{H,t}^F)^n (C_{F,t}^F)^{1-n}}{n^n(1-n)^{1-n}}$,

and with the corresponding price index;

$P_t^F \equiv (P_{H,t}^F)^n (P_{F,t}^F)^{1-n}$.

Here $C_{i,t}^{j}$ denotes the consumption by households in country j of goods produced in country i and $P_{i,t}^{j}$ is the price paid by the consumer in country j when purchasing one unit of the good produced by country i, where $i, j = H, F$.

Assuming no transportation costs of exporting consumer goods and given that prices are set considering the monetary union as a common market, the identical preferences imply that the law of one price holds.

That is, $P_{i,t}^{H} = P_{i,t}^{F}$ for $i = H, F$ and we can denote the general price level in both countries by P_t.

We further define the terms of trade as relative prices of consumer goods, that is:

$$T_t \equiv P_{F,t}/P_{H,t}$$

Given the Cobb-Douglas type preferences for consumption goods, utility maximization implies that expenditure on each good is proportional to the size of the economy in which the product is produced. Since the consumer price indices of both countries are the same, i.e. $P_t^H = P_t^F = P_t = P_{H,t}^{n} P_{F,t}^{1-n} = T_t^{-n} P_{F,t} = T_t^{1-n} P_{H,t}$, it follows that:

$$C_{H,t}^{H} = nT^{1-n}C_t^H \quad \textit{and} \quad C_{F,t}^{H} = (1-n)T^{-n}C_t^H \tag{1.2}$$

A set of analogous conditions holds for foreign country consumption, C_t^F.[5]

To simplify, financial markets are assumed to offer complete risk diversification both domestically and internationally, as in Devereux & Engel (2000). Benigno (2004) assumes

5 For foreign country we have: $C_{H,t}^{F} = nT^{1-n}C_t^F$ and $C_{F,t}^{F} = (1-n)T^{-n}C_t^F$.

complete financial markets domestically but allows for households of the two countries to take a position in a union bond. It is shown that under reasonable assumptions this bond is redundant and that there is perfect risk sharing of consumption across countries also in that case.[6] Hence, our assumption simplifies the analysis without being overly restrictive.

It follows that consumption is guaranteed to be equal for all consumers across the monetary union at any given date, that is $C_t^H = C_t^F = C_t$.

Let $A_{t,t+1}$ denote the random payoff in period $t+1$ from the portfolio of assets purchased at time t and define the associated stochastic discount factor as $Q_{t,t+1}$. Households pay lump sum taxes of Υ_t and obtain returns from ownership of profit making intermediate firms, F_t. The representative domestic household is thus faced with the following budget constraint:

$$\begin{aligned} & E_t(Q_{t,t+1}A_{t,t+1}(h)) + \frac{M_t(h)}{P_t} + C_t(h) \\ \leq \; & \Gamma_t(h) + W_{H,t}(h)N_{H,t}(h) + F_t - \Upsilon_t \end{aligned} \tag{1.3}$$

Where $\Gamma_t(h) \equiv A_{t-1,t}(h) + \frac{M_{t-1}(h)}{P_{t-1}}$, and where foreign households face an analogous constraint.

Given that households are monopolistically competitive suppliers of labour, demand for labour is given by the constant elasticity function:

6 Appendix A of Benigno (2004) shows that, assuming no initial holdings of the internationally traded bond, it becomes redundant.

$$N_{H,t}(h) = \left(\frac{W_{H,t}(h)}{W_{H,t}}\right)^{-\eta_{H,t}} N_{H,t} \tag{1.4}$$

This is consistent with cost minimization given that the basket of labour hours employed is a CES function. The parameter $\eta_{H,t}$ is the wage elasticity of labour hours demanded and is assumed to be the same for all households in country H, but can vary across countries as well as over time.

Each household seeks to maximize its lifetime utility function (1.1) by choosing consumption levels, holdings of real money balances and its supply of labour subject to the budget constraint (1.3) and demand for its labour (1.4). In addition to equation (1.2), which gives the optimal amount to spend on each consumer good for a given level of consumption, we have the following first order conditions:

$$\frac{W_{H,t}(h)}{P_t} = (1 + \mu^w_{H,t})\frac{V'(N_{H,t}(h))}{U'(C_t)} \tag{1.5}$$

The market power of households as monopolistically competitive suppliers of labour is reflected by the mark-up of real wages, where $\mu^w_{H,t} \equiv 1/(\eta_{H,t} - 1)$. Since wages are flexible and preferences identical for all households, it follows that all households within a country work the same amount of hours and hence earn the same wage, i.e. $W_{H,t}(h) = W_{H,t}$ and $N_{H,t}(h) = N_{H,t}$ for $h\epsilon[0, n]$, and $W_{F,t}(h) = W_{F,t}$ and $N_{F,t}(h) = N_{F,t}$ for $h\epsilon[n, 1]$.

Defining the price of a one-period discount bond as $R_t^{-1} = E_t(Q_{t,t+1})$, optimal consumption planning is determined by the familiar Euler equation:

$$U_C(C_t) = \beta R_t E_t \left\{ U_C(C_{t+1}) \frac{P_t}{P_{t+1}} \right\} \tag{1.6}$$

Finally, we have the marginal utility of holding one unit of real money being equal to its opportunity cost, that is:

$$L_{M/P}\left(\frac{M_t}{P_t}\right) = \left(1 - R_t^{-1}\right) U_C(C_t) \tag{1.7}$$

Note that the money supply is set to satisfy this condition given the interest rate chosen by the union central bank, R_t. Furthermore, since consumption is guaranteed for all households across the union, it follows that the above two equations (1.6) and (1.7) are identical for all households within the monetary union, while (1.5) is identical for all households within a given country.

1.2.2 Final goods firms

Each country has a continuum of final goods firms that produce the goods consumed by households. The number of these firms is equal to the number of households and they are hence distributed over the interval $[0, n)$ in home country and $(n, 1]$ in foreign country. Final goods firms operate under perfect competition and the consumer goods differ only depending on the country in which they are produced. In contrast to CGG (2002), we assume that firms use input baskets supplied by both countries' intermediate firms to produce final goods. The production function of home country firms is given by:

$$Y^s_{H,t} = A^H_t \left((1-\lambda^H)^{\frac{1}{v}} (X^H_{H,t})^{1-\frac{1}{v}} + (\lambda^H)^{\frac{1}{v}} (X^H_{F,t})^{1-\frac{1}{v}} \right)^{\frac{v}{v-1}} \tag{1.8}$$

Where $Y^s_{H,t}$ is the output produced by the representative final goods firm in home country, while $X^H_{i,t}$ is the basket of inputs produced in country i and used by home country firms, all in per capita terms. The technology parameter A^H_t, determines the efficiency of the production process and v gives the level of substitutability between domestic and foreign input baskets. We have defined $\lambda^H \equiv (1-n)\omega$, where ω takes a value between zero and one and measures the degree of home bias in technology.

When there is no home bias, then $\omega = 1$ and firms use a proportion of inputs from home country equal to its economic size n. The other benchmark case is when there is complete home bias in technology and $\omega = 0$. In this case firms only use domestically produced intermediate goods in their production process. Allowing for ω to take a value between 0 and 1 also enables us to fully identify the specific effect that trade in inputs has on our model.

The level of home bias in technology and the level of trade in intermediate goods are inversely related, so that when there is complete home bias then there is no trade in inputs, and when there is no home bias then there is complete openness in trade of intermediate goods.

Benigno & Theonissen (2008) use one traded and one non-traded intermediate good in a production process isomorphic to ours and use this together with incomplete financial markets to explain the consumption-real exchange rate anomaly. There is empirical evidence to support the presence of home bias within the European Union.[7]

Country F's final goods firms are distributed over the interval $[n, 1]$ and face an analogous problem with the production technology:

$$Y^s_{F,t} = A^F_t \left((1-\lambda^F)^{\frac{1}{v}} (X^F_{H,t})^{1-\frac{1}{v}} + (\lambda^F)^{\frac{1}{v}} (X^F_{F,t})^{1-\frac{1}{v}} \right)^{\frac{v}{v-1}} \tag{1.9}$$

Where $1 - \lambda^F \equiv n\omega$, the technology parameter is given by A^F_t and where $X^F_{H,t}$ and $X^F_{F,t}$ are the home and foreign countries' input baskets used by foreign country's firms to produce consumer goods $Y^s_{F,t}$.

The input baskets are made up of inputs produced by a continuum of intermediate firms in both countries according to the following CES functions:

$$X^H_{H,t} \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\epsilon}} \int_0^n x^H_{H,t}(f)^{1-\frac{1}{\epsilon}} df \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{1.10}$$

$$X^H_{F,t} \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\epsilon}} \int_n^1 x^H_{F,t}(f)^{1-\frac{1}{\epsilon}} df \right]^{\frac{\epsilon}{\epsilon-1}}$$

[7] Head & Mayers (2000) find evidence of a preference for domestic products within the EU. The European countries have been free from trade tariffs since 1969 and the European Single Market Act helped to eliminate non-tariff barriers of trade by the end of 1992. Nevertheless, biases in trade are still evident and hence attributed to differences in preferences.

With similar functions for foreign country, and where $\epsilon > v > 1$ so that input goods of the same country are closer substitutes than the country H and country F input baskets are. Cost minimization, taking the input baskets demanded $X_{H,t}^{d,i}$ and $X_{F,t}^{d,i}$ as given, implies the following demand functions for intermediate firm f´s products:

$$x_{H,t}^{d,i}(f) = \left(\frac{IP_{H,t}(f)}{IP_{H,t}}\right)^{-\epsilon} X_{H,t}^{d,i} \text{ and } x_{F,t}^{d,i}(f) = \left(\frac{IP_{F,t}(f)}{IP_{F,t}}\right)^{-\epsilon} X_{F,t}^{d,i} \tag{1.11}$$

Where the price indices of Country H and F input baskets, $IP_{H,t}$ and $IP_{F,t}$ respectively, are given by:

$$IP_{H,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n IP_{H,t}(f)^{\epsilon-1} df\right]^{\frac{1}{\epsilon-1}}, \tag{1.12}$$

$$IP_{F,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}} \int_n^1 IP_{F,t}(f)^{\epsilon-1} df\right]^{\frac{1}{\epsilon-1}}$$

Note that we assume, as with consumer prices, that the law of one price holds so that final goods firms of both countries pay the same price for input baskets.

Minimizing costs, while taking the price $P_{H,t}(P_{F,t})$ of the consumer good $Y_{H,t}(Y_{F,t})$ in country $H(F)$ as given and noting that all final goods firms within a country are identical gives the aggregate demand functions for the input baskets as functions of their prices and given levels of output:[8]

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[8] The aggregate demand functions are given by:

$$X^d_{H,t} = \left(\frac{IP_{H,t}}{P_{H,t}}\right)^{-v}(1-\lambda^H)\frac{Y^s_{H,t}}{A^H_t} + \left(\frac{IP_{H,t}}{P_{F,t}}\right)^{-v}\lambda^H\frac{Y^s_{F,t}}{A^F_t}, \tag{1.13}$$

$$X^d_{F,t} = \left(\frac{IP_{F,t}}{P_{H,t}}\right)^{-v}(1-\lambda^F)\frac{Y^s_{H,t}}{A^H_t} + \left(\frac{IP_{F,t}}{P_{F,t}}\right)^{-v}\lambda^F\frac{Y^s_{F,t}}{A^F_t}$$

Where the aggregate demand for import baskets in per capita terms is defined as $X^d_{i,t} \equiv X^{d,H}_{i,t} + X^{d,F}_{i,t}$ and where output $Y^s_{i,t}$ is also given in per capita terms. Finally, we have the price of one unit of country $H(F)$'s consumption good being determined by the index:

$$P_{H,t} = \left[(1-\lambda^H)IP^{v-1}_{H,t} + \lambda^H IP^{v-1}_{F,t}\right]^{\frac{1}{v-1}}, \tag{1.14}$$

$$P_{F,t} = \left[(1-\lambda^F)IP^{v-1}_{H,t} + \lambda^F IP^{v-1}_{F,t}\right]^{\frac{1}{v-1}}$$

Hence, when there is no trade in inputs and $\omega = 0$, final goods prices in both countries are equal to the prices of the domestic intermediate good indices. When $\omega = 1$ and home bias is absent, final goods prices in both countries are equal. In general however, the higher ω is, the lower is the degree of home bias and the closer are the price levels of the two countries.

$nX^d_{H,t} = \left(\frac{IP_{H,t}}{P_{H,t}}\right)^{-v}(1-\lambda^H)\frac{nY^s_{H,t}}{A^H_t} + \left(\frac{IP_{H,t}}{P_{F,t}}\right)^{-v}(1-\lambda^F)\frac{(1-n)Y^s_{F,t}}{A^F_t}$

and

$(1-n)X^d_{F,t} = \left(\frac{IP_{F,t}}{P_{H,t}}\right)^{-v}\lambda^H\frac{nY^s_t}{A^H_t} + \left(\frac{IP_{F,t}}{P_{F,t}}\right)^{-v}\lambda^F\frac{(1-n)Y^s_{F,t}}{A^F_t}$

Dividing these through by n and $(1-n)$ respectively, to get the functions in per capita terms gives (1.13).

1.2.3 Intermediate goods firms

Intermediate firms use domestically supplied labour to produce inputs or intermediate goods. These inputs are traded internationally and are used in the production process of final goods firms in both home and foreign country, depending on the level of home bias in technology as discussed in the previous section.

We assume, without loss of generality, that the number of intermediate firms in each country is the same as the number of households and consumer goods firms. That is, country H has a continuum of firms over $[0, n)$ and country F over the $(n, 1]$ interval.

A representative firm f of country H and F respectively, faces the following linear technology:

$$x^s_{H,t}(f) = B^H_t N_{H,t}(f) \quad \textit{and} \quad x^s_{F,t}(f) = B^F_t N_{F,t}(f) \tag{1.15}$$

Here, $B^H_t (B^F_t)$ is a technology parameter of intermediate firms in country $H(F)$, while the basket of labour hours employed $N_{H,t}(f)(N_{F,t}(f))$ by firm f in home (foreign) country is given by:

$$N_{H,t}(f) \equiv \left[\frac{1}{n}\int_0^n N_{H,t}(h)^{1-\frac{1}{\eta_{H,t}}} dh\right]^{\frac{\eta_{H,t}}{\eta_{H,t}-1}} \textit{ and } N_{F,t}(f) \equiv \left[\frac{1}{1-n}\int_n^1 N_{F,t}(h)^{1-\frac{1}{\eta_{F,t}}} dh\right]^{\frac{\eta_{F,t}}{\eta_{F,t}-1}}$$

The wage indices are given by:

$$W_{H,t} \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\eta}} \int_0^n W_t(h)^{1-\frac{1}{\eta_{H,t}}} dh\right]^{\frac{\eta_{H,t}}{\eta_{H,t}-1}} \tag{1.16}$$
$$W_{F,t} \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{1-\eta}} \int_n^1 W_t(h)^{1-\frac{1}{\eta_{F,t}}} dh\right]^{\frac{\eta_{F,t}}{\eta_{F,t}-1}}$$

Where the parameter $\eta_{i,t}$ is the wage elasticity of labour hours demanded.

Choosing labour to minimize costs conditional on output yields the real marginal cost:

$$MC_{H,t} = \frac{(1-\tau^H)W_{H,t}/P_{H,t}}{B_t^H} = \frac{(1-\tau^H)(W_{H,t}/P_t)T_t^{1-n}}{B_t^H} \tag{1.17}$$
$$MC_{F,t} = \frac{(1-\tau^F)(W_{F,t}/P_t)T_t^{-n}}{B_t^F}$$

where τ^i is a government subsidy provided to intermediate firms of country $i = H, F$.

Following Calvo (1983), we assume staggered price setting so that firms in country $H(F)$ get to adjust their price every period with a fixed probability of $1-\rho^H$ $(1-\rho^F)$ and hence keep prices fixed while adjusting output to accommodate demand with a probability of $\rho^H(\rho^F)$.

Domestic firms setting their price in period t, choose the price $IP_{H,t}^0$ by solving the following program:

$$\underset{\{IP_{H,t}^0\}}{Max} : E_t \sum_{s=t}^{\infty} (\rho^H)^{s-t} Q_{t,s} x_{H,s}(f) \left[IP_{H,t}^0(f) - P_{H,s}MC_{H,s}(f)\right] \tag{1.18}$$

$$subject\ to\ x^d_{H,s}(f) = \left(\frac{IP_{H,s}(f)}{IP_{H,s}}\right)^{-\epsilon} X^d_{H,s}$$

with intermediate firms in country F optimizing an analogous program.

If firms could set their price in each period with certainty, that is if $\rho = 0$, they would choose:

$$IP^0_{H,t}(f) = \left(1 + \mu^p_{H,t}\right) MC_{H,t} P_{H,t} \tag{1.19}$$

That is, they would choose a price that is a mark-up over the nominal marginal cost, where $\mu^p_{H,t} \equiv \left(\frac{1}{\epsilon-1}\right)$ is a result of the product differentiation of inputs and the market power of intermediate firms. As ϵ increases and intermediate goods become closer substitutes, the mark-up approaches zero.

For a positive ρ, we have domestic firms adjusting their price $IP^0_{H,t}$ in period t to satisfy:

$$E_t \sum_{s=t}^{\infty} (\rho^H)^{s-t} Q_{t,s} x_{H,s}(f) \left[IP^0_{H,t}(f) - (1 + \mu^p_{H,t}) P_{H,s} MC_{H,s}(f)\right] = 0 \tag{1.20}$$

Given an analogous problem and solution for intermediate firms located in country F, we have the price indices of the two countries' input baskets determined by:

$$IP_{H,t} \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \left[\rho^H (nIP_{H,t-1})^{1-\epsilon} + (1-\rho^H)(nIP^0_{H,t})^{1-\epsilon}\right]\right]^{\frac{1}{1-\epsilon}}, \tag{1.21}$$

$$IP_{F,t} \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\epsilon}} \left[\rho^F ((1-n) IP_{F,t-1})^{1-\epsilon} + (1-\rho^F)((1-n) IP^0_{F,t})^{1-\epsilon} \right] \right]^{\frac{1}{1-\epsilon}}$$

1.3 Equilibrium

This section characterizes the rational expectations equilibrium of the monetary union model outlined above. The equilibrium conditions for households, intermediate firms and final goods firms are all simultaneously satisfied in this general equilibrium. As is standard for New Keynesian DSGE models used for policy analysis, we define three different specifications for the REE that differ only due to their assumptions about the presence of exogenous shocks and the flexibility of prices.[9] In the defined steady state equilibrium, there are no shocks and the economy is at a state of rest. The flexible price equilibrium then introduces the presence of exogenous shocks while maintaining the assumption of flexible prices. Finally, we introduce price rigidities à la Calvo (1983) into the model with exogenous shocks. In this case, prices are sluggish to adjust to economic shocks, causing distortions to output. It is then the role of monetary policy to steer the economy towards the equilibrium that would have prevailed under flexible prices and in the absence of distortions to output. Consequently, we focus on the sticky price equilibrium of the model when looking at monetary policy in the following section. Nevertheless, it is important to clearly

[9] See Woodford (2003, Ch. 3 and Ch. 4) and Galí (2008, Ch. 1 and Ch. 2) for an overview of New Keynesian DSGE models.

identify the steady state and flexible price equilibrium to which the sticky price equilibrium is compared.

We start by specifying consumption, the terms of trade and the marginal cost of intermediate firms as functions of domestic and foreign levels of output. We then derive output as a function of exogenous parameters under the assumption of flexible prices. We further specify the deterministic steady state of our model in which there are no shocks to the economy. A log-linearization around this steady state under the assumption of sticky prices then gives us familiar IS schedules and New Keynesian Phillips Curves (NKPCs). These are used in the following section to identify optimal discretionary monetary policy.

We define the per capita aggregate demand for home (foreign) country's final goods as $Y^d_{H,t}(Y^d_{F,t})$ and use (1.2) to obtain the market clearing conditions:[10]

$$Y^d_{H,t} = T^{1-n}_t C_t \quad \textit{and} \quad Y^d_{F,t} = T^{-n}_t C_t \tag{1.22}$$

Hence, although consumption is guaranteed output can vary across countries due to disturbances to the terms of trade.

The equilibrium of our model is given by the aggregate demand block described in section 1.2.1 and the supply side of the economy is determined by intermediate firms of section 1.2.3, the final goods firms outlined in section 1.2.2 and households´ labour

10 The aggregate demand for home goods $nY^d_{H,t}$ is from (1.2) given by:
$nY^d_t = nC^H_{H,t} + (1-n)C^F_{H,t} = n^2T^{1-n}C_t + (1-n)nT^{1-n}C_t = nT^{1-n}C_t$
Since consumption is guaranteed. Dividing throug by n gives country H´s market clearing condition in per capita terms: $Y^d_t = T^{1-n}C_t$. The foreign market clearing condition is derived analogously.

supply decisions. Specifically, the Euler equation (1.6), the condition for money market clearing (1.7) and the above market clearing conditions (1.22) together with the condition; $C_t^H = C_t^F = C_t$, determine the sequence for $\left\{C_t, M_t, R_t\right\}$ given the sequence of consumer prices $\{P_t\}$ and the initial condition P_{-1}.

By combining the market clearing conditions for home and foreign country and by imposing the equilibrium condition $Y_t^d = Y_t^s = Y_t$, we can determine per capita consumption as a function of output:

$$C_t = Y_{H,t}^n Y_{F,t}^{1-n} \tag{1.23}$$

We also have the terms of trade stabilizing differences between output per capita in the two regions, that is:

$$T_t = \frac{Y_{H,t}}{Y_{F,t}} \tag{1.24}$$

To simplify our analysis in what follows we impose iso-elastic preferences on households:

$$U(C_t) - V(N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\phi}}{1+\phi} \tag{1.25}$$

Where σ is the relative risk aversion of households, while ϕ measures workers' disutility from working.

On the supply side, the sequence for prices is determined by the optimal price setting condition for intermediate firms (1.20), the condition determining the input prices (1.21) and the relationship between input prices and the prices of final goods (1.14). Furthermore, combining labour supply (1.5) and intermediate firms condition for cost minimization (1.17), yields the condition for marginal cost:[11]

$$MC_{H,t} = (1-\tau^H)(1+\mu^w_{H,t})\frac{N^\phi_{H,t}C^\sigma_t T^{1-n}_t}{B^H_t} \tag{1.26}$$

and

$$MC_{F,t} = (1-\tau^F)(1+\mu^w_{F,t})\frac{N^\phi_{F,t}C^\sigma_t T^{-n}_t}{B^F_t} \tag{1.27}$$

We aggregate the labour hours used as inputs by intermediate firms in home country:

$$N_{H,t} = n^{-1}\int_0^n N_t(f)df = \frac{n^{-1}\int_0^n x_t(f)df}{B^H_t} \tag{1.28}$$

we further make use of;

$$x_{H,t}(f) = \left(\frac{IP_{H,t}(f)}{IP_{H,t}}\right)^{-\epsilon} X_{H,t}$$

define the dispersion of intermediate goods prices as:

[11] Recall that $\mu^w_{i,t} \equiv 1/(\eta_{i,t}-1)$ where $\eta_{i,t}$ is the wage elasticity of labour.

$$V_t \equiv n^{-1}\int_0^n \left(\frac{IP_{H,t}(f)}{IP_{H,t}}\right)^{-\epsilon} df$$

The aggregate production function is then given by:

$$X_{H,t}^S = \frac{N_t B_t^H}{V_t} \tag{1.29}$$

We combine this aggregate production function (1.29) with the demand function for input baskets (1.13) and the equilibrium condition $Y_t^d = Y_t^s = Y_t$ to obtain N_t as a function of output. Further using the above conditions for consumption (1.23) and the terms of trade (1.24), gives the marginal cost as a function of output, prices and exogenous variables:

$$\begin{aligned} MC_{H,t} \quad = \quad & (1-\tau^H)(1+\mu_{H,t}^w){Y_{H,t}}^{(1-n)+n\sigma}{Y_{F,t}}^{(\sigma-1)(1-n)}\left(B_t^H\right)^{-(1+\phi)} \qquad (1.30) \\ & V_t^\phi\left(\left(\frac{IP_{H,t}}{P_{H,t}}\right)^{-v}(1-\lambda^H)Y_{H,t}+\left(\frac{IP_{H,t}}{P_{F,t}}\right)^{-v}\lambda^H Y_{F,t}\right)^\phi \end{aligned}$$

With an analogous condition holding for foreign country.[12] Note that we have made the simplifying assumption; $A_t^H = A_t^F = 1$.

The marginal cost is affected by both domestic and foreign output, as in CGG (2002).

[12] Specifically we have $MC_{F,t} = (1-\tau^F)(1+\mu_{F,t}^w)V_{F,t}^\phi\left(B_t^F\right)^{-(1+\phi)}{Y_{H,t}}^{n(\sigma-1)}{Y_{F,t}}^{\sigma-n(\sigma-1)}$ $\left(\left(\frac{IP_{H,t}}{P_{F,t}}\right)^{-v}(1-\lambda^F)Y_{H,t}+\left(\frac{IP_{F,t}}{P_{F,t}}\right)^{-v}\lambda^F Y_{F,t}\right)^\phi$

Output affects the marginal cost and hence prices through three different channels: *(i)* the wealth effect, *(ii)* the terms of trade effect and *(iii)* through workers' required compensation for working longer hours.

As output increases, households become wealthier and consume more, which increases the marginal rate of substitution between consumption and labour and thus has an incremental effect on wages and the marginal cost. Because of consumption risk sharing across the union, both domestic and foreign output affect the marginal cost through this channel. Since a proportion of the wealth effect $(1-n)$ now comes from foreign output, the effect from domestic output is smaller here than in the closed economy case (n times as large).

Domestic and foreign output affect the marginal cost in different directions through the terms of trade. While the terms of trade effect of domestic output on the domestic marginal cost is positive, an increase in foreign output depreciates the terms of trade and hence also the real marginal cost facing domestic intermediate firms.

Finally, an increase in output requires that workers work longer hours, which increases their disutility from working and thereby their required compensation, i.e. their wages. Our model differs from CGG (2002) in that foreign output affects the domestic marginal costs through this last channel. Only in the special case of complete home bias $(\omega = 0)$ does our marginal cost reduce to that of CGG (2002).

With trade in intermediate goods, foreign final goods firms use domestically produced intermediate goods as inputs. As domestic intermediate firms increase their use of domestic labour to accommodate a higher demand for their products from foreign final goods firms, their marginal cost increases. Consequently, with $\omega > 0$ we have a positive correlation between foreign output and the domestic marginal cost even with logarithmic utility $(\sigma = 1)$. This is in contrast to the model of CGG (2002), in which logarithmic utility of consumption implies that foreign output is neutral on the marginal cost as the wealth effect and the terms of trade effect cancel each other out.

1.3.1 Flexible price equilibrium

In this section we consider the case of flexible prices $(\rho = 0)$ and assume that the wage mark-up is fixed at ${\mu_i}^w$, where $i = H, F$, as in CGG (2002). This is practical for two reasons: it allows us to consider fluctuations in output that are not due to changes in the degree of labour market efficiency and it also makes sense if we consider variations in the wage mark-up as a proxy for wage rigidities, since wages are assumed flexible here.

Note that in the flexible price equilibrium all intermediate firms within a region charge the same price and produce the same amount of output, that is $V_t = 1$. The aggregate production function then becomes $X^S_{H,t} = N_t B_t$.

We consider the symmetric case in which both the subsidies and the technology of intermediate firms are the same in both union countries, i.e. $\tau^H = \tau^F = \tau$ and $B^H_t = B^F_t = B_t$.[13] It follows from (1.17) that the nominal marginal costs are the same in both

[13] See *Appendix 1B* for derivation of the level of subsidy supporting welfare maximisation of union households. The subsidy $\tau^H = \tau^F$ is taken to be $(1+\mu^w)(1+\mu^p)-1$ and is hence effectively a subsidy that offsets the distortions to production arising from mark-up pricing by intermediate firms and the mark-up wages of households.

countries, i.e. $P_{H,t}MC_{H,t} = P_{F,t}MC_{F,t}$. This implies from (1.19) that intermediate firms across the union set the same price in each period, so $IP_t^0(f) = IP_{H,t} = IP_{F,t}$.

Finally, having used (1.21), equation (1.14) implies that consumer goods prices are equal in the two countries $IP_{H,t} = IP_{F,t} = P_{H,t} = P_{F,t}$. The marginal cost is then given by:

$$\overline{MC}_{H,t} = \overline{MC}_{F,t} = (1+\mu_t^p)^{-1} \tag{1.31}$$

Combining (1.39) and (1.31) yields the natural level of output as a function of exogenous parameters:

$$\begin{aligned}
((1-\lambda^H)\bar{Y}_{H,t} + \lambda^H \bar{Y}_{F,t})^{\phi} \bar{Y}_{H,t}{}^{(1-n)+n\sigma} &= \frac{\bar{Y}_{F,t}{}^{(1-\sigma)(1-n)} B_t^{(1+\phi)}}{(1-\tau)\,(1+\mu_t^p)\,(1+\mu_H^w)} \\
((1-\lambda^F)\bar{Y}_{H,t} + \lambda^F \bar{Y}_{F,t})^{\phi} \bar{Y}_{F,t}^{\sigma(1-n)+n} &= \frac{\bar{Y}_{H,t}{}^{n(1-\sigma)} B_t^{(1+\phi)}}{(1-\tau)\,(1+\mu_t^p)\,(1+\mu_F^w)}
\end{aligned} \tag{1.32}$$

The simultaneous equations in (1.32) solve for equilibrium output of both countries. Although we cannot offer an explicit solution for output we can get some insight into the relationship between output and the exogenous variables as well as the relationship between foreign and domestic output.

The effect of foreign output on domestic is ambiguous and depends on the relative magnitudes of σ, ϕ, ω and n. Again we have the three effects by which foreign output affects the domestic marginal cost at work.

The likelihood of foreign output having a negative effect on domestic output increases with the level of trade in inputs. This is because an increase in foreign output now increases the demand for and hence the cost of inputs that are also used by domestic final goods firms, causing a reduction in domestic production, ceteris paribus.

An increase in the government subsidy τ or intermediate firms' productivity B_t raises output, while an increase in workers' or intermediate firms' monopolistic market power lowers it.

1.3.2 The steady state

Here we characterize the symmetric and deterministic steady state equilibrium of our model. We drop the time subscript when considering the steady state value of variables.

In the steady state there are no shocks to the economy and the technology parameters B^H and B^F are both equal to one. Inflation rates in both countries are zero and the common central bank anchors the interest rate at the inverse of the household discount factor, so that $R^{-1} = \beta$.

In the steady state the terms of trade is equal to one $(T = 1)$, consumption is constant at C by the Euler equation (1.6) and real money balances are determined by (1.7). This implies, by the market clearing conditions (1.22), that the steady state level of output per capita is equal to consumption per capita in both union countries, i.e. $Y_H = Y_F = C$.

It is straightforward to solve for the steady state level of output as:

$$Y_H = Y_F = \left[(1-\tau)(1+\mu^p)(1+\mu^w)\right]^{-\frac{1}{\phi+\sigma}} \tag{1.33}$$

Note that since the level of subsidy in the steady state is given by $\tau = 1 - (1 + \mu^w)^{-1}(1+\mu^p)^{-1}$ (see *appendix 1B*) it follows that the steady state level of output and hence also consumption is equal to one.

We now proceed to consider the case of staggered prices setting as in Calvo (1983), allowing for small perturbations in the vicinity of our steady state.

1.3.3 Log-linear equilibrium fluctuations

In this section we reduce our model to a system consisting of a union IS schedule and New Keynesian Phillips Curves (NKPC) for each union country. We do so by allowing for small deviations from our steady state equilibrium under the assumption of sticky prices, as suggested by Calvo (1983). Define x_t as the log-deviation of the generic variable X_t from its steady state value X. The deviation from our defined steady state arises because we are allowing for both the presence of shocks and for prices to be sticky. That is, $x_t = \bar{x}_t + \widetilde{x}_t$ where $\bar{x}_t$ is the log-deviation from the steady state arising under flexible prices while $\widetilde{x}_t$ denotes the percentage fluctuation from the flexible price equilibrium due to prices being sticky. We define world variables as the union average; $X^W \equiv nX^H + (1-n)X^F$ and relative variables as the difference between foreign and domestic, i.e. $X^R \equiv X^F - X^H$.

Consider first aggregate demand, while the money market clears according to (1.7) given the interest rate set by the union central bank, the log-linear version of the Euler equation (1.6) is given by:[14,15]

$$c_t = E_t(c_{t+1}) - \sigma^{-1}\left[r_t - E_t(\pi^W_{t+1})\right] \tag{1.34}$$

Where the interest rate is defined as $r_t \equiv (R_t - 1)$ and π^W_t is the union average domestic inflation rate or equivalently the individual countries' c.p.i. inflation rates and is given by $\ln\left(\frac{P_t}{P_{t-1}}\right)$ or $n\pi^H_t + (1-n)\pi^F_t$, which are the same in this model due to the identical preferences of country H and country F consumers and the law of one price.

We now make use of the log-linearized versions of equation (1.23); $c_t = ny^H_t + (1-n)y^F_t = y^W_t$. Substituting this expression into (1.34) yields the union IS schedule:

$$\widetilde{c}_t = \widetilde{y}^W_t = E_t(\widetilde{y}^W_{t+1}) - \sigma^{-1}\left[r_t - E_t(\pi^W_{t+1}) - rr_t\right] \tag{1.35}$$

Where the natural rate of interest is a function of changes to the natural rate of output, specifically $rr_t = \sigma E_t(\Delta \overline{y}^W_{t+1})$. This is similar to the cooperative case in CGG (2002) in that we make the output gaps of both countries endogenous when linearizing. It differs from their results under Nash competition in which they take foreign variables as given.

14 See Friedman (2003) for a discussion about the abandonment of the LM curve in the macroeconomic literature.

15 Generally speaking, σ is the relative risk aversion of households, that is $\sigma = -\frac{U'(C)\bar{C}}{U''(C)}$.

Equation (1.35) resembles an IS curve of a closed economy if you consider the monetary union as one large country and it is also equivalent to the IS schedules in the monetary union model of Benigno (2004).[16]

We now turn to the supply side of our framework and derive the NKPCs. Combining the optimal price setting condition for intermediate firms (1.20) with the input price index evolving according to the Calvo (1983) model (1.21) and log-linearizing around the steady state yields:[17,18]

$$\pi_{H,t} = \delta mc_t + \beta E_t(\pi_{H,t+1}) \tag{1.36}$$

Where $\pi_{H,t}$ is the price inflation of country H's input goods and where:

$$\delta \equiv \left[(1-\rho)(1-\beta\rho)\right]/\rho$$

as the discount rate used by firms R^{-1} is assumed to be the same as for households, i.e. β (recall our defined steady state).[19]

Log-linearising the expression for marginal cost (1.30) using a linear approximation of the aggregate production function (1.29), where $x_t = n_t + b_t$ and combining this with

16 Key to obtaining this result is the assumption of complete financial markets leading to guaranteed consumption for all households within the monetary union. In the model of Benigno (2004), consumption is also guaranteed as mentioned in section 1.2.1.

17 We focus on the domestic case here, but the case for foreign country follows analogously.

18 Note here the difference in notation, inflation of input prices $IP_{i,t}$ is denoted by $\pi_{i,t}$, while inflation of final goods prices $P_{i,t}$ is denoted by π_t^i

19 See Woodford (2003, Ch. 3) for derivation and proof of the general case.

the linearized version of the demand for the input baskets (1.13), yields the expression for the marginal cost:

$$mc_{H,t} = \mu_t^w + \kappa^H \tilde{y}_t^H + \kappa_0^H \tilde{y}_t^F \tag{1.37}$$

where we have defined $\kappa^H \equiv (1-n)+n\sigma+\phi(1-\lambda^H)$ and $\kappa_0^H \equiv (\sigma-1)(1-n)+\lambda^H\phi$. Note that we have made use of the log-linearized version of the domestic natural level of output (1.33);

$$\hat{y}_t^H = \left\{(1+\phi)b_{H,t} - \kappa_0 \hat{y}_t^F\right\}\left(\kappa^H\right)^{-1}$$

Combining (1.36) and (1.37) gives the New Keyenesian Phillips curve for domestic intermediate goods inflation:

$$\pi_{_{H,t}} = \gamma^H \tilde{y}_t^H + \gamma_0^H \tilde{y}_t^F + \beta E_t(\pi_{H,t+1}) + u_{H,t} \tag{1.38}$$

where $\gamma^H \equiv \delta\kappa^H$, $\gamma_0^H \equiv \delta\kappa_0^H$ and where the cost push shock is a function of variations in the wage mark-up, $u_{H,t} = \delta\mu_{H,t}^w$.

The analogous case for country F's input inflation is given by:

$$\pi_{F,t} = \gamma^F \tilde{y}_t^H + \gamma_0^F \tilde{y}_t^F + \beta E_t(\pi_{F,t+1}) + u_{F,t} \tag{1.39}$$

where $u_{F,t} \equiv \delta\mu_{F,t}$, $\gamma^F \equiv \delta\kappa^F$, $\gamma_0^F \equiv \delta\kappa_0^F$, with $\kappa^F = n(\sigma-1) + (1-\lambda^F)\phi$ and $\kappa_0^F = \sigma(1-n) + n + \lambda^F\phi$.

We assume that the labour market shocks follow a stable *AR(1)* process;

$$\mu_{i,t} = \varrho\mu_{i,t-1} + \varepsilon_t \qquad\qquad i = H, F$$

where $0 < \varrho < 1$ and where ε_t is an *i.i.d* random shock with zero mean.

To obtain inflation in output prices from inflation in input prices, we make use of the log-linearized version of the price indices (1.14):

$$\pi_t^H = (1-\lambda^H)\pi_{H,t} + \lambda^H\pi_{F,t} \quad \textit{and} \quad \pi_t^F = (1-\lambda^F)\pi_{H,t} + \lambda^F\pi_{F,t} \tag{1.40}$$

Hence, we have the supply schedules for the two union members given by:

$$\pi_t^H = \theta^H\tilde{y}_t^H + \theta_0^H\tilde{y}_t^F + \beta E_t(\pi_{t+1}^H) + u_t^H \tag{1.41}$$

$$\pi_t^F = \theta^F\tilde{y}_t^H + \theta_0^F\tilde{y}_t^F + \beta E_t(\pi_{t+1}^F) + u_t^F$$

where we have defined:

$\theta^H \equiv (1-\lambda^H)\gamma^H + \lambda^H\gamma^F, \qquad \theta^F \equiv (1-\lambda^F)\gamma^H + \lambda^F\gamma^F,$

$\theta_0^H \equiv (1-\lambda^H)\gamma_0^H + \lambda^H\gamma_0^F, \qquad \theta_0^F \equiv (1-\lambda^F)\gamma_0^H + \lambda^F\gamma_0^F,$

$u_t^H \equiv (1-\lambda^H)u_{H,t} + \lambda^H u_{F,t}, \quad u_t^F = (1-\lambda^F)u_{H,t} + \lambda^F u_{F,t}.$

As the level of trade in inputs increases ($\omega \uparrow$), λ^H and λ^F approach $(1-n)$. There is hence convergence of consumer inflation levels, with increasing trade. Furthermore, the foreign country labour market shock $u_{F,t}$ now has a direct effect on domestic inflation through the domestic supply shock u_t^H. This is because a shock to domestic wages changes the price of domestic intermediate goods, which are used in the production process of foreign final goods firms. More generally, we have the following result.

Proposition 1 *The correlation between domestic and foreign supply shocks, u_t^H and u_t^F, increases with the level of trade in inputs, i.e.* $\frac{\delta Corr(u_t^H, u_t^F)}{\delta\omega} > 0$.

Proof. *See appendix 1A.* ■

Figure 1.1 illustrates how the correlation coefficient between supply shocks increases with the level of input trade. With complete home bias and no trade in inputs the supply shocks are completely independent of one another, while with complete openness in trade they are equal to each other.

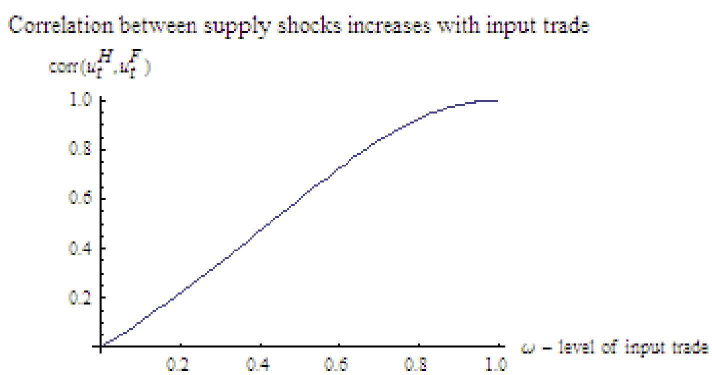

Fig. 1.1. The graph shows that the correlation coefficient between supply shocks increases with the level of trade in intermediate goods (ω).

This result is significant since evidence suggests that trade between euro zone countries has increased since the launch of the EMU.[20] If this higher level of trade, as concluded

[20] According to Ottaviano et al. (2009), the value of exports and imports of goods within the euro zone increased from 26% to 33% of GDP between 1998 and 2007. In the same period, that of services increased from 5% to 7%. Even when controlling for exogenous effects, trade seems to have had a modest but significant increase.

here, increases the correlation between exogenous shocks, the cost of relinquishing independence of monetary policy could decrease over time.[21] Hence, even if the benefits of joining a monetary union do not exceed the costs ex ante, they may still do so after having joined the monetary union.[22]

Our result also suggests that labour markets can have spill over effects on output and prices of final goods of other countries and that this effect increases with the level of trade, as long as the intermediate goods sector is labour intensive.

It is clear from equation (1.40) that also inflation levels become more correlated and consequently business cycles become more synchronized with a higher level of trade in intermediate goods. In the following section, we show that inflation and output can be expressed as functions of the supply shocks.

Looking at the supply schedules, it is useful to consider two benchmark cases, the first with complete home bias and no input trade ($\omega = 0$), and the second with no home bias and complete openness for trade ($\omega = 1$). We then illustrate graphically how the effect that the output gap has on inflation changes with the level of home bias.

1 *Complete home bias*

With no trade in intermediate goods we have:

$\omega = 0,$

[21] This is essentially what Frankel & Rose (1998) refers to as the endogeneity of the optimum currency area criteria.

[22] Of course there may also be other benefits from increased trade in terms of efficient resource allocation etc.

$$\lambda^H = \left(1 - \lambda^F\right) = 0,$$

$$\left(1 - \lambda^H\right) = \lambda^F = 1.$$

It follows that the supply schedules of the two countries reduces to that of CGG (2002), where international policy cooperation is assumed. Specifically, we have:

$$\theta^H = \delta\left[(1-n) + n\sigma + \phi\right], \qquad \theta_0^H = \delta\left[(\sigma - 1)(1-n)\right],$$

$$\theta^F = \delta\left[n(\sigma - 1)\right] \qquad \textit{and} \qquad \theta_0^F = \delta\left[\sigma(1-n) + n + \phi\right]$$

so the wealth and terms of trade effects cross borders, but there is no effect on the domestic marginal cost and hence not on domestic inflation, from the disutility of foreign workers. Given that final goods firms are perfectly competitive, intermediate prices are equal to consumer goods prices by (1.14);

$$\pi_t^H = \pi_{H,t}, \qquad \pi_t^F = \pi_{F,t} \qquad \text{and} \qquad u_t^H = u_{H,t}, \qquad u_t^F = u_{F,t},$$

so supply shocks are now completely uncorrelated and there are no direct spill-over effects of domestic labour markets on foreign output or inflation.

2 *No Home bias*

Consider now the case in which there is no home bias in production and complete openness in trade of intermediate goods, so that:

$$\omega = 1$$

$$\lambda^H = \lambda^F = 1 - n$$

$$\left(1 - \lambda^H\right) = \left(1 - \lambda^F\right) = n.$$

Since firms face the same technology and are perfectly competitive, by (1.14) final goods prices will be equal in both countries:

$$\pi_t^H = \pi_t^F = \pi_t^W \qquad and \qquad u_t^H = u_t^F = u_t^W$$

The supply schedules in (1.41) now become a union supply schedule of the form:[23]

$$\pi_t^W = \zeta_Y \widetilde{y}_t^W + \beta E_t(\pi_{t+1}^W) + u_t^W \tag{1.42}$$

where $\zeta_Y \equiv \delta\left[\sigma + \phi\right]$. This is identical to a closed economy supply schedule. Hence, a perfectly symmetric monetary union with identical production technology and preferences can be considered to be one large economy.

Since prices of final goods are equal in both countries, the terms of trade is equal to one. This implies that there are no disturbances to the terms of trade or relative prices and that the monetary authorities can focus on stabilizing union-wide shocks. In this case there is no cost of giving up independence of monetary policy.

3 *Looking at the general case using calibrated parameter values*

We now examine how the level of home bias affects the parameter values in the more general case of $0 < \omega < 1$. To do this we use the suggested calibration of Benigno (2004), which is similar to that of Rotemberg & Woodford (1998) but applied to the Euro zone economy rather than the US economy and therefore better suits our purpose.

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[23] Note that in the general case of $0 < \omega < 1$ the union NKPC can still be obtained, but must be combined with a condition for relative inflation; $\pi_t^R = -\zeta_T \widetilde{T}_t + \beta E_t(\pi_{t+1}^R) + u_t^R$, where $\zeta_T \equiv \delta(1+\phi(1-\omega))(1-\omega)$, to fully represent (1.41). See chapter 2.2.1 for derivation of this condition. When $\omega = 1$, then $\pi_t^R = 0$ and (1.42) suffices to represent the supply side of the monetary union.

We set the intertemporal rate of substitution β to 0.99 and since these are assumed to be quarterly values this implies an annual equilibrium interest rate of 4.1%.[24] The coefficient of risk aversion σ is set to a conventional value of $1/6$. Furthermore, assuming that the elasticity of the average real wage with respect to variation in production is slightly higher in Europe than in the US and by assuming that the labour share of total income is equal to 0.75 we have that the elasticity $\phi = 0.67$.[25] We let the sizes of the two regions of the monetary union be equal, that is $n = 0.5$.

The average length of a union price contract is set to four quarters, so that $\rho = \frac{3}{4}$. Benigno & Lopez-Salido (2006) provide empirical evidence that support this assumption for the major European economies.

In *figure 1.2* below we show how the Philips curve parameter values vary with the openness of input trade for the above calibration. With no trade, foreign output initially has a negative effect on domestic inflation as the terms of trade effect exceeds the wealth effect. With input trade increasing however, the effect of households' disutility from working increases, which eventually leads to a positive foreign output gap coefficient θ_0^H.

The domestic output gap always has a positive effect on domestic consumer goods inflation, however its magnitude decreases when domestic firms start to use foreign inputs in their production process as this decreases the relevance of domestic wages on the domestic marginal cost.

24 This is approximately the 20 year German average (see Benigno (2004)).

25 Chang et al. (2007) find empirical estimates of labour supply elasticity ranging between 0.4 and 1.

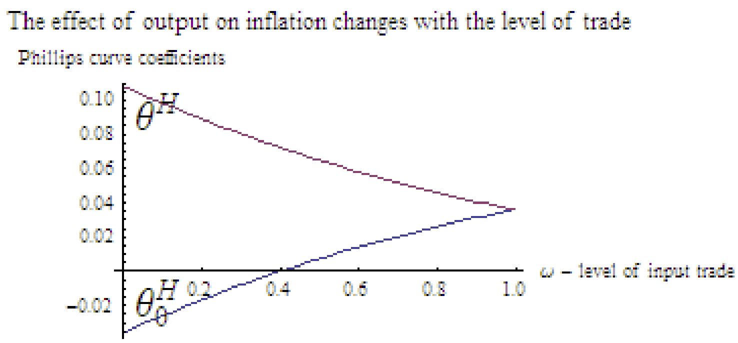

Fig. 1.2. The graph shows that the effect of domestic output on domestic inflation decreases, while that of foreign output on domestic inflation increases, with the level of trade in inputs (ω), as determined by the Phillips curve coeffiecents.

Note that with the countries being of equal size, i.e. $n = 0.5$ we have that $\theta^H = \theta_0^F$ and $\theta_0^H = \theta^F$, hence an analogous result is obtained for foreign country's supply coefficients.

In the following section we present the central bank loss function and derive optimal discretionary monetary policy.

1.4 Welfare and Optimal Monetary Policy

Given that the union economy is subject to exogenous shocks and given that prices are slow to adjust to such shocks, there will be distortions to output, consumption and the welfare of union households. The role for monetary policy in our framework is then to minimize the reduction in household utility caused by distortions to the economy due to prices being sticky rather than flexible. As such, optimal monetary policy maximizes the expected aggregate utility of households across the monetary union. Comparing the optimal policy in our monetary union to that of a closed economy gives some insight into the costs of having only one central bank instead of two (one for each country). Furthermore, we are particularly interested in the effect that the level of trade in inputs has on our results. Although useful in examining the costs, our model does not allow us to consider the benefits of a monetary union.

While monetary policy seeks to offset the effect that price rigidities has on output, fiscal policy acts to neutralize distortions to output arising from mark-ups in prices and wages due to imperfectly competitive markets. Specifically, the optimal subsidy paid to intermediate firms is given by $\tau^H = \tau^F = 1 - (1+\mu^w)^{-1}(1+\mu^p)^{-1}$.[26]

We assume that liquidity from holding real money balances is small and hence consider the cashless limiting case in which money holdings do not affect welfare. Keeping in mind that consumption is guaranteed and taking a second order approximation of the aggregate loss in household utility due to prices being sticky rather than flexible gives the central bank loss function:[27]

$$E_0\left(L^{CB}\right) = -\frac{\epsilon}{2\delta} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \left(\pi_t^W\right)^2 + n(1-n)(1-\omega)^{-2}\left(\pi_t^R\right)^2 \\ +n\alpha^H \left(\tilde{y}_t^H\right)^2 + (1-n)\alpha^F \left(\tilde{y}_t^F\right)^2 \\ +\Phi 2n(1-n)\tilde{y}_t^H \tilde{y}_t^F \end{array} \right\} \tag{1.43}$$

Where we have defined:[28]

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[26] See *appendix 1B* for derivation of this.

[27] *Appendix 1B* derives this expression by a second order approximation of the sum of households' utility functions.

[28] Recall the definition of relative inflation; $\pi_t^R \equiv \pi_t^F - \pi_t^H$.

$$\alpha^H \equiv \tfrac{\delta}{\epsilon}\left[(1+\phi)\left((1-\lambda^H)^2+(1-n)n\omega^2\right)-n(1-\sigma)\right]$$
$$\alpha^F \equiv \tfrac{\delta}{\epsilon}\left[(1+\phi)((1-n)n\omega^2+\left(\lambda^F\right)^2)-(1-n)(1-\sigma)\right]$$
$$\Phi \equiv \tfrac{\delta}{\epsilon}\left[(1+\phi)\omega((1-\lambda^H)+\lambda^F)-(1-\sigma)\right]$$

In the special case of complete home bias and no trade in inputs ($\omega = 0$) this function reduces to that of the cooperative case in CGG (2002). When there is no home bias ($\omega = 1$), then equation (1.43) is not a valid expression and the loss function becomes that of a closed economy (see equation $(1.A10)$ of *appendix 1B*).

We now consider the case in which the union central bank seeks to optimize the loss function (1.43) by setting the interest rate r_t in each period. That is, we look at time consistent or discretionary welfare optimizing monetary policy.

Proceeding with the optimization problem of the central bank, we initially ignore the constraint presented by the IS schedule (1.35). This procedure is often used in both open and closed economy models as the IS curve is then satisfied by setting the appropriate interest rate.[29] However, in a monetary union we implicitly have the additional constraint that interest rates in the two countries must be the same and hence that the central bank only has one policy instrument at its disposal. Clearly, this is not sufficient to stabilize both union-wide and relative shocks. Nonetheless, proceeding with the same method allows us to easily compare our results to the case with two independent central banks.

[29] See for example Clarida et al. (1999) for a closed economy case and Clarida et al. (2002) for a two-country model.

Choosing $\left\{\pi_t^H, \pi_t^F, \tilde{y}_t^H, \tilde{y}_t^F\right\}$ to maximize (1.43) subject to the supply schedules in (1.41), while taking expectations of future values and exogenous variables as given, yields the FOCs:

$$-\frac{\epsilon}{\delta}\left[n\alpha^H \tilde{y}_t^H + n(1-n)\Phi \tilde{y}_t^F\right] = -\psi_1\theta^H - \psi_2\theta^F$$

$$-\frac{\epsilon}{\delta}\left[(1-n)\alpha^F \tilde{y}_t^F + n(1-n)\Phi \tilde{y}_t^H\right] = -\psi_1\theta_0^H - \psi_2\theta_0^F$$

$$-\frac{\epsilon}{\delta}\left[n(n+(1-n)(1-\omega)^{-2})\pi_t^H + n(1-n)\left(1-(1-\omega)^{-2}\right)\pi_t^F\right] = \psi_1$$

$$-\frac{\epsilon}{\delta}\left[n(1-n)\left(1-(1-\omega)^{-2}\right)\pi_t^H + (1-n)((1-n)+n(1-\omega)^{-2})\pi_t^F\right] = \psi_2$$

Where ψ_1 and ψ_2 are the Lagrangian multipliers. Further combining these into two equations gives the relationship between country output gaps and inflation levels as:

$$\begin{aligned} \tilde{y}_t^H &= -\Psi^H\pi_t^H + \Psi_0^H\pi_t^F \\ \tilde{y}_t^F &= -\Psi_0^F\pi_t^F + \Psi^F\pi_t^H \end{aligned} \tag{1.44}$$

where $\Psi^H \equiv \frac{\epsilon(1+(1-\omega)\phi - n\omega((2-\omega)+(1-\omega)\phi))}{(1-\omega)^2(1+\phi)}$, $\Psi_0^H \equiv \frac{\epsilon((1-n)\omega((2-\omega)+(1-\omega)\phi))}{(1-\omega)^2(1+\phi)}$, $\Psi_0^F \equiv \frac{\epsilon(1+(1-\omega)\phi-(1-n)\omega((2-\omega)+(1-\omega)\phi))}{(1-\omega)^2(1+\phi)}$ *and* $\Psi^F \equiv \frac{\epsilon(n\omega((2-\omega)+(1-\omega)\phi))}{(1-\omega)^2(1+\phi)}$.

The conditions in (1.44) show the welfare optimizing relationship between country inflation and output gap levels. With no input trade ($\omega = 0$) we have $\Psi^H = \epsilon$ and $\Psi_0^H = 0$, so the optimal policy reaction to an increase in domestic inflation would be to contract the domestic output gap below its natural level, while keeping the foreign output gap unchanged. As the level of trade increases, the optimal policy suggests adjusting output in both countries following a change in domestic inflation. This is because there is now a direct spill-over effect of the domestic labour market on the foreign economy. It should be noted that the total effect of inflation on output is independent of the level of trade, since $(-\Psi^H + \Psi_0^H) = -\epsilon$.

The optimal policy of (1.44) is not feasible for the central bank of a monetary union since it has no instrument at its disposal to contract only domestic output, while leaving foreign output unchanged. This is clear from the IS schedule (1.35), since a change in the interest rate affects only the average union output gap and has no effect on relative variables. The central bank of a monetary union can only affect relative variables in the case of asymmetric price rigidities, as we will show in *chapter 3*.

Combining the equations in (1.44) gives the following feasible condition for monetary policy:

$$\tilde{y}_t^W = -\epsilon\pi_t^W \tag{1.45}$$

Thus, the central bank should adopt a lean against the wind type of policy to tackle union-wide shocks. This type of rule is common in the literature and the aggressiveness with which the central bank should contract economic activity in the union following a rise in inflation depends positively on the degree of competitiveness of intermediate goods, ϵ.

Equation (1.45) suggests that there is a cost of having a monetary union since country specific shocks cannot be stabilized. We rewrite (1.44) to illustrate that this cost decreases with the level of trade in inputs:

$$
\begin{aligned}
\tilde{y}_t^H &= -\vartheta\left(1+\left(1-\omega\right)\phi\right)\pi_t^H + \vartheta\omega\left(\left(\left(2-\omega\right)+\left(1-\omega\right)\phi\right)\right)\pi_t^W \qquad (1.46)\\
\tilde{y}_t^F &= -\vartheta\left(1+\left(1-\omega\right)\phi\right)\pi_t^F + \vartheta\omega\left(\left(\left(2-\omega\right)+\left(1-\omega\right)\phi\right)\right)\pi_t^W
\end{aligned}
$$

where $\vartheta \equiv \frac{\epsilon}{(1-\omega)^2(1+\phi)}$. It is evident from (1.46) that the weight and hence relevance of union inflation on country output gaps is increasing in ω. As trade in intermediate goods increases, domestic inflation levels converge and approach the union average level. The magnitude of country-specific shocks then decrease, since with a higher level of trade labour shocks increasingly have cross border effects. Consequently, with no home bias in production, the welfare optimizing condition in the case of two independent central banks becomes condition (1.45), that of the monetary union central bank. This leads us to our second proposition.

Proposition 2 *There is a welfare loss of having a monetary union vis-a-vis each country having monetary independence, because the union central bank has no means of stabilizing relative or country-specific shocks. This loss decreases as the level of trade in inputs increases.*

The optimal union inflation level can be written as a function of the exogenous supply shocks u_t^H and u_t^F. Combining the two supply schedules in (1.41) gives the union average NKPC (1.42), by then substituting (1.45) while assuming rational expectations, we have:

$$\pi_t^W = \left(1-\varrho\beta+\epsilon\delta\left(\sigma+\phi\right)\right)^{-1}u_t^W \qquad (1.47)$$

So union inflation, or equivalently domestic inflation in consumer prices, is a function of union supply shocks.

1.4.1 Specifications for the optimal policy rule

We will now derive three different specifications for the optimal interest rate rule under discretion. In the following chapter we will show that the specification of the policy rule is crucial for both determinacy and E-stability of the union economy.

Substituting (1.45) and (1.42) into the IS curve (1.35) and rearranging, gives the optimal interest rate rule:

$$r_t = rr_t + \sigma E_t\left(\tilde{y}^{W}_{t+1}\right) + \left[1 + \frac{\beta\sigma}{(\zeta_Y + \epsilon)}\right] E_t\left(\pi^W_{t+1}\right) + \frac{\sigma}{(\zeta_Y + \epsilon)} E_t\left(u^W_{t+1}\right) \tag{1.48}$$

where $\zeta_Y \equiv \delta\left[\sigma + \phi\right]$. We will refer to (1.48) as the expectations based optimal policy rule. Note that the expectations of agents enter explicitly in this policy rule.

An alternative specification is obtained by substituting (1.45) into the IS curve (1.35):

$$r_t = rr_t + \left[1 + \frac{(1-\varrho)\,\sigma\epsilon}{\varrho}\right] E_t\left(\pi^W_{t+1}\right) \tag{1.49}$$

Hence, we now have the optimal interest rate rule as a function of only expected future inflation. Note that we have imposed the assumption of rational expectations to transform the current value of the output gap in (1.35) into its expected future value. The RE assumption is maintained throughout this chapter and it is crucial for the derivation of all policy rules considered here. However, the RE assumption was imposed explicitly when transforming the optimal policy rule into the form of (1.49), while this was not the case for (1.48). This implies that the two rules are not necessarily equivalent when relaxing the RE assumption even though they are under RE.[30]

Since $\frac{(1-\varrho)\sigma\epsilon}{\varrho} > 0$, the interest rate policy rule satisfies the Taylor principle, i.e. the interest rate is increased by more than one-for-one to a rise in expected inflation. Interestingly, the optimal interest rate rule is independent of the level of home bias in production.

Finally, combining (1.47) with (1.48), or otherwise, gives the optimal interest rate feedback rule as a function of the exogenous supply shocks:

$$r_t = rr_t + \left[\frac{\varrho + (1-\varrho)\,\sigma\epsilon}{1 - \varrho\beta + \epsilon\delta\,(\sigma+\phi)}\right] u_t^W \tag{1.50}$$

We will refer to this policy rule as the fundamentals based rules since interest rates are not adjusted to changes in expected future values of output or inflation. Note that the optimal policy is to increase the interest rate to tackle union-wide supply shocks, while not reacting to relative shocks. The larger relative shocks are, the higher is then the cost of having a monetary union. Since the magnitude of relative shocks decreases with the level of trade, by *proposition 1*, we conjecture that a higher level of trade is welfare improving in the sense that it reduces the cost of having a common central bank.

[30] We note that these policy rules are optimal under RE, this is not necessarily the case when the RE assumption is relaxed.

In *chapter 2* we will examine determinacy and E-stability of the alternative specifications for the interest rate rule. We will show that both determinacy and E-stability of the policy rule hinges on the specification of the rule considered.

1.5 Conclusion

This chapter has presented a baseline two-country monetary union model with home bias in the production technology of final goods firms. We have shown that as this home bias disappears and consequently as trade in inputs increases, the union economy behaves more like a large closed economy.

Specifically, trade in inputs causes domestic labour market shocks to have direct spillover effects on the market for foreign final goods and vice versa. This has the effect of increasing the correlation between countries' supply shocks, which in turn synchronizes the movements of the two countries' consumer price inflation levels.

A higher level of trade also reduces the volatility of countries' supply shocks and hence of inflation in consumer prices. Furthermore, with no home bias and complete openness in trade, price levels become equal in both countries and there is no welfare loss in the monetary union arising from the inability of the central bank to stabilize relative or country-specific shocks. In general, a higher level of trade leads to a higher level of union welfare.

The optimal monetary policy rule under discretion puts a weight on each countries' inflation that is equal to its economic size, i.e. n for home country and $(1-n)$ for foreign country. It also satisfies the Taylor principle and hence the real interest rate is increased to a rise in expected inflation. The union central bank stabilizes union-wide shocks, as the tackling of relative shocks is not feasible. In the following chapter we will examine determinacy and E-stability of the optimal policy rules derived here and of some more general Taylor rules. A key difference between our framework and the closed economy models is that there are relative shocks that cannot be stabilized by the union central bank. The key question that we consider in *chapter 2* is then whether this factor can lead to excess volatility and an unstable system under recursive least squares learning.

1.A Appendix: *Proof of proposition 1*

Recall the following definitions;

$$\lambda^H \equiv (1-n)\,\omega \qquad \textit{and} \qquad (1-\lambda^F) \equiv n\omega$$

The supply shocks are given by (see section 1.3.3);

$$u_t^H = (1-\lambda^H)u_{H,t} + \lambda^H u_{F,t} \qquad \textit{and} \qquad u_t^F = (1-\lambda^F)u_{H,t} + \lambda^F u_{F,t}$$

where; $\qquad u_{H,t} = \delta\mu_{H,t}^w \qquad \textit{and} \qquad u_{F,t} = \delta\mu_{F,t}^w,$

this implies the correlation coefficient:

$$Corr(u_t^H, u_t^F) = \frac{(1-\lambda^H)(1-\lambda^F)Var(\mu_{H,t}^w)+\lambda^H\lambda^F Var(\mu_{F,t}^w)}{\sqrt{(1-\lambda^H)^2 Var(\mu_{H,t}^w)+\left(\lambda^H\right)^2 Var(\mu_{F,t}^w)}\sqrt{(1-\lambda^F)^2 Var(\mu_{H,t}^w)+\left(\lambda^F\right)^2 Var(\mu_{F,t}^w)}}$$

which is increasing in the level of trade, ω.

Specifically, the derivative of the correlation coefficient with respect to ω is given by:

$$\frac{\delta Corr(u_t^H, u_t^F)}{\delta\omega} =$$

$$\frac{(1-\omega)Var(\mu_{H,t}^w)Var(\mu_{F,t}^w)\Big((1-n)\big(n(1-\omega)^2+(1-n)\big)Var(\mu_{F,t}^w)+n\big((1-n)(1-\omega)^2+n\big)Var(\mu_{H,t}^w)\Big)}{\Big((1-n\omega)^2 Var(\mu_{F,t}^w)+n^2\omega^2 Var(\mu_{H,t}^w)\Big)^{\frac{3}{2}}\Big((1-n)^2\omega^2 Var(\mu_{F,t}^w)+n^2(1-(1-n)\omega)^2 Var(\mu_{H,t}^w)\Big)^{\frac{3}{2}}} > 0.$$

1.B Appendix: *Derivation of loss function and subsidies*

In this appendix we derive the central bank loss function (1.43) and the welfare optimizing subsidy rates of the fiscal governments. The policy maker seeks to minimize the aggregate loss in utility of union households caused by distortions to the union economy due to prices being sticky rather than flexible.

We hence derive the loss function by taking a second-order approximation of the sum of households' utility functions around the flexible price steady state equilibrium, corresponding to the optimal choice of the subsidy rates τ^H and τ^F. In doing so we maintain the assumption of rational expectations. As in Benigno (2004) we assume that the liquidity from holding real money balances is small and hence consider the cashless limiting case in which money holdings do not affect welfare. This implies that the utility of the representative agent of home and foreign country, at a given date t, is given by:

$$\mathbf{w}_t^H = U(C_t) - V(N_{H,t}) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{{N_{H,t}}^{1+\phi}}{1+\phi} \; and \; \mathbf{w}_t^F = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{{N_{F,t}}^{1+\phi}}{1+\phi} \qquad (1.A1)$$

The central bank assigns a weight to each country proportional to its economic size so that overall union welfare is:

$$\mathbf{w}_t^W = U(C_t) - nV(N_{H,t}) - (1-n)V(N_{F,t}) \qquad (1.A2)$$

Note that consumption is the same for all households in the union, given the assumption of perfect risk sharing. Before deriving the second-order approximation of the period utility function around the steady state, we determine the subsidy rate τ^i required for welfare optimization. This is obtained by maximizing:

$$U(C) - nV(N_H) - (1-n)V(N_F)$$

subject to:

$C = Y_H^n Y_F^{1-n}; \qquad N_H = (1-\lambda)Y^H + \lambda Y^F$ *and* $N_F = (1-\lambda^F)Y^H + \lambda^F Y^F$

where we have made use of the production function of intermediate firms, given that the technology parameters are given by; $B_H = B_F = 1$, in our defined steady state. By substituting the constraints into the objective function and maximizing with respect to Y^H and Y^F, or otherwise, we obtain the first order condition:

$$U'(C)\frac{C}{Y} = V'(N) \tag{1.A3}$$

Since per capita consumption is equal to per capita output in the steady state, given that the terms of trade is equal to one, we have: $U'(C) = V'(N)$

We use the condition for the marginal cost:

$$MC = \frac{(1+\mu^w)V'(N)}{(1+\tau)U'(C)}$$

substituting $(1.A3)$ and recalling from section $1.3.1$ that in the absence of price rigidities the marginal cost is equal to $(1+\mu^p)^{-1}$, gives the optimal subsidy solving:

$(1-\tau) = (1+\mu^w)^{-1}(1+\mu^p)^{-1}$

Hence the subsidy serves to offset the distortion caused by the mark-ups in wages and prices of inputs, specifically:

$$\tau^H = \tau^F = 1 - (1+\mu^w)^{-1}(1+\mu^p)^{-1} \tag{1.A4}$$

We now proceed to approximate the period union welfare function $(1.A2)$ about the value that would prevail under flexible prices. This will then allow the central bank to implement monetary policy to drive the economy towards this flexible price equilibrium and offset the distortion to the union economy arising due to staggered price setting conditions of intermediate firms. We start by linearly approximating the first term, the utility of consumption, by a second order expansion:

$$\begin{aligned} U(C_t) &= U(\bar{C}_t) + U'(\bar{C}_t)\bar{C}_t\left[\tilde{c}_t + \frac{1}{2}(1-\sigma)\tilde{c}_t^2\right] + o(\| a \|^3) \qquad (1.A5) \\ &= U(\bar{C}_t) + U'(\bar{C}_t)\bar{C}_t\left[\begin{array}{c} n\tilde{y}_t^H + (1-n)\tilde{y}_t^F + \frac{1}{2}(1-\sigma) \\ \left(\begin{array}{c} n^2\left(\tilde{y}_t^H\right)^2 + (1-n)^2\left(\tilde{y}_t^F\right)^2 \\ +2n(1-n)\tilde{y}_t^H\tilde{y}_t^F \end{array}\right) \end{array}\right] + o(\| a \|^3) \end{aligned}$$

where we have made use of the log-linearized version of the market clearing condition for consumption (1.23) and where $o(\| a \|^n)$ represents terms that are of order higher than the n^{th}, in the bound $\| a \|$ on the amplitude of the relevant shocks. We further linearize $U'(\bar{C}_t)\bar{C}_t$ around the steady state:

$$\begin{aligned} U'(\bar{C}_t)\bar{C}_t &= U'(C)C + C\bar{c}_t\left[U''(C)C + U'(C)\right] + o(\| a \|^2) \\ &= U'(C)\left(1 + (1-\sigma)\bar{c}_t\right) + o(\| a \|^2) \end{aligned}$$

Combining this with $(1.A5)$ above gives:

$$
\begin{aligned}
U(C_t) - U(\bar{C}_t) &= U'(C)C + U'(\bar{C}_t)C \left[\begin{array}{c} (1+(1-\sigma)\,\bar{c}_t)\left(n\overset{\sim H}{y_t} + (1-n)\overset{\sim F}{y_t}\right) \\ +\frac{1}{2}(1-\sigma)\left(\begin{array}{c} n^2\left(\overset{\sim H}{y_t}\right)^2 + (1-n)^2\left(\overset{\sim F}{y_t}\right)^2 \\ +2n(1-n)\overset{\sim H}{y_t}\overset{\sim F}{y_t} \end{array} \right) \end{array} \right] \\
+t.i.p. + o(\ \| \ a \|^3) & \\
&= U'(C)C + U'(\bar{C}_t)C \left[\begin{array}{c} (1+(1+\phi)\,\bar{n}_t^w)\left(n\overset{\sim H}{y_t} + (1-n)\overset{\sim F}{y_t}\right) \\ +\frac{1}{2}(1-\sigma)\left(\begin{array}{c} n^2\left(\overset{\sim H}{y_t}\right)^2 + (1-n)^2\left(\overset{\sim F}{y_t}\right)^2 \\ +2n(1-n)\overset{\sim H}{y_t}\overset{\sim F}{y_t} \end{array} \right) \end{array} \right] \\
+t.i.p. + o(\ \| \ a \|^3) &
\end{aligned}
$$

where $t.i.p.$ contains the terms that are independent of policy. Note that under flexible prices $(1+\phi)\bar{n}_t^W = (1-\sigma)\bar{c}_t$, since in this case the linearized marginal cost of home country gives:

$$\sigma\bar{c}_t + \phi\bar{n}_t^H + (1-n)\bar{T}_t = b_t^H$$

where we are assuming that the optimal subsidy is implemented. Similarly, the marginal cost of foreign country implies:

$$\sigma\bar{c}_t + \phi\bar{n}_t^F - n\bar{T}_t = b_t^F$$

A linear combination of these two with country weights gives:

$$\sigma\bar{c}_t + \phi\bar{n}_t^W = b_t^W$$

Combining (1.13) and (1.29) and linearizing about the flexible price equilibrium gives:

$$\bar{n}_t^H + b_t^H = (1-\lambda^H)\bar{y}_t^H + \lambda^H \bar{y}_t^F \qquad \textit{and} \qquad \bar{n}_t^F + b_t^F = (1-\lambda^F)\bar{y}_t^H + \lambda^F \bar{y}_t^F$$

A linear combination of these two with weights equal to country sizes gives:

$$\bar{n}_t^W + b_t^W = \bar{y}_t^W$$

which can be combined with the linearized version of the equilibrium condition for consumption; $\bar{c}_t = \bar{y}_t^W$, to obtain: $b_t^W = \bar{c}_t - \bar{n}_t^W$.

Finally, we substitute this into: $\sigma\bar{c}_t + \phi\bar{n}_t^W = b_t^W$, and obtain $(1+\phi)\bar{n}_t^W = (1-\sigma)\bar{c}_t$. We now turn to the remaining components of the union welfare function and log-linearize the disutility of households from working about the level resulting under flexible prices. For country H, we have:

$$V(N_{H,t}) = V(\bar{N}_{H,t}) + V'(\bar{N}_{H,t})\bar{N}_t\left(\tilde{n}_t^H + \frac{1}{2}(1+\phi)\left(\tilde{n}_t^H\right)^2\right)V(N_{H,t}) + o(\| a \|^3)$$

where: $\qquad V'(\bar{N}_{H,t})\bar{N}_{H,t} = V'(N)N\left(1 + (1+\phi)\bar{n}_t^H\right) + o(\| a \|^2)$

keeping in mind that $\frac{V''(N)N}{V'(N)} = \phi$. We substitute the latter equation into the former and obtain:

$$V(N_t) - V(\bar{N}_t) \;=\; V'(N)N\left(\tilde{n}^H{}_t + \frac{1}{2}(1+\phi)\left(\tilde{n}_t^H\right)^2 + (1+\phi)\bar{n}_t^H\tilde{n}_t^H\right) \quad (1.A6)$$
$$+t.i.p. + o(\ \|\ \ a\ \|^3)$$

The log-linearized version of the aggregate production function for intermediate goods (1.29) is:

$$\tilde{n}_t^H = \tilde{x}_t^H + v_t^H$$

where $v_t^H \equiv \log\left(n^{-1}\int_0^n \left(\frac{(IP_{H,t}(f))}{(IP_{H,t})}\right)^{-\epsilon} df\right)$ and $IP_{H,t}$ is the price of the input basket produced by home country. We combine this with the first order approximation of the aggregate demand for inputs (1.13) keeping in mind that $y_t^H = \bar{y}_t^H + \tilde{y}_t^H$, hence $\tilde{n}_t^H = (1-\lambda^H)\tilde{y}_t^H + \lambda^H\tilde{y}_t^F + v_t^H$. We make use of the following lemma.

Lemma

Define the cross-sectional dispersion of prices;
$\sigma^2_{IP_{H,t}} = \int_0^n \left(\log(IP_{H,t}(f)) - \log(IP_{H,t})\right)^2 df,$
then up to a second order approximation: $v_t^H \simeq (\epsilon/2)\sigma^2_{IP_{H,t}}$.

Proof

Let; $\hat{IP}_{H,t}(f) = \log(IP_{H,t}(f)) - \log(IP_{H,t})$, then:
$\left(\frac{IP_{H,t}(f)}{IP_{H,t}}\right)^{1-\epsilon} = \exp\left[(1-\epsilon)\hat{IP}_{H,t}(f)\right]$
$= 1 + (1-\epsilon)\hat{IP}_{H,t}(f) + \frac{(1-\epsilon)^2}{2}\hat{IP}_{H,t}(f)^2 + o(\|\ a\ \|^3)$

Furthermore the definition of the price of the input basket implies:

$1 = \left[\left(\frac{1}{n}\right)^{\frac{1}{1-\epsilon}} \int_0^n \left(\frac{IP_{H,t}(f)^{\epsilon-1}}{IP_{H,t}}\right) df\right]^{1-\epsilon}.$

We combine these two results and take expectations by averaging across firms:

$$E_f\left\{\overset{\wedge}{IP}_{H,t}(f)\right\} = \frac{(\epsilon-1)}{2}E_f\left\{\overset{\wedge}{IP}_{H,t}(f)^2\right\} = \frac{(\epsilon-1)}{2}Var_f\left\{\log(IP_{H,t}(f))\right\}.$$

Finally, a second order approximation of $\left(\frac{IP_{H,t}(f)}{IP_{H,t}}\right)^{-\epsilon}$ gives:

$$\left(\frac{IP_{H,t}(f)}{IP_{H,t}}\right)^{1-\epsilon} = 1 - \epsilon\left(\overset{\wedge}{IP}_{H,t}(f)\right) + \frac{\epsilon^2}{2}\overset{\wedge}{IP}_{H,t}(f)^2 + o(\parallel a \parallel^3).$$

Combining the above results implies:

$$n^{-1}\int_0^n \left(\frac{(IP_{H,t}(f))}{(IP_{H,t})}\right)^{-\epsilon} df = 1 + \frac{\epsilon}{2}Var_f\left\{IP_{H,t}(f)\right\}.$$

Taking logs gives our *lemma*: $v_t \simeq (\epsilon/2)\sigma^2_{IP_{H,t}} + o(\parallel a \parallel^3)$ $\square$

Using this lemma we have: $\tilde{n}_t^H = (1-\lambda^H)\tilde{y}_t^H + \lambda^H\tilde{y}_t^F + (\epsilon/2)\sigma^2_{IP_{H,t}}$, further substituting this into $(1.A6)$ gives:

$$\begin{array}{rcl} V(N_t) - V(\bar{N}_t) & = & V\acute{}(N)N\left(\begin{array}{c} (1-\lambda)\tilde{y}_t^H + \lambda\tilde{y}_t^F + \frac{\epsilon}{2}\sigma^2_{IP^H,t} \\ +\frac{1}{2}(1+\phi)\left(\begin{array}{c} (1-\lambda)^2\left(\tilde{y}_t^H\right)^2 + \lambda^2\left(\tilde{y}_t^F\right)^2 \\ +\lambda(1-\lambda)\tilde{y}_t^H\tilde{y}_t^F \end{array}\right) \\ +(1+\phi)\bar{n}_t((1-\lambda)\tilde{y}_t^H + \lambda\tilde{y}_t^F \end{array}\right) \\ +t.i.p. + o(\parallel a \parallel^3) & & \end{array} \quad (1.A7)$$

Where we have used the above lemma to substitute:

$$\tilde{n}_t^H = \bar{n}_t^H\left((1-\lambda^H)\tilde{y}_t^H + \lambda^H\tilde{y}_t^F\right) + o(\parallel a \parallel^3)$$

Similarly to $(1.A7)$, we can approximate the disutility from working in foreign country:

$$V(N_{F,t}) - V(\bar{N}_{F,t}) = V'(N_F)N_F \left(\begin{array}{c} \widetilde{n}_t^F + \frac{1}{2}(1+\phi)\left(\widetilde{n}_t^F\right)^2 \\ +(1+\phi)\bar{n}_t^F \widetilde{n}_t^F) \end{array} \right) + t.i.p. + o(\| a \|^3)$$

We substitute $\widetilde{n}_t^F = (1-\lambda^F)\widetilde{y^H}_t + \lambda^F \widetilde{y}^F_t + \frac{\epsilon}{2}\sigma^2_{IP_{F,t}}$ into the above expression, giving:

$$\begin{aligned} V(N_{F,t}) - V(\bar{N}_{F,t}) &= V'(N_F)N_F \left(\begin{array}{c} (1-\lambda^F)\widetilde{y^H}_t + \lambda^F \widetilde{y}^F_t + \frac{\epsilon}{2}\sigma^2_{IP_{F,t}} \\ +\frac{1}{2}(1+\phi)\left(\begin{array}{c} (1-\lambda^F)^2 \widetilde{y}_t^2 + \left(\lambda^F\right)^2 \left(\widetilde{y}_t^F\right)^2 \\ +\lambda^F(1-\lambda^F)\widetilde{y^H}_t \widetilde{y}_t^F \end{array} \right) \\ +(1+\phi)\bar{n}_t(1-\lambda^F)\widetilde{y}_t^H + \lambda^F \widetilde{y}^F_t \end{array} \right) \\ &+ t.i.p. + o(\| a \|^3) \end{aligned} \quad (1.A8)$$

Noting that $n(1-\lambda^H) + (1-n)\lambda^H = n$ and that $n(1-\lambda^F) + (1-n)\lambda^F = (1-n)$, linearly combining $(1.A7)$ and $(1.A8)$ and substituting this together with $(1.A5)$ into $(1.A2)$, and recalling that in our defined steady state equilibrium $U'(C)C = V'(N)N$, we obtain:

$$
\begin{aligned}
\widetilde{\mathbf{w}}_t^W &= U(\widetilde{C}_t) - nV(\widetilde{N}_{H,t}) - (1-n)V(\widetilde{N}_{F,t}) \\
&= -\frac{\epsilon}{2}\left(n\sigma^2_{IP_{H,t}} + (1-n)\sigma^2_{IP_{F,t}}\right) + \frac{1}{2}(1-\sigma)\left(\begin{array}{c} n^2\left(\widetilde{y}_t^H\right)^2 + (1-n)^2\left(\widetilde{y}_t^F\right)^2 \\ +2n(1-n)\left(\widetilde{y}_t^H\right)^2\left(\widetilde{y}_t^F\right)^2 \end{array}\right) \\
&\quad -\frac{1}{2}(1+\phi)\left(\begin{array}{c} \left(n(1-\lambda^H)^2 + (1-n)(1-\lambda^F)^2\right)\left(\widetilde{y}_t^H\right)^2 \\ +\left(n\left(\lambda^H\right)^2 + (1-n)\left(\lambda^F\right)^2\right)\left(\widetilde{y}_t^F\right)^2 + \\ \left(n\lambda^H(1-\lambda^H) + (1-n)\lambda^F(1-\lambda^F)\right)\widetilde{y}_t^H\widetilde{y}_t^F \end{array}\right)
\end{aligned}
$$

From the above we have:

$$
\widetilde{\mathbf{w}}_t^W = -\frac{\epsilon}{2\delta}\sum_{t=0}^{\infty}\left\{\begin{array}{c} \left(n\pi^2_{H,t} + (1-n)\pi^2_{F,t}\right) + n\alpha^H\left(\widetilde{y}_t^H\right)^2 \\ +(1-n)\alpha^F\left(\widetilde{y}_t^F\right)^2 + \Phi 2n(1-n)\widetilde{y}_t^H\widetilde{y}_t^F \end{array}\right\}
$$

where $\alpha^H \equiv \frac{\delta}{\epsilon}\left[(1+\phi)\left((1-\lambda^H)^2 + (1-n)n\omega^2\right) - n(1-\sigma)\right]$

$\alpha^F \equiv \frac{\delta}{\epsilon}\left[(1+\phi)((1-n)n\omega^2 + \left(\lambda^F\right)^2) - (1-n)(1-\sigma)\right]$

$\Phi \equiv \frac{\delta}{\epsilon}\left[(1+\phi)\omega((1-\lambda^H) + \lambda^F) - (1-\sigma)\right]$

Note that since $n(1-\lambda^H) + (1-n)\lambda^H = n$ and $n(1-\lambda^F) + (1-n)\lambda^F = (1-n)$, we have:

$\pi_t^W \equiv n\pi_t^H + (1-n)\pi_t^F = n\pi_{H,t} + (1-n)\pi_{F,t}$

Furthermore,

$\left(\pi_t^W\right)^2 = (n\pi_{H,t} + (1-n)\pi_{F,t})^2 = n^2\pi^2_{H,t} + (1-n)^2\pi^2_{F,t} + 2n(1-n)\pi_{H,t}\pi_{F,t}$

$= n\pi^2_{H,t} + (1-n)\pi^2_{F,t} - n(1-n)(\pi_{H,t} - \pi_{F,t})^2$

We note that $\pi_{H,t} - \pi_{F,t} = \frac{\pi_t^H - \pi_t^F}{1-\omega}$ from (1.40), where $\frac{\delta\left(\frac{\pi_t^H - \pi_t^F}{1-\omega}\right)}{\delta\omega} = 0$, since $\left(\pi_t^H - \pi_t^F\right)$ is decreasing in ω to the same extent as the denominator. Hence the central bank should react to changes in relative inflation $\pi_t^R \equiv \pi_t^H - \pi_t^F$ only when these are due to changes in the relative inflation of the inputs. As trade in intermediate goods increases, the importance of reacting to changes in relative input inflation diminishes. With no home bias, relative inflation becomes irrelevant since a change in input inflation affects both countries in the same way. We can finally write the central bank loss function as a function of the output gaps and inflation of final goods,

$$E_0\left(L^{CB}\right) = -\frac{\epsilon}{2\delta} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \left(\pi_t^W\right)^2 + n(1-n)(1-\omega)^{-2}\left(\pi_t^R\right)^2 \\ +n\alpha^H \left(\overset{\sim H}{y_t}\right)^2 + (1-n)\alpha^F \left(\overset{\sim F}{y_t}\right)^2 \\ +\Phi 2n(1-n)\overset{\sim H}{y_t}\overset{\sim F}{y_t} \end{array} \right\} \tag{1.43}$$

It is illustrative to consider two special cases:

No trade in inputs ($\omega = 0$)

With no trade we get the result of the cooperative case in Clarida et al. (2002) where the loss function becomes:

$$E_0\left(L^{CB}\right) = -\frac{\epsilon}{2\delta} E_0 \sum_{t=0}^{\infty} \left\{ \begin{array}{c} \left(\pi_t^W\right)^2 + n(1-n)\left(\pi_t^R\right)^2 + n\frac{\gamma}{\epsilon}\left(\overset{\sim H}{y_t}\right)^2 \\ +(1-n)\frac{\gamma_0^F}{\epsilon}\left(\overset{\sim F}{y_t}\right)^2 - 2n(1-n)\frac{\delta(1-\sigma)}{\epsilon}\overset{\sim H}{y_t}\overset{\sim F}{y_t} \end{array} \right\} \tag{1.A9}$$

The other special case is that in which final goods firms of both country face the same technology.

No home bias in trade ($\omega = 1$)

With complete openness in trade the union economy becomes isomorphic to a large closed economy and the resulting loss function is:

$$E_0\left(L^{CB}\right) = -\frac{\epsilon}{2\delta} E_0 \sum_{t=0}^{\infty} \left\{ \left(\pi_t^W\right)^2 + \left(\frac{\delta\left(\phi+\sigma\right)}{\epsilon}\right) \left(\tilde{y}_t^W\right)^2 \right\} \qquad (1.A10)$$

Chapter 2
Learning in a Monetary Union with Home Bias in Production

Chapter Overview

This chapter examines determinacy and E-stability of different interest rate rules in the two-country monetary union model with home bias in input trade, presented in chapter 1.

We consider three different specifications of welfare optimizing discretionary monetary policy rules derived under RE and find that the way in which the interest rate rule is formulated has crucial implications for both determinacy and E-stability. Specifically, a fundamentals based policy rule, in which the interest rate is adjusted to changes in union-wide supply shocks, is found to be indeterminate and unstable under RLS learning. On the other hand, an expectations based policy rule is both determinate and E-stable.

We also look at general Taylor rules in which the interest rate is adjusted to either contemporaneous or expected future values of union inflation and the union output gap. We find that even though the union central bank fails to stabilize shocks to relative variables, the necessary and sufficient conditions for a unique stationary rational expectations equilibrium and RLS learning stability known from the closed economy literature, also hold for our monetary union model.

In following chapters we will consider monetary union models with asymmetries that alter the results found in this chapter.

2.1 Introduction

In *chapter 1*, we presented a two-country monetary union model with trade in both final and intermediate goods. We derived the central bank loss function by aggregating the loss in household utility arising due to prices being sticky à la Calvo (1993), rather than flexible. Under the assumption of rational expectations we then derived optimal discretionary monetary policy, in which the union central bank stabilizes union-wide shocks. However, monetary policy was found not have an impact on relative variables and it can hence not offset relative shocks.

A key assumption used in the derivation of optimal policy in the previous chapter is that agents have *rational expectations* (RE). This implies that they both know the structure of the economy and the values of all of its parameters. However, there are two main issues with the RE assumption that need to be addressed for the analysis in *chapter 1* to be complete.

First, as pointed out by e.g. Clarida et al. (2000), Bernanke & Woodford (1997) and Woodford (1999), even if a policy rule could lead to a welfare optimizing equilibrium, it is not certain that this is the unique stationary equilibrium associated with the given policy rule. If policy is not *determinate* we could then either have excess volatility as agents fail to cooperate towards a specific equilibrium or we could have the economy reaching a less desirable suboptimal equilibrium. To avoid this, the monetary authorities should only implement a welfare optimizing interest rate rule if it guarantees *determinacy*. Consequently, in this chapter we supplement the analysis of the previous chapter by identifying determinacy conditions for the different specifications of the optimal interest rate rule as well as for some more general specifications of a Taylor rule.

Second, as pointed out by e.g. Howitt (1992), interest rate rules derived under the assumption of RE may be unstable if economic agents instead use an *adaptive learning* algorithm to update their beliefs about the economy. Howitt therefore recommended that any monetary policy analysis assuming RE should also be complemented by an investigation of the stability of such policy under the assumption of learning. The E-stability principle states that a *rational expectations equilibrium* (REE) is locally asymtpotically stable under recursive least squares learning if it is *E-stable*. We will hence use the methodology of Evans & Honkapohja (2001) to examine the E-stability of the different monetary policy rules.

Evans & Honkaphoja (2003) use the closed economy model of Clarida et al. (1999) to show that the specification of the optimal interest rate rule derived under RE has implications for both determinacy and learning stability. They find that a fundamentals based rule, in which the central bank adjusts interest rates to exogenous shocks, is neither determinate nor E-stable. However, an interest rate rule that also reacts to expected future changes in inflation and output is found to govern both a unique stationary REE and stability under RLS learning. Evans & Honkaphoja (2003) argue that this result is an argument for the monetary authorities to explicitly consider the expectations of the private sector when setting policy. Bullard & Schaling (2006) show that this result also holds for the open economy by looking at the two-country model of Clarida, Gali & Gertler (2002).[1] In this chapter we find that the result also extends to our monetary union.

The seminal paper by Bullard & Mitra (2002) examines determinacy and E-stability for different specifications of a Taylor rule in the closed economy model of Woodford (1999). They find that the Taylor principle, in which interest rates are increased by more than one-for-one following a rise in inflation so as to increase real interest rates, plays an important role for both determinacy and E-stability of monetary policy.

In recent years, an increasing number of papers have considered learning in open economy models. This differs from the closed economy case in several ways.

For one, the issue of what inflation the central bank should target, that in producer prices or that in consumer prices, now becomes of relevance. Findings by Bullard & Schaling (2006) and Llosa & Tuesta (2008) suggest that the definition of the target inflation rate has implications for E-stability of monetary policy. In our model, as in a closed economy, the average union CPI and PPI are the same and hence this is not an issue. This is generally true for monetary union models.[2]

The interest rate rule in an open economy can also react to the terms of trade or the exchange rate. Bullard & Schaling (2006) and Wang (2006) look at two-country models and find that if the home country pegs its exchange rate to that of the foreign country,

[1] A short coming of Bullard & Schaling (2006) is that they use the supply schedule derived in Clarida et al. (2002) under the assumption of Nash competition in policy, even when they consider optimal policy under monetary policy cooperation.

[2] It would be an interesting exercise to put forward a monetary union model in which average union consumer prices are not equal to average union producer prices and examine how determinacy and E-stability of monetary policy is affected by this.

then the world economy is determinate and E-stable if the monetary policy of the foreign country meets determinacy and E-stability conditions. We draw a parallel between this case and our monetary union model.

Finally, the degree of openness in trade now matters. Bullard & Schaling (2006) interpret the size of home country as being its degree of openness so that when it approaches one it becomes a closed economy, while as it approaches zero in size it becomes a small open economy. They find that the smaller a country is in size, i.e. the more open it is in trade, the more aggressive does monetary policy have to be to guarantee determinacy and RLS learning stability. The findings of Llossa & Tuesta (2008) support this result. In this chapter we show that the level of trade within a monetary union has no effect on the stability conditions for monetary policy. Furthermore, we find that the results from the closed economy literature hold for our monetary union framework, even though relative shocks cannot be stabilized by monetary policy. In following chapters we will look at cases in which this result breaks down.

In the following section we present the environment for our analysis, including an explanation of the methodology for finding the determinacy and E-stability conditions for monetary policy. *Section 2.3* presents the results, while *section 2.4* concludes the chapter.

2.2 The Environment

In this section we set up the environment for our analysis. We start by summarizing the model presented in *chapter 1* in its reduced form and emphasize the key elements needed for the analysis in this chapter. We then present the interest rate rules of the union central bank, that are to be combined with the log-linearized equations of the model to make the complete system. This is followed by an explanation of the methodology used to examine determinacy and E-stability. Finally, for convenience we also re-present the calibration of the underlying parameter values of the model. These are used in following sections as we complement the analytical representation of our results by also depicting them graphically.

2.2.1 The baseline model

The monetary union consists of two countries; home and foreign. Each country has three different types of agents; households, intermediate goods firms and final goods firms. Each type of agent in the monetary union is spread over the unit interval, with $[0, n)$ residing in the home country and $(n, 1]$ residing in the foreign country. That is, the size of the home country is n, while that of the foreign country is $(1 - n)$.

Households across the monetary union have the same preferences and consume goods produced in both countries according to the Cobb-Douglas function:

$$C_t \equiv \frac{(C_{H,t})^n (C_{F,t})^{1-n}}{n^n (1-n)^{1-n}}$$

Furthermore, financial markets are assumed to be complete, implying that all households across the union consume the same amount. Output in the two countries can still vary however, due to fluctuations in the terms of trade, as suggested by the market clearing conditions:

$$Y_{H,t}^{d} = T_t^{1-n} C_t \qquad \textit{and} \quad Y_{F,t}^{d} = T_t^{-n} C_t \tag{2.1}$$

Where the terms of trade has been defined as the ratio of foreign final goods prices over domestic, i.e. $T_t \equiv P_{F,t}/P_{H,t}$.

Intermediate firms use domestically supplied labour to produce inputs under imperfect competition. They also face a fixed probability $(1-\rho)$ of changing there prices in each period as in the Calvo (1983) model. Hence the price rigidities in our framework arise from the sector of intermediate goods.

Final goods firms are perfectly competitive and use baskets of intermediate goods produced in both countries according to the production function:

$$Y_{i,t}^{s} = A_t^i \left((1-\lambda^i)^{\frac{1}{v}} (X_{H,t}^i)^{1-\frac{1}{v}} + (\lambda^i)^{\frac{1}{v}} (X_{F,t}^i)^{1-\frac{1}{v}} \right)^{\frac{v}{v-1}} \tag{2.2}$$

Where $X_{j,t}^i$ is the basket of inputs supplied by intermediate firms in country j and used by final goods firms in country i, for $i, j = H, F$. There is home bias in production so that domestic firms generally use domestically produced inputs more efficiently than they use the input basket produced abroad. Specifically, we have:

$$\lambda^H \equiv (1-n)\,\omega \qquad \textit{and} \qquad \left(1-\lambda^F\right) \equiv n\omega$$

where ω takes a value between zero and one and measures the degree of home bias. The degree of openness in trade of intermediate goods has an inverse relationship with the level of home bias in production. When $\omega = 1$, there is no home bias in production and complete openness in trade and when $\omega = 0$, there is complete home bias and no trade in intermediate goods.

We define union average variables as a linear combination of home and foreign values where each country is given a weight equal to its size:

$X^W \equiv nX^H + (1-n)\,X^F$

Relative variables are defined as the difference between foreign and domestic:

$X^R \equiv X^F - X^H$.

The reduced form of the system is described by the following conditions:[3]

$$\widetilde{y}_t^W = E_t(\widetilde{y}_{t+1}^W) - \sigma^{-1}\left[r_t - E_t(\pi_{t+1}^W) - rr_t\right] \tag{2.6}$$

$$\pi_t^H = \zeta_Y \widetilde{y}_t^W - (1-n)\zeta_T \widetilde{y}_t^R + \beta E_t(\pi_{t+1}^H) + u_t^H \tag{2.3}$$

$$\pi_t^F = \zeta_Y \widetilde{y}_t^W + n\zeta_T \widetilde{y}_t^R + \beta E_t(\pi_{t+1}^F) + u_t^F \tag{2.4}$$

$$\widetilde{T}_t = -\widetilde{y}_t^R \tag{2.5}$$

$$\widetilde{T}_t = \widetilde{T}_{t-1} + \pi_t^R \tag{2.9}$$

Where we have defined $\zeta_Y \equiv \delta\,(\sigma + \phi)$ and $\zeta_T \equiv \delta(1 + \phi(1-\omega))(1-\omega)$ and where β is the household discount factor, σ is the coefficient of relative risk aversion and

[3] Note that we have rearranged the supply schedules in (1.41) by using the fact that for the generic variable X we have: $a_H X^H + a_F X^F = a_W X^W + a_R X^R$, where the coefficients solve $a_W = a_H + a_F$ and $a_R = na_F - (1-n)a_H$.

ϕ measures workers' disutility from working. The sensitivity of inflation to changes in the marginal cost is determined by $\delta \equiv \left[(1-\rho)(1-\beta\rho)\right]/\rho$ and the natural rate of interest measures expected changes to the natural level of union output, i.e. $rr_t \equiv \sigma E_t(\Delta \bar{y}^W_{t+1})$.

In order to obtain analytical results for determinacy and learning stability in following sections, we need to partition our system into two independent subsystems. To do this we first rewrite the two supply schedules (2.3) and (2.4) by using the definitions for union and relative variables and by substituting (2.5). The endogenous variables $\left\{\tilde{y}^W_t, \pi^W_t, \pi^R_t, \tilde{T}_t\right\}$ are then determined by:

$$\tilde{y}^W_t = E_t(\tilde{y}^W_{t+1}) - \sigma^{-1}\left[r_t - E_t(\pi^W_{t+1}) - rr_t\right] \tag{2.6}$$

$$\pi^W_t = \zeta_Y \tilde{y}^W_t + \beta E_t(\pi^W_{t+1}) + u^W_t \tag{2.7}$$

$$\pi^R_t = -\zeta_T \tilde{T}_t + \beta E_t(\pi^R_{t+1}) + u^R_t \tag{2.8}$$

$$\tilde{T}_t = \tilde{T}_{t-1} + \pi^R_t \tag{2.9}$$

Note that there are two independent subsystems; (2.6) and (2.7) determine union average variables $\tilde{y}^W_t$ and π^W_t, while (2.8) and (2.9) determine the relative variables π^R_t and $\tilde{T}_t$. Because of this, monetary policy has no impact on relative variables. This distinguishes our model from a closed economy, and the purpose of this chapter is to examine whether this alters the results from the closed economy literature on learning.

To further simplify our analysis, in what follows we combine the two equations describing the relative system, (2.8) and (2.9), into the following condition for the terms of trade:

$$\widetilde{T}_t = \psi \widetilde{T}_{t-1} + \beta \psi E_t \left(\widetilde{T}_{t+1} \right) + \psi u_t^R \tag{2.10}$$

where $\psi \equiv \frac{1}{1+\beta+\zeta_T}$. We now present the Taylor rules that are to be combined with (2.6), (2.7) and (2.10) when we look at determinacy and E-stability in following sections.

2.2.2 The monetary policy rules

In addition to looking at determinacy and E-stability of the optimal policy rules derived in *chapter 1*, we will also consider some more generally specified interest rate rules. Following Taylor (1993), interest rate feedback rules have received much attention in the literature on monetary policy. Bullard & Mitra (2002) examine determinacy and E-stability of Taylor rules that react to output and inflation. They consider four different specifications; an interest rate rule with contemporaneous data, lagged data, expected contemporaneous data and expected future data. In this chapter, as in Llosa & Tuesta (2008), we focus on Taylor rules with contemporaneous and expected future data. As discussed in the introduction to this thesis, these type of rules are of empirical merit.

The contemporaneous Taylor rule takes the following form:

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \widetilde{y}_t^W \tag{2.11}$$

Where interest rates are increased to contract the economy if either inflation or output rises above its natural level, i.e. $\varphi_\pi, \varphi_Y > 0$. Of course, we do not consider an interest rate rule that includes the terms of trade since interest rates have no effect on the terms of trade, nor on any other relative variables in this framework.[4] We also note that inflation in consumer prices and the union average inflation in producer prices are the same in this framework. This is because the number of households is assumed to be equal to the number of final goods firms in each country. Hence, we conjecture that these two definitions of inflation remain equivalent even with home bias in consumer preferences.

The forward looking Taylor rule is given by:

$$r_t = \varphi_\pi E_t \pi_{t+1}^W + \varphi_Y E_t \widetilde{y}_{t+1}^W \tag{2.12}$$

In *section 2.3.3.* we consider different specifications of the optimal policy rule derived in *chapter 1.*

2.2.3 Methodology

In this section we outline the methodology used throughout this thesis to obtain the determinacy and E-stability conditions for economic policy. We show a method for obtaining these results for partitioned systems that will simplify the analysis significantly and enables us to obtain analytical results.

[4] When price rigidities are asymmetric in the two countries, the system can no longer be partitioned and interest rates do affect relative variables. We will hence consider a Taylor rule including the terms of trade in *chapter 3*, when we relax the assumption of homogeneous price rigidities.

Equilibrium Determinacy

To examine uniqueness of stationary REE, we write the system in the general form:

$$E_t x_{t+1} = B x_t + \Lambda w_t \tag{2.13}$$

Where B and Λ are $n \times n$ matrices of coefficients, x_t is an $n \times 1$ vector of the state variables and w_t is the $n \times 1$ vector of exogenous variables or the shocks in our model. In order to get all variables with the same time index, we have introduced an auxiliary variable in the place for the terms of trade so that $x_t = \left[\widetilde{y}_t^W, \pi_t^W, \pi_t^R, \varkappa_t\right]'$, where we have defined $\varkappa_t \equiv \widetilde{T}_{t-1}$. The terms of trade will be the sole predetermined state variable in all models considered in this thesis, so this methodology is applied throughout. A standard result is that a unique stationary rational expectations equilibrium exists when the number of eigenvalues of B that are outside the unit circle coincides with the number of free state variables. If the number of eigenvalues outside the unit circle is smaller than the number of free state variables, then there are multiple stationary REE or indeterminacy. On the other hand, if the number of eigenvalues outside the unit circle exceeds the number of free state variables, then there is no stationary REE and the system is explosive.

In order to obtain analytical results, we partition our system into two subsystems. Let,

$$x_t = [x_{1,t}, x_{2,t}]', \; w_{2,t} = [w_{1,t}, w_{2,t}]', \; B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}, \text{ and } \Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix}$$

We initially consider the system given by (2.6), (2.7), (2.8) and (2.9) and we then have $x_{1,t} = \left[\widetilde{y}_t^W, \pi_t^W\right]'$ and $x_{2,t} = \left[\pi_t^R, \varkappa_t\right]'$. If either or both of B_{12} and B_{21} are equal to zero, then the eigenvalues of B are those of B_{11} and B_{22}. It follows that for the whole system to be determinate, we need the number of eigenvalues of B_{11} and B_{22} that are outside the unit circle to correspond to the number of free variables in their respective subsystems. That is to say, we need each of the two subsystems to be determinate for the whole system to be determinate. Furthermore, since (2.8) and (2.9) can be combined into (2.10), it follows from this that determinacy of B_{22} is satisfied when the univariate condition for the terms of trade (2.10) induces determinacy. Evans & McGough (2005) consider a univariate difference equation of the general form:

$$y_t = \alpha E_t\left(y_{t+1}\right) + \delta y_{t-1} + v_t \tag{2.14}$$

where y_t is the endogenous variable and v_t the exogenous variable. They show that when $|\alpha + \delta| < 1$ the system is determinate, but if $|\alpha + \delta| > 1$ it is either indeterminate and has multiple stationary solutions when $|\delta| < |\alpha|$ or it has no stationary solution and is explosive when $|\delta| > |\alpha|$.[5]

Learning Stability[6]

Rather than assuming that agents are fully rational and that they know the values of the economic parameters, we assume that they know the general structure of the economy

[5] See *figure 1* of Evans & McGough (2005),

[6] This section closely follows Evans & Honkapohja (2001, Ch. 10.3)

but update their beliefs about the parameter values in each period using recursive least squares. Hence, the agents of the economy act like econometricians using available data to run regressions, i.e. they are *learning* about the economy as more data becomes available. We assume that the monetary authorities update their beliefs about the economy in the same way as private agents, so that the expectations entering the Taylor rules are the same as those of the private sector.[7] In this thesis, we use the concept of E-stability to derive conditions for local stability of REE under recursive least squares learning. In particular, we focus on the E-stability of minimal state variable (MSV) solutions. Our methodology follows that of Evans & Honkapohja (2001, Ch. 10.3).

We proceed by writing the economic system in the general form:[8]

$$x_t = \Gamma + \Omega E_t x_{t+1} + \Phi x_{t-1} + \Theta w_t \tag{2.15}$$

$$w_t = \varrho w_{t-1} + \varepsilon_t \tag{2.16}$$

Where Γ is the $n \times 1$ vector of constants, Ω, Φ, Θ and ϱ are $n \times n$ matrices of coefficients, x_t is an $n \times 1$ vector of the state variables and w_t is the $n \times 1$ vector of exogenous variables. We assume that w_t follows a stationary VAR, so that ϱ has all of its eigenvalues inside the unit circle, while ε_t is an $n \times 1$ vector of white noise. Agents are assumed to have contemporaneous data in their information set so that $E_t x_{t+1}$ is formed as a linear function of $(1, x_t', w_t')'$. Following McCallum (1983), this implies an MSV solution

[7] Honkapohja & Mitra (2005) consider the case in which the central bank and private agents have heterogeneous forecasting and show that this could be an additional source of instability under adaptive learning.

[8] In line with the methodology of Bullard & Mitra (2002), we initially include the vector of constants Γ and write (2.15) in a general form, eventhough this is zero for the models considered in this thesis. We note that even when Γ is equal to zero, agents do not necessarily know this under learning.

of the form:

$$x_t = a + bx_{t-1} + cw_t \tag{2.17}$$

Where the $n \times 1$ vector a, and the two $n \times n$ matrices b and c are all determined by the method of undetermined coefficients. We will refer to (2.17) as the perceived law of motion (PLM). Taking expectations of equation (2.17) and iterating forward yields:

$$E_t x_{t+1} = a + bx_t + c\varrho w_t \tag{2.18}$$

substituting this into equation (2.15) and rearranging then gives the actual law of motion (ALM):

$$x_t = (I - \Omega b)^{-1}(\Gamma + \Omega a) + (I - \Omega b)^{-1}\Phi x_{t-1} + (I - \Omega b)^{-1}(\Theta + \Omega c\varrho)w_t \tag{2.19}$$

Under rational expectations, agents know the true parameter values of the economy so that the PLM and the ALM coincide. This implies that a, b and c must be such that the following conditions are satisfied:

$$(I - \Omega\bar{b} - \Omega)\bar{a} = \Gamma \tag{2.20}$$

$$\Omega\bar{b}^2 - \bar{b} + \Phi = 0 \tag{2.21}$$

$$(I - \Omega\bar{b})\bar{c} - \Omega\bar{c}\varrho = \Theta \tag{2.22}$$

However, under RLS learning these equations do not generally hold and $\xi' = (a, b, c)$ changes over time. Indeed, for real time RLS learning we have $\xi_t' = (a_t, b_t, c_t)$ being updated according to:

$$\xi_t = \xi_{t-1} + t^{-1}R_t^{-1}z_{t-1}\epsilon_t'$$
$$R_t = R_{t-1} + t^{-1}\left(z_{t-1}z_{t-1}' - R_{t-1}\right)$$

where $z_t' = \left(1, x_{t-1}', w_t'\right)$ and $\epsilon_t = y_{t-1} - \xi_{t-1}'z_{t-1}$. We are interested in the circumstances under which the estimates $\xi_t' = (a_t, b_t, c_t)$ converge locally asymptotically to the stationary MSV solution $\bar{\xi}' = \left(\bar{a}, \bar{b}, \bar{c}\right)$ that satisfies $(2.20) - (2.22)$ and has all roots of $\bar{b}$ inside the unit circle. *Proposition 10.4* of Evans & Honkapohja (2001) states that this is the case when the MSV solution $\bar{\xi}'$ is E-stable. Hence in this thesis we derive E-stability conditions for economic policy to examine local asymptotic stability of REE under RLS learning. To examine E-stability we consider the mapping from the PLM (2.17) to the ALM (2.19) as:

$$T(a,b,c) = \left((I - \Omega b)^{-1}(\Gamma + \Omega a), (I - \Omega b)^{-1}\Phi, (I - \Omega b)^{-1}(\Theta + \Omega c \varrho)\right) \tag{2.23}$$

E-stability is then determined by the following matrix differential equation in notional time (τ):

$$\frac{d}{d\tau}(a,b,c) = T(a,b,c) - (a,b,c) \tag{2.24}$$

An MSV solution $\left(\bar{a}, \bar{b}, \bar{c}\right)$ is then E-stable if the system (2.24) is locally asymptotically stable at this solution. To analyze local stability of (2.24) at the RE solution $\left(\bar{a}, \bar{b}, \bar{c}\right)$ satisfying equations $(2.20) - (2.22)$, we linearize the system of differential equations at this

REE. Using the rules for vectorization of matrix products, we have:

$$DT_a(\bar{a}, \bar{b}) = (I - \Omega\bar{b})^{-1}\Omega \tag{2.25}$$

$$DT_b(\bar{b}) = \left[(I - \Omega\bar{b})^{-1}\Phi\right]' \otimes \left[(I - \Omega\bar{b})^{-1}\Omega\right] \tag{2.26}$$

$$DT_c(\bar{b}, \bar{c}) = \varrho' \otimes \left[(I - \Omega\bar{b})^{-1}\Omega\right] \tag{2.27}$$

Proposition 10.3 of Evans & Honkapohja (2001) states that an MSV solution $\left(\bar{a}, \bar{b}, \bar{c}\right)$ is E-stable if all eigenvalues of the matrices $DT_a(\bar{a}, \bar{b})$, $DT_b(\bar{b})$ and $DT_c(\bar{b}, \bar{c})$ have real parts less than 1. If any eigenvalue has a real part larger than 1, then the solution is not E-stable.

Let us define; $A \equiv \left[(I - \Omega\bar{b})^{-1}\Phi\right]$ and $F \equiv \left[(I - \Omega\bar{b})^{-1}\Omega\right]$. The eigenvalues of Kronecker products are the products of the eigenvalues of the multiplied matrices.[9] This implies that the eigenvalues of F and the cross products of these eigenvalues with those of A and ϱ need to have real parts smaller than one for E-stability. We assume that the exogenous variables are uncorrelated so that ϱ is diagonal. Furthermore, given that it is already assumed to be stable and assuming that all its entries are positive, it follows that ϱ has all of its eigenvalues with positive real parts less than one. Therefore, the necessary and sufficient condition for E-stability is that the eigenvalues of F, and the cross products of these eigenvalues with those of A, all have real parts less than one.

Our methodology here is similar to that of Wang (2006). However, Wang (2006) argues that E-stability is assured when both A and F have eigenvalues with real parts less

9 See Magnus and Neudecker (1998).

than one, but this is not necessarily true. It could for instance be the case that A and F have large negative eigenvalues. By themselves, they seem to induce E-stability, but when multiplied they become positive and larger than one. It could also be that some eigenvalues of A are larger than one. This could still imply E-stability if the eigenvalues of F are negative or sufficiently small.

As for the case of determinacy, we need to partition our system in order to obtain analytical results. Noting that we do not have constants in our system, we have:

$$\begin{aligned} \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} &= \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \begin{bmatrix} E_t x_{1,t+1} \\ E_t x_{2,t+1} \end{bmatrix} + \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} \\ &+ \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} \end{aligned} \tag{2.28}$$

Where $x_{1,t} = \left[\tilde{y}_t^W, \pi_t^W \right]'$ and $x_{2,t} = \widetilde{T}_t$. For the cases we consider in this thesis in which the economic system can be partitioned, it will be either block-diagonal or block-triangular. That is, we have either $(\Omega_{21}, \Phi_{21}, \Theta_{21})$, or $(\Omega_{12}, \Phi_{12}, \Theta_{12})$, or both being equal to zero. Assume without loss of generality that $(\Omega_{21}, \Phi_{21}, \Theta_{21})$ is equal to zero. This yields:

$$\begin{aligned} x_{1,t} &= \Omega_{12} E_t x_{2,t+1} + \Omega_{11} E_t x_{1,t+1} + \Phi_{12} x_{2,t-1} \\ &+ \Phi_{11} x_{1,t-1} + \Theta_{12} w_{2,t} + \Theta_{11} w_{1,t} \end{aligned} \tag{2.29}$$

$$x_{2,t} = \Omega_{22} E_t x_{2,t+1} + \Phi_{22} x_{2,t-1} + \Theta_{22} w_{2,t} \tag{2.30}$$

Where the second subsystem (2.30) is independent of the first. The MSV solution of this partitioned system is:

$$\begin{aligned}\begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} &= \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} \\ &\quad + \begin{bmatrix} c_{11} & c_{21} \\ 0 & c_{22} \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix}\end{aligned} \tag{2.31}$$

To find the rational expectations solution we then use the partitioned version of $(2.20) - (2.22)$. For $\bar{b}$ this implies:

$$\Omega \bar{b}^2 - \bar{b} + \Phi = 0$$

$$\Rightarrow \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} \bar{b}_{11} & \bar{b}_{12} \\ 0 & \bar{b}_{22} \end{bmatrix}^2 - \begin{bmatrix} \bar{b}_{11} & \bar{b}_{12} \\ 0 & \bar{b}_{22} \end{bmatrix} + \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ 0 & \Phi_{22} \end{bmatrix} = 0$$

which gives the following equations solving for $\bar{b}$:

$$\Omega \bar{b}_{11}^2 - \bar{b}_{11} + \Phi_{11} = 0 \tag{2.32}$$

$$\Omega_{12}^2 \bar{b}_{22} + \Omega_{11} \bar{b}_{12} \bar{b}_{22} + \Omega_{11} \bar{b}_{12} \bar{b}_{22} - \bar{b}_{12} + \Phi_{12} = 0 \tag{2.33}$$

$$\Omega_{22} \bar{b}_{22}^2 - \bar{b}_{22} + \Phi_{22} = 0 \tag{2.34}$$

For a we have $(I - \Omega\bar{b} - \Omega)\bar{a} = \Gamma$ giving the solution $\bar{a} = 0$ in all cases since Γ equals zero. Finally, for c we have $(I - \Omega\bar{b})\bar{c} - \Omega\bar{c}\varrho = \Theta$ or equivalently:

$$\left[\begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix} - \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} \bar{b}_{11} & \bar{b}_{12} \\ 0 & \bar{b}_{22} \end{bmatrix} \right] \begin{bmatrix} \bar{c}_{11} & \bar{c}_{12} \\ 0 & \bar{c}_{22} \end{bmatrix}$$

$$- \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} \begin{bmatrix} \varrho_{11} & \varrho_{12} \\ 0 & \varrho_{22} \end{bmatrix} = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ 0 & \Theta_{22} \end{bmatrix}$$

The relevant matrices for E-stability become:

$$
\begin{aligned}
A &\equiv \left[(I-\Omega\bar{b})^{-1}\Phi\right] \\
&= \left[\begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix} - \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{bmatrix}\right]^{-1}\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \\
&= \begin{bmatrix} (I_1-\Omega_{11}b_{11})^{-1}\Omega_{11} & Z\Omega_{22}+(I_1-\Omega_{11}b_{11})^{-1}\Omega_{12} \\ 0 & (I_2-\Omega_{22}b_{22})^{-1}\Omega_{22} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}
\end{aligned}
$$

and

$$
\begin{aligned}
F &\equiv \left[(I-\Omega\bar{b})^{-1}\Omega\right] \\
&= \begin{bmatrix} (I_1-\Omega_{11}b_{11})^{-1}\Phi_{11} & Z\Phi_{22}+(I_1-\Omega_{11}b_{11})^{-1}\Phi_{12} \\ 0 & (I_2-\Omega_{22}b_{22})^{-1}\Phi_{22} \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ 0 & F_{22} \end{bmatrix}
\end{aligned}
$$

where $Z \equiv -(I_2-\Omega_{22}b_{22})^{-1}(\Omega_{21}b_{11}+\Omega_{22}b_{22})(I_1-\Omega_{11}b_{11})^{-1}$, using block-wise inversion as suggested by Banachiewicz (1937). The eigenvalues of A are the eigenvalues of A_{11} and A_{22}, while the eigenvalues of F are those of F_{11} and F_{22}. Therefore, for E-stability we need the eigenvalues of F_{11} and F_{22}, and the cross products of these with the eigenvalues of A_{11} and A_{22}, to all have real parts less than one.

2.2.4 Calibration

We use the calibration outlined in *chapter 1*, which is summarized here for convenience.[10] The intertemporal rate of substitution β is set to 0.99. The coefficient of relative risk aversion σ is set to a conventional value of $1/6$ and that of labour elasticity ϕ is assumed to be 0.67. We assume that price contracts on average last for 4 quarters so that $\rho = \frac{3}{4}$. This implies an output gap coefficient of $\zeta_Y = 7.\,181\,1 \times 10^{-2}$. Finally, without loss of generality, we let both countries be of equal size, i.e. $n = 0.5$.

[10] See page 55 of *section 1.3* for further motivation for these values.

2.3 The Results

In this section we present the determinacy and E-stability conditions for monetary policy. We first look at the contemporaneous Taylor rule (2.11), we then examine the forward looking rule (2.12) before considering stability of optimal policy in *section 2.3.3*. In each case, we proceed by first examining determinacy and then E-stability.

2.3.1 The contemporaneous Taylor rule

The system is now given by the IS schedule (2.6), the NKPC (2.7), the Taylor rule (2.11) and the condition for the terms of trade (2.10). We start by looking at determinacy.

Equilibrium Determinacy

For determinacy of the relative subsystem we require (2.10) to induce a unique equilibrium for the terms of trade. Writing the union average subsystem in the form of (2.13) gives:

$$B_{11} = \begin{bmatrix} \sigma^{-1}\left(\varphi_Y + \beta^{-1}\zeta_Y\right) + 1 & \sigma^{-1}\left(\varphi_\pi - \beta^{-1}\right) \\ -\beta^{-1}\zeta_Y & \beta^{-1} \end{bmatrix}$$

For a unique stationary REE we then need B_{11} to have both of its eigenvalues outside of the unit circle, in addition to (2.10) being determinate. This implies the following result.

Proposition 1 *The necessary and sufficient condition for determinacy of the contemporaneous Taylor rule;*

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \tilde{y}_t^W \tag{2.11}$$

is that the following Taylor principle condition holds:

$$\zeta_Y \left(\varphi_\pi - 1\right) + (1-\beta)\varphi_Y > 0 \tag{2.35}$$

Proof. *See appendix 2A.* ■

So the familiar result from the closed economy literature also holds for our monetary union model. Clarida et al. (2000) present evidence that monetary policy by the Federal Reserve did not satisfy the Taylor principle in the pre-Volcker period (pre-October 1979) and attribute excess economic volatility to this fact.

Interestingly, neither the relative size of the two member countries (n), nor the level of trade in inputs (ω) has an effect on the condition for determinacy. Our results thus differ from those of Bullard & Schaling (2006) and Llosa & Tuesta (2008), who find that a higher level of trade reduces the determinacy region of monetary policy. The determinate region is depicted in *figure 2.1* for different values of φ_Y and φ_π.

Learning Stability

We now relax the assumption of RE and consider the local stability of monetary policy under RLS learning. To examine E-stability we use the partitioned version of (2.28). We emphasize that unlike for determinacy, E-stability of each subsystem does not necessarily guarantee E-stability of the whole system. The derivation of the following result is given in *appendix 2B*.

Proposition 2 *When monetary policy follows the contemporaneous Taylor rule;*

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \tilde{y}_t^W \tag{2.11}$$

then the necessary and sufficient condition for E-stability is that the Taylor principle holds:

$$\zeta_Y (\varphi_\pi - 1) + (1 - \beta)\varphi_Y > 0 \tag{2.35}$$

Proof. *See appendix 2B.* ■

As for the closed economy case considered by Bullard & Mitra (2002), the Taylor principle is the necessary condition for both determinacy and E-stability. McCallum (2007) shows that when contemporaneous data is in the information set of economic agents, then determinacy is sufficient but not necessary for E-stability. Our framework falls into this category and for a contemporaneous interest rate rule all unique stationary REE are learnable under RLS, while no multiple stationary equilibria are E-stable. Although there could be cases where multiple equilibria are learnable under RLS, this is normally not the case for contemporaneous Taylor rules. The E-stability region is plotted in *figure 2.1*.

2.3.2 The forward looking Taylor rule

In this section we consider the case in which the monetary authorities adjust interest rates to expected future levels of the output gap and inflation. The system is now represented by the IS schedule (2.6), the NKPC (2.7), the condition for the terms of trade (2.10) and the Taylor rule (2.12). We first proceed by examining determinacy and then E-stability.

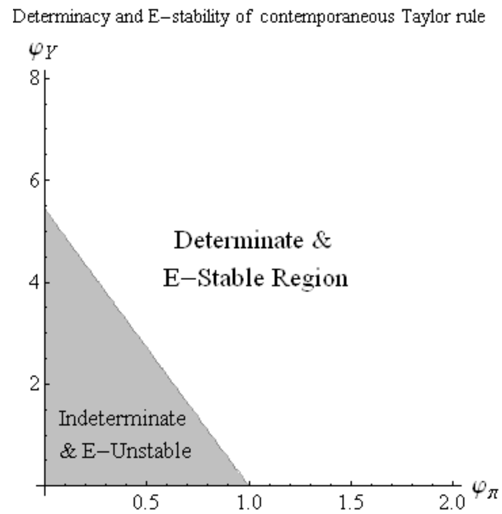

Fig. 2.1. The above graph shows the Taylor principle as the necessary and sufficient condition for determinacy and E-stability of a contemporaneous interest rate rule.

Equilibrium Determinacy

As above, determinacy of the union economy is determined by the determinacy of the two partitioned systems. While the determinacy of (2.10) is independent of policy, combining (2.6), (2.7) and (2.12) and writing this in the form of (2.13) implies the following key matrix:

$$B_{11} = \begin{bmatrix} \frac{1+\beta^{-1}\zeta_Y\sigma^{-1}(1-\varphi_\pi)}{(1-\sigma^{-1}\varphi_Y)} & -\frac{\sigma^{-1}(1-\varphi_\pi)\beta^{-1}}{(1-\sigma^{-1}\varphi_Y)} \\ -\beta^{-1}\zeta_Y & \beta^{-1} \end{bmatrix}$$

For a unique stationary rational expectations equilibrium we require B_{11} to have both of its eigenvalues outside of the unit circle, since $\tilde{y}_t^W$ and π_t^W are both free variables. We have the following result.

Proposition 3 *The necessary and sufficient conditions for determinacy when the union central bank employes a forward looking interest rate rule of the form:*

$$r_t = \varphi_\pi E_t\left(\pi^W_{t+1}\right) + \varphi_Y E_t\left(\overset{\sim W}{y}_{t+1}\right) \tag{2.12}$$

is that the following two conditions are met:

$$\zeta_Y\left(\varphi_\pi - 1\right) + (1-\beta)\varphi_Y > 0 \tag{2.35}$$

and

$$2\sigma(1+\beta) > \zeta_Y\left(\varphi_\pi - 1\right) + \varphi_Y(1+\beta) \tag{2.36}$$

Proof. *See appendix 2A.* ■

The Taylor principle is still a necessary condition for determinacy, but it is not sufficient. An overly aggressive policy, in particular to expected values of output, can induce multiple equilibria even if the Taylor principle holds. It has been suggested that a low weight should be given to output due to frequent revisions to GDP data. Here we give another reason for not reacting aggressively to changes in output.

Our results replicate those of the closed economy presented by Bullard & Mitra (2002).[11] Similarly, Llosa & Tuesta (2008) find the same determinacy conditions for the open economy when the central bank targets domestic price inflation. In their model, parameters are affected by open economy factors however. Furthermore, they find an additional constraint for the coefficient on inflation (φ_π), when the central bank targets inflation in consumer prices. The determinacy conditions are plotted in *figure 2.2*.

[11] Note that although Bullard & Mitra (2002) have a third determinacy condition, this is redundant as it is satisfied when (2.35) and (2.36) both hold.

Learning Stability

To examine E-stability of REE, we write the system in the form (2.28). The following result is obtained.

Proposition 4 *For the contemporaneous Taylor rule;*

$$r_t = \varphi_\pi E_t \pi^W_{t+1} + \varphi_Y E_t \tilde{y}^W_{t+1} \tag{2.12}$$

the necessary and sufficient condition for E-stability of an MSV solution is that the Taylor principle holds:

$$\zeta_Y \left(\varphi_\pi - 1\right) + (1-\beta)\varphi_Y > 0 \tag{2.35}$$

Proof. See *appendix 2B*. ■

So, the Taylor principle is both sufficient and necessary for E-stability of a forward looking Taylor rule. This means that in the case where monetary policy reacts aggressively, in particular to the output gap, we could have two equilibria that are both learnable. The E-stable region is depicted below in *figure 2.2*.

2.3.3 Optimal discretionary monetary policy

In *section 1.4.1* of the previous chapter, we derived optimal discretionary monetary policy of the union central bank and presented three alternative specifications of the welfare maximizing interest rate rule. Here, we revisit each of these three in turn to examine determinacy and E-stability.[12]

[12] See *section 1.4.1* for details on the derivation of these rules.

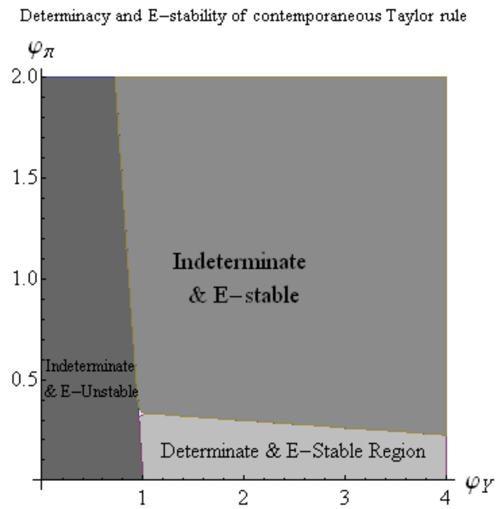

Fig. 2.2. The above figure plots the regions for determinacy and E-stability of a forward looking Taylor rule. There is now the possibility of multiple stable REE.

Consider first our version of what Evans & Honkapohja (2003) refer to as the fundamentals based policy rule:

$$r_t = rr_t + \left[\frac{\varrho + (1-\varrho)\sigma\epsilon}{1 - \beta\varrho + \epsilon\zeta_Y} \right] u_t^W \tag{2.37}$$

This type of feedback rule, that reacts only to exogenous shocks, has been known to induce multiple equilibria. Evans & Honkapohja (2003) show that this type of policy rule is also unstable under learning. This result also holds for an open economy, as shown by Bullard & Schaling (2006). Here we find that it extends to our monetary union framework as well.

Proposition 5 *A fundamentals based optimal policy rule of the form:*

$$r_t = rr_t + \left[\frac{\varrho + (1-\varrho)\sigma\epsilon}{1 - \beta\varrho + \epsilon\zeta_Y} \right] u_t^W \tag{2.37}$$

is both indeterminate and E-unstable in the monetary union framework considered here.

Proof. See *appendix 2C.* ■

The policy rule (2.37) was derived under the assumption of RE. When instead allowing the expectations of private agents to enter explicitly in the policy rule, it becomes both determinate and locally asymptotically stable under RLS learning as stated in *proposition 6* below. Evans & Honkapohja (2003) thus argue that private sector expectations should enter the monetary policy rule explicitly to take into consideration that economic agents could use some learning algorithm to update their beliefs about the economy. Central banks have in practice shown to take expectations into consideration when forming policy. For example, Jean-Claude Trichet, the president of the European Central Bank has stated that "the ECB also monitors a range of indicators that provide more direct evidence on inflationary expectations in the euro area."[13]

Proposition 6 *An expectations based optimal policy rule of the form:*

$$r_t = rr_t + \sigma E_t\left(\tilde{y}_{t+1}^W\right) + \left[1 + \frac{\beta\sigma}{(\zeta_Y + \epsilon)}\right] E_t\left(\pi_{t+1}^W\right) + \left[\frac{\sigma}{\zeta_Y + \epsilon}\right] u_t^W \qquad (2.38)$$

induces a unique stationary REE that is also E-stable.

Proof. See *appendix 2C.* ■

Preston (2008) shows that when economic agents form their decisions based on forecasts into the indefinite future rather than one period ahead, a policy rule in the form of

[13] Speech by Jean-Claude Trichet, 25 February (2005), *Monetary policy and private expectations*, Zolotas lecture at the Bank of Greece, www.ecb.int/press.

(2.38) is not necessarily stable under learning. He finds that in general, a price targeting policy rule is more likely to induce learning stability than an inflation targeting rule. It is argued that price level anchoring better restrains private sector expectations and thus more likely prevents self-fulfilling expectations.

Finally, we consider a policy rule in which interest rates are adjusted only to changes in expected inflation.

Proposition 7 *An inflation targeting policy rule of the form:*

$$r_t = rr_t + \left[1 + \frac{(1-\varrho)\,\sigma\epsilon}{\varrho}\right] E_t\left(\pi_{t+1}^W\right) \tag{2.39}$$

is always E-stable, but not always determinate.

Proof. See *appendix 2C.* ■

Hence, for our monetary union framework, the interest rate rule in the form of (2.38) is the more desirable of the three specifications.

The determinacy and E-stability results of the different specifications of the policy rule; (2.37), (2.38) and (2.39) support the results presented by Evans & Honkapohja (2003) and Bullard & Schaling (2006).

2.4 Conclusion

We have considered determinacy and expectational stability of monetary policy in a two-country monetary union model with trade in inputs. It has been shown that the results known from the closed economy literature hold for our framework and that the presence of relative or country-specific shocks has no impact on this result. Furthermore, as opposed to open economy models, the level of intra-union trade or relative country sizes has no effect on the regions for determinacy and E-stability.

In accordance with Bullard & Mitra (2002) we find that the Taylor principle is both necessary and sufficient for contemporaneous or forward looking interest rate rules to induce E-stability of MSV solutions. While it also guarantees determinacy of a contemporaneous Taylor rule, a very aggressive forward looking policy rule could lead to multiple stationary REE.

We have also concluded that as in the case considered by Evans & Honkapohja (2003), a fundamentals based optimal policy rule is both indeterminate and E-unstable, while an expectations based rule guarantees a unique stationary REE that is locally asymptotically stable under RLS learning.

Our finding suggests that the monetary authorities should consider the monetary union as a large closed economy when setting policy. However, we will show in the following chapters that this result does not hold when we introduce monetary union frameworks with additional asymmetries.

2.A Appendix: *Proof of determinacy results*

In this appendix we derive the determinacy conditions stated in the above propositions. For determinacy we need each of the two subsystems in our model to induce a unique stationary REE. While the first subsystem is given by (2.6), (2.7) and the relevant interest rate rule, the relative subsystem is determined by (2.10). Following the determinacy conditions for (2.14) of *section 2.2.3*, we have that (2.10) is determinate when:

$\left|\frac{1+\beta}{1+\beta+\zeta_T}\right| < 1$

which always holds since $\zeta_T = \delta(1+\phi(1-\omega))(1-\omega) > 0$ for $0 < \omega < 1$.[14] Hence for determinacy of REE in the union economy, we need (2.6), (2.7) and the relevant Taylor rule to guarantee determinacy of the first subsystem. Since this forms a 2×2 system, the characteristic polynomials will be of the form:

$$p(\lambda) = \lambda^2 + a_1\lambda + a_0 \tag{2.A1}$$

For determinacy we then need both of the solutions for λ to be outside the unit circle. This occurs when:[15]

$|a_0| > 1$ and $|a_1| < |1 + a_0|$. We use this result below to derive the determinacy results for the different monetary policy rules.

---

[14] Note that when $\omega = 1$, inflation in both countries is the same and there is no relative subsystem.

[15] See LaSalle (1986, p.28)

2.A.1 Proof of proposition 1

Combining (2.6), (2.7) and (2.11) and writing the system in the form of (2.13) yields:

$$B_{11} = \begin{bmatrix} \sigma^{-1}\left(\varphi_Y + \beta^{-1}\zeta_Y\right) + 1 & \sigma^{-1}\left(\varphi_\pi - \beta^{-1}\right) \\ -\beta^{-1}\zeta_Y & \beta^{-1} \end{bmatrix}$$

Writing the eigenvalues of B_{11} in the form of $(2.A1)$ then solves:

$$a_0 = \beta^{-1}\sigma^{-1}(\varphi_Y + \zeta_Y\varphi_\pi) + \beta^{-1}$$

and

$$a_1 = -\sigma^{-1}\left(\varphi_Y + \beta^{-1}\zeta_Y\right) - 1 - \beta^{-1}$$

Hence we have:

$|a_0| > 1 \Longrightarrow \sigma^{-1}(\varphi_Y + k_Y\varphi_\pi) > -(1-\beta)$, which is satisfied trivially, while:

$|a_1| < |1 + a_0| \Longrightarrow$

$$\zeta_Y\left(\varphi_\pi - 1\right) + (1-\beta)\varphi_Y > 0 \tag{2.35}$$

Proof of Proposition 3

For the system given by (2.6), (2.7), (2.10) and (2.12), determinacy requires the following matrix to have its eigenvalues inside the unit circle:

$$B_{11} = \begin{bmatrix} \frac{1+\beta^{-1}\zeta_Y\sigma^{-1}(1-\varphi_\pi)}{(1-\sigma^{-1}\varphi_Y)} & -\frac{\sigma^{-1}(1-\varphi_\pi)\beta^{-1}}{(1-\sigma^{-1}\varphi_Y)} \\ -\beta^{-1}\zeta_Y & \beta^{-1} \end{bmatrix}$$

This matrix is obtained by substituting (2.12) into (2.6) and combining this with (2.7) in the form of (2.13). While B_{11} determines the system for union average variables, (2.10) determines that for relative variables. As shown above, (2.10) is independent of policy and is always determinate. Hence, the determinacy of the monetary union depends on B_{11}. As in the *proof of proposition 1* we let the characteristic polynomials be in the form of $(2.A1)$, where:

$$a_0 = \beta^{-1}(1-\sigma^{-1}\varphi_Y)^{-1}$$

$$a_1 = -\left(1+\beta^{-1}\zeta_Y\sigma^{-1}\left(1-\varphi_\pi\right)\right)\left(1-\sigma^{-1}\varphi_Y\right)^{-1} - \beta^{-1}$$

For determinacy we need; $|a_0| > 1$ and $|a_1| < |1 + a_0|$. The first condition $|a_0| > 1$ gives the following constraint for when a_0 is negative:

$$\varphi_Y < \sigma\left(\beta^{-1}+1\right) \tag{2.A2}$$

The condition $|a_1| < |1 + a_0|$ implies:

$$2\sigma(1+\beta) > \zeta_Y\left(\varphi_\pi - 1\right) + \varphi_Y(1+\beta) \tag{2.36}$$

if $(a_0 + 1)$ and a_1 are both negative or positive, and:

$$0 < (1-\beta)\,\varphi_Y + \zeta_Y\left(\varphi_\pi - 1\right) \tag{2.35}$$

in the case where $(a_0 + 1)$ is positive and a_1 negative, or vice versa. Rearranging (2.35) gives: $-(1-\beta)\,\varphi_Y < \zeta_Y\left(\varphi_\pi - 1\right)$, substituting this into (2.36) and rearranging yields: $\varphi_Y < \sigma\left(\beta^{-1}+1\right)$. That is, combining (2.35) and (2.36) gives $(2.A2)$, making it redundant. The necessary and sufficient conditions for determinacy are then (2.35) and (2.36).

2.B Appendix: *Proof of E-stability results*

In this appendix we derive the E-stability results of the above propositions, using the methodology outlined in *section 2.2.3*. In this chapter, F and A are block-diagonal and F_{11} and A_{11} are determined by (2.6), (2.7) and the interest rate rule, while F_{22} and A_{22} are determined by (2.10). F_{22} and A_{22} are independent of policy and we will derive these for the generic case here. Solving (2.34) using (2.10) yields:

$b_{22} = \frac{1 \mp \sqrt{1-4\beta\psi}}{2\psi\beta}$ where; $\psi \equiv \frac{1}{1+\beta+\zeta_T}$

We note that $(1-4\beta\psi)$ is between zero and one and that the negative conjugate of b_{22} is always stable, while the positive conjugate never is. For the negative conjugate we have:

$$F_{22} = \frac{2\beta\psi}{(1+\sqrt{1-4\beta\psi^2})} \qquad \text{and} \qquad A_{22} = \frac{2\psi}{(1+\sqrt{1-4\beta\psi^2})}$$

It suffices to show that A_{22} is less than one, since $F_{22} = \beta A_{22}$ and $0 < \beta < 1$.

$$A_{22} = \frac{2\psi}{(1+\sqrt{1-4\beta\psi^2})} < 1$$

$$(2\psi - 1)^2 < 1 - 4\beta\psi^2$$

$$0 < 1 + 2\psi\left(1 - (1+\beta)\,\psi\right)$$

Since $1 - 2\,(1+\beta)\,\psi > 0$

It hence follows that F_{22} and the product of F_{22} and A_{22} are both less than one. Furthermore, since A_{22} is between zero and one, it follows that the eigenvalues of $F_{11}A_{22}$ have real parts less than one if those of F_{11} have real parts less than one. Consequently, for

E-stability of monetary policy, we need the interest rate rule to induce learning stability of the first subsystem, as determined by F_{11} and A_{11}.

Proof of Proposition 2

Given what is explained above, expectational stability of the union economy is determined by A_{11} and F_{11} as defined in *section 2.3.3*. It follows from (2.33) that $b_{11} = 0$ since $\Phi_{11} = 0$. This in turn gives:

$$F_{11} = \begin{bmatrix} \frac{\sigma}{\varphi_Y + \varphi_\pi \zeta_Y + \sigma} & \frac{1-\beta\varphi_\pi}{\varphi_Y + \varphi_\pi \zeta_Y + \sigma} \\ \frac{\sigma\zeta_Y}{\varphi_Y + \varphi_\pi \zeta_Y + \sigma} & \frac{\beta(\varphi_Y + \sigma) + \zeta_Y}{\varphi_Y + \varphi_\pi \zeta_Y + \sigma} \end{bmatrix}, \qquad A_{11} = 0$$

For E-stability we require the eigenvalues of F_{11} to have real parts less than one. This is equivalent to the matrix $(F_{11} - I)$ having negative real parts, where I is the identity matrix. Let the eigenvalues be given by:

$$p(\lambda) = \lambda^2 + c_1\lambda + c_0 \tag{2.A3}$$

then these are negative if and only if $c_0 > 0$ *and* $c_1 > 0$.

Here we have:

$$F_{11} - I = \frac{1}{(\varphi_Y + \varphi_\pi \zeta_Y + \sigma)} \begin{bmatrix} -(\varphi_Y + \varphi_\pi \zeta_Y) & (1 - \beta\varphi_\pi) \\ \sigma\zeta_Y & -(1-\beta)(\varphi_Y + \sigma) - (\varphi_\pi - 1)\zeta_Y \end{bmatrix}$$

hence*:*

$$c_0 = \frac{(\varphi_\pi - 1)\zeta_Y + \varphi_Y(1-\beta)}{(\varphi_Y + \varphi_\pi \zeta_Y + \sigma)}$$

$$c_1 = c_0 + \frac{\varphi_\pi \zeta_Y + \varphi_Y + \sigma(1-\beta)}{(\varphi_Y + \varphi_\pi \zeta_Y + \sigma)}$$

The system is then E-stable when $c_0 > 0$, which gives the Taylor principle as the necessary and sufficient condition for E-stability:

$$0 < (1-\beta)\,\varphi_Y + \zeta_Y\,(\varphi_\pi - 1) \tag{2.35}$$

Proof of Proposition 4

We follow the above methodology and note that $\Phi_{11} = 0$, implies from (2.33) that $b_{11} = 0$ and hence also that $A_{11} = 0$. For E-stability we therefore require that both of the eigenvalues of F_{11} have real parts less than one, where;

$$F_{11} = \begin{bmatrix} (1-\sigma^{-1}\varphi_Y) & \sigma^{-1}\,(1-\varphi_\pi) \\ \zeta_Y(1-\sigma^{-1}\varphi_Y) & \beta + \zeta_Y\sigma^{-1}\,(1-\varphi_\pi) \end{bmatrix}$$

Writing the eigenvalues of the matrix $[F_{11} - I]$ in the form of $(2.A3)$ then yields:

$$c_0 = \sigma^{-1}\varphi_Y\,(1-\beta) + \zeta_Y\sigma^{-1}\,(\varphi_\pi - 1)$$

and

$$c_1 = \sigma^{-1}\varphi_Y + (1-\beta) + \zeta_Y\sigma^{-1}\,(\varphi_\pi - 1)$$
$$= c_0 + \beta\sigma^{-1}\varphi_Y + (1-\beta)$$

As stated in the above *proof of proposition 2*, the eigenvalues of the matrix F_{11} have real parts less than one when those of $[F_{11} - I]$ have negative real parts, which occurs when $c_{0,}c_1 > 0$. Since $\beta\sigma^{-1}\varphi_Y + (1-\beta) > 0$, we then require $c_0 > 0$ for E-stability. This solves for the Taylor principle as the necessary and sufficient condition for E-stability:

$$0 < (1-\beta)\,\varphi_Y + \zeta_Y\,(\varphi_\pi - 1) \tag{2.35}$$

2.C Appendix: *Determinacy and E-stability of optimal policy rules*

Proof of Proposition 5

The economic system is now given by equations (2.6), (2.7), (2.37) and (2.10). We show in the above proofs of *propositions 1* and *2*, that (2.10) independently determines the evolution of the terms of trade and the relative subsystem. Furthermore, the condition is both determinate and E-stable. Hence, for determinacy we substitute (2.37) into (2.6) and combine this with (2.7) and write it in the form of (2.13):

$$B_{11} = \begin{bmatrix} 1+\sigma^{-1}\beta^{-1}\zeta_Y & -\sigma^{-1}\beta^{-1} \\ -\beta^{-1}\zeta_Y & \beta^{-1} \end{bmatrix}$$

The eigenvalues of B_{11} are then:

$$\lambda_{1,2} = \frac{(\sigma(1+\beta)+\zeta_Y)\pm\sqrt{(\sigma(1+\beta)+\zeta_Y)^2-4\beta\sigma^2}}{2\beta\sigma}$$

It can easily be verified that while the negative conjugate is between zero and one, the positive is always larger than one. Hence, the policy rule (2.37) is indeterminate.

Considering E-stability we note that $\Phi_{11}=0$, implying that $b_{11}=0$ and that $A_{11}=0$. For E-stability we therefore need both eigenvalues of F_{11} to have real parts less than one, where:

$$F_{11} = \begin{bmatrix} 1 & \sigma^{-1} \\ \zeta_Y & \beta+\zeta_Y\sigma^{-1} \end{bmatrix}$$

and the eigenvalues are given by:

$$\lambda_{1,2} = \frac{(\sigma(1+\beta)+\zeta_Y)\pm\sqrt{(\sigma(1+\beta)+\zeta_Y)^2-4\beta\sigma^2}}{2\sigma}$$

As above, the positive conjugate is always larger than one and the policy rule (2.37) is therefore not locally asymptotically stable under RLS learning .

Proof of Proposition 6

Combining (2.6), (2.7) and (2.38) and writing this in the form of (2.13) gives the matrix:

$$B_{11}^{-1} = \begin{bmatrix} 0 & -\frac{\beta}{(\zeta_Y+\epsilon)} \\ 0 & \frac{\beta\epsilon}{(\zeta_Y+\epsilon)} \end{bmatrix}$$

Note that B_{11}^{-1} is singular and we hence do not obtain B_{11}. Equivalently to having the eigenvalues of B_{11} outside the unit circle is having those of B_{11} inside the unit circle, The eigenvalues are equal to zero and $\frac{\beta\epsilon}{(\zeta_Y+\epsilon)}$, which is clearly between zero and one. So, the policy rule (2.38) is always determinate.

Since $\Phi_{11} = 0$, we have: $b_{11} = 0$ and $A_{11} = 0$. This implies that $F_{11} = B_{11}^{-1}$ and hence that the policy rule (2.38) is stable under learning.

Proof of proposition 7

We note that the policy rule (2.39) is a special case of (2.12), in which $\varphi_Y = 0$ and $\varphi_\pi = \left[1 + \frac{(1-\varrho)\sigma\epsilon}{\varrho}\right]$. From *proposition 3*, we have that the two conditions (2.35) and (2.36), must hold for determinacy. Of these, the Taylor principle is always satisfied since $\varphi_\pi = \left[1 + \frac{(1-\varrho)\sigma\epsilon}{\varrho}\right] > 1$ given our assumption of $0 < \varrho < 1$, while the second constraint is satisfied when:

$$\varphi_\pi < \frac{2\sigma(1+\beta)}{\zeta_Y} + 1$$

$\Rightarrow \varrho < \frac{\epsilon\zeta_Y}{2\sigma(1+\beta)+\epsilon\zeta_Y}$

From *proposition 4*, the necessary and sufficient condition for E-stability of a forward looking interest rate rule is the Taylor principle (2.35). Hence the policy rule (2.39) is always locally asymptotically stable under RLS learning.

Chapter 3
Learning in a Monetary Union with Heterogeneous Price Contracts

Chapter Overview

In this chapter we consider determinacy and E-stability of a two country monetary union model that allows for asymmetries in price rigidities. While the results of chapter 2 hold for this model when price contracts are of equal length in both countries, the determinate and E-stable regions for monetary policy increase when introducing heterogeneous price rigidities. Specifically, the Taylor principle is no longer a required condition. Furthermore, an interest rate rule reacting to the terms of trade, in addition to output and inflation, is more likely to induce determinacy and local stability under RLS learning.

3.1 Introduction

3.1.1 Overview

In *chapter 1*, we introduced a monetary union model and demonstrated that the union central bank has no means of stabilizing asymmetric shocks, when there is homogeneity in price rigidities. The previous chapter concluded that the results from the closed economy literature on learning also hold for a monetary union model with homogeneous price rigidities.

Here, we consider a monetary union model in which the assumption of homogeneous price rigidities is relaxed. This implies that monetary policy now has an effect on relative variables as a change in the interest rate has different impacts on the inflation of different countries. We examine the effect that this has on determinacy and E-stability by allowing for the degree of asymmetry in price rigidities to vary. We also consider a Taylor rule that acts to stabilize the terms of trade in addition to inflation and output.

The assumption of asymmetric price rigidities means that the system cannot be partitioned as in the previous chapter and consequently we do not obtain analytical results. Instead, we provide a numerical analysis that produces some clear and interesting results that are presented graphically.

3.1.2 The model framework

The model put forward in *chapter 1* is rather complex in structure and to simplify analysis we use the model of Benigno (2004) to examine learning in a monetary union with heterogeneous price rigidities. The monetary union is made up of two countries; (H)ome and (F)oreign. Each country has a sovereign fiscal government but they share a common central bank. The monetary union consists of a continuum of economic agents or households spread along the unit interval, of which $[0, n)$ reside in home country and $(n, 1]$ in foreign country. Each household produces a unique good monopolistically that makes part of its country's consumption bundle, while consuming the consumption bundles of both union countries.

Following Calvo (1983), each producer of country i faces a fixed probability α^i of keeping their price fixed in each period, for $i = H, F$. The average length of a contract, $\left(\frac{1}{1-\alpha^i}\right)$ is therefore the same for all producers within a given country, but generally different for producers in country H than for those in country F. We keep the union average length of a price contract fixed, while allowing the average contract length in the two member countries to differ. This allows us to examine the specific effect that asymmetric price rigidities has on determinacy and E-stability.

3.1.3 Related literature and main results

Using the forward looking closed economy framework of Woodford (1999), Bullard & Mitra (2002) examine determinacy and E-stability of monetary policy. They consider interest rate rules reacting to inflation and the output gap, and find that the Taylor principle, where the central bank reacts to movements in inflation by more than one-for-one in order to adjust the real interest rate, is closely linked with determinacy and E-stability. Bullard & Mitra (2007) find that; including lagged values of the interest rate in the Taylor rule, so as to smoothen the path of interest rates, increases the determinate and E-stable region for monetary policy.

Wang & Wong (2005) examine the effect of inertia in the Phillips curve on determinacy and learning stability of monetary policy and find that policy needs to be more aggressive to ensure stability, when the degree of inertia is higher. In our framework, the terms of trade brings inertia into the system. While the terms of trade has no effect on union output and inflation when price rigidities are homogeneous, the whole system exhibits inertia when there is heterogeneity in price rigidities. Nonetheless, we find that the larger the difference between the average length of a price contract in the two countries, i.e. the larger the asymmetry in price rigidities, the larger is the set of policy rules that induce determinacy and E-stability.

In the open economy, the central bank has the option of stabilizing exchange rates in addition to output and inflation. Using the small open economy framework of Galì & Monacelli (2005), Llosa & Tuesta (2008) find that the chosen exchange rate regime has crucial implications for determinacy and E-stability of monetary policy. Specifically, when interest rates are adjusted to fluctuations in the exchange rate, then the Taylor principle condition is relaxed. Llosa & Tuesta (2008) point out that the nominal exchange rate changes one for one with changes in the lagged interest rate by the interest rate parity condition. Thus including the exchange rate in the Taylor rule has a similar effect as using a smoothened interest rate rule of the type in Bullard & Mitra (2007). However, when policy is forward looking and stabilizing the exchange rate, the determinate and E-stable region does not necessarily increase, as further restrictions on the aggressiveness of monetary policy must be satisfied. Looking at two country open economy models, Wang (2006) and Bullard & Schaling (2006) find that when one country pegs its currency to the second country, stability hinges upon the policy of the second country.

Here, we show that when monetary policy stabilizes the terms of trade in addition to inflation and the output gap, a less aggressive response to the latter two is required to guarantee a unique stationary REE and local stability under RLS learning. It is as if the Taylor principle condition shifts inwards, thus increasing the region for stable policy as in Llosa & Tuesta (2008). In contrast to their case however, the stability region in our case also increases when the policy rule is forward looking.

Bullard & Schaling (2006) interpret the size of a country as its degree of openness so that as it approaches one it becomes a closed economy while when it approaches zero in size it becomes a small open economy. They find that the smaller a country is in size, i.e. the more open it is in trade, the more aggressive monetary policy has to be to guarantee determinacy and learning stability. Findings by Llosa & Tuesta (2008) support this result. In this chapter we find that the determinate and E-stable region for monetary policy increases with the size of the country with the higher degree of price rigidity, keeping the average level of price rigidity across the union fixed.

3.1.4 Organization

In the following section, we set up the environment for our analysis. This includes a presentation of the reduced form of the Benigno (2004) model, the interest rates rules to be considered, a brief summary of the methodology used to obtain the determinacy and E-stability results and a calibration of the model's underlying parameters. In *section 3.3* we then present the results for the contemporaneous interest rate rules, while *section 3.4* presents those for the forecast based policy rules. *Section 3.5* concludes the chapter.

3.2 The Environment

We first look at the underlying structure of the Benigno (2004) model and then present it in its reduced form. The monetary policy rules are then presented, which together with the core equations of the model give our monetary union system. Finally, we put forward a calibration for the models underlying parameters, which will be used in following sections when presenting our results.

3.2.1 The baseline model

Here, we outline the main features of the Benigno (2004) model.[1] Consider a monetary union consisting of two countries; (H)ome and (F)oreign. The union contains a continuum of households spread over the unit interval, where those residing in home country are spread over $[0, n)$ while residents of the foreign country occupy $(n, 1]$. That is, the size of country H is n and that of country F is $(1 - n)$. Each household is both a consumer and a monopolistically competitive producer. For simplicity, the population sizes are assumed to equate to the economic sizes of the two regions.

The consumer problem

Each household consumes the bundles produced in both countries and the elasticity of substitution between these two bundles is assumed to be one. In addition, the sovereign governments of the two regions each consume bundles produced in their respective

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[1] In this section, we summarize the main features of the framework. This serves our purpose and readers are refered to Benigno (2004) for a more detailed description.

countries. They also provide nominal lump sum transfers to households while imposing a proportional tax on nominal income.

Asset markets are complete domestically while positions can be taken in an internationally traded bond. However, given the assumption of consumption preferences and assuming no initial holdings of the international bond, it becomes redundant. A corollary of this is that there is perfect risk sharing of consumption between regions, i.e. $C_t^H = C_t^F = C_t$, for all t.[2] Eventhough consumers in both countries are guaranteed the same level of consumption, output can still vary due to fluctuations in the terms of trade and different levels of government spending. The market clearing conditions are:[3]

$$Y_t^H = T_t^{1-n} C_t + G_t^H \quad \textit{and} \quad Y_t^F = T_t^{-n} C_t + G_t^F \tag{3.1}$$

Where the terms of trade is defined as the price of the foreign consumption bundle over the price of the home country consumption bundle, i.e. $T_t \equiv \frac{P_{F,t}}{P_{H,t}}$.

Firms and price setting

Households produce a differentiated product that makes part of the domestic consumption bundle, where the elasticity of substitution across goods produced within a country σ is greater than one, as in Dixit and Stiglitz (1977).

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2 See the companion *Appendix A* of Benigno (2004).

3 This expression differs from the market clearing conditions of the previous two chapters since there is now government consumption. In this chapter we assume that government expenditure is not actively used to stabilize economic fluctuations but is instead set at a steady state value. In the following chapter we consider the case in which fiscal policy is used in tandem with monetary policy to stabilize shocks.

Following Calvo (1983), each producer of country i faces a fixed probability $(1-\alpha^i)$ of changing their price in each period, for $i = H, F$. The average length of a contract $\left(\frac{1}{1-\alpha^i}\right)$ is therefore the same for all agents in country i but not necessarily so for agents in different countries. We then keep the average price contract length fixed for the union as a whole (we define the union average length of a contract as $\left(\frac{1}{1-\alpha^H}\right)^n \left(\frac{1}{1-\alpha^F}\right)^{1-n}$), while letting the relative contract length between the two countries vary, we can isolate the effect that heterogeneous price rigidities has on our results. In this chapter , we will without loss of generality assume that the home country has a higher degree of price rigidities than the foreign country.

Log-linearisation around steady state

We now look at the log-linearized form of the model under the assumption of sticky prices. For the generic variable X, we have defined the log deviation from its steady state value as $x = \widetilde{x} + \bar{x}$, where $\bar{x}$ is the flexible price value and $\widetilde{x}$ the deviation from this due to prices being sticky. We further define union average variables as; $X^W \equiv nX^H + (1-n)\,X^F$ and relative variables as: $X^R \equiv X^F - X^H$.

The system is reduced to the following four equations:

$$\widetilde{y}_t^W = E_t(\widetilde{y}_{t+1}^W) - \rho^{-1}\left[(r_t - E_t(\pi_{t+1}^W)) - rr_t\right] \tag{3.2}$$

$$\pi_t^H = (1-n)k_T^H(\widetilde{T}_t) + k_C^H \widetilde{y}_t^W + \beta E_t(\pi_{t+1}^H) \tag{3.3}$$

$$\pi_t^F = -nk_T^F\left(\widetilde{T}_t\right) + k_C^F \widetilde{y}_t^W + \beta E_t(\pi_{t+1}^F) \tag{3.4}$$

$$\widetilde{T}_t = \widetilde{T}_{t-1} + \pi_t^F - \pi_t^H - E_t \Delta \bar{T}_t \tag{3.5}$$

Where $\widetilde{y}_t^W$ is the union output gap, π_t^W, π_t^H and π_t^F are the union, home and foreign country's respective inflation levels, rr_t is the natural rate of interest familiar from the closed economy literature and can be interpreted as a government expenditure shock over the natural level of output: $rr_t \equiv \frac{\rho\eta}{\rho+\eta} E_t(\Delta \bar{y}_{t+1}^W - \Delta g_t^W)$. The deviation of the terms of trade from its natural level is denoted by $\widetilde{T}_t$.

The inverse of the relative risk aversion is given by; $\rho \equiv -\frac{U_{CC}\bar{C}}{U_C}$, while the inverse of the elasticity of producing goods is defined as: $\eta \equiv -\frac{V_{yy}\bar{C}}{V_y}$. The supply schedule coefficients are then given by:

$k_C^i \equiv [(1-\alpha^i\beta)(1-\alpha^i)/\alpha^i] \times [(\rho+\eta)\,/\,(1+\sigma\eta)]$

and

$k_T^i \equiv k_C^i\,[(1+\eta)\,/\,(\rho+\eta)] \qquad \textit{for } i = H, F$

where σ is the elasticity of substitution across goods produced within a region. Equation (3.2) is an IS type curve for the union as a whole and is similar to those in the closed economy literature. Equations (3.3) and (3.4) are the supply schedules of the two union members and the last equation (3.5), is the condition for the terms of trade.

Because the terms of trade enter the supply schedules and hence affects inflation, there are three different distortions to the economy; *(i)* the deadweight loss from the inefficient price and production levels caused by monopolistic competition, *(ii)* the rigidity in prices caused by staggered price setting and *(iii)* the distortions to the terms of trade causing disequilibrium of relative prices. Benigno (2004) assumes a subsidy in accordance with the framework to neutralize the first of these three distortions.[4] Monetary policy can then achieve an efficient outcome only in the case where nominal rigidities are equal across the two regions. This is because only homogenous price rigidities implies that deviations of the terms of trade from equilibrium are neutral for the region as a whole. When this is the case we have that $k_T^H = k_T^F = k_T$ and that $k_C^H = k_C^F = k_C$, thus using the definition of a union variable $x^W \equiv nx^H + (1-n)x^F$ we can combine equations (3.3) and (3.4) to get a union and a relative supply schedule:

$$\pi_t^W = k_C \widetilde{y}_t^W + \beta E_t(\pi_{t+1}^W) \tag{3.6}$$

$$\pi_t^R = -k_T \left(\widetilde{T}_t\right) + \beta E_t(\pi_{t+1}^R) \tag{3.7}$$

In this case the system can be partitioned, where (3.2) and (3.6) determine the union variables $\left\{\widetilde{y}_t^W, \pi_t^W\right\}$ and (3.5) and (3.7) determine $\left\{\pi_t^R, \widetilde{T}_t\right\}$. We show below that as in the previous chapter, the results from the closed economy literature on learning hold for this model when price rigidities are equal in the two countries.

In the more general case, when the average length of price contracts differ in the two regions, it is useful to write equation (3.2) as:

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4 This implies imposing the subsidy: $\tau^H = \tau^F = (1-\sigma)^{-1}$.

$$\widetilde{y}_t^W = E_t(\widetilde{y}_{t+1}^W) - \rho^{-1}\left[(r_t - nE_t(\pi_{t+1}^H) - (1-n)E_t(\pi_{t+1}^F)) - rr_t\right] \qquad (3.2')$$

This yields a system with four equations determining the four state variables; $\left\{\widetilde{y}_t^W, \pi_t^H, \pi_t^F, \widetilde{T}_t\right\}$. While the first three of these are free, the terms of trade is predetermined.

3.2.2 Specifications for the interest rate rules

In this chapter and throughout this thesis, we will consider both contemporaneous and forward looking interest rate rules. In practice the European Central Bank (ECB) has the mission to govern price stability in its member countries and thus safeguard the value of the single European currency, the euro.[5] However, "without prejudice to the objective of price stability, the European System of Central Banks[6] shall support the general economic policies in the Community with a view to contributing to the achievement of the objectives of the Community".[7] Hence, there is scope for stabilization of output or economic activity in the EMU. We therefore focus on Taylor rules that react to both inflation and the output gap. Furthermore, the inflation target of the ECB is currently the harmonized index of consumer prices (HICP) of the member countries, where the weight given to each country is equal to its proportion of union consumption. This is defined as π_t^W in our model and hence our baseline Taylor rule is given by:

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[5] Laid down in *article 2* of the *Treaty on European Union*.

[6] The European systems of Central Banks consists of the ECB and the national Central Banks of all the EU members.

[7] *Article 105.1* of the Treaty on European Union.

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \tilde{y}_t^W \tag{3.8}$$

Note that it is convenient to rewrite this using the definition of a union variable:

$$r_t = \varphi_\pi \left(n\pi_t^H + (1-n)\pi_t^F\right) + \varphi_Y \tilde{y}_t^W \tag{3.8$'$}$$

We also examine the case where a higher weight is put on home country's inflation, this yields:

$$r_t = \varphi_\pi \left(\gamma\pi_t^H + (1-\gamma)\pi_t^F\right) + \varphi_Y \tilde{y}_t^W \tag{3.9}$$

Where $\gamma\epsilon[n, 1]$ gives a higher weight to home country than its size n. Note that changing the country weights is equivalent to keeping them fixed at country sizes while introducing a new term in the Taylor rule, relative inflation $\pi_t^R \equiv \pi_t^F - \pi_t^H$. If we define the interest rate rule as: $r_t = \varphi_\pi \left(n\pi_t^H + (1-n)\pi_t^F\right) + \varphi_y \tilde{y}_t^W - \varphi_{\pi^R}\pi_t^R = \varphi_\pi \left(\gamma\pi_t^H + (1-\gamma)\pi_t^F\right) + \varphi_y \tilde{y}_t^W$, it follows that $\gamma = n + \frac{\varphi_{\pi^R}}{\varphi_\pi}$.

As shown in the companion *appendix B* of Benigno (2004), interest rates do not affect the terms of trade when price rigidities are the same in the two countries, since inflation in both countries would be equally sensitive to movements in interest rates. However, with heterogeneous price rigidities, monetary policy does have an effect on the terms of trade. We hence consider a Taylor rule of the form:

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \tilde{y}_t^W + \varphi_T \tilde{T}_t \tag{3.10}$$

The coefficient on the terms of trade φ_T is positive because an increase in the interest rate deteriorates the terms of trade. A rise in r_t contracts union output $\widetilde{y}_t^W$, which has the effect of lowering inflation in the two countries through the supply schedules (3.3) and (3.4). Assuming that home country has more rigid prices than foreign and that $\alpha^H > \alpha^F$, it follows that $k_c^H < k_c^F$. This means that the fall in foreign country inflation is greater than the fall in domestic inflation and consequently that the terms of trade depreciates with contractionary monetary policy.[8] Hence, monetary policy is stabilizing of the terms of trade.

In addition to the above Taylor rules, we consider two forward looking rules of the types:

$$r_t = \varphi_\pi E_t\left(\pi_{t+1}^W\right) + \varphi_Y E_t\left(\widetilde{y}_{t+1}^W\right) \tag{3.11}$$

and

$$r_t = \varphi_\pi E_t\left(\pi_{t+1}^W\right) + \varphi_Y E_t\left(\widetilde{y}_{t+1}^W\right) + \varphi_T E_t\left(\widetilde{T}_{t+1}\right) \tag{3.12}$$

3.2.3 Calibration of parameter values

Using the methodology outlined in *section 2.2.3* of the previous chapter, we obtain numerical results that are represented graphically. To obtain these results, we make use of

[8] Taken future expectations as given, the relationship between the interest rate and the terms of trade is $\frac{dT}{dr} = \frac{\delta\pi^F}{\delta y^W}\frac{\delta y^W}{\delta r} - \frac{\delta\pi^H}{\delta y^W}\frac{\delta y^W}{\delta r} = \frac{d\pi^F}{\delta r} - \frac{d\pi^H}{\delta r} = \rho^{-1}(k_c^H - k_c^F) < 0$ assuming that country H has more rigid prices than country F. Hence, to offset deviations in the terms of trade from its natural level we have a positive reaction of interest rates.

the model parameter values suggested by Benigno (2004). This allows us to examine how determinacy and E-stability is altered by changes in the policy coefficients. The analysis was also carried out using the calibration of Rotemberg & Woodford (1998), but as the qualitative results remain unchanged, these are omitted in what follows.

The parameter values in Benigno (2004) are similar to those suggested by e.g. Rotemberg & Woodford (1998), but are more applicable to the European economy rather than to the US economy. The intertemporal rate of substitution β is assumed to have a value of 0.99, since these are assumed to be quarterly values this implies an annual equilibrium interest rate of 4.1%.[9] The producer mark-up is assumed to be 15%, which suggests that the elasticity of substitution between goods produced within a region σ takes a value of 7.66. The coefficient of relative risk aversion ρ is set to a conventional value of $1/6$. Furthermore, assuming that the elasticity of the average real wage with respect to variation in production is slightly higher in Europe than in the US and by assuming that labours' share of total income is equal to 0.75, we have that the elasticity η is equal to 0.67. For simplicity, we assume in what follows that the sizes of the two regions of the monetary union are equal, that is $n = 0.5$, unless otherwise specified.

Finally, we let the average length of a union contract be four quarters throughout the chapter, but the results hold for other levels of average price rigidities as well. Benigno & Lopez-Salido (2006) provide empirical evidence supporting this assumption, looking at the major European economies. Recall that the probability facing producers in country i of

[9] This is approximately the 20 year German average, as pointed out by Benigno (2004).

keeping their price fixed, is given by α^i. Thus the average length of a price contract is given by: $\left(\frac{1}{1-\alpha^i}\right)$. We define the union average length of a contract as the geometric average:

$$\left(\frac{1}{1-\alpha^H}\right)^n \left(\frac{1}{1-\alpha^F}\right)^{1-n} = 4 \tag{3.13}$$

3.3 Contemporaneous Taylor Rules

In this section we examine determinacy and E-stability of our monetary union framework when the central bank employs one of the contemporaneous interest rate rules outlined in *section 3.2.2*. We first consider the benchmark case of homogeneous price rigidities across the monetary union. This is then compared to our results under the assumption of heterogeneous price rigidities.

3.3.1 Homogeneous price rigidities

As a point of reference, we first consider the case where price contracts in both countries of the monetary union last for four quarters on average. That is, $\left(\frac{1}{1-\alpha^H}\right) = \left(\frac{1}{1-\alpha^F}\right) = 4$. This implies that the supply curve coefficients outlined in *section 3.2.1* take the same value for both countries, so that $k_T^H = k_T^F = k_T$ and $k_C^H = k_C^F = k_C$. As explained in *section 3.2.1* above, this implies that the system is given by the following equations in addition to the employed monetary policy rule:

$$\tilde{y}_t^W = E_t(\tilde{y}_{t+1}^W) - \rho^{-1}\left[(r_t - E_t(\pi_{t+1}^W)) - rr_t\right] \tag{3.2}$$

$$\pi_t^W = k_C \widetilde{y}_t^W + \beta E_t(\pi_{t+1}^W) \tag{3.6}$$

$$\pi_t^R = -k_T \left(\widetilde{T}_t\right) + \beta E_t(\pi_{t+1}^R) \tag{3.7}$$

$$\widetilde{T}_t = \widetilde{T}_{t-1} + \pi_t^F - \pi_t^H - E_t \Delta \bar{T}_t \tag{3.5}$$

Note that we have two independent subsystems; (3.2) and (3.6) determine union variables $\left\{\widetilde{y}_t^W, \pi_t^W\right\}$, while (3.5) and (3.7) determine the relative variables $\left\{\pi_t^R, \widetilde{T}_t\right\}$. Monetary policy only affect the first of these two subsystems and has no effect on relative variables. That is, interest rates have no effect on either relative inflation nor on the terms of trade. We consider the contemporaneous Taylor rule;

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \widetilde{y}_t^W \tag{3.8}$$

and combine this with the system above, which yields the following result.

Proposition 1 *Assume that price rigidities in the monetary union are homogeneous. If monetary policy follows the contemporaneous interest rate rule:*

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \widetilde{y}_t^W \tag{3.8}$$

then the necessary and sufficient condition for both determinacy and E-stability is that the Taylor principle is satisfied:

$$k_C(\varphi_\pi - 1) + (1 - \beta)\varphi_Y > 0 \tag{3.14}$$

Proof. See *appendix 3A*. ■

Hence, for this monetary union model, as for the monetary union model of the previous two chapters, the condition for a unique stationary and expectationally stable REE is that the Taylor principle holds. This is also the determinacy and E-stability condition for the closed economy model considered by Bullard & Mitra (2002). The stability region for monetary policy under equal price rigidities is depicted by the solid line in *figure 3.1* below.

We now relax the assumption of homogenous price contracts and consider the case in which the home country has a higher degree of price rigidity than the foreign country.

3.3.2 Heterogeneous price rigidities

In this section, we consider the case in which the average length of price contracts is different in the two countries of the monetary union. As in the previous section, we assume that the average length of a contract in the union is fixed at 4 quarters, so that:

$$\left(\frac{1}{1-\alpha^H}\right)^n \left(\frac{1}{1-\alpha^F}\right)^{1-n} = 4$$

However, we now assume that $\alpha^H > \alpha^F$, or put differently;

Assumption: *Home country has more rigid prices than foreign country.*

For example if the average contract in home country is 5 quarters then that in foreign country will be approximately 3 quarters.[10,11]

[10] This is only approximately true since we have defined the average union contract length geometrically.

[11] Benigno and Lopez-Salido (2006) find that price rigidities vary across countries, with Italy and the Netherlands having more flexible prices than Germany, France and Spain. Their findings suggest that differences in the average length of price contracts could be around 2 quarters. Further evidence of differences in price rigidities is provided by e.g. Nickel (1997), who looks at wage rigidities in major European countries.

Taylor rule reacting to output and inflation

Let the economic system be given by (3.2), (3.3), (3.4), (3.5) and the contemporaneous Taylor rule (3.8). Using the methodology outlined in *section 2.2.3*, numerical results are obtained in what follows. The relevant matrices of the system are presented in *appendix 3B*.

We first consider how the determinacy and E-stability region changes as the degree of heterogeneity in price rigidities ($\alpha^H - \alpha^F$) increases. The results are illustrated in *figure 3.1*, and clearly, a higher level of asymmetry in price rigidities increases both the determinate and the E-stable region.

Result 1 *When the central bank conducts monetary policy according to the contemporaneous Taylor rule;*

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \tilde{y}_t^W \tag{3.8}$$

and when the union average length of a price contract is fixed at four quarters;

$\left(\frac{1}{1-\alpha^H}\right)^n \left(\frac{1}{1-\alpha^F}\right)^{1-n} = 4,$

then the region for determinacy and E-stability increases with the level of heterogeneity in price rigidities.

As is common for contemporaneous interest rate rules, the determinate and E-stable regions coincide.

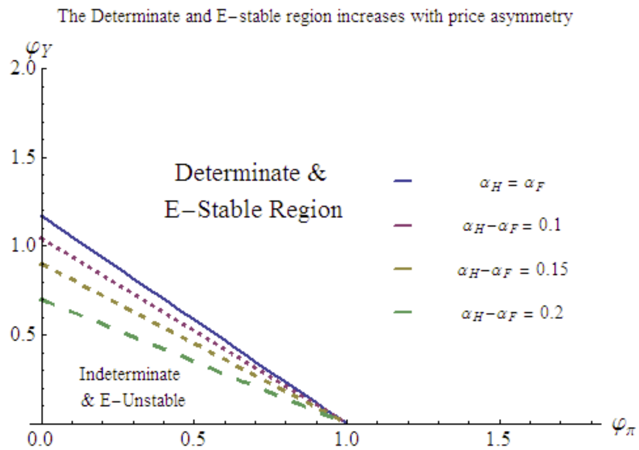

Fig. 3.1. The graph shows that the determinate and E-stable region for monetary policy increases with the level of asymmetry in price rigidities.

Our result could potentially have implications not only for monetary union models, but also for closed economy models. As in the model of Aoki (2001), closed economy models could have sectors with different levels of price rigidities. Not recognizing this, or simplifying by assuming one sector with the average level of price rigidity, could lead to underestimates of the stability regions for monetary policy.

We note that the stability region in *figure 3.1* changes for the output coefficient φ_Y but not for the inflation coefficient φ_π. With homogeneous price rigidities, we have from the Taylor principle (3.14) that for stability the output gap has to satisfy: $\varphi_Y > \frac{k_C}{(1-\beta)}$ when there is no stabilization of inflation, $\varphi_\pi = 0$. Since the coefficient k_C is convex in α, the average supply curve coefficient;

$$k_C^W \equiv n k_C^H + (1-n)\, k_C^F$$

increases with the level of asymmetry in price rigidities. Hence, based on this, the Taylor principle suggests that the stability region should decrease as we impose heterogeneous price rigidities. But, in fact the opposite occurs. In *figure 3.2*, we assume that $\varphi_\pi = 0$ and compare the actual determinate and E-stable region for φ_Y when the level of asymmetry in price rigidities changes, to that suggested by the Taylor principle of the form:

$$\varphi_Y > \frac{k_C^W}{(1-\beta)}$$

It is clear that the difference between the Taylor principle and the actual stability region increases with asymmetry. We conjecture that this result is due to the spill-over effect of the relative system on the subsystem for union variables. We speculate that the inertia in the terms of trade could make the system more likely to be determinate and E-stable. Wang & Wong (2005) suggest that inertia of inflation in the supply schedule reduces rather than increases the determinacy and E-stability region. However, their case is different in that past inflation and expected future inflation are a convex combination and inertia therefore reduces the sensitivity of inflation to expected future inflation. In our case the inertia in itself does not alter the values of other coefficients.

We fixed the level of asymmetry in price rigidities ($\alpha^H - \alpha^F$) and changed the weight attached to the inflation level of home country γ in the policy rule (3.9) and examined if this had an impact on determinacy and E-stability. As explained in *section 3.2.2*, this is equivalent to the case in which the central bank stabilizes relative inflation (π_t^R) in addition to union-wide inflation (π_t^W). Our findings were that this has no effect on the determinacy and E-stability region for monetary policy.

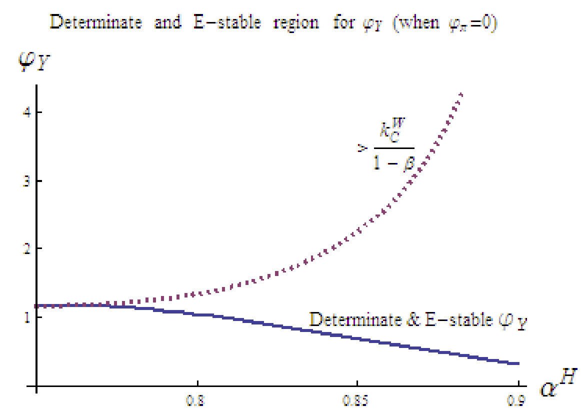

Fig. 3.2. The average length of a price contract across the monetary union is held fixed at 4 quarters, while interest rates are assumed to only be adjusted to changes in the output gap. The determinate and E-stable region is given by the solid line. However, the Taylor principle only holds above the dotted line, suggesting that it does not have to be satisfied for determiancy and E-stability under heterogeneous price rigidities.

However, with heterogeneous price rigidities, the relative sizes of the two countries now matter. As the size of home country (n) increases, so does the region for determinate and E-stable values for the policy coefficients. This is illustrated in *figure 3.3*. Llosa & Tuesta (2008) and Bullard & Schaling (2006) find that the determinacy and E-stability region for monetary policy in an open economy increases with its size, where the size of a country is interpreted as the inverse of its openness in trade.

Result 2 *Keeping the length of an average union price contract fixed, the region for determinate and E-stable monetary policy increases with the size of the country with the higher level of price rigidity.*

Note that in both *figure 3.1* and *figure 3.3*, it is the output gap coefficient φ_Y that changes the determinacy and E-stability region. The cut-off for an interest rate rule reacting

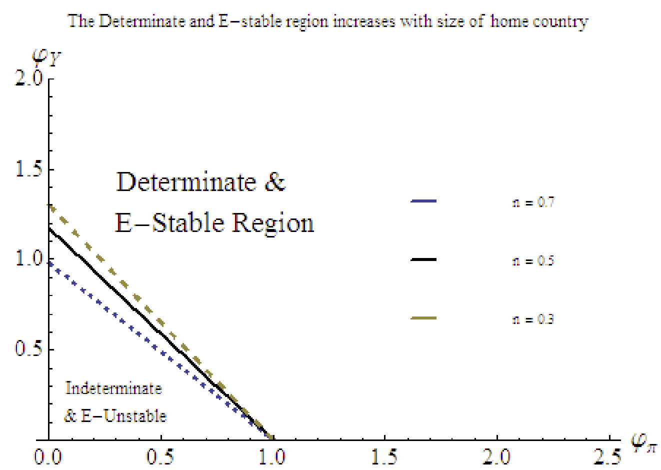

Fig. 3.3. The figure shows that the determinate and E-stable region increases with the size of the country with a higher degree of price rigidities. The average duration of a price contract in the monetary union is fixed at 4 and that of home country at 5 quarters, allowing duration in the foreign country to vary.

only to inflation to be stable is still that interest rates are adjusted sufficiently to change the real interest rate, i.e. $\varphi_\pi > 1$.

Interest rate rule with the terms of trade

This section investigates the case where the central bank adjusts interest rates to stabilise the terms of trade in addition to inflation and the output gap. Llosa & Tuesta (2008) examine determinacy and RLS learning stability for a Taylor rule with inflation, output and the nominal exchange rate. They find that when reacting to the nominal exchange rate, the standard Taylor principle of the form (3.14) does no longer need to hold for determinacy and E-stability. In fact, their results suggest a parallel shift of the indeterminate and E-unstable region in $\{\varphi_\pi, \varphi_Y\}$ space towards the origin as the stabilization of exchange rates becomes more aggressive.

The system is given by the IS schedule (3.2), the two supply schedules (3.3) and (3.4), the condition for the terms of trade (3.5), and the interest rate rule:

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \widetilde{y}_t^W + \varphi_T \widetilde{T}_t \tag{3.10}$$

Using the methodology of *section 2.2.3*, we find the restrictions on the policy coefficients for a unique stationary REE and local stability under recursive least squares learning.[12] We illustrate our results for the case where the sizes of the economies are equal ($n = 0.5$) and where the difference in the average length of price contracts is 2 quarters. *Figure 3.4* shows the combinations of the policy coefficients φ_Y and φ_π that induce stability, given different values of the policy coefficient on the terms of trade φ_T.

As in Llosa & Tuesta (2008), the determinate and E-stable region increases by a parallel shift towards the origin as φ_T increases. The magnitude of these shifts also increase with the degree of heterogeneity in price rigidities. If the stabilisation of the terms of trade is sufficiently aggressive, then interest rates do not need to react to inflation nor to the output gap to achieve stability. That is, if φ_T is large enough, then φ_Y and φ_π can be zero and monetary policy will still be both determinate and E-stable. Again, the determinate and E-stable regions exactly coincide.

Result 3 *When the monetary union is subject to heterogeneous price rigidities, while monetary policy stabilizes the terms of trade in addition to output and inflation accord-*

12 The matrices describing the system are presented in *appendix 3B*.

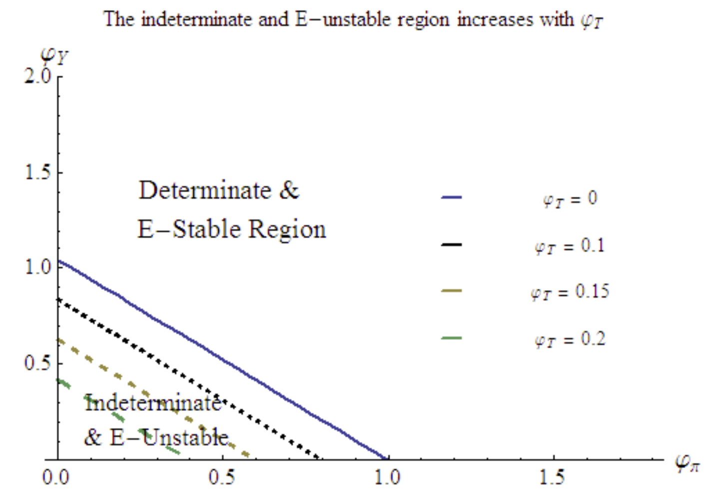

Fig. 3.4. The figure shows the determinate and E-stable region for the output gap and inflation policy coefficients in the Taylor rule (3.10) for different values of the policy coefficient on the terms of trade φ_T. Note that we are assuming heterogeneous price rigidities for this policy rule.

ing to;

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \widetilde{y}_t^W + \varphi_T \widetilde{T}_t \tag{3.10}$$

then the larger φ_T is, the smaller will the policy coefficients φ_π and φ_Y need to be to induce determinacy and E-stability. If φ_T is sufficiently large, then determinacy and E-stability is guaranteed for all non-negative values of φ_Y and φ_π.

Bullard & Mitra (2007) find that including the lagged interest rate in the Taylor rule increases the determinate and E-stability region. Here, the terms of trade is determined by the condition:

$$T_t = T_{t-1} + \pi_t^R$$

As mentioned above, adjusting interest rates to changes in relative inflation (π_t^R) has no effect on determinacy and E-stability. The increased stability must therefore be a result of the first term (T_{t-1}) in the above condition indirectly entering the interest rate rule via the contemporaneous terms of trade (T_t). We hence conjecture that it is the inertia in the interest rate rule, brought about by the inertia in terms of trade, that increases the stability region with φ_T. Llosa & Tuesta (2008) draw a similar conclusion for the Taylor rule including the nominal exchange rate, with the motivation that the current exchange rate has a one-for-one relationship with the lagged interest rate by the uncovered interest parity.

We now examine if our results are altered when the central bank is forward looking in its policy.

3.4 Forward Looking Taylor Rules

In this section, we look at determinacy and E-stability of the forward looking Taylor rules presented in *section 3.2.2*. As above, we first consider the case of homogeneous price rigidities and then impose the assumption of home country having a higher degree of price rigidity than foreign country.

3.4.1 Homogeneous price rigidities

In order to assess the effect that heterogeneity in price rigidities has on the determinacy and E-stability of forward looking monetary policy, we must first establish the stability conditions under symmetric price rigidities. As in *section 3.3.1*, we can partition the system into two independent subsystems, since $\alpha^H = \alpha^F$ implies that $k_C^H = k_C^F = k_C$ and that

$k_T^H = k_T^F = k_T$. The union variables $\left\{ \widetilde{y}_t^W, \pi_t^W \right\}$ are determined by (3.2), (3.6) and the Taylor rule:

$$r_t = \varphi_\pi E_t \left(\pi_{t+1}^W \right) + \varphi_Y E_t \left(\widetilde{y}_{t+1}^W \right) \tag{3.11}$$

The relative variables $\left\{ \pi_t^R, \widetilde{T}_t \right\}$ are independent of policy and determined by (3.5) and (3.7). The subsystem for the union variables is isomorphic to a closed economy model and as in *section 3.3.1*, we obtain the same conditions for determinacy and E-stability as Bullard & Mitra (2002).

Proposition 2 *When both union countries have the same level of price rigidity, while monetary policy is forward looking according to:*

$$r_t = \varphi_\pi E_t \left(\pi_{t+1}^W \right) + \varphi_Y E_t \left(\widetilde{y}_{t+1}^W \right) \tag{3.11}$$

Then, the necessary and sufficient condition for E-stability is that the Taylor principle holds:

$$k_C(\varphi_\pi - 1) + (1 - \beta)\varphi_Y > 0 \tag{3.14}$$

In addition to (3.28)*, the following condition must hold for determinacy:*

$$2\rho(1 + \beta) > k_C \left(\varphi_\pi - 1 \right) + \varphi_Y(1 + \beta) \tag{3.15}$$

Proof. See *appendix 3C.* ■

It can thus be concluded that for quite general specifications of the Taylor rule employed by the central bank, a monetary union model with homogeneous price rigidities has the same determinacy and E-stability conditions for monetary policy as a closed economy model. We note that this result was obtained in *chapter 2* for a model that is similar to the one treated here in terms of its reduced form, but quite different in terms of its foundations. The determinacy and E-stability regions for homogeneous price rigidities are depicted by the solid lines in *figure 3.5*.

3.4.2 Heterogeneous price rigidities

We have shown above that the Taylor principle (3.14) is not necessary for determinacy and E-stability when price rigidities are asymmetric and policy follows a contemporaneous interest rate rule. However, when policy is forward looking there is an additional constraint on determinacy, as stated in *proposition 2*. Here, we examine how both constraints in *proposition 2* are affected by heterogeneity in price rigidities. The union average length of a price contract is fixed at 4 quarters as in equation (3.13), but we set that of home country at 5 quarters making that of foreign country approximately 3 quarters.

Taylor rule reacting to output and inflation

Our monetary union framework is now given by (3.2), (3.3), (3.4), (3.5) and the forward looking Taylor rule (3.11). The methodology of *section 2.2.3* is used to obtain numerical results, with the relevant matrices presented in *appendix 3B*.

We find that the E-stable region is the same as in the case of a contemporaneous interest rate rule. That is, the E-stability region pivots around the horizontal axis as the level of asymmetry in price rigidities increases. This is illustrated by the dotted blue line in *figure*

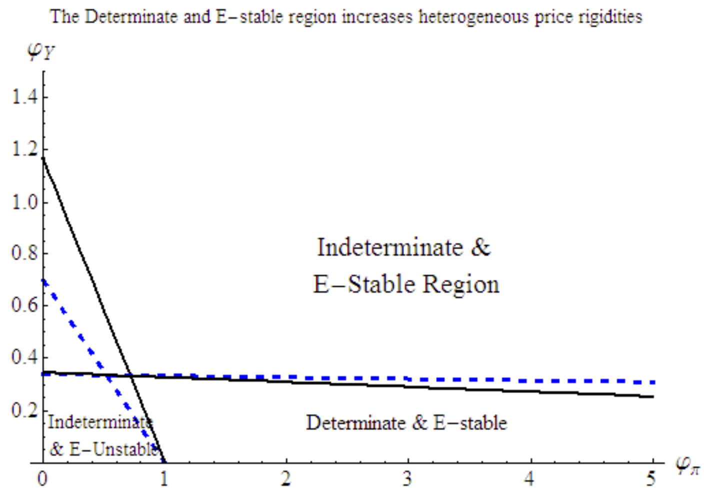

Fig. 3.5. The solid lines shows the determinacy and E-stability regions for monetary policy under homogeneous price rigidities, while the dotted blue lines show the stability regions under heterogeneous price rigidities.

3.5. Interestingly, the additional constraint on determinacy (3.15) is relaxed when heterogeneity in price rigidities is introduced, although only marginally so. It can be concluded that the determinate and E-stable region increases with different levels of price rigidities also for a forward looking Taylor rule.

Result 4 *For the forward looking Taylor rule:*

$$r_t = \varphi_\pi E_t \left(\pi_{t+1}^W\right) + \varphi_Y E_t \left(\tilde{y}_{t+1}^W\right) \tag{3.11}$$

The determinate and E-stable region increases with the degree of asymmetry in price rigidities.

Interest rate rule with the terms of trade

The central bank now employs the forecast based interest rate rule that stabilizes the terms of trade in addition to output and inflation (3.12). This Taylor rule together with equations (3.2), (3.3), (3.4) and (3.5) describe the union economy. The key matrices of *section 3.2.3* for this case are given in *appendix 3B*.

In *section 3.3.2*, we proposed that an interest rate rule that stabilizes the terms of trade in addition to output and inflation is more likely to induce determinacy and E-stability of REE. This finding is similar to a result by Llosa & Tuesta (2008) who find that the Taylor principle is relaxed when including the exchange rate in the interest rate rule of a small open economy. However, the authors find that the determinacy region decreases when policy is forward looking as stabilizing the exchange rate tightens condition (3.15). As a result, the indeterminate and E-stable region increases in their case.

The results are plotted in *figure 3.6*, where the difference in the length of price contracts is fixed at 2 quarters. The blue dotted lines show the determinacy and E-stability regions for the policy rule (3.12) when $\varphi_T = 0.15$ and the solid back lines show those for $\varphi_T = 0$. As above, the Taylor principle shifts in towards the origin when the terms of trade is stabilized by policy. In contrast to the findings by Llosa & Tuesta (2008) however, including the terms of trade in the Taylor rule has no effect on the second determinacy condition (3.15). Stabilizing the terms of trade can therefore help eliminate multiple stationary REE even when policy is forward looking.

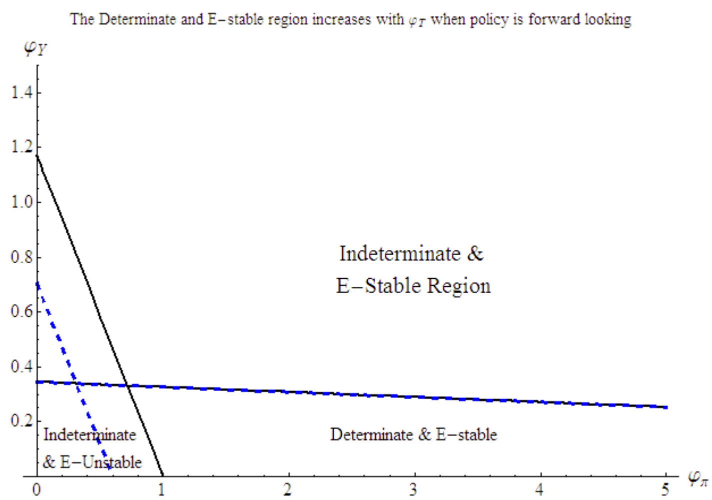

Fig. 3.6. The graph compares the stability regions of forward looking monetary policy when $\varphi_T = 0$ (black solid line) to the case where $\varphi_T = 0.15$ (blue dotted line). Stabilization of the terms of trade increases the determinate and E-stable region for monetary policy in $(\varphi\pi, \varphi_Y)$ space.

Result 5 When there is heterogeneity in price rigidities in the monetary union, while policy is conducted according to the rule;

$$r_t = \varphi_\pi E_t\left(\pi^W_{t+1}\right) + \varphi_Y E_t\left(\widetilde{y}^W_{t+1}\right) + \varphi_T E_t\left(\widetilde{T}_{t+1}\right) \tag{3.12}$$

then the set of determinate and E-stable combinations of $\{\varphi_\pi, \varphi_Y\}$ increases with φ_T.

3.5 Conclusion

We have examined determinacy and E-stability of monetary policy in the two-country monetary union model of Benigno (2004). With homogeneous price rigidities, we replicated the results presented for the closed economy case by Bullard & Mitra (2002).

However, keeping the average price rigidity across the union fixed, while allowing the level of asymmetry in price rigidities to vary, the results from the closed economy are altered. Due to a spill-over effect of relative variables on union average variables, the determinate and E-stable region increases with heterogeneity in price rigidities. This is true regardless of whether policy reacts to contemporaneous data or whether it is forecast based. Furthermore, the larger the country with the more rigid prices, the larger is the determinacy and E-stability region for monetary policy, ceteris paribus.

Finally, with heterogeneous price rigidities it is possible to use monetary policy to stabilize fluctuations in the terms of trade. We find that a monetary policy rule that stabilizes the terms of trade is more likely to induce a unique and E-stable stationary rational expectations equilibrium. If the adjustment of interest rates to disturbances in the terms of trade is sufficiently aggressive, then policy can achieve determinacy and E-stability without stabilizing output and inflation. Llosa & Tuesta (2008) find that an interest rate rule that stabilizes the exchange rate, relaxes the Taylor principle and hence increases the determinate and E-stable region for a contemporaneous policy rule. However, when policy is forward looking, stabilization of the exchange rate can actually reduce the set of determinate and E-stable policy combinations in their model. In our framework, the determinacy and E-stablity region becomes larger even when policy is forecast based. Bullard & Mitra (2006) show that inertia in the interest rate rule increases the determinate and E-stable region for monetary policy. We hence conjecture that it is the inertia in the terms of trade, bringing inertia into the interest rate rule, that causes the stability region for policy to increase in our case.

3.A Appendix: *Proof of proposition 1*

Determinacy

To examine determinacy we write the system in the partitioned form of (2.13) of *section 2.2.3*. As in *chapter 2*, we then need both subsections to induce determinacy for the whole system to be determinate. Consider first the system for union variables; equations (3.2), (3.6) and the interest rate rule (3.8). We have the following matrix:

$$B_{11} = \begin{bmatrix} \rho^{-1}\left(\varphi_Y + \beta^{-1}k_C\right) + 1 & \rho^{-1}\left(\varphi_\pi - \beta^{-1}\right) \\ -\beta^{-1}k_C & \beta^{-1} \end{bmatrix}$$

For determinacy, B_{11} must have both of its eigenvalues outside of the unit circle. Using the methodology of *appendix 2A*, we have:

$$a_0 = \beta^{-1}\rho^{-1}(\varphi_Y + k_C\varphi_\pi) + \beta^{-1}$$

and

$$a_1 = -\rho^{-1}\left(\varphi_Y + \beta^{-1}k_C\right) - 1 - \beta^{-1}$$

where:

$$|a_1| < |1 + a_0| \Longrightarrow$$

$$k_C\left(\varphi_\pi - 1\right) + (1-\beta)\varphi_Y > 0 \tag{3.14}$$

Consider now the subsystem for relative variables given by (3.5) and (3.7). Combining these two yields:

$$\widetilde{T}_t = \frac{\beta}{(1+\beta+k_T)}E_t\left(\widetilde{T}_{t+1}\right) + \frac{1}{(1+\beta+k_T)}\widetilde{T}_{t-1} - \frac{(1-\beta\varrho)}{(1+\beta+k_T)}\Delta\bar{T}_t \tag{3.A1}$$

Given the determinacy condition for (2.14), the general version of $(3.A1)$ described in *section 2.2.3*, we need the following condition to hold:

$$\left|\frac{1}{(1+\beta+k_T)}+\frac{\beta}{(1+\beta+k_T)}\right|<1$$

which always holds since $k_T>0$. The necessary and sufficient condition for a unique stationary REE is therefore that the Taylor principle (3.14) holds.

E-stability

As in *chapter 2*, we partition the system in the form of (2.28) and for E-stability we then need the eigenvalues of F_{ii} and the products of these and those of A_{jj} to have real parts less than one, for $i,j=1,2$. The second subsystem is given by $(3.A1)$, hence solving (2.34) using this yields:

$$b_{22}=\frac{1\mp\sqrt{1-4\beta\psi}}{2\psi\beta} \qquad \text{where; } \psi\equiv\frac{1}{1+\beta+k_T}$$

We refer back to *appendix 2B* where it is shown that this implies that:

$$F_{22}=\frac{2\beta\psi}{(1+\sqrt{1-4\beta\psi^2})} \qquad \text{and} \qquad A_{22}=\frac{2\psi}{(1+\sqrt{1-4\beta\psi^2})}$$

Further, since $F_{22}=\beta A_{22}$ and $0<\beta<1$, it follows that A_{22} is less than one if F_{22} is less than one. *Appendix 2A* shows that this is the case since $1-2(1+\beta)\psi>0$.

Hence, need the eigenvalues of F_{11} and the products of these with those of A_{11} to have real parts less than one. For the subsystem of union variables, made up of (3.2), (3.6) and (3.8) there are no lagged variables and $\Phi_{11}=0$. This implies that $b_{11}=0$, giving the following matrices:

$$F_{11}=\begin{bmatrix}\frac{\rho}{\varphi_Y+\varphi_\pi k_C+\rho} & \frac{1-\beta\varphi_\pi}{\varphi_Y+\varphi_\pi k_C+\rho}\\ \frac{\rho k_C}{\varphi_Y+\varphi_\pi k_C+\rho} & \frac{\beta(\varphi_Y+\sigma)+k_C}{\varphi_Y+\varphi_\pi k_C+\rho}\end{bmatrix},\qquad A_{11}=0$$

For E-stability we require the eigenvalues of F_{11} to have real parts less than one. Using the method of *appendix 2B* gives:

$$c_0=\frac{(\varphi_\pi-1)k_C+\varphi_Y(1-\beta)}{(\varphi_Y+\varphi_\pi k_C+\rho)}$$

$$c_1=c_0+\frac{\varphi_\pi k_C+\varphi_Y+\rho(1-\beta)}{(\varphi_Y+\varphi_\pi k_C+\rho)}$$

Hence, the system is E-stable when $c_0>0$, which gives the Taylor principle as the necessary and sufficient condition for E-stability:

$$0<(1-\beta)\,\varphi_Y+k_C\,(\varphi_\pi-1) \tag{3.14}$$

3.B Appendix: *The key matrices*

In this appendix, we present the matrices describing the economic system for the different interest rate rules. We specify B needed to examine determinacy, and Ω and Φ needed to examine E-stability. These matrices are defined for the general case in *section 2.2.3*.

For the policy rule: $r_t=\varphi_\pi\pi_t^W+\varphi_Y\tilde{y}_t^W$

$$B=\begin{bmatrix}\rho & n & 1-n & 0\\ 0 & \beta & 0 & 0\\ 0 & 0 & \beta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}\rho+\varphi_Y & \varphi_\pi n & \varphi_\pi(1-n) & 0\\ -k_C^H & 1+(1-n)k_T^H & -(1-n)k_T^H & -(1-n)k_T^H\\ -k_C^F & -nk_T^F & 1+nk_T^F & nk_T^F\\ 0 & -1 & 1 & 1\end{bmatrix}$$

$$\Omega=\begin{bmatrix}\rho+\varphi_Y & \varphi_\pi n & \varphi_\pi(1-n) & 0\\ -k_{CH} & 1 & 0 & -(1-n)k_{TH}\\ -k_{CF} & 0 & 1 & nk_{TF}\\ 0 & 1 & -1 & 1\end{bmatrix}^{-1}\begin{bmatrix}\rho & n & 1-n & 0\\ 0 & \beta & 0 & 0\\ 0 & 0 & \beta & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$\Phi = \begin{bmatrix} \rho+\varphi_Y & \varphi_\pi n & \varphi_\pi(1-n) & 0 \\ -k_{CH} & 1 & 0 & -(1-n)k_{TH} \\ -k_{CF} & 0 & 1 & nk_{TF} \\ 0 & 1 & -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For the policy rule: $r_t = \varphi_\pi \pi_t^W + \varphi_Y \widetilde{y}_t^W + \varphi_T \widetilde{T}_t$

$$B = \begin{bmatrix} \rho & n & 1-n & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \rho+\varphi_Y & \varphi_\pi n-\varphi_T & \varphi_\pi(1-n)\varphi_T & \varphi_T \\ -k_C^H & 1+(1-n)k_T^H & -(1-n)k_T^H & -(1-n)k_T^H \\ -k_C^F & -nk_T^F & 1+nk_T^F & nk_T^F \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

$$\Omega = \begin{bmatrix} \rho+\varphi_Y & \varphi_\pi n & \varphi_\pi n & \varphi_T \\ -k_{CH} & 1 & 0 & -(1-n)k_{TH} \\ -k_{CF} & 0 & 1 & nk_{TF} \\ 0 & -1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \rho & n & 1-n & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \rho+\varphi_Y & \varphi_\pi \gamma & \varphi_\pi(1-\gamma) & \varphi_T \\ -k_{CH} & 1 & 0 & -(1-n)k_{TH} \\ -k_{CF} & 0 & 1 & nk_{TF} \\ 0 & 1 & -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For the policy rule: $r_t = \varphi_\pi E_t\left(\pi_{t+1}^W\right) + \varphi_Y E_t\left(\widetilde{y}_{t+1}^W\right)$

$$B = \begin{bmatrix} \rho-\varphi_Y & n\,(1-\varphi_\pi) & (1-n)\,(1-\varphi_\pi) & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}$$
$$\begin{bmatrix} \rho & 0 & 0 & 0 \\ -k_C^H & 1+(1-n)k_T^H & -(1-n)k_T^H & -(1-n)k_T^H \\ -k_C^F & -nk_T^F & 1+nk_T^F & nk_T^F \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

$$\Omega = \begin{bmatrix} \rho & 0 & 0 & 0 \\ -k_{CH} & 1 & 0 & -(1-n)k_{TH} \\ -k_{CF} & 0 & 1 & nk_{TF} \\ 0 & 1 & -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \rho-\varphi_Y & n\,(1-\varphi_\pi) & (1-n)\,(1-\varphi_\pi) & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \rho & 0 & 0 & 0 \\ -k_{CH} & 1 & 0 & -(1-n)k_{TH} \\ -k_{CF} & 0 & 1 & nk_{TF} \\ 0 & 1 & -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For the policy rule: $r_t = \varphi_\pi E_t\left(\pi_{t+1}^W\right) + \varphi_Y E_t\left(\widetilde{y}_{t+1}^W\right) + \varphi_T E_t\left(\widetilde{T}_{t+1}\right)$

$$B = \begin{bmatrix} \rho - \varphi_Y & n\left(1-\varphi_\pi\right)+\varphi_T & \left(1-n\right)\left(1-\varphi_\pi\right)-\varphi_T & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}$$
$$\begin{bmatrix} \rho & \varphi_\pi n - \varphi_T & \varphi_\pi(1-n)+\varphi_T & \varphi_T \\ -k_C^H & 1+(1-n)k_T^H & -(1-n)k_T^H & -(1-n)k_T^H \\ -k_C^F & -nk_T^F & 1+nk_T^F & nk_T^F \\ 0 & -1 & 1 & 1 \end{bmatrix}$$
$$\Omega = \begin{bmatrix} \rho & 0 & 0 & 0 \\ -k_{CH} & 1 & 0 & -(1-n)k_{TH} \\ -k_{CF} & 0 & 1 & nk_{TF} \\ 0 & -1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \rho-\varphi_Y & n\left(1-\varphi_\pi\right) & \left(1-n\right)\left(1-\varphi_\pi\right) & -\varphi_T \\ 0 & 0 & \beta & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\Phi = \begin{bmatrix} \rho & 0 & 0 & 0 \\ -k_{CH} & 1 & 0 & -(1-n)k_{TH} \\ -k_{CF} & 0 & 1 & nk_{TF} \\ 0 & 1 & -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.C Appendix: *Proof of proposition 2*

In this appendix we prove *proposition 5* of *chapter 3*. The economy is now described by two independent subsystems. Union variables $\left\{\widetilde{y}_t^W, \pi_t^W\right\}$ are determined by (3.2), (3.6) and the policy rule (3.11), while relative variables $\left\{\pi_t^R, \widetilde{T}_t\right\}$ are determined by (3.5) and (3.7) or equivalently $(3.A1)$. As shown in the above *proof of proposition 1* in *appendix 3A*, the relative subsystem induces both determinacy and E-stability independently of monetary policy. Hence, for determinacy and E-stability of the monetary union, we derive the determinacy and E-stability conditions for the subsystem given by (3.2), (3.6) and (3.11).

Determinacy

To examine determinacy we write the system in the partitioned form of (2.13) as in *section 2.2.3*. We have:

$$B_{11} = \begin{bmatrix} \frac{1+\beta^{-1}k_C\rho^{-1}(1-\varphi_\pi)}{(1-\rho^{-1}\varphi_Y)} & -\frac{\rho^{-1}(1-\varphi_\pi)\beta^{-1}}{(1-\rho^{-1}\varphi_Y)} \\ -\beta^{-1}k_C & \beta^{-1} \end{bmatrix}$$

Using the methodology of above we have:

$a_0 = \beta^{-1}(1-\rho^{-1}\varphi_Y)^{-1}$

$a_1 = -\left(1+\beta^{-1}k_C\rho^{-1}\left(1-\varphi_\pi\right)\right)(1-\rho^{-1}\varphi_Y)^{-1} - \beta^{-1}$

From $|a_1| < |1+a_0|$, we have the two determinacy conditions:

$$0 < k_C\left(\varphi_\pi - 1\right) + \left(1-\beta\right)\varphi_Y \tag{3.14}$$

and

$$2\sigma(1+\beta) > k_C\left(\varphi_\pi - 1\right) + \varphi_Y(1+\beta) \tag{3.15}$$

E-stability

Proceeding as in *appendix 3A*, we have:

$$F_{11} = \begin{bmatrix} (1-\rho^{-1}\varphi_Y) & \rho^{-1}\left(1-\varphi_\pi\right) \\ k_C(1-\rho^{-1}\varphi_Y) & \beta + k_C\rho^{-1}\left(1-\varphi_\pi\right) \end{bmatrix}, A_{11} = 0$$

For E-stability F_{11} needs both of its eigenvalues to have real parts less than one. We then need: $c_0 > 0$ *and* $c_1 > 0$ for E-stability, where:

$c_0 = \rho^{-1}\varphi_Y(1-\beta) + k_C\rho^{-1}\left(\varphi_\pi - 1\right)$

and

$$c_1 = \rho^{-1}\varphi_Y + (1-\beta) + k_C\rho^{-1}(\varphi_\pi - 1)$$
$$= c_0 + \beta\rho^{-1}\varphi_Y + (1-\beta)$$

Hence, the monetary union is E-stable when $c_0 > 0$. This solves for the Taylor principle:

$$0 < k_C(\varphi_\pi - 1) + (1-\beta)\varphi_Y \tag{3.14}$$

Chapter 4
Learning in A Monetary Union with Fiscal and Monetary Policy

Chapter Overview

So far, we have considered determinacy and E-stability of monetary policy under the assumption that fiscal policy has no role in stabilizing shocks. In this chapter, we study learning when fiscal policy stabilizes relative or country-specific shocks, while monetary policy stabilizes union-wide shocks. We use the general equilibrium sticky price model put forward by Beetsma & Jensen (2005) to consider three different specifications of a government spending rule and the combinations of each of these with either a contemporaneous or a forward looking Taylor rule.

It is found that when fiscal policy reacts to the terms of trade, it needs to satisfy determinacy and E-stability conditions independently of monetary policy. On the other hand, when the fiscal authorities stabilize domestic output gaps, this has a direct effect on the determinacy and E-stability of monetary policy. Finally, a relative inflation fiscal policy rule needs to satisfy stability conditions independently, but could at the same time also affect the E-stability of monetary policy.

4.1 Introduction

4.1.1 Overview

In joining a monetary union, countries relinquish independence of monetary policy as an instrument for stabilizing domestic shocks. We showed in *chapter 1* that the union central bank has no means of stabilizing relative shocks, when price rigidities are homogeneous across the monetary union. Even in the heterogeneous case, there is a trade off between stabilizing relative and union-wide shocks. Furthermore, with several countries in the monetary union, monetary policy becomes a futile tool for tackling country-specific shocks.

Given this, fiscal policy becomes more important in a monetary union as an instrument for stabilizing domestic shocks. Consequently, in this chapter we consider a monetary union model in which the union central bank implements policy to stabilize union-wide shocks, while fiscal policy is employed to support the domestic economies in the event of relative shocks.

We examine determinacy and E-stability of a monetary union model in which the central bank sets interest rates according to either a contemporaneous or a forward looking Taylor rule, while the national fiscal authorities adjust government spending to either the terms of trade, relative inflation or domestic output. This hence gives six different policy combinations to consider. While a fiscal policy reacting to the output gap aims to offset shocks, this is not the case for a policy rule reacting to the terms of trade or relative inflation. Rather, such a rule acts to support domestic demand when the terms of trade or relative prices make domestic goods less competitive. This has the effect of reinforcing shocks to competitiveness and differs from the interest rate rule considered in *chapter 3*, which actually stabilized the terms of trade by offsetting shocks to competitiveness.

4.1.2 The model framework

The model considered here is that of Beetsma & Jensen (2005) and is an extension of the Benigno (2004) model to allow for fiscal policy to actively stabilize macroeconomic shocks.

The monetary union is made up of a continuum of households spread over the unit interval. Of these we have $[0, n)$ residing in home country and $(n, 1]$ residing in foreign country. Households consume baskets of goods produced in both countries, while producing differentiated goods under monopolistic competition. Fiscal governments only consume goods produced domestically. We assume that Ricardian equivalence holds, as public spending is financed by either debt issuance or by lump sum taxes. Financial markets are complete internationally, so that consumption is guaranteed for all households within the monetary union. However, output can still differ in the two member countries due to fluctuations in the terms of trade and government expenditure.

Producers face a fixed probability $(1 - \alpha)$ of changing their prices in each period, following Calvo (1983). We assume that both countries have the same level of price rigidity.

4.1.3 Related literature and main results

Sargent & Wallace (1981) emphasize the importance of fiscal and monetary policy coordination and show that an unbalanced fiscal budget can lead to speculative hyperinflation. In light of this, the convergence criteria of the EMU puts upper bounds on the budget deficits and government debt levels of its member countries.[1]

Leeper (1991) considers combinations of fiscal and monetary policy rules in which policy is either active or passive. He defines passive fiscal policy as one where the fiscal authorities adjust taxes to finance spending, while active fiscal policy does not adhere to a balanced budget.[2] Similarly, passive monetary policy adjusts the money supply to support fiscal spending, while active monetary policy pursues aggressive inflation stabilization independently of fiscal policy, so as to satisfy the Taylor principle. It is concluded that determinacy results when either policy is active while the other is passive. So, the Taylor principle does not have to be satisfied by the monetary authorities if fiscal policy is sufficiently aggressive in stabilizing prices. If both monetary and fiscal policy are passive, the system becomes indeterminate with multiple stationary equilibria. On the other hand, if the interest rate rule satisfies the Taylor principle while fiscal policy ignores the budget constraint, then all solutions to the system are explosive.

As pointed out by Krugman (1998, 2000), fiscal policy has been regarded as a useful tool to excerpt economies from liquidity traps. This idea is based on the notion that a

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1 *Article 104* of the *convergence criteria* states that budget deficits may not normally exceed 3%, while government debt is limitied to 60% of GDP (see *www.ecb.int*).

2 Under active fiscal policy, taxes are increased to at least cover interest payments on newly issued debt.

liquidity trap is an inferior low-inflation steady state, where monetary policy is ineffective since interest rates are restricted to being non-negative. Evans & Honkapohja (2005) introduce learning into a flexible price model to analyse liquidity traps. They find that while the superior of two equilibria is stable under learning, the liquidity trap is not. Large negative shocks can therefore lead to deflationary spirals with falling prices and output. Switching to a more aggressive monetary policy rule at low inflation levels was found to be an effective measure for preventing this. However, when prices are sticky rather than flexible, Evans, Guse & Honkapohja (2008) conclude that relying solely on aggressive monetary policy is not sufficient to avoid a downward spiral of output and inflation, following a large negative shock. Instead, aggressive monetary policy must be accompanied by an aggressive fiscal policy rule.[3,4]

In this chapter, government consumption increases the demand for domestic goods and thereby increases domestic output, while the government budget constraints are assumed to be satisfied so that spending is financed by either debt issuance or lump sum taxes. The fiscal policy rules considered here do not have the objective of stabilizing neither the terms of trade nor relative inflation. Rather, fiscal policy supports demand for domestic goods following shocks to relative prices. This implies that following a deterioration of the terms of trade, government expenditure is increased, which deteriorates the terms of trade further. Consequently, in contrast to Evans, Guse & Honkapohja (2008), we find that an excessively aggressive fiscal policy rule reacting either to relative inflation

[3] The fiscal policy rule considered is a tax feedback rule that Leeper (1991) classifies as passive.

[4] As pointed out by Evans (2008), an aggressive fiscal policy is by its own sufficient to prevent the economy from reaching the liquidity trap.

or the terms of trade could eliminate all stable equilibria regardless of the monetary policy pursued by the central bank.[5] This also contrasts our findings in the previous chapter, where a Taylor rule that stabilizes the terms of trade was found to both help induce determinacy and E-stability. Furthermore, determinacy and E-stability conditions in this case must be met simultaneously and independently by both fiscal and monetary policy. The stability conditions for monetary policy replicate those of *chapter 2*, with one exception.[6] If monetary policy is forward looking, while fiscal spending is aggressively adjusted to fluctuations in relative inflation, but not sufficiently so to induce an explosive system, then the indeterminate and E-stable region for monetary policy could become unstable under RLS learning.

In addition to the aforementioned fiscal policy rules, we consider one in which government spending is used as a means to stabilize the domestic output gaps. In this case, determinacy and E-stability hinges upon monetary policy, but fiscal policy has a direct impact on the stability of monetary policy. The more aggressive fiscal policy is, the more aggressive does monetary policy need to be to guarantee E-stability.

4.1.4 Organization

The following section presents the monetary union model in its reduced form as well as the economic policy rules. We then outline the results for each fiscal policy rule in turn; *section*

[5] Note that in the Leeper (1991) model, an aggressive (or active) fiscal policy rule eliminates all stable equilibria only when monetary policy is aggressive as well. Here, the system becomes explosive regardless of monetary policy.

[6] Recall that these are the closed economy results presented by Bullard & Mitra (2002). *Chapter 3* concluded that these results also hold for the Benigno (2004) model when price rigidities are homogeneous in the monetary union. An assumption that is maintained throughout this chapter.

4.3 looks at the terms of trade fiscal policy rule, *section 4.4* examines the relative inflation fiscal policy rule and finally we present the results for the output gap fiscal policy rule in *section 4.5*. In each of these cases we first consider determinacy and then E-stability and present our results both algebraically and graphically. The chapter is concluded by *section 4.6*.

4.2 The Environment

This section summarizes the main features of the monetary union model and presents it in its reduced form. This is followed by a description of the set of monetary and fiscal policy rules that we examine in following sections. We also provide a calibration for the model parameters in order to complement our analytical results with graphical illustrations.

4.2.1 The baseline model

We make use of the two country monetary union model put forward by Beetsma & Jensen (2005). This extends the framework of Benigno (2004), by allowing the two countries' sovereign governments to actively use fiscal policy to support the domestic economy following country-specific shocks. The monetary union is inhabited by a continuum of agents, or households spread over the unit interval. Of these, $n\epsilon\,[0,1]$ reside in the home country while $(1-n)$ reside in the foreign country. Each household produces a unique product under imperfect competition, while consuming bundles of consumer goods produced both domestically and abroad. The distortions to the economy from imperfect competition in production are offset by a government subsidy paid to producers. Financial markets are assumed to be complete internationally, implying perfect risk sharing of consumption for all agents across the monetary union. Public spending is financed by either debt issuance or by lump sum taxes so that Ricardian equivalence holds. Each government consumes only domestically produced goods so that even with perfect risk sharing of consumption, output in the two countries can differ both due to government expenditure or the terms of trade.

In this chapter we only outline the log-linearized equilibrium dynamics of the model under sticky prices à la Calvo (1983). This will serve our purpose and we refer readers wanting to examine the model in more detail to Beetsma & Jensen (2005). We denote home(foreign) country variables by superscript $H(F)$ and define union and relative variables as: $X^W \equiv nX^H + (1-n)X^F$ and $X^R \equiv X^F - X^H$, respectively. We further define $\tilde{x}$ as the log deviation of X under sticky prices from the value that would prevail under flexible prices $\bar{x}$. That is; $\tilde{x} = x - \bar{x}$, where x is the log deviation of X from its steady state value.[7]

Following Beetsma & Jensen (2005, p. 329) market clearing implies:

$$\tilde{y}_t^H = \xi_c\Big[(1-n)\tilde{T}_t + \tilde{c}_t^W\Big] + (1-\xi_c)\,\tilde{G}_t^H \tag{4.1}$$

and

$$\tilde{y}_t^F = \xi_c\Big[-n\tilde{T}_t + \tilde{c}_t^W\Big] + (1-\xi_c)\,\tilde{G}_t^F$$

[7] We have simplified the notation here in that $\tilde{x}$ is what Beetsma & Jensen (2005) refer to as $\left(\hat{X} - \tilde{X}\right)$. Note that we will for convenience denote the terms of trade and government spending by capital letters, although these have the same definition as $\tilde{x}$.

Where $\widetilde{y}_t$ is the output gap, $\widetilde{c}_t^W$ is consumption, $\widetilde{G}_t$ government expenditure and where ξ_c is the steady state consumption share of output. The terms of trade is defined as the ratio of the foreign price index over the domestic, i.e. $T \equiv \frac{P^F}{P^H}$.

The IS schedule, the new Keynesian Phillips curves and the condition for the terms of trade are given by:

$$\widetilde{c}_t^W = E_t(\widetilde{c}_{t+1}^W) - \rho^{-1}\xi_c \left[\left(r_t - E_t(\pi_{t+1}^W) \right) - rr_t \right] \tag{4.2}$$

$$\begin{aligned} \pi_t^H &= \beta E_t(\pi_{t+1}^H) + k^H(1+\eta\xi_c)(1-n)\widetilde{T}_t \\ &\quad + k^H \xi_c^{-1}(\rho+\eta\xi_c)\widetilde{c}_t^W + k^H \eta(1-\xi_c)\widetilde{G}_t^H \end{aligned} \tag{4.3}$$

$$\begin{aligned} \pi_t^F &= \beta E_t(\pi_{t+1}^F) - k^F(1+\eta\xi_c) n \widetilde{T}_t \\ &\quad + k^F \xi_c^{-1}(\rho+\eta\xi_c)\widetilde{c}_t^W + k^F \eta(1-\xi_c)\widetilde{G}_t^F \end{aligned} \tag{4.4}$$

$$\widetilde{T}_t = \widetilde{T}_{t-1} + \pi_t^R - \Delta\bar{T}_t \tag{4.5}$$

Here, the interest rate set by the common central bank is given by r_t, while π_t denotes inflation. The natural level of the nominal interest rate is given by:

$$rr_t = \rho E_t \left[\left(\bar{c}_{t+1}^W - \bar{c}_t^W \right) + \left(D_{t+1}^W - D_t^W \right) \right],$$

where D_t^W is proportional to the demand shock. The coefficient of relative risk aversion is given by $\rho \equiv U_{CC}(\bar{C},0)\bar{C}/U_C(\bar{C},0)$, and $\eta \equiv v_{YY}(\bar{Y},0)\bar{Y}/v_Y(\bar{Y},0)$ is a function of workers disutility from working. The discount factor is defined as β and σ is the elasticity of substitution across goods produced within a country.[8] Finally, we have the Phillips curve coefficients: $k^H \equiv \frac{(1-\alpha^H\beta)(1-\alpha^H)}{\alpha^H(1+\eta\sigma)}$ and $k^F \equiv \frac{(1-\alpha^F\beta)(1-\alpha^F)}{\alpha^F(1+\eta\sigma)}$ where $(1-\alpha^i)$ is the probability with which a firm in country i adjusts its price in any given period.[9]

We combine the two market clearing conditions (4.1) and obtain an expression for consumption as a function of output and government expenditure:

$$\tilde{c}_t^W = \xi_c^{-1}\tilde{y}_t^W - n\xi_c^{-1}(1-\xi_c)\tilde{G}_t^H - (1-n)\xi_c^{-1}(1-\xi_c)\tilde{G}_t^F \tag{4.6}$$

We now substitute (4.6) into $(4.2)-(4.5)$ to eliminate consumption. Then, by rewriting the two supply schedules making use of the definitions for union inflation $\pi_t^W \equiv n\pi_t^H + (1-n)\pi_t^F$ and relative inflation $\pi_t^R \equiv \pi_t^F - \pi_t^H$, we obtain the reduced form of the system:[10]

$$\tilde{y}_t^W = E_t(\tilde{y}_{t+1}^W) - \rho^{-1}\xi_c\left[\left(r_t - E_t(\pi_{t+1}^W)\right) - rr_t\right] - (1-\xi_c)E_t(\tilde{G}_{t+1}^W - \tilde{G}_t^W) \tag{4.7}$$

$$\pi_t^W = \beta E_t(\pi_{t+1}^W) + k_Y\tilde{y}_t^W - k_{G^W}\tilde{G}_t^W \tag{4.8}$$

$$\pi_t^R = \beta E_t(\pi_{t+1}^R) - k_T\tilde{T}_t + k_{G^R}\tilde{G}_t^R \tag{4.9}$$

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8 As σ is the source of the distortion to production, caused by mark-up pricing, it is assumed to be offset by subsidies. Specifically, $\frac{\sigma}{(\sigma-1)(1-\tau^H)} = \frac{\sigma}{(\sigma-1)(1-\tau^F)} = 1$ where τ^H and τ^F are the tax rates. This leaves economic policy in the form of monetary policy and government expenditure to tackle inefficiencies arising due to staggered price setting and disturbances to the terms of trade.

9 In this chapter we assume that $\alpha^H = \alpha^F$, so that $k^H = k^F$.

10 Substituting (4.6) eliminates consumption and the two market clearing conditions, while rewriting the supply schedules into a condition for union and one for relative inflation allows for the system to be partitioned, as shown in *section 4.2.3*. This is necessary to obtain analytical results.

$$\widetilde{T}_t = \widetilde{T}_{t-1} + \pi_t^R - \Delta\bar{T}_t \tag{4.10}$$

Where we have assumed that price rigidities in the two countries are equal, that is $\alpha^H = \alpha^F$ and hence that $k^H = k^F = k$. Furthermore we have defined $k_{G^W} \equiv k(1-\xi_c)\xi_c^{-1}\rho$, $k_{G^R} \equiv k(1-\xi_c)\eta$, $k_T \equiv k(1+\eta\xi_c)$ and $k_Y \equiv k(\xi_c^{-1}\rho+\eta)$. We can regard the above system as two subsystems; equations (4.7) and (4.8) contains the union variables $\left\{\widetilde{y}_t^W, \pi_t^W\right\}$, while equations (4.9) and (4.10) only include relative variables $\left\{\widetilde{T}_t, \pi_t^R\right\}$. We will refer to these as the union and relative subsystems throughout this chapter. These four equations, together with the interest rate policy rule of the union central bank and the fiscal policy rules of the national governments, define our system. We now proceed to present the policy rules.

4.2.2 The policy rules

In this section we present the Taylor rules employed by the union central bank and the government spending rules of the fiscal authorities. We consider both contemporaneous and forward looking Taylor rules that react to union inflation and the union output gap. While the monetary authorities seek to offset union wide shocks, fiscal policy reacts to relative shocks by either adjusting expenditure to fluctuations in the terms of trade, relative inflation or the domestic output gap.

Monetary policy rules

We consider two standard Taylor rules where the central bank adjusts interest rates to fluctuations in the output gap and inflation. The contemporaneous Taylor rule is given by:

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \tilde{y}_t^W \tag{4.11}$$

In this chapter we only consider Taylor rules that react to union variables, since as explained in *chapter 1*, in the absence of asymmetric price rigidities monetary policy has no impact on relative variables. Furthermore, *article 105* of the *Treaty on European Union* states that maintaining price stability is the Eurosystem of Central Banks' main objective. The target inflation of the ECB is the *Harmonized Index of Consumer Prices* in which each country is given a weight equal to its proportion of union consumption, which is equivalent to π_t^W in our model. Furthermore, the ECB has some scope to support economic activity of the Euro Area as long as this is not done at the expense of price stability. Hence the above policy rule includes the union output gap in addition to union inflation.

In addition to (4.11), as in previous chapters we consider an interest rate rule that is forecast based:

$$r_t = \varphi_\pi E_t\left(\pi_{t+1}^W\right) + \varphi_Y E_t\left(\tilde{y}_{t+1}^W\right) \tag{4.12}$$

The relative importance the central bank attaches to inflation and output is determined by the relative magnitudes of φ_π and φ_Y.

In what follows, we will denote the cases where (4.11) is employed by the superscript C for contemporaneous, while the superscript F will be used when monetary policy is forward looking according to (4.12).

Fiscal policy rules

While restrictions on EMU members' fiscal policy have been put in place to ensure stability of the euro-zone economy, fiscal policy can still be used, to some extent, to offset country-specific or relative shocks. The greater the asymmetry of shocks in a monetary union, the more important is the role of fiscal policy as an instrument for stabilization.

Below we consider some simple government spending rules for the two countries of our monetary union model. These policy rules differ from the tax rule considered by Evans et al. (2008), in that fiscal policy is fully financed in our case.[11]

We consider three fiscal policy rules in turn, each to be combined with each of the two Taylor rules outlined above, giving six different policy combinations.

Consider first the case in which government expenditure reacts to the terms of trade. A deterioration of the terms of trade makes goods produced in home country less competitive in relation to those produced in the foreign country. Fomestic government expenditure is therefore increased following such shocks. The policy rule is given by:

$$\widetilde{G}_t^H = -g_T^H \widetilde{T}_t \qquad and \qquad \widetilde{G}_t^F = g_T^F \widetilde{T}_t \tag{4.13}$$

[11] Note that Evans et al. (2008) use the tax rule of Leeper (1991).

We stress that (4.13) does not stabilize the terms of trade, but rather hinders stabilization. This is in contrast to the interest rate rule considered in *chapter 3*, which stabilized the terms of trade.

As an alternative to the terms of trade fiscal policy rule, we consider a rule in which the fiscal authorities react to relative inflation. As with the terms of trade, rather than offsetting relative inflation, governments use expenditure to support economic activity altered by a shock to relative inflation. The fiscal policy rule takes the form:

$$\widetilde{G}_t^H = -g_{\pi^R}^H \pi_t^R \qquad and \qquad \widetilde{G}_t^F = g_{\pi^R}^F \pi_t^R \tag{4.14}$$

Finally, we consider the case in which the fiscal authorities use government expenditure to close the domestic output gap. This is perhaps the more realistic case as governments have been known to use expansionary budgets in downturns while improving public finances during booms.

$$\widetilde{G}_t^H = -g_Y^H \widetilde{y}_t^H \qquad and \qquad \widetilde{G}_t^F = -g_Y^F \widetilde{y}_t^F \tag{4.15}$$

Because the state variables of our partitioned system $(4.7)-(4.10)$ are $\left\{\widetilde{y}_t^W, \pi_t^W, \pi_t^R, \widetilde{T}_t\right\}$, we must manipulate (4.15) to obtain it in terms of these variables. Using the definition of union and relative variables to rewrite the conditions in (4.15) yields:

$$\begin{aligned}
\widetilde{G}_t^W &= -\left[n g_Y^H + (1-n) g_Y^F\right] \widetilde{y}_t^W - \left[n(1-n)\left(g_Y^F - g_Y^H\right)\right] \widetilde{y}_t^R \\
&= -g_Y^W \widetilde{y}_t^W - n(1-n) g_Y^R \widetilde{y}_t^R
\end{aligned}$$

and

$$\begin{aligned}\widetilde{G}_t^R &= -\left[g_Y^F - g_Y^H\right]\widetilde{y}_t^W - \left[ng_Y^H + (1-n)g_Y^F\right]\widetilde{y}_t^R \\ &= -g_Y^R\widetilde{y}_t^W - g_Y^W\widetilde{y}_t^R\end{aligned}$$

Furthermore, combining the market clearing conditions (4.1) and rearranging gives:
$\widetilde{y}_t^R = -\xi_c\widetilde{T}_t + (1-\xi_c)\widetilde{G}_t^R$

We substitute this expression back into the fiscal policy rules and rearrange before solving the resulting two expressions simultaneously to obtain union and relative fiscal policy as a function of the union output gap and the terms of trade. The resulting policy rules are:

$$\widetilde{G}_t^W = -\psi_Y^W\widetilde{y}_t^W + \psi_T^W\widetilde{T}_t \qquad and \qquad \widetilde{G}_t^R = -\psi_Y^R\widetilde{y}_t^W + \psi_T^R\widetilde{T}_t \tag{4.15a}$$

where we have defined:

$\psi_Y^W \equiv \left[g_Y^W - \frac{n(1-n)(1-\xi_c)\left(g_Y^R\right)^2}{1+g_Y^W(1-\xi_c)}\right]$, $\psi_T^W \equiv \left[\frac{n(1-n)(1-\xi_c)\left(g_Y^R\right)}{1+g_Y^W(1-\xi_c)}\right]$, $\psi_Y^R \equiv \left[\frac{g_Y^R}{1+g_Y^W(1-\xi_c)}\right]$ and
$\psi_T^R \equiv \left[\frac{g_Y^W\xi_c}{1+g_Y^W(1-\xi_c)}\right]$

To obtain analytical results we consider a special case of $(4.15a)$ in which $g_Y^H = g_Y^F$ and consequently $\psi_T^W = \psi_Y^R = 0$.

In sum, the fiscal policy rules dictate that government spending is either adjusted to the terms of trade (4.13), relative inflation (4.14) or fluctuations in the domestic output gap (4.15). We will denote these cases by the superscripts *TT*, *RI* and *DY*, respectively.

4.2.3 Calibration

To illustrate our results graphically, we assign values to the structural parameters of our model using the calibration suggested by Beetsma & Jensen (2005). For robustness we have also examined our results using two alternative calibrations by Benigno (2004) and Rotemberg & Woodford (1998). We found that our qualitative findings were not altered, so in what follows we will present only the parameter values of Beetsma & Jensen (2005). We set the intertemporal rate of substitution β to 0.99 and the coefficient of relative risk aversion ρ is set to a value of 2.5.[12] Given that 0.6 and 0.2 are reasonable approximations of the consumption and government expenditure shares of output respectively, we set the steady state consumption to be three times larger than the steady state government expenditure, that is $\xi_c = 0.75$. In choosing the labour supply elasticity η and the mark-up σ, there is a trade off between getting reasonable values for these and getting a realistic response of inflation to changes in the real variables. By setting $\eta = \sigma = 3$, we get a Phillips curve coefficient of $k = 0.0086$, implying that the elasticity of inflation with respect to the consumption gap is around 0.04.[13] Price contracts are assumed to last for a year on average (4 quarters) and hence we set $\alpha = 0.75$.[14] Finally, we assume, where not otherwise specified, that both countries are of equal size, i.e. $n = 0.5$.

We now proceed to present our results. We consider each fiscal policy rule in turn by first combining it with the contemporaneous and then with the forward looking interest rate rule. In each case, we first examine determinacy and then E-stability.

12 See Beetsma and Schotman (2001).

13 See Beetsma & Jensen (2005, p. 334).

14 This is consistent with empirical evidence by Benigno & Lopez-Salido (2006) for Europe‘s main economies.

4.3 Terms of Trade Fiscal Policy Rule

In this section we look at determinacy and E-stability of the monetary union when the fiscal authorities use the terms of trade spending rule:

$$\tilde{G}_t^H = -g_T^H \tilde{T}_t \qquad and \qquad \tilde{G}_t^F = g_T^F \tilde{T}_t \tag{4.13}$$

We will in turn consider the cases where this rule is used in combination with the contemporaneous (4.11) and forward looking (4.12) Taylor rules of *section 4.2.2*. These cases are denoted as *TT-C* and *TT-F* respectively.

4.3.1 Terms of trade fiscal policy rule with contemporaneous Taylor rule

In this section we look at determinacy and E-stability of the monetary union when the terms of trade fiscal policy rule (4.13) is combined with the contemporaneous Taylor rule:

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \tilde{y}_t^W \tag{4.11}$$

We refer to this case as *TT-C* and first proceed to look at determinacy and then at E-stability.

Determinacy: case *TT-C*

To examine determinacy, use the methodology outlined in *section 2.2.3* and the appendices of the two previous chapters. The system is now given by equations $(4.7)-(4.10)$, the interest rate rule (4.11) and the fiscal policy rule (4.13). This system is block-triangular, since combining (4.9), (4.10) with the government spending rule (4.13), gives an independent subsystem that determines relative variables $\left\{\pi_t^R, \widetilde{T}_t\right\}$. This subsystem must be determinate, while the subsystem made up of (4.8), (4.8), (4.11) and (4.13) must be determinate in $\left\{\widetilde{y}_t^W, \pi_t^W\right\}$, for the union economy to have a unique stationary REE. Considering first the relative subsystem, we substitute the fiscal policy rule (4.13) into (4.9) and combine this with (4.10) to eliminate relative inflation. We have the univariate condition for the terms of trade:

$$\widetilde{T}_t = \beta\theta^{-1}E_t\left(\widetilde{T}_{t+1}\right) + \theta^{-1}\widetilde{T}_{t-1} - (1-\beta\varrho)\theta^{-1}\Delta\bar{T}_t \tag{4.16}$$

where $\theta \equiv \left(1+\beta+k_T - k_{G^R}(g_T^H + g_T^F)\right)$. For equilibrium determinacy of the union economy, (4.16) must then be determinate in $\widetilde{T}_t$, while the following matrix has both of its eigenvalues outside the unit circle:

$$B_{11}^{TT-C} = \begin{bmatrix} \rho^{-1}\xi_c\left(\varphi_Y + \beta^{-1}k_Y\right)+1 & \rho^{-1}\xi_c\left(\varphi_\pi - \beta^{-1}\right) \\ -\beta^{-1}k_Y & \beta^{-1} \end{bmatrix}$$

This gives our first result.

Proposition 1 *When fiscal policy reacts to the terms of trade according to:*

$$\widetilde{G}_t^H = -g_T^H\widetilde{T}_t \qquad \textit{and} \qquad \widetilde{G}_t^F = g_T^F\widetilde{T}_t \tag{4.13}$$

while monetary policy follows a contemporaneous Taylor rule of the form:

$$r_t = \varphi_\pi\pi_t^W + \varphi_Y\widetilde{y}_t^W \tag{4.11}$$

then the necessary and sufficient conditions for determinacy are:

$$0 < (1-\beta)\,\varphi_Y + k_Y\,(\varphi_\pi - 1) \tag{4.17}$$

and either

$$g_T^H + g_T^F < \frac{k_T}{k_{G^R}} \tag{4.18}$$

or

$$\frac{2(1+\beta)+k_T}{k_{G^R}} < g_T^H + g_T^F \tag{4.19}$$

Proof. See *appendix 4A*. ■

We first note that monetary and fiscal policy must both meet determinacy conditions independently of one another, for the union economy to be determinate. The determinacy condition for monetary policy is the Taylor principle found as a key condition in previous chapters. The fiscal authorities must either only modestly adjust government spending to changes in the terms of trade or it must be extremely aggressive in its response to fluctuations in the terms of trade. When government expenditure is aggressive but not sufficiently so to induce determinacy, the system is explosive and no stationary REE exists. Leeper (1991) also finds that an aggressive fiscal policy rule can eliminate all stationary equilibria. However, in his model this occurs when monetary policy is aggressive as well, to the extent that it satisfies the Taylor principle. This is because the government budget constraint is violated by the authorities in that case, making agents unwilling to hold government bonds. Here, the budget constraint is satisfied and fiscal policy destabilizes the system by reinforcing shocks to the terms of trade.

For example, in the absence of fiscal policy, a deterioration of the terms of trade lowers demand for domestic goods from consumers in both countries, which eventually lowers domestic prices and re-stabilizes the terms of trade . If the fiscal authorities instead use government expenditure to support demand for domestic goods in this case, to the extent that the boost in demand from government spending exceeds the reduction in consumption due to the terms of trade shock, then rather than stabilizing, the terms of trade will deteriorate further. This in turn increases government spending, leading to an explosive equilibrium path.

It should be noted that (4.19) implies an extreme and perhaps unrealistic fiscal response to the terms of trade. Specifically, it would require combined fiscal spending to change by more than six hundred percent to a one percent change in the terms of trade, given the calibration of *section 4.2.3*. On the other hand, for (4.18) to be satisfied, government expenditure cannot change by more than four percent to a one percent change in the terms of trade. We regard the latter as more plausible and hence interpret the result as a caveat against using government expenditure as an aggressive policy instrument.

This could have implications for the potential costs of joining a monetary union, since it further restricts the tools available to tackle asymmetric shocks. Llosa & Tuesta (2008) and Wang (2006) find that a monetary policy rule that stabilizes the exchange rate in addition to output and inflation can help induce determinacy. This result holds for a Taylor rule that stabilizes the terms of trade in a monetary union with heterogeneous price rigidities, as shown in *chapter 3*. Fiscal policy does not bring this added stability to the system, but rather the contrary.

Figure 4.1 plots the determinacy region for monetary policy, while *figure 4.2* shows that for fiscal policy, given the calibration outlined in *section 4.2.3*.

E-stability: case *TT-C*

To examine E-stability we substitute (4.11) and (4.13) into $(4.7)-(4.10)$ and write the system in the form of equation (2.28) of *section 2.2.3*. As in the above case for determinacy, we simplify the system by eliminating relative inflation. The system is then given by the independent relative subsystem determining the terms of trade (4.16) and the subsystem determining union output and inflation; (4.7), (4.8), (4.11) and (4.13). The key matrices of (2.28) become:

$$\Omega_{11}^{TT-C} = \begin{bmatrix} \frac{\xi_c^{-1}\rho}{\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho} & \frac{1-\beta\varphi_\pi}{\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho} \\ \frac{\xi_c^{-1}\rho k_Y}{\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho} & \frac{\beta\left(\varphi_Y+\xi_c^{-1}\rho\right)+k_Y}{\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho} \end{bmatrix}, \Phi_{11}^{TT-C} = 0$$

$$\Omega_{22}^{TT-C} = \beta\theta^{-1}, \qquad \text{and} \qquad \Phi_{22}^{TT-C} = \theta^{-1}.$$

It follows that the conditions for E-stability coincide with the above criteria for determinacy, as shown in *appendix 4B*.

Proposition 2 *Suppose that agents use recursive least squares to update their beliefs about the economy and that the government spending rule is given by:*

$$\widetilde{G}_t^H = -g_T^H \widetilde{T}_t \qquad \textit{and} \qquad \widetilde{G}_t^F = g_T^F \widetilde{T}_t \tag{4.13}$$

while the monetary policy rule is:

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \tilde{y}_t^W \tag{4.11}$$

then an MSV solution is locally asymptotically stable under learning when the following conditions hold:

$$0 < (1-\beta)\,\varphi_Y + k_Y\,(\varphi_\pi - 1) \tag{4.17}$$

and either

$$g_T^H + g_T^F < \frac{k_T}{k_{G^R}} \tag{4.18}$$

or

$$\frac{2(1+\beta) + k_T}{k_{G^R}} < g_T^H + g_T^F \tag{4.19}$$

Proof. See *appendix 4B.* ■

So all unique stationary equilibria are locally stable under RLS learning, while no multiple equilibria are E-stable. Again, fiscal and monetary policy must independently satisfy the stability conditions for the union economy to be E-stable. This implies that the whole union economy could become unstable if only one of its member countries uses an unstable fiscal policy rule.

Sargent & Wallace (1981) show that consistent or excessive budget deficits lead to seigniorage that in turn destabilizes the economy through hyper inflation. The EMU has therefore put restrictions on both the debt levels and the deficit levels of its member countries, as stipulated in the *convergence criteria*. Our findings strengthen the argument for restricting the flexibility of fiscal spending, in particular; an excessive response of government expenditure to fluctuations in the terms of trade should be avoided. This makes fiscal policy less of a substitute for independent monetary policy of individual countries. *Figure 4.1* depicts the determinate and E-stable region for monetary policy, while *figure 4.2* shows that for fiscal policy. These two must be satisfied simultaneously for the union economy to be stable under learning. Note that if fiscal policy violates stability conditions there exists no stable equilibrium regardless of whether monetary policy induces stability or not.[15] In the case where monetary policy does not satisfy the Taylor principle while fiscal policy satisfies stability conditions, there exist multiple equilibria that are unstable under learning.

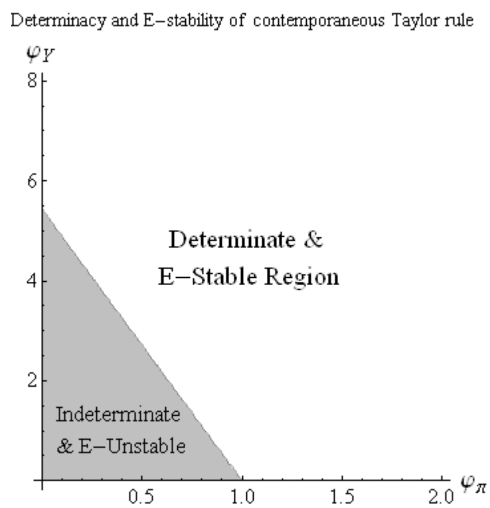

Fig. 4.1. The Taylor principle is both necessary and sufficent for determinacy and E-stability of a contemporaneous interest rate rule. The fiscal policy rule must simultaneously satisfy the appropriate conditions for the economy to be determinate and E-stable.

[15] We do not consider the E-stability of explosive solutions here, since these policies are to be avoided by policy makers even if they are learnable under recursive least squares. For a discussion on the E-stability of explosive solutions see Evans & Honkapohja (2001, *Ch 9.6*).

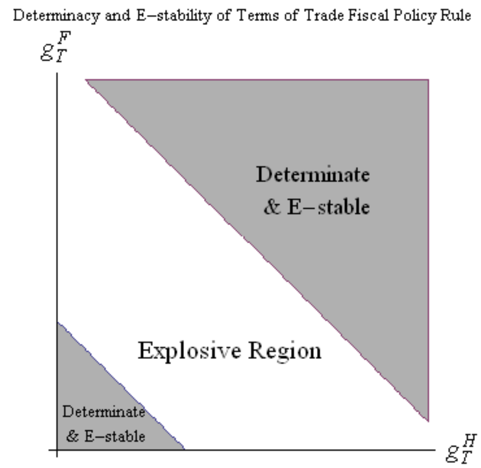

Fig. 4.2. Government expenditure needs to be either only modestly adjusted to changes in the terms of trade or it needs to be extremely aggressive. Anything in between will lead to an explosive system with no stationary REE. Note that the outer stability region requires unrealistically large responses of fiscal spending to fluctuaitons in the terms of trade.

4.3.2 Terms of trade fiscal policy rule with forward looking Taylor rule

As in the above case, the system is given by $(4.7)-(4.10)$ and the fiscal policy rule reacting to the terms of trade (4.13), however we now assume that the central bank is forward looking in its interest rate policy according to:

$$r_t = \varphi_\pi E_t \left(\pi^W_{t+1}\right) + \varphi_Y E_t \left(\widetilde{y}^W_{t+1}\right) \tag{4.12}$$

The conditions for determinacy and E-stability under this policy combination is outlined in what follows. Note that the case of a terms of trade fiscal policy rule and a forward looking Taylor rule is denoted by *TT-F*.

Determinacy: case *TT-F*

Since the interest rate does not affect relative variables, we can use of equation (4.16) to describe the relative subsystem. The subsystem determining the union variables $\left\{\widetilde{y}_t^W, \pi_t^W\right\}$ is now given by (4.7), (4.8), (4.11) and (4.13), giving the key matrix:

$$B_{11}^{TT-F} = \begin{bmatrix} \frac{1+\beta^{-1}k_Y\rho^{-1}\xi_c(1-\varphi_\pi)}{(1-\rho^{-1}\xi_c\varphi_Y)} & -\frac{\rho^{-1}\xi_c(1-\varphi_\pi)\beta^{-1}}{(1-\rho^{-1}\xi_c\varphi_Y)} \\ -\beta^{-1}k_Y & \beta^{-1} \end{bmatrix}$$

Since the condition determining the terms of trade (4.16) has not changed and since determinacy of the economy is determined by each subsystem independently, the constraints on fiscal policy are the same as above. However, because monetary policy now is forecast based, the determinacy conditions for the Taylor rule are the same as for the forward looking interest rate rules of previous chapters. That is, as in Bullard & Mitra (2002), the Taylor principle must be satisfied while monetary policy cannot be overly aggressive, in particular in its stabilization of the output gap.[16] We have the following proposition.

Proposition 3 *When fiscal policy reacts to the terms of trade:*

$$\widetilde{G}_t^H = -g_T^H \widetilde{T}_t \qquad \textit{and} \qquad \widetilde{G}_t^F = g_T^F \widetilde{T}_t \tag{4.13}$$

while the interest rate stabilizes expected future inflation and output:

$$r_t = \varphi_\pi E_t\left(\pi_{t+1}^W\right) + \varphi_Y E_t\left(\widetilde{y}_{t+1}^W\right) \tag{4.12}$$

then the necessary and sufficient conditions for determinacy are:

$$0 < (1-\beta)\varphi_Y + k_Y(\varphi_\pi - 1) \tag{4.17}$$

[16] Note that although Bullard & Mitra (2002) have an additional condition $\varphi_Y < \rho\xi_c^{-1}\left(\beta^{-1}+1\right)$, for the forward looking Taylor rule, this is redundant as shown in *appendix 4A*.

$$2\rho\xi_c^{-1}(1+\beta) > k_Y\left(\varphi_\pi - 1\right) + \varphi_Y(1+\beta) \tag{4.20}$$

and either

$$g_T^H + g_T^F < \frac{k_T}{k_{G^R}} \tag{4.18}$$

or

$$\frac{2(1+\beta)+k_T}{k_{G^R}} < g_T^H + g_T^F \tag{4.19}$$

Proof. See *appendix 4A.* ■

Hence, regardless of the interest rate rule employed, monetary and fiscal policy must meet stability conditions independently, when fiscal policy supports the economy following shocks to the terms of trade. The conditions for determinacy of fiscal policy are depicted in *figure 4.2*, while those for monetary policy are shown in *figure 4.3*.

E-stability: case *TT-F*

For E-stability of REE under a terms of trade fiscal policy rule and a forward looking Taylor rule, we write the system in the form of (2.28), where:

$$\Omega_{11}^{TT-F} = \begin{bmatrix} (1-\rho^{-1}\xi_c\varphi_Y) & \rho^{-1}\xi_c\left(1-\varphi_\pi\right) \\ k_Y(1-\rho^{-1}\xi_c\varphi_Y) & \beta + k_Y\rho^{-1}\xi_c\left(1-\varphi_\pi\right) \end{bmatrix}, \Phi_{11}^{TT-F} = 0$$

$$\Omega_{22}^{TT-F} = \beta\theta^{-1}, \qquad \text{and} \qquad \Phi_{22}^{TT-F} = \theta^{-1}.$$

We have the following result.

Proposition 4 *Given the government spending rule:*

$$\widetilde{G}_t^H = -g_T^H\widetilde{T}_t \qquad \textit{and} \qquad \widetilde{G}_t^F = g_T^F\widetilde{T}_t \tag{4.13}$$

and the Taylor rule:

$$r_t = \varphi_\pi E_t\left(\pi^W_{t+1}\right) + \varphi_Y E_t\left(\widetilde{y}^W_{t+1}\right) \tag{4.12}$$

the economy is E-stable when the following conditions hold:

$$0 < (1-\beta)\,\varphi_Y + k_Y\left(\varphi_\pi - 1\right) \tag{4.17}$$

and either

$$g^H_T + g^F_T < \frac{k_T}{k_{G^R}} \tag{4.18}$$

or

$$\frac{2(1+\beta) + k_T}{k_{G^R}} < g^H_T + g^F_T \tag{4.19}$$

Proof. See *appendix 4B.* ■

We conclude that for a fiscal policy rule that reacts to the terms of trade, fiscal and monetary policy must satisfy stability conditions independently, regardless of the monetary policy rule of the central bank. Furthermore, the determinacy and E-stability conditions for fiscal policy coincide. This is not the case for monetary policy however. For a forward looking Taylor rule, there is a policy region that induces multiple stationary equilibria that are all stable under RLS learning. The constraints on monetary policy are the same as in the homogeneous price rigidity case of *chapters 2* and *3*. That is, for a monetary union model with a terms of trade fiscal policy rule, the stability conditions for monetary policy are the same as for a closed economy.

We emphasize that for this policy combination, each subsystem being E-stable is sufficient for an E-stable monetary union. While this is always true about determinacy, it is not necessarily true for RLS learning stability. We will show in the following section that when fiscal spending is adjusted to relative inflation while the interest rate rule is forward looking, there is interdependency of the learning stability of fiscal and monetary policy.

Figure 4.2 shows the E-stability conditions for fiscal policy and *figure 4.3* shows the Taylor principle as the necessary and sufficient condition for E-stability of monetary policy.

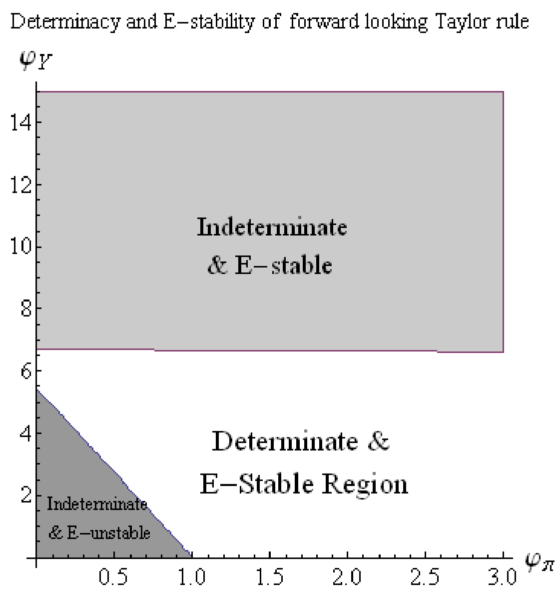

Fig. 4.3. The Taylor principle is not sufficient for determinacy, but it is sufficient for E-stability. With an aggressive forward looking Taylor rule there may be multiple stationary and learnable equilibria. Note that fiscal policy must also be determinate and E-stable for the economy to be so.

4.4 Relative Inflation Fiscal Policy Rule

Since there is no exchange rate to restore competitiveness following a shock to relative inflation in a monetary union, such a shock leads to larger distortions than it would in an open economy. As suggested by Beetsma & Jensen (2005), fiscal policy can assist the economy in this case by supporting domestic demand. In this section we consider a government spending rule that reacts to relative inflation according to:[17]

$$\widetilde{G}_t^H = -g_{\pi^R}^H \pi_t^R \qquad and \qquad \widetilde{G}_t^F = g_{\pi^R}^F \pi_t^R \tag{4.14}$$

As in the previous section, we combine the fiscal policy rule with each of the two Taylor rules of *section 4.2.2* in turn. In each case, determinacy is examined before E-stability.

4.4.1 Relative inflation fiscal policy rule with contemporaneous Taylor rule

Here we consider the system $(4.7)-(4.10)$, the fiscal policy rule (4.14) and the contemporaneous Taylor rule (4.11). We establish a set of conditions that fiscal and monetary policy must satisfy for determinacy and local stability under recursive least squares learning.

Determinacy: case *RI-C*

We substitute the fiscal policy rule (4.14) into (4.9) and combine the resulting condition with (4.10). This eliminates relative inflation and gives an independent condition determining the terms of trade:

$$\begin{aligned} \widetilde{T}_t \quad = \quad & \beta\theta^{-1}E_t\left(\widetilde{T}_{t+1}\right) + \left(1 - k_{G^R}\left(g_{\pi^R}^H + g_{\pi^R}^F\right)\right)\theta^{-1}\widetilde{T}_{t-1} \\ & - \left(1 - k_{G^R}\left(g_{\pi^R}^H + g_{\pi^R}^F\right) - \beta\varrho\right)\theta^{-1}\Delta\bar{T}_t \end{aligned} \tag{4.21}$$

[17] Recall that relative inflation is defined as: $\pi_t^R \equiv \pi_t^F - \pi_t^H$.

where we have defined $\theta \equiv \left(1 - k_{G^R}\left(g^H_{\pi^R} + g^F_{\pi^R}\right) + \beta + k_T\right)$. As explained in *section 4.2.3*, since we have a triangular system, it will be determinate when both of the subsystems are determinate. Combining the policy rules (4.11) and (4.14) with the IS schedule (4.7) and the NKPC (4.8) yields:

$$B^{RI-C}_{11} = \begin{bmatrix} \rho^{-1}\xi_c\left(\varphi_Y + \beta^{-1}k_Y\right) + 1 & \rho^{-1}\xi_c\left(\varphi_\pi - \beta^{-1}\right) \\ -\beta^{-1}k_Y & \beta^{-1} \end{bmatrix}$$

For determinacy of the case *RI-C* we hence require equation (4.21) to induce determinacy. while B^{RI-C}_{11} has both of its eigenvalues outside of the unit circle. Noting that $B^{RI-C}_{11} = B^{TT-C}_{11}$, it is clear that both eigenvalues are outside the unit circle if and only if the Taylor principle is satisfied. Consequently, as in the case of a terms of trade fiscal policy rule and a contemporaneous Taylor rule, this is the determinacy condition for monetary policy. However, the determinacy region for fiscal policy is now different, as stated in the following proposition.

Proposition 5 *When government expenditure is dictated by a policy rule reacting to relative inflation:*

$$\tilde{G}^H_t = -g^H_{\pi^R}\pi^R_t \qquad \text{and} \qquad \tilde{G}^F_t = g^F_{\pi^R}\pi^R_t \tag{4.14}$$

and the union central bank adjusts interest rates according to:

$$r_t = \varphi_\pi \pi^W_t + \varphi_Y \tilde{y}^W_t \tag{4.11}$$

then there exists a unique stationary REE when the following inequalities are satisfied:

$$0 < (1-\beta)\varphi_Y + k_Y(\varphi_\pi - 1) \tag{4.17}$$

$$\frac{1+\beta+\frac{1}{2}k_T}{k_{G^R}} > g^H_{\pi^R} + g^F_{\pi^R} \tag{4.22}$$

Proof. See *appendix 4A*. ■

When the above condition for fiscal policy is not met, all MSV solutions are explosive. The intuition for this is the same as for the terms of trade fiscal policy rule above. Following a negative shock to relative inflation, fiscal policy is implemented to support demand for domestic goods. However, if the fiscal expansion is sufficiently large, this will further increase relative prices of domestic goods, leading to an inflationary spiral.

The determinacy condition for fiscal policy here is not as restrictive as condition (4.18) for the terms of trade fiscal policy rule. Violation of condition (4.22) requires joint government expenditure to change by more than three hundred percent to a one percent change in inflation differentials, given the calibration of Beetsma & Jensen (2005) used in this chapter. On the other hand, here there is no determinacy region for an extremely aggressive fiscal policy as there was for the terms of trade policy rule when (4.19) is satisfied. Nonetheless, given the unrealistically aggressive response to shocks required for (4.19) to hold, we regard the fiscal policy rule reacting to relative inflation as more likely to induce a unique stationary REE than the terms of trade fiscal policy rule.

As in the previous section, fiscal and monetary policy both need to be determinate independently of one another for the economy to be determinate. The determinacy condition for monetary policy is illustrated in *figure 4.1*, while that for the relative inflation fiscal policy rule is depicted in *figure 4.4*.

E-stability: case RI-C

We write the system $(4.7)-(4.10)$, (4.12) and (4.14) in the form of (2.28). Again, we reduce the relative subsystem to equation (4.21). We have the following partitioned matrices:

$$\Omega_{11}^{RI-C} = \begin{bmatrix} \frac{\xi_c^{-1}\rho}{\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho} & \frac{1-\beta\varphi_\pi}{\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho} \\ \frac{\xi_c^{-1}\rho k_Y}{\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho} & \frac{\beta\left(\varphi_Y+\xi_c^{-1}\rho\right)+k_Y}{\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho} \end{bmatrix}, \Phi_{11}^{RI-C} = 0$$

$$\Omega_{22}^{RI-C} = \beta\theta^{-1}, \qquad \text{and} \qquad \Phi_{22}^{RI-C} = \theta^{-1}.$$

Appendix 4B derives the E-stability conditions stated in the following proposition.

Proposition 6 *When the fiscal policy rule:*

$$\tilde{G}_t^H = -g_{\pi^R}^H \pi_t^R \qquad \text{and} \qquad \tilde{G}_t^F = g_{\pi^R}^F \pi_t^R \tag{4.14}$$

is combined with the contemporaneous Taylor rule:

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \tilde{y}_t^W \tag{4.11}$$

then the following conditions are both necessary and sufficient for E-stability:

$$0 < (1-\beta)\,\varphi_Y + k_Y\,(\varphi_\pi - 1) \tag{4.17}$$

$$\frac{1+\beta+\frac{1}{2}k_T}{k_{G^R}} > g_{\pi^R}^H + g_{\pi^R}^F \tag{4.22}$$

Proof. See *appendix 4B*. ■

This is consistent with the proposition of McCallum (2007), stating that determinacy is sufficient for E-stability. Furthermore, as is often the case for contemporaneous rules, the determinacy and E-stability regions coincide, leaving no region of multiple stationary REE that are stable under learning.

As in the case of a terms of trade fiscal policy rule, an aggressive government spending rule by either country can destabilize the union economy. Given that $\left(g^H_{\pi^R} + g^F_{\pi^R}\right)$ in (4.22) is independent of country sizes, an unstable policy by even a small country can have implications for the stability of a large monetary union.

The E-stability region for monetary policy is shown in *figure 4.1*, while *figure 4.4* shows the determinate and E-stable region for the relative inflation fiscal policy rule.

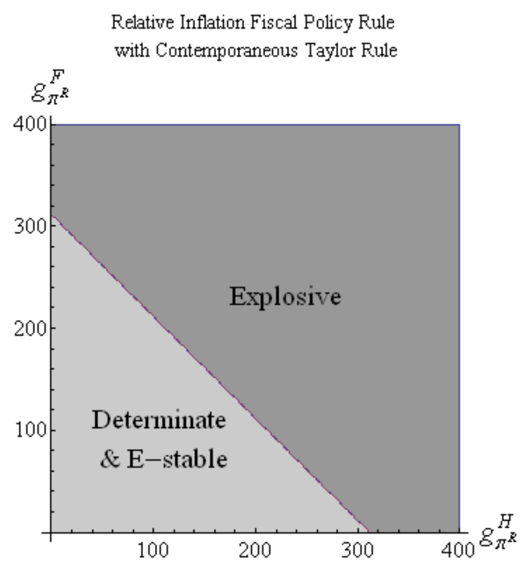

Fig. 4.4. An aggressive fiscal policy rule could lead to an explosive system even if monetary policy meets stability requirements. In addition to a stable fiscal policy, the Taylor principle (see *figure 4.1*) has to hold for *RI-C* to have a unique stationary REE that is stable under RLS learning.

4.4.2 Relative inflation fiscal policy rule with forward looking Taylor rule

This section looks at determinacy and E-stability under a relative inflation government spending rule and a forward looking interest rate rule. The assumption of the central bank being forward looking while fiscal policy reacts to contemporaneous data is reasonable as central banks have shown to take forecasts of future values into consideration when setting interest rates. The system is now given by $(4.7) - (4.10)$, the fiscal policy rule (4.14) and the Taylor rule (4.12).

Determinacy: case *RI-F*

For determinacy of REE we partition the system as in *section 2.2.3* and reduce the system further by using (4.21). The model economy has a unique stationary REE when (4.21) is determinate and the matrix B_{11}^{RI-F} has its eigenvalues outside the unit circle.

$$B_{11}^{RI-F} = \begin{bmatrix} \frac{1+\beta^{-1}k_Y\rho^{-1}\xi_c(1-\varphi_\pi)}{(1-\rho^{-1}\xi_c\varphi_Y)} & -\frac{\rho^{-1}\xi_c(1-\varphi_\pi)\beta^{-1}}{(1-\rho^{-1}\xi_c\varphi_Y)} \\ -\beta^{-1}k_Y & \beta^{-1} \end{bmatrix}$$

Noting that $B_{11}^{RI-F} = B_{11}^{TT-F}$ it is evident that the union subsystem is determinate when (4.17) and (4.20) are satisfied (see *appendix 4A*). The relative subsystem (4.21) is the same as in the above case of a contemporaneous Taylor rule and is thus determinate when (4.22) is satisfied. This means that as in previous cases, monetary and fiscal policy need to satisfy determinacy conditions independently of one another for the union economy to be determinate, which leads us to the following result.

Proposition 7 *When fiscal policy reacts to relative inflation according to:*

$$\tilde{G}{}_t^H = -g_{\pi^R}^H \pi_t^R \qquad and \qquad \tilde{G}{}_t^F = g_{\pi^R}^F \pi_t^R \tag{4.14}$$

while the interest rate is adjusted to expected future values of inflation and the output gap:

$$r_t = \varphi_\pi E_t \left(\pi_{t+1}^W\right) + \varphi_Y E_t \left(\tilde{y}{}_{t+1}^W\right) \tag{4.12}$$

then, the necessary and sufficient conditions for determinacy are:

$$0 < (1 - \beta)\, \varphi_Y + k_Y \left(\varphi_\pi - 1\right) \tag{4.17}$$

$$2\rho\xi_c^{-1}(1 + \beta) > k_Y \left(\varphi_\pi - 1\right) + \varphi_Y (1 + \beta) \tag{4.20}$$

and

$$\frac{1 + \beta + \frac{1}{2}k_T}{k_{G^R}} > g_{\pi^R}^H + g_{\pi^R}^F \tag{4.22}$$

Proof. See *appendix 4A*. ■

So neither fiscal policy nor monetary policy can be too aggressive if the system is to have a unique stationary REE. *Figures 4.5* and 4.*6* show the determinacy region for monetary and fiscal policy respectively. These two need to be satisfied simultaneously for the economy to be determinate. If monetary policy is such that it is in the indeterminate region of *figure 4.5* and fiscal policy is in the determinate region of *figure 4.6*, then there exists two stationary rational expectations equilibria. On the other hand, when fiscal policy is not determinate, there exist no stationary solution regardless of the monetary policy employed by the central bank.

E-stability: case *RI-F*

For learning stability we partition the system into a union and a relative subsystem as in previous sections, giving the matrices for (2.28):

$$\Omega_{11}^{RI-F} = \left[\begin{array}{cc} (1-\rho^{-1}\xi_c\varphi_Y) & \rho^{-1}\xi_c\,(1-\varphi_\pi) \\ k_Y(1-\rho^{-1}\xi_c\varphi_Y) & \beta + k_Y\rho^{-1}\xi_c\,(1-\varphi_\pi) \end{array} \right], \Phi_{11}^{RI-F} = 0$$

$\Omega_{22}^{RI-F} = \beta\theta^{-1}$, and $\Phi_{22}^{RI-F} = \theta^{-1}$.

As opposed to the above cases of E-stability, here it is not sufficient for each of the subsystems to be stable under learning for the whole system to be so. In fact, each subsystem directly affects the E-stability of the other subsystem. That is, the E-stability of monetary policy depends not only on the interest rate rule, but also on fiscal policy and vice versa. Although learning stability of each subsystem is not sufficient for the monetary union to be E-stable, it is a necessary condition, as explained in *appendix 4B*. We have the following result.

Proposition 8 *When the fiscal policy rule:*

$$\tilde{G}{}_t^H = -g_{\pi^R}^H \pi_t^R \qquad and \qquad \tilde{G}{}_t^F = g_{\pi^R}^F \pi_t^R \tag{4.14}$$

is combined with the monetary policy rule:

$$r_t = \varphi_\pi E_t\left(\pi_{t+1}^W\right) + \varphi_Y E_t\left(\tilde{y}{}_{t+1}^W\right) \tag{4.12}$$

then the necessary and sufficient conditions for E-stability are:

$$0 < (1-\beta)\,\varphi_Y + k_Y\,(\varphi_\pi - 1) \tag{4.17}$$

and either

$$g^H_{\pi^R} + g^F_{\pi^R} < \frac{1}{k_{G^R}} \tag{4.23}$$

or

$$\frac{1}{k_{G^R}} < \left(g^H_{\pi^R} + g^F_{\pi^R}\right) < \frac{1+\beta+\frac{1}{2}k_T}{k_{G^R}} \tag{4.24}$$

and

$$\varphi_Y + k_Y\left(\varphi_\pi - 1\right) < \rho\xi_c^{-1}\left(3+\beta\right) \tag{4.25}$$

Proof. See *appendix 4B*. ■

As discussed above, for a forward looking interest rate rule there is an indeterminate and E-stable region. This has been found for the closed economy by Bullard & Mitra (2002), for an open economy by Llosa & Tuesta (2008) and for a monetary union model in this thesis. *Proposition 8* implies that if fiscal policy is sufficiently aggressive to violate inequality (4.23) but not (4.24), then a subset of this indeterminate and E-stable region instead becomes unstable under RLS learning. The interdependence of E-stability for fiscal and monetary policy is illustrated in *figures 4.5* and *4.6*. Note that, when $\left(g^H_{\pi^R} + g^F_{\pi^R}\right) > \frac{1+\beta+\frac{1}{2}k_T}{k_{G^R}}$, all solutions are explosive regardless of the policy coefficients of monetary policy.

4.5 Domestic Output Fiscal Policy Rule

In previous sections we have considered fiscal policy rules that support domestic demand in the event of shocks affecting the competitiveness of domestic goods. In this section

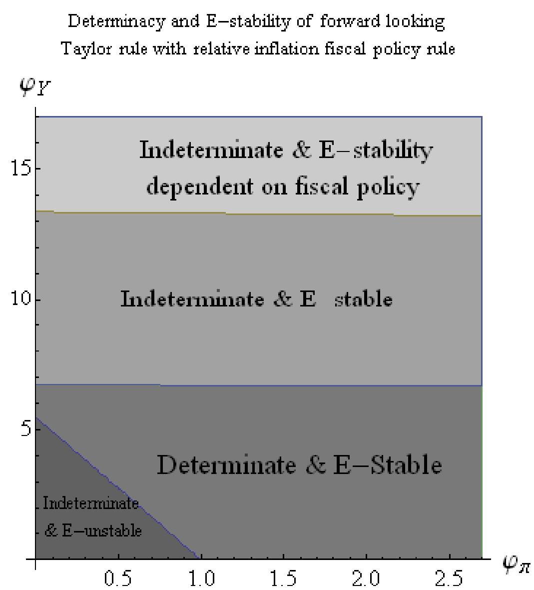

Fig. 4.5. The above graph shows the determinacy and E-stability of a forward looking monetary policy rule when fiscal policy reacts to relative inflation. An aggressive fiscal policy rule can make an aggressive monetary policy unstable under RLS learning if it is also indeterminate. This graph is to be considered jointly with *figure 4.6*.

we consider the case in which national governments seek to stabilize their economies by directly adjusting the level of government expenditure to offset shocks to the domestic output gap. This is perhaps the most realistic and observable government spending policy in practice. We use the rule outlined in *section 4.2.2*:

$$\tilde{G}{}_t^H = -g_Y^H \tilde{y}{}_t^H \quad \text{and} \quad \tilde{G}{}_t^F = -g_Y^F \tilde{y}{}_t^F \tag{4.15}$$

Thus government consumption is increased following a negative shock to domestic output. This policy rule allows fiscal policy to stabilize country-specific shocks, while the monetary authorities stabilize union-wide shocks. In order to obtain analytical results, we assume that $g_Y^H = g_Y^F$, implying the following special case for the policy rule (4.15):

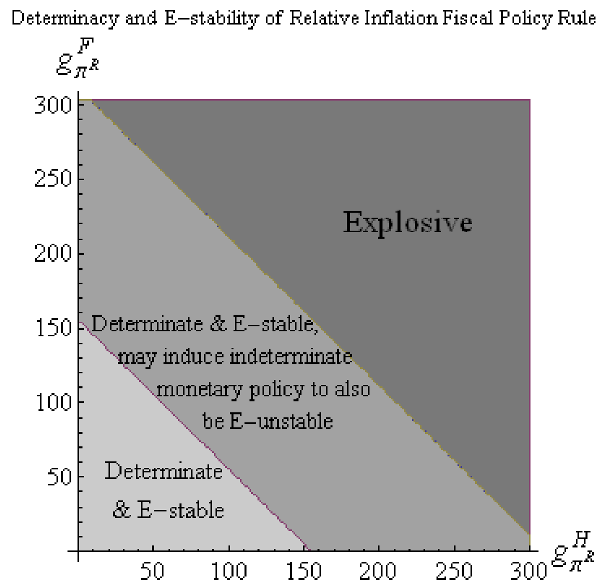

Fig. 4.6. A very aggressive fiscal policy eliminates all stationary REE. Even if fiscal policy is less aggressive, it could cause an aggressive monetary policy rule to become unstable under RLS learning if that rule is also indeterminate (see *figure 4.5*).

$$\tilde{G}_t^W = -g_Y^W \tilde{y}_t^W \qquad and \qquad \tilde{G}_t^R = \left[\frac{g_Y^W \xi_c}{1 + g_Y^W (1 - \xi_c)}\right] \tilde{T}_t \tag{4.15a}$$

where $g_Y^W \equiv n g_Y^H + (1-n) g_Y^F$. Note that this fiscal policy rule stabilizes the economy by offsetting shocks. This was not the case for the previous rules considered. We complement the fiscal spending rule with an interest rate rule that stabilizes union-wide shocks.

4.5.1 Domestic output fiscal policy rule with contemporaneous Taylor rule

We now combine the domestic output fiscal policy rule $(4.15a)$ with the contemporaneous Taylor rule (4.11) and the economic system $(4.7)-(4.10)$. We in turn consider determinacy and E-stability of this system.

Determinacy: case *DY-C*

We combine $(4.9), (4.10)$ and $(4.15a)$ into:

$$\widetilde{T}_t = \gamma^{-1}\beta E_t(\widetilde{T}_{t+1}) + \gamma^{-1}\widetilde{T}_{t-1} - \gamma^{-1}(1-\beta\varrho)\Delta\bar{T}_t \tag{4.26}$$

where we have defined $\gamma \equiv \left[1+\beta+k_T-k_{G^R}\left(\frac{g_Y^W\xi_c}{1+g_Y^W(1-\xi_c)}\right)\right]$. The union subsystem obtained by substituting (4.11) and $(4.15a)$ into (4.7) and (4.8) respectively gives:

$$B_{11}^{DY-C} = \begin{bmatrix} 1+\frac{\rho^{-1}\xi_c\left(\varphi_Y+\beta^{-1}\left(k_Y+k_{G^W}g_Y^W\right)\right)}{1+(1-\xi_c)g_Y^W} & \frac{\rho^{-1}\xi_c\left(\varphi_\pi-\beta^{-1}\right)}{1+(1-\xi_c)g_Y^W} \\ -\beta^{-1}\left(k_Y+k_{G^W}g_Y^W\right) & \beta^{-1} \end{bmatrix}$$

Output now affects inflation through two different channels; directly and through its effect on government expenditure. In this case, fiscal policy does not only have an impact on the relative variables but also on the union average. Consequently it has a direct impact on the determinacy of monetary policy where it decreases the determinacy region as stated in the following proposition.

Proposition 9 *When monetary policy stabilizes union-wide shocks according to the Taylor rule:*

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \tilde{y}_t^W \tag{4.11}$$

while national governments stabilize the domestic output gap according to:

$$\tilde{G}_t^H = -g_Y^H \tilde{y}_t^H \quad \text{and} \quad \tilde{G}_t^F = -g_Y^F \tilde{y}_t^F \tag{4.15}$$

then the following version of the Taylor principle is both necessary and sufficient for determinacy:

$$0 < \left(k_Y + k_{G^W} g_Y^W\right)\left(\varphi_\pi - 1\right) + \left(1 - \beta\right)\varphi_Y \tag{4.27}$$

Proof. See *appendix 4A*. ■

Unless the central bank adjusts interest rates by more than one-for-one to fluctuations in inflation, an aggressive fiscal policy could lead to indeterminacy even if the standard Taylor principle (4.17) is satisfied by monetary policy. In fact, when $\varphi_\pi < 1$, the more aggressively government spending is adjusted to fluctuations in domestic output gaps, the more aggressive will monetary policy have to react to union-wide output shocks for the system to be determinate.

In contrast to the fiscal policy rules considered above, for this policy combination, there are no independent determinacy conditions for fiscal policy. This is because only a proportion of the stabilization of the domestic output gap affects relative variables, the rest feeds through to union variables. Furthermore, the coefficient on the terms of trade in $(4.15a)$, $\left[\frac{g_Y^W \xi_c}{1+g_Y^W(1-\xi_c)}\right]$ is concave in g_Y^W and there does not exist a sufficiently large value for g_Y^W to destabilize the relative subsystem. Consequently, for this policy combination there always exists at least one stationary REE. That is, there exists no policy parameter values for which all equilibria are explosive.

Leeper (1991) finds that it is the combination of fiscal and monetary policy that determines if equilibria are stationary in a closed economy. In his framework, the more aggressive either of monetary and fiscal policy is, the less aggressive the other has to be for determinacy. Here we find just the opposite, the more aggressive fiscal policy is, the more aggressive monetary policy needs to be to induce determinacy. The determinacy region for economic policy is plotted in *figure 4.7* in $\{\varphi_\pi, \varphi_Y, g_Y^W\}$ space.

E-stability: case *DY-C*

Combining the fiscal policy rule $(4.15a)$, the monetary policy rule (4.11) and (4.7) – (4.10) and then writing the system in the form of (2.28) yields the following:

$$\Omega_{11}^{DY-C} =$$
$$\begin{bmatrix} \zeta^{-1}(1+(1-\xi_c)g_Y^W) & \zeta^{-1}\rho^{-1}\xi_c(1-\beta\varphi_\pi) \\ \zeta^{-1}\left(k_Y+k_{Gw}g_Y^W\right)\left(1+(1-\xi_c)g_Y^W\right) & \beta+\zeta^{-1}\left(k_Y+k_{Gw}g_Y^W\right)\rho^{-1}\xi_c(1-\beta\varphi_\pi) \end{bmatrix}$$

$$\Phi_{11}^{DY-C} = 0, \qquad \Omega_{22}^{DY-C} = \gamma^{-1}\beta \qquad \text{and} \qquad \Phi_{22}^{DY-C} = \gamma^{-1}$$

The following result is obtained.

Proposition 10 *The requirements for E-stability of the system with the fiscal policy rule:*

$$\tilde{G}_t^H = -g_Y^H \tilde{y}_t^H \qquad \textit{and} \qquad \tilde{G}_t^F = -g_Y^F \tilde{y}_t^F \tag{4.15}$$

and the interest rate rule:

$$r_t = \varphi_\pi \pi_t^W + \varphi_Y \tilde{y}_t^W \tag{4.11}$$

is that the modified Taylor principle holds:

$$0 < \left(k_Y + k_{G^W} g_Y^W\right)(\varphi_\pi - 1) + (1-\beta)\,\varphi_Y \tag{4.27}$$

Proof. See *appendix 4B*. ■

The E-stability condition is the same as the condition for determinacy derived above. If the central bank is sufficiently aggressive in tackling inflation, then fiscal policy will not destabilize the system under RLS learning. However, if the central bank instead puts a high weight on the output gap, then aggressive fiscal policy stabilization by either union member may destabilize the union economy, as shown in *figure 4.7*. Note that since it is the magnitude of $g_Y^W \equiv n g_Y^H + (1-n) g_Y^F$ that influences determinacy and E-stability, a small country is less likely to destabilize the union economy than a large country. This was not the case for the terms of trade and relative inflation fiscal policy rules, since in those cases the system was destabilized through the relative subsystem. Here fiscal policy has an effect through the subsystem for union variables.

4.5.2 Output gap fiscal policy rule with forward looking Taylor rule

Here we examine the case in which the fiscal authorities react to current values of domestic output according to $(4.15a)$, while the monetary authorities are forward looking in their policy:

$$r_t = \varphi_\pi E_t\left(\pi_{t+1}^W\right) + \varphi_Y E_t\left(\widetilde{y}_{t+1}^W\right) \tag{4.12}$$

The determinacy and E-stability results for this case are presented below.

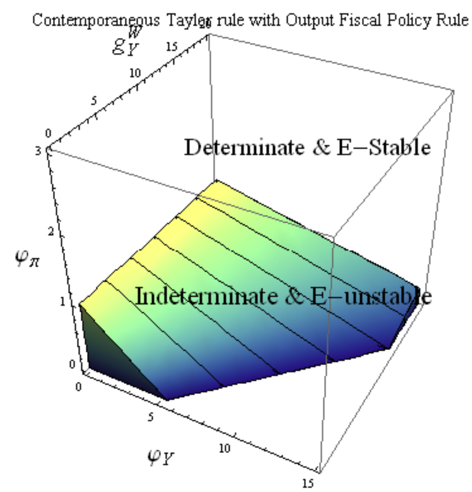

Fig. 4.7. Fiscal policy reduces the determinate and E-stable region for monetary policy. The unshaded region shows the set of combinations for the policy coefficients that induce determinacy and E-stability. The shaded region shows policy that is indeterminate and unstable under RLS learning.

Determinacy: case *DY-F*

The system is now given by $(4.7) - (4.10)$, the fiscal spending rule $(4.15a)$ and the monetary policy rule (4.12). We partition the system into two subsystems for union and relative variables, where the latter is given by (4.26) and the former by the following matrix:

$$B_{11}^{DY-F} = \begin{bmatrix} \frac{\left(1+(1-\xi_c)g_Y^W+\beta^{-1}\rho^{-1}\xi_c(1-\varphi_\pi)\left(k_Y+k_{G^W}g_Y^W\right)\right)}{(1-\rho^{-1}\xi_c\varphi_Y+(1-\xi_c)g_Y^W)} & -\frac{\beta^{-1}\rho^{-1}\xi_c(1-\varphi_\pi)}{(1-\rho^{-1}\xi_c\varphi_Y+(1-\xi_c)g_Y^W)} \\ -\beta^{-1}\left(k_Y+k_{G^W}g_Y^W\right) & \beta^{-1} \end{bmatrix}$$

We have the following result.

Proposition 11 *When the central bank of the monetary union is forward looking in its policy according to:*

$$r_t = \varphi_\pi E_t\left(\pi_{t+1}^W\right) + \varphi_Y E_t\left(\widetilde{y}_{t+1}^W\right) \tag{4.12}$$

while the sovereign fiscal authorities adjust government spending to stabilize fluctuations in the domestic output gap:

$$\widetilde{G}_t^H = -g_Y^H \widetilde{y}_t^H \qquad and \qquad \widetilde{G}_t^F = -g_Y^F \widetilde{y}_t^F \tag{4.15}$$

then the determinacy conditions are given by:

$$0 < \left(k_Y + k_{G^W} g_Y^W\right)(\varphi_\pi - 1) + (1 - \beta)\,\varphi_Y \tag{4.27}$$

and

$$(\varphi_\pi - 1)\left(k_Y + k_{G^W} g_Y^W\right) + (1 + \beta)\,\varphi_Y < 2\rho\xi_c^{-1}(1 + \beta)\left(1 + (1 - \xi_c)\,g_Y^W\right) \tag{4.28}$$

Proof. See *appendix 4A*. ■

As for the contemporaneous Taylor rule, an aggressive fiscal policy rule may lead to an indeterminate system if interest rates do not react by more than one-for-one to changes in inflation. However, if the central bank puts a high weight on output in its policy rule, then an aggressive fiscal policy can help induce determinacy. This is illustrated in *figure 4.8* where the determinacy region shifts out with increases in g_Y^W. Hence, fiscal policy could have desirable effects on the determinacy in a monetary union. The policy combination here is plausible since governments do use spending to smoothen domestic business cycles, while central banks tend to be forward looking in their policy.[18]

[18] Recall that the evidence of Molodtsova et al. (2009) suggests that a forward looking Taylor rule of the form (4.12) best explains the past behaviour of the ECB policy rate.

E-stability: case *DY-F*

The system is given by $(4.7)-(4.10)$, (4.12) and $(4.15a)$. Partitioning the system as in (2.28), we have:

$$\Omega_{11}^{DY-F}=\left[\begin{array}{cc}\frac{(1-\rho^{-1}\xi_c\varphi_Y+(1-\xi_c)g_Y^W)}{(1+g_Y^W(1-\xi_c))} & \frac{\rho^{-1}\xi_c(1-\varphi_\pi)}{(1+g_Y^W(1-\xi_c))}\\ \frac{(k_Y+k_{GW}g_Y^W)(1-\rho^{-1}\xi_c\varphi_Y+(1-\xi_c)g_Y^W)}{(1+g_Y^W(1-\xi_c))} & \beta+\frac{(k_Y+k_{GW}g_Y^W)\rho^{-1}\xi_c(1-\varphi_\pi)}{(1+g_Y^W(1-\xi_c))}\end{array}\right],$$

$$\Phi_{11}^{DY-F}=0,\qquad \Omega_{22}^{DY-F}=\gamma^{-1}\beta\qquad\text{and}\qquad\Phi_{22}^{DY-F}=\gamma^{-1}$$

The modified Taylor principle presented in the previous section is then the required condition for E-stability of REE.

Proposition 12 *For the policy rules:*

$$r_t=\varphi_\pi E_t\left(\pi_{t+1}^W\right)+\varphi_Y E_t\left(\tilde{y}_{t+1}^W\right)\tag{4.12}$$

and

$$\tilde{G}_t^H=-g_Y^H\tilde{y}_t^H\qquad \text{and}\qquad \tilde{G}_t^F=-g_Y^F\tilde{y}_t^F\tag{4.15}$$

the modified Taylor principle is both necessary and sufficient for E-stability:

$$0<\left(k_Y+k_{GW}g_Y^W\right)(\varphi_\pi-1)+(1-\beta)\varphi_Y\tag{4.39}$$

Proof. See *appendix 4B*. ■

Hence, the stability of the system depends on the coordination of fiscal and monetary policy. For a contemporaneous interest rate rule or a less aggressive forward looking interest rate rule, fiscal policy implemented to stabilize shocks is more likely to destabilize

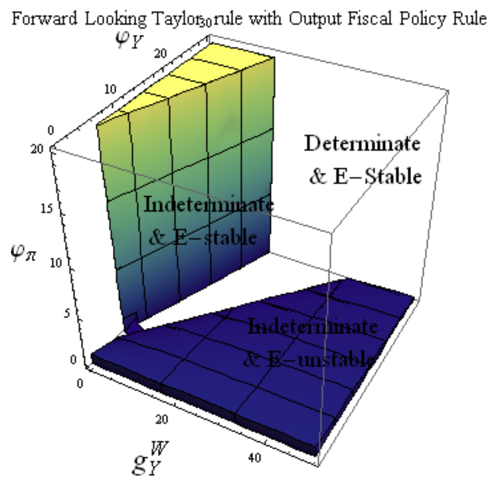

Fig. 4.8. The unshaded area shows the determinate and E-stable region for policy. The dark blue area shows the indeterminate and E-unstable region, while the coloured region has multiple E-stable equilibria. Fiscal policy changes the stability region for monetary policy. If the Taylor principle is satisfied, then fiscal policy allows an interest rate rule to react more aggressively to fluctuations in output.

the economy by inducing indeterminacy and instability under RLS learning. On the other hand, when the Taylor rule is forward looking and attaches a high weight to stabilizing fluctuations in the union output gap, then fiscal policy can actually help stabilize the union economy. We interpret this case as giving a clear message to agents that no deviations in output, country-specific or union-wide, contemporaneous or future values, are tolerated by policy makers. A less aggressive monetary policy on the other hand could imply that the efforts of the fiscal and monetary authorities are offsetting, thus leading to an ambiguous change in the union output gap and a destabilization of the union economy.

4.6 Conclusion

In *chapter 1*, we showed that monetary policy is ineffective in tackling asymmetric shocks in a monetary union with homogeneous price rigidities. Hence, in this chapter we introduced fiscal policy as an instrument that actively supports the domestic economy in the event of asymmetric shocks. Employing the model proposed by Beetsma & Jensen (2005), we have examined how the combinations of different monetary and fiscal policy rules affect determinacy and E-stability of REE in a monetary union.

We find that fiscal policy reacting to either relative inflation or the terms of trade, must satisfy determinacy and E-stability conditions independently of monetary policy, for the union economy to be determinate and E-stable. The stability conditions for monetary policy in this case are those known from the closed economy model considered by Bullard & Mitra (2002). For fiscal policy to induce stability, it need not be overly aggressive as this can eliminate all stationary equilibria regardless of what monetary policy rule is implemented.

Leeper (1991) finds that an aggressive fiscal policy rule can lead to an explosive system if monetary policy is aggressive as well. For his framework this is because it would imply a violation of the government budget constraint, making agents unwilling to hold government bonds. In our case the instability arises when the fiscal authorities reinforce shocks to the competitiveness of domestic goods, by increasing government expenditure to the extent that this increases domestic prices, making goods even less competitive.

We showed in *chapter 3* that in contrast to a fiscal policy rule that reacts to the terms of trade, an interest rate rule including the terms of trade can actually help induce both determinacy and E-stability. This result also holds for open economy models with Taylor rules that stabilize the exchange rate, as shown by Llosa & Tuesta (2008). The reason for this is that monetary policy rules that react to shocks in relative prices actually stabilize these shocks by restoring prices. On the other hand, fiscal policy reinforces these shocks by supporting demand for domestic goods. Sargent &Wallace (1981) emphasize the importance of prudent fiscal spending when an economy prioritizes price stability. Thus, the convergence criteria of the EMU imposes restrictions on the flexibility of the fiscal spending of its member countries. Our results serve as additional motivation for these restrictions.

When a fiscal policy rule reacting to relative inflation is combined with a forward looking Taylor rule, then fiscal policy directly affects the learning stability of monetary policy. Specifically, if monetary policy is indeterminate but E-stable, then an aggressive fiscal policy rule can make monetary policy unstable under learning.

Finally, when fiscal policy stabilizes the domestic output gaps while monetary policy reacts to shocks in union output and inflation, then fiscal policy does not have to satisfy any determinacy or E-stability conditions independently of monetary policy. Instead, fiscal policy increases the indeterminate and E-unstable region of the Taylor principle. Consequently, the stability region for a contemporaneous interest rate rule is smaller, the more aggressive fiscal policy is in its stabilization efforts. However, if monetary policy is forward looking and sufficiently aggressive to satisfy the Taylor principle, then an aggressive fiscal policy rule can actually help induce determinacy. Evans et al. (2008) find that an aggressive fiscal policy rule can help prevent the economy from reaching a deflationary spiral, when a liquidity trap exists. Our result suggests that in some cases it could also help eliminate multiplicity of stationary rational expectations equilibria.

4.A Appendix: *Proof of determinacy results*

In this appendix we prove the determinacy conditions of the paper. We employ the methodology outlined in *section 2.2.3*.

Proof of proposition 1

For determinacy we first need both of the eigenvalues of

$$B_{11}^{TT-C} = \begin{bmatrix} \rho^{-1}\xi_c\left(\varphi_Y + \beta^{-1}k_Y\right) + 1 & \rho^{-1}\xi_c\left(\varphi_\pi - \beta^{-1}\right) \\ -\beta^{-1}k_Y & \beta^{-1} \end{bmatrix}$$

to be outside of the unit circle. Using the method of *appendix 2A*, we need $\left|a_0^{TT-C}\right| > 1$ and $\left|a_1^{TT-C}\right| < \left|1 + a_0^{TT-C}\right|$, where:

$$a_0^{TT-C} = \beta^{-1}\rho^{-1}\xi_c(\varphi_Y + k_Y\varphi_\pi) + \beta^{-1}$$

and

$$a_1^{TT-C} = -\rho^{-1}\xi_c\left(\varphi_Y + \beta^{-1}k_Y\right) - 1 - \beta^{-1}$$

Hence we have:

$\left|a_0^{TT-C}\right| > 1 \Longrightarrow \rho^{-1}\xi_c(\varphi_Y + k_Y\varphi_\pi) > -(1-\beta)$, which is satisfied trivially, while:

$\left|a_1^{TT-C}\right| < \left|1 + a_0^{TT-C}\right| \Longrightarrow$

$$0 < (1-\beta)\,\varphi_Y + k_Y\left(\varphi_\pi - 1\right) \tag{4.17}$$

So the Taylor principle is our first determinacy condition and puts restrictions on the interest rate rule of the union central bank. We consider now the relative subsystem given by (4.16). Using the results presented in *section 2.2.3* for the generic condition (2.14), we have:

$\alpha + \delta = \frac{1+\beta}{1+\beta+k_T-k_{G^R}(g_T^H+g_T^F)}$

and it follows that $\left|\frac{1+\beta}{1+\beta+k_T-k_{G^R}(g_T^H+g_T^F)}\right| < 1$ when the following conditions hold:

$$\frac{k_T}{k_{G^R}} > g_T^H + g_T^F \tag{4.18}$$

or

$$g_T^H + g_T^F > \frac{2(1+\beta)+k_T}{k_{G^R}} \tag{4.19}$$

Hence one of these two conditions must hold together with the Taylor principle for the interest rate rule, for the system to be determinate. If one of these holds while the Taylor principle is not satisfied, the system is indeterminate with multiple stationary, while if none of the two conditions for the fiscal policy rule holds then the system has no stationary solution and is explosive. This is easily seen since $\delta = \theta^{-1}$ and $\alpha = \beta\theta^{-1}$, while $0 < \beta < 1$, implying that $\delta > \alpha$.

Proof of proposition 3

The system under a terms of trade fiscal policy rule and a forward looking Taylor rule (*TT-F*) is given by (4.7), (4.8), (4.12) and (4.16). Substituting the Taylor rule (4.12) into (4.7) and combining this with (4.8) we get the representation of the union system by the matrix:

$$B_{11}^{TT-F} = \begin{bmatrix} \frac{1+\beta^{-1}k_Y\rho^{-1}\xi_c(1-\varphi_\pi)}{(1-\rho^{-1}\xi_c\varphi_Y)} & -\frac{\rho^{-1}\xi_c(1-\varphi_\pi)\beta^{-1}}{(1-\rho^{-1}\xi_c\varphi_Y)} \\ -\beta^{-1}k_Y & \beta^{-1} \end{bmatrix}$$

Since the relative subsystem, given by (4.16) is independent of $\left\{\widetilde{y}_t^W, \pi_t^W\right\}$, we need it to be determinate for the relative subsystem to be determinate. In the above *proof of proposition 1* we show that (4.27) induces a unique stationary equilibrium when the fiscal policy rules satisfies one of the two conditions:

$$\frac{k_T}{k_{G^R}} > g_T^H + g_T^F \tag{4.18}$$

or

$$g_T^H + g_T^F > \frac{2\left(1+\beta\right) + k_T}{k_{G^R}} \tag{4.19}$$

For the whole system to be determinate, B_{11}^{TT-F} must have both of its eigenvalues outside the unit circle. We have:

$$a_0^{TT-F} = \beta^{-1}(1 - \rho^{-1}\xi_c\varphi_Y)^{-1}$$

$$a_1^{TT-F} = -\left(1 + \beta^{-1}k_Y\rho^{-1}\xi_c\left(1-\varphi_\pi\right)\right)\left(1 - \rho^{-1}\xi_c\varphi_Y\right)^{-1} - \beta^{-1}$$

For the eigenvalues of B_{11}^{TT-F} to be outside the unit circle it must be that both

$\left|a_0^{TT-F}\right| > 1$ and $\left|a_1^{TT-F}\right| < \left|1 + a_0^{TT-F}\right|$.

The first condition $\left|a_0^{TT-F}\right| > 1$ gives the following constraint for when a_0^{TT-F} is negative:

$$\varphi_Y < \rho\xi_c^{-1}\left(\beta^{-1} + 1\right) \tag{4.A1}$$

When $a_0^{TT-F} + 1$ and a_1^{TT-F} are both negative or positive condition $\left|a_1^{TT-F}\right| < \left|1 + a_0^{TT-F}\right|$ implies:

$$2\rho\xi_c^{-1}(1+\beta) > k_Y(\varphi_\pi - 1) + \varphi_Y(1+\beta) \tag{4.20}$$

In the case where $a_0^{TT-F}+1$ is positive and a_1^{TT-F} negative or vice versa, we have:

$$0 < (1-\beta)\varphi_Y + k_Y(\varphi_\pi - 1) \tag{4.17}$$

If we rearrange (4.17) we have $-(1-\beta)\varphi_Y < k_Y(\varphi_\pi - 1)$, which can be substituted into (4.20) and then rearranged to get $\varphi_Y < \rho\xi_c^{-1}\left(\beta^{-1}+1\right)$. That is, combining (4.17) and (4.20) gives $(4.A1)$, making it redundant. Hence, (4.17), (4.20) and either (4.18) or (4.19) must hold for the system *TT-F* to be determinate, which proves *proposition 3*.

Proof of proposition 5

The system *RI-C* of *section 4.4.1* is determinate when both of its subsystems are determinate given that the second or relative subsystem (4.21) is independent from the first. The first subsystem contains the variables $\left\{\tilde{y}_t^W, \pi_t^W\right\}$ and written in the form of (4.13) is described by the matrix:

$$B_{11}^{RI-C} = \begin{bmatrix} \rho^{-1}\xi_c\left(\varphi_Y + \beta^{-1}k_Y\right)+1 & \rho^{-1}\xi_c\left(\varphi_\pi - \beta^{-1}\right) \\ -\beta^{-1}k_Y & \beta^{-1} \end{bmatrix}$$

For monetary policy to be determinate B_{11}^{RI-C} must have both of its eigenvalues outside the unit circle. Noting that $B_{11}^{RI-C} = B_{11}^{TT-C}$, we refer to the *proof of proposition 1* to show that this is the case if and only if the Taylor principle (4.17) holds. For the relative subsystem to be determinate we require equation (4.21) to be determinate. We have;

$$\alpha + \delta = \frac{1+\beta-k_{GR}\left(g_{\pi R}^H + g_{\pi R}^F\right)}{1+\beta+k_T-k_{GR}\left(g_{\pi R}^H+g_{\pi R}^F\right)}$$

and it follows that $\left|\frac{1+\beta-k_{GR}\left(g^H_{\pi R}+g^F_{\pi R}\right)}{1+\beta+k_T-k_{GR}\left(g^H_{\pi R}+g^F_{\pi R}\right)}\right| < 1$ when the following condition holds:

$$\frac{1+\beta+\frac{1}{2}k_T}{k_{GR}} > g^H_{\pi^R} + g^F_{\pi^R} \tag{4.22}$$

It is straightforward to see that when this condition is violated it must be that:
$\left|\frac{\beta}{1+\beta+k_T-k_{GR}\left(g^H_{\pi R}+g^F_{\pi R}\right)}\right| < \left|\frac{1-k_{GR}\left(g^H_{\pi R}+g^F_{\pi R}\right)}{1+\beta+k_T-k_{GR}\left(g^H_{\pi R}+g^F_{\pi R}\right)}\right|$

and hence the system is explosive and has no stationary solutions in that case.

Proof of proposition 7

The system *RI-F* is partitioned according to the methodology of *section 2.2.3*, giving the key matrix :
$B_{11}^{RI-F} = \begin{bmatrix} \frac{1+\beta^{-1}k_Y\rho^{-1}\xi_c(1-\varphi_\pi)}{(1-\rho^{-1}\xi_c\varphi_Y)} & -\frac{\rho^{-1}\xi_c(1-\varphi_\pi)\beta^{-1}}{(1-\rho^{-1}\xi_c\varphi_Y)} \\ -\beta^{-1}k_Y & \beta^{-1} \end{bmatrix}$
and equation (4.21). The above *proof of proposition 5* shows that determinacy of (4.21) implies:

$$\frac{1+\beta+\frac{1}{2}k_T}{k_{GR}} > g^H_{\pi^R} + g^F_{\pi^R} \tag{4.22}$$

We note that $B_{11}^{RI-F} = B_{11}^{TT-F}$, the *proof of proposition 3* shows that this matrix induces determinacy when these conditions are satisfied:

$$0 < (1-\beta)\,\varphi_Y + k_Y\,(\varphi_\pi - 1) \tag{4.17}$$

and

$$2\rho\xi_c^{-1}(1+\beta) > k_Y\,(\varphi_\pi - 1) + \varphi_Y(1+\beta) \tag{4.20}$$

This gives *proposition 7*.

Proof of proposition 9

The system with a contemporaneous Taylor rule and a fiscal policy rule stabilizing the domestic output gap (*DY-C*) is represented by the relative subsystem:

$$\widetilde{T}_t = \gamma^{-1}\beta E_t(\widetilde{T}_{t+1}) + \gamma^{-1}\widetilde{T}_{t-1} - \gamma^{-1}(1-\beta\varrho)\Delta\bar{T}_t \tag{4.26}$$

where we have defined $\gamma \equiv \left[1+\beta+k_T - k_{G^R}\left(\frac{g_Y^W \xi_c}{1+g_Y^W(1-\xi_c)}\right)\right]$, and the world system written in the form of equation (2.13);

$$B_{11}^{DY-C} = \begin{bmatrix} 1+\frac{\rho^{-1}\xi_c\left(\varphi_Y+\beta^{-1}\left(k_Y+k_{G^W}g_Y^W\right)\right)}{1+(1-\xi_c)g_Y^W} & \frac{\rho^{-1}\xi_c\left(\varphi_\pi-\beta^{-1}\right)}{1+(1-\xi_c)g_Y^W} \\ -\beta^{-1}\left(k_Y+k_{G^W}g_Y^W\right) & \beta^{-1} \end{bmatrix}$$

For the model to have a unique stationary REE under the policy rules of *DY-C*, both subsystems need to be determinate. For the relative system to be determinate it must be that

$$\left|\frac{1+\beta}{1+\beta+k_T-k_{G^R}\left(\frac{g_Y^W\xi_c}{1+g_Y^W(1-\xi_c)}\right)}\right| < 1$$

following the methodology of the general case outlined above. It is straightforward to show that this inequality always holds by using the definitions of the Phillips curve coefficients given in *section 4.2.1*; $k_{G^R} \equiv k(1-\xi_c)\eta$ and $k_T \equiv k(1+\eta\xi_c)$. We have that: $k_T - \frac{k_{G^R}g_Y^W\xi_c}{1+g_Y^W(1-\xi_c)} = \frac{k_T+g_Y^W(1-\xi_c)k}{1+g_Y^W(1-\xi_c)} > 0$ and hence $\frac{1+\beta}{1+\beta+k_T-k_{G^R}\left(\frac{g_Y^W\xi_c}{1+g_Y^W(1-\xi_c)}\right)}$ is always positive and less than one and the relative subsystem is always determinate. For union determinacy we further need B_{11}^{DY-C} to have both of its eigenvalues inside the unit circle. Here we have:

$$a_0^{DY-C} = \beta^{-1}\rho^{-1}\xi_c\left\{\frac{\varphi_\pi(k_Y+k_{G^W}g_Y^W)+\varphi_Y}{1+(1-\xi_c)g_Y^W}+\rho\xi_c^{-1}\right\}$$

and

$a_1^{DY-C} = -\frac{\rho^{-1}\xi_c\left(\varphi_Y+\beta^{-1}\left(k_Y+k_{G^W}g_Y^W\right)\right)}{1+(1-\xi_c)g_Y^W} - \beta^{-1} - 1$

a_0^{DY-C} is always positive and larger than one. From $\left|a_1^{DY-C}\right| < \left|1 + a_0^{DY-C}\right|$ we have the inequality:

$$0 < (1-\beta)\,\varphi_Y + \left(k_Y + k_{G^W}g_Y^W\right)(\varphi_\pi - 1) \tag{4.27}$$

Proof of proposition 11

We show in the above *proof of proposition 9* that the relative subsystem is always stable under the fiscal policy rule $(4.15a)$. Determinacy of the union subsystem is hence both necessary and sufficient for determinacy of the whole system. We require that both eigenvalues of B_{11}^{DY-F} are outside the unit circle;

$$B_{11}^{DY-F} = \begin{bmatrix} \frac{\left(1+(1-\xi_c)g_Y^W+\beta^{-1}\rho^{-1}\xi_c(1-\varphi_\pi)\left(k_Y+k_{G^W}g_Y^W\right)\right)}{(1-\rho^{-1}\xi_c\varphi_Y+(1-\xi_c)g_Y^W)} & -\frac{\beta^{-1}\rho^{-1}\xi_c(1-\varphi_\pi)}{(1-\rho^{-1}\xi_c\varphi_Y+(1-\xi_c)g_Y^W)} \\ -\beta^{-1}\left(k_Y+k_{G^W}g_Y^W\right) & \beta^{-1} \end{bmatrix}$$

This yields:

$a_0^{DY-F} = \frac{\beta^{-1}\left(1+(1-\xi_c)g_Y^W\right)}{\left(1-\rho^{-1}\xi_c\varphi_Y+(1-\xi_c)g_Y^W\right)}$

and

$a_1^{DY-F} = -\frac{\left(1+(1-\xi_c)g_Y^W+\beta^{-1}\rho^{-1}\xi_c(1-\varphi_\pi)\left(k_Y+k_{G^W}g_Y^W\right)\right)}{(1-\rho^{-1}\xi_c\varphi_Y+(1-\xi_c)g_Y^W)} - \beta^{-1}$

As explained above, for both eigenvalues to be outside the unit circle we must have: $\left|a_0^{DY-F}\right| > 1$ and $\left|a_1^{DY-F}\right| < \left|1 + a_0^{DY-F}\right|$. When either a_1^{DY-F} is negative and $\left(a_0^{DY-F}+1\right)$ is positive or vice versa, the condition $\left|a_1^{DY-F}\right| < \left|1 + a_0^{DY-F}\right|$ implies:

$$0 < (\varphi_\pi - 1)\left(k_Y + k_{G^W} g_Y^W\right) + \varphi_Y (1 - \beta) \tag{4.27}$$

If both a_1^{DY-F} and $\left(a_0^{DY-F} + 1\right)$ are negative or if both are positive we have:

$$(\varphi_\pi - 1)\left(k_Y + k_{G^W} g_Y^W\right) + \varphi_Y (1 + \beta) < 2\rho\xi_c^{-1}(1 + \beta)\left(1 + (1 - \xi_c)g_Y^W\right) \tag{4.28}$$

Hence these are the two conditions required for determinacy under a forward looking Taylor rule and government spending rules stabilizing the domestic output gap. Note that the condition $\varphi_Y < \rho\xi_c^{-1}\left(\beta^{-1} + 1\right)\left(1 + (1 - \xi_c)g_Y^W\right)$, obtained from $\left|a_0^{DY-F}\right| > 1$ in the case where $a_0^{DY-F} < 0$, is redundant as it can be obtained by combining (4.27) and (4.28).

4.B Appendix: *Proof of E-stability results*

In this appendix we derive the E-stability conditions of *chapter 4*, using the methodology outlined in *section 2.2.3.*

Proof of proposition 2

Where $F \equiv (I - \Omega\bar{b})^{-1}\Omega$ and $A \equiv (I - \Omega\bar{b})^{-1}\Phi$ and where: $F = \begin{bmatrix} F_{11} & 0 \\ F_{21} & F_{22} \end{bmatrix}$ and $A = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}$, E-stability results when the eigenvalues of F_{11}, F_{22}, and the cross products of these and the eigenvalues of A_{11} and A_{22}, all have eigenvalues with real parts less than one as explained in *section 2.2.3*.

For the case of a terms of trade fiscal policy rule and a contemporaneous Taylor rule (*TT-C*) the MSV solution gives the following: $b_{11}^{TT-C} = 0$ and $b_{22}^{TT-C} = \frac{1 \mp \sqrt{1 - \frac{4\beta}{\theta^2}}}{2\theta^{-1}\beta}$, recalling that $\theta \equiv \left(1+\beta+k_T-k_{G^R}(g_T^H+g_T^F)\right)$. The first of these follows from (2.32), given that $\Phi_{11}^{TT-C}=0$. The solution for b_{22}^{TT-C} is obtained using (2.34) and only the negative conjugate is ever stable and that is when the determinacy conditions of *proposition 1* are met. This gives;

$$F_{11}^{TT-C}=\begin{bmatrix} \frac{\xi_c^{-1}\rho}{\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho} & \frac{1-\beta\varphi_\pi}{\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho} \\ \frac{\xi_c^{-1}\rho k_Y}{\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho} & \frac{\beta\left(\varphi_Y+\xi_c^{-1}\rho\right)+k_Y}{\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho} \end{bmatrix}, A_{11}^{TT-C}=0$$

$$F_{22}^{TT-C}=\frac{2\beta}{\theta(1+\sqrt{1-4\beta\theta^{-2}})} \qquad \text{and} \qquad A_{22}^{TT-C}=\frac{2}{\theta(1+\sqrt{1-4\beta\theta^{-2}})}$$

Note that $F_{11}^{TT-C}=\Omega_{11}^{TT-C}$ and $A_{11}^{TT-C}=0$, both as a result of $b_{11}^{TT-C}=0$. Also, since the relative subsystem was reduced to one equation (4.16), F_{22}^{TT-C} and A_{22}^{TT-C} are scalars. We now need F_{11}^{TT-C} and $A_{22}^{TT-C}\cdot F_{11}^{TT-C}$ to have eigenvalues with real parts less then one, while F_{22}^{TT-C} and $A_{22}^{TT-C}\cdot F_{22}^{TT-C}$ need to have real parts less than one. Considering first F_{11}^{TT-C}, its characteristic polynomials has real parts less than one when $\left(F_{11}^{TT-C}-I\right)$ has negative real parts. Let the eigenvalues be given by $p(\lambda)=\lambda^2+c_1\lambda+c_0$, then these are negative when $c_0>0$ *and* $c_1>0$. Here we have:

$$F_{11}^{TT-C}-I=\frac{1}{\left(\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho\right)}\begin{bmatrix} -(\varphi_Y+\varphi_\pi k_Y) & (1-\beta\varphi_\pi) \\ \xi_c^{-1}\rho k_Y & -(1-\beta)\left(\varphi_Y+\xi_c^{-1}\rho\right)-(\varphi_\pi-1)k_Y \end{bmatrix}$$

$$c_0^{TT-C}=\frac{(\varphi_\pi-1)k_Y+\varphi_Y(1-\beta)}{(\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho)}$$

$$c_1=c_0^{TT-C}+\frac{k_Y\varphi_\pi+\varphi_Y+\xi_c^{-1}\rho(1-\beta)}{(\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho)}$$

Hence, we require $c_0^{TT-C}>0$, which gives the Taylor principle:

$$0<(1-\beta)\,\varphi_Y+k_Y\,(\varphi_\pi-1) \tag{4.17}$$

which is our first E-stability condition. We now impose the restriction $F_{22}^{TT-C} = \frac{2\beta}{\theta(1+\sqrt{1-4\beta\theta^{-2}})} < 1$. This condition always holds for a real and stationary solution b_{22}^{TT-C}. Similarly, $A_{22}^{TT-C} \cdot F_{22}^{TT-C} < 1$ always holds when a stationary MSV solution exists. This implies that all stationary MSV solutions are learnable under RLS and we have the following conditions:

$$\frac{k_T}{k_{G^R}} > g_T^H + g_T^F \tag{4.18}$$

or

$$g_T^H + g_T^F > \frac{2\left(1+\beta\right) + k_T}{k_{G^R}} \tag{4.19}$$

In this paper we do not consider the E-stability of explosive solutions since although these may be learnable, they are undesirable for obvious reasons. Finally, we use the result that $0 < A_{22}^{TT-C} = \frac{2}{\theta(1+\sqrt{1-4\beta\theta^{-2}})} < 1$ holds given b_{22}^{TT-C}, and that the Taylor principle implies that the eigenvalues of F_{22}^{TT-C} have real parts less than one, to conclude that the eigenvalues of $A_{22}^{TT-C} \cdot F_{11}^{TT-C}$ also have real parts less then one. This follows since the eigenvalues of $A_{22}^{TT-C} \cdot F_{11}^{TT-C}$ are each of the eigenvalues of F_{11}^{TT-C} multiplied by the scalar A_{22}^{TT-C}.

Proof of proposition 4

We proceed as in the *proof of proposition 2* noting that the relative subsystem is again represented by (4.16). We have:

$$F_{11}^{TT-F} = \begin{bmatrix} (1-\rho^{-1}\xi_c\varphi_Y) & \rho^{-1}\xi_c\left(1-\varphi_\pi\right) \\ k_Y(1-\rho^{-1}\xi_c\varphi_Y) & \beta + k_Y\rho^{-1}\xi_c\left(1-\varphi_\pi\right) \end{bmatrix}, A_{11}^{TT-F} = 0$$

$F_{22}^{TT-F} = \frac{2\beta}{\theta(1+\sqrt{1-4\beta\theta^{-2}})}$ and $A_{22}^{TT-F} = \frac{2}{\theta(1+\sqrt{1-4\beta\theta^{-2}})}$

Where the MSV solution, $b_{11}^{TT-F} = 0$ implies that $F_{11}^{TT-F} = \Omega_{11}^{TT-F}$. For E-stability we require that F_{11}^{TT-F} and $A_{22}^{TT-F} \cdot F_{11}^{TT-F}$ have their eigenvalues with real parts less then one, while F_{22}^{TT-F} and $A_{22}^{TT-F} \cdot F_{22}^{TT-F}$ need to have real parts less than one. It is shown in the *proof of proposition 2* that $F_{22}^{TT\_C}$ and $A_{22}^{TT\_C} \cdot F_{22}^{TT\_C}$ are always less than one when one of the determinacy conditions (4.18) or (4.19) is met. Noting that $F_{22}^{TT-F} = F_{22}^{TT-C}$ and that $A_{22}^{TT-F} = A_{22}^{TT-C}$, it is clear that this is also the case for F_{22}^{TT-F} and $A_{22}^{TT-F} \cdot F_{22}^{TT-F}$. Given that A_{22}^{TT-F} is smaller than one and non-negative it follows that if the eigenvalues of F_{11}^{TT-F} have real parts less than one, then so will $A_{22}^{TT-F} \cdot F_{22}^{TT-F}$. Hence, for E-stability it is necessary and sufficient that either (4.18) or (4.19) holds while the eigenvalues of F_{11}^{TT-F} both have real parts less than one, or equivalently that $\Omega_{11}^{TT\_F} - I$ have negative real parts. For the first subsystem we have:

$c_0^{TT-F} = \rho^{-1}\xi_c\varphi_Y(1-\beta) + k_Y\rho^{-1}\xi_c\left(\varphi_\pi - 1\right)$

and

$c_1^{TT-F} = \rho^{-1}\xi_c\varphi_Y + (1-\beta) + k_Y\rho^{-1}\xi_c\left(\varphi_\pi - 1\right)$

$= c_0^{TT-F} + \beta\rho^{-1}\xi_c\varphi_Y + (1-\beta)$

Hence we need $c_0^{TT-F} > 0$ to hold since this implies that $c_1^{TT-F} > 0$ as well. It is straight forward to show that $c_0^{TT-F} > 0$ gives the Taylor principle. Our conditions for E-stability of the combination of policy rules *TT-F* are hence (4.17) and (4.18) or (4.19).

Proof of proposition 6

As in the above *proof of proposition 2* we have the REE solution for the union subsystem solving for; $b_{11}^{RI-C} = 0$, following from equation (2.32) given that $\Phi_{11}^{RI-C} = 0$. It hence follows, equivalently to above that $F_{11}^{RI-F} = \Omega_{11}^{RI-F}$ and that $A_{22}^{RI-F} = 0$. This implies that we need the following conditions for E-stability: all eigenvalues of the matrices F_{11}^{RI-F} and $A_{22}^{RI-F} \cdot F_{11}^{RI-F}$ must have real parts less than one , while the scalars F_{22}^{RI-F} and $A_{22}^{RI-F} \cdot F_{22}^{RI-F}$ must have real parts less than one. Considering first F_{11}^{RI-F}, we note that $F_{11}^{RI-F} = F_{11}^{TT-F}$ and hence as shown in the above *proof of proposition 2*, it has eigenvalues with real parts less than one when the Taylor principle (4.17) is satisfied. This is hence our first requirement for stability under RLS learning. The RE solution of (4.21) yields; $b_{22}^{TT-C} = \frac{1 \mp \sqrt{1 - \frac{4\beta\left(1 - k_{GR}(g_T^H + g_T^F)\right)}{\theta^2}}}{2\theta^{-1}\beta}$, while the negative conjugate is stable when (4.22) is satisfied, the positive conjugate is never stable. We now have:

$$F_{22}^{RI-C} = \frac{2\beta}{\theta\left(1 + \sqrt{1 - 4\beta\left(1 - k_{GR}(g_T^H + g_T^F)\right)\theta^{-2}}\right)}$$

and

$$A_{22}^{RI-C} = \frac{2\left(1 - k_{GR}(g_T^H + g_T^F)\right)}{\theta\left(1 + \sqrt{1 - 4\beta\left(1 - k_{GR}(g_T^H + g_T^F)\right)\theta^{-2}}\right)}$$

where we have used the stationary solution of b_{22}^{TT-C}. It can easily be shown that F_{22}^{RI-C} and $A_{22}^{RI-C} \cdot F_{22}^{RI.C}$ are always less than one regardless of the value of the fiscal policy coefficients $(g_T^H + g_T^F)$. Finally, we need the matrix $A_{22}^{TT-C} \cdot F_{11}^{RI-C}$ to have real parts less than one for the system to be E-stable. We know that F_{11}^{RI-C} has eigenvalues with real parts less than one when (4.17) is satisfied and it follows that $A_{22}^{TT-C} F_{11}^{RI-C}$ has eigenvalues with real parts less than one if $0 < A_{22}^{TT-C} < 1$. If instead $-1 < A_{22}^{TT-C} < 0$, we need F_{11}^{RI-C} to have eigenvalues with real parts larger than -1. When (4.22) is satisfied and there is a stationary REE, then it can be shown that:

$$-1 < A_{22}^{TT\_C} < 1$$

This means that we need to prove that the eigenvalues of F_{11}^{RI-C} always have real parts larger than negative one, given that they are less than one. The eigenvalues of F_{11}^{RI-C} are given by:

$$\lambda_{11}^{RI-C} = \frac{\rho(1+\beta)+\xi_c(k_Y+\varphi_Y\beta)\pm\sqrt{(\rho(1+\beta)+\xi_c(k_Y+\varphi_Y\beta))^2-4\beta\rho(\rho+\xi_c(k_Y\varphi_\pi+\varphi_Y))}}{2(\rho+\xi_c(k_Y\varphi_\pi+\varphi_Y))}$$

since $(\rho(1+\beta)+\xi_c(k_Y+\varphi_Y\beta))$ and $(\rho+\xi_c(k_Y\varphi_\pi+\varphi_Y))$ are positive, it follows that the positive conjugate has positive real parts. Furthermore, since the positive conjugate has real parts less than one by the Taylor principle, it must be that the negative conjugate has a real part larger than negative one. This implies that $A_{22}^{TT-C}\cdot F_{11}^{RI-C}$ has eigenvalues with real parts less than one when the Taylor principle (4.17) and the condition for fiscal policy (4.22) are satisfied, which concludes the proof of *proposition 6.*

Proof of proposition 8

The proof of this proposition partly follows from the *proofs of propositions 4* and *6*. Since $\Omega_{11}^{RI-F} = \Omega_{11}^{TT-F}$ and $\Phi_{11}^{RI-F} = \Phi_{11}^{TT-F} = 0$, it follows as above that $b_{22}^{RI-F} = 0$ and hence that $\Omega_{11}^{RI-F} = F_{11}^{RI-F}$ and that $A_{11}^{RI-F} = 0$. We also have the relative subsystem described by (4.21) as in the case with a contemporaneous Taylor rule (*RI-C*). This implies that; $b_{22}^{RI-F} = b_{22}^{RI-C}$, $F_{11}^{RI-F} = F_{11}^{RI-C}$ and that $F_{11}^{RI-F} = F_{11}^{RI-C}$. Because $A_{11}^{RI-F} = 0$, it is necessary and sufficient for E-stability that F_{11}^{RI-F} and $A_{22}^{RI-F}\cdot F_{11}^{RI-F}$ have eigenvalues with real parts less than one , while the scalars F_{22}^{RI-F} and $A_{22}^{RI-F}\cdot F_{22}^{RI-F}$ are less than one. We show in the above *proof of proposition 6* that $F_{22}^{RI-F}(= F_{22}^{RI-C})$ and $A_{22}^{RI-F} \cdot F_{22}^{RI-F}$ $(= A_{22}^{RI-C} \cdot F_{22}^{RI-C})$ are less than one when a stationary MSV solution exists under rational expectations, i.e. when the inequality (4.21) is satisfied. The *proof of proposition 4* shows that $F_{11}^{RI-F}(= F_{11}^{TT-F})$ has eigenvalues with real parts less than one when the Taylor principle (4.17) is satisfied. For E-stability we then require that these conditions are met and in addition that the eigenvalues of $A_{22}^{RI-F} \cdot F_{11}^{RI-F}$ have real parts less than one. As shown above, when (4.22) holds, then $-1 < A_{22}^{RI-F} < 1$. Specifically,

$$0 < \left(g_T^H + g_T^F\right) < \tfrac{1}{k_{GR}} \Longrightarrow 0 < A_{22}^{RI-F} < 1$$

and

$$0 < \left(g_T^H + g_T^F\right) < \tfrac{(1+\beta)+\frac{1}{2}k_T}{k_{GR}} \Longrightarrow -1 < A_{22}^{RI-F} < 0$$

Since the eigenvalues of $A_{22}^{RI-F} \cdot F_{11}^{RI-F}$ are the products of the eigenvalues of F_{11}^{RI-F} and the scalar A_{22}^{RI-F}, it follows that when $-1 < A_{22}^{RI-F} < 0$, F_{11}^{RI-F} must have real parts between negative one and one. We know by the Taylor principle that real parts are less than one, but we also need the conditions for the eigenvalues having real parts greater than minus one. For the matrix F_{11}^{RI-F} to have characteristic roots with real parts greater than negative one, it must that the matrix $\left(F_{11}^{RI-F} + I\right)$ has eigenvalues with positive real parts. For the general case, the polynomial: $p(\lambda) = \lambda^2 + a_1\lambda + a_0$, has characteristic roots less than one when $a_1 < 0$ and $a_0 > 0$. In this case we have the matrix:

$$\left(F_{11}^{RI-F} + I\right) = \begin{bmatrix} (1-\rho^{-1}\xi_c\varphi_Y)+1 & \rho^{-1}\xi_c\left(1-\varphi_\pi\right) \\ k_Y(1-\rho^{-1}\xi_c\varphi_Y) & \beta + k_Y\rho^{-1}\xi_c\left(1-\varphi_\pi\right)+1 \end{bmatrix}$$

and

$$a_0^{RI-F} = \tfrac{2(1+\beta)\rho+\xi_c(k_Y(\varphi_\pi-1)+(1+\beta)\varphi_Y)}{\rho}$$

$$a_1^{RI-F} = -(1 - \rho^{-1}\xi_c\varphi_Y) - \beta - k_Y\rho^{-1}\xi_c\,(1 - \varphi_\pi) - 2$$

$a_0^{RI-F} > 0$ is always satisfied when the Taylor principle holds, but $a_1^{RI-F} < 0$ implies the following additional constraint:

$$\varphi_Y + k_Y\,(\varphi_\pi - 1) < \rho\xi_c^{-1}\,(3 + \beta) \tag{4.25}$$

This condition always holds when the determinacy condition (4.20) holds and this is hence not a violation of McCallum's (2007) result stating that with contemporaneous data in the information set, determinacy is sufficient but not necessary for stability under learning RLS. E-stability is then guaranteed by the Taylor principle (4.17) and either

$$\left(g_T^H + g_T^F\right) < \frac{1}{k_{G^R}} \tag{4.23}$$

or

$$\left(g_T^H + g_T^F\right) < \frac{(1+\beta) + \frac{1}{2}k_T}{k_{G^R}} \tag{4.22}$$

and (4.25). When $\frac{1}{k_{G^R}} < \left(g_T^H + g_T^F\right) < \frac{(1+\beta)+\frac{1}{2}k_T}{k_{G^R}}$ and $\varphi_Y + k_Y\,(\varphi_\pi - 1) > \rho\xi_c^{-1}\,(3+\beta)$, the system may or may not be E-stable. We do not obtain analytical results in this case, but the non-linear relationship between the policy parameters that needs to be satisfied in this case for E-stability to hold is:

$$\frac{\left(1-k_{G^R}(g_T^H+g_T^F)\right)\left(\rho(1+\beta)-\xi_c(\varphi_Y+k_Y(\varphi_\pi-1))-\sqrt{-4\beta\rho(\rho-\xi_c\varphi_Y)+(1+\beta)\rho-\xi_c(\varphi_Y+k_Y(\varphi_\pi-1))}\right)}{\rho\theta\left(1+\sqrt{1-4\beta\left(1-k_{G^R}(g_T^H+g_T^F)\right)\theta^{-2}}\right)} < 1$$

where this is the negative conjugate of the eigenvalues of F_{11}^{RI-F} multiplied by A_{22}^{RI-F}.

Proof of proposition 10

First we note that as in previous cases of this chapter the MSV solution gives: $b_{11}^{DY-C} = 0$, resulting from $\Phi_{11}^{DY-C} = 0$. This also implies:

$$F_{11}^{DY-C} = \Omega_{11}^{DY-C} =$$
$$\begin{bmatrix} \zeta^{-1}(1+(1-\xi_c)g_Y^W) & \zeta^{-1}\rho^{-1}\xi_c(1-\beta\varphi_\pi) \\ \zeta^{-1}\left(k_Y + k_{G^W}g_Y^W\right)\left(1+(1-\xi_c)g_Y^W\right) & \beta+\zeta^{-1}\left(k_Y + k_{G^W}g_Y^W\right)\rho^{-1}\xi_c(1-\beta\varphi_\pi) \end{bmatrix}$$

Further,

$$\widetilde{T}_t = \gamma^{-1}\beta E_t(\widetilde{T}_{t+1}) + \gamma^{-1}\widetilde{T}_{t-1} - \gamma^{-1}(1-\beta\varrho)\Delta\bar{T}_t \tag{4.26}$$

has the MSV solution:

$b_{22}^{DY-C} = \frac{1\mp\sqrt{1-\frac{4\beta}{\gamma^2}}}{2\gamma^{-1}\beta}$ where $\gamma \equiv \left[1+\beta+k_T - k_{G^R}\left(\frac{g_Y^W\xi_c}{1+g_Y^W(1-\xi_c)}\right)\right]$. The negative conjugate is always stable while the positive never is. To consider E-stability of the stable solution we compute:

$F_{22}^{DY-C} = \frac{2\beta}{\gamma\left(1+\sqrt{1-\frac{4\beta}{\gamma^2}}\right)}$ and $A_{22}^{DY-C} = \frac{2}{\gamma\left(1+\sqrt{1-\frac{4\beta}{\gamma^2}}\right)}$

The E-stability conditions are then that F_{11}^{DY-C} and $A_{22}^{DY-C}F_{11}^{DY-C}$ have eigenvalues with real parts less than one and that F_{22}^{DY-C} and $A_{22}^{DY-C}F_{22}^{DY-C}$ are less than one. Considering first F_{11}^{DY-C}, we need $\Omega_{11}^{DY-C} - I_{11}$ to have eigenvalues with negative real parts. Given the roots $p(\lambda) = \lambda^2 + c_1^{DY-C}\lambda + c_0^{DY-C}$, we need $c_0^{DY-C} > 0 \;\; and \;\; c_1^{DY-C} > 0$, where:

$$c_0^{DY-C} = \frac{(1-\beta)\varphi_Y + (\varphi_\pi - 1)\left(k_Y + k_{G^W}g_Y^W\right)}{\varphi_Y + \varphi_\pi\left(k_Y + k_{G^W}g_Y^W\right) + \rho\xi_c^{-1}(1+(1-\xi_c)g_Y^W)}$$

and

$$c_1^{DY-C} = c_0^{DY-C} + \frac{\rho\xi_c^{-1}(1-\beta)(1+(1-\xi_c)g_Y^W) + \varphi_Y + \varphi_\pi\left(k_Y + k_{G^W}g_Y^W\right)}{\varphi_Y + \varphi_\pi\left(k_Y + k_{G^W}g_Y^W\right) + \rho\xi_c^{-1}(1+(1-\xi_c)g_Y^W)}$$

Since $\frac{k_Y\varphi_\pi+\varphi_Y+\xi_c^{-1}\rho(1-\beta)}{(\varphi_Y+\varphi_\pi k_Y+\xi_c^{-1}\rho)} > 0$, c_1^{DY-C} will always be positive when c_0^{DY-C} is positive. We have; $\frac{(1-\beta)\varphi_Y+(\varphi_\pi-1)\left(k_Y+k_{Gw}g_Y^W\right)}{\varphi_Y+\varphi_\pi\left(k_Y+k_{Gw}g_Y^W\right)+\rho\xi_c^{-1}(1+(1-\xi_c)g_Y^W)} > 0$, giving the E-stability condition:

$$0 < (\varphi_\pi - 1)\left(k_Y + k_{GW}g_Y^W\right) + (1-\beta)\,\varphi_Y \tag{4.27}$$

Given that $k_T > \frac{k_{GR}g_Y^W\xi_c}{1+g_Y^W(1-\xi_c)}$, as shown in the *proof of proposition 9*, and given that $0 < \beta < 1$ implies that $(1+\beta)^2 > 4\beta$, it can easily be verified that $0 < A_{22}^{DY-C} < 1$. Since $F_{22}^{DY-C} = \beta A_{22}^{DY-C}$, it follows that $0 < F_{22}^{DY-C}\cdot A_{22}^{DY-C} < 1$. Finally, this also implies that when the eigenvalues of F_{11}^{DY-C} have real parts less than one, so do those of $F_{22}^{DY-C}\cdot A_{11}^{DY-C}$, which concludes the proof.

Proof of proposition 12

As above, $\Phi_{11}^{TT-F} = 0$ implies that $b_{11}^{DY-C} = 0$ and hence that $\Omega_{11}^{DY-F} = F_{11}^{DY-F}$ and that $A_{11}^{DY-F} = 0$. The E-stability conditions of *section 2.2.3* then implies that F_{11}^{DY-F} and $A_{22}^{DY-F}\cdot F_{11}^{DY-F}$ must have eigenvalues with real parts less than one while that F_{22}^{DY-F} and $A_{22}^{DY-F}F_{22}^{DY-F}$ are less than one. From the *proof of proposition 10* we have that $0 < A_{22}^{DY-F} < 1$, and that $0 < F_{22}^{DY-F}\cdot A_{22}^{DY-F} < 1$, since $F_{22}^{DY-F} = F_{22}^{DY-C}$ and $A_{22}^{DY-F} = A_{22}^{DY-C}$ as determined by equation (4.26). E-stability then requires that $F_{11}^{DY-F} - I_{11}$ has eigenvalues with negative real parts.

$$\Omega_{11}^{DY-F} - I_{11} = \begin{bmatrix} \frac{(1-\rho^{-1}\xi_c\varphi_Y+(1-\xi_c)g_Y^W)}{(1+g_Y^W(1-\xi_c))} - 1 & \frac{\rho^{-1}\xi_c(1-\varphi_\pi)}{(1+g_Y^W(1-\xi_c))} \\ \frac{(k_Y+k_{GW}g_Y^W)(1-\rho^{-1}\xi_c\varphi_Y+(1-\xi_c)g_Y^W)}{(1+g_Y^W(1-\xi_c))} & \frac{(k_Y+k_{GW}g_Y^W)\rho^{-1}\xi_c(1-\varphi_\pi)}{(1+g_Y^W(1-\xi_c))} - (1-\beta) \end{bmatrix},$$

In line with the above methodology we then have:

$$c_0^{DY-F} = \frac{(1-\beta)\rho^{-1}\xi_c\varphi_Y + (k_Y + k_{G^W} g_Y^W)\rho^{-1}\xi_c(\varphi_\pi - 1)}{(1+g_Y^W(1-\xi_c))}$$
$$c_1^{DY-F} = \frac{\rho^{-1}\xi_c\varphi_Y + (1-\beta)(1+g_Y^W(1-\xi_c)) + (k_Y + k_{G^W} g_Y^W)\rho^{-1}\xi_c(\varphi_\pi - 1)}{(1+g_Y^W(1-\xi_c))}$$

$$= c_0^{DY-F} + \frac{\beta\rho^{-1}\xi_c\varphi_Y + (1-\beta)(1+g_Y^W(1-\xi_c))}{(1+g_Y^W(1-\xi_c))}$$

Since $\frac{\beta\rho^{-1}\xi_c\varphi_Y + (1-\beta)(1+g_Y^W(1-\xi_c))}{(1+g_Y^W(1-\xi_c))} > 0$, it suffices to derive the conditions for $\left|c_0^{DY-F}\right| > 0$. It is clear that this is the Taylor principle:

$$(1-\beta)\varphi_Y + (k_Y + k_{G^W} g_Y^W)(\varphi_\pi - 1) > 0 \tag{4.27}$$

This concludes the proof.

Conclusion

This thesis has examined determinacy and E-stability of economic policy in monetary union models. The aim has been to compare stability of policy to the closed and open economy cases and highlight the effects that the key features of monetary unions have on the determinacy and E-stability of economic policy.

We have shown that in the absence of intrinsic asymmetries, a monetary union can be regarded as a closed economy for monetary policy considerations. In *chapter 1*, we demonstrated that an optimal policy rule in such a union is of the same form as those found for closed economy models. Hence, the union central bank stabilizes shocks to union-wide variables, while having no instrument at its disposal to stabilize shocks to relative variables. This result holds even when there is home bias in input trade.

While the failure to stabilize relative shocks has a negative impact on welfare RE, it has no effect on determinacy and E-stability of monetary policy. In *chapter 2*, we showed that the results from the closed economy literature on learning also hold for quite a general specification of a monetary union model. Specifically, as in the closed economy considered by Evans & Honkapohja (2003), a fundamentals based policy rule is neither determinate nor E-stable, while an expectations based policy rule leads to a unique stationary REE that is stable under recursive leas squares learning. Similarly, the determinacy and E-stability conditions for general Taylor rules coincide with those presented by Bullard & Mitra (2002) for the closed economy model of Woodford (1999). That is, the Taylor principle is both necessary and sufficient for E-stability of contemporaneous and forward looking interest rate rules. While this is true for determinacy of a contemporaneous Taylor rule, a forecast based monetary policy rule may lead to multiple stationary REE if it is very aggressive. However, when introducing asymmetries into a monetary union, these results no longer hold.

In *chapter 3*, we used the monetary union model of Benigno (2004) to examine determinacy and E-stability in a monetary union with asymmetries in price rigidities. We find that the higher the degree of asymmetry in price rigidities, the larger is the determinacy and E-stability region for monetary policy. Furthermore, an interest rate rule that reacts to the terms of trade in addition to the output gap and inflation is more likely to induce stability in this case. Bullard & Mitra (2007) find that inertia in the interest rate rule has a positive effect on stability of policy. Given that the terms of trade is an autoregressive process, we hence conjecture that the terms of trade induces stability because of the inertia it brings to the interest rate rule. Llosa & Tuesta (2008) draw a similar conclusion for a Taylor rule that stabilizes the exchange rate of a small open economy. They point out that the exchange rate is a function of lagged values of the interest rate, by the interest rate parity condition.

As discussed in the introduction of this thesis, fiscal policy has an important role in stabilizing country-specific shocks in a monetary union. However, we show in *chapter 4* that if fiscal policy is too aggressive, it can make all rational expectations equilibria explosive. Leeper (1991) finds that when fiscal policy is not well coordinated with monetary policy, so that both policies ignore the government budget constraint, then this could lead to an explosive economy. Our results give reason for even more caution; if fiscal policy is overly aggressive following a shock to competitiveness, then the economy could be destabilized regardless of monetary policy. Furthermore, aggressive fiscal spending by even a small union member can destabilize the whole union economy. We hence argue for caution when using fiscal policy as a stabilization instrument.

When fiscal policy directly stabilizes domestic output gaps, while monetary policy offsets shocks to the union economy, then fiscal policy tightens the Taylor principle condition for monetary policy and hence potentially destabilizes the union economy. However, if monetary policy is forward looking and sufficiently aggressive to satisfy the Taylor principle, then an aggressive fiscal policy rule can actually help induce determinacy. Evans et al. (2008) find that an aggressive fiscal policy rule can help prevent the economy from reaching a deflationary spiral, when a liquidity trap exists. Our result suggests that in some cases it could also help eliminate multiplicity of stationary REE.

In sum, policy markers in a monetary union need to carefully consider asymmetries that may alter the stability of policy. In particular, monetary and fiscal policy should be well coordinated and restrictions on the flexibility of fiscal spending should be abided. Specifically, the union central bank and the national fiscal governments should have a common and clear policy objective. In this case asymmetries can help guarantee stability rather than destabilize the economy.

Appendix A
Glossary of Symbols

Variables and Functions

Variable	Description
$A_{t,t+1}$	the pay off in period t from the asset portfolio acquired in period t
A_t^i	the technology parameter facing final goods firms in country i
$C_t^i(h)$	consumption bundle of household in country i
D_t^W	demand shock *(Ch. 4)*
F_t	household's returns from ownership of intermediate firms
G_t^i	country i government expenditure on the domestic consumption good
$IP_{i,t}$	the price of country i's basket of input goods
$IP_{H,t}^0(f)$	the input price chosen by firm f in the period it gets to adjust price
$L(\cdot)$	household utility derived from holding real money balances
L^{CB}	the loss function of the union central bank
$M_s(h)$	household's holdings of money
$MC_{i,t}$	the real marginal cost for intermediate firms in country i
$N_{i,s}(h)$	labour supply of household in country i
P_s^i	the price of consumer goods produced in country i
$Q_{t,t+1}$	the stochastic discount factor
r_t	the nominal interest rate
rr_t	the natural rate of interest
T_t	the terms of trade

$U_t(h)$	the utility function of representative household h
$Q_{t,t+1}$	the stochastic discount factor
r_t	the nominal interest rate
rr_t	the natural rate of interest
T_t	the terms of trade
$U_t(h)$	the utility function of representative household h
$u_{i,t}$	cost push shock to country i's inflation in input prices
u_t^i	cost push shock to country i's inflation in consumer prices
V_t	the dispersion of intermediate goods prices
v_t	the exogenous variable in the univariate difference equation *(Ch. 2)*
$W_{H,t}(h)$	the wage received by the representative household of country H
w_t	the vector of exogenous variables in the difference equation used to analyse determinacy and E-stability *(section 2.2.3)*
X	generic variable
x_t	the vector of state variables in the difference equation used to analyse determinacy and E-stability *(section 2.2.3)*
$X_{i,t}^j$	the basket of input goods produced by country i's intermediate firms and sold to country j's final goods firms
$x_{i,t}^j$	the input good produced by the representative intermediate firm of country i's and sold to country j's final goods firms
$Y_{i,t}$	Country i's output in per capita terms
y_t	generic variable in the univariate difference equation *(section 2.2.3)*
$\tilde{y}_t^i$	the output gap of country i under sticky prices
$\overset{-}{y}_t^i$	the output gap of country i under flexible prices

z_t	vector of regressors in the learning process
Γ_t	household's wealth
Υ_t	lumpsum taxes paid by households
ϵ_t	the forecast error of agents under RLS learning
ε_t	random $i.i.d$ zero-mean shock
$\eta_{H,t}$	the wage elasticity of labour hours demanded
$\mu^w_{H,t}$	mark-up of real wages
$\mu^p_{H,t}$	the mark-up of input prices over the nominal marginal cost of intermediate firms
π^i_{t+1}	the price inflation of country i's consumer goods
$\pi_{i,t+1}$	the price inflation of country i's producer goods
$\varkappa_t$	the auxiliary variable defined as the lagged terms of trade

Coefficients and Parameters

Coefficient	Description
A	matrix whose eigenvalues determine E-stability
A^H_t	the technology coefficient for final goods firms
a	The vector of constants in the PLM
$a_0,\ a_1$	coefficients in the characteristic polynomial for determinacy *(appendices)*
B	The matrix of coefficients in the equation used to examine determinacy

B_t^H the technology of intermediate firms

b the matrix of coefficients on lagged endogenous variables in the PLM

b_t^H log-linearized technology coefficient of intermediate firms *(Ch. 1)*

c matrix of coefficients on exogenous variables in the PLM *(Ch. 2)*

$c_0,\ c_1$ coefficients in the characteristic polynomial for E-stability *(appendices)*

F matrix whose eigenvalues determine E-stability

g_T^i the coefficient on the terms of trade in country i's government spending rule

g_Y^i the coefficient on the country i's output gap in country i's government spending rule

$g_{\pi^R}^i$ the coefficient on the relative inflation in country i's government spending rule

k^i coefficient in country i's supply curve

k_C^i coefficient on the union output gap in the union supply schedule *(Ch. 3)*

k_{G^W} coefficient on union government expenditure in the union supply schedule

k_{G^R} coefficient on relative government expenditure in the relative supply schedule

k_T coefficient on the terms of trade in the relative supply schedule

k_Y the coefficient on the union output gap in the union supply schedule *(Ch. 4)*

n the size of home country

v the level of substitutability between domestic and foreign input baskets

α the coefficient on expected future values in the univariate difference equation *(section 2.2.3 and appendices)*

α the probability with which producers in country i keep their prices fixed (*Chs. 3 & 4)*

α^i the weight attached to country i's per capita output gap in the loss function *(Ch. 1)*

β the household discount factor

Γ the vector of constants in the difference equation used when looking at E-stability

γ coefficient in the univariate condition for the terms of trade under a terms of trade fiscal policy rule *(Ch. 4)*

γ^i the effect of a change in country H's output gap on country i's inflation in producer prices

γ_0^i the effect of a change in country F's output gap on country i's inflation in producer prices

δ the sensitivity of inflation to changes in the marginal cost *(Ch. 1)*

δ the coefficient on lagged values in the generic univariate difference equation *(section 2.2.3 and appendices)*

ϵ the constant elasticity of substitution between domestic intermediate goods

ζ_Y the output gap coefficient in the union supply schedule

ζ_T the coefficient on the terms of trade in the relative supply schedule

η the inverse of the elasticity of producing goods

Θ the matrix of coefficients on exogenous variables in the difference equation used when looking at E-stability

θ coefficient in the univariate condition for the terms of trade

θ_0^H	the effect of a change in country H's output on country i's inflation in consumer prices
θ_0^H	the effect of a change in country F's output on country i's inflation in consumer prices
ϑ	measures the sensitivity of inflation to changes in union-wide and domestic inflation under the optimal discretionary monetary policy rule
κ^i	the effect of a change in country H's output on country i's marginal cost
κ_0^i	the effect of a change in country F's output on country i's marginal cost
λ	the characteristics polynomial *(appendices)*
λ^i	the weight attached to the basket of home country input, in the production function of final goods firms in country i *(Ch. 1)*
$1-\lambda^i$	the weight attached to the basket of foreign country input, in the production function of final goods firms in country i *(Ch. 1)*
ξ_c	the steady state consumption share of output *(Ch. 4)*
ρ^i	the probability with which firms in country i get to keep their price fixed *(Ch. 3 & 4)*
σ	the relative risk aversion of households *(Ch. 1 & 2)*
τ^H	a subsidy paid to intermediate firms
Φ	the weight given to the product of both countries' output gaps in the central bank loss function *(Ch. 1)*
ϕ	coefficient measuring workers' disutility from working *(Ch. 1 & 2)*
φ_π	the coefficient on inflation in the interest rate rule
φ_Y	the coefficient on the output gap in the interest rate rule
φ_T	the coefficient on the terms of trade in the interest rate rule

Ψ^i the effect of foreign output on country i's inflation under optimal discretionary monetary policy

Ψ_0^i the effect of foreign output on country i's inflation under optimal discretionary monetary policy

ψ coefficient in the univariate condition for the terms of trade

ψ_Y^W the coefficient on the union output gap in the union government spending rule

ψ_Y^R the coefficient on the relative output gap in the relative government spending rule

ψ_T^W the coefficient on the terms of trade in the union government spending rule

ψ_T^R the coefficient on the terms of trade in the relative government spending rule

ω the level of trade in intermediate goods, decreases with the level of home bias (Ch. 1 & 2)

ϱ the matrix of coefficients on lagged values in the autoregressive condition for exogenous variables

References

Aoki, K., (2001). "Optimal monetary responses to relative price changes", *Journal of Monetary Economics* 48, 55– 80.

Babetskii, I., (2005). "Trade integration and synchronization of shocks", *The Economics of Transition*, The European Bank for Reconstruction and Development, 13(1), 105-138.

Banachiewicz, T., (1937). "Zur Berechnung der Determinante, wie auch der Inversen, und zur darauf basierten Auflosung der Systeme linearer Gleichungen", *Acta Astronomica*, Ser C 3, 41–67.

Bank of England, U.K., (2003). *Inflation Report*, Bank of England.
www.bankofengland.co.uk/publications/inflationreport/2003.htm (3 Jan 2010).

Benigno, P., (2004). "Optimal Monetary Policy in a Currency Area", *Journal of International Economics*, 63, 293-320.

Benigno, G & Benigno, P., (2006). "Designing Targeting Rules for International Monetary Policy Cooperation", *Journal of Monetary Economics, 53*, 473-506.

Benigno, P. & Lopez-Salido, J. D., (2006). "Inflation Persistence and Optimal Monetary Policy in the Euro Area", *Journal of Money, Credit and Banking*, 38(3), 587-614.

Benigno, G. & Thoenissen, C., (2008). "Consumption and real exchange rates with incomplete markets and non-traded goods", *Journal of International Money and Finance*, Elsevier, 27(6), 926-948, October.

Bernanke, B. & Woodford, M., (1997). "Inflation forecasts and monetary policy", *Journal of Money, Credit, and Banking,* 29, 653–684.

Beetsma, R. M.W.J. & Jensen, H., (2005). "Monetary and fiscal policy interactions in a micro-founded model of a monetary union", *Journal of International Economics*, Elsevier, 67(2), 320-352, December.

Beetsma, R. M. W. J. & Schotman, P. C., (2001). "Measuring Risk Attitudes in a Natural Experiment: Data from the Television Game Show Lingo", *Economic Journal*, Royal Economic Society, 111(474), 821-48, October.

Bofinger, P. & Mayer, E., (2007). "Monetary and Fiscal Policy Interaction in the Euro Area with Different Assumptions on the Phillips Curve", *Open Economies Review*, Springer US, 18(3), 291-305.

Bullard, J. & Mitra, K. (2002). "Learning About Monetary Policy Rules", *Journal of Monetary Economics*, 49, 1105-1129.

Bullard, J. & Mitra, K., (2007). "Determinacy, Learnability, and Monetary Policy Inertia", *Journal of Money, Credit and Banking*, Blackwell Publishing, 39(5), 1177-1212, 08.

Bullard, J. & Schaling, E. (2006). "Monetary Policy, Determinacy, and Learnability in the Open Economy",*Working paper series No. 611*, The European Central Bank.

Calvo, G. (1983). 'Staggered prices in a utility maximising framework.' *Journal of Monetary Economics* 12, 383-398.

Chang, Y., Doh, T. & Schorfheide F. (2007). "Non-stationary Hours in a DSGE Model", *Journal of Money, Credit and Banking*, Blackwell Publishing, 39(6),1357-1373, 09.

Clarida, R., Gali, J. & Gertler, M. (1999). "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, American Economic Association, 37(4), 1661-1707, December.

Clarida, R., Gali, J., & Gertler, M. (2000). "Monetary policy rules and macroeconomic stability: evidence and some theory." *Quarterly Journal of Economics* 115, 147–180.

Clarida, R., Gali, J., & Gertler, M. (2002). "A Simple Framework for International Monetary Policy Analysis", *Journal of Monetary Economics,* 49, 879-904.

Clausen, V. & Wohltmann, H., (2005). "Monetary and Fiscal Policy in a Large Asymmetric Monetary Union - A Dynamic Three-Country Analysis", *Economics working papers*, 18, Christian-Albrechts-University of Kiel, Department of Economics.

Devereux, M. B. & Engel, C., (2000). "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility", *NBER Working Paper No. 7655*.

Dixit, A. K. & Stiglitz, J. E. (1977). "Monopolistic Competition and Optimum Product Diversity." *American Economic Review*, 67(3), 297-308.

European Central Bank, E.U., (2003). *Monthly Bulletin*. European Central Bank, *www.ecb.int/pub/mb/html/index.en.html (3 Jan 2010)*.

European Commission (1990). "One market, one money: An evaluation of the potential benefits and costs of forming an economic and monetary union", *European Economy*, 44, October. Brussels: Commission for the European Communities.

European Union (1992), "Treaty on European Union", Maastricht,
www.eurotreaties.com/maastricht.pdf (5 Feb 2010).

Eusepi, S., (2007). "Learnability and monetary policy: A global perspective," *Journal of Monetary Economics*, Elsevier, vol. 54(4), 1115-1131, May.

Evans, G., (2008). "Monetary and Fiscal Policy under Learning in the Presence of a Liquidity Trap", Monetary and Economic Studies, Institute for Monetary and Economic Studies, Bank of Japan, 26, 59-86, December.

Evans, G. W., Guse, E. & Honkapohja, S., (2008). "Liquidity traps, learning and stagnation", *European Economic Review*, Elsevier, 52(8), 1438-1463, November.

Evans, G. & Honkapohja, S. (1999) "Learning Dynamics." in *Handbook of Macroeconomics*, Taylor, J. and Woodford, M., Elsevier, Amsterdam, 1, 449-542.

Evans, G. & Honkapohja, S. (2001) "Learning and Expectations in Macroeconomics." *Princeton University Press*, Princeton, New Jersey.

Evans, G. & Honkapohja, S. (2003) "Expectations and the stability problem for optimal monetary policies", *Review of Economic Studies* 70, 807–824.

Evans, G. W., & Honkapohja, S., (2005). "Policy interaction, expectations and the liquidity trap", *Review of Economic Dynamics* 8, 303–323.

Evans,G. & Honkapohja, S., (2006). "Monetary Policy, Expectations and Commitment", *Scandinavian Journal of Economics*, Blackwell Publishing, 108(1), 15-38, 03.

Evans, G. W. & McGough, B., (2005). "Stable sunspot solutions in models with predetermined variables," *Journal of Economic Dynamics and Control*, Elsevier, 29(4), 601-625, April.

Frankel, J. A. & Rose, A. K., (1998). "The endogeneity of the optimum currency area criteria", *Economic Journal*, 108 (449), July, 1009–1025.

Friedman, B. J., (2003). "The LM Curve: A Not-So-Fond Farewell", *NBER Working Paper* No. W10123

Gali, J., (2008). "Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework", *Princeton University Press.*

Gali, J. & Monacelli, T. (2005). "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", *Review of Economic Studies*, 72, 707-734.

Head, K. & Mayer, T. (2000). "Non-Europe: The Magnitude and Causes of Market Fragmentation i the EU", *Weltwirtschafliches Archiv*, 136(2), 284-314.

Hillberry, R., & Hummels, D. (2002). "Explaining home bias in consumption: The role of intermediate input trade", *NBER working paper 9020*.

Honkapohja, S. & Mitra, K. (2005). "Performance of monetary policy with internal central bank forecasting", *Journal of Economic Dynamics and Control*, 29(4), 627-658.

Honkapohja, S. & Mitra, K., (2006). "Learning Stability in Economies with Heterogeneous Agents", *Review of Economic Dynamics*, Elsevier for the Society for Economic Dynamics, 9(2), 284-309, April.

Howitt, P., (1992). "Interest Rate Control and Nonconvergence to Rational Expectations", J*ournal of Political Economy*, University of Chicago Press, 100(4), 776-800, August.

Hurwicz, L. (1946). "Theory of the Firm and of Investment", *Econometrica*, (14), 109-136.

Johnson, H. G. & Swoboda. A. K., (1973). "The Economics of Common Currencies", *George Allen and Unwin*, London.

Krugman, P. (1993). "Lessons of Massachusetts for EMU", in Torres, F. and Giavazzi, F. (eds.), *Adjustment and Growth in the European Monetary Union*, 241–61. Cambridge, UK, Cambridge University Press and CEPR.

Krugman, P. R., (1998). "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, 2, 137–205.

Krugman, P. R., (2000). "Thinking About the Liquidity Trap", *Journal of the Japanese and International Economies* 14(4), December, 221-237.

LaSalle, J. P., (1986). "The Stability and Control of Discrete Processes", *Applied Mathematical Science*, Vol. 62. Springer, New York.

Leeper, E. M., (1991). "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies," *Journal of Monetary Economics*, 27, 129–147.

Llosa, L. G. & Tuesta, V., (2008). "Determinacy and Learnability of Monetary Policy Rules in Small Open Economies", *Journal of Money, Credit and Banking*, Blackwell Publishing, 40(5), 1033-1063, 08.

Lopez, R. A., E. Pagoulatos, & M. A. Gonzalez. (2006). “Home Bias and U.S. Imports of Processed Food Products.” *North American Journal of Trade and Finance*, 17, 363-373.

Magnus, J. & Neudecher, H. (1998). "*Matrix Differential Calculus with Applications in Statistics and Econometrics*." *John Wiley & Sons, Inc.*

McCallum, B. T., (1983). "On non-uniqueness in rational expectations models: an attempt at perspective", *Journal of Monetary Economics 11*, 139–168.

McCallum, J. (1995). "National borders matter: Canada–U.S. regional trade patterns", *American Economics Review*, 85, 615–623.

McCallum, B. T., (2007). "E-stability vis-a-vis determinacy results for a broad class of linear rational expectations models", *Journal of Economic Dynamics and Control*, Elsevier, 31(4), 1376-1391, April.

Mélitz, J., (1997). "Some cross-country evidence about debt, deficits and the behavior of monetary and fiscal authorities", *CEPR Discussion Paper No. 1653*.

Molodtsova, T., Nikolsko-Rzhevskyy, A. & Papell, D., (2009). "Taylor Rules and the Euro", *MPRA Paper 11348*, University Library of Munich, Germany.

Mundell, R. A. (1961). "A Theory of Optimum Currency Areas", *The American Economic Review*, LI, No. 4 (November), 509-517.

Mundell, R. A. (1973). "Uncommon Arguments for Common Currencies", in *Johnson and Swoboda* [1973], *chapter 7*.

Murray, J., Schembri, L. & St-Amant, P. (2003). "Revisiting the case for flexible exchange rates in North America", *The North American Journal of Economics and Finance*, Elsevier, 14(2), 207-240, August.

Muth, J. F., (1961). "Rational Expectations and the Theory of Price Movements", *Econometrica 29*, 315-335.

Nickell, S., (1997). "Unemployment and labor market rigidities: Europe versus North America", *Journal of Economic Perspectives,* (11), 55–74.

Obstfeld, M., & Rogoff, K., (1998). "Risk and Exchange Rates", *NBER Working Paper No. 6694*.

Ottaviano, G. I. P., Taglioni, D. & di Mauro, F. (2009). "The euro and the competitiveness of European firms", *Economic Policy*, CEPR, CES, MSH, 24, 5-53, 01.

Preston, B., (2008). "Adaptive learning and the use of forecasts in monetary policy," *Journal of Economic Dynamics and Control*, Elsevier, 32(11), 3661-3681, November.

Rotemberg, J. & Woodford, M., (1998) "An optimization-based framework for the evaluation of monetary policy." *NBER Technical Working Paper No. 233*, May.

Rotemberg, J. & Woodford, M. (1999). "Interest-rate rules in an estimated sticky-price model". In: Taylor, J. (Ed.), *Monetary Policy Rules*. University of Chicago Press, Chicago.

Sargent, T.J. & Wallace, N., (1981). "Some unpleasant monetarist arithmetic", *Quarterly Review*, Federal Reserve Bank of Minneapolis, issue Fall.

Taylor, J. (1993). "Discretion versus Policy Rules in Practice". *Carnegie-Rochester Series on Public Policy*, 39, 195-214.

Taylor, J. (1999). "A Historical Analysis of Monetary Policy Rules," NBER Chapters, in: Monetary Policy Rules, pages 319-348 National Bureau of Economic Research, Inc.

Trichet, J., (2005). "Monetary policy and private expectations", *Zoleta lectures at the Bank of Greece (25 Feb 2005)*.

Wang, M. & Wong, M.C. S. (2005). "Learning dynamics in monetary policy: The robustness of an aggressive inflation stabilizing policy", *Journal of Macroeconomics*, 27(1), 143-151.

Wang, Q. (2006). "Learning Stability for Monetary Policy Rules in a Two-Country Model", *Cambridge Working Papers in Economics* No. 0659.

Wolf, H. C. (2000). Intranational home bias in trade. Review of Economics and Statistics, 82, 555–563.

Woodford, M. (1999). "Optimal Monetary Policy Inertia", *NBER Working Paper* No. 7261.

Woodford, M. (2003). "Interest & Prices: Foundations of a Theory of Monetary Policy", Princeton University Press.

Zanna, L. (2004). "PPP rules, macroeconomic (In)stability and learning", *International Finance Discussion Papers 814*, Board of Governors of the Federal Reserve System (U.S.).